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Dynamic Banking and the Value of Deposits

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[Preliminary and Incomplete]

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Abstract

Deposits are inside money issued by banks, serving as means of payment for the rest of the economy. Depositors value the payment function and assign a money premium to deposits, which reduces banks' cost of financing. Therefore, deposits create value for banks. However, driven by payment flows, deposits are essentially debts with random maturities that cannot be fully controlled. Banks adjust deposit flows by changing the deposit rate, but given depositors' right to withdraw, banks cannot set too negative a rate. Once the rate hits this lower bound, banks lose control of leverage. Under equity issuance costs, deposits destroy value for banks that are significantly undercapitalized and eager to deleverage. Outside money issued by the government can liberate undercapitalized banks by absorbing the money demand.

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1 Introduction

Deposits are inside money – stores of value and means of payment issued by banks to depositors. Because deposits can be withdrawn at any time, they are generally treated as short-term debts in macro-finance models (e.g., Drechsler, Savov, and Schnabl, 2018) with interest rates below the prevailing rate by a money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2017; Begenau, 2019).

In contrast, we argue that deposits are not short-term debts. As means of payment, they are essentially bank debts with *stochastic maturity*. We develop a dynamic model of banking, where deposit liability plays the central role, and characterize a bank's decisions on deposit-taking, lending, reserve holding, short-term borrowing, equity issuance, and dividend payout in the presence of financing costs and regulatory constraints. The paper also discusses the model's implications on the financial stability effect of outside money, the importance of safe asset supply by the government, the transmission of monetary policy in the presence of frictional interbank market.

Brunnermeier and Sannikov (2016) is a notable exception to the practice of modelling deposits as short-term debts. They model deposits as long-term nominal liabilities, whose endogenous price fluctuation causes vicious interactions in crises between the fire sale of bank assets and Fisherian deflationary spiral. Our paper focuses instead on a single bank's problem, and therefore, do not analyze the implications of nominal contracts and macroeconomic feedback effects. Our focus is on modelling deposits as term debts with stochastic maturity and controllable increments.

Banks are at the center of payment system (Bianchi and Bigio, 2014; Piazzesi and Schneider, 2016; Denbee, Julliard, Li, and Yuan, 2018). When a depositor makes payments with deposits, her bank debits her deposit account, and if the payee's account is at a different bank, the payer's bank has to send reserves to the payee's bank to settle the payment, losing deposits on the liability side and reserves on the asset side of its balance sheet. As such, depositors' payment leads to an effective withdrawal of deposits. Depositors may also withdraw dollar bills to settle transactions outside of the banking system. Therefore, the maturity of deposit liability is not chosen by the bank. It depends on depositors' payment needs that are uncertain. With a diversified depositor base, the bank essentially views deposits as debts that retire at a stochastic rate.

Modelling deposits as debts with stochastic maturity is in line with the classic three-dates models (see, e.g., Diamond and Dybvig, 1983; Allen and Gale, 2004b).[1] The second feature of deposits in our model is a lower bound on deposit rate. A bank cannot set the deposit rate that is too negative, because depositors will withdraw en masse to earn the zero return of holding dollar bills. The third feature of deposits is that a bank faces an elastic demand for its deposit liabilities, reflecting its deposit market power (Drechsler, Savov, and Schnabl, 2017), and, by adjusting the deposit rate, it can partially control the deposit inflows and outflows.

These three features of deposits are at the core of our model. We model a bank's decision-making environment in continuous time, so the shocks to loan returns and to deposit flows are diffusive shocks. As a result, the bank does not default because it can always adjust its balance sheet locally. Without default, we can focus on the key implications of the three defining features of deposits. In addition to choosing the deposit rate, the bank also make decisions on the size of risky loan portfolio, reserve holdings, short-term risk-free debts (or risk-free investments), equity issuance, and dividend payout. The model features equity issuance costs following Bolton, Chen, and Wang (2011) and, on the regulatory side, capital requirement and liquidity requirements.

Next, we summarize the main results. The bank's problem is Markovian and exhibits convenient homogeneity properties. The ratio of bank capital (equity) to deposit stock, "k", emerges as the key state variable that drives the bank's decision making. Solving the bank's problem is equivalent to solving an ordinary differential equation (ODE) given by the Hamilton–Jacobi–Bellman equation. We calibrate the parameters and solve the ODE numerically.

The optimality conditions for the bank's payout and equity issuance decisions lead to a upper (payout) boundary and a lower (issuance) boundary for k: when bank capital is sufficiently large relative to deposit liabilities, the bank pays out dividends; when bank capital is sufficiently small relative to deposit liabilities, the bank pay the issuance costs and raise equity.

The equity issuance costs cause the marginal value of equity capital to be larger than one, and depending on the value of k, the marginal value of capital varies in a wide range from one (at the payout boundary) and above four (at the issuance boundary) even though we calibrate the

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[1] In these models, agents' random preferences over consumption timing translate into uncertainty in deposit outflow.

proportional and fixed issuance costs to conservative values in the empirical literature. The large variation of marginal value of capital causes the bank to be endogenously risk-averse. The bank reduces lending significantly as k declines, approaching the costly recapitalization. As k increases, the bank increases its leverage dramatically, via both deposits and short-term debts. Near the peak of stationary density of k, the marginal value of equity is only slightly above one, so for the majority of time, the bank does not seem to be financially constrained. However, the shadow value of equity shoots up dramatically when equity is significantly depleted relative to the deposit liabilities (i.e, k is low), showing a sharp contrast between normal time and under-capitalized period.

The capital requirement does not always bind, consistent with the finding of Begenau, Bigio, Majerovitz, and Vieyra (2019). It binds only when bank capital is sufficiently *high* relative to deposits (i.e, k is high) and the risk-taking incentive is strong, which it happens around 10% of the time according to the stationary density of k generated by the numeric solution. A non-binding capital requirement does not mean that bank capital is cheap. In the presence of equity issuance cost, the shadow value of one dollar capital can move significantly above one especially when k is low when the capital requirement is not binding. There is an active debate on how expensive bank capital is. Our model shows that, when valuing bank capital, the other components of the liability structure matter, especially the deposits.

Deposits are valuable because, enjoying the convenience of payment services, depositors are willing to accept a deposit rate that is below the prevailing risk-free rate. When the bank has sufficient capital to buffer risk (i.e., k is sufficiently away from the lower boundary of costly equity issuance), deposits create value by allowing the bank to finance risky lending with relatively cheap sources of funds. The deposit stock thus serves as a form of productive capital for the bank.

However, when the bank's capital is significantly depleted, the marginal value of deposits turns sharply negative. The reason is that near the costly issuance boundary, deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage that amplifies the shock impact, increasing the likelihood of costly issuance. The bank wants to deleverage, turning away deposits by lowering the deposit rate. However, the deposit rate cannot be set too negative without triggering massive withdrawal of deposits and the shut-down of the bank. We normalize

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the lower bound on deposit rate to be zero but acknowledge that in reality, banks may set a negative rate explicitly or implicitly by charging fees. Therefore, in our model, once the deposit rate hits the zero lower bound, the bank loses control of its leverage.

The equity issuance costs are the key ingredients that lead to the state-dependent value of deposits and other results. We show that without the equity issuance costs, the bank's value is linear in its equity capital and deposit stock, and the bank does not exhibit endogenous risk aversion, always setting its leverage to the maximum allowed by the capital requirement. Moreover, the loan risk and the risk in deposit flow uncertainty both disappear from the bank's value function.

Deposits are very different from short-term debts. For short-term debts, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who do not hold accounts at the bank. As long as depositors are willing to hold deposits, the bank cannot turn away the existing depositors.

Overall, deposits add value only if the bank has sufficient amount of capital. When capital is significantly low following bad shocks to the loan portfolio and when the zero lower bound on deposit rate binds, deposits become burden. The bank can be liberated if depositors decide to withdraw at a faster rate. One way to achieve this is to provide depositors alternative monetary assets. The government can increase its supply of short-term government bonds, which have long been recognized as money-like especially when held through the money-market mutual funds.[2] As long as government securities are not perfect substitutes of deposits in terms of the payment convenience, depositors will not withdraw en masse to pursue the positive yield on government securities but only gradually rebalance their portfolio by reducing the weight on deposits. Therefore, when banks are undercapitalized at the deposit zero lower bound, government debt issuance stabilizes banks by reducing their leverage to a more desirable level.

When capital is abundant relative to deposits (i.e., k is high), the bank raises funds from short-term debts for risky lending, but when capital is relatively scarce (i.e., k is low), the bank

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[2]The monetary value of government liabilities is an old theme (Patinkin, 1965; Friedman, 1969). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jørgensen (2012), Greenwood, Hanson, and Stein (2015), Bolton and Huang (2016), Nagel (2016), and Li (2017).

switches its short-term debt position, holding risk-free debts instead to reduce the overall riskiness of its asset portfolio. When k is small and the deposit rate hits the zero lower bound, the bank loses control of its leverage, so it has to work on the asset-side of its balance sheet, tuning down its risk exposure by holding risk-free assets, in order to reduce the likelihood of costly equity issuance. Such portfolio rebalancing creates a demand for safe assets in financial crises. The government is in a unique position to supply such assets. Banks' demand for safe assets pushes downward the interest rate, reducing the government's financing cost. The government can thus issue more debts to meet banks' demand, and then use the proceeds from debt issuance to stimulate the economy.

The bank switches from short-term borrowing to short-term lending when its capital is too low relative to deposit liabilities. This suggests that as long as banks are creditworthy, the interbank market can serve as a counter-balancing force against the collapse of lending in crisis. Because loan creation simultaneously creates deposits, lending incurs the costs of settling payments associated with the newly created deposits.[3] Such costs are mitigated when the interbank market has abundant funding. Therefore, when the undercapitalized banks are eager to lend on a risk-free basis, banks can easily borrow in the interbank market to reduce the settlement costs, which stimulates lending. Note that this mechanism is active only if banks are creditworthy and interbank lending does not involve counterparty default risk. In times of financial stress, the impact of government guarantee can be amplified by this channel, because by taking the bank default risk off the table, government guarantee activates this positive effect of an active interbank market on bank lending.

2   Model

We model a single bank's decisions under the risk-neutral measure, effectively assuming no arbitrage and taking as exogenous the pricing kernel (stochastic discount factor) that depends on the aggregate dynamics of the broader economy. Let r denote the risk-free rate, which is also the

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[3]Bank lending is essentially a debt swap. The borrower gives the lending bank its debt, i.e., the loan. The lending bank gives the borrower newly issued deposits (Tobin, 1963; Bianchi and Bigio, 2014; Donaldson, Piacentino, and Thakor, 2018). When the borrower spends such deposits, the bank changes credit the deposits to the payees' accounts if the payees hold accounts at this bank, and if the payees hold accounts at a different bank, the lending bank dispenses reserves to the payee's bank and the payee's bank credits deposits in the payee's account.

expected return of all financial assets under the risk-neutral measure.

Risky assets. We use A_t to denote the value of the bank's holdings of loans and other investments at time t.[4] It has the following law of motion:

$$dA_t = A_t \left(r + \alpha_A \right) dt + A_t \sigma_A d\mathcal{W}_t^A, \tag{1}$$

The parameter α_A reflects the return from the bank's expertise. Because we set up our model under the risk-neutral measure, α_A is the risk-adjusted value-added.[5] The second term in (1) describes the Brownian shock, where σ_A is the diffusion-volatility parameter and $\mathcal{W}^A$ is a standard Brownian motion. Examples of these shocks include unexpected charge-offs of delinquent loans. At any time t, the bank may adjust its risky assets and the liability structure (i.e., deposits, bonds, and equity).

