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Moderate economic inequality boosts AI

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Abstract. This study is an empirical exploration of the economic incentives for developing artificial intelligence (AI). Standard economic principles predict that high wages would encourage labor-saving innovations such as AI-based technologies. To test this hypothesis, we use hourly wage and AI investment data from 27 countries. The results show that the relationship between wages and AI is statistically insignificant. Labor-saving innovations, however, are strongly related to the distribution of income. According to our estimates, there is indeed a statistically significant and robust relationship between AI investments and income inequality. Moreover, the relationship is hump-shaped, implying that per capita AI investment achieves its peak at a certain level of inequality, measured by the Gini coefficient. The available data shows that this optimal degree of economic inequality (0.389) is very close to the moderate inequality levels observed among the global top-three countries in AI investment per capita— the US (0.382), Singapore (0.386), and Israel (0.350).

JEL: D30; O31

Keywords: Artificial intelligence; economic inequality; wages

1. Introduction

This study investigates how wages and income inequality shape the incentives of investing in AI from an empirical standpoint. Theoretically, the basic economic prediction is simple: if wages are sufficiently high, then replacing labor with AI would be profitable. That is, focusing on incentives suggests that high wages would boost demand for AI, reminiscent of the analysis of the Industrial Revolution in the 18th century by Allen (2009, 2011). According to Allen, during the 18th century, high wages and cheap capital in Britain encouraged labor-saving innovations, and thus, became the root cause of the Industrial Revolution.

Based on this historical experience, and simple economic inference, can we deduce that higher wages would help countries to get ahead in the ongoing AI revolution? To empirically test this hypothesis against real-world observations, we use data from 26 countries, including major

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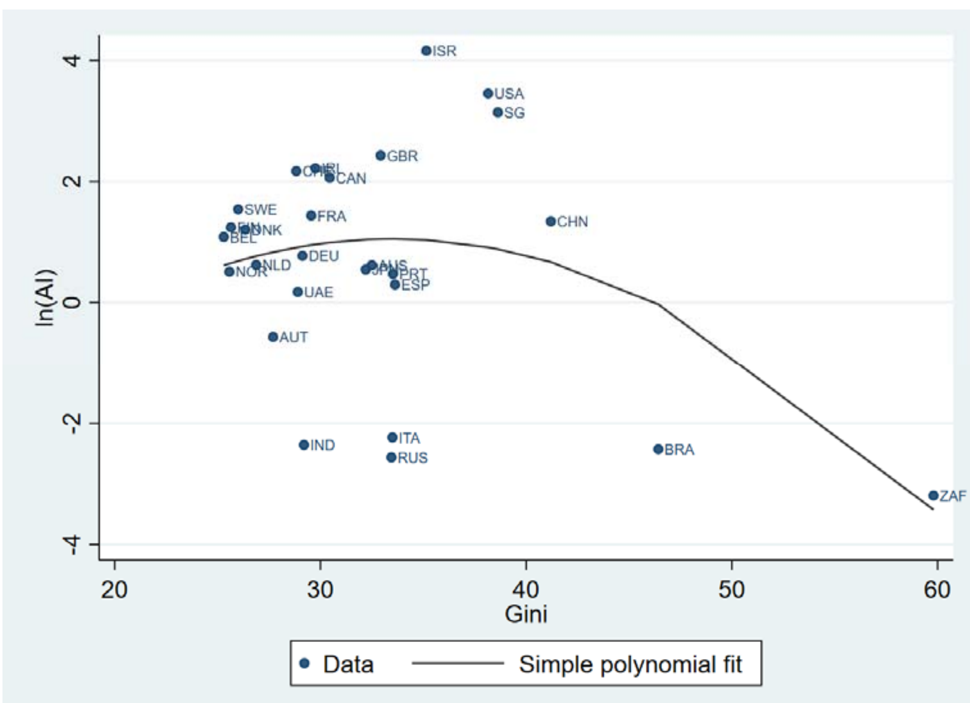
OECD countries such as the US, Germany, UK, and Canada. Our dataset also includes major developing economies such as India and China. Due to the availability of the AI data, which we obtained from the Stanford Institute for Human-Centered Artificial Intelligence, the time span of the data is from 2015 to 2018. This gives us 104 data points. We also control for human capital, physical capital per worker, and total factor productivity. The estimation results suggest a statistically insignificant relationship between AI investments and wages.

