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Abstract

We study the welfare impact of rules of origin in free trade agreements where final-good producers source customized inputs from suppliers within the trading bloc. We employ a property-rights framework that features hold-up problems in suppliers' decisions to invest. Moreover, underinvestment is more severe for higher productivity firms. In this context, a rule of origin offers preferred market access for final goods if a sufficiently high fraction of inputs used in the production process is sourced within the trading bloc. Such a rule alters behavior for only a subset of suppliers, as some (very-high-productivity) suppliers comply with the rule in an unconstrained way and some (very-low-productivity) suppliers choose not to comply. For those suppliers it does affect, the rule increases investment, but it also induces excessive sourcing (for given investment) within the trading bloc. From a social standpoint, it is best to have a rule that affects high-productivity suppliers, since their original underinvestment problem is more severe. This calls for a relatively *strict* rule, which requires a high share of within-bloc inputs. The reason is that the marginal net welfare gain from tightening the rule increases with productivity. Therefore, when industry productivity is high, a strict rule of origin is socially desirable; in contrast, when industry productivity is low, no rule of origin is likely to help. Regardless of the case, a sufficiently strict rule ensures welfare gains.

JEL Classification: F13; F15; L22; D23.
Keywords: Hold-up problem; Sourcing; Incomplete contracts; Regionalism.

†Ornelas: Sao Paulo School of Economics-FGV, CEPR, CESifo and CEP-LSE; eaornelas@gmail.com. Turner: Department of Economics, Terry College of Business, University of Georgia, Athens, GA 30602; jlturner@uga.edu. We thank Bruno Daré for competent research assistance and thank Bernardo Guimaraes, Kamal Saggi and seminar participants at the 2021 NBER conference "Trade and Trade Policy in the 21st Century," the Sao Paulo School of Economics-FGV and the University of Georgia for helpful comments.

1 Introduction

Since the conclusion of the Uruguay Round of multilateral negotiations in 1994, the main mode of trade liberalization has been through the formation of free trade agreements (FTAs). In tandem with this phenomenon, rules of origin (ROOs) have multiplied. Under these rules, goods whose inputs come substantially from within-bloc countries are traded freely, but otherwise face tariffs. Their formal goal is to prevent the trans-shipment of imported goods from low-tariff to high-tariff countries within the same trading bloc, but the stringency of the rules vary substantially across agreements and across products within agreements. Generally, economists take a dim view of ROOs, as they tend to lower welfare in competitive models (Grossman, 1981; Krishna, 2006). The typical view is that ROOs are distortionary because they "prevent final good producers from choosing the most efficient input suppliers around the world" (Conconi et al., 2018, p. 2336). Accordingly, the empirical literature customarily interprets findings that ROOs have a negative effect on imports from third countries as evidence of welfare-reducing trade diversion.

While this is true in some contexts, it may not be when within-FTA sourcing is inefficiently low absent ROOs. That is the case when firms need to make relationship-specific, unrecoverable investments that cannot be fully contracted upon, a situation that typically arises when inputs are customized. And in the modern era of global sourcing, customization is indeed prevalent in many industries. This is particularly common in the context of FTAs, which both promote and are promoted by global/regional value chains (Baldwin, 2011; Johnson and Noguera, 2017; Ruta, 2017). In such cases, terms of trade are determined via bargaining and input suppliers may face hold-up problems. But then, if a ROO induces changes in input mixes, there is not necessarily a welfare loss, and there may be welfare gains. We capture and study these effects in this paper, where we provide the first analysis of ROOs using a property-rights model.

Building on Ornelas, Turner and Bickwit (2021) – henceforth OTB – we consider an environment where specialized input suppliers with heterogeneous productivity form vertical chains with producers of final goods. Both firms are in countries that belong to the same FTA. Within each vertical chain, the supplier invests in technology (marginal cost curve reduction) prior to bargaining with the producer over inputs; those investments are non-contractible. The producer also has the option to purchase generic inputs from a competitive world market, and the model is set up so that the producer always sources a mix of specialized and generic inputs in equilibrium. Under free trade, investments are inefficiently low due to the possibility of hold up. Crucially, because investment affects more units of inputs in more productive firms, the underinvestment problem is more severe for higher productivity firms.

To this basic setup we add a rule such that the final goods produced by a vertical chain enjoy preferred market access as long as the fraction of inputs produced within the bloc exceeds a prespecified level.[1] We find that this rule may increase or decrease aggregate welfare, but that welfare is more likely to rise when it is *stricter*. The reason is twofold. First, a rule of origin will typically affect choices made by just a subset of suppliers, and a stricter rule targets higher productivity firms (under a strict rule, those with low productivity choose to forgo the benefits from compliance). Second, for such high-productivity suppliers, a relatively strict rule is desirable because the marginal net welfare gain from tightening the rule increases with productivity. This happens because the extra investment that the rule induces is more socially beneficial when it comes from the suppliers more affected by the original underinvestment problem – i.e., those with higher productivity.

The heterogeneous incidence echoes both early and new work on ROOs (e.g., Grossman, 1981; Head et al., 2021), which show that a ROO matters only for particular levels of the supply curve and there are three cases to consider. In our model, we can classify the cases according to supplier productivity. The highest-productivity suppliers have the lowest marginal cost curves and produce very high levels of intermediate inputs. Their vertical chains comply with the ROO without altering investments, so the rule has no effect on their behavior. At the opposite extreme, the lowest-productivity suppliers have the highest marginal cost curves and produce very low levels of intermediate inputs. Because compliance would be too costly for them, their vertical chains are unwilling to comply with the ROO, so the rule does not affect their sourcing behavior either. Finally, vertical chains with suppliers within an intermediate productivity range find it optimal to source extra specialized inputs

[1] There are different ways to determine "origin" (see the detailed discussion by Inama, 2009), but Kniahin and Melo (2022), studying 370 trade agreements, find that the most common definition is a minimal regional value content.

within the FTA to gain preferential access for final goods. Naturally, this split is mediated by the level of the final-good tariff in the FTA partner, which determines the extent of the gains from compliance with the ROO.[2]

For the suppliers that comply in a constrained way, we say the rule is *binding*. The extra inputs serve as a commitment device for the supplier to invest more. This additional investment tends to improve efficiency. However, for given investment, there will also be excessive sourcing within the FTA – the force on which most of the literature has concentrated. A socially optimal rule should trade off the benefits from mitigating the hold-up problem against the costs due to excessive within-FTA sourcing for the population of suppliers. Because the marginal gain from tightening the rule is higher, and the corresponding marginal cost is lower, for high-productivity firms, a hypothetical optimal rule set at the firm level would be increasing in productivity. Thus, a relatively strict rule is both binding for more productive suppliers and is particularly beneficial (from a social standpoint) when it affects their behavior.

Considering the whole distribution of suppliers, analogous results obtain. A stricter ROO is more likely to generate a positive welfare effect for all affected suppliers because it binds for more productive suppliers, and the potential welfare gains are higher when those suppliers are affected. Now, whether aggregate welfare rises or falls with the stringency of a ROO does depend on the specific distribution of supplier productivity. Nevertheless, a very lenient rule is likely to be harmful, because it affects the behavior primarily of firms whose original underinvestment problem is mild and whose excessive FTA sourcing due to the rule would be severe. In contrast, a sufficiently strict rule *ensures* a welfare gain, as it affects only the behavior of firms whose original underinvestment problem is severe and whose excessive FTA sourcing due to the rule would be mild. We highlight this with an example where productivity follows a Pareto(k) distribution. For any k, the welfare effect is negative for low r and positive for high r.