Deposits. Deposits are at the core of our model. Let X_t denote the value of deposits at time t on the liability side of the bank's balance sheet. It has the following law of motion:

$$dX_t = -X_t \left(\delta_X dt - \sigma_X d\mathcal{W}_t^X \right) + X_t n \left(i_t \right) dt. \tag{2}$$

where $\mathcal{W}_t^X$ is a standard Brownian motion. Given a diversified depositor base, a $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X \right)$ fraction are withdrawn in dt because depositors may need cash or pay agents who hold accounts at other banks. If $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X \right) > 0$, the bank's own depositors receive payments into their accounts. Deposits thus have an average *effective duration* of $1/\delta_X$, and σ_X captures payment flow uncertainty.[6] The stochastic withdrawal is in line with the three-dates models (see, e.g., Diamond and Dybvig, 1983; Allen and Gale, 2004b), where agents' stochastic preferences over early and

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[4]The bank's assets include not only loans but also other assets that generate revenues of trading and services such as cash management, trade credit, derivatives, structured products, and underwriting of securities (Bolton, 2017).

[5]The bank may have expertise in loan screening (Ramakrishnan and Thakor, 1984), monitoring (Diamond, 1984), relationship lending (Boot and Thakor, 2000), restructuring (Bolton and Freixas, 2000), and serving local markets (Gertler and Kiyotaki, 2010). More generally, in the macro-finance literature, banks are often modelled as agents with expertise in asset management (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014, 2016).

[6]The value of δ_X largely depends on where the bank sits in the payment network, and the payment flow uncertainty σ_X can be significant in data (see, e.g., Denbee, Julliard, Li, and Yuan, 2018).

late consumption translate into uncertainty in the deposit outflow. The deposit flow shock, $d\mathcal{W}_t^X$, is likely to be positively correlated with the loan repayment shock, $d\mathcal{W}_t^A$, as a healthy asset portfolio can attract depositors. Let ϕdt denote the instantaneous covariance between $d\mathcal{W}_t^X$ and $d\mathcal{W}_t^A$.

In the presence of diffusive shocks (instead of jump shocks), the bank can avoid default by adjusting the balance sheet locally and thus preserve a positive continuation value for equityholders. Therefore, deposits are risk-free for depositors. The deposit rate is i_t, chosen by the bank. The spread, $r - i_t$, can be positive if agents value the convenience of deposits as means of payment (e.g., DeAngelo and Stulz, 2015; Nagel, 2016; Piazzesi and Schneider, 2016; Li, 2017, 2018) or value the secondary-market liquidity due to deposits' information-insensitivity (e.g., Gorton and Pennacchi, 1990; Holmström, 2012; Dang, Gorton, Holmström, and Ordonez, 2014).

The bank can adjust the growth rate of deposit stock by setting i_t via $n(i_t)\, dt$, where the deposit demand elasticity depends on the bank's deposit market power (Drechsler, Savov, and Schnabl, 2017). When the deposit rate is very low, we can have $n(i_t) < 0$.

To maintain the existing deposits (and customer relationships) and attract new deposits, the bank pays a flow cost $C(n(i_t), X_t)\, dt$, for example, from operating branches.

Deposits are essentially long-term debts with stochastic maturity and controllable increments. Not all depositors withdraw at the same time, and withdrawal depends on depositors' payment needs. Therefore, a diversified depositor base implies an effective duration of deposits that depends on the average rate of withdrawal.

Our treatment of deposits stands in contrast with the macro-finance literature that generally treats deposits simply as short-term debts because depositors can withdraw at any time. We emphasize that the right to withdrawal does not necessarily translate into a low duration of deposits but rather imposes a lower bound on the feasible deposit rate – the bank cannot set a negative deposit rate because depositors will withdraw en masse and earn the zero return on dollar bills. We assume that in such a bank run, the shareholders' equity is wiped out, so the bank always avoids such scenario. We will show that the zero lower bound on deposit rate generates a rich set of results. In reality, depositors may tolerate a negative rate because the return on dollar bills is negative due to information and because holding a large volume of dollar bills is costly, but as long as there exists

a lower bound on the deposit rate, our qualitative results still hold.

Reserves and Settlement Costs.   Equation (2) gives the *net* deposit flow in from t to $t + dt$, which is of order dt. However, within a period dt, the bank can face large deposit outflows due to depositors' payment activities, and thus, has to hold reserves. When depositors instruct payments to recipients who hold accounts at other banks, the bank settles the payments by transferring reserves to the recipients' banks.[7] The bank may receive payments (reserve inflows), and therefore, by the end of the period, has a *net* flow of deposits that is much smaller than the intra-period gross exposure. However, if the outflows significantly exceeds the inflows at some point within the period, the bank exhausts its reserves and is forced to borrow reserves from other banks or the central bank, incurring costs. To avoid such costs, the bank holds a buffer stock of reserves, R_t, on the asset side of its balance sheet. The bank's reserves pay an interest rate ι set by the central bank.[8]

We specify the settlement costs as $S\left(R_t, X_t, A_t\right)$, which decreases in reserves and increases in deposits and loans. The payment imbalance increases in deposits because the more deposits the bank has, the more payments it settles. The imbalance also increases in the size of loan portfolio because when the bank extends loans, it creates an equal amount of deposits for borrowers to make payments, and such payments may be sent to other banks, resulting in reserve outflows (Tobin, 1963; Bianchi and Bigio, 2014; Donaldson, Piacentino, and Thakor, 2018).[9] By explicitly modelling interbank credit markets, Bigio and Sannikov (2019) provide a microfoundation for the

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[7]We consider real-time gross settlement (RTGS) that has been adopted in most developed countries, such as the Eurosystem, Japan, the U.K., and the U.S. In contrast to deferred net settlement (DNS), RTGS minimizes counterparty risks by requiring banks to settle transactions in real time using reserves and does not accommodate the netting of payment inflows and outflows.

[8]In the U.S., The Financial Services Regulatory Relief Act of 2006 authorized the Federal Reserve Banks to pay interest on balances held by or on behalf of depository institutions at Reserve Banks, subject to regulations of the Board of Governors, effective October 1, 2011. The effective date of this authority was advanced to October 6, 2008 by the Emergency Economic Stabilization Act of 2008. Sweden and Denmark have paid negative interest on excess reserves (effectively taxing banks for exceeding their reserve requirements) as an expansionary monetary policy measure. The Bank of England started to pay interest of 0.5% on reserves on 5 March 2009.

[9]Bank lending is essentially a debt swap. The borrower gives the lending bank its debt, i.e., the loan. The lending bank gives the borrower newly issued deposits. When the borrower spends such deposits, the bank changes credit the deposits to the payees' accounts if the payees hold accounts at this bank, and if the payees hold accounts at a different bank, the lending bank dispenses reserves to the payee's bank and the payee's bank credits deposits in the payee's account. McLeay, Radia, and Thomas (2014) describe the mechanics of loan and deposit creation.

settlement costs and connect deposits to a bank's need to hold reserves. We emphasize that loan creation also leads to payment outflows, highlighting a necessary link between the bank's lending decision and the choice of reserves.

Finally, the bank also holds reserves to meet the regulatory requirement of reserve holdings:

$$R_t \geq \xi_R X_t. \tag{3}$$

This regulatory constraint can also be motivated by the more recent requirement on liquidity coverage ratio – banks must hold a sufficient amount of high-quality liquid assets to cover cash outflows (Basel Committee on Banking Supervision, 2013).

Bonds. The bank issues short-term bonds (e.g., financial commercial papers), and it is costless to do so. Let B_t denote the value of bonds issued at t that will mature at $t + dt$. Without default risk, the contractual rate of return for short-term debt initiated at t is the risk-free rate r. The bank's bond interest payment over time interval dt is $B_t r dt$. The bank may choose not to issue bonds but instead invest in risk-free bonds issued by other entities in the economy (e.g., the government). In this case, we have $B_t < 0$. Whether the bank issues or holds risk-free bonds will depend on its risk-taking capacity, which in turns depends on the existing deposit liabilities and equity capital.

Equity, Dividend, and Costly Issuance. Let K_t denote the bank's equity (or "capital"), so the following identities summarizes all the balance-sheet items:

$$K_t = \underbrace{A_t + R_t}_{assets} - \underbrace{(B_t + X_t)}_{debts}. \tag{4}$$

The bank can pay out dividends that reduce K_t. We use U_t to denote the (undiscounted) cumulative dividends, so the amount of (non-negative) incremental payout is dU_t. However, the bank must meet a capital requirement. For example, the Basel III accords stipulate that banks must back a specific percentage of risk-weighted assets with equity.[10] As in Begenau (2019), Davydiuk (2017),

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[10]See Thakor (2014) for a review of the debate on bank capital and its regulations.

Nguyen (2015), and Van den Heuvel (2018), we introduce

$$K_t \geq \xi_K A_t \,. \tag{5}$$

The bank may find it optimal to issue external equity. In reality, banks face significant external financing costs due to asymmetric information and incentive issues.[11] A large empirical literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.[12] Let F_t denote the bank's (undiscounted) cumulative net external equity financing up to time t and H_t to denote the corresponding (undiscounted) cumulative costs of external equity financing up to time t. Following Bolton, Chen, and Wang (2011), we assume that the bank incurs both fixed and proportional costs of issuing equity. To preserve the model's homogeneity property for tractability purposes, we further assume that the fixed cost is proportional to X_t, so that $\psi_0 X_t$ denotes the fixed equity-issuance cost, and $\psi_1 M_t$ refers to the proportional equity-issuance cost, where M_t is the amount raised.

The bank's equityholders are protected by limited liability. Let τ denote the stochastic stopping time when the bank defaults. Therefore, the bank maximizes the equityholders' value,

$$V_0 = \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t} \left(dU_t - dF_t - dH_t\right)\right].^{13} \tag{6}$$

Because the bank only faces (locally continuous) diffusive shocks, it can avoid default as long as the continuation value is positive. In our numeric solution, this is indeed the case, so $\tau = +\infty$. We assume that the discount rate ρ, i.e., the equityholders' required return, is greater than r. This

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[11]Explicitly modeling informational asymmetry would result in a substantially more involved analysis. Lucas and McDonald (1990) provides a tractable analysis by making the simplifying assumption that the informational asymmetry is short lived, i.e. it lasts one period.

[12]Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was -3% and the loss in equity value as a percentage of the size of the new equity issue was as high as -31% (see Eckbo, Masulis, and Norli, 2007, for a survey).

[13]We assume that the bank manager's incentive is aligned with equityholders. Becht, Bolton, and Röell (2011) discuss the issues of corporate governance in the banking sector.

impatience can be microfounded by a Poisson death rate that is equal to $\rho - r$. In our numeric solution, we calibrate the spread to banks' exit rate. Note that K_t is the book value of equity and V_t, the equityholders' value at t, is the market value of equity. In perfect capital markets à la Modigliani and Miller (1958), $V_t = K_t$. However, as we sill show shortly, $V_t > K_t$ due to the equity issuance cost. The wedge between V_t and K_t measures the value of internal capital.

3   Dynamic Banking without Equity Issuance Costs

A key friction in our model is the equity issuance cost. Next, we show that without such costs, the value function is linear in the deposit stock, X, and capital, K. As a result, the bank does not exhibit endogenous risk aversion and the marginal value of deposits is constant.

Without the issuance costs, the marginal value of capital is equal to one, i.e., $V_K(X, K) = 1$, because if $V_K(X, K) > 1$, the bank will raise equity, and, as previously discussed, the bank pays out dividend if $V_K(X, K) \leq 1$. In Appendix A, we show that there exists a constant Q such that

$$V(X, K) = QX + K. \tag{7}$$

A key result is that Q does not depend on any of the risk parameters, i.e., σ_A and σ_X. Without the equity issuance costs, the bank is not concerned about risks because when it needs capital following adverse shocks, it can always raise capital. The bank's optimal lending is proportional to equity capital and the capital requirement always binds, i.e., $A/K = 1/\xi_K$, as long as the marginal cost of payment settlement is smaller than α_A. Moreover, the bank sets a constant deposit rate. I will compare these properties with those of the solution under the equity issuance costs.

