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Simon Martin, W. Benedikt Schmal

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Simon Martin † W. Benedikt Schmal ‡

Abstract:

Sophisticated collusive compensation schemes such as assigning future market shares or direct transfers are frequently observed in detected cartels. We show formally why these schemes are useful for dampening deviation incentives when colluding firms are temporary asymmetric. The relative attractiveness of each of these schemes is shaped by firms' ability to predict future market conditions, possibly aided by algorithms. Prices and profits are inverse u-shaped in prediction ability. Assigning future market shares is optimal when prediction ability is intermediate, and otherwise direct transfers are optimal. Competition authority's limited resources should be utilized to respond to these changing market conditions.

JEL Classification: D21, L41, L51

Keywords: algorithmic collusion, market forecasting, prediction ability, firm asymmetry, compensation schemes

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†Düsseldorf Institute for Competition Economics (DICE) and CESifo. Universitätsstraße 1, 40225 Düsseldorf, Germany. Contact: simon.martin@dice.hhu.de

‡Düsseldorf Institute for Competition Economics (DICE). Universitätsstraße 1, 40225 Düsseldorf, Germany. Contact: schmal@dice.hhu.de

1 Introduction

When investigating leaked-out cartels, one consistently finds sophisticated mechanisms to organize the cartel (Hyytinen et al., 2019 Levenstein and Suslow, 2006, Levenstein and Suslow, 2011, Harrington, 2006). Involved firms do not only coordinate offers, quantities or prices. They also use payment schemes to compensate cartel members that came off badly in an auction or time period. For instance, in the vitamin cartel firms with an output above the collusive quotas directly bought vitamins from the other cartel members to compensate them. The Austrian construction cartel used a particular fund for compensation payments (Oberster Gerichtshof der Republik Österreich, 2017). This construction cartel is a very recent example for complicated cartel structures that arranged complex market sharing and compensation mechanisms. Despite its widespread usage, there is little theoretical literature on the exact motives for employing certain collusive compensation schemes.

Additionally, competition authorities worldwide are concerned about the increasing usage of algorithms and machine learning tools to support firms' pricing (OECD, 2017). Algorithms might 'learn' to behave cooperatively (Calvano et al., 2020a, 2020b, 2021, Assad et al., 2020, 2021, Brown and MacKay, 2020, Johnson et al., 2020, Harrington, 2021, Normann and Sternberg, 2021, Klein, 2019). Moreover, better algorithms fed by ever increasing amounts of collected data may enhance firms' ability for more tailored pricing to certain users (Peiseler et al., 2018) or entire demand environments (Miklós-Thal & Tucker, 2019). This last aspect seems to be particularly relevant. A prominent example for this is 'Amazon forecast', an algorithm based demand prediction software which is not only used by Amazon shopping, but also offered globally to third party firms. The service applies machine learning techniques to time series data from the past and "cause an improvement in forecast quality by up to 50%".[1] The software has prominent users: Foxconn, a contract manufacturer for, e.g., Apple, Microsoft, and Nintendo; Clearly, a large online eye wear retailer, or Axiom Telecom, the largest

[1] https://aws.amazon.com/forecast/?nc1=h_ls

telecommunications operator in the Near East.[2]

On top of the aforementioned reasons, the question of collusive compensation is also likely to become more relevant once we take into account the social structures in which economic behaviour is embedded (Granovetter, 1985). Trust is a crucial ingredient into cooperation of any kind, which may be established fairly easily when decision makers are relatively homogeneous. Since managerial boards become increasingly diverse, additional necessity to sustain collusive agreements through sophisticated schemes might arise.

What are the incentives for colluding parties to use certain collusion compensation schemes? How do these incentives depend on better prediction ability? Which of these collusive schemes should we expect to increasingly emerge in the future? Although these are very pressing issues both for theorists and competition authorities, the existing literature does not yet offer any answers. This is the first paper to explicitly address these questions.

We consider an infinitely repeated game between two firms. Each period consists of two stages. In the first stage, firms are temporarily asymmetric. Consumers have a strict preference for one firm, but the identity of the preferred firm is stochastic and varies over time. Firms compete in prices and receive a noisy signal about the second stage, in which valuations are symmetric and either high or low for both firms. Firms have to pay an entry cost in order to offer their products in the second stage. Joint profits are highest when only the preferred firm sells at monopoly prices in the first stage and only one firms enters the second stage if it can at least cover its entry cost in expectation. Collusion can be sustained if neither firm has an incentive to deviate. The temporary asymmetry in stage one creates very high incentives for the currently disadvantaged firm to deviate, in particular when it received a signal that also the second stage will be disappointing.

We identify two collusive compensation schemes that allow to dampen the incentives

[2] `https://aws.amazon.com/forecast/customers/`

to deviate for the disadvantaged firm. First, the disadvantaged firm may be assigned 'future market shares', i.e. the exclusive right to enter into the second stage. Note that this can still be achieved with an implicit understanding of all parties involved, i.e., it could still be viewed as a form of 'tacit collusion', which is not forbidden in most jurisdictions. Second, the disadvantaged firm might receive a 'bribe' in the form of a direct transfer from the other firm. This is more effective since the optimal transfer can be chosen such that there is no slack left in the incentive compatibility constraint. However, it entails the disadvantage that this is clearly illegal behaviour which will be fined in case of detection.

We analyze the most profitable collusive equilibrium for different compensation schemes, a given discount factor δ and the signal precision ρ, i.e., the firms' ability to accurately predict the state in stage two already upon receiving a signal in stage one. Intuitively, advanced collusive scheme that dampen incentives to deviate enable higher collusive prices. Only one firm enters the stage-two market in any collusive agreement, unless a sufficiently precise signal ($\rho > \bar{\rho}$) indicates that expected profits are so low that not even the entry costs can be recovered.

We find that all three compensation schemes are harder to sustain when ρ increases as long as $\rho < \bar{\rho}$, but easier to sustain when ρ increases when $\rho > \bar{\rho}$. Collusive schemes that are easier to sustain enable higher collusive prices. In the first case, the currently disadvantaged firm is increasingly convinced that the future will not be great. Hence, cheating on the competitor and reaping high profits right away becomes more appealing. Once $\rho > \bar{\rho}$, firms understand that after receiving a bad signal, entry is not worthwhile, but conditional on entry after a good signal, the payoff will be substantial, and more so the more precise is the signal. Thus, the disadvantaged firm has lower incentives to deviate. This implies that the relationship between prices and prediction ability is inverse u-shaped.

Finally, we consider the relative attractiveness of each of these compensation schemes depending on the signal precision ρ and the fines imposed upon conviction in case of collusive compensation through direct transfers. The extent to which firms can benefit from better prediction ability depends on the collusive compensation scheme. We find that assigning future market shares is optimal when prediction ability is intermediate, and otherwise direct transfers are optimal. Since competition authorities need to utilize limited resources efficiently, our analysis sheds light onto which schemes probably demand closest attention in the future.

This paper is structured as follows. We relate to existing literature in Section 1.1 and describe our model in Section 2. We provide an analysis of collusive compensation schemes in Section 3, we relate to welfare aspects in Section 4 and we characterize optimal collusive compensation in Section 5. We present a numerical example in Section 6. In Section 7, we discuss policy implications and conclude.

1.1 Related Literature

Our paper relates to several strands of the literature. Closest to our paper is Miklós-Thal and Tucker (2019), whose basic setup is similar to ours. Symmetric firms receive a noisy signal about consumer valuations and might adjust their prices accordingly. Higher prediction accuracy may lower profits and increase consumer surplus, casting doubt on aforementioned concerns about enhanced ability to collude through algorithms. In contrast to their paper, our firms are temporary asymmetric.[3] The signal is not about consumer valuations, but about market size in the next sub-period. This channel gives rise to interesting non-monotonic affects. Moreover, we consider additional collusive compensation schemes and relate their relative attractiveness to prediction accuracy.

Collusion under temporary asymmetry appears in the literature on bidding rings (McAfee and McMillan, 1992, Pesendorfer, 2000, Skrzypacz and Hopenhayn, 2004, Blume and Heidhues, 2008). Indeed, asymmetry is an inherent ingredient in all auction

[3] See, e.g., Miklós-Thal (2011) and Athey and Bagwell (2008) for a setting with permanent cost asymmetries, and Häckner (1994), Bos and Marini (2019), and Bos et al. (2020) for settings with vertically differentiated products..

models, in which the degree of asymmetry is private information. Thus, colluding parties first of all need to ensure efficient revelation of the private information. In our set-up, there is no private information, and we focus on the interactive effect between collusive compensation and prediction accuracy instead. Our 'baseline tacit collusion' and 'assignment of future market shares' can be thought of as *weak* cartels in the terminology of McAfee and McMillan, 1992, whereas 'direct transfers' are akin to a *strong* cartel (Bos & Pot, 2012).

Regarding empirical research, Levenstein and Suslow (2006, 2011, 2012), Harrington (2006) and Hyytinen et al., 2019 summarize detected cartels and illustrate some of the compensation schemes formally considered in this paper, e.g., the industrial tubes cartel and tacit collusion over market entries in the French telecommunication sector. In the former, tube manufacturers coordinated market shares and regions. When a customer requested an offer from the 'wrong' firm, it set an unrealistically high price to avoid winning the contract, i.e. it basically left the market.[4] In the French telecommunication sector, Bourreau et al. (2021) provide evidence that the incumbent operators to have been tacitly colluding over market entry to avoid cannibalization and to ensure high mark-ups. Our framework addresses these issues and shows how algorithmic collusion might enable more stable collusion in the future.

Concerns similar to the ones described in this paper arise in the literature on algorithmic pricing (Calvano et al., 2020a, 2020b, 2021, Assad et al., 2020, 2021, Brown and MacKay, 2020, Johnson et al., 2020, Harrington, 2021) and experiments thereof (Normann and Sternberg, 2021, Klein, 2019). We add to this literature by providing a theoretical foundation for the incentives and mechanics of different collusive compensation schemes, and how these incentives depend on prediction accuracy. Existing research mainly focuses on the usage of algorithms for price setting, while we analyze the use of algorithms to forecast markets. In that sense, it corresponds to the seminal

[4]See "Commission Decision of 16 December 2003 relating to a proceeding pursuant to Article 81 of the EC Treaty and Article 53 of the EEA Agreement – Case COMP/E-1/38.240 - Industrial tubes: `https://ec.europa.eu/competition/antitrust/cases/dec_docs/38240/38240_29_1.pdf`

early work of Green and Porter (1984) as well as Rotemberg and Saloner (1986) on collusion with varying demand. Recent literature in the field of demand uncertainty comes from O'Connor and Wilson (2021) and Bajari et al. (2019). The application of artificial intelligence to decision making in general has been examined by, e.g., Agrawal et al. (2019b, 2019c).

The usage of algorithms and, more broadly, artificial intelligence in industrial contexts raises many policy issues. Among others, this is examined by Assad et al. (2021) and Agrawal et al. (2019a).

2 Model

We consider an infinitely repeated interaction between two firms with a common discount factor δ. Each period consists of two stages. In each stage, there is a mass 1 of homogeneous consumers with unit demand. In the first stage, consumer valuations are given by (v_H, v_L) for firm 1 and firm 2, respectively, or by (v_L, v_H), with $v_H > v_L > 0$. Ex-ante, each of these two states is equally likely, and the states are $i.i.d.$ over time. In that sense, firms are temporary asymmetric. We refer to the firm with v_H as *preferred firm* and to the firm with v_L as *disadvantaged firm*.

In the second stage, firms are symmetric, but again, there are two states which are ex-ante equally likely and $i.i.d.$ over time, denoted by H and L. In state H, consumers' valuation for both firms is $v > 0$, and in state L, consumers' valuation for both firms is zero. Firms have to pay a fixed entry cost F in order to offer their product in the second stage. This payment is publicly observable. Throughout, marginal costs of production are normalized to zero.

The timing of the stage game is as follows. Firms observe the realization of the state in stage 1 and receive a common signal $s \in \{h, l\}$ about the second stage.[5] The

[5]The model could also be adjusted to allow for an additional signal concerning the first stage. However, in our context such a signal would have little relevance since we are primarily interested in the interactive effect of the ability to predict on different collusive compensation schemes. This only matters once temporary asymmetries have already materialized.

signal has precision ρ, i.e., $Pr(h|H) = Pr(l|L) = \rho \in \left[\frac{1}{2}, 1\right]$. Since ex-ante both states are equally likely, also the posterior $Pr(H|h) = Pr(L|l) = \rho$, so the probability that the predicted state of the world realizes is also given by ρ. For $\rho = 1/2$, the algorithm has no predictive power, i.e. the posterior after receiving any signal equals the prior: $Pr(H|h) = Pr(H|l) = Pr(H)$. For $\rho = 1$, the algorithm has perfect prediction ability. Upon receiving signals, firms simultaneously set period-1 prices and make their entry decision for the second stage, both of which are publicly observable. In the second stage, those firms that entered the market observe the state realization and set a price for the second stage. For the infinitely repeated game, we are interested in the most profitable subgame-perfect equilibrium supported by grim trigger strategies.

We make the following additional assumptions on the parameters of the model.

Assumption 1. *(i)* $\frac{v}{2} - F > 0$*, (ii)* $v_L > v - F$ *and (iii)* $2(v - 2F) > v_L$*.*

The rationale behind these assumptions is as follows. Part (i), $\frac{v}{2} - F > 0$, ensures that, absent any signal, a monopolist would find it profitable to enter in the second stage. In that sense, our model can be interpreted as capturing *low-entry-cost industries*. In *high-entry-cost industries*, this inequality would be reversed and a monopolist would ex-ante not enter.

Part (ii), $v_L > v - F$, originates from the collusive scheme with future market shares. It contrasts the short-term gain from deviation in a collusive agreement with monopoly prices, given by v_L, with the profit the firm obtains if the signal is perfectly informative ($\rho = 1$). This condition ensures that there is an incentive to deviate from a collusive agreement with monopoly prices. If this condition fails, than there is only very little at stake for the disadvantaged firm in the first stage to start with, rendering all considerations on collusion irrelevant. This assumption could easily be relaxed and is made for easy of exposition only.

Part(iii), $2(v-2F) > v_L$, establishes an upper bound on v_L, which is relevant in the first stage, relative to $v/2 - F$, which are relevant in the second stage. This condition ensures that the second stage is sufficiently important, relative to the first stage. If this condition fails, then potential profits accrued in the second stage are irrelevant, and hence there is little scope for the collusive compensation schemes we contemplate. This assumption is made for clarity only and could easily be dispensed with.

Taken together, our assumptions can be written as:

$$2(v - 2F) > v_L > v - F > 0 \tag{1}$$

where (i) $\frac{v}{2} - F > 0$ is implied by $v - F > 0$.

Example: We consider the outlined setting as fairly general, but it applies in particular to creative industries such as film or video game production, as popularity is always somewhat random there.[6] Consider gaming enterprises that have developed a new action game. They could either compete or jointly evaluate which game has the higher valuation in the market and agree to sell this only (at price v_H in our stage 1). Both firms do not know next year's demand for action games but have to make costly investments to develop a new game (entry costs F in our model). To add zest to a withdrawal, the preferred studio in stage 1 could take measures such as bribes or dedicate the 'action game market' in stage 2 to the disadvantaged firm.