An external tariff on intermediate inputs alters the welfare consequences of a ROO.

[2] Most obviously, if that tariff is close to zero, any ROO will be innocuous because vertical chains will have nothing to gain by altering their sourcing choices. In contrast, if the final-good tariff is huge, then every supplier complies regardless of the rule. The tripartite case arises when the final-good tariff is not extreme.

Without an external tariff on inputs, we show that a sufficiently strict ROO will raise welfare for *any* distribution of suppliers. But a positive input tariff increases investment by all suppliers, leading some to overinvest. An even tighter ROO is then needed to boost welfare, as it is necessary to tailor the ROO to affect higher-productivity suppliers, who are still underinvesting. However, when the external tariff on inputs is sufficiently high, all suppliers overinvest and any binding ROO would only worsen welfare, because it would be ill-suited to address overinvestment problems.

If we consider limiting cases of our environment, then it is possible to find other situations where a ROO always worsens welfare. First, if there is no investment decision by the supplier, then there is no underinvestment problem and the ROO leads just to trade diversion. This is the case usually considered in the literature, which justifies the interpretation of many empirical analyses. Second, if suppliers have full bargaining ability, then investments are efficient without external tariffs and ROOs, and any rule will lead to trade diversion and excessive investments. Third, if suppliers have no bargaining ability, then ROOs do not affect investments and, again, yield only trade diversion. Put together, these cases make clear that ROOs can be beneficial only if (1) hold-up problems are present and (2) investment responds to policy.

The paper is organized as follows. After discussing the related literature in the next subsection, we set up the basic model in section 2. In section 3, we study the choice of input mix for given level of investment. We then move to understand firms' choice of investment in section 4. This allows us to analyze the welfare impacts of ROOs in section 5. In section 6, we extend the analysis to the case where there is a strictly positive external tariff on inputs, and in section 7 we discuss modeling alternatives and extensions of our model. We conclude in section 8.

1.1 Related Literature

The theoretical literature on ROOs goes back to Grossman (1981), who studies the consequences of local content requirements rules, including the case where access to preferential treatment for exports requires a minimum level of domestic value added. For that situation, in a competitive setting, he identifies the three cases of non-compliance, unconstrained compliance and constrained compliance, as we also find in our environment. Krishna (2006) offers a similar perspective. Also in a competitive model, Falvey and Reed (1998) show how ROOs can distort allocative efficiency and underline the many ways in which "origin" can be defined. In a recent paper, Chung and Perroni (2021) depart from the competitive benchmark to study the effects of ROOs when markets are oligopolistic and show that stricter ROOs tend to lead to higher prices for intermediate goods.

Recently, Head et al. (2021) evaluate the effect of NAFTA's ROOs for auto parts. They use a structural model that yields a "ROO Laffer curve" with respect to the share of intra-bloc inputs used. The curve reflects the tripartite separation of firms regarding compliance depending on their cost structures. Intuitively, at low levels, tightening the rule induces more within-bloc sourcing, but eventually the effect reverses, as more firms choose to not comply under a strict rule. A similar pattern emerges in our model as well.

Naturally, the motives behind ROOs are diverse, and raising aggregate welfare may be far from the main goal in some circumstances. In particular, as Krueger (1999) forcefully notes, ROOs can be imposed for protectionist reasons and constitute a source of economic inefficiency in FTAs. We do not study how or why ROOs are actually chosen, but we show that a relatively strict rule can be welfare improving, regardless of its motivations.[3]

Empirically, several papers evaluate the impact of ROOs on trade flows from a reduced-form perspective (see, for example, the studies in Cadot et al., 2006). In contrast, Cherkashin et al. (2015) structurally estimate an heterogeneous-firm model with firm-market specific demand shocks to study the impact of changing the costs of meeting ROOs faced by Bangladeshi exports in apparel. They find that fewer requirements for meeting ROOs are associated with greater exports and more entry in the long run. Conconi et al. (2018) provide an insightful product-level analysis of NAFTA, finding that ROOs induce a relocation of sourcing from outside to inside the bloc.[4] That type of result is often taken as evidence that ROOs are

[3]Tsirekidze (2021), building on the theoretical analysis of FTA formation by Saggi and Yildiz (2010), shows that ROOs could be useful to facilitate the achievement of global free trade. Although in a very distinct context, our analysis connects with Tsirekidze's in that he also uncovers a potentially beneficial role for ROOs, in contrast with the largely negative view from most of the literature.

[4]Sytsma (2022) provides a related analysis for the impact of relaxing ROOs in the European Union's Generalized System of Preferences, finding similar results. Bombarda and Gamberoni (2013) show how diagonal cumulation slackens the restrictiveness of ROOs.

distortionary. Yet here we show that greater within-FTA sourcing need not be distortionary and can be welfare enhancing. Thus, an implication of our analysis is that market structure and firm organization must be considered in the interpretation of empirical results about the effects of ROOs.

Other empirical papers document the incomplete use of preferences in FTAs, and often associate that to poorly designed ROOs. Recently, Crivelli et al. (2021) reveal that preferences in the European Union FTAs, although widely used, are still far from being used by all potential beneficiaries. They provide a detailed account of product-specific ROOs that are too strict to be useful by some firms. Our analysis makes clear that partial utilization is generally inevitable, as firms with different levels of productivity will have different incentives to comply. In particular, the vertical chains with *low-productivity* suppliers in the FTA will choose not to comply. From a social welfare perspective, this tends to be beneficial, because they are the least affected by hold-up problems. Related to this point, Krishna et al. (2021) show how documentation costs to satisfy ROOs prevent the full use of preferences in FTAs. Interestingly, they find that firms learn how to satisfy ROOs as they accumulate experience exporting a product to an FTA partner, implying that the fixed cost of documentation falls over time at the firm level. We do not consider fixed costs to use ROOs in our main analysis, but we discuss how they could be incorporated in section 7.

Finally, our modeling approach is related to that of other papers studying trade in intermediates under incomplete contracts (e.g., Antràs and Helpman, 2004; Ornelas and Turner, 2008, 2012), especially those in the context of trade agreements (e.g., Antràs and Staiger, 2012). As indicated above, it is methodologically closest to OTB. The main difference is the focus. In OTB we study the welfare effects of an FTA due to within-bloc reduction of input tariffs. Considering the institutional design of FTAs around the world, here we take the next logical step. Given free trade in inputs within the FTA, we analyze how free trade in final goods within the bloc, when mediated by ROOs – as in all existing FTAs – affect the desirability of the trading bloc. Thus, this paper helps toward a fuller understanding of the welfare implications of free trade agreements.[5]

[5]For a recent review of models of international trade featuring incomplete contracts, bargaining and specialized components in the context of global value chains, see Antràs (2020).

2 The Model

We build on OTB's model, with the key departure being the introduction of a ROO requirement to make the trade of final goods tariff-free within an FTA. Except for that, we keep the final good's market as simple as possible, focusing instead on the inputs market. As we will see, the ROO alters sourcing decisions which, in turn, alter investment decisions. This has implications for the incidence of hold-up problems, for the efficiency of sourcing and, therefore, for the welfare consequences of FTAs.