4 Dynamic Banking under Equity issuance Costs

4.1 Bank Optimization

In this subsection, we derive the optimality conditions for the bank's control variables and the Hamilton-Jacobi-Bellman (HJB) equation for the bank's value function. In the next subsections, we parameterize the functions of payment settlement cost and deposit maintenance cost and provide intuitive characterizations of the bank's optimal policies.

State and control variables. The bank solves a dynamic optimization problem with two state variables, the deposit stock X_t and the equity capital K_t. We denote the shareholders' value at time t as V_t. This present value results from the bank's optimal control of the stochastic processes of loan portfolio size A_t, reserve holdings R_t, short-term borrowing B_t, the deposit rate i_t, the payout of dividends dU_t, and the value of newly issued equity shares dF_t:

$$V_t = V(X_t, K_t) = \max_{\{A,R,B,i,U,F\}} \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t}(dU_t - dF_t - dH_t)\right]. \tag{8}$$

The value function is a function of the state variables, i.e., $V_t = V(X_t, K_t)$. Every instant, given the state variables, X_t and K_t, the bank optimizes the control variables before the realization of diffusion shocks, taking into consideration the impact on the evolution of state variables (and through such impact, the continuation value). To solve the bank's optimal choices and value function, we need the laws of motion of state variables that show how the choice variables affect their evolution. The law of motion for X_t is given by (2). For the equity capital K_t, we have

$$dK_t = A_t\left[(r + \alpha_A)dt + \sigma_A d\mathcal{W}_t^A\right] - B_t r dt + R_t \iota dt - S(R_t, X_t, A_t)$$
$$- X_t i_t dt - C(N_t, X_t)dt - dU_t + dF_t. \tag{9}$$

The first three terms on the right side record the return on loans, bond interest expenses, and interest on reserves. The fourth term is the payment settlement cost. The fifth term is deposit interest expenses, and the sixth term is the operation cost associated with adjusting and maintaining the

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deposit stock. The last two terms are the dividend payout and capital raised via the equity issuance.

Given X_t and K_t, the bank's choices of A_t, R_t, and B_t resemble a portfolio problem (Merton, 1969).[14] Let π_t^A denote the portfolio weight on loans, i.e., $\pi_t^A (X_t + K_t) = A_t$, and π_t^R denote the portfolio weight on reserves, i.e., $\pi_t^R (X_t + K_t) = R_t$, so the weight on bonds is $\left(\pi_t^A + \pi_t^R - 1\right)$ because $B_t = A_t + R_t - (X_t + K_t)$. Note that if $A_t > X_t + K_t$, the bank issues bonds, $B_t > 0$, paying the interest rate r; if $A_t < X_t + K_t$, the bank lends in the short-term debt market (i.e., $B_t < 0$) and earns the interest rate r. We can rewrite the law of motion for K_t in (9) as

$$
\begin{aligned}
dK_t = {} & (X_t + K_t) \left[r + \pi_t^A \alpha_A - \pi_t^R (r - \iota)\right] dt + (X_t + K_t) \pi_t^A \sigma_A d\mathcal{W}_t^A \\
& - S\left(\pi_t^R (X_t + K_t), X_t, \pi_t^A (X_t + K_t)\right) - X_t i_t dt - C(N_t, X_t) dt - dU_t + dF_t. \quad (10)
\end{aligned}
$$

Given the Markov nature of the bank's problem, we suppress the time subscript of X and K going forward to simplify the notations wherever it does not cause confusion.

Payout and Equity Issuance. The bank pays out dividends, i.e, $dU_t > 0$, only if the decrease of continuation value is equal to or less than the consumption value of dividends,

$$
V(X, K) - V(X, K - dU_t) \leq dU_t, \quad (11)
$$

i.e.,

$$
V_K(X, K) \leq 1. \quad (12)
$$

The optimality of payout also requires the following smooth-pasting condition:

$$
V_{KK}(X, K) = 0. \quad (13)
$$

The bank raises equity, i.e., $dF_t > 0$, only if the increase of shareholders' value after issuance

---

[14]The bank may adjust the loan amount A_t by selling loans in the secondary market. The technological progress on the reduction of information asymmetries between loan buyers and loan sellers facilitate the trading of loans, and and the design of contract between the loan buyers and originators can alleviate the moral hazard problem (reduced monitoring incentive) on the part of loan originators (e.g., Pennacchi, 1988; Gorton and Pennacchi, 1995).

is equal to or greater than the cost,

$$V(X, K + dF_t) - V(X, K) \geq dF_t + dH_t = \psi_0 X + (1 + \psi_1) M_t. \tag{14}$$

where $dF_t = M_t$ is the capital raised and, as previously discussed, the issuance costs have a fixed and a proportional components, $dH_t = \psi_0 X + \psi_1 M_t$. The fixed cost is set to be proportional to X_t, i.e., the size of the bank to avoid it being negligible as all the balance-sheet variables grow exponentially on the optimal path. The optimal amount of issuance is given by the following first-order condition:

$$V_K(X, K + M_t) = 1 + \psi_1. \tag{15}$$

HJB Equation. Given the laws of motion (2) for X and (10) for K, in the interior region where $dU_t = 0$ and $dF_t = 0$, the bank's HJB equation is

$$
\begin{aligned}
\rho V(X, K) = \max_{\{\pi^A, \pi^R, i\}} \ & V_X(X, K) X \left[-\delta_X + n(i) \right] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \\
& + V_K(X, K)(X + K) \left[r + \pi^A \alpha_A - \pi^R(r - \iota) \right] + \frac{1}{2} V_{KK}(X, K)(X + K)^2 \left(\pi^A \sigma_A \right)^2 \\
& - V_K(X, K) \left[S\left(\pi^R(X + K), X, \pi^A(X + K) \right) + Xi + C(n(i), X) \right] \\
& + V_{XK}(X, K) X(X + K) \pi^A \sigma_A \sigma_X \phi.
\end{aligned}
\tag{16}
$$

Lending. The first-order condition for π^A gives the following solution:

$$\pi^A = \min \left\{ \frac{\alpha_A + \epsilon(X, K) \sigma_A \sigma_X \phi - S_A(R, X, A)}{\gamma(X, K) \sigma_A^2 \left(\frac{X+K}{K} \right)}, \ \frac{K}{\xi_K(X + K)} \right\} \tag{17}$$

where $S_A(R, X, A)$ is the marginal settlement cost from loans, the second term in $\min\{\cdot, \cdot\}$ is given by the capital requirement, and we define

$$\gamma(X, K) \equiv \frac{-V_{KK}(X, K) K}{V_K(X, K)}, \tag{18}$$

the endogenous relative risk aversion, and

$$\epsilon(X, K) \equiv \frac{V_{XK}(X, K) X}{V_K(X, K)}, \tag{19}$$

the elasticity of marginal value of capital, $V_K(X, K)$, to deposits. Even though the bank evaluates the equityholders' payoffs with a risk-neutral objective in (6), it can be effectively risk-averse, i.e., $\gamma(X, K) > 0$, due to the equity issuance cost. When $\epsilon(X, K) > 0$, deposits and capital are complementary in creating value for banks' shareholders.

While setting up $\pi^A = A/(X + K)$ as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e., $A/K = \pi^A(X + K)/K$:

$$\frac{A}{K} = \min \left\{ \frac{\alpha_A + \epsilon(X, K)\sigma_A\sigma_X\phi - S_A(R, X, A)}{\gamma(X, K)\sigma_A^2}, \ \frac{1}{\xi_K} \right\} \tag{20}$$

This solution resembles Merton's solution of portfolio choice. In the numerator, a higher risk-adjusted excess return, α_A, increases lending. The bank's incentive to lend is also strengthened when deposits are natural hedge – the asset-side shock, $d\mathcal{W}^A$, and the liability-side (deposit) shock, $d\mathcal{W}^A$ are positively correlated ($\phi > 0$) and more deposits make capital more valuable (i.e., $\epsilon(X, K) > 0$).[15] The marginal settlement cost, $S_A(R, X, A)$, drags down the effective expected return from lending. It captures the concern that lending, by creating deposits for the borrowers to make payments, leads to reserve outflows and potential costs of interbank or discount-window borrowing. In the denominator, the risk aversion, $\gamma(X, K)$ is multiplied by the lending risk, σ_A^2, as we typically see in the solutions of portfolio optimization problems.

Reserve Holdings. When the reserve requirement (3) does not bind, the optimality condition for π^R equates the marginal cost of holding reserves, i.e., accepting the below-r rate of return ι, and

---

[15]While different in mechanism, this feature of our model echoes the literature on the synergy between lending and deposit-taking (see, e.g., Calomiris and Kahn, 1991; Berlin and Mester, 1999; Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006; Hanson, Shleifer, Stein, and Vishny, 2015)

the marginal benefit of holding reserves to reduce the payment settlement cost:

$$r - \iota = -S_R \left(\pi^R \left(X + K \right), X, \pi^A \left(X + K \right) \right) . \tag{21}$$

The reserve requirement can be rewritten as the following restriction on π^R:

$$\pi^R \geq \frac{\xi_R X}{(X + K)} . \tag{22}$$

Deposit rate. The bank sets the deposit rate, i, to equate the marginal value of new deposits, $V_X (X, K) N_i (i, X)$, and the marginal costs from reducing the shareholders' profits (i.e., return on equity capital) by paying interests on the existing deposits, $V_K (X, K) X$, and by paying the costs of maintaining a larger deposit franchise, $V_K (X, K) C_N (N (i, X), X) N_i (i, X)$:

$$V_X (X, K) N_i (i, X) = V_K (X, K) \left[X + C_N (N (i, X), X) N_i (i, X) \right] . \tag{23}$$

4.2   Optimal Reserve Holdings

We specify the functional form of payment settlement cost, $S(R, X, A)$ to obtain an intuitive expression for the bank's optimal reserve holdings and for our numeric solution of the bank's value function and optimal policies. The settlement cost is increasing and convex in deposits and loans and is decreasing in reserves, so we adopt a quadratic form often used in the Q-theory literature:

$$S(R, X, A) = \frac{1}{2} \left(\frac{\chi_1 X + \chi_2 A}{R} \right)^2 R . \tag{24}$$

The parameter $\chi_1 (\in (0, 1))$ captures the fraction of deposits that are used as means of payment by depositors and the associated costs to the bank due to the reserve flow imbalances and the consequent interbank or discount-window borrowing. The parameters $\chi_2 (\in (0, 1))$ can be interpreted as the convolution of how much of loan value A_t that is from the new loans that lead to deposit creation and, among the new loans (and new deposits issued to finance such loans), what is the fraction that cause payment outflows and the associated costs to the bank.

This functional form gives rise to a Baumol-Tobin style optimal policy of reserve holding. First, we obtain the marginal reduction of settlement cost from reserves

$$S_R(R, X, A) = -\frac{1}{2}\left(\frac{\chi_1 X + \chi_2 A}{R}\right)^2 .$$ (25)

Therefore, the optimality condition (21) for π_t^R implies that $r - \iota = \frac{1}{2}\left(\frac{\chi_1 X + \chi_2 A}{R}\right)^2$, so rearranging the equation we obtain the following reserve holding policy

$$R = \frac{\chi_1 X + \chi_2 A}{\sqrt{2(r - \iota)}}$$ (26)

This optimal reserve holding policy is in the spirit of Baumol (1952) and Tobin (1956) who show that the demand for means of payment is equal to the product of transaction costs (mapping to χ_1 and χ_2) and transaction needs (mapping to X and A) divided by the square root of two times the cost of holding means of payment. As previously discussed, lending and deposit-taking require interbank settlement as the borrowers and depositors make payments. Reserves settle transactions among banks, but holding reserves incurs a carry cost because the interest on reserve is below r.