2.1 Analysis - Stage game

In order to illustrate the mechanics of the model, we first analyze the stage game in case firms compete with each other. As usual, we proceed with backwards induction. Since we are eventually interested in the most severe punishment through grim trigger strategies, we focus on the least profitable subgame-perfect equilibrium.

In the second stage, firms are aware of the entry decision of the other firm and the state realization. Since this is a standard Bertrand game, profits for both firms are zero in case both entered, whereas in case of a single entrant this firm acts as a monopolist and makes profit v when the state is H and zero otherwise.

[6] According to Statista research, the video game market in the US alone has a size of 65.5 Billion USD: `https://www.statista.com/topics/868/video-games/#dossierSummary`

Taking the signal $s \in \{h, l\}$ and expected second stage profits as given, firms simultaneously set first-stage prices and decide on entry for stage 2. If expected profits given the signal make entry for a single firm profitable (i.e., it holds that $Pr(H|s)v - F > 0$), then the market entry game has a hawk-and-dove game flavor as shown in Figure 1. There are two asymmetric Nash equilibria in pure strategies and one in mixed strategies, which results in zero expected profits for both firms. Thus, irrespective of the signal s, expected profits for stage 2 are zero in the least profitable subgame-perfect equilibrium.

		firm 2 Enter	*firm 2* Do not enter
firm 1	Enter	$-F, -F$	$Pr(H\|s)v - F, 0$
firm 1	Do not enter	$0, Pr(H\|s)v - F$	$0, 0$

Figure 1: Stage 2 market entry game conditional on signal s

In stage 1, valuations are v_H and v_L, respectively, so in equilibrium the preferred firm obtains a profit of $v_H - v_L$. Since ex-ante each firm is equally likely to be the preferred one, expected profits per period are $\frac{v_H - v_L}{2} + 0$. Hence, the discounted expected profit for each firm in the least profitable Nash equilibrium is given by:

$$E(\pi)_N = \left(\frac{v_H - v_L}{2}\right)\frac{1}{1-\delta}. \tag{2}$$

3 Collusive Compensation Schemes

Instead of competing with each other, both firms could collude. We are interested in the most profitable outcome that can be supported in a subgame-perfect equilibrium for a given collusive compensation scheme and the parameters of the model. In contrast to the literature on endogenous cartel formation (Bos & Harrington, 2010; Bloch, 2018; Belleflamme & Bloch, 2004; Harrington & Chang, 2009), we take the potential collusive structure as given and investigate the most collusive outcomes given this structure.

A standard efficiency argument dictates that in the most profitable collusive equilibrium, only the preferred firm is active in the first stage and charges a signal-dependent price p_s, $s \in \{h, l\}$, and the disadvantaged is not active (or alternatively, charges any prohibitively high price). Thus, we are interested in the pair price (p_l, p_h) that maximizes joint expected profits, subject to state- and signal-dependent incentive compatibility constraints.

The most efficient collusion is joint-profit maximization, i.e. setting monopoly prices after both signals and making a monopoly-like entry decision. A monopolist would optimally sell only the high valued good at price $p_1 = v_H$ in stage 1. If monopoly entry into the second stage is profitable in expectation, then it is accompanied by a stage-two monopoly price $p_2 = v$. This expectation crucially depends on the signal precision, so using a predictive algorithm can change the decision making compared to the uninformed case. In low-cost industries, monopoly entry is always profitable after signal h since the expected revenue based on the prior exceeds the costs ($\frac{v}{2} \geq F$), and it is also profitable after signal l as long as $\rho \leq \bar{\rho}$, where $\bar{\rho}$ is given by:

$$\begin{aligned} (1-\rho)v &\geq F \\ \rho \leq \bar{\rho} &= \frac{v-F}{v}. \end{aligned} \tag{3}$$

In the following, we examine the interaction of prediction accuracy with several cartel compensation mechanisms. Under this term we subsume all actions taken within a cartel. While, for example, price or quantity fixing is an external action as it impacts the market, internal actions are those that incentivize or compensate firms of a cartel for some behaviour. These compensation schemes are useful as we have by construction asymmetries that lead to differences in the incentive constraints. The first scheme is (baseline) tacit collusion, i.e. we assume behaviour of the two firms that is somehow coordinated to optimize joint profits but do not require direct communication (Harrington & Skrzypacz, 2011). The second scheme is the assignment of the market in stage 2 to the firm facing the low valuation in stage 1, i.e. it is guaranteed that the low state firm gets the stage 2 market as long as the posterior-weighted expected profit is positive. The last scheme we examine uses direct transfers between the two firms. The high state firm in stage 1 pays the low state firm not to deviate, which possibly enables higher collusive prices. This comes at the prize of a higher detection likelihood and possible fines as direct payments leave marks.

For all compensation schemes, collusion at monopoly prices, i.e., $p_h = p_l = v_H$, is only possible when firms are sufficiently patient. Otherwise, a lower price $p_s^*(\rho)$, $s \in \{h, l\}$, needs to be chosen such that currently disadvantaged firm has no incentive to deviate.

When the signal h was received, continuation values are higher and hence punishment after a deviation is higher. Therefore even moderate prices decreases suffice to deter deviations. Thus, for each collusive compensation scheme and given δ and ρ, either (i) $p_1 = v_H$ is sustainable after both signals, (ii) v_H remains sustainable after the h signal but the prices needs to be distorted downwards after the l signal or (iii) after both signals, prices need to be distorted downwards.

We introduce the following notation for describing the critical thresholds on δ. First, we differentiate between the two intervals of ρ. For $\rho \leq \bar{\rho}$, we use $\widehat{CDF}$, and for $\rho > \bar{\rho}$, we use $\widetilde{CDF}$. To separate the CDF by compensation schemes, we use the subscripts $\{base, f.m.s., d.t.\}$ for baseline tacit collusion, assigning future market shares, and direct transfers, respectively.

3.1 Baseline tacit collusion

We first describe baseline tacit collusion. In any efficient collusive scheme, the disadvantaged firm in stage 1 has an incentive to deviate since it does not make any profits in stage 1 otherwise. Under baseline tacit collusion, the disadvantaged firm is only kept in check by the threat of reversion to Nash equilibrium play forever in case of a deviation. In case the low valuation firm obeys to the collusive agreement, the firms randomize the decision who gets the whole market in the following second stage.

Conditional on period one valuations $v_1 \in \{H, L\}$ and signal $s \in \{h, l\}$, we denote the respective continuation values by $V_{v_1,s}$. Since all of these are ex-ante equally likely, the ex-ante expected discounted profits under collusion are:

$$V_{base}(\rho) = \frac{V_{Hh}(\rho) + V_{Hl}(\rho) + V_{Lh}(\rho) + V_{Ll}(\rho)}{4}. \tag{4}$$

Given the signal structure, entry in stage 2 is always profitable after signal h, but only when $\rho \leq \bar{\rho}$ after signal l. So the total expected profit from stage 2 is

$$\pi_{2,h} = \frac{\rho v - F}{2}$$

$$\pi_{2,l} = \begin{cases} \frac{(1-\rho)v - F}{2} & \text{if } \rho \leq \bar{\rho} \\ 0 & \text{if } \rho > \bar{\rho} \end{cases}$$

Under baseline collusion, the identity of the potential entrant in stage 2 is determined randomly. Thus, the respective continuation values are given by

$$\begin{aligned} V_{Hh} &= p_h + \frac{\pi_{2,h}}{2} + \delta V \\ V_{Hl} &= p_l + \frac{\pi_{2,l}}{2} + \delta V \\ V_{Lh} &= 0 + \frac{\pi_{2,h}}{2} + \delta V \\ V_{Ll} &= 0 + \frac{\pi_{2,l}}{2} + \delta V \end{aligned} \tag{5}$$

Collusion at monopoly prices is sustainable as long as neither firm has an incentive to deviate for all possible signals. Clearly, the currently preferred firm never has an incentive to deviate, so we only need to consider deviation incentives for the currently disadvantaged firm. The most profitable deviation, given that the other firm is charging a price p_s, is charging a price p' sufficiently low such that the entire demand is attracted, which is given by $p' = v_L - v_H + p_s$. This will be met immediately with the most severe punishment in stage 2, resulting in 0 profits, and Nash equilibrium punishment in all future periods. Thus, upon receiving signal h, the incentive constraint (IC) is given by

$$(IC_h):\ V_{Lh} \geq v_L - v_H + p_h + \delta E(\pi)_N \tag{6}$$

and upon signal l, the constraint is given by

$$(IC_l):\ V_{Ll} \geq v_L - v_H + p_l + \delta E(\pi)_N. \tag{7}$$

For collusion at monopoly prices, $p_h = p_l = v_H$, so $V_{Lh} > V_{Ll}$, so IC_l is binding. When δ is sufficiently high, monopoly prices are sustainable. Otherwise, first p_l needs to be adjusted, which relaxes the IC_l but tightens IC_h. When δ is very small, both p_h and p_l need to be adjusted. The following proposition shows how these adjustments are made optimally.

It will be convenient to define several thresholds of the critical discount factor (CDF) and interior price functions. For ease of reference, these are collected in Appendix A, along with several useful properties collected in Lemma 3, which will be repeatedly used in the following.

We next characterize the highest sustainable sets of prices under baseline tacit collusion.

Proposition 1. *For baseline tacit collusion, the highest sustainable prices p_l^* and p_h^* are as follows. If $\rho \leq \bar{\rho}$, then $(p_l^*, p_h^*) =$*

$$\begin{cases} (v_H, v_H) & \delta \geq \widehat{CDF}_{base,1} \\ \left(\hat{p}^{l,1}_{base}(\rho), v_H\right) & \widehat{CDF}_{base,1} > \delta \geq \widehat{CDF}_{base,2} \\ \left(\hat{p}^{l,2}_{base}(\rho), \hat{p}^{h}_{base}(\rho)\right) & otherwise \end{cases}$$

If $\rho > \bar{\rho}$*, then* $(p_l^*, p_h^*) =$

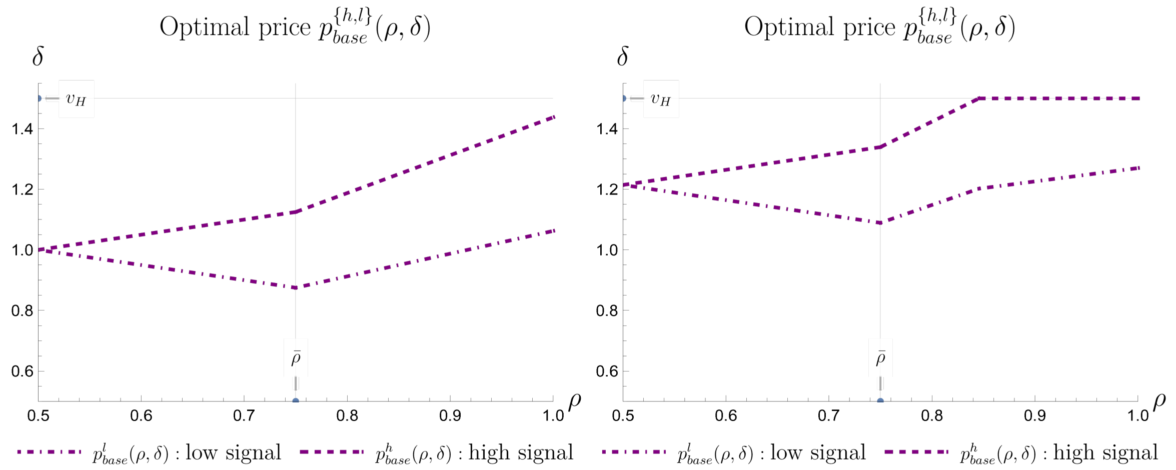
$$\begin{cases} (v_H, v_H) & \delta \geq \widetilde{CDF}_{base,1} \\ \left(\tilde{p}_{base}^{l,1}(\rho), v_H\right) & \widetilde{CDF}_{base,1} > \delta \geq \widetilde{CDF}_{base,2} \\ \left(\tilde{p}_{base}^{l,2}(\rho), \tilde{p}_{base}^{h}(\rho)\right) & otherwise \end{cases}$$

Prices and critical discount factors can be found in Appendix A.

Proof. See appendix. □

Proposition 1 shows the highest sustainable prices under baseline collusion. Depending on the prediction ability ρ and the discount factor δ, different prices are sustainable. By lowering the price the currently preferred firm offers, also undercutting becomes less attractive since less is at stake to start with, which lowers the incentive to deviate and, by that, the CDF necessary for stable collusion. We need to take into account that for $\rho < \bar{\rho}$, entry into the second stage if profitable after both signals, but for $\rho > \bar{\rho}$ it is no longer profitable after a bad signal. This affects the disadvantaged firm's incentive to deviate and hence modulates the strength of the pricing distortion that successfully deters such a deviation. The incentive to deviate is always stronger after a bad signal, because in that case the second-stage punishment is relatively less damaging since also equilibrium payoffs are relatively low. Hence, prices p_l need to be distorted more frequently and more severely than prices p_h. Both when $\rho \leq \bar{\rho}$ and when $\rho > \bar{\rho}$, there are three possible cases, depending on the discount factor δ. When δ is sufficiently high, then monopoly prices v_H are sustainable after both signals. When δ is intermediate, then prices p_l after the bad signal need to be distorted downwards and prices p_h after the good signal are still sustainable. When δ is very low, then both prices need to be distorted downward relative to monopoly levels, but more so after a bad signal.

As an illustration of optimal prices in baseline tacit collusion, consider Figure 2 (see Section 6 for a deeper numerical illustration), using both a low (left panel) and a high (right panel) discount factor δ. Throughout, higher prices are sustainable after a high signal (p_h) then after a low signal (p_l), since the incentives to deviate are lower when the future is promising. This effect is stronger the more precise the signal (for $\rho < \bar{\rho}$), which increases the wedge between p_h and p_l. When the signal is already sufficiently precise ($\rho > \bar{\rho}$), then anyways there is no longer entry after the bad signal and there is only a positive effect of increasing signal precision. When δ is low (left panel), then monopoly prices $p_h = v_H = \frac{3}{2}$ can never be sustained, but when δ is high (right panel), then monopoly prices $p_h = v_H = \frac{3}{2}$ can be sustained when the signal is sufficiently precise.



This figure shows optimal prices under baseline tacit collusion for $\delta = \frac{1}{2}$ (left panel) and $\delta = \frac{11}{20}$ (right panel). Other parameter values: $v = 1$, $v_H = \frac{3}{2}$, $v_L = 1$, $F = \frac{1}{4}$.

Figure 2: Baseline tacit collusion: Optimal Prices

3.2 Assigning Future Market Shares

An alternative to simple tacit coordination is the assignment of future market shares. While the former provides no immediate incentive for good cartel conduct but solely the prospect of shared monopoly profits, the latter has a direct gratification. Now we assume that the firm currently facing the high valuation promises the other firm the market of stage 2. This is in contrast to our baseline tacit collusion, where there is randomization for the entry into stage 2. Now the low state firm is guaranteed the right the enter into stage 2, still facing the risk of low demand. This promise expires after the current period.