The lack of a significant impact of wages on AI investments is indeed not very surprising if we remind ourselves that wages paid to employees do not fully capture the cost of labor. For example, many firms invest in AI-based technologies to reduce their payroll tax burden, according to an op-ed published by the New York Times in 2019. Moreover, the cost of labor is not the only factor that determines the demand for labor-saving technologies and related AI-based systems. The net rate of return on investment–collectively determined by interest rates, price markups, taxes, and subsidies–is also potentially relevant. As it happens, all these variables are very crucial for the distribution of disposable income and economic inequality, typically measured by the post-tax Gini coefficient.

Figure 1: AI vs Gini



Notes: Logarithm of AI investment per capita and Gini data on average from 2015 to 2018 for 27 countries. The solid line depicts the simple 2^{nd} order polynomial fitting.

Consequently, we investigate the relationship between AI investments and the Gini coefficient. According to our estimations, there is a statistically significant relationship between income inequality and AI investments. The most remarkable aspect of this relationship is its non-linearity. Specifically, the estimated relationship between AI and Gini is hump-shaped; that is, a certain degree of inequality leads to a maximum of level investments in AI.

Incidentally, the Gini coefficients of the global leaders in per capita AI investments are very close to this optimal level: 0.389. These figures are 0.382 in the US, 0.386 in Singapore, and 0.350 in Israel, the global leader. It is safe to say that these figures represent moderate income inequality levels in the world as the range of the Gini coefficient in our data set is 0.25-0.60. Lastly, it is noteworthy that China, which attracted global attention with its significant leap in AI development, also has a Gini coefficient (0.412) in the proximity of this optimal level.

We interpret our results as follows. In the US, Singapore, and Israel, where rates of return are higher due to more capital-friendly tax code than the Western European countries, income inequality is also higher. This incentivizes AI investments more strongly in the top-three. Due to higher levels of payroll taxes, however, it is more profitable to substitute labor with AI, for example, in Western European countries compared to Brazil and South Africa where higher income inequality levels are observed. This mechanism explains why a moderate level of income inequality would yield the right conditions for AI, as is empirically observed among the top three countries.

The estimation results are, nonetheless, asymmetric in the sense that the detrimental impact of economic inequality on AI investments is clearly larger than that of equality. So the unequal income distribution levels that can be observed in Brazil and South Africa have a stronger negative impact on AI investment than the egalitarian income distribution levels in the Scandinavian countries. This asymmetry can be visually verified by inspecting Figure 1 above. The numerical simulations of our theoretical model also produce a similar asymmetric relationship between AI and income inequality.

We also conduct robustness checks by invoking different econometric techniques and specifications. The results are almost identical to our original findings. We also seek for outliers in the data and find no evidence in this regard. Finally, to demonstrate the theoretical foundations of our empirical results, we present a toy model in the appendix where firms invest in AI in pursuit of profit maximization. Consistent with our empirical findings, we numerically show that there is indeed an optimal inequality level that yields the highest level of AI in our theoretical model.

The paper is organized as follows. The next section discusses the data. Section 3 explains the econometric model and results. We conduct robustness checks in Section 4. The final section concludes.

2. Data

We use annual data from 26 countries, covering the period from 2015 to 2018. These data include AI investments, wages, post-tax Gini coefficients, and several control variables, such as physical capital, human capital, and total factor productivity. In the following, we provide further details of the variables we use in the analysis.

2.1 AI Investments

We obtain the AI investment data from The Global AI Vibrancy Tool provided by the Human-Centered AI Institute at Stanford University, Stanford, CA (see vibrancy.aiindex.org). This dataset provides several country-specific AI metrics ranging from the number of unique AI occupations to the number of AI patents per capita. Due to our focus on AI investments, we use the private AI investment per capita data from this data set. The investment data covers 26 countries and is available from 2015 to 2018.