4.3 Optimal Deposit Rate

First, we specify the deposit demand as

$$n(i) = \omega i ,$$ (27)

where, as shown in (2), ω is the semi-elasticity of deposits X_t with respect to i, which we will calibrate to the estimate from Drechsler, Savov, and Schnabl (2017) in our numeric solution. Next, we specify the deposit maintenance/adjustment cost as follows,

$$C(n(i), X) = \left(\theta_0 + \frac{\theta_1}{2}n(i)^2\right)X .$$ (28)

17

The cost is increasing in the existing amount of deposits, X_t, and is increasing and convex in the flow of new deposits $n(i)$, reflecting the increasing marginal cost of expanding the depositor base.

This functional form leads to a Hayashi style optimal policy of deposit rate. In Hayashi (1982), firms make investments in productive capital, while, in our model, the bank attracts depositors by raising the deposit rate, building up its customer capital. Using (23), we obtain

$$i = \frac{\frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega}}{\theta_1} . \tag{29}$$

Consistent with the evidence in Drechsler, Savov, and Schnabl (2017), the deposit rate is higher when the demand is more elastic, i.e., ω is high. The bank also sets a higher rate to attract more deposits when the marginal adjustment cost increases slowly, i.e., θ is low.

The bank sets a high deposit rate when the marginal value of deposits, $V_X(X, K)$, is high relative to the marginal value of equity capital, $V_K(X, K)$. Paying a higher deposit rate attracts more deposits but paying more interest expenses reduce earnings and equity. Section 3 presents the solution of the bank's problem without the equity issuance costs. In that case, the marginal value of equity is always equal to one and the marginal value of deposits is a constant Q. Therefore, the optimal rate is a constant:

$$i = \frac{Q - \frac{1}{\omega}}{\theta_1} . \tag{30}$$

In the presence of equity issuance cost, the optimal policy of deposit rate depends on X and K.[16]

An interesting feature of the optimal deposit rate is that it hits the zero lower bound when

$$\frac{V_X(X, K)}{V_K(X, K)} \leq \frac{1}{\omega} . \tag{31}$$

Once the deposit rate reaches zero, the bank cannot further decrease the deposit rate to reduce deposits. Later we show that this restriction makes deposits undesirable, especially when the bank is undercapitalized, and thus, is afraid that a high leverage due to large deposit liabilities amplifies

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[16]The difference between (29) and (30) is akin to the difference in a firm's optimal investment in Hayashi (1982) and Bolton, Chen, and Wang (2011). In Bolton, Chen, and Wang (2011), the cost of raising equity induces a state-dependent value of liquidity, so the ratio of marginal value of capital to that of liquidity drives the firm's investment.

the impact of negative shocks on equity, increasing the likelihood of costly equity issuance. When the deposit demand is more elastic, i.e., ω is high, the bank has to pay a higher deposit rate, as shown in (29), which turns to decrease the shareholders' value. However, given the value function, it is less likely for the condition (31) to hold, because a high demand elasticity allows the bank to control the deposit flow more effectively and thereby to avoid hitting the zero lower bound. Our numeric solution will show which force dominates given the calibrated parameter values.

4.4 Optimal Risk-Taking

Given the functional forms of payment settlement costs and deposit maintenance costs, the bank's problem is homogeneous in X and its value function $V(X, K) = v(k) X$, where

$$k = \frac{K}{X}, \tag{32}$$

Therefore, instead of working with X and K as the state variables, we will work with X and k. The capital-to-deposit ratio, k, captures the composition of long-term funding on the liability side of the bank's balance sheet. We will show that the choice variables are functions of k only.

Next, we simplify the expression of loan-to-capital ratio, a measure of the bank's risk-taking. First, note that from (26), we obtain the marginal payment settlement cost from lending:

$$S_A(R, X, A) = \chi_2 \left(\frac{\chi_1 X + \chi_2 A}{R} \right) = \chi_2 \sqrt{2(r - \iota)}, \tag{33}$$

Next, we simplify the expressions of the effective risk aversion in (18)

$$\gamma(k) = \frac{-V_{KK}(X, K) K}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \tag{34}$$

and the elasticity of marginal value of capital to deposits in (19)

$$\epsilon(k) = \frac{V_{XK}(X, K) X}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \tag{35}$$

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which happens to be equal to $\gamma(k)$.

Using (33) and $\epsilon(k) = \gamma(k)$, we simplify the optimal loan-to-capital ratio from (20):

$$\frac{A}{K} = \min\left\{\frac{\alpha_A - \chi_2\sqrt{2(r-\iota)}}{\gamma(k)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi, \ \frac{1}{\xi_K}\right\}, \tag{36}$$

The bank's risk-taking is state-dependent and only depends on k through $\gamma(k)$. When the effective risk aversion is low, the bank chooses a high loan-to-capital ratio; when the effective risk aversion is high, the bank reduces its risk exposure. In our numeric solution, we show that $\gamma(k)$ decreases in k, so the loan-to-capital ratio increases when the bank has a high equity buffer relative to its deposit liabilities. The correlation between the loan return shock and the deposit flow shock, ϕ, induces a hedging demand. The risk of deposit flow is essentially the bank's background risk from the perspective of portfolio management. The constant marginal settlement cost, $\chi_1\sqrt{2(r-\iota)}$, drags down the expected return from lending and the deposit-hedging term becomes a constant $\frac{\sigma_X}{\sigma_A}\phi$. Accordingly, we impose the parameter restriction

$$\alpha_A > \chi_2\sqrt{2(r-\iota)}. \tag{37}$$

4.5 Solving the Value Function

The Value Function ODE. To solve the bank's value function, we simplify the HJB equation to obtain an ordinary differential equation for $v(k)$. First, given that $V(X, K) = v(k)X$, we obtain

$$V_K(X, K) = v'(k) , \ V_X(X, K) = v(k) - v'(k)k$$

$$V_{KK}(X, K) = v''(k)\frac{1}{X}, \ V_{XX}(X, K) = v''(k)\frac{k^2}{X}, \ V_{XK}(X, K) = -v''(k)\frac{k}{X}. \tag{38}$$

Using these expressions, we can rewrite the HJB equation (16) as

$$\rho v\left(k\right) = \max_{\pi^A, \pi^R, i} \; \left[v\left(k\right) - v'\left(k\right)k\right]\left(-\delta_X + \omega i\right) + \frac{1}{2}v''\left(k\right)k^2\sigma_X^2 \tag{39}$$

$$+ v'\left(k\right)\left(1+k\right)\left[r + \pi^A\alpha_A - \pi^R\left(r - \iota\right)\right] + \frac{1}{2}v''\left(k\right)\left(1+k\right)^2\left(\pi^A\sigma_A\right)^2$$

$$- v'\left(k\right)\left[\frac{1}{2}\left(\frac{\frac{\chi_1}{1+k} + \chi_2\pi^A}{\pi^R}\right)^2\pi^R\left(1+k\right) + i + \theta_0 + \frac{\theta_1}{2}\left(\omega i\right)^2\right]$$

$$- v''\left(k\right)k\left(1+k\right)\pi^A\sigma_A\sigma_X\phi\,.$$

To show that (39) is an ODE for $v\left(k\right)$, we need to show that the control variables only depend on k and the level and derivatives of $v\left(k\right)$. First, by definition, $\pi^A = A/\left(X + K\right)$, so we obtain the following simplified expression for π^A from (36):

$$\pi^A = \left(\frac{A}{K}\right)\left(\frac{K}{K+X}\right) = \min\left\{\frac{\alpha_A - \chi_2\sqrt{2\left(r - \iota\right)}}{\gamma\left(k\right)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi, \; \frac{1}{\xi_K}\right\}\left(\frac{k}{1+k}\right)\,. \tag{40}$$

Rearranging (26), we can solve π^R as a linear function of π^A and the state variable k:

$$\pi^R = \frac{\chi_2}{\sqrt{2\left(r - \iota\right)}}\pi^A + \frac{\chi_1}{\left(1+k\right)\sqrt{2\left(r - \iota\right)}}\,, \tag{41}$$

so it also only depends on k and the level and derivatives of $v\left(k\right)$. Finally, the optimal deposit rate given by (29) only depends on $V_X\left(X, K\right) = v\left(k\right) - v'\left(k\right)k$ and $V_K\left(X, K\right) = v'\left(k\right)$. Then we can substitute these optimal choices into the HJB equation to obtain an second-order ODE for $v\left(k\right)$ that contains only k and the level and derivatives of $v\left(k\right)$. Fully solving the model then takes two steps, first, solving the ODE to obtain $v\left(k\right)$, and second, using the solved $v\left(k\right)$ and its derivatives to solve the bank's optimal choices.

Boundary conditions. Let $\overline{k}$ and $\underline{k}$ denote respectively the dividend payout and issuance boundaries, and let $m \equiv M/X$ denote the amount financing raised via issuance (scaled by X). The

boundary conditions implied the optimality condition on payout (12) and (13) are

$$v'\left(\overline{k}\right) = 1 \,, \tag{42}$$

and the smooth-pasting condition,

$$v''\left(\overline{k}\right) = 0 \,. \tag{43}$$

The boundary conditions implied by the optimality condition on issuance (14) and (15) are

$$v\left(\underline{k} + m\right) - v\left(\underline{k}\right) = \psi_0 + \left(1 + \psi_1\right) m \,, \tag{44}$$

and

$$v'\left(\underline{k} + m\right) = 1 + \psi_1 \,. \tag{45}$$

Our numerical solution of $v\left(k\right)$ features global concavity, so (42) and (45) imply that $\overline{k} > \underline{k}$. Given $\underline{k}$, the four boundary conditions above solve the second-order ODE for $v\left(k\right)$ (i.e., the HJB equation), the upper boundary $\overline{k}$, and the endogenous issuance amount m. However, we still need one condition to pin down $\underline{k}$. In our numerical solution, $v\left(k\right)$ is globally concave, so $\underline{k} = 0$, i.e., the bank does not pay the issuance costs unless its capital drops to zero. However, in the presence of reserve requirement, $\underline{k}$ can be positive. Substituting (41) into the reserve requirement (22), we have

$$\frac{\chi_2}{\sqrt{2\left(r - \iota\right)}}\pi^A + \frac{\chi_1}{\left(1 + k\right)\sqrt{2\left(r - \iota\right)}} \geq \frac{\xi_R}{\left(1 + k\right)} \,, \tag{46}$$

Using (40) to substitute out π^A and rearranging the equation, we have

$$\min\left\{\frac{\alpha_A - \chi_2\sqrt{2\left(r - \iota\right)}}{\gamma\left(k\right)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi, \ \frac{1}{\xi_K}\right\}k \geq \frac{\xi_R\sqrt{2\left(r - \iota\right)} - \chi_1}{\chi_2} \,. \tag{47}$$

In our numeric solution, the right side increases in k (as $\gamma\left(k\right)$ increases in k). Therefore, (47) imposes a lower bound of k, and if it is above zero, it pins down $\underline{k}$ (otherwise, $\underline{k} = 0$).

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. Whenever applicable, parameter values are annualized.