Intuitively, such a promise enables firms to sustain higher collusive prices since the incentives to deviate are weakened. The continuation values then look as follows:

$$\begin{aligned} V_{Hh} &= p_h + 0 + \delta V \\ V_{Hl} &= p_l + 0 + \delta V \\ V_{Lh} &= 0 + \pi_{2,h} + \delta V \\ V_{Ll} &= 0 + \pi_{2,l} + \delta V \end{aligned} \tag{8}$$

In contrast to the continuation values from baseline tacit collusion in (5), the continuation values of the currently preferred firm (V_{Hs}) are reduced, whereas the continuation values of the disadvantaged firm (V_{Ls}) are increased. Since the binding constraint originates from deviating incentives of the disadvantaged firm, this shifts slack of a non-binding IC and relaxes the binding IC. We next characterize the highest sustainable prices when future market shares are used.

Proposition 2. *For collusion via assigning future market shares, the highest sustainable prices p_l^* and p_h^* are as follows. If $\rho \leq \bar{\rho}$, then $(p_l^*, p_h^*) =$*

$$\begin{cases} (v_H, v_H) & \delta \geq \widehat{CDF}_{f.m.s.,1} \\ \left(\hat{p}^{l,1}_{f.m.s.}(\rho), v_H\right) & \widehat{CDF}_{f.m.s.,1} > \delta \geq \widehat{CDF}_{f.m.s.,2} \\ \left(\hat{p}^{l,2}_{f.m.s.}(\rho), \hat{p}^{h}_{f.m.s.}(\rho)\right) & otherwise \end{cases}$$

If $\rho > \bar{\rho}$, then $(p_l^*, p_h^*) =$

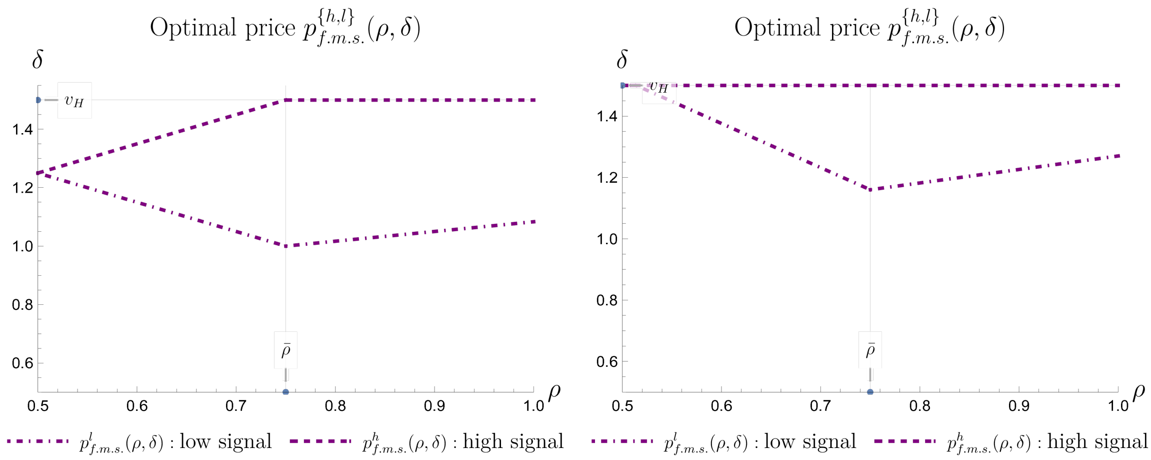
$$\begin{cases} (v_H, v_H) & \delta \geq \widetilde{CDF}_{f.m.s.,1} \\ \left(\tilde{p}^{l,1}_{f.m.s.}(\rho), v_H\right) & \widetilde{CDF}_{f.m.s.,1} > \delta \geq \widetilde{CDF}_{f.m.s.,2} \\ \left(\tilde{p}^{l,2}_{f.m.s.}(\rho), \tilde{p}^{h}_{f.m.s.}(\rho)\right) & otherwise \end{cases}$$

Prices and critical discount factors can be found in Appendix A.

Proof. See appendix. □

The price structure when assigning future market shares are used (Proposition 2) is very similar to baseline tacit collusion (Proposition 1). Both for low and high levels of ρ, there are three different cases depending on δ. Monopoly prices after both signals are only sustainable when δ is sufficiently high. Otherwise, prices need to be distorted downwards, and more so after a bad signal because than the incentive to deviate is stronger.

Figure 3 illustrates optimal prices when future market shares are used, using the same parameter values as in Section 6. In the left panel, where δ is low, monopoly prices v_H after the high signal are only sustainable when ρ is high. In the right panel δ is high, so monopoly prices v_H after always sustainable after the high signal. In this example, expected prices and profits decrease in ρ when $\rho < \bar{\rho}$, because p_h cannot exceed v_H, but the incentive to deviate after the bad signal increases and hence p_l needs to decrease.



This figure shows optimal prices when future market shares are used for $\delta = \frac{1}{2}$ (left panel) and $\delta = \frac{11}{20}$ (right panel). Other parameter values: $v = 1$, $v_H = \frac{3}{2}$, $v_L = 1$, $F = \frac{1}{4}$.

Figure 3: Collusion via future market shares: Optimal prices

3.3 Direct Transfers

The collusive schemes we considered so far do not require any communication. For future market shares, some explicit coordination might be helpful, but, in principle, not necessary. Even then, this can be done verbally, i.e. the cartel members can in principle avoid (nearly) any records. In the next step, we formally describe a compensation scheme with direct transfers between the firms, which is often understood as a 'strong cartel' (McAfee & McMillan, 1992; Bos & Pot, 2012). Now, there exist by construction some records regarding the payment of the transfer. Even if it happens in cash, someone has to withdraw it from some account, which again leaves some traces.

We model the possibility for direct transfers as follows. As in the baseline model, firms still simultaneously set prices p_s upon receiving the signal $s \in \{h, l\}$. After stage 1 prices are set, the currently preferred firm can make a transfer t_s to the disadvantaged firm, and then the stage 2 entry decisions are made as before. Reflecting the possibility of detection by the competition authority, we assume that a per-period fine of Φ has to be paid when firms collude.

Qualitatively, these model adjustments affects incentives in the following way. As we have an asymmetry in stage 1, the firm currently facing the low valuation has a binding incentive constraint. In order to relax this constraint, the high state firm could pay a signal-contingent transfer t_s to the low state firm in order to incentivize it to stick to the collusive agreement, i.e. to stay away from the market in stage 1. This transfer needs to be sufficiently small such that it is also in the preferred firm's interest to actually pay it.

Additionally, we now include the possibility for fines, e.g., through a competition authority. For collusive compensation via tacit collusion and via the assignment of future market shares, we assume that both schemes can theoretically happen without explicit coordination. In theory, firms could not only reach tacit collusion without communication but they could also coordinate silently on withdrawing from the market in some stages. In contrast, there is no doubt that direct transfers need some communication and leave traces on bank accounts. Such a behaviour is clearly illegal. Hence, firms have to take a detection risk and subsequent punishment into account. For the sake of simplicity, we assume in our setting a punishment term Φ that has to be paid every round by each firm. We consider the payment as welfare neutral, e.g., because the regulator uses collected fines efficiently.[7]

The continuation values then look as follows:

$$
\begin{aligned}
V_{Hh} &= p_h - t_h + \frac{\pi_{2,h}}{2} - \Phi + \delta V \\
V_{Hl} &= p_l - t_l + \frac{\pi_{2,l}}{2} - \Phi + \delta V \\
V_{Lh} &= 0 + t_h + \frac{\pi_{2,h}}{2} - \Phi + \delta V \\
V_{Ll} &= 0 + t_h + \frac{\pi_{2,l}}{2} - \Phi + \delta V
\end{aligned}
\tag{9}
$$

In contrast to the continuation values from baseline tacit collusion in (5), the disadvantaged firm (V_{Ls}) now additionally expects a signal-contingent transfer t_s as long it does not deviate, but also payment of a fine Φ. This relaxes the binding IC_l. However, now also the preferred firm (V_{Hs}) may have an incentive to deviate, namely simply refusing to pay the transfer. Note that we assume that the transfer is paid only after the pricing decision has been made (and is observable by both firms) such that the firms cannot react to a potential deviation in stage 1 anymore.

We now characterize the optimal transfers and then the maximal sustainable prices using these transfers.

Lemma 1. *If firms collude via direct transfers, the optimal transfer depends on the signal $s \in \{h, l\}$ and the price p_s and is given by*

$$t_s^* = \frac{p_s - v_H + v_L}{2}.$$

Proof. See appendix. □

[7]We take the fine Φ as exogenously given. The optimal design of fines and leniency are beyond the scope of this paper and addressed, e.g., in Bos and Harrington, 2015, Bos et al., 2018, Aubert et al., 2006, Spagnolo, 2004, and Harrington, 2008.

As Lemma 1 shows, the optimal transfers are linear in the collusive prices. These transfers are optimal in the sense that they do not leave any slack in any IC. This is achieved through equating the currently preferred and the currently disadvantaged firms' incentives to deviate. A very small transfer does not achieve anything and the disadvantaged firm is still as inclined to deviate as it was before. Also a very high transfer cannot be optimal since then the currently preferred firm may deviate and simply refuse to pay it. The optimal transfers exactly balance these two forces such that both firms' incentive to deviate is equally strong. The precise level of the transfer that achieves this depends on how much is to be gained from deviation, which in turn depends on the prices p_l and p_h being set in equilibrium.

Note that deviation and subsequent punishment of the other player through grim trigger strategies now also entails another advantage, namely avoiding being caught colluding by the competition authority and paying fines. If the detection probability, or the associated fine, is sufficiently high, then collusion with direct transfers are not sustainable at all. We formally derive the upper bound on fines such that collusion with direct transfers are feasible in the following lemma.

Lemma 2. Direct transfers *as a collusive compensation scheme is sustainable as long as* $\Phi < \bar{\Phi}$, *where*

$$\bar{\Phi} = \begin{cases} \frac{v-2F}{4} & \rho \leq \bar{\rho} \\ \frac{\rho v - F}{4} & \rho > \bar{\rho} \end{cases}$$

See appendix.

We are now ready to characterize the highest sustainable prices when such transfers are used.

Proposition 3. *For collusion with direct transfers, collusion is sustainable as long as* $\Phi < \bar{\Phi}$. *In that case, the highest sustainable prices* p_l^* *and* p_h^* *are as follows. If* $\rho \leq \bar{\rho}$, *then* $(p_l^*, p_h^*) =$

$$\begin{cases} (v_H, v_H) & \delta \geq \widehat{CDF}_{d.t.,1} \\ \left(\hat{p}^{l,1}_{d.t.}(\rho), v_H\right) & \widehat{CDF}_{d.t.,1} > \delta \geq \widehat{CDF}_{d.t.,2} \\ \left(\hat{p}^{l,2}_{d.t.}(\rho), \hat{p}^{h}_{d.t.}(\rho)\right) & otherwise \end{cases}$$

If $\rho > \bar{\rho}$, *then* $(p_l^*, p_h^*) =$

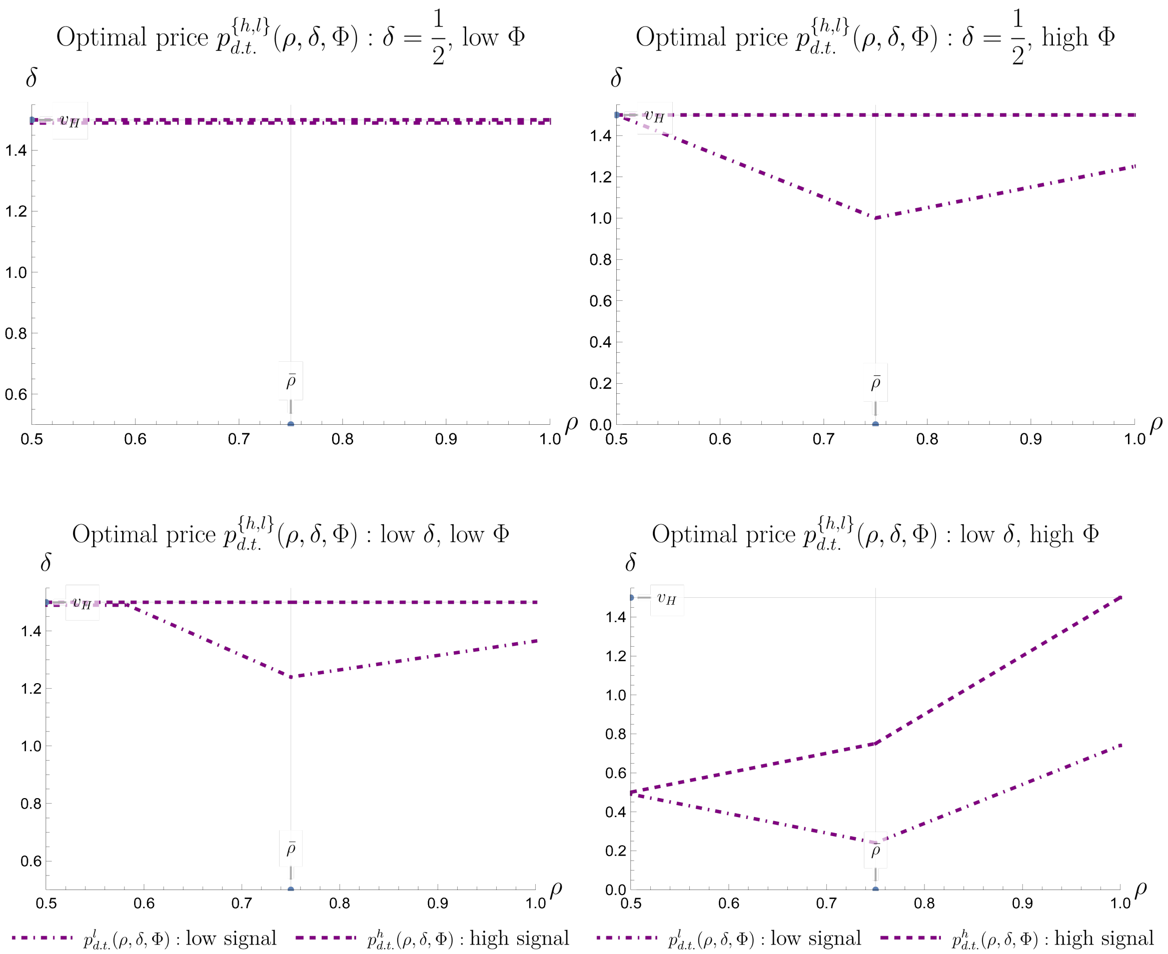
$$\begin{cases} (v_H, v_H) & \delta \geq \widetilde{CDF}_{d.t.,1} \\ \left(\tilde{p}^{l,1}_{d.t.}(\rho), v_H\right) & \widetilde{CDF}_{d.t.,1} > \delta \geq \widetilde{CDF}_{d.t.,2} \\ \left(\tilde{p}^{l,2}_{d.t.}(\rho), \tilde{p}^{h}_{d.t.}(\rho)\right) & otherwise \end{cases}$$

Throughout, the optimal transfers $t_s = t_s^*$, $s \in \{h, l\}$, *as shown in Lemma 1 are used. Prices and critical discount factors can be found in Appendix A.*

Proof. See appendix. □

The basic structure from pricing with direct transfers (Proposition 3) is similar to baseline tacit collusion and future market shares. For a discount factor high enough to maintain collusion with monopolistic pricing, the optimal transfer equals $t_s = \frac{v_L}{2}$ independently of the signal. Reducing equilibrium prices also lowers the transfer. The loss caused by lowering the price is split evenly between both firms. Both firms are ex-ante symmetric and hence equally likely to become preferred or disadvantaged, so, in expectation, it does not matter how high the transfers are and whether a firm is giving or receiving them.