There is a final good x whose production is carried out by final-good producers (F) – or simply *producers*, for brevity – located in the *Home* country. Those firms transform intermediate inputs into good x. Consumption of good x increases consumers' utility at a decreasing rate. Its world price is p_w^x. We consider that *Home* is small in world markets, and therefore its producers take p_w^x as given. Under free trade, *Home* is an exporter of x. There is also an homogeneous numéraire good y that enters consumers' utility function linearly. Thus, if they purchase any y, extra income will be directed to the consumption of that good. We assume relative prices are such that consumers always purchase some y. Production of one unit of y requires one unit of labor, the market for y is perfectly competitive and y is traded freely. This sets the wage rate in the economy to unity and effectively shuts down general equilibrium effects.

Home is in a free trade agreement with *Foreign*, which is an importer of x. This means that trade between them is free provided that rules of origin, when present, are respected. The FTA's ROO requires that fraction $r \in (0, 1)$ of the inputs used to produce x come from within the FTA. If final-good producers fail to comply with the ROO, they must pay *Foreign*'s MFN specific tariff $\tau > 0$ on final goods sold to *Foreign*. Hence, compliance yields savings of τ times *Home*'s exports of x to *Foreign*.

When sourcing, each producer may purchase generic inputs z available in the world market (ROW) and/or specialized inputs q from a *Home* supplier (S).[6] Generic inputs are priced in the world market at p_w^z, and decisions in the FTA do not affect p_w^z. In our baseline

[6]It would make no difference for the analysis if specialized suppliers were located in *Home*'s FTA partner, *Foreign*, provided that they could sell to producers in *Home* without incurring tariffs.

model, there are no tariffs assessed on z. We relax this assumption in section 6.

As in Grossman (1981), we assume that intermediate goods are perfect substitutes. We define units so that one unit of generic input and one of specialized input have the same revenue-generating value for a producer. Under this normalization, F's revenue depends on just the total number of intermediate inputs he purchases, Q, and not its composition. Note that the ROO requires $q \geq rQ$.

Now, to acquire customized inputs, F and S must first specialize their technologies toward each other, constituting a *vertical chain*. This implies that a producer purchases specialized inputs from only one supplier. All producers are identical, whereas each supplier is identified by ω, an heterogeneity parameter that indexes (the inverse of) her productivity. The distribution of suppliers follows a continuous and strictly increasing distribution $G(\omega)$, with an associated density $g(\omega)$, where ω lies on $[0, p_w^z]$.[7]

Once F and S are specialized toward each other, S makes a non-contractible relationship-specific investment that lowers her marginal cost. The investment is observed by both F and S, but is not verifiable in court. The analysis would remain analogous if the producer also made an analogous ex-ante investment.

After investment is sunk, the firms bargain over how much to trade and at what price. The specialized inputs are not traded on an open market, and have no scrap value. Furthermore, the parties cannot use contracts to affect their trading decisions.[8] If bargaining breaks down, S produces the numéraire good and earns zero (ex post) profit, while F purchases only generic inputs. If bargaining is successful, F imports z from ROW and purchases q from S.

The timing of events within each vertical chain is therefore as follows. First, (i) S makes

[7] Here we take the FTA and the structure of matching as given, so that we can focus on the first-order welfare effects of ROOs. Our specification is consistent with the "large natural trading partner" case from OTB, where all specialized suppliers are in the same country and changes to FTA characteristics do not affect matches. Note that OTB also fully characterizes how an FTA alters the matching equilibrium for cases where suppliers are located in multiple countries. In section 7.4, we discuss how endogenizing matching would affect our results. See Grossman and Helpman (2021) for a detailed analysis of how discriminatory tariffs affect the structure of global value chains in a setting where matching and bargaining take center stage.

[8] This is the same approach used by Antràs and Staiger (2012), among others analyzing related environments. It can be formally justified if, for example, quality were not verifiable in a court and the supplier could produce either high-quality or low-quality specialized inputs, with low-quality inputs entailing a negligible production cost for the supplier but being useless to the producer.

an irreversible relationship-specific investment. Once the investment is sunk, (*ii*) F and S bargain over price and quantity of q. If bargaining is successful, production and trade of q takes place; otherwise, $q = 0$ and S produces the numéraire good. Subsequently, (*iii*) F purchases z. Then, (*iv*) production occurs and final good x is sold, with payments dependent on whether the ROO is satisfied. We solve the game by backward induction, from the perspective of a single vertical chain. As in OTB, we use the term *Y-chain* when referring to the entire supply chain, as distinct from the F-S vertical chain.[9]

2.1 Cost and Production Functions

To produce q customized inputs, which requires only labor, S incurs cost $C(q, i, \omega)$, where i is the level of her relationship-specific investment. Investment reduces both cost and marginal cost of production, while ω has the opposite effects: $C_i < 0, C_{qi} < 0; C_\omega > 0, C_{q\omega} > 0$. The marginal cost curve is positively sloped ($C_{qq} > 0$) and the cost of investment, $I(i)$, is increasing and convex ($I' > 0, I'' > 0$). We assume a linear-quadratic specification, so that third derivatives of functions $C(.)$ and $I(.)$ are nil.

While this set of assumptions is sufficient for some results, to generate closed-form analytical solutions we adopt the following specific functional forms:

$$\begin{aligned} C(q, i, \omega) &= (\omega - bi)q + \tfrac{c}{2}q^2, \\ I(i) &= i^2. \end{aligned} \tag{1}$$

Here, ω is the intercept of the marginal cost curve; c is the slope of the marginal cost curve; and b represents the effectiveness of investment in reducing production costs. We assume that $2c > b^2$ to ensure that the choice of investment is always finite.

We focus on the case where F purchases both generic and specialized inputs in equilibrium.[10] This will always happen provided that the marginal cost of the most productive supplier ($\omega = 0$) is high enough so that F wants to purchase some generic inputs even when matched with that best supplier. This simplifies the analysis significantly, but the important

[9] We use *Y-chain* because visually the supply chain resembles a Y: it includes two sources of upstream supply and one downstream final-goods producer.

[10] This is in line with the findings of Boehm and Oberfield (2020), who document that mixing customized and standardized inputs is a common practice for Indian manufacturing plants.

requisite is that the producer must have the option of buying generic inputs when negotiating with his specialized supplier, because that establishes the threat point in the bargaining process.

We adopt the following specification for the production function of x:

$$x = \begin{cases} Q & \text{if } Q \leq \bar{Q} \\ \bar{Q} & \text{if } Q > \bar{Q}. \end{cases} \tag{2}$$

That is, F can transform Q into x under constant returns to scale until capacity $\bar{Q}$ is reached; beyond that, additional units of Q are no longer useful. We further assume that $(p_w^x - p_w^z)$ is sufficiently large to ensure that $Q = \bar{Q}$ is always optimal.[11] As will become clear in section 5, this specification makes it possible to isolate the welfare generated in the inputs market. This setup is, obviously, artificial. We know, since Grossman (1981), that ROOs can have either a positive or a negative impact on final good production and exports. We choose to ignore that possibility to focus on a pedagogically more useful setup, which allows us to concentrate on the first-order effects stemming from the market for inputs. Nevertheless, we indicate in section 7 how our analysis and results would change once one allows final-good production to vary as well.

3 The Choice of Inputs Conditional on Investment

After S chooses her investment, F and S bargain over the number and price of the specialized intermediate inputs. We assume the outcome follows Generalized Nash Bargaining and specify the supplier as having bargaining ability $\alpha \in (0, 1)$. The two firms jointly choose the number of specialized inputs q and their price p^s according to

$$\max_{\{q, p^s\}} \left[U_S^T - U_S^0\right]^{\alpha} \left[U_F^T - U_F^0\right]^{1-\alpha},$$

where U_j^m is the gross profit (i.e., profit absent transfers) that firm $j = F, S$ would receive under scenario m. The two possible scenarios are either bargaining and trading ($m = T$) or not reaching a bargain and thus not trading ($m = 0$).