Parameters	Symbol	Value
risk-free rate	r	2%
discount rate	ρ	13%
bank excess return	α_A	1.2%
asset return volatility	σ_A	10%
average duration of deposits	$1/\delta_X$	20 years
deposit flow volatility	σ_X	20%
correlation between deposit and asset shocks	ϕ	0.6
deposit demand semi-elasticity	ω	5.3
deposit maintenance cost (linear component)	θ_0	-0.03
deposit maintenance cost (quadratic component)	θ_1	0.5
deposit coefficient in payment settlement cost	χ_1	0.3%
loan coefficient in payment settlement cost	χ_2	3.6%
capital requirement	ξ_K	7%
reserve requirement	ξ_R	5%
interest on reserves	ι	0.68%
equity issue fixed cost	ψ_0	1.0%
equity issue propositional cost	ψ_1	5.0%

5 Quantitative Analysis

5.1 Parameter Choices

One unit of time is set to one year. We choose r equal to 2% in line with the historic average of fed fund rate since 1990s. The spread between shareholders' required return and the risk-free rate, $\rho - r$, has several components. First, as previously discussed, it includes a Poisson-arriving death rate. We set this component to 7% following Gertler, Kiyotaki, and Prestipino (2019). Second, the shareholders require a risk premium, which we set to 4%, consistent with a beta of banking sector equal to 0.7 (Fahlenbrach, Prilmeier, and Stulz, 2012, 2017) and a historic average of equity premium around 6%. Therefore, ρ is equal to 13%, which is the sum of risk-free rate (2%), the

exit rate (7%), and the shareholders' required compensation for systematic risk (4%).

We set α_A to 1.28% following Begenau (2019) who calculate that the average pre-tax excess return on the aggregate bank loan portfolio is around 0.32% per quarter higher than a maturity- and credit-matched replicating portfolio based on investment-grade corporate bonds from Vanguard. This parameter captures banks' expertise. We set asset-value volatility, σ_A, to 10% following Sundaresan and Wang (2014) who in turn refer to the calculation of Moody's KMV Investor Service.

On the deposit dynamics, we set δ_X to 5% and σ_X to 20% so the mean and volatility of deposit growth rate are in line with the calibration of Bianchi and Bigio (2014). We set ω, the semi-elasticity of deposits with respect to the deposit rate, to 5.3, which is the estimate from Drechsler, Savov, and Schnabl (2017). We set θ_0, the cost of maintaining existing deposits, to -0.03 so the model generates an average deposit-to-capital ratio in line with data (FRED). We adjust θ_1 to 0.5 so the model generates an average deposit rate that matches data (Driscoll and Judson, 2013). The correlation between asset-side shock and liability-side (deposit) shock is set to 0.6 so that the model generates an average loan-to-deposit ratio that matches the data (FRED). The two parameters of settlement cost function, χ_1 and χ_2, are chosen so that the averages of reserve-to-deposit ratio and reserve-to-loan ratio match data (FRED).

In accordance with Basel III capital standards, FDIC-insured depository institutions are required to maintain the leverage ratio (Tier 1 capital divided by total assets) of 4% (e.g., Nguyen, 2015) and the Tier 1 capital ratio (Tier 1 capital divided by total risk-weighted assets) of 6% (increased to 7% from 2019 onward) (see, e.g., Begenau, 2019; Davydiuk, 2017). While these two ratios are different in the data, but are the same in the model, ξ_K. We set ξ_K equal to 7%. Begenau (2019) and Davydiuk (2017) set ξ_K to be the sample average of the ratio of Tier 1 equity to risky assets in their models for the reason that banks typically maintain a buffer over the regulatory thresholds in order to prevent regulatory corrective action.[17] Note that in our model, the buffer arises endogenously, driven by banks' precaution to avoid paying the equity issuance costs, so we set ξ_K to the regulatory threshold. The Board mandated a zero reserve requirement for banks with

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[17]In other studies on banking regulations, De Nicolò, Gamba, and Lucchetta (2014) calibrates the capital requirements to 4% and 12%, Hugonnier and Morellec (2017) calibrates the thresholds to 4% , 7%, 9%, and 20% to investigate the effects of the proposal by Admati and Hellwig (2013), and Phelan (2016) calibrates the threshold to 7.7% and 10.6% in a macroeconomic model.
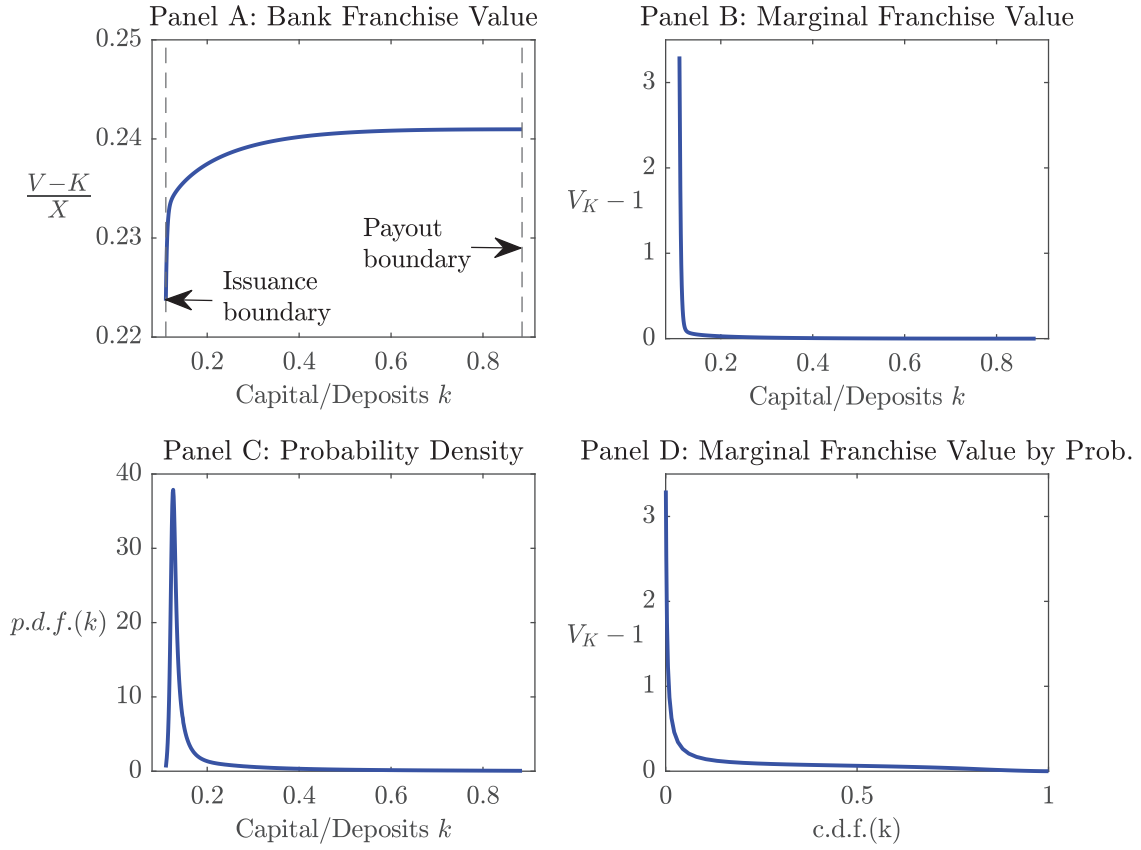
Figure 1: The Shareholders' Value and Book Value of Bank Capital

eligible deposits up to $ 16 million, 3% for banks up to $ 122.3 million, and 10% thereafter, and, in March 2020, set the reserve requirements for all sizes of deposits to zero. Instead, the liquidity coverage ratio imposes a requirement on banks to hold liquid assets. We set $\xi_R = 5\%$. We set the interest on reserves equal to 0.68%, the historic average.

Finally, we set the proportional issuance cost is 5% (Boyson, Fahlenbrach, and Stulz, 2016) and the fixed cost 1% following Bolton, Chen, and Wang (2011).[18]

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5.2 Franchise Value

In perfect capital markets, the bank shareholders' value, V_t, is equal to book equity, K_t. In the presence of equity issuance costs, the bank has to maintain a positive level of profits whose present value justifies paying the cost to raise equity. The issuance costs thus create a wedge between V_t and K_t, which we call the franchise value. In Panel A of Figure 1, we plot the wedge (scaled by the deposit stock X_t) against k, the ratio of bank capital to deposits. As capital accumulates relative to deposits, the franchise value increases. The comovement between franchise value and equity capital is consistent with the evidence from Boyson, Fahlenbrach, and Stulz (2016).

The bank pays the issuance costs at the lower bound of k, $\underline{k}$, when capital is significantly depleted relative to deposits. The further away it is from the issuance boundary, the lower the likelihood of hitting the boundary and paying the issuance costs. Therefore, the franchise value increase in k. The curve ends on the right side at the dividend payout boundary $\overline{k}$. Because the likelihood of paying the issuance cost is low near $\overline{k}$, the franchise value is relatively insensitive to the variation of k. Our results have several implications on the empirical analysis of bank valuation (e.g., Minton, Stulz, and Taboada, 2019). First, instead of book equity as the nominator of valuation metric, deposit stock emerges as the natural denominator. Second, scaled by deposits, the shareholders' value increases in the capital-to-deposit ratio, reaching its highest level when the bank pays out dividends and falling to its lowest level when the bank raises equity. This is consistent with the evidence that bank value is procyclical while equity issuance is countercyclical (Adrian, Boyarchenko, and Shin, 2015; Baron, 2020).

Panel B of Figure 1 plots the marginal franchise value of bank capital, $V_K - 1$. Without financial frictions, this variable should always be equal to zero as $V_t = K_t$. However, near the issuance boundary, the marginal value of equity capital shoots up dramatically. At $\underline{k}$, a value of $V_K - 1$ larger than three means that one dollar of equity is worth four dollars because the imminence of costly equity issuance. Note that even though the proportional cost of issuance is only 5%, due to the fixed cost (0.01), the value of one dollar equity can be more than 300% higher than one.

Panel C of Figure 1 plots the stationary probability density of the state variable k. It shows

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[18]Nguyen (2015) introduces a proportional cost of equity issuance and calibrates it to 2.5% following Gomes (2001).

that in the long run, how much time the bank spends in various regions of k. The probability mass is highly concentrated in the area where k is still sufficiently large and the marginal value of equity is low. In fact, the density function peaks at the k where the marginal value of equity is only 1.0851, i.e., $V_K - 1$ equal to 0.0851. Therefore, even though for the majority of time, the bank does not seem to be financially constrained, the shadow value of equity shoots up dramatically when equity is significantly depleted, showing a sharp contrast between normal time and crisis period. Panel C of Figure 1 is likely to have underestimated the shadow value of internal capital for banks because, in crisis, the equity issuance costs are likely to increase (Bolton, Chen, and Wang, 2013) but in our model, we only consider the constant issuance costs.

In Panel D of Figure 1, we plot the marginal franchise value of bank capital against the cumulative distribution function of k. In the graph, the horizontal span of the curve represents the amount of time the bank spends in this region on the long run. For example, the bank spends half of the time in the region to the right of 0.5 with a marginal franchise value of capital below 0.0634, i.e., one dollar of equity worth below 1.0634 dollars. However, on the left side (from 0 and 0.01), the bank spends 1% of time with the one dollar of equity worth more than 1.8702 dollars, i.e., a marginal franchise value of 0.8702 or higher. Crisis is rare event but its impact is significant.

In Figure 2, we analyze the bank's risk-taking. Panel A plots the ratio of loan value to capital. It cannot exceeds the regulatory upper bound from the capital requirement. Risk-taking is strongly procyclical. As capital accumulates relative to deposits, i.e., k, increases, the bank levers up quickly through deposits and the issuance of short-term bonds and invest in risky loans. If capital is depleted relative to deposits, the bank deleverages. This is consistent with the findings of Ben-David, Palvia, and Stulz (2020) that distressed banks deleverage and decrease observable measures of riskiness, in contrast to the prediction of moral hazard or gambling for resurrection.