2.2 Wages

The annual wages per worker per hour across countries were calculated using data from Penn World Table (PWT, v.9.1; Feenstra *et al.*, 2015). To calculate the wages, we used data on shares of labor compensation in the gross domestic product (GDP) at current national prices, population, real GDP, and average annual hours per worker:

$$\text{Hourly wage } = \frac{\text{Labor share} \times \text{GDP}}{\text{Employment} \times \text{Average working hours}}.$$

The PWT data is available up to year 2017. As for 2018, we extrapolate the country-specific hourly wages by estimating a first-order autoregressive process with a linear trend:

$$W_{i,t} = \alpha_{i,0} + \alpha_{i,1} W_{i,t-1} + \alpha_{i,2} t + u_{i,t},$$

where W_{it} denotes hourly wages in country i and year $t = \{2000, \ldots, 2017\}$, and $u_{i,t}$ is the country-specific error term. As we shall see when we present our robustness checks, we also estimate the model without extrapolated data, and the results virtually stay the same.

2.3 Income inequality

In this study, the main indicator for income inequality is the post-tax Gini coefficient in the countries that we are interested in. The post-tax Gini coefficients are obtained from Standardized World Income Inequality Database (Solt, 2019). The Gini coefficient is a measure that varies between zero and one. When it is zero, this indicates perfect equality of income distribution where everyone has the same income. When the Gini coefficient is equal to one, all income is earned by a single individual. The corresponding data is available up to 2017. Nonetheless, because the Gini coefficient exhibits very little variation over time, we used 2017's coefficients for 2018.

2.4 Control variables

The variables that we control for are physical capital, human capital, and total factor productivity (TFP). Notice that human capital is typically associated with successful developments in technology (Lucas, 1988), science, and computer-related fields (Caselli and Coleman, 2001; He *et al.*, 2019). By the same token, TFP is also clearly relevant.

These data are obtained from the PWT and are available on an annual basis. The physical capital is at current purchasing power parities (PPPs, in mil. 2011US$). The human capital index is based on years of schooling and returns to education. The TFP is also calculated at current PPPs and normalized by taking the US TFP equal to one.

As the PWT data is available up to the year 2017, we also extrapolate these country-specific data for 2018 using the same strategy that we use for extrapolating the wage data. As we mentioned earlier, we also conduct our analysis without extrapolation by using lagged values as a robustness check. Our results are not affected by this alternative approach.

3. Model and results

In this section, we present the estimated models and the results. We use the standard linear regression model with a time trend. The first model that we analyze involves wage as an explanatory variable:

$$\ln AI_{it} = \beta_0 + \beta_1 t + \beta_2 W_{it} + \beta_3 W_{it}^2 + \boldsymbol{\beta_4 X_{it}} + \epsilon_{it}$$

where AI_{it} is per capita investments in AI in country i in year t, W_{it} stands for the level of wages, and $\boldsymbol{X}_{it}$ stands for the country specific set of control variables.

Note that our econometric model includes the square of the wage rate. This is to capture the non-linear relationship between AI investments and cost of labor. The results are provided in Table 1. Note that in model 1 (M1) we simply estimate a quadratic relationship between wages and the per capita AI investments with a time trend. We then add all other variables one by one to the estimation, until all available variables are used in M4.

Table 1

Estimation Results: AI and Wage

	M1	M2	M3	M4
Constant	-4.169***	-8.033***	-7.990***	-10.447***
	0.856	1.376	1.458	1.752
Trend	0.539***	0.501***	0.503***	0.528***
	0.156	0.149	0.151	0.148
Wage	0.221***	0.083	0.092	-0.071
	0.067	0.075	0.122	0.137
Wage^2	-0.003**	-0.001	-0.001	0.0001
	0.001	0.001	0.002	0.002
Human Capital		1.840***	1.809***	2.023***
		0.529	0.621	0.613
Capital			-2.11E-07	5.46E-08
			2.24E-06	2.19E-06
TFP				6.307**
				2.618
R^2	0.31	0.39	0.39	0.42

Notes: Standard errors are in smaller fonts. *P<0.1, **P<0.05, ***P<0.01.

Our results indicate that the relationship between AI investments and wage rates is hump-shaped if other economic variables are not controlled for. This outcome is in line with economic reasoning which predicts that high wages could encourage labor saving technology, such as AI. Due to increasing cost of production, however, this relationship is not linear but strictly concave. Nevertheless, once we control for other economic variables that are expected to affect AI investments, such as human and physical capita and TFP, the relationship between AI investments and wage becomes statistically insignificant. Comparing models M1 and M2 suggests that wages act as a proxy for human capital. Further analyses conducted using M3 and M4 show that we cannot reject the hypothesis that TFP has a statistically significant impact on AI, while the coefficient of physical capital is statistically insignificant.