[11] See section 7 and Appendix B for discussion of the case where firms might choose total inputs $Q < \bar{Q}$.

Conditional on inverse productivity ω, investment i, specialized inputs q and total inputs $\bar{Q}$, producer utilities are:

$$U_F^T = \begin{cases} (p_w^x + \tau)\bar{Q} - p_w^z z - p^s q & \text{if } q \geq q_r \\ p_w^x \bar{Q} - p_w^z z - p^s q & \text{else;} \end{cases}$$

$$U_F^0 = \begin{cases} (p_w^x + \tau - p_w^z)\bar{Q} & \text{if } q \geq q_r \\ (p_w^x - p_w^z)\bar{Q} & \text{else,} \end{cases}$$

where $q_r \equiv r\bar{Q}$. Note that the producer's utility depends on whether the ROO is satisfied. If it is, then he obtains an extra surplus of $\tau\bar{Q}$. If the bargain fails, $q = 0$, $z = \bar{Q}$, and the $q \geq q_r$ constraint is satisfied only if $r = 0$.

For supplier utilities, we have

$$U_S^T = p^s q - C(q, i, \omega);$$
$$U_S^0 = 0.$$

Observe that, while the supplier's utility under a bargain, U_S^T, does not depend directly on whether the ROO is satisfied, the supplier shares part of the additional producer utility through the input price p^s.

For $r > 0$, the bargaining surplus $\Sigma \equiv \left[U_F^T - U_F^0\right] + \left[U_S^T - U_S^0\right]$ is

$$\Sigma = \begin{cases} \Sigma^{RC} \equiv \tau\bar{Q} + p_w^z q - C(q, i, \omega) & \text{if } q \geq q_r \\ \Sigma^{NC} \equiv p_w^z q - C(q, i, \omega) & \text{else.} \end{cases}$$

The RC superscript denotes rule-of-origin compliance, while the NC superscript denotes non-compliance.[12] In an efficient bargain, the vertical chain solves:

$$\max_q \ \Sigma.$$

Ignoring the $q \geq q_r$ constraint on Σ^{RC} for a moment, we see that the same choice of inputs that maximizes Σ^{RC} also maximizes Σ^{NC}. This choice, denoted q_0, equalizes the marginal cost of generic and specialized inputs:

$$p_w^z \equiv C_q(q_0, i, \omega).$$

[12] In the special case of $r = 0$, the bargaining surplus always equals Σ^{NC} because compliance is assured regardless of bargaining.

Using the implicit function theorem, it follows from the properties of $C(.)$ that q_0 is increasing in investment.

Reimposing the constraint, if q_0 complies with the ROO ($q_0 \geq q_r$), then it clearly optimizes Σ and yields bargaining surplus $\Sigma^{RC}(q_0, i, \omega)$. This holds for a sufficiently high investment,

$$i \geq i_{UC}(\omega), \tag{3}$$

where the *unconstrained-compliance investment threshold* $i_{UC}(\omega)$ solves $q_0(i_{UC}, \omega) \equiv q_r$. This threshold is increasing in ω.[13] With lower productivity, investment must be higher for the vertical chain to comply with the ROO in an unconstrained way.

Now, if $i < i_{UC}(\omega)$, then input level q_0 does not comply with the ROO and would yield bargaining surplus $\Sigma^{NC}(q_0, i, \omega)$. The vertical chain then has an additional consideration. By choosing $q \geq q_r$, it can earn the extra surplus $\tau\bar{Q}$ while sacrificing some efficiency by producing inputs at a marginal cost higher than p_w^z. Since Σ^{RC} is strictly decreasing in q as q rises above q_0, the best choice satisfying the constraint is $q = q_r$ and yields bargaining surplus $\Sigma^{RC}(q_r, i, \omega)$. This is then compared to the optimal bargaining surplus under non-compliance, $\Sigma^{NC}(q_0, i, \omega)$. If $\Sigma^{NC}(q_0, i, \omega) > \Sigma^{RC}(q_r, i, \omega)$, then non-compliance is optimal. This holds for a sufficiently low investment,

$$i < i_{NC}(\omega, \tau), \tag{4}$$

where the *non-compliance investment threshold* $i_{NC}(\omega, \tau)$ solves $\Sigma^{RC}(q_r, i_{NC}, \omega) = \Sigma^{NC}(q_0, i_{NC}, \omega)$. Intuitively, chains choose not to comply when investment is so low that producing q_r would require pushing marginal cost to an excessively high level. For $i \in [i_{NC}(\omega, \tau), i_{UC}(\omega))$, however, that distortion is worth incurring and $q = q_r$ is optimal. The $i_{NC}(\omega, \tau)$ threshold is also increasing in ω.[14]

The following lemma summarizes optimal sourcing for given investment levels. All proofs are in the Appendix.

[13] Because it follows from the properties of $C(.)$ that q_0 is decreasing in ω, we have that $\frac{di_{UC}}{d\omega} = -\frac{\partial q_0/\partial \omega}{\partial q_0/\partial i} > 0$. Under functional forms (1), $i_{UC} = \frac{\omega - p_w^z + cq_r}{b}$.

[14] We have that $\frac{di_{NC}}{d\omega} = \frac{C_\omega(q_0) - C_\omega(q_r)}{C_i(q_r) - C_i(q_0)} > 0$, where the positive sign follows because $C_{qi} < 0, C_{q\omega} > 0$, and $q_r > q_0$ at this point. Under functional forms (1), $i_{NC} = i_{UC} - \frac{\sqrt{2c\tau\bar{Q}}}{b}$.

Lemma 1 *Conditional on ω and i, there exist investment thresholds $i_{NC}(\omega,\tau)$ and $i_{UC}(\omega)$, with $i_{NC}(\omega,\tau) \leq i_{UC}(\omega)$, such that the equilibrium level of inputs q_i satisfies the following:*

$$q_i = \begin{cases} q_0 = \frac{p_w^z - \omega + bi}{c} & \text{if } i < i_{NC}(\omega,\tau) \\ q_r = r\bar{Q} & \text{if } i \in [i_{NC}(\omega,\tau), i_{UC}(\omega)] \\ q_0 = \frac{p_w^z - \omega + bi}{c} & \text{if } i > i_{UC}(\omega). \end{cases}$$

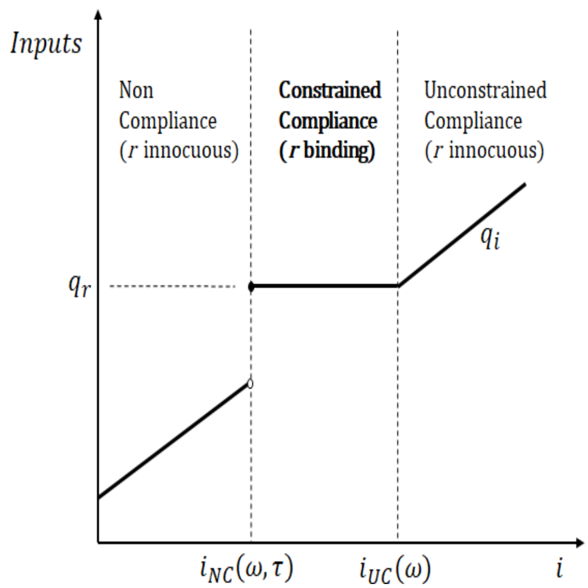
In the subsequent analysis, the following definitions are useful.