In Figure 4, we illustrate optimal prices when direct transfers are used, using low and high levels of δ (top and bottom panels, respectively) and low and high levels of punishment Φ (left and right panels, respectively). When δ is low but Φ is high (top right panel), then monopoly prices after the good signal are always sustainable and there is a u-shaped relationship between prices after the bad signal and signal precision.



These figures show optimal prices when direct transfers are used for $\delta = \frac{1}{2}$ (top panels) and $\delta = \frac{2}{5}$ (bottom panels), for zero punishment ($\Phi = 0$, left panels) and for high punishment ($\Phi \approx \bar{\Phi}$, right panels). Other parameter values: $v = 1$, $v_H = \frac{3}{2}$, $v_L = 1$, $F = \frac{1}{4}$. As these figures illustrate, monopoly prices are easily sustainable when firms use sophisticated collusive compensation schemes.

Figure 4: Collusion via direct transfers: Optimal prices

4 Welfare analysis

Given our characterization of optimal prices, we can now investigate how expected prices, profits, consumer surplus, and total welfare change depending on prediction ability ρ for different collusive schemes.

In our model, both stage-1 states are ex-ante equally likely, and hence the expected stage-1 prices are given by

$$E(p) = \frac{p_l^* + p_h^*}{2}$$

in any collusive compensation scheme. Consumers have unit demand and thus stage-1 prices are welfare-neutral transfers from consumers to firms. In any collusive equilibrium, there is stage-2 entry by at most one firm, so consumers can never extract positive surplus from stage 2. Thus, from a consumer welfare point of view, only stage-1 prices matter. Ex-ante expected consumer surplus (CS) is given by

$$\begin{aligned} CS &= \left[Pr(h)\left[v_H - p_h^*\right] + Pr(l)\left[v_H - p_l^*\right]\right]\frac{1}{1-\delta} \\ &= \left[v_H - \frac{1}{2}\left[p_h^* + p_l^*\right]\right]\frac{1}{1-\delta}\,. \end{aligned}$$

Concerning expected profits, there is an additional effect through stage-2 entry costs and profits. Since both firms are ex-ante symmetric, we define producer surplus (PS) as the sum of expected profits. In any collusive equilibrium, there is entry into stage 2 after both signals when $\rho \leq \bar{\rho}$, but only after signal h when $\rho > \bar{\rho}$. The fine Φ only has to be paid in the *direct transfers* (d.t.) compensation scheme, so producer surplus is given by

$$PS = \begin{cases} \Big[\underbrace{\frac{1}{2}\left(p_h^* + p_l^*\right)}_{\textit{stage 1}} + \underbrace{\frac{1}{2}v - F}_{\textit{stage 2}} \quad -2\Phi \cdot \mathbb{1}_{d.t.}\Big]\frac{1}{1-\delta} & \text{if } \rho \leq \bar{\rho} \\ \Big[\overbrace{\frac{1}{2}\left(p_h^* + p_l^*\right)} + \overbrace{\frac{1}{2}(\rho v - F)} -2\Phi \cdot \mathbb{1}_{d.t.}\Big]\frac{1}{1-\delta} & \text{if } \rho > \bar{\rho} \end{cases}$$

Finally, we define total surplus (TS) as the sum of consumer and producer surplus and the punishment term Φ in case direct transfers are used. In that sense, we view Φ not simply as a form of 'burning money' to keep firms in check, but used by a benevolent social planner in the interest of society. Thus, total surplus is given by

$$TS = CS + PS + 2\Phi \cdot \mathbb{1}_{d.t.} \, .$$

We now present our main comparative statics results.

Proposition 4. *For all collusive compensation schemes, when ρ increases then*

(i) expected prices, profits and producer surplus weakly decrease, consumer welfare weakly increases and total surplus is constant when $\rho \leq \bar{\rho}$ but

(ii) expected prices, profits, and producer surplus weakly increase, consumer welfare weakly decreases, and total surplus weakly increase when $\rho > \bar{\rho}$.

Proof. See appendix. □

The key insights from Proposition 4 are twofold. First, expected prices are u-shaped in prediction ability ρ, and second, total welfare is initially constant, and then increasing in ρ. Profits and producer surplus follow the comparative statics of prices, whereas consumer surplus is indirectly proportional to prices. All these results hold for all our collusive schemes.

Regarding prices, the intuition is as follows. When ρ increases when it is initially low, then deviations become more attractive for the currently disadvantaged firm. Thus, collusive prices have to be distorted downwards in order to deter deviations. When ρ is already high, then after the low signal, entry into stage 2 no longer takes place, so there is no immediate punishment from deviating. But staying in the collusive agreement becomes more profitable because entry costs are spent more efficiently, so it becomes more attractive to adhere to the collusive agreement, allowing sustainability of higher collusive prices.

The results on consumer surplus then follow readily from the definition.

The total welfare considerations in Proposition 4 serve as a relevant benchmark of the overall role of prediction accuracy in our setting. Note that prices in this model are simply welfare-neutral transfers between consumers and firms. Since there is unit demand and the market is always covered, there are no welfare effects of pricing (no dead weight loss). The only welfare-relevant quantity is F, reflecting the fixed costs of setting up a business for the second stage. As long as $\rho \leq \bar{\rho}$, the entry decision remains unaffected (there is entry after both signals), and hence there is also no welfare effect. Once ρ exceeds $\bar{\rho}$, firms no longer make socially inefficient investments after bad signals, which increases total welfare (through increasing profits). This effect increases in ρ and hence total welfare is maximal when prediction accuracy is perfect ($\rho = 1$). In this sense, our results are reminiscent of results in Bos and Pot, 2012.

5 Optimal collusive compensation

With our benchmark results on welfare in mind, we now turn attention to our main object of interest. Suppose potentially colluding firms additionally cooperatively choose a collusive compensation scheme. Depending on the parameters of the model, which collusive compensation schemes are most profitable? And how does the answer depend on the prediction accuracy ρ? The following propositions answer these questions.

We first show collusion using future market shares is always weakly more profitable than baseline tacit collusion.

Proposition 5. *For any ρ, assigning future market shares yields weakly higher profits than baseline tacit collusion, i.e., $E[\Pi_{f.m.s.}(\rho)] \geq E[\Pi_{base}(\rho)]$.*

Proof. See appendix. □

Proposition 5 shows that assigning future market shares is always weakly more profitable. In both collusive compensation schemes, price setting is constrained by deviation incentives of the disadvantaged firm. Assigning future market shares relaxes this constraint through the promise of higher market shares in the stage 2. This comes essentially for free, since it is ex-ante profit neutral and hence does not affect deviation incentives of the preferred firm. Thus, unless firms are sufficiently patient such that monopoly prices are sustainable either way, higher prices can be sustained when future market shares are used, which yields higher profits.

However, as we show next, the comparison between future market shares and direct transfers is more ambiguous. Of course, in the absence of fines Φ, direct transfers would always be optimal, since they optimally remove any slack in the IC constraint of the preferred firm. For interior levels of Φ, and crucially depending on ρ, this outcome may be reversed.

Proposition 6. *For any v, v_L, v_H, F, there exists a δ and Φ, such that direct transfers are most profitable when ρ is sufficiently low but future market shares are most profitable when ρ is sufficiently high, i.e., $\Pi_{f.m.s.}\left(\frac{1}{2}\right) < \Pi_{d.t.}\left(\frac{1}{2}\right)$, but $\Pi_{f.m.s.}(\rho_1) > \Pi_{d.t.}(\rho_1)$ for some ρ_1 that satisfies $\frac{1}{2} < \rho_1 < \bar{\rho}$.*

Proof. See appendix. □

Before we interpret this result, we show that it also naturally emerges when $\rho > \bar{\rho}$:

Proposition 7. *For any v, v_L, v_H, F, there exists a δ and Φ, such that direct transfers are most profitable when ρ is sufficiently high but future market shares are most profitable when ρ is sufficiently low, i.e., $\Pi_{f.m.s.}(1) < \Pi_{d.t.}(1)$, but $\Pi_{f.m.s.}(\rho_2) > \Pi_{d.t.}(\rho_2)$ for some ρ_2 that satisfies $\bar{\rho} < \rho_2 < 1$.*

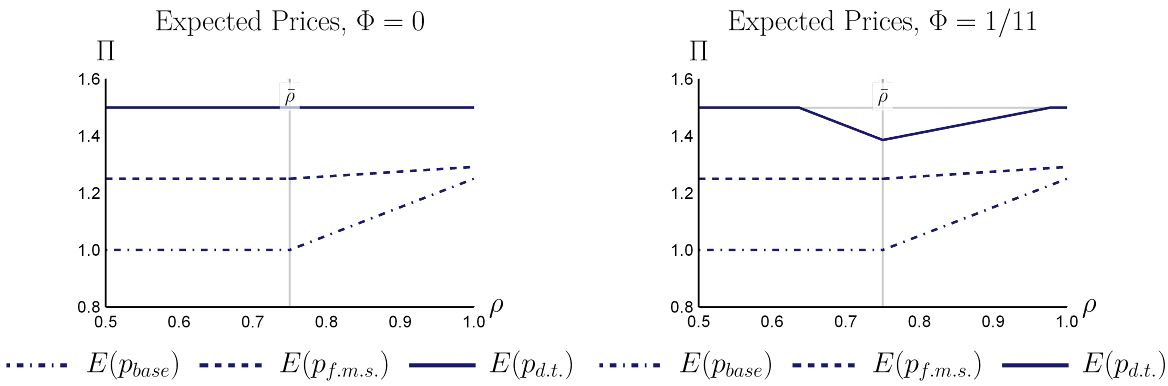
Proof. See appendix. □

Proposition 6 and Proposition 7 show that in any industry, the relative attractiveness of different collusive schemes crucially depends on the predication accuracy ρ. Of course, when direct transfers are not penalized at all ($\Phi = 0$) or penalized extremely harsh in expectation, then direct transfers are always, respectively never, optimal. For intermediate levels of Φ, however, direct transfers are only optimal when signal precision is sufficiently small (Proposition 6) or sufficiently high (Proposition 7). As signal precision increases, future market shares become relatively more attractive, and maybe become even the preferred collusive scheme. The reason is the following. For all parameter values, expected prices (and profits) under future market shares are constant in ρ for $\rho < \bar{\rho}$ for certain values of δ, as long as both p_h and p_l are distorted. For precisely these values of δ, however, p_h under direct transfers is capped at v_H and cannot increase any further; but increasing ρ increases the incentives for deviations after bad signal, which reduces p_l and hence expected prices (and profits) from direct transfers. This is less concern when ρ is small, such that direct transfers are optimal. When ρ is sufficiently high, however, this negative effect, combined with the possibility of fines Φ, make collusion through future market shares more attractive. Therefore, competition authorities' limited resources should be optimally utilized depending on the institutional context.

6 Numerical example

We illustrate our main results through a numerical example. In particular, we set $v = 1$, $v_H = \frac{3}{2}$, $v_L = 1$, $F = \frac{1}{4}$, and $\delta = \frac{1}{2}$. We show how prices, profits, consumer surplus and total surplus react to changes in prediction ability ρ, depending on whether the punishment term Φ is low or high, and for our three collusive compensation schemes.

We first investigate expected prices in Figure 5. For all collusive compensation schemes, prices are u-shaped in prediction accuracy. When ρ increases when it was initially low, the disadvantaged firm becomes more inclined to deviate when a high signal was received, so prices need to be distorted downwards to deter deviations. Conversely, when ρ increases from already high levels, there is no entry in stage 2 when a low signal was received, so entry costs are less frequently wasted which increases expected profits. This makes it more important to adhere to the collusive agreement, making higher prices sustainable. Moreover, the figure illustrates that expected prices are highest when direct transfers are used.

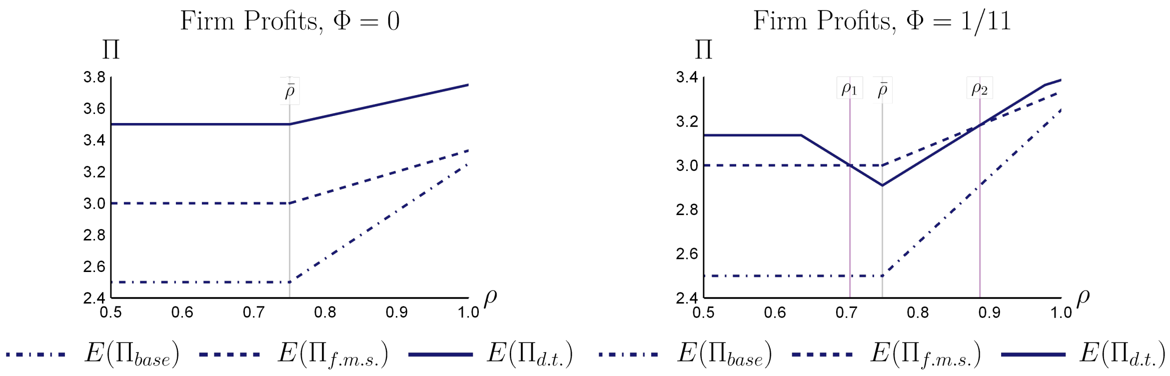


This figure shows expected prices as a function of prediction ability ρ for different collusive compensation schemes, both for low punishment ($\Phi = 0$, left panel) and for high punishment ($\Phi = 1/11$, right panel). For all collusive schemes, prices are u-shaped in prediction accuracy. Expected prices are always highest when direct transfers are used.

Figure 5: Expected prices depending on ρ.

Figure 6 depicts profits as a function of prediction ability ρ. On top of the effect of expected prices, profits additionally depend on entry costs (which are no longer paid after a bad signal was received in case $\rho > \bar{\rho}$), and the punishment term Φ in case direct transfers are used. Thus, when Φ is low (left panel), the ranking of profits across collusive compensation schemes follows exactly the expected prices, as in Figure 5. Additionally, all profits increase in ρ when $\rho > \bar{\rho}$ because entry costs are wasted less frequently.

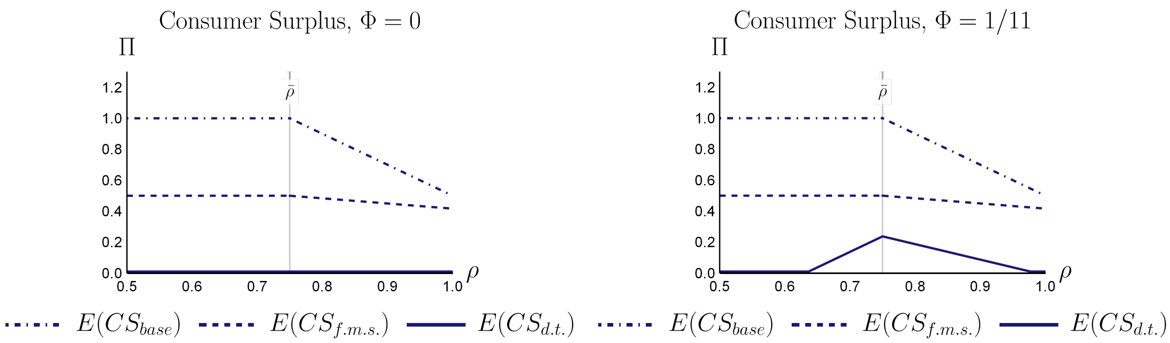
The right panel of Figure 6 illustrates our main result. When Φ is high, there is the possibility for a double regime switch, i.e., the optimal collusive compensation scheme changes depending on ρ. Assigning future market shares is optimal when ρ is intermediate, and direct transfers are optimal otherwise. The extent to which firms are able to take advantage of better forecasting ability crucially depends on the compensation scheme employed.