An intuitive measure of the risk-taking incentive is $\gamma(k)$, the relative risk aversion coefficient of the bank's value function that is defined in (18). As shown in Panel B of Figure 2, $\gamma(k)$ shoots up dramatically as the bank becomes undercapitalized. In Panel C and D, we plot the loan-to-capital ratio and $\gamma(k)$ against the c.d.f. of k shows how much time the bank spends in different regions. In Panel D, more than 50% of the time, the bank's relative risk aversion is below 0.2 (in

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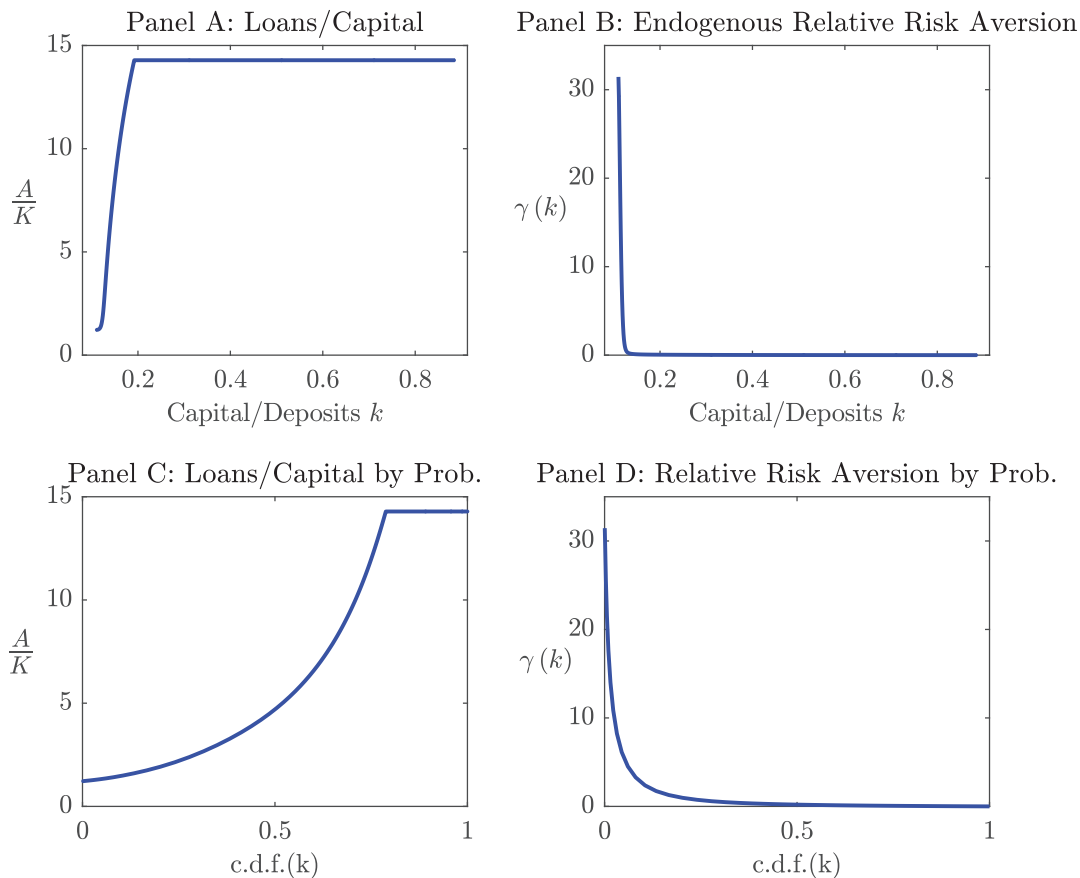Figure 2: Risk-Taking

the region right to 0.5), but in 1% of the time (the region from 0 to 0.01), the bank' relative risk aversion is beyond 17.6.

Panel A of Figure 3 plots the marginal value of deposits, i.e., $V_X(X, K) = v(k) - v'(k)k$, which we call *deposit Q*. When the bank has ample capital relative to deposits, i.e., k is large, the deposit Q is positive. According to Panel C where the deposit Q is plotted against the c.d.f. of k, the deposit Q is above 0.19 more than 97% of the time (i.e., to the right of 0.1 on the X-axis). Deposit financing is cheaper than short-term borrowing at the rate r. In Panel B of Figure 3, we compare the risk-free rate and the deposit rate. The spread reflects the bank's deposit market power (Drechsler, Savov, and Schnabl, 2017) and depositors valuing deposits for the convenience
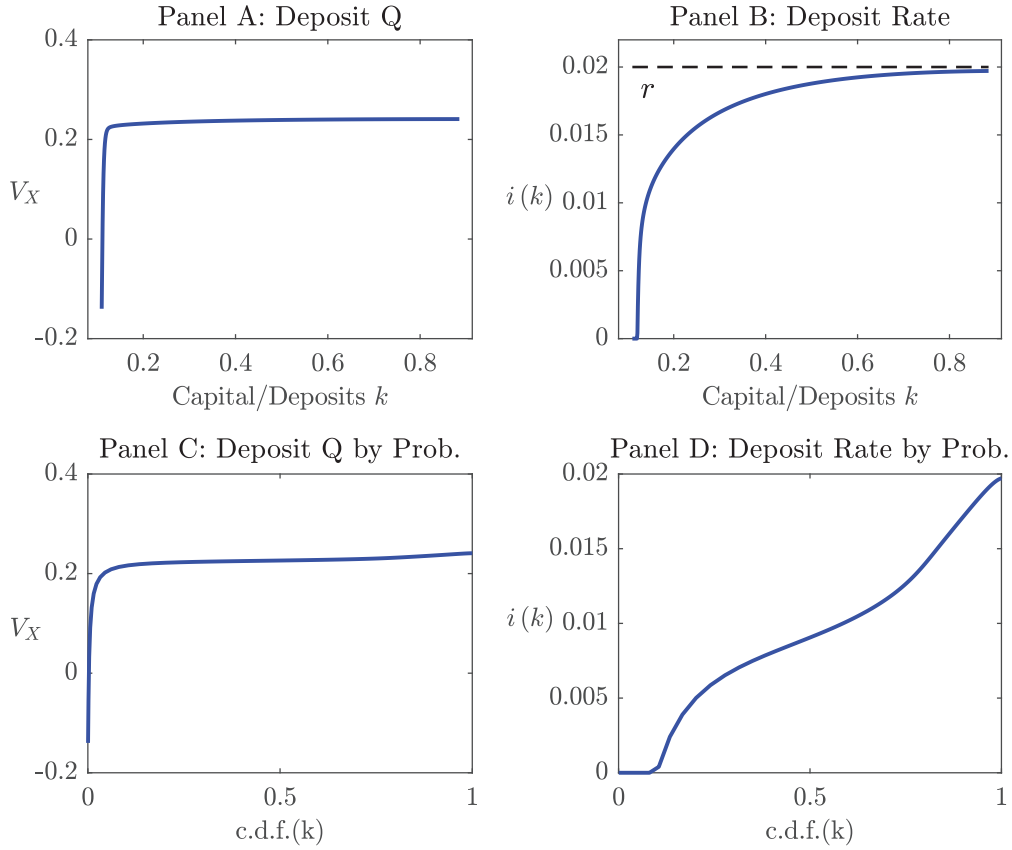
Figure 3: The Value of Deposits

of making payments. When the bank has sufficient capital to buffer risk, deposits create value by allowing the bank finance risky lending with relative cheap sources of funds. The deposit stock serves as a form of productive capital for the bank. When k is sufficiently large, the bank is willing to set a high deposit rate to attract depositors. The comovement of loan growth (Panel A of Figure 2) and deposit rate increase (Panel B of Figure 3 is consistent with the finding of Ben-David et al. (2017). As shown in Panel D of Figure 3, the bank spends relative equal amount of time across different values of deposit rate, except that at the zero lower bound, there is a point mass.

A very interesting finding is that the deposit Q can turn negative when bank capital is low relative to the deposit stock. The reason is that when k is near the equity issuance boundary,

deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage that amplifies the impact of shocks on bank capital and makes it more likely to incur costly equity issuance. The bank may want to delever, turning away deposits by lowering the deposit rate. However, as shown by Panel B of Figure 3, doing has a limit, that is the zero lower bound of deposit rate. Setting a deposit rate below zero causes depositors to withdraw deposits en masse and hoard dollar bills (which has a zero return). While we do not explicitly model the consequence of a run, the value destroyed through liquidation of loans and fire sale is likely to make the zero lower bound a binding constraint for the bank. Therefore, to reduce leverage, the best that the bank can do is to set deposit rate to zero.

Deposits are very different from short-term debts. For short-term debts, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who do not hold accounts at the bank. As long as depositors are willing to hold deposits, the bank cannot turn away the existing depositors. After hitting the zero lower bound, the bank loses control of its leverage.

Figure 4 analyzes the the bank's debt structure. Panel A plots the ratio of short-term bond to deposits, and Panel C reports the ratio over the c.d.f. of the state variable k. When capital is abundant relative to deposits, the bank raises funds from short-term debts for risky lending, while when capital is relatively scarce, the bank switches its short-term debt position, holding risk-free debts to reduce the overall riskiness of its asset portfolio. When k is small and the deposit rate reaches the zero lower bound, the bank loses control of its leverage, and therefore, has to work on the asset-side of its balance sheet, tuning down its risk exposure by holding risk-free assets, in order to reduce the likelihood of costly equity issuance. As shown in Panel B of Figure 4, once the bank has stopped borrowing short-term debts and deposits become the only type of debts, a further decline of $k = K/X$ induces a lock-step increase of leverage X/K.
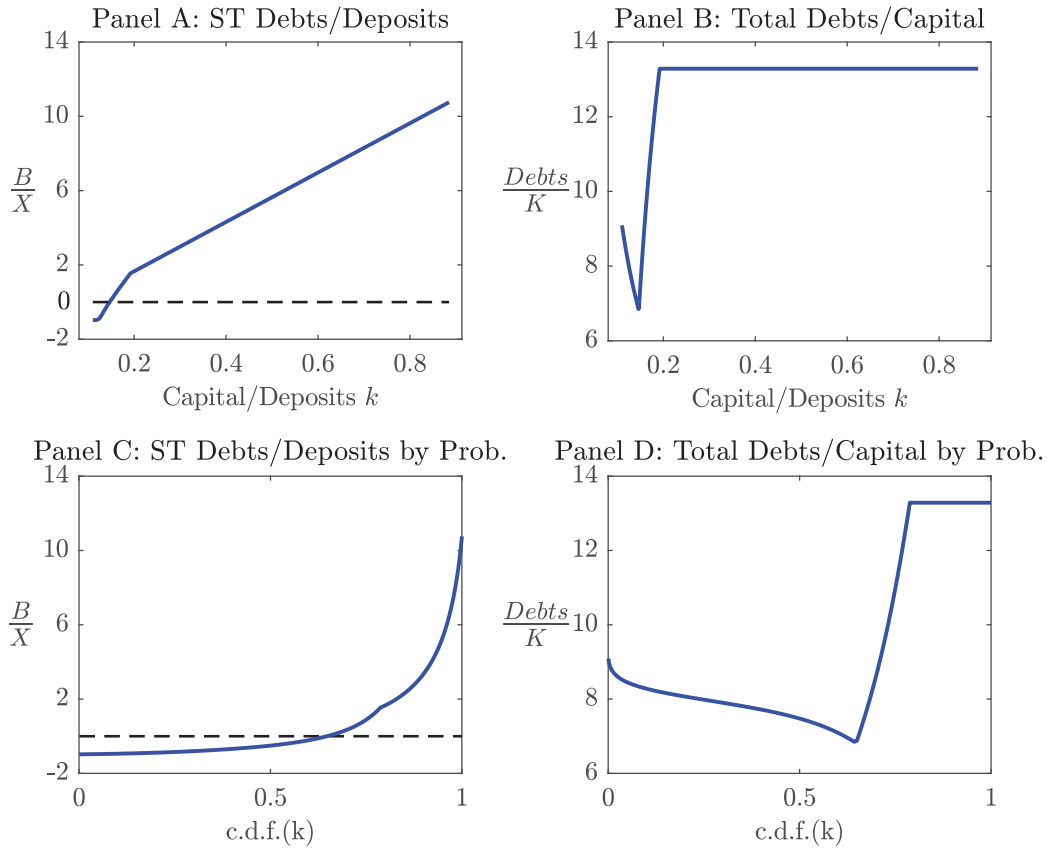
Figure 4: Debt and Leverage