Definitions (ROO Effectiveness)

1. *A rule of origin that yields equilibrium output choice $q_r > q_0$ is* **binding***.*

2. *A rule of origin that yields equilibrium output choice q_0 is* **innocuous***.*

Figure 1: *Optimal Specialized Inputs Conditional on Investment*



Note: Conditional on a given ω, this diagram illustrates the relationship between investment i and the choice of specialized inputs q_i that obtains under Generalized Nash Bargaining.

According to these Definitions, a ROO is binding for $i \in [i_{NC}(\omega,\tau), i_{UC}(\omega))$ and is innocuous for other levels of i. Figure 1 highlights, for a fixed ω, these definitions and the relationship between investment and the vertical chain's choice of inputs. Note that an increase in the final-goods tariff τ slacks the non-compliance investment threshold $i_{NC}(\omega,\tau)$,[15] but has no effect on the unconstrained-compliance investment threshold $i_{UC}(\omega)$. And with no final-goods tariff, the investment thresholds coincide: $i_{NC}(\omega,0) = i_{UC}(\omega)$. Naturally, if $\tau = 0$ then there is no reason to deviate from q_0 to achieve compliance; any ROO would be innocuous.

The finding that the ROO typically binds for some but not all producers resembles early findings from the literature (Grossman, 1981; Krishna, 2006). However, as we will see, our model yields quite novel implications for welfare. Before that, we study the investment decision.

4 The Choice of Investment

Now consider the supplier's choice of investment. She solves

$$max_i \quad U_S(i) \equiv \alpha\Sigma(i,\omega) - I(i), \tag{5}$$

where

$$\Sigma(i,\omega) = \begin{cases} \Sigma^{NC}(q_0,i,\omega) & \text{if} \quad i < i_{NC}(\omega,\tau) \\ \Sigma^{RC}(q_r,i,\omega) & \text{if} \quad i \in [i_{NC}(\omega,\tau), i_{UC}(\omega)] \\ \Sigma^{RC}(q_0,i,\omega) & \text{if} \quad i > i_{UC}(\omega) \end{cases}$$

incorporates ROO-effectiveness. Just as the choice of inputs conditional on investment is tripartite, the investment decision as a function of inverse productivity ω is also tripartite.

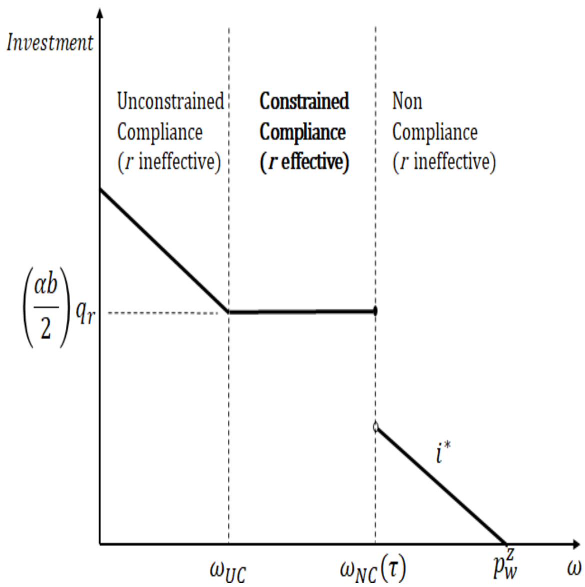
Proposition 1 *Optimal investment i^* is defined implicitly by $-\alpha C_i(q_{i^*}, i^*, \omega) = I'(i^*)$. There exist thresholds ω_{UC} and $\omega_{NC}(\tau)$, with $0 \leq \omega_{UC} \leq \omega_{NC}(\tau) \leq p_w^z$, such that equilibrium investment i^* satisfies:*

$$i^* = \begin{cases} i_0^* = \left(\frac{\alpha b}{2c-\alpha b^2}\right)(p_w^z - \omega) & \textit{if } \omega < \omega_{UC} & \textit{High Productivity;} \\ i_r^* = \frac{\alpha b q_r}{2} & \textit{if } \omega \in [\omega_{UC}, \omega_{NC}(\tau)] & \textit{Medium Productivity;} \\ i_0^* = \left(\frac{\alpha b}{2c-\alpha b^2}\right)(p_w^z - \omega) & \textit{if } \omega > \omega_{NC}(\tau) & \textit{Low Productivity.} \end{cases}$$

Figure 2 highlights the highly-intuitive pattern to equilibrium investment. For a given r, suppliers can be grouped into high, low and medium productivity categories, partitioned

[15]This follows from $\frac{di_{NC}}{d\tau} = \frac{\bar{Q}}{C_i(q_r)-C_i(q_0)} < 0$.

Figure 2: *Equilibrium Investment Conditional on* ω



Note: The relationship between inverse productivity ω and equilibrium investment i^*.

by the cutoffs ω_{UC} and $\omega_{NC}(\tau)$. For high-productivity suppliers, the ROO is innocuous because they comply in an unconstrained way. For low-productivity suppliers, the ROO is similarly innocuous, but because they fail to comply. The ROO is binding only for medium-productivity suppliers. By complying, they produce more inputs than they otherwise would, so they also invest more than they otherwise would.

Intuitively, the supplier's investment always equalizes its marginal cost (I') with fraction α of its marginal benefit $-C_i$. If the supplier expects to produce extra inputs due to constrained compliance with the ROO, then the marginal benefit of investment is higher so the supplier invests more. The $I'(i^*) = -\alpha C_i(q_{i^*}, i^*, \omega)$ condition also ensures a fundamental hold-up problem in our setting; when $\alpha < 1$, the supplier under-invests in the absence of policies.

With our functional forms, the choice of investment satisfies

$$i^* = \frac{\alpha b}{2} q_{i^*}. \tag{6}$$

Substituting, we can then write equilibrium inputs as

$$q_{i^*} = \begin{cases} q_0^* = \frac{2(p_w^z - \omega)}{2c - \alpha b^2} & \text{if } \omega < \omega_{UC} \\ q_r^* = r\bar{Q} & \text{if } i \in [\omega_{UC}, \omega_{NC}(\tau)] \\ q_0^* = \frac{2(p_w^z - \omega)}{2c - \alpha b^2} & \text{if } \omega > \omega_{NC}(\tau). \end{cases}$$

When the thresholds ω_{UC} and $\omega_{NC}(\tau)$ are interior (as in Figure 2),[16] they satisfy

$$\begin{aligned} \omega_{UC} &= p_w^z - \tfrac{r\bar{Q}(2c-\alpha b^2)}{2}; \text{ and} \\ \omega_{NC}(\tau) &= p_w^z - \tfrac{r\bar{Q}(2c-\alpha b^2)}{2} + \sqrt{(2c - \alpha b^2)\tau\bar{Q}}. \end{aligned}$$

An increase in r shifts the range of affected suppliers to the left, while an increase in τ shifts $\omega_{NC}(\tau)$ to the right and widens the range.

In the interior case,

$$\omega_{NC}(\tau) - \omega_{UC} = \sqrt{(2c - \alpha b^2)\tau\bar{Q}},$$

so that the width of the range of supplier productivity affected by the ROO is independent of the stringency of the rule, r. Unsurprisingly, the width is increasing in $\tau\bar{Q}$, the bargaining-surplus bonus from compliance. Moreover, it is decreasing in both supplier bargaining ability, α, and in the effectiveness of investment, b. When investment is very effective to reduce marginal cost (b is high) and the investing party is very responsive (α is high), the range narrows. Intuitively, in that case investment is a "key decision" for suppliers, and therefore most of them are unwilling to distort it to reap the gain from compliance. Conversely, for low α and low b, the role of investment is diminished for suppliers, and therefore most of them become willing to distort it to earn $\tau\bar{Q}$.