This figure shows firm profits as a function of prediction ability ρ for different collusive compensation schemes, both for low punishment ($\Phi = 0$, left panel) and for high punishment ($\Phi = 1/11$, right panel). When Φ is low, direct transfers always yield the highest profit. When Φ is high, assigning future market shares is optimal when ρ is intermediate.

Figure 6: Expected profits depending on ρ.

Next, consider Figure 7 for an illustration of consumer surplus. Since direct transfers enable higher sustainable prices, consumer surplus is the lowest in that case. The effect on consumer welfare is indirectly proportional to first-stage prices, so it is inverse u-shaped in prediction ability ρ.

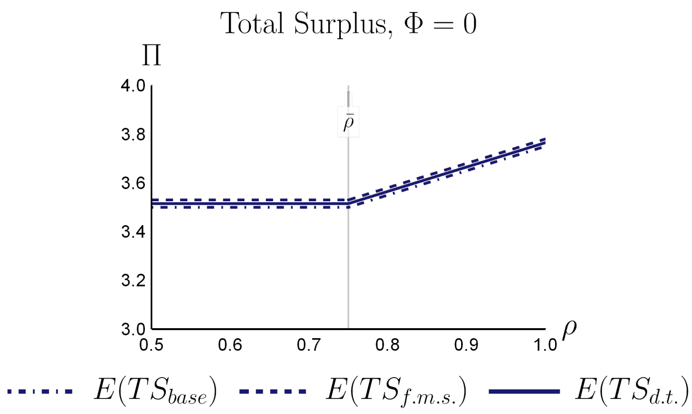


This figure shows consumer surplus as a function of prediction ability ρ for different collusive compensation schemes, both for low punishment ($\Phi = 0$, left panel) and for high punishment ($\Phi = 1/11$, right panel).

Figure 7: Expected consumer surplus depending on ρ.

Combining the profits of both firms and customer welfare, we get total surplus as shown in Figure 8. Up to the threshold $\bar{\rho}$, social welfare is flat in the prediction ability

ρ, because only prices are affected, which are welfare-neutral. Once the $\bar{\rho}$-threshold is reached, total welfare increases in ρ because less inefficient resources are wasted on unprofitable markets (no more entry after signal l). Note that by definition, the punishment is a pure transfer from firms to the state and hence welfare-neutral. Thus, the figure looks identical for $\Phi = 1/11$ and hence is omitted. Finally, note that total surplus is identical across all collusive compensation schemes.



This figure shows total surplus as a function of prediction ability ρ for different collusive compensation schemes and $\Phi = 0$. Total surplus increases in ρ because fewer resources are wasted on inefficient stage-2 entry. Total surplus is identical across all collusive compensation schemes.

Figure 8: Expected total surplus depending on ρ.

7 Conclusion

In the last two decades, firms started accumulating unprecedented amounts of data, allowing them to better forecast the economic environment in which they will operate tomorrow. This trend is bound to continue in the foreseeable future. At the same time, managerial boards become increasingly more diverse.

Combining these two observations triggers the obvious question how firms are expected to sustain collusion in the twenty-first century. Our paper provides answers to these questions by considering a model with temporary asymmetry. We show that assigning future market makes it easier to sustain collusion by reducing the incentives to deviate for firms that are temporarily disadvantaged. Direct transfers are even more powerful but may come at the expense of detection through competition authorities. As prediction ability increases, direct transfers become relatively less attractive when prediction ability was initially low. Levenstein and Suslow (2011) find that strong cartels using transfers tend to be more stable. This is consistent with our finding the direct transfers become more attractive when prediction ability increases from already high levels.

Tacit collusive schemes could, in principle, operate without any personal interaction between firm representatives. As Jaspers (2017) emphasizes, cartels operate beyond the legal confines, and, by that, rely heavily on social networks and pressure among the cartel members. This might become more difficult as managerial boards of firms get more diverse in terms of nationalities, genders and cultural backgrounds. Hence, algorithmic market forecasting does not only makes tacit collusion more attractive, it also addresses potentially lower cartel strength due to higher diversity in the responsible boards.

Our results reveal that the relative attractiveness of different compensation schemes crucially depends on the firms' ability to forecast future market conditions. Competitive authorities should thus increasingly focus their resources on detecting and fighting such collusive schemes. On the other hand, tacit collusion is by definition not illegal, as conscious and documented arrangements that could be punishable do not exist. An indirect measure might be lowering entry barriers. As documented in Levenstein and Suslow, 2006, many cartels broke down once new competitors entered the market.

Collusion is total-welfare neutral in our setting, and better prediction ability increases total welfare. Since we focused on the relative attractive of different collusive compensation schemes from a firm's perspective, we abstracted from additional aspects such as demand reductions incurred through collusion. These seem to be a fruitful area left for future research.

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Appendix

A Additional definitions

For the analysis of *baseline tacit collusion*, we define the following quantities:

$$\widehat{CDF}_{base,1} = \frac{2(F + (\rho - 1)v + 2v_L)}{(2\rho - 1)v + 6v_L}$$
$$\widehat{CDF}_{base,2} = \frac{4(F - \rho v + 2v_L)}{3(v - 2\rho v + 4v_L)}$$
$$\widetilde{CDF}_{base,1} = \frac{4v_L}{\rho v + 6v_L - F}$$
$$\widetilde{CDF}_{base,2} = \frac{4(F - \rho v + 2v_L)}{3(F - \rho v + 4v_L)}$$
$$\hat{p}^{l,1}_{base}(\rho) = \frac{v(\delta - 2(\delta\rho - \rho + 1)) + 5\delta v_H - 6\delta v_L + 2F - 4(v_H - v_L)}{5\delta - 4}$$
$$\hat{p}^{l,2}_{base}(\rho) = \frac{v(3\delta - 6\delta\rho + 4(\rho - 1)) + 4(3\delta - 2)(v_H - v_L) + 4F}{12\delta - 8}$$
$$\hat{p}^{h}_{base}(\rho) = \frac{3\delta(2\rho v - v + 4v_H - 4v_L) + 4F - 4\rho v - 8v_H + 8v_L}{12\delta - 8}$$
$$\tilde{p}^{l,1}_{base}(\rho) = \frac{\delta(F - \rho v + 5v_H - 6v_L) - 4v_H + 4v_L}{5\delta - 4}$$
$$\tilde{p}^{l,2}_{base}(\rho) = \frac{3\delta(F - \rho v + 4v_H - 4v_L) - 8v_H + 8v_L}{12\delta - 8}$$
$$\tilde{p}^{h}_{base}(\rho) = \frac{(4 - 3\delta)F + (3\delta - 4)\rho v + 4(3\delta - 2)(v_H - v_L)}{12\delta - 8}$$

For the analysis of collusion with *assigning future market shares*, we define the following quantities:

$$\widehat{CDF}_{f.m.s.,1} = \frac{4(F + (\rho - 1)v + v_L)}{2F + (4\rho - 3)v + 6v_L}$$
$$\widehat{CDF}_{f.m.s.,2} = \frac{2(F - \rho v + v_L)}{F - 3(\rho v - v_L) + v}$$
$$\widetilde{CDF}_{f.m.s.,1} = \frac{4v_L}{\rho v + 6v_L - F}$$
$$\widetilde{CDF}_{f.m.s.,2} = \frac{2(F - \rho v + v_L)}{2F - 2\rho v + 3v_L}$$
$$\hat{p}^{l,1}_{f.m.s.}(\rho) = \frac{v(4(\rho - 1 - \delta\rho) + 3\delta) - 2(\delta - 2)F - \delta v_H + (6\delta - 4)(v_H - v_L)}{5\delta - 4}$$

$$\hat{p}^{l,2}_{f.m.s.}(\rho) = \frac{v(2(\delta+\rho-1)-3\delta\rho)-(\delta-2)F}{3\delta-2} + v_H - v_L$$
$$\hat{p}^{h}_{f.m.s.}(\rho) = \frac{(2-\delta)F-\delta(-3\rho v+v-3v_H+3v_L)-2(\rho v+v_H-v_L)}{3\delta-2}$$
$$\tilde{p}^{l,1}_{f.m.s.}(\rho) = \frac{\delta(F-\rho v+5v_H-6v_L)-4v_H+4v_L}{5\delta-4}$$
$$\tilde{p}^{l,2}_{f.m.s.}(\rho) = \frac{\delta(F-\rho v)+(3\delta-2)(v_H-v_L)}{3\delta-2}$$
$$\tilde{p}^{h}_{f.m.s.}(\rho) = \frac{-2(\delta-1)F+2(\delta-1)\rho v+(3\delta-2)(v_H-v_L)}{3\delta-2}$$

For the analysis of collusion with *direct transfers*, we define the following quantities:

$$\widehat{CDF}_{d.t.,1} = \frac{2(F+2\Phi+(\rho-1)v+v_L)}{(2\rho-1)v+4v_L}$$
$$\widehat{CDF}_{d.t.,2} = \frac{F+2\Phi-\rho v+v_L}{2v_L-2\rho v+v}$$
$$\widetilde{CDF}_{d.t.,1} = \frac{2(2\Phi+v_L)}{4v_L+\rho v-F}$$
$$\widetilde{CDF}_{d.t.,2} = \frac{F+2\Phi-\rho v+v_L}{F-\rho v+2v_L}$$
$$\hat{p}^{l,1}_{d.t.}(\rho) = \frac{v(-2\delta\rho+\delta+2\rho-2)+3\delta v_H-4\delta v_L+2F+4\Phi-2v_H+2v_L}{3\delta-2}$$
$$\hat{p}^{l,2}_{d.t.}(\rho) = \frac{v(-2\delta\rho+\delta+\rho-1)+(2\delta-1)(v_H-v_L)+F+2\Phi}{2\delta-1}$$
$$\hat{p}^{h}_{d.t.}(\rho) = \frac{\delta(2(\rho v+v_H-v_L)-v)+F+2\Phi-\rho v-v_H+v_L}{2\delta-1}$$
$$\tilde{p}^{l,1}_{d.t.}(\rho) = \frac{\delta(F-\rho v+3v_H-4v_L)+2(2\Phi-v_H+v_L)}{3\delta-2}$$
$$\tilde{p}^{l,2}_{d.t.}(\rho) = \frac{\delta(F-\rho v+2v_H-2v_L)+2\Phi-v_H+v_L}{2\delta-1}$$
$$\tilde{p}^{h}_{d.t.}(\rho) = \frac{(\delta-1)(\rho v-F)+(2\delta-1)(v_H-v_L)+2\Phi}{2\delta-1}$$

Lemma 3. *Given our assumptions and in the relevant parameter range, for all cartel compensation schemes k, $k \in \{base, f.m.s., d.t.\}$, it always holds that:*

(i) $\widehat{CDF}_{k,1} < \frac{2}{3}$ *(for $\rho \leq \bar{\rho}$)*

(ii) $\widetilde{CDF}_{k,1} < \frac{2}{3}$ *(for $\rho > \bar{\rho}$)*

Proof. We separately show the result for each of the three cartel compensation schemes $k \in \{base, f.ms., d.t.\}$.

First, consider baseline tacit collusion, $k = base$.

(i) The constraint is only relevant for $\rho \leq \bar{\rho}$. Note that $\widehat{CDF}_{base,1}$ increases in ρ:

$$\frac{\partial \widehat{CDF}_{base,1}(\rho)}{\partial \rho} = \frac{2v(\overbrace{-2F+v}^{>0}+2v_L)}{(-2\rho v + v - 6v_L)^2} > 0$$

and hence the constraint is binding at $\rho = \bar{\rho}$. The result then follows readily since

$$\widehat{CDF}_{base,1}(\bar{\rho}) = \frac{4v_L}{6v_L + v - 2F} < \frac{2}{3}$$
$$6v_L < 6v_L + v - 2F$$
$$v - 2F > 0$$

where the last inequality is always satisfied by assumption.

(ii) The constraint is only relevant for $\rho > \bar{\rho}$. Note that $\widetilde{CDF}_{base,1}$ decreases in ρ since

$$\frac{\partial \widetilde{CDF}_{base,1}(\rho)}{\partial \rho} = \frac{-4vv_L}{(F - \rho v - 6v_L)^2} < 0$$

and hence the constraint is binding at $\rho = \bar{\rho}$. Since $\widetilde{CDF}_{base,1}(\bar{\rho}) = \frac{4v_L}{6v_L+v-2F} = \widehat{CDF}_{base,1}(\bar{\rho})$, the result follows readily from (i).

Next, consider collusion using future market shares, $k = f.m.s.$

(i) The constraint is only relevant for $\rho \leq \bar{\rho}$. Note that $\widehat{CDF}_{base,1}$ increases in ρ:

$$\frac{\partial}{\partial \rho}\widehat{CDF}_{f.m.s.,1} = \frac{4v(\overbrace{-2F+v}^{>0}+2v_L)}{(2F + (4\rho - 3)v + 6v_L)^2} > 0 \tag{10}$$

and hence the constraint is binding at $\rho = \bar{\rho}$. The result then follows readily since

$$\widehat{CDF}_{f.m.s.,1}(\bar{\rho}) = \frac{4v_L}{6v_L + v - 2F} < \frac{2}{3}$$

This is the same as before for baseline tacit collusion.

(ii) The constraint is only relevant for $\rho > \bar{\rho}$. As $\widetilde{CDF}_{base.,1} = \widetilde{CDF}_{f.m.s.,1}$, the respective derivation for basleine tacit collusion beforehand applies as well.

Finally, consider collusion using direct transfers, $k = d.t.$

By construction, $CDF_{d.t.,1}(\rho, \Phi) < \frac{2}{3} \, \forall \rho$ holds as we define the upper bound of Φ such that it satisfies exactly this condition, see the proof of Lemma 2. □

B Additional proofs

Proof of Proposition 1:

Proof. The proof proceeds as follows. We first consider the case where $\rho \leq \bar{\rho}$, and then the case where $\rho > \bar{\rho}$. For both cases, we characterize conditions such that monopoly prices $p_h = p_l = v_H$ are sustainable, as well as the highest sustainable prices in cases they are not.

Suppose that $\rho \leq \bar{\rho}$. Then in equilibrium, there is entry after both signals, so the respective continuation values are given by

$$V_{Hh} = p_h + \frac{\rho v - F}{2} + \delta V$$
$$V_{Hl} = p_l + \frac{(1-\rho)v - F}{2} + \delta V$$
$$V_{Lh} = 0 + \frac{\rho v - F}{2} + \delta V$$
$$V_{Ll} = 0 + \frac{(1-\rho)v - F}{2} + \delta V$$

Since there is no incentive to deviate for the currently preferred firm, we only need to investigate incentive compatibility for the currently disadvantaged firm. Given that the preferred firm charges p_s after signal $s \in \{h, l\}$, the most profitable deviation is to a price $p' = v_L - v_H + p_s$. When $p_s = v_H$, then $p' = v_L$. Hence the two incentive compatibility constraints, upon receiving a high and a low signal, respectively, need to be satisfied:

$$(IC_h) : V_{Lh}(p_h, p_l) \geq v_L - v_H + p_h + \delta E(\pi)_N$$
$$(IC_l) : V_{Ll}(p_h, p_l) \geq v_L - v_H + p_l + \delta E(\pi)_N$$

When $p_h = p_l = v_H$, then $V_{Lh} > V_{Ll}$ and the RHS is the same in both conditions, so IC_l is binding. We can readily solve for δ and obtain that collusive prices $p_h = p_l = v_H$

are sustainable when

$$\delta \geq \widehat{CDF}_{base,1} = \frac{2(F + (\rho - 1)v + 2v_L)}{(2\rho - 1)v + 6v_L} \tag{11}$$

holds.