6 Policy Implications

6.1 Deposit Demand Elasticity, Cost of Deposits, and Risk-Taking

There is an unsettled debate on how competition affects banks' lending on both the theoretical and empirical fronts (Keeley, 1990; Petersen and Rajan, 1995; Jayaratne and Strahan, 1996; Allen and Gale, 2004a; Boyd and De Nicoló, 2005; Bertrand, Schoar, and Thesmar, 2007; Erel, 2011; Scharfstein and Sunderam, 2016; Drechsler, Savov, and Schnabl, 2017; Liebersohn, 2017). Our model shows a particular force that suggests more competition in the deposit market results in more caution in risk-taking, especially through the delaying of payout to shareholders.
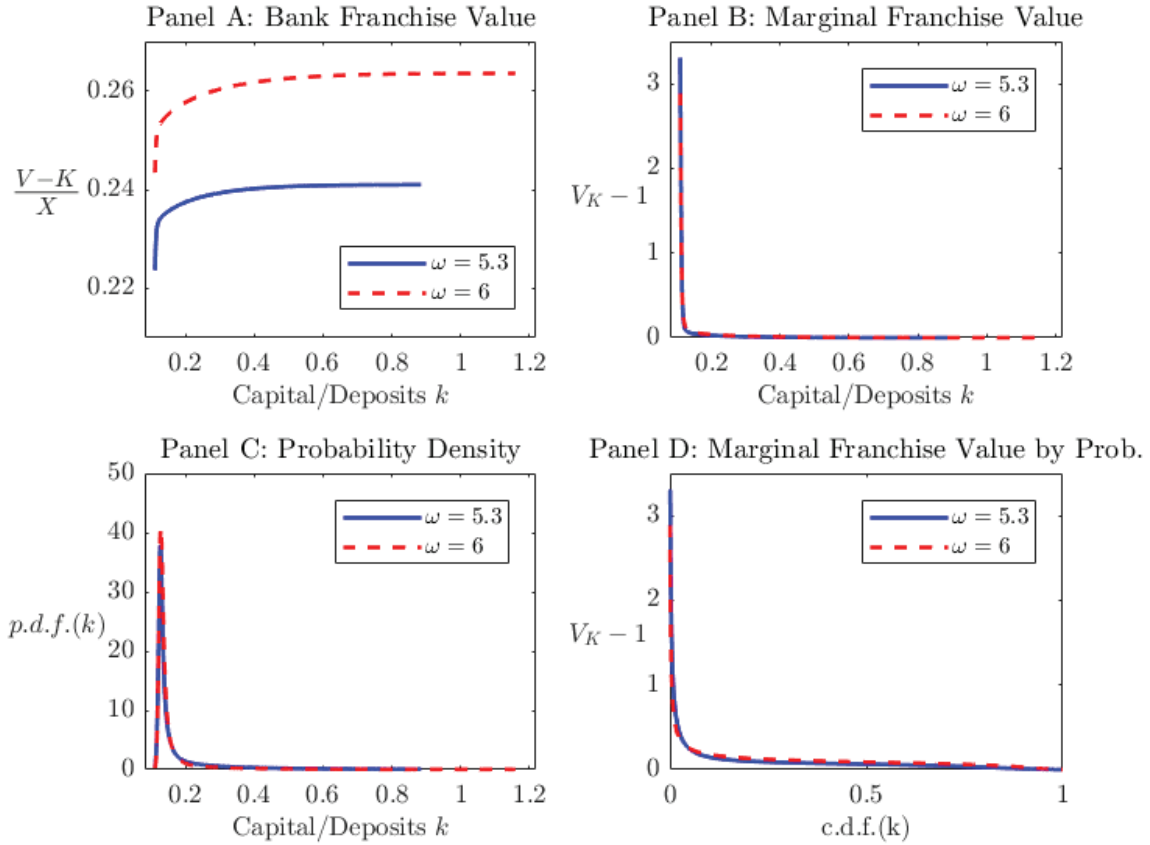
Figure 5: Deposit Demand Elasticity and the Value of Bank Capital

In our baseline solution, we set ω to 5.3, an estimate from Drechsler, Savov, and Schnabl (2017). In Figure 5, we compare the franchise value and the marginal value of bank capital of the baseline case and the solution with a higher deposit demand elasticity, $\omega = 6$. Under a more elastic demand for deposits, the bank can adjust the deposit flow more easily by setting the deposit rate. As shown in Panel A of Figure 5, a more elastic demand for deposits increases the bank's value because a prominent problem of deposit liabilities is that the maturities are largely out of the bank's control, especially so when the deposit rate has already hit the zero lower bound.

Panel B and D of Figure 5 shows that the marginal value of bank capital is quite insensitive to the change of ω. Therefore, the increase of bank value in Panel A can be largely attributed to a
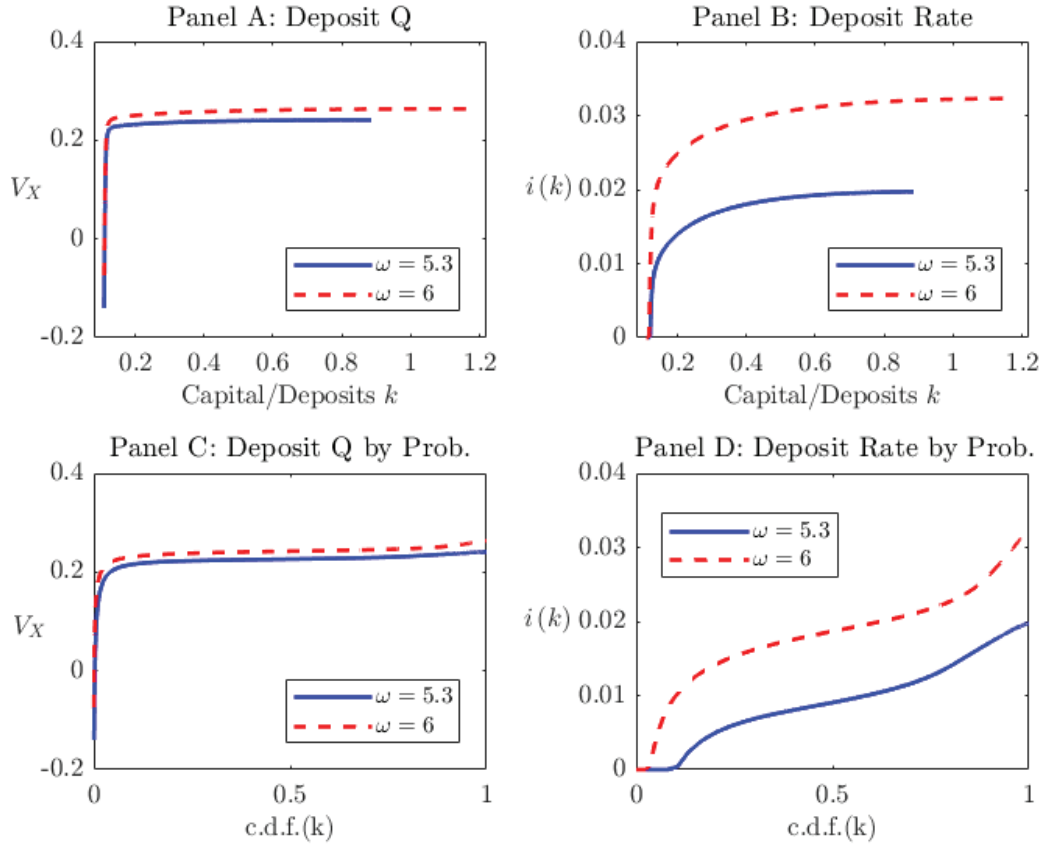
Figure 6: Deposit Demand Elasticity and the Value of Deposits

higher value of deposits as the bank's value function depends on only two state variables, capital K and deposit X. In Panel C of Figure 5, we show that the distribution of state variable k is extended to the right. Accordingly, in Panel A and B, the curves end at higher values of k.

The bank pays out dividends at a higher value of capital-to-deposit ratio when the deposit demand is more elastic. The bank is more eager to preserve capital because deposit is now more valuable. In Panel A of Figure 6, the marginal value of deposits is higher when the demand elasticity is higher and the deposit flow is more sensitive to the deposit rate set by the bank. To take advantage of the more flexible and valuable deposits, the bank preserves capital and delays payout in order to stay away from the left (low k) region where the zero lower bound on deposit

rate binds and the marginal value of deposit turns negative.

Consistent with the findings of Drechsler, Savov, and Schnabl (2017), a more elastic demand for deposits leads to higher deposit rate in Panel B and D of Figure 6 – when depositors are more price sensitive, the bank needs to set higher rates to attract depositors. This certainly implies a higher financing cost for banks, but, as shown in Panel A of Figure 6, the value of deposits actually increases because now the bank can better control the flow of deposits.

Under a higher elasticity of deposit demand, the bank is willing to pay a deposit rate that is above the risk-free rate (2% in our numeric solution). This seems puzzling because, if the bank can borrow cheaper in the short-term debt markets, why resorts to more expensive deposit financing? The bank pays higher deposit rates when it can afford to do so, i.e., when its capital is high relative to the deposit liability, so by building up a larger depositor base, the bank can enjoy a lower cost of financing when it is undercapitalized, i.e., when k is low. According Panel D of Figure 6, the deposit rate is still below the risk-free rate more than 50% of the time. Overall, the cost of deposits is higher when the demand elasticity is higher.

Figure 7 compares the bank's risk-taking behavior in the baseline case and the case with higher deposit demand elasticity. We see that in Panel A of Figure 7, given k, a higher cost of deposits does not significantly discourage the bank from lending, because this negative impact is offset by the positive impact of the bank having more adjustable deposit liabilities. Panel B of Figure 7 shows that given k, the bank's endogenous risk aversion is largely insensitive to the deposit demand elasticity. Panel C of Figure 7 shows that because the distribution of k changes, the bank engages in less risk-taking over the long run, consistent with the finding in Liebersohn (2017), though conditional on k, the bank's risk-taking is not significantly affected by the deposit demand elasticity as shown in Panel A.