If r is sufficiently high, then $\omega_{UC} = 0$ and the ROO binds for the very highest-productivity suppliers. If r is sufficiently low, then $\omega_{NC}(\tau) = p_w^z$ and ROO binds for the very lowest-productivity suppliers. A high τ has a similar effect, pushing $\omega_{NC}(\tau)$ up. Thus, we could have situations where r and τ are high enough so that the ROO binds for all suppliers because the gain from compliance is sizable (τ is very high) and compliance requires a great share

[16] The interior case holds for $r \in \left(2\sqrt{\frac{\tau}{\bar{Q}(2c-\alpha b^2)}}, \frac{2p_w^z}{\bar{Q}(2c-\alpha b^2)}\right)$.

of within-FTA inputs (r is very high). We can think of this as the limiting case when the tripartite equilibrium described in Proposition 1 collapses to one where the ROO is binding for every vertical chain.[17]

5 Welfare

We now consider the welfare effects of a rule of origin r. The welfare generated by a single Y-chain has potentially several components: (1) F's profit; (2) S's profit; (3) consumer surplus (CS) from the final good in $Home$; (4) CS from the final good in $Foreign$; (5) tariff revenue (TR) from imports of x in $Foreign$; (6) TR from imports of z in $Home$ (in section 6). Thus, there is welfare due to actions both in the inputs and the final-good markets.

The literature on ROOs has concentrated on how they affect the inputs market. As the discussion in the introduction highlights, ROOs are often associated with welfare-reducing trade diversion in the inputs market, because they can induce excessive sourcing within the bloc. We follow the literature by concentrating the analysis on the inputs market.[18] To do so, we shut down welfare effects in the final-goods market.

Specifically, we consider a variation of Grossman and Helpman's (1995) "enhanced protection" case. In that scenario, final demand in *Foreign* is large enough relative to supply in *Home*, so that *Home* can sell all it produces in *Foreign* without affecting prices there, which is given by the marginal imported unit, which comes from *ROW*. Thus, in *Foreign* the only welfare change is lost tariff revenue on everything it imports from *Home* under the agreement. Each producer in *Home* will sell all of his production in *Foreign*. $Home$ will import everything it consumes, but since the price does not change (it is given by the world price p_w^x before and after the FTA), consumers are indifferent. The implication is that the welfare components (3) and (4) above are constant, unaffected by the FTA and its ROOs.

In Grossman and Helpman's (1995) model, where the goal is to study the final-goods market, under enhanced protection the net welfare effect is negative because the gain for

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[17] The other limiting case is when $r = 0$, in which case $p_w^z = \omega_{UC} < \omega_{NC}(\tau)$ and every Y-chain (trivially) complies unconstrained to the rule.

[18] This is not without loss of generality. For example, a ROO could generate trade diversion in inputs but induce trade creation in final goods, or vice-versa.

Home producers is less than the loss of tariff revenue in *Foreign*. This happens because *Home* producers will work up in their marginal cost curves. The difference, negative, is the cost of trade diversion for the countries. Here, this does not happen because of production function (2). Under that specification, in the final-good market we have trade creation = trade diversion = 0. That is, there is no net gain or loss from trade in the final good x with the FTA, only a transfer between the two countries. Specifically, *Home* benefits from the free internal trade in final goods because each of its final-good producers earns $\tau\bar{Q}$ with the higher price earned by exporting to *Foreign*'s market. Meanwhile, *Foreign* loses exactly $\tau\bar{Q}$ in tariff revenue for each variety of final good x imported from *Home* under the FTA. The net effect for the bloc is therefore nil with respect to the trade of the final good.

Thus, for each Y-chain, the welfare component (5) above is fully offset by the gain in the final-good market of $Home$'s producers, which is part of the welfare component (1). That is, from the bloc's viewpoint, *the FTA is welfare-neutral with respect to the trade of good x.* It follows that, in our baseline model, the welfare changes due to an FTA between $Home$ and $Foreign$ stem *only* from changes in S's profit and in F's profit due to expenditures on inputs.

Such joint welfare due to a single Y-chain can be written as the sum of producer utility (net of the private gain from ROO compliance) and supplier utility, if we ignore terms that do not change with the FTA. Specifically:

$$\hat{\Psi}(q,i,\omega) = [p_w^x\bar{Q} - p_w^z(\bar{Q}-q) - p^s q] + [p^s q - C(q,i,\omega) - I(i)].$$

Subtracting the constant $(p_w^x - p_w^z)\bar{Q}$ and rearranging, we obtain

$$\Psi(q,i,\omega) = p_w^z q - C(q,i,\omega) - I(i). \tag{7}$$

Thus, ignoring constant terms, the contribution of a single Y-chain to aggregate welfare corresponds to the savings due to the production of q units of inputs by the specialized supplier, rather than importing those units of generic inputs, net of investment costs. Note that Ψ does not depend directly on the rule of origin r. However, equilibrium welfare does depend on r through its effects on equilibrium investment and input choice.

Recall that, if $r = 0$, then $i^* = i_0^*$ and $q_{i^*} = q_0^*$ for all ω. We denote $\Delta\widetilde{\Psi}(r,\omega) \equiv \Psi(q_{i^*}, i^*, \omega) - \Psi(q_0^*, i_0^*, \omega)$ as the welfare effect of moving from no rule of origin to rule of origin r for a single chain involving a supplier with parameter ω – the "Y-chain-level welfare effect." The aggregate welfare effect integrates this term over all levels of productivity:

$$\Delta W(r) = \int_0^{p_w^z} \Delta\widetilde{\Psi}(r,\omega) g(\omega) d\omega. \tag{8}$$

In subsection 5.1, we focus on the Y-chain-level welfare effect for a given ω. This analysis identifies complete welfare effects for the case of a degenerate $g(\omega)$ distribution – that is, absent firm heterogeneity. Comparative statics analysis also yields insights that are useful for the aggregate welfare analysis. We turn to aggregate welfare effects with non-degenerate distributions of suppliers in subsection 5.2. Finally, to highlight the role of the hold-up problem in our results, in subsection 5.3 we consider the special cases where investment is useless ($b = 0$) and where the hold-up problem is either unsolvable ($\alpha = 0$) or nonexistent ($\alpha = 1$).

5.1 Homogeneous Suppliers

Fix $\omega \in [0, p_w^z)$ and focus on the investment choice. Conditional on q_i satisfying the optimal bargaining condition, $p_w^z = C_q(q_i, i, \omega)$, the *first-best* investment that maximizes (7) is

$$i^{fb} = \frac{b(p_w^z - \omega)}{2c - b^2}.$$

However, first-best welfare is not achievable generally due to the hold-up problem. This is illustrated in the simplest way for the case of no ROO. Equilibrium investment is

$$i^* = i_0^* = \frac{\alpha b(p_w^z - \omega)}{2c - \alpha b^2} < i^{fb},$$

inefficiently low because the supplier captures only share α of the returns from the investment but pays all investment costs.