Now suppose that $\delta < \widehat{CDF}_{base,1}$ so collusion at monopoly prices is not sustainable. Decreasing p_l relaxes IC_l but tightens IC_h since also V decreases, whereas decreasing p_h relaxes IC_h and tightens IC_l. When δ is just slightly below $\widehat{CDF}_{base,1}$ and $p_h = p_l = v_H$, then IC_l fails but there is slack in IC_h. Hence, we can decrease p_l and keep p_h fixed such that IC_l is satisfied. Hence, we solve

$$V_{Ll}(v_H, p_l) = v_L - v_H + p_l + \delta E(\pi)_N$$

for p_l while holding $p_h = v_H$ fixed, which yields

$$p_l = \hat{p}^{l,1}_{base} = \frac{v(-2\delta\rho + \delta + 2\rho - 2) + 5\delta v_H - 6\delta v_L + 2F - 4v_H + 4v_L}{5\delta - 4}.$$

Since decreasing p_l tightens IC_h, we need to check whether IC_h is still satisfied. This holds as long as

$$V_{Lh}(v_H, \hat{p}^{l,1}_{base}) \geq v_L - v_H + \hat{p}^{l,1}_{base} + \delta E(\pi)_N$$

which holds whenever

$$\delta \geq \widehat{CDF}_{base,2} = \frac{4(F - \rho v + 2v_L)}{3(v - 2\rho v + 4v_L)}.$$

Thus, for $\widehat{CDF}_{base,1} > \delta \geq \widehat{CDF}_{base,2}$, $(p_l, p_h) = (\hat{p}^{l,1}_{base}, v_H)$ are the highest sustainable prices.

Next, we consider $\delta < \widehat{CDF}_{base,2}$. Then both p_l and p_h need to be decreased in order to simultaneously satisfy IC_l and IC_h with equality, which leads to

$$\begin{aligned} p_l = \hat{p}^{l,2}_{base} &= \frac{v(-6\delta\rho + 3\delta + 4\rho - 4) + 4(3\delta - 2)(v_H - v_L) + 4F}{12\delta - 8} \\ p_h = \hat{p}^{h}_{base} &= \frac{3\delta(2\rho v - v + 4v_H - 4v_L) - 4\rho v - 8(v_H - v_L) + 4F}{12\delta - 8}. \end{aligned}$$

By construction, these prices satisfy all IC constraints.

Now suppose that $\rho > \bar{\rho}$. The proof is analogous to the case where $\rho \leq \bar{\rho}$. The only difference is that now, in equilibrium there is only entry after signal h but not after signal l, which changes the on-equilibrium continuation values:

$$\begin{aligned} V_{Hh} &= p_h + \frac{\rho v - F}{2} + \delta V \\ V_{Hl} &= p_l + \delta V \\ V_{Lh} &= 0 + \frac{\rho v - F}{2} + \delta V \end{aligned}$$

$$V_{Ll} = 0 + \delta V$$

and hence, indirectly, also the foregone profits from deviation. The punishment payoffs and hence the RHS of the incentive compatibility constraints remain unchanged. The remainder of the proof is analogous to the case where $\rho \leq \bar{\rho}$ and hence not repeated. Monopoly prices $p_h = p_l = v_H$ are sustainable when δ is sufficiently high. For intermediate levels of δ, p_l needs to be decreased. When such a decrease would eventually violate IC_h, also p_h needs to be decreased. Making these adjustments optimally yields exactly the price schedule stated in the proposition.

□

Proof of Proposition 2:

Proof. This proof proceeds in the same manner as the proof for Proposition 1. When $\rho \leq \bar{\rho}$, there is entry of the disadvantaged firm after both signals and hence the continuation values are given by

$$\begin{aligned} V_{Hh} &= p_h + \delta V \\ V_{Hl} &= p_l + \delta V \\ V_{Lh} &= 0 + \rho v - F + \delta V \\ V_{Ll} &= 0 + (1-\rho)v - F + \delta V \end{aligned}$$

whereas for $\rho > \bar{\rho}$, the disadvantaged firm enters only after the good signal, which yields continuation values

$$\begin{aligned} V_{Hh} &= p_h + \delta V \\ V_{Hl} &= p_l + \delta V \\ V_{Lh} &= 0 + \rho v - F + \delta V \\ V_{Ll} &= 0 + \delta V \end{aligned}$$

Proceeding analogously to the proof for Proposition 1, we obtain the threshold levels of δ and the optimal price schedule as stated in the proposition. □

Proof of Lemma 1:

Proof. After signal $s \in \{h, l\}$ and given that prices p_s and transfers t_s are expected, the optimal deviation for the preferred firm is not paying the transfer while still charging the same price; and for the disadvantaged firm to undercut. Using the continuation values in (9), this yields the following IC constraints:

$$\begin{aligned} IC_{Hs}(t_s) &= V_{Hs}(t_s) - \left(\frac{\delta(v_H - v_L)}{2(1-\delta)} + p_s \right) \\ IC_{Ls}(t_s) &= V_{Ls}(t_s) - \left(\frac{\delta(v_H - v_L)}{2(1-\delta)} + p_s - v_H + v_L \right) \end{aligned}$$

The optimal transfer t_s^* equates these two ICs and hence

$$\begin{aligned} IC_{Hs}(t_s) &= IC_{Ls}(t_s) \\ -t_s &= t_s - (p_h - v_H + v_L) \\ t_s &= \frac{p_h - v_H + v_L}{2} \end{aligned}$$

□

Proof of Proposition 3:

Proof. The proof proceeds similar to the proof or Proposition 1, using optimal transfer as described in Lemma 1 and the continuation values in (9). Depending on ρ and δ, monopoly prices after both signal signals are sustainable or alternatively prices need to be adjusted. The exact thresholds on δ and prices are the stated in the proposition text. □

Proof of Lemma 2:

Proof. Setting a price $p_s^* < v_H$ aims at lowering the critical discount factor necessary to sustain collusion. Direct transfers introduce Φ as additional variable. As both the optimal prices $p_{d.t.}(\Phi)$ and the critical discount factors $CDF_{d.t.}(\Phi)$ are functions of the punishment term, we need to define an interval for Φ in which prices and $CDFs$ are well-behaved. Hence, we have to ensure that the the optimal prices do not exceed the monopolistic price v_H as this is the highest possible price to sell a positive quantity. We evaluate the partial derivative of $p_{d.t.}^*$ as shown in Prop. 3 with respect to Φ. Differentiating the optimal price for $\rho \leq \bar{\rho}$ as well as for $\rho > \bar{\rho}$ leads to the same derivatives, namely

$$\frac{\partial p_l^*}{\partial \Phi} = \begin{cases} 0 & \delta > CDF_{d.t.,1} \quad \forall \rho \\ \frac{4}{3\delta - 2} & CDF_{d.t.,1} \geq \delta > CDF_{d.t.,2} \quad \forall \rho \\ \frac{2}{2\delta - 1} & CDF_{d.t.,2} \geq \delta \quad \forall \rho \end{cases}$$

$$\frac{4}{3\delta - 2} \begin{cases} \geq 0 & \delta > \frac{2}{3} \\ \leq 0 & \delta < \frac{2}{3} \\ - & \delta = \frac{2}{3} \end{cases}$$

$$\frac{\partial p_h^*}{\partial \Phi} = \begin{cases} 0 & \delta > \widehat{CDF}_{d.t.,2} \quad \forall \rho \\ \frac{2}{2\delta - 1} & o/w \quad \forall \rho \end{cases}$$

$$\frac{2}{2\delta - 1} \begin{cases} \geq 0 & \delta > \frac{1}{2} \\ \leq 0 & \delta < \frac{1}{2} \\ - & \delta = \frac{1}{2} \end{cases}$$

Hence, to satisfy all four incentive constraints (a firm can get either the high or the low valuation in stage 1 and the firms receive either a good or a bad signal regarding stage 2), the price does not react negatively in response to an increase in Φ anymore. If Φ gets too large, the respective price pair 2 (p_l', v_H) or else pair 3 (p_l'', p_h'') must rise above the monopolistic price v_H, which would actually cause zero demand as the price would exceed consumer valuation. Therefore, we restrict the punishment term Φ to ensures such a price cannot happen. We do that by ensuring that $CDF_{d.t.,1} \ngeq \frac{2}{3}$. As the critical discount factor differs in ρ, we evaluate first the condition for $\widehat{CDF}_{d.t.,1}$ and then $\widetilde{CDF}_{d.t.,1}$ at the threshold $\frac{2}{3}$. For $\rho \leq \bar{\rho}$, this yields

$$\frac{2}{3} = \widehat{CDF}_{d.t.,1} = \frac{2(F + 2\Phi + (\rho - 1)v + v_L)}{(2\rho - 1)v + 4v_L}$$

and hence

$$\widehat{\Phi}_1 = \frac{1}{6}\left(v(2-\rho) - 3F + v_L\right)$$

and for $\rho > \bar{\rho}$, we obtain

$$\frac{2}{3} = \widetilde{CDF}^{l,m}_{d.t.} = \frac{2(2\Phi + v_L)}{4v_L + \rho v - F}$$

and hence

$$\widetilde{\Phi}_1 = \frac{1}{6}(\rho v - F + v_L).$$

Equivalently, we evaluate the upper bound on Φ for price pair 3 $-(p_l'', p_h'')$ $-$, such that $CDF_{d.t.,2} < \frac{1}{2}$ holds. So for $\rho \leq \bar{\rho}$, we set

$$\frac{1}{2} = \widehat{CDF}_{d.t.,2} = \frac{F + 2\Phi - \rho v + v_L}{2v_L - 2\rho v + v}$$

and obtain

$$\widehat{\Phi}_2 = \frac{1}{4}(v - 2F)$$

and for $\rho > \bar{\rho}$, we set

$$\frac{1}{2} = \widetilde{CDF}_{d.t.,2} = \frac{F + 2\Phi - \rho v + v_L}{2v_L - \rho v + v}$$

and obtain

$$\widetilde{\Phi}_2 = \frac{1}{4}(\rho v - F).$$

One can easily see that the upper bound of Φ depends on the case in which ρ is. To ensure that the all of the former equalities are satisfied, we have to set the upper bound of the punishment terms – depending on ρ such that $\widehat{\widehat{\Phi}} = min\{\widehat{\Phi}_1, \widehat{\Phi}_2\}$ and $\widetilde{\widetilde{\Phi}} = min\{\widetilde{\Phi}_1, \widetilde{\Phi}_2\}$. As $\frac{\partial \widehat{\Phi}_1}{\partial \rho} < 0$, we set $\rho = \bar{\rho}$ to ensure that even $\underset{\rho}{min}\{\widehat{\Phi}_1\}$ exceeds $\widehat{\Phi}_2$.

Next, note that

$$\widehat{\Phi}_2 = \frac{v-2F}{4} < \widehat{\Phi}_1(\bar{\rho}) = \frac{1}{6}\left[\left(2 - \frac{v-F}{v}\right) v - 3F + v_L\right]$$

can we simplified to

$$\frac{v-2F}{4} < v - 2F + v_L,$$

which holds by assumption.

Now we show that for $\rho > \bar{\rho}$ $\widetilde{\Phi}_1(\rho)$ is smaller than $\widetilde{\Phi}_2(\rho)$ since

$$\widetilde{\Phi}_1(\rho) = \frac{\rho v - F}{4} < \widetilde{\Phi}_2(\rho) = \frac{1}{6}(\rho v - F + v_L)$$
$$v_L > \frac{1}{2}(\rho v - F),$$

which holds for all $\rho > \bar{\rho}$.

Thus, collusion with direct transfers is sustainable as long as $\Phi < \bar{\Phi}$. □

Proof of Proposition 4:

Proof. We proceed as follows. We first show the comparative static results of prices for both $\rho \leq \bar{\rho}$ and $\rho > \bar{\rho}$, followed by results for consumer surplus, producer surplus, and total surplus.

Recall the definitions of prices as price pairs as done in the the first three propositions with $k \in \{base, f.m.s., d.t.\}$:

$$\text{For } \rho \leq \bar{\rho}: \begin{cases} (v_H, v_H) & \textit{price pair } 1 \\ \left(\hat{p}_k^{l,1}(\rho), v_H\right) & \textit{price pair } 2 \\ \left(\hat{p}_k^{l,2}(\rho), \hat{p}_k^{h}(\rho)\right) & \textit{price pair } 3 \end{cases}$$

$$\text{For } \rho > \bar{\rho}: \begin{cases} (v_H, v_H) & \textit{price pair } 1 \\ \left(\tilde{p}_k^{l,1}(\rho), v_H\right) & \textit{price pair } 2 \\ \left(\tilde{p}_k^{l,2}(\rho), \tilde{p}_k^{h}(\rho)\right) & \textit{price pair } 3 \end{cases}$$

The expected price $E(p)$ is the average of every price pair.

Consider first $\rho \leq \bar{\rho}$.

For baseline tacit collusion, we obtain:

$$E\left[(v_H, v_H)\right] = v_H$$
$$E\left[\left(\hat{p}_{base}^{l,1}(\rho), v_H\right)\right] = \frac{v(\delta - 2\delta\rho + 2\rho - 2) - 6\delta v_L + 2F + 4v_L}{10\delta - 8} + v_H$$
$$E\left[\left(\hat{p}_{base}^{l,2}(\rho), \hat{p}_{base}^{h}(\rho)\right)\right] = \frac{2F - v}{6\delta - 4} + v_H - v_L$$

where the first and the third quantity are clearly constant in ρ, and

$$\frac{\partial}{\partial\rho}E\left[\left(\hat{p}^{l,1}_{base}(\rho), v_H\right)\right] = \frac{(1-\delta)v}{5\delta-4} < 0$$

where the last inequality follows from the fact that $\delta < \frac{2}{3} < \frac{4}{5}$ whenever we are in that case by Lemma 3.

Similarly, for the assignment of future market shares, we obtain

$$E\left[(v_H, v_H)\right] = v_H$$
$$E\left[\left(\hat{p}^{l,1}_{f.m.s.}(\rho), v_H\right)\right] = \frac{2F(2-\delta)+v(4\rho-4\delta\rho+3\delta-4)-6\delta v_L+4v_L}{10\delta-8} + v_H$$
$$E\left[\left(\hat{p}^{l,2}_{f.m.s.}(\rho), \hat{p}^{h}_{f.m.s.}(\rho)\right)\right] = \frac{(v-2F)(\delta-2)}{6\delta-4} + v_H - v_L$$

where the first and third quantity are constant in ρ and

$$\frac{\partial}{\partial\rho}E\left[\left(\hat{p}^{l,1}_{f.m.s.}(\rho), v_H\right)\right] = \frac{2(1-\delta)v}{5\delta-4} < 0$$

where the last inequality follows from the fact that $\delta < \frac{2}{3} < \frac{4}{5}$ whenever we are in that case by Lemma 3.