One interesting finding is that due to the shift of probability mass towards higher values of k (and the bank's delay of paying dividends), the risk-taking has been shifted towards regions where banks are better capitalized (i.e., higher value of k). A higher demand elasticity is often associated with more intense competition for deposits among banks, so our result suggests that competition can stabilize banks by discouraging banks from payout to shareholders and taking less
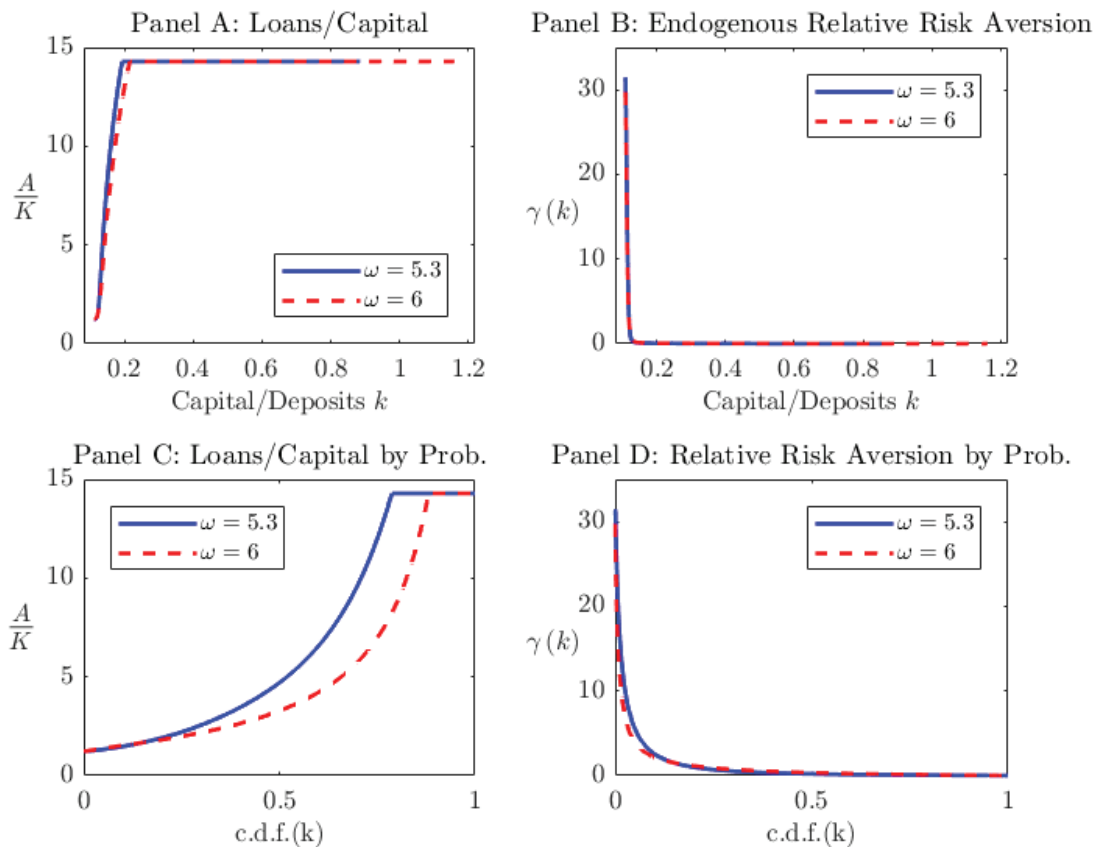
Figure 7: Deposit Demand Elasticity and Risk-Taking

risks. This result is achieved without bank default. It complements the result of Keeley (1990) that competition erodes bank charter values, which in turn caused banks to increase default risk through increases in asset risk and reductions in capital.

6.2 Outside Money for Financial Stability

Depositors hold deposits as means of payment. The payment functionality of deposits has two impacts on the bank. First, the bank has to hold reserves to buffer intra-period payment flow imbalances, which drags down the effective expected return on lending and the effective cost of maintaining deposits. Second, as shown in Panel B of Figure 3, depositors enjoy the payment

convenience, and thus, are willing accept a deposit rate that is below the prevailing interest rate r, so the bank can finance lending cheaply. Overall, deposits add value *if* the bank is well capitalized. When capital is significantly depleted due to bad shocks to the loan portfolio, deposits become burden and the marginal value of deposits turn negative as shown in Panel A of Figure 3.

A undercapitalized bank loses its control over leverage once it hits the zero lower bound on the deposit rate. The variation of deposit stock on the liability side of its balance sheet is completely at the mercy of depositors. The bank can be liberated if depositors decide to withdraw faster, which, through the lens of the model, translates into a higher rate of δ_X. When the deposit Q, $V_X(X, K)$ is negative, deposit outflow actually adds value to the bank, i.e., $V_X(X, K)\delta_X dt > 0$, by reducing the leverage and the risk of costly equity issuance.

One way to achieve faster deposit withdrawal rate is to provide depositors alternative monetary assets to hold. The government may increase its supply of short-term government bonds, which have long been recognized as money-like especially when investors hold them through the money-market mutual funds.[19] As long as government securities are not perfect substitutes of deposits in terms of the payment convenience, depositors will not withdraw en masse to pursue the positive yield on government securities but only rebalance their portfolio by reducing the weight on deposits. Therefore, an increase of supply of government securities can actually stabilize banks by reducing their leverage when banks are undercapitalized relative to their deposit stock.

This implication of our model stands in contrast with Li (2017) who shows that the money-like securities issued by the government crowd out banks' profits from issuing money-like securities, and thereby, delay the accumulation of bank capital, prolonging financial crises. Our model differs from Li (2017) in three aspects. First, we differentiate the short-term debts and deposits that feature a zero lower bound on the interest rate and stochastic maturity out of the bank's control, while Li (2017) studies banks' standard short-term debt with a money premium (or convenience yield) attached to it. Second, Li (2017) endogenizes the risk-free rate while we pursue a partial equilibrium analysis with r fixed.

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[19]The monetary service of government liabilities is an old theme (e.g., Patinkin (1965); Friedman (1969)). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jørgensen (2012), Greenwood, Hanson, and Stein (2015), Bolton and Huang (2016), and Nagel (2016).

Our result that government securities, by crowding out banks' money-like liabilities, stabilize the banks echoes Greenwood, Hanson, and Stein (2015) and Krishnamurthy and Vissing-Jørgensen (2015). However, we obtain our result in a fully dynamic setting and our result is derived from the special contractual features of deposits: (1) deposits have stochastic maturity that depends on depositors' payment activities; (2) deposit rate has a zero lower bound.

6.3   Banks' Demand of Safe Assets

Banks lose control of their leverage once they hit the deposit zero lower bound, so, in order to reduce the risk exposure of bank capital, banks rebalance their asset portfolio towards risk-free assets (as shown in Panel A of Figure 4). Such portfolio rebalancing of undercapitalized banks creates a demand for safe assets in financial crises. The government is in a unique position to supply such assets. In a general equilibrium setting, the interest rate r is endogenous, so banks' demand for safe assets is likely to push downward the interest rate, reducing the government's financing cost. The government can thus issue more debts, meeting the banks' demand, and then use the proceeds from debt issuance to stimulate the economy. During the height of the global financial crisis, the supply of U.S. Treasury bills tripled, and banks significantly increased their holdings of Treasury securities, which were then sold to the Federal Reserve (Fed) in exchange for interest-paying reserves as the Fed conducted quantitative easing by purchasing Treasury securities.

Undercapitalized banks' portfolio rebalancing towards safe assets is necessary because of the special features of deposits. Effectively, banks are performing a liquidity transformation for the non-financial sector. They hold risk-free assets that do not directly serve as means of payment, while have on the liability side of their balance sheets the deposits that the non-financial sector use to settle transactions. The unique position of banks in the payment system is the root of banks' demand for safe assets in crises. As discussed in the previous subsection, if depositors have close substitutes to hold in stead of deposits as means of payment, banks can off-load deposit liabilities, and thereby, having a weaker need for safe assets. In fact, the modern reforms of payment system facilitate such transition. Instead of having banks holding safe assets and issuing deposits, money market funds allow the non-financial sector to hold safe assets directly and then use money-market

fund shares as close substitutes of deposits to make payments. A more recent development is central bank digital currency.

6.4 Monetary Policy Transmission and Interbank Markets

In this paper, we model a single bank's decision problem, but the bank's choices have implications on how things play out in a more general equilibrium setting. So far, we have discussed the market of safe and money-like securities. Next, we discuss our model's implications on the transmission of monetary policies and the interbank credit market.

A direct implication on monetary policy is that when the central bank increases the interest on reserves, ι, banks hold more reserves and the reduced the settlement costs associated with loan creation leads to more lending. As shown in (36), bank lending increases in ι. Changing ι may have other effects through its impact on the interbank credit market (Bigio and Sannikov, 2019).

As shown in Panel A of Figure 4, the bank switches from short-term borrowing to short-term lending when its capital is too low relative to deposit liabilities. This suggests that as long as bank are creditworthy, the interbank market is a counter-balancing force against the collapse of lending in crisis. Given that undercapitalized banks are eager to lend on a risk-free basis, banks can easily borrow in the interbank market to cover intraday (or intra-period) payment flow imbalance, effectively facing lower costs of payment settlement, which in turn stimulates lending, as shown in (40). Therefore, endogenizing the interbank market can lead to a potential mechanism that sustains lending when banks are undercapitalized. The increased demand for safe assets translates into abundant interbank credit, which stimulates lending by reducing payment settlement costs. Note that this mechanism is active *only if* banks are creditworthy and interbank lending does not involve exposure to counterparty default risks. In times of financial stress, the impact of government guarantee can be amplified by this channel, because by taking the bank default risk off the table, government guarantee activates the positive effect of interbank market on bank lending.

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A   Solving the Model without Equity Issuance Costs

By inspecting the HJB equation (16), we know that when the value function is linear in K, the bank maximizes π^A so the capital requirement binds, i.e., $A/K = 1/\xi_K$, as long as $S_A(R, X, A) < \alpha_A$, which we assume to hold in Section 4. The optimal deposit rate is given by (23). The optimal reserve holding is given by (26), which does not depend on the value function.

Next we solve Q given the functional forms of payment settlement costs and deposit maintenance costs in Section 4. To determine the constant Q, we substitute the value function, its derivatives, and the optimal control variables into the HJB equation (16):

$$\frac{\omega}{\theta_1}\left(1 - \frac{\omega}{2}\right)Q^2 - \left(\frac{2}{\theta_1} + \delta_X + \rho\right)Q + r - \chi_1\sqrt{2(r - \iota)} - \theta_0 + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta_1}$$
$$+ \left[r + \frac{1}{\xi_K}\left(\alpha_A - \chi_2\sqrt{2(r - \iota)}\right) - \rho\right]k = 0.$$

For this equation to hold, we need

$$\frac{\omega}{\theta_1}\left(1 - \frac{\omega}{2}\right)Q^2 - \left(\frac{2}{\theta_1} + \delta_X + \rho\right)Q + r - \chi_1\sqrt{2(r - \iota)} - \theta_0 + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta_1} = 0, \quad (48)$$

which solves Q, and the coefficient on k is equal to zero, i.e.,

$$\rho = r + \frac{1}{\xi_K}\left(\alpha_A - \chi_2\sqrt{2(r - \iota)}\right). \quad (49)$$

Equation (49) requires that the bank is indifferent between paying out dividend and retaining equity. If $\rho > r + \frac{1}{\xi_K}\left(\alpha_A - \chi_2\sqrt{2(r - \iota)}\right)$, the bank prefers paying out dividends because the expected return on equity capital is below the shareholders' required rate of return. If $\rho < r + \frac{1}{\xi_K}\left(\alpha_A - \chi_2\sqrt{2(r - \iota)}\right)$, the bank never pays dividend and prefers to raise an infinite amount of equity because the expected return on equity is greater than the shareholders' required return.

Under the condition,

$$\left(\frac{2}{\theta_1} + \delta_X + \rho\right)^2 \geq \frac{4\omega}{\theta_1}\left(1 - \frac{\omega}{2}\right)\left[r - \chi_1\sqrt{2(r - \iota)} - \theta_0 + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta_1}\right], \quad (50)$$

the roots of Equation (48) exist and are given by

$$Q = \frac{\left(\frac{2}{\theta_1} + \delta_X + \rho\right) \pm \sqrt{\left(\frac{2}{\theta_1} + \delta_X + \rho\right)^2 - \frac{4\omega}{\theta_1}\left(1 - \frac{\omega}{2}\right)\left[r - \chi_1\sqrt{2(r - \iota)} - \theta_0 + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta_1}\right]}}{\frac{2\omega}{\theta_1}\left(1 - \frac{\omega}{2}\right)}.$$
$$(51)$$