Following OTB, we define the extent of the hold-up problem as

$$HUP_0 \equiv i^{fb} - i_0^*.$$

Noting that HUP_0 decreases in ω, we establish a result that holds here and is likely to apply more generally:

Remark 1 *The fundamental hold-up problem is more severe for higher-productivity suppliers.*

The inefficiency in the supplier's investment choice increases with the share $(1-\alpha)$ of the returns to investment.[19] In turn, those returns are increasing in supplier productivity, because higher-productivity suppliers produce more, and therefore investment lowers the cost of more units when productivity is higher.

Now, because a ROO can affect investment decisions, it can also affect the severity of the hold-up problem. This is a potential source of gain. However, there are three potential difficulties. First, the ROO may not be binding. Second, when the ROO is binding, it will distort the choice of inputs. Third, the resulting investment may exceed the first best. For future use, it is useful to define the (potential) excess of investment under a binding ROO as

$$EXC_r \equiv i_r^* - i^{fb}.$$

Considering all dimensions, the following lemma shows that the welfare impact of a rule of origin is tripartite in nature.

Lemma 2 *For any $\omega \in [0, p_w^z)$, there exist values*

$$\underline{r}(\omega) \equiv \frac{2(p_w^z-\omega)}{\bar{Q}(2c-\alpha b^2)} \text{ and}$$

$$\bar{r}(\omega,\tau) \equiv \frac{2(p_w^z-\omega)}{\bar{Q}(2c-\alpha b^2)} + \frac{2\sqrt{(2c-\alpha b^2)\tau\bar{Q}}}{\bar{Q}(2c-\alpha b^2)}$$

such that the rule of origin is binding if and only if $r \in (\underline{r}(\omega), \bar{r}(\omega,\tau)]$. The Y-chain-level welfare effect of the rule of origin is

$$\Delta\widetilde{\Psi}(r,\omega) = \begin{cases} 0 & \text{if } r \leq \underline{r}(\omega) \\ \Delta\Psi(r,\omega) \equiv (p_w^z-\omega)(q_r^*-q_0^*) - \frac{\left(2c-2\alpha b^2+\alpha^2 b^2\right)\left(q_r^{*2}-q_0^{*2}\right)}{4} & \text{if } r \in (\underline{r}(\omega), \bar{r}(\omega,\tau)] \\ 0 & \text{if } r > \bar{r}(\omega,\tau). \end{cases}$$

[19] Specifically, $HUP_0 = \frac{2(1-\alpha)bc(p_w^z-\omega)}{(2c-b^2)(2c-\alpha b^2)}$. If $\alpha = 1$, then $i_0^* = i^{fb}$ and $HUP_0 = 0$.

Lemma 2 identifies the levels in which the rule of origin is binding and shows its welfare effect for a given ω, which is non-zero only if the ROO is binding. The lower cutoff, $\underline{r}(\omega)$, comes from setting $\omega = \omega_{UC}$ and solving for r. Any $r < \underline{r}(\omega)$ requires a level of within-FTA sourcing lower than what a vertical chain with parameter ω chooses to do regardless of the ROO, which is therefore redundant. The higher cutoff, $\bar{r}(\omega, \tau)$, stems from setting $\omega = \omega_{NC}(\tau)$ and solving for r. Any $r > \bar{r}(\omega, \tau)$ requires a level of within-FTA sourcing higher than what a vertical chain with parameter ω is willing to meet to avoid paying tariff τ on its final-good exports.

Clearly, when τ is small, the difference between $\underline{r}(\omega)$ and $\bar{r}(\omega, \tau)$ is also small: if the private gain from compliance is modest, most levels of r will be unable to affect investment and sourcing decisions. In the limit when $\tau = 0$, $\underline{r}(\omega) = \bar{r}(\omega, 0)$ and there is no binding rule of origin. Observe also that both $\underline{r}(\omega)$ and $\bar{r}(\omega, \tau)$ decrease with ω. This happens because higher-productivity suppliers always invest more and sell more inputs; thus, it takes a higher r to match the fraction of inputs they produce absent a ROO.

Relying on Lemma 2, the following proposition characterizes the welfare effects of r for the case of homogeneous suppliers.

Proposition 2 *Let the distribution of inverse productivity be degenerate and centered on any $\omega \in [0, p_w^z)$. There exist*

$$\begin{aligned} \underline{\tau}(\omega) &\equiv \tfrac{1}{\bar{Q}(2c-\alpha b^2)} \left[\tfrac{(p_w^z-\omega)\alpha(1-\alpha)b^2}{2c-2\alpha b^2+\alpha^2 b^2} \right]^2 \quad \text{and} \\ \hat{r}(\omega) &\equiv \tfrac{2(p_w^z-\omega)}{\bar{Q}(2c-2\alpha b^2+\alpha^2 b^2)} \end{aligned}$$

such that:

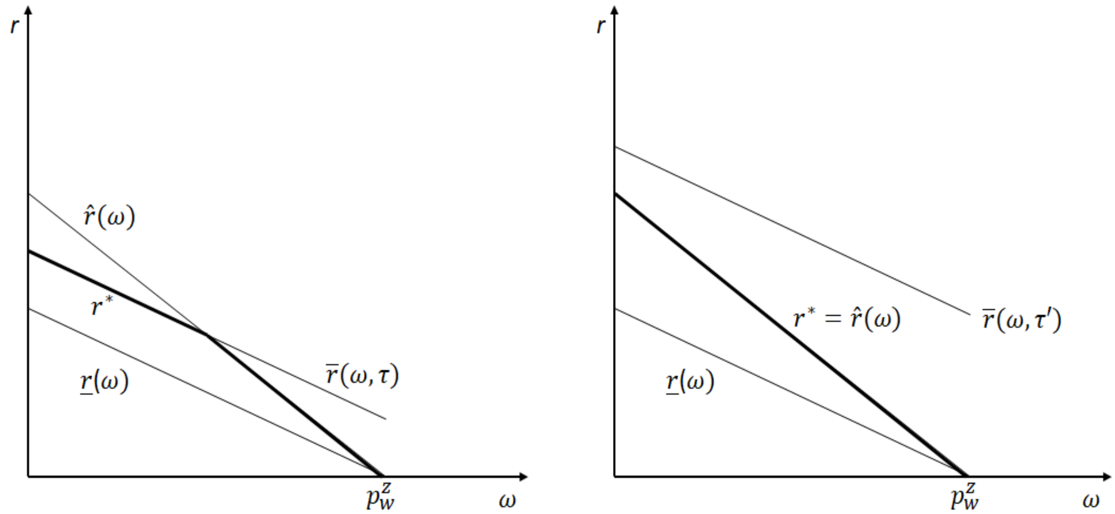
(i) If $\tau > \underline{\tau}$, then $\Delta\Psi(r, \omega)$ is an inverted-U function of r and is maximized with rule of origin $r^* = \hat{r}(\omega)$;

(ii) If $\tau \leq \underline{\tau}$, then $\Delta\Psi(r, \omega)$ is a strictly increasing function of r and is maximized with rule of origin $r^* = \bar{r}(\omega, \tau)$;

(iii) $r^* = \min\{\hat{r}(\omega), \bar{r}(\omega, \tau)\}$.

The function $\Delta\Psi(\omega, r)$ is an inverted-U function of r with a unique maximizer $\hat{r}(\omega)$. But $\hat{r}(\omega)$ maximizes welfare only if it is binding for Y-chains with inverse productivity ω.