We believe that these results shed light on the production technology of AI, as it seems that the most crucial factors of production are an educated workforce (human capital) and accumulated knowledge (TFP). In contrast, physical capital has a negligible role.

Nonetheless, there would be no room for relative prices to play a role in AI investments according to these results as wages are out of the picture due to insignificant coefficient estimates. Yet, there are other crucial economic factors and variables, such as interest rates, taxes, and market power, which can have significant impacts on AI investments. As it happens, these variables play a decisive role in determining income inequality and distribution. The standard measure that collectively captures the effects of these variables on income inequality is the Gini coefficient. Consequently, in the following, we investigate the relationship between the post-tax Gini coefficient and AI investments, while controlling for the same set of variables. In particular, we estimate the following model:

$$\ln AI_{it} = \beta_{0i} + \beta_1 t + \beta_2 Gini_{it} + \beta_3 Gini_{it}^2 + \boldsymbol{\beta_4 X_{it}} + u_{it}$$

where $Gini_{it}$ is the level of income inequality measured by the Gini coefficient. The results are presented in Table 2.

Table 2

Estimation Results: AI and Gini coefficient

	M1	M2	M3	M4
Constant	-7.844*	-14.461***	-15.095***	-18.832***
	3.157	3.002	3.108	3.195
Year	0.561***	0.501***	0.493***	0.536***
	0.157	0.140	0.141	0.136
Gini (×100)	0.445***	0.398***	0.420***	0.586***
	0.164	0.144	0.147	0.150
Gini^2	-0.007***	-0.006***	-0.006***	-0.008***
	0.002	0.002	0.002	0.002
Human Capital		2.171***	2.076***	1.325***
		0.369	0.388	0.440
Capital			1.01E-06	-1.73E-06
			1.26E-06	1.48E-06
TFP				4.836***
				1.515
R^2	0.26	0.45	0.45	0.50

Notes: Standard errors are in smaller fonts. *P<0.1, **P<0.05, ***P<0.01.

According to these results, we find a statistically significant relationship between the Gini coefficient and AI investments. In all specifications, this relationship between income inequality and AI is non-linear and concave. Particularly, there is a certain level of Gini coefficient that maximizes the AI investments even if we control for the other economic variables. Notice also that the signs of the coefficients of the control variables and their statistical significances have stayed the same in comparison to our first analysis based on wages. Furthermore, as can be seen from the R^2 values, there is a significant increase in the explanatory power of the models with the Gini coefficient in the presence of the economic control variables.

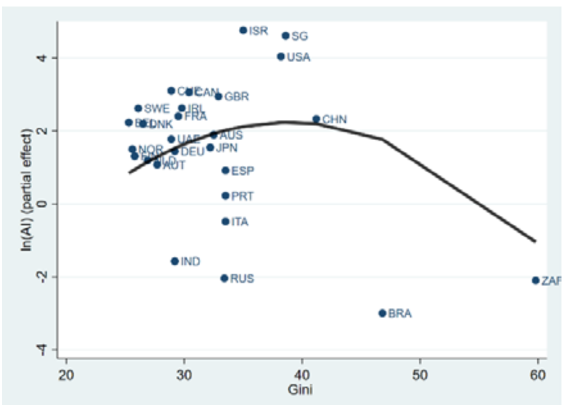
The results summarized in Table 2 suggest that moderate-income inequality levels provide the most suitable economic environment and incentives for AI investments. According to the benchmark case (M4), the Gini coefficient, which maximizes the AI investment per capita, is estimated as 0.389. We see that the US and Singapore, with the Gini coefficients 0.382 and 0.386, respectively, are in the vicinity of this optimal level. Inspection shows that these two countries are among the top three in AI investments per capita, following Israel. The global leader, Israel, has a Gini coefficient of 0.350, which is also close to the optimal Gini coefficient level. To explain this empirical result in a rigorous fashion, we also develop a simple economic model and its results in the appendix.

Our preferred explanation for our results is based on the opposite forces that high income taxes exert on the incentives of investing in AI. First of all, a high payroll tax would encourage AI investments by raising labor costs, while reducing income inequality by financing redistributive policies. Indeed, substantially higher AI investments per capita figures are observed in egalitarian Western European and Nordic countries compared to Brazil and South Africa, which are notoriously famous for their income inequality.