For direct transfers, we obtain:

$$E\left[(v_H, v_H)\right] = v_H$$
$$E\left[\left(\hat{p}^{l,1}_{d.t.}(\rho), v_H\right)\right] = \frac{v(\delta-2\delta\rho+2\rho-2)-4\delta v_L+2F+4\Phi+2v_L}{6\delta-4} + v_H$$
$$E\left[\left(\hat{p}^{l,2}_{d.t.}(\rho), \hat{p}^{h}_{d.t.}(\rho)\right)\right] = \frac{2F+4\Phi-v}{4\delta-2} + v_H - v_L$$

where the first and third quantity are constant in ρ and

$$\frac{\partial}{\partial\rho}E\left[\left(\hat{p}^{l,1}_{d.t.}(\rho), v_H\right)\right] = \frac{(1-\delta)v}{3\delta-2} < 0$$

where the last inequality follows from the fact that $\delta < \frac{2}{3}$ whenever we are in that case by Lemma 3.

Thus, for all collusive compensation schemes, prices weakly decrease in ρ and $\rho \leq \bar{\rho}$.

Lemma 3 shows that price pair 2 implies $\delta < \frac{2}{3}$ for all three compensation schemes, such that $\frac{\partial}{\partial\rho}E\left[\left(\hat{p}^{l,1}_{k}(\rho), v_H\right)\right] < 0$ always holds.

Second, consider $\rho > \bar{\rho}$.

For baseline tacit collusion, the expected prices and respective derivatives become:

$$E\left[(v_H, v_H)\right] = v_H$$
$$E\left[\left(\tilde{p}^{l,1}_{k}(\rho), v_H\right)\right] = \frac{\delta(F-\rho v-6v_L)+4v_L}{10\delta-8} + v_H$$

$$E\left[\left(\tilde{p}_k^{l,2}(\rho),\tilde{p}_k^{h}(\rho)\right)\right] = \frac{F-\rho v}{6\delta-4}+v_H-v_L$$
$$\frac{\partial}{\partial\rho}E\left[(v_H,v_H)\right]=0$$
$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_k^{l,1}(\rho),v_H\right)\right] = \frac{\delta v}{8-10\delta}>0 \quad \forall\delta\in\left[0,\frac{4}{5}\right)$$

$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_{base}^{l,2}(\rho),\tilde{p}_{base}^{h}(\rho)\right)\right] = \frac{v}{4-6\delta}>0 \quad \forall\delta\in\left[0,\frac{2}{3}\right)$$

For the assignment of future market shares, the expected prices and respective derivatives become:

$$E\left[(v_H,v_H)\right]=v_H$$
$$E\left[\left(\tilde{p}_k^{l,1}(\rho),v_H\right)\right] = \frac{\delta(F-\rho v+10v_H-6v_L)-8v_H+4v_L}{10d-8}$$
$$E\left[\left(\tilde{p}_{f.m.s.}^{l,2}(\rho),\tilde{p}_{f.m.s.}^{h}(\rho)\right)\right] = \frac{(\rho v-F)(\delta-2)}{6\delta-4}+v_H-v_L$$
$$\frac{\partial}{\partial\rho}E\left[(v_H,v_H)\right]=0$$
$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_{f.m.s.}^{l,1}(\rho),v_H\right)\right] = \frac{\delta v}{8-10\delta}>0 \quad \forall\delta\in\left[0,\frac{4}{5}\right)$$
$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_{f.m.s.}^{l,2}(\rho),\tilde{p}_{f.m.s.}^{h}(\rho)\right)\right] = \frac{(2-\delta)v}{4-6\delta}>0 \quad \forall\delta\in\left[0,\frac{2}{3}\right)$$

For collusion via direct transfers the expected prices and respective derivatives become:

$$E\left[(v_H,v_H)\right]=v_H$$
$$E\left[\left(\tilde{p}_{d.t.}^{l,1}(\rho),v_H\right)\right] = \frac{\delta(F-\rho v-4v_L)+2(2\Phi+v_L)}{6d-4}+v_H$$
$$E\left[\left(\tilde{p}_k^{l,2}(\rho),\tilde{p}_k^{h}(\rho)\right)\right] = \frac{F+4\Phi-\rho v}{4d-2}+v_H-v_L$$
$$\frac{\partial}{\partial\rho}E\left[(v_H,v_H)\right]=0$$
$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_{d.t.}^{l,1}(\rho),v_H\right)\right] = \frac{\delta v}{4-6\delta}>0 \quad \forall\delta\in\left[0,\frac{2}{3}\right)$$
$$\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_{d.t.}^{l,2}(\rho),\tilde{p}_{d.t.}^{h}(\rho)\right)\right] = \frac{v}{2-4\delta}>0 \quad \forall\delta\in\left[0,\frac{1}{2}\right)$$

Lemma 3 shows that price pair 2 implies $\delta<\frac{2}{3}$ for all three compensation schemes, such that $\frac{\partial}{\partial\rho}E\left[\left(\hat{p}_k^{l,1}(\rho),v_H\right)\right]>0$ always holds. As price pair 3 requires a weakly lower critical discount factor, $\delta<\frac{2}{3}$ is implied as well, such that $\frac{\partial}{\partial\rho}E\left[\left(\tilde{p}_k^{l,2}(\rho),\tilde{p}_k^{h}(\rho)\right)\right]>0$ holds for baseline tacit collusion and collusion via future market shares. Collusion via direct transfers requires $\delta < \frac{1}{2}$. Hence, we need to show that $\frac{1}{2} > \widetilde{CDF}_{d.t.,2}$:

$$\frac{1}{2} > \widetilde{CDF}_{d.t.,2}(\rho) = \frac{F + 2\Phi - \rho v + v_L}{F - \rho v + 2v_L}$$

always hold. Since $\widetilde{CDF}_{d.t.,2}$ decreases in ρ:

$$\frac{\partial}{\partial \rho}\widetilde{CDF}_{d.t.,2} = \frac{\overbrace{v\,(2\Phi - v_L)}^{<0 \text{ for } \bar{\Phi}}}{(F - \rho v + 2v_L)^2} < 0,$$

the constraint is binding at $\rho = \bar{\rho}$ and $\Phi = \bar{\Phi}$. Since

$$\widetilde{CDF}_{d.t.,2}(\bar{\rho}, \bar{\Phi}) = \frac{1}{2}$$

the constraint is always satisfied. Hence, $\frac{\partial}{\partial \rho} E\left[\left(\tilde{p}^{l,2}_{d.t.}(\rho), \tilde{p}^{h}_{d.t.}(\rho)\right)\right] > 0$ holds as well.

All results and consumer surplus, producer surplus, and total welfare then follow straight away from the respective definitions.

□

Proof of Proposition 5:

Proof. To show that the assignment of future market shares is weakly preferred to tacit collusion in terms of expected profits, consider first how the collusive pricing works in the given model. For monopolistic pricing (price pair 1, (v_H, v_H)), expected profits for all three schemes are the same. Once the exogenous discount factor is below the respective critical factor, firms need to lower prices to maintain collusion. Hence, this proof proceeds in two steps: First, it is shown that the respective $CDFs$ of *f.m.s.* are below those for baseline tacit collusion, i.e. lowering prices is only necessary for a lower δ (see Appendix A for the exact CDF values). Second, it is shown that once prices for both schemes need to be adjusted downwards, expected prices for *f.m.s.* are above prices for tacit collusion.

For $\rho \leq \bar{\rho}$, this becomes:
First, we compare $\widehat{CDF}_{f.m.s.,1}$ and $\widehat{CDF}_{base,1}$:

$$\widehat{CDF}_{f.m.s.,1} - \widehat{CDF}_{base,1} = \frac{2\overbrace{((1-\rho)v - F)}^{\geq 0}\overbrace{(2F - v - 2v_L)}^{<0}}{\underbrace{((2\rho - 1)v + 6v_L)}_{>0}\underbrace{(2F + (4\rho - 3)v + 6v_L)}_{>0}} \leq 0$$

The LHS of the numerator becomes smallest at $\rho = \bar{\rho}$:

$$\left(1 - \frac{v - F}{v}\right) v - F = 0$$

By continuity, any $\rho < \bar{\rho}$ leads to $(1 - \rho)v - F > 0$. As $v_L > v - F$ holds by assumption, it is easy to see that booth terms in the denominator must be negative.

Next, we compare $\widehat{CDF}_{f.m.s.,2}$ and $\widehat{CDF}_{base,2}$:

$$\widehat{CDF}_{f.m.s.,2} - \widehat{CDF}_{base,2} = \frac{\overbrace{2(v-2F)}^{>0}\overbrace{(F-\rho v - v_L)}^{<0}}{3(-2\rho v + v + 4v_L)(F - 3\rho v + v + 3v_L)} < 0$$

Both parts of the denominator becomes smallest at $\rho = \bar{\rho}$. Hence, we ensure that both LHS and RHS exceed 0 at $\rho = \bar{\rho}$:

$$\begin{aligned}
\text{LHS:}\quad \left(1 - 2\frac{v-F}{v}\right) v + 4v_L &> 0 \\
4v_L &> \left(2\frac{v-F}{v} - 1\right) v \\
4v_L &> v - F \\
\text{RHS:}\quad \left(1 - 3\frac{v-F}{v}\right) v + F + 3v_L &> 0 \\
3v_L &> 2(v-F)
\end{aligned}$$

The inequalities for both LHS and RHS always hold. Hence, for $\rho \leq \bar{\rho}$, the CDFs of *f.m.s.* are lower than those of baseline collusion.

For $\rho > \bar{\rho}$:

As $\widetilde{CDF}_{f.m.s.,1} = \widetilde{CDF}_{base,1}$ holds, we directly compare $\widetilde{CDF}_{f.m.s.,2} = \widetilde{CDF}_{base,2}$:

$$\widetilde{CDF}_{f.m.s.,2} - \widetilde{CDF}_{base,2} = \frac{2\overbrace{(\rho v - F)}^{>0}\overbrace{(F - \rho v - v_L)}^{<0}}{3\underbrace{(2F - 2\rho v + 3v_L)}_{>0}\underbrace{(F - \rho v + 4v_L)}_{>0}} < 0$$

Thus, for all 'pairs' of equivalent CDFs holds $CDF_{f.m.s.} \leq CDF_{base}$. By that, every *f.m.s.* price pair is stable within a larger range of discount factors than baseline tacit collusion price pairs.

Second, we compare actual prices of both schemes:

For monopolistic pricing – price pair 1, (v_H, v_H) – prices are the same. Hence, we compare the expected adjusted prices (price pairs 2 and 3):
For $\rho \leq \bar{\rho}$:

$$E\left[\left(\hat{p}^{l,1}_{f.m.s.}(\rho), v_H\right)\right] - E\left[\left(\hat{p}^{l,1}_{base}(\rho), v_H\right)\right] = \frac{(1-\delta)\overbrace{(F + (\rho - 1)v)}^{<0}}{5\delta - 4} > 0\,\forall \delta \in \left[0, \frac{4}{5}\right)$$

$$E\left[\left(\hat{p}^{l,2}_{f.m.s.}(\rho), \hat{p}^{h}_{f.m.s.}(\rho)\right)\right] - E\left[\left(\hat{p}^{l,2}_{base}(\rho), \hat{p}^{h}_{base}(\rho)\right)\right] = \frac{(1-\delta)(v-2F)}{4 - 6\delta} > 0\,\forall \delta \in \left[0, \frac{2}{3}\right)$$

As shown in Lemma 3, $\delta < \frac{2}{3}$ holds, such that in both cases the adjusted f.m.s. price exceeds the adjusted baseline price.

For $\rho > \bar{\rho}$:

$$E\left[\left(\tilde{p}^{l,1}_{f.m.s.}(\rho), v_H\right)\right] - E\left[\left(\tilde{p}^{l,1}_{base}(\rho), v_H\right)\right] = 0$$
$$E\left[\left(\tilde{p}^{l,2}_{f.m.s.}(\rho), \tilde{p}^{h}_{f.m.s.}(\rho)\right)\right] - E\left[\left(\tilde{p}^{l,2}_{base}(\rho), \tilde{p}^{h}_{base}(\rho)\right)\right] = \frac{(1-\delta)(\rho v - F)}{4-6\delta} > 0 \, \forall \delta \in \left[0, \frac{2}{3}\right)$$

As shown in Lemma 3, $\delta < \frac{2}{3}$ also holds for critical discount factors in the $\rho > \bar{\rho}$ case.

Hence, the assignment of future market shares is weakly better than tacit collusion.

□

Proof of Proposition 6:

Proof. For firm i, the expected profit is given by

$$E(\Pi_i) = \left[\frac{1}{2}\Big[\underbrace{\frac{p_h^* + p_l^*}{2}}_{\text{stage 1}} + \underbrace{\frac{1}{2}v - F}_{\text{stage 2}}\Big] - \Phi \cdot \mathbb{1}_{d.t.}\right]\frac{1}{1-\delta} \qquad \text{if } \rho \leq \bar{\rho}\,.$$

As stage 2 profits are independent of ρ, variation can only stem from stage 1. As shown in the previous propositions on the optimal prices (props. 1, 2, 3), there exist three price pairs. To separate them, we use a running index $\{1, 2, 3\}$ to describe the respective pairs in descending order for $\rho \leq \bar{\rho}$. Pair 1 is (v_H, v_H), in which fully monopolistic pricing is possible. Pair 2 is (p'_l, v_H), in which the price is lowered following a low signal $(p'_l < v_H)$ but p_h remains at the monopolistic price v_H. Pair 3 is (p''_l, p''_h), in which both prices p_l and p_h are adjusted downwards.

Assume the setting in which a firm optimally applies price pair 2 within a future market shares scheme and price pair 3 within a direct transfer compensation scheme. For a regime switch from scheme $d.t.$ to $f.m.s.$ being optimal, we need $E[\Pi_{d.t.}(\rho)] > E[\Pi_{f.m.s.}(\rho)]$. To sustain the choice of price pair 3 within the $f.m.s.$ scheme, $\delta < \widehat{CDFf.m.s.}, 2$ must be satisfied. Second, $\widehat{CDF}_{d.t.,1} \geq \delta \geq \widehat{CDF}_{d.t.,2}$ must hold to ensure the firm optimally applies price pair 2 within the $d.t.$ compensation scheme. For a regime switch expected profits necessarily need to intersect. Hence, we define the difference of the expected profits functions for $\rho \leq \bar{\rho}$ as:

$$\hat{\Delta}(\rho, \Phi) = E[\Pi_{f.m.s.} - \Pi_{d.t.}(\rho, \Phi)] = \frac{F - 6\Phi - \rho v + v_L}{2(3\delta - 2)} \tag{12}$$

In a very first step we evaluate expected profits for stage 1 at the lower bound of ρ, i.e. $\rho = \frac{1}{2}$ and for zero punishment ($\Phi = 0$). It has to be shown that $E[\Pi_{d.t.}(\frac{1}{2}, 0)] > E[\Pi_{f.m.s.}(\frac{1}{2})]$ holds to ensure that without prediction accuracy, direct transfers lead to higher expected profits than assigning future market shares. For $\Phi = 0$, the difference in eq. (12) must become negative, i.e.

$$\hat{\Delta}(\rho, 0) = \frac{F - \rho v + v_L}{2(3\delta - 2)} < 0$$
$$\Leftrightarrow v_L > \rho v - F \text{ for } \delta < \frac{2}{3}$$
$$\Leftrightarrow v_L > \frac{v}{2} - F,$$

which holds by assumption (see eq. 1). The restriction on $\delta < \frac{2}{3}$ is not binding as the upper bound on the exogenous discount factor, $\widehat{CDF}_{d.t.,1}$ is below $\frac{2}{3}$:

$$\widehat{CDF}_{d.t.,1} = \frac{2(F + 2\Phi + (\rho - 1)v + v_L)}{(2\rho - 1)v + 4v_L} \overset{!}{<} \frac{2}{3}$$

$\widehat{CDF}_{d.t.,1}(\rho, \Phi)$ becomes largest at $\Phi = \bar{\Phi}$ and $\rho = \bar{\rho}$. Plugging in leads to

$$\widehat{CDF}_{d.t.,1}(\bar{\rho}, \bar{\Phi}) = \frac{2\Big(F + 2\overbrace{\frac{v - 2F}{4}}^{\bar{\Phi}} + \Big(\overbrace{\frac{v - F}{v}}^{\bar{\rho}} - 1\Big)v + v_L\Big)}{\Big(2\underbrace{\frac{v - F}{v}}_{\bar{\rho}} - 1\Big)v + 4v_L} < \frac{2}{3}$$
$$\frac{2F - v - 2v_L}{2F - v - 4v_L} < \frac{2}{3}$$
$$2F - v + 2v_L > 0$$
$$v_L > \frac{v}{2} - F,$$

which holds by assumption.