Figure 3: *Optimal ROO, Homogeneous Suppliers, Varying* τ



(a) *Final Goods Tariff* τ (b) *Final Goods Tariff* $\tau' > \tau$

Note: These diagrams highlight the optimal ROO for the case where all suppliers have the same level of inverse productivity. The x-axis is the level of inverse productivity ω that all suppliers have, while the bold-face function is the optimal ROO $r^*(\omega)$ for that set of suppliers. In the left panel, τ is relatively small, which constrains $r^*(\omega)$ for low ω. In the right panel, τ is high enough so that $r^*(\omega)$ is unconstrained.

Because of the hold-up problem, $\hat{r}(\omega)$ is always above $\underline{r}(\omega)$. If $\hat{r}(\omega)$ is below $\bar{r}(\omega, \tau)$, then $\hat{r}(\omega)$ is optimal and is independent of τ. But if $\hat{r}(\omega)$ is above $\bar{r}(\omega, \tau)$, then $\bar{r}(\omega, \tau)$ is the highest r that yields constrained compliance and is optimal. Specifically, the threshold $\underline{\tau}$ is the level of τ that implicitly solves $\hat{r}(\omega) = \bar{r}(\omega, \tau)$. For any $\tau < \underline{\tau}$, $\hat{r}(\omega) > \bar{r}(\omega, \tau)$, and hence setting $r = \hat{r}(\omega)$ in that case would induce non-compliance by a vertical chain with parameter ω. The best choice of r is therefore $\bar{r}(\omega, \tau)$, which is the highest r that is binding and induces more investment.

Regardless of the case, it follows from Lemma 2 and Proposition 2 that both $\bar{r}(\omega, \tau)$ and $\hat{r}(\omega)$ decrease with ω. Therefore, the optimal ROO in the homogeneous case is increasing in productivity.

Corollary 1 *When suppliers are homogeneous, the optimal rule of origin is increasing in productivity.*

Figures 3a-b highlight the optimal $r^*(\omega)$ and its complementarity with τ. In Figure 3a,

$r^*(\omega)$ equals $\bar{r}(\omega,\tau)$ if ω is low and $\hat{r}(\omega)$ if ω is high. In Figure 3b, τ is high enough so that $\hat{r}(\omega)$ is always binding, so $r^*(\omega) = \hat{r}(\omega)$ everywhere. Generally, welfare under the optimal r^* increases with τ and strictly increases with τ on $[0, \underline{\tau}]$, i.e., whenever $\hat{r}(\omega) > \bar{r}(\omega,\tau)$. Intuitively, a higher τ provides greater scope of action for a ROO, which allows for higher welfare if r is chosen optimally. Thus, if the FTA-importing country has high tariffs on the final good, the ROO has the potential to be more useful to mitigate hold-up inefficiencies and increase welfare.

To see why $\Delta\Psi(r,\omega)$ is an inverted-U function of r and why $r^*(\omega)$ is decreasing in ω, we analyze two opposing effects of the ROO on welfare. On the one hand, it induces more investment, helping to alleviate the hold-up problem (although it can also induce too much investment). On the other hand, it yields a socially excessive number of specialized inputs because, for any i, the supplier's marginal cost of the final unit exceeds the unit price of generic inputs, $C_q(q_r, i) > C_q(q_0, i) = p_w^z$. Analogously to OTB, we call the former effect *relationship strengthening* and the latter effect *sourcing diversion*.[20]

More precisely, we define relationship strengthening as

$$\Delta\Psi_{RS} = p_w^z\,(q_1 - q_0^*) + [C(q_0^*, i_0^*, \omega) - C(q_1, i_r^*, \omega)] - [I(i_r^*) - I(i_0^*)]\,,$$

where q_1 denotes the level of specialized inputs that equalizes the marginal costs of the two types of inputs when $i = i_r^*$. That is, q_1 solves $C_q(q_1, i_r^*) = p_w^z$. Using our functional forms and manipulating, this expression can be rewritten as

$$\Delta\Psi_{RS} = \left(\frac{2c - b^2}{2c}\right)(i_r^* - i_0^*)\,(HUP_0 - EXC_r)\,. \tag{9}$$

This effect is positive if and only if the ROO moves the level of investment *closer* to the first-best level, relative to the situation without ROOs.[21] Differentiating equation (9) with

[20] OTB define those forces to describe the effects of tariff preferences, without any ROO. Here they are defined to describe the effects of a ROO, without any tariff preference (until section 6, when we consider both). Indeed, those two terms can be used to describe the effects of any policy that affects firm choice of investment and its subsequent impact on production. However, the precise form of the two effects vary with the specifics of the policy in analysis.

[21] It can also be expressed as $\Delta\Psi_{RS} = \left(\frac{\alpha b(2c-b^2)}{4c}\right)(q_r^* - q_0^*)\,(HUP_0 - EXC_r)$.

respect to r, while noticing that r affects it only via i_r^* and EXC_r (through its effect on i_r^*), one finds that

$$\frac{\partial \Delta\Psi_{RS}}{\partial r} = \left(\frac{2c - b^2}{2c}\right) \alpha b \bar{Q} \left(i^{fb} - i_r^*\right),$$

which is positive if and only if $i_r^* < i^{fb}$. That is, increasing r improves welfare through the relationship-strengthening effect provided that it does not induce investment in excess of i^{fb}. Since $\partial \Delta\Psi_{RS}/\partial r^2 < 0$, it follows that $\Delta\Psi_{RS}$ is concave in r.

In turn, we define sourcing diversion as

$$\Delta\Psi_{SD} = C(q_1, i_r^*, \omega) - C(q_r, i_r^*, \omega) + p_w^z(q_r^* - q_1),$$

With our functional forms, this expression can be rewritten as

$$\Delta\Psi_{SD} = -\frac{c}{2}\left(q_r^* - q_1\right)^2, \tag{10}$$

which is negative because $q_r^* > q_1$. This effect is always negative[22] and increases in magnitude as ω rises. The reason is that, with lower productivity, i_0^* is lower, so ROO compliance generates a bigger increase in investment above i_0^*, which yields a greater sourcing distortion, $q_r^* - q_1^*$. Differentiating equation (10) with respect to r, we have that

$$\Delta\Psi_{SD} = -c\bar{Q}\left(q_r^* - q_1\right),$$

so increasing r makes sourcing diversion monotonically worse. Furthermore, since $\partial^2 \Delta\Psi_{SD}/\partial r^2 < 0$, $\Delta\Psi_{SD}$ is also concave in r, and so is the full $\Delta\Psi$.

Note that, if $r = \underline{r}(\omega)$, then $i_r^* = i_0^*$ and the ROO is innocuous. For slightly higher r, i_r^* rises and $\Delta\Psi_{SD}$ grows in magnitude, but the loss due to sourcing diversion is of second order. By contrast, the relationship-strengthening effect is of first order. However, as r increases further, the loss due to sourcing diversion grows larger, while the relationship-strengthening effect starts to decrease eventually. Hence, the net effect is an inverted-U curve, reaching its maximum at $\hat{r}(\omega)$.

[22] It can also be expressed as $\Delta\Psi_{SD} = -\frac{(2c-\alpha b^2)^2}{8c}(q_r^* - q_0^*)^2$.

The level of investment that obtains when $r = \hat{r}(\omega)$ is binding is

$$i^{sb} = \frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2}.$$

This level of investment is, essentially, a *second-best* level and satisfies the following lemma.

Lemma 3 *$i^{fb} > i^{sb} > i_0^*$. Moreover, the differences $(i^{fb} - i^{sb})$ and $(i^{sb} - i_0^*)$ decrease with ω.*

We have that $i^{sb} > i_0^*$ because i^{sb} is the level of investment that optimally trades off the gains from relationship strengthening against the losses from sourcing diversion. In turn