On the other hand, despite its positive impact on income equality, a high income tax on capital gains would reduce profitability, discouraging risky investments in innovation and technology. Thus, due to higher income taxes on capital earnings, the Western European and Nordic countries lose their edge in AI investments to Israel, the US, and Singapore –the global leaders in per capita AI investments. To put it succinctly, income taxes on labor and capital unambiguously reduce income inequality while creating two opposite forces on automation investments. This explains the hump-shaped relationship between income inequality and per-capita AI investments in our estimations.

One can notice that the results are asymmetric since the negative impact of economic inequality on AI investments is larger than that of economic equality. Hence the unequal income distribution levels observed in Brazil and South Africa have a more substantial negative impact on AI investments than the egalitarian income distribution levels in the Scandinavian countries. This asymmetry can be visually verified in figures 2a and 2b, which depict the partial effect of income inequality on per capita AI investments for years 2018 and 2015-2018, respectively.

Figure 2



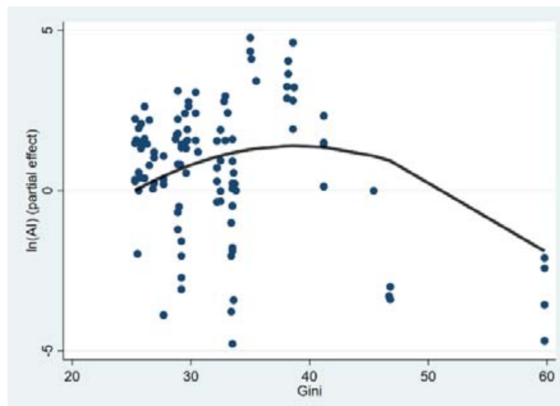


Figure 2a Figure 2b

Notes: The graphs correspond to the regression system in Table 2/M4. The curve in the figure to the left (right) shows the partial relation between the Gini coefficient and the log of per capita AI investments for year(s) 2018 (2015-2018), holding fixed the estimated effects of the explanatory variables other than the Gini coefficient and its square.

It is noteworthy that China's AI ambitions have recently attracted global attention and have frequently been covered in the media. According to our results, indeed, China has a Gini coefficient that is very close to the optimal level, indicating that it has a rather ideal level of income inequality for AI investments. Note, however, that we use per capita AI investments in our analysis. In levels, China is indeed the second largest investor in AI. Nevertheless, in per capita terms, China is lagging behind.

In light of our results, it seems that China must refrain from any policies that can deteriorate the income inequality to preserve its competitive position in the ongoing AI revolution. This is because higher income inequality is more damaging for per capita AI investments than lower-levels of the Gini coefficient.

4. Robustness

In this section we re-estimate our model using different techniques to test the robustness of the coefficient estimates generated by the least square regressions. These estimation methods are random effects models – including maximum likelihood (MLE) and generalized least squares (GLS) –and population averaged model (PA). The results with these three alternative estimation techniques are presented in Table 3.

Table 4

Estimation Results: Random-effects and population-averaged linear models

	GLS	MLE	PA	GLS	MLE	PA
Constant	-5.030*	-5.677*	-5.677**	-15.859**	-16.682***	-16.682***
	3.008	2.988	2.758	6.912	5.509	5.484
Year	0.513***	0.513***	0.513***	0.503***	0.511***	0.511***
	0.085	0.082	0.082	0.076	0.075	0.075
Wage	0.216	0.187	0.187			
	0.216	0.207	0.201			
Wage^2	-0.002	-0.002	-0.002			
	0.003	0.003	0.003			
Gini (×100)				0.569*	0.555**	0.555**
				0.325	0.258	0.258
Gini^2				-0.008**	-0.007**	-0.007**
				0.004	0.003	0.003
Human Capital	0.669	0.876	0.876	1.173	1.330*	1.330*
	1.114	1.081	1.020	0.846	0.703	0.700
Capital	2.79E-07	2.04E-07	2.04E-07	2.27E-06	1.02E-06	1.02E-06
	3.72E-06	3.44E-06	3.44E-06	2.80E-06	2.45E-06	2.33E-06
TFP	-2.036	-1.355	-1.355	0.612	1.773	1.773
	3.338	3.429	3.180	2.361	2.230	2.130
R^2 (within)	0.46			0.45		

Notes: Standard errors are in smaller fonts. **P*<0.1, ***P*<0.05, ****P*<0.01.