Having shown that the difference in eq. (12) is indeed negative, such that $d.t.$ is more attractive for zero punishment and no predictive ability, we can solve for the value $\hat{\Phi} > 0$ that allows for a regime switch. Setting $\hat{\Delta}(\rho, \Phi) = 0$, we obtain

$$\frac{F - 6\Phi - \rho v + v_L}{2(3\delta - 2)} \overset{!}{=} 0$$
$$\Leftrightarrow \hat{\Phi}(\rho) = \frac{1}{6}(F - \rho v + v_L)$$

Plugging the interval boundaries of ρ into $\hat{\Phi}(\rho)$, we get

$$\hat{\Phi}\left(\frac{1}{2}\right) = \frac{1}{6}(F - \frac{v}{2} + v_L) \equiv \Phi''$$
$$\hat{\Phi}(\bar{\rho}) = \frac{1}{6}(2F - v + v_L) \equiv \Phi'$$

This, however, has to satisfy the conditions on Φ as shown in Lemma 2, i.e. $\hat{\Phi}(\rho) \in$

$(0, \bar{\Phi})$. As $2F - v < \frac{v}{2} - F$, Φ'' is the binding constraint at that point. Hence, we need

$$\frac{1}{6}(F - \frac{v}{2} + v_L) \leq \frac{1}{4}(v - 2F)$$
$$\Leftrightarrow v_L \leq 4\left(\frac{v}{2} - F\right),$$

which is an upper bound for v_L we consider as not too restrictive.

Consider first Φ''.
For Φ'' to cause a regime switch, it must be ensured that the boundaries for δ are never violated. Hence, we have to evaluate, whether $\widehat{CDF}_{d.t.,1}(\Phi'') \geq \widehat{CDF}_{d.t.,2}(\Phi'')$ and whether $\widehat{CDF}_{f.m.s.,2} \geq \widehat{CDF}_{d.t.,2}(\Phi'')$. First, we evaluate, whether $\widehat{CDF}_{f.m.s.,2} \geq \widehat{CDF}_{d.t.,2}$ holds. For $\rho = \frac{1}{2}$, this becomes

$$\widehat{CDF}^{h,a}_{f.m.s.}\left(\frac{1}{2}\right) - \widehat{CDF}^{h,a}_{d.t.}\left(\frac{1}{2}, \Phi''\right) = \frac{1}{3}\Big(\underbrace{\frac{v - 2F}{v_L}}_{>0} + \underbrace{\frac{8F - 4v}{2F - v + 6v_L}}_{<0}\Big)$$
$$\frac{v - 2F}{v_L} \overset{!}{>} \frac{4(v - 2F)}{2F - v + 6v_L}$$
$$2F - v + 6v_L > 4v_L$$
$$v_L > \frac{v}{2} - F,$$

which holds by assumption. Second, we evaluate whether the second condition on δ is satisfiable. Hence, we evaluate whether

$$\widehat{CDF}_{d.t.,1}(\rho, \Phi'') \geq \widehat{CDF}_{d.t.,2}(\rho, \Phi'')$$
$$\frac{2(F + 2\Phi + (\rho - 1)v + v_L)}{(2\rho - 1)v + 4v_L} \geq \frac{F + 2\Phi - \rho v + v_L}{-2\rho v + v + 2v_L}$$
$$\Leftrightarrow \frac{2(F + 2\Phi + (\rho - 1)v + v_L)}{(2\rho - 1)v + 4v_L} - \frac{F + 2\Phi - \rho v + v_L}{-2\rho v + v + 2v_L} \geq 0$$

Again, we evaluate the difference at $\rho = \frac{1}{2}$.

$$\widehat{CDF}_{d.t.,1}\left(\frac{1}{2}, \Phi''\right) - \widehat{CDF}_{d.t.,2}\left(\frac{1}{2}, \Phi''\right)$$
$$= \frac{2F - v + 2v_L}{3v_L} - \frac{2F - v + 2v_L}{3v_L} = 0$$

However, differentiating both terms by ρ leads to

$$\frac{8v(-2F + v + v_L)}{3(-2\rho v + v - 4v_L)^2} > \frac{2v(4F - 2v + v_L)}{3(-2\rho v + v + 2v_L)^2}$$

For $\rho \in \left(\frac{1}{2}, \bar{\rho}\right]$ and the usual parameter restrictions, it can be shown that the change in $\widehat{CDF}_{d.t.,1}\left(\frac{1}{2}, \Phi''\right)$ exceeds the change in $\widehat{CDF}_{d.t.,2}\left(\frac{1}{2}, \Phi''\right)$, such that the difference is weakly larger than zero and satisfies the condition imposed.

Consider now the case of Φ', i.e. a regime switch happens at $\rho_1 \lesssim \bar{\rho}$ for $\Phi \approx \Phi'$. Again, we evaluate, whether $\widehat{CDF}_{f.m.s.,2} \geq \widehat{CDF}_{d.t.,2}$ holds. For $\Phi = \Phi'$ and $\rho = \bar{\rho}$, this can be expressed as

$$\widehat{CDF}_{f.m.s.,2} - \widehat{CDF}_{d.t.,2}(\bar{\rho}, \Phi') > 0$$
$$\Leftrightarrow \frac{2\overbrace{(v - 2F)}^{>0}\overbrace{(2F - v + v_L)}^{>0}}{3\underbrace{(2F - v + 2v_L)}_{>0}\underbrace{(4F - 2v + 3v_L)}_{>0}} > 0$$

Second, we check for Φ', whether also $\widehat{CDF}_{d.t.,1} \geq \widehat{CDF}^{h,a}_{d.t.}$ holds.To fo this, we evaluate again, whether the difference of them is positive:

$$\widehat{CDF}_{d.t.,1}(\Phi', \bar{\rho}) - \widehat{CDF}_{d.t.,2}(\Phi', \bar{\rho}) > 0$$
$$\Leftrightarrow \frac{2\overbrace{(v - 2F)^2}^{>0}}{\underbrace{(v - 2F + 4v_L)(2F - v + 2v_L)}_{>0}} > 0$$

Eventually, we can solve for ρ_1 causing the regime switch. For that, we use eq. (12), set it equal to zero, set $\Phi = \hat{\Phi} \in \{\Phi', \Phi''\}$ and solve for ρ:

$$\frac{F - 6\hat{\Phi} - \rho v + v_L}{2(3\delta - 2)} \stackrel{!}{=} 0$$
$$\Leftrightarrow \rho = \frac{F - 6\hat{\Phi} + v_L}{v} \equiv \rho_1$$

This leads to a non-empty set of δ that satisfies the regime switch conditions, namely $\delta \in \left(\widehat{CDF}_{d.t.,2}(\rho, \hat{\Phi}), \min\left\{\widehat{CDF}_{d.t.,1}(\rho, \hat{\Phi}), \widehat{CDF}_{f.m.s.,2}(\rho)\right\}\right]$ with $\hat{\Phi} \in [\Phi'', \Phi']$. □

Proof of Proposition 7:

Proof. Suppose δ is such that for both *f.m.s.* and *d.t.*, $p_h = v_H$ is sustainable, but $p_l = v_H$ is not sustainable, which requires that $p_l < v_H$ is set in the most profitable equilibrium. This requires that $\widetilde{CDF}_{f.m.s.,1} > \delta > \widetilde{CDF}_{f.m.s.,2}$ as well as $\widetilde{CDF}_{d.t.,1} > \delta > \widetilde{CDF}_{d.t.,2}$

Conditional on this case distinction, we can define a function $\tilde{\Delta}(\rho; \Phi)$ as

$$\tilde{\Delta}(\rho; \Phi) = E[\Pi_{f.m.s.}(\rho)] - E[\Pi_{d.t.}(\rho, \Phi)] = \frac{\delta(v_L + \rho v + 30\Phi - F) - 24\Phi}{(22 - 15\delta)\delta - 8}$$

where we are interested in parameter values δ and Φ such that $\tilde{\Delta}(\bar{\rho}; \Phi) > 0$ and $\Delta(1; \Phi) < 0$. We can readily observe that Δ increases linearly in Φ and decreases linearly in ρ. Note that, clearly, for $\Phi = 0$ we always have $\Delta < 0$ for any ρ.

$$\frac{\partial}{\partial\rho}\tilde{\Delta}(\rho;\Phi) = \underbrace{\frac{\delta v}{(22-15\delta)\delta - 8}}_{<0\,\forall\,\delta<\frac{2}{3}} < 0$$

$$\frac{\partial}{\partial\Phi}\tilde{\Delta}(\rho;\Phi) = \frac{6}{2-3\delta} > 0 \quad \forall\,\delta < \frac{2}{3}$$

$$\tilde{\Delta}(\rho;0) = \frac{\delta(v_L + \rho v - F)}{(22-15\delta)\delta - 8} < 0.$$

Given this, we can simply find a value Φ such that both regimes are equally profitable for a given level of signal precision ρ. So we define Φ''' as the solution to

$$\tilde{\Delta}(\bar{\rho};\Phi''') = 0$$

Solving for Φ, this leads to:

$$\Phi'''(\rho) = \kappa\frac{v_L + \rho v - F}{6} \quad \text{where } \kappa = \frac{\delta}{4-5\delta} < 1\,\forall\,\delta \in \left[0, \frac{4}{5}\right).$$

Hence, Φ''', evaluated at $\rho = \bar{\rho}$, becomes

$$\Phi'''(\bar{\rho}) = \kappa\frac{v_L + v - 2F}{6}.$$

We now establish that a regime switch arbitrarily close to $\bar{\rho}$, using $\Phi = \Phi'''$, can always be found. This requires showing that $\bar{\Phi} > \Phi''' > 0$, and that moreover we remain in the relevant case for both *f.m.s.* and *d.t.* . First, note that $\Phi''' > 0$ holds whenever $0 < \kappa < 1$, since $\rho v > F\ \forall\rho$ holds. As stated in Lemma 3, price pair 2 implies $\delta < \frac{2}{3}$, such that $\Phi_1 > 0$ is always satisfied. Second, we need to ensure that $\Phi''' \leq \bar{\Phi}$ is satisfied. For $\rho > \bar{\rho}$, this becomes:

$$\frac{\rho v - F}{4} \geq \kappa\frac{v_L + \rho v - F}{6}$$
$$\Leftrightarrow v_L \leq \frac{3-2\kappa}{2\kappa}(\rho v - F)$$

To ensure this condition on v_L is satisfied, we minimize the RHS by setting $\rho = \bar{\rho}$:

$$v_L \leq \frac{3-2\kappa}{2\kappa}(v - 2F), \text{ which holds if } \frac{3-2\kappa}{2\kappa} \geq 2$$
$$2 \leq \frac{3-2\kappa}{2\kappa} \Leftrightarrow \kappa \leq 1,$$

which holds by construction, see above.

Next, we show that we can always pick a δ such that we are indeed in the relevant cases for both compensation schemes. For assigning future market shares, this is the case as long as

$$\widetilde{CDF}_{f.m.s.,1}(\bar{\rho}) = \frac{4v_L}{v + 6v_L - 2F} > \widetilde{CDF}_{f.m.s.,2}(\bar{\rho}) = \frac{2(2F - v + v_L)}{4F - 2v + 3v_L}$$

which can be written as

$$2(v - 2F)(vL + v - 2F) > 0$$

which always holds since both terms in the last inequality are positive by assumption. For direct transfer, a relevant δ is feasible as long as

$$\widetilde{CDF}_{d.t.,1}(\bar{\rho}, \Phi''') = \frac{2\left(\frac{1}{3}\kappa(v - 2F + v_L) + v_L\right)}{v - 2F + 4v_L} > \widetilde{CDF}_{d.t.,2}(\bar{\rho}, \Phi''') = \frac{(v - 2F)(\kappa - 3) + (\kappa + 3)v_L}{6F - 3v + 6v_L}$$

which can be written as

$$\frac{\overbrace{(\kappa - 1)}^{<0}\overbrace{(2F - v)}^{<0}\overbrace{(2F - v - v_L)}^{<0}}{\underbrace{(2F - v - 4v_L)}_{<0}\underbrace{(2F - v + 2v_L)}_{>0}} > 0$$

which always holds by assumption.

Finally, it has to be ensured that the relevant CDF intervals overlap such that there is an admissible δ in between that guarantees the relevant cases for both schemes. This holds as long as

$$\widetilde{CDF}_{d.t.,1}(\bar{\rho}, \Phi''') = \frac{2\left(\frac{1}{3}\kappa(v - 2F + v_L) + v_L\right)}{v - 2F + 4v_L} > \widetilde{CDF}_{f.m.s.,2}(\bar{\rho}) = \frac{2(2F - v + v_L)}{4F - 2v + 3v_L}$$

which holds as long as $v_L > \frac{2}{3}(v - 2F)$, which is satisfied by assumption. By continuity, any value for Φ arbitrarily close to Φ''' leads to an interior regime switch between the expected profits of *f.m.s.* and *d.t.* as stated in the proposition. Equivalently to the derivation of Φ''', we can set $\Delta(\rho, \Phi) = 0$ and solve for ρ:

$$\Delta(\rho, \Phi) = \frac{\delta(v_L + \rho v + 30\Phi - F) - 24\Phi}{22\delta - 15\delta^2 - 8} = 0$$
$$\Leftrightarrow \rho(\Phi) = \frac{\delta(F - 30\Phi - v_L) + 24\Phi}{\delta v} \equiv \rho_2.$$

$\rho_2 \in (\bar{\rho}, 1]$ captures the point of the regime switch. The regime switch happens for a punishment term $\hat{\Phi} \approx \Phi'''$ and an exogenous discount factor δ in the non-empty set $\delta \in \left[\max\left\{\widehat{CDF}_{d.t.,2}(\rho, \hat{\Phi}), \widehat{CDF}_{f.m.s.,2}(\rho)\right\}, \min\left\{\widehat{CDF}_{d.t.,1}(\rho, \hat{\Phi}), \widehat{CDF}_{f.m.s.,1}(\rho)\right\}\right)$. □