Despite some limited disparities in significance levels, the results in Table 3 are qualitatively identical to our original findings. To be specific, the coefficients estimates wages are statistically insignificant. On the other hand, the statistically significant Gini coefficient estimates further imply a hump-shaped relationship.

As an additional robustness check, we test for the existence of outliers. Our approach is to re-run the benchmark model (M4) after omitting each country one by one and to seek for any marked difference. This exercise is repeated for both the wage model and the Gini model, giving us 2×26 different estimations as there are 26 countries in our dataset. Once again, the results for the Gini model exhibit no significant differences from our earlier findings which show that the AI investments would follow an inverted-U shape as inequality rises. The most notable impact of this exercise is that the coefficient estimates of human capital and capital become statistically insignificant and significant, respectively, when the Singaporean data is omitted. Moreover, the coefficient estimate of the capital becomes statistically significant at 10% level when the Portuguese data is omitted.

Likewise, in the first model that involves wage as the explanatory variable, wage coefficients are still statistically insignificant according to our analysis to detect outliers. However, the coefficient of TFP becomes insignificant when we omit the Israeli and Irish data once at a time. Omitting the Singaporean data, on the other hand, results in a statistically significant coefficient estimate of capital (at the 10% significance level) alongside with an insignificant human capital coefficient.

One may argue that our econometric specification succumbs to the endogeneity problem due to reverse causality. To be specific, wages and income inequality could be considered as the dependent variable as AI reduces labor demand, affecting wages and income distribution. To address this issue and to avoid reverse causality, we estimate two alternative models where we take the lagged values of the dependent variables, i.e. wages, Gini coefficients, and control variables. In formal terms, we estimate

$$\ln AI_{it} = \beta_0 + \beta_1 t + \beta_2 W_{it-1} + \beta_3 W_{it-1}^2 + \boldsymbol{\beta_4 X_{it-1}} + \epsilon_{it}$$

and

$$\ln AI_{it} = \beta_{0i} + \beta_1 t + \beta_2 Gini_{it-1} + \beta_3 Gini_{it-1}^2 + \boldsymbol{\beta_4 X_{it-1}} + u_{it}.$$

where $t = 2015, \dots, 2018$. Another interesting feature of this alternative approach is that this specification does not require data extrapolation as the original values of wages, Gini coefficients, and the control variable are available for 2017. We take M4 as the benchmark case where all available control variables are included. The results are presented in Table 5.

Table 5

Estimation Results with Lagged Dependents (no extrapolation)

	I	II
Constant	-11.1***	-18.997***
	1.759	3.217
Year_{t-1}	0.5***	0.535***
	0.146	0.136
Wage_{t-1}	-0.114	
	0.131	
Wage_{t-1}^2	-0.0003	
	0.0018	
Gini_{t-1} (×100)		0.599***
		0.149
Gini_{t-1}^2		-0.0076***
		0.0018
Human Capital $_{t-1}$	2.186***	1.287***
	0.63	0.452
Capital_{t-1}	7.12E-07	1.45E-07
	2.12E-06	1.47E-07
TFP_{t-1}	7.541***	4.74***
	2.622	1.552
R^2	0.42	0.5

According to Table 5, the results are almost identical to the corresponding findings presented in Table 1 and 2 (M4). This suggests that our main results are not affected by using the lagged values of the dependent variables, a strategy that avoids endogeneity due to reverse causality or other problems related to data extrapolation.

To sum up, after conducting these robustness checks, we failed to find any evidence or alternative specification that would jeopardize our results presented in the main section. In particular, different econometric techniques yield practically identical results compared to our original findings. We find no evidence for outliers that could discredit the hump-shaped relationship between per capita AI investments and Gini coefficients or the statistically insignificant coefficients of wages. Our results are also robust with respect to using lagged values of the dependent variables that tackle the issue of reverse causality and allow us to dispense with data extrapolation.

5. Conclusion

In this study, we empirically analyze how wages and income inequality affect AI, a key labor-saving technology. Our results suggest that the impact of wages on AI is statistically insignificant. Nonetheless, we cannot reject the hypothesis that income inequality measured by the standard Gini coefficient has a statistically significant effect on per capita AI investments. According to our results, the relationship between income inequality and AI is nonlinear. In particular, we find that per capita AI investment reaches its peak point at a certain economic inequality level, corresponding to 0.389 as the Gini coefficient. Supporting our empirical findings; the Gini coefficient of Israel (0.350), the US (0.386), and Singapore (0382)—the global leaders in AI per capital--are clustered around this particular optimal level of economic inequality.

As the range of Gini coefficient is 0.25-0.60 in our dataset, these figures suggest that a moderate level of income inequality yields the most suitable environment for developing AI technologies. Notice that another country leading the AI revolution is China, which is only second to the US in total AI investments. Indeed, the economic inequality in China (0.412) is very close to our estimated optimal level of the Gini coefficient. This result reveals that China has a valuable edge in the global AI race. Yet, for the years to come, it is crucial that China avoids worsening income inequality to protect its advantageous position.

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Appendix: A Toy Economic Model

Consider an economy with two individuals: a firm owner and a worker. In this simple economic model we shall demonstrate the hump-shaped relationship between the optimal AI investments and the disposable income inequality, measured by the post-tax Gini coefficient. In particular, we vary the tax rate and plot its impact on AI and income inequality levels.

Now suppose that a consumption good is produced using human labor, H, supplied by the worker and artificial intelligence, AI. So the production technology is

$$Q = F(H + AI)$$

where Q is the level of output of the final consumption good, and F is a concave increasing function. The AI development requires physical resources denoted by X:

$$AI = X^a$$

where $a \in (0,1)$.

The government imposes sales/excise taxes $t \in [0,1]$ so that the revenue of producing Q net of taxes is $(1-t)Q$ for the firm owner. The collected taxes, tQ, is redistributed to each individual as lump-sum transfers, denoted by T. Therefore, the government budget constraint is $2T = tQ$.

Assuming in-house development of AI, the firm solves the profit maximization problem:

$$\pi \stackrel{\text{def}}{=} \max(1-t)Q - wH - X$$

by choosing (H, X) subject to Eq (1-2) above. The first order conditions of profit maximization are

$$(1-t)F' = w$$

$$(1-t)aX^{a-1}F' = 1$$

where F' is the first order derivative of $F(\cdot)$.

The labor supply of the worker is assumed to be

$$h = 0.5(1 - T/w)$$

so that the labor supply increases with wages and decreases with unconditional government transfers.‡ The firm owner does not supply labor so that $H = h$ at the labor market equilibrium.

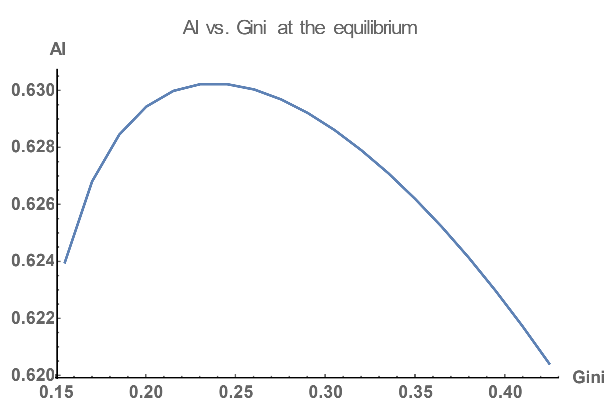
To numerically simulate the model; let $a = 0.1$, and $F = (H + AI)^{\beta}$ where $\beta = 0.15$. At these parameter values, we solve the model and obtain the equilibrium level of AI and income distribution measured by the Gini coefficient at each $t \in [0,1]$. The general formula for the Gini coefficient is

$$Gini = \frac{2\sum i \times y_i}{n \times \sum y_i} - \frac{n+1}{n}$$

where $y_i \leq y_{i+1}$ is the income level of individual $i = 1, \dots, n$. In this toy model, in which $n = 2$, this expression boils down to

$$Gini = \frac{\pi + \mathrm{T}}{Q} - \frac{1}{2}.$$

The graph of the optimal AI and the Gini couple as taxes vary starting between $t = 0$ and $t = 0.9$ is plotted below:



‡ To be rigorous, the labor supply is the solution to the utility maximization problem: $\max\{u(c,h)$ s.t. $c = w_H h + T\}$ where $u(c,l)$ represents the preferences of the individual over consumption, c, and labor supply, h. Obviously, $c = w_H h + T$ is the budget of the individual. The most widely used and known preference is Cobb-Douglas, $u(c,h) = c \times (1-h)$ which yields exactly the supply curve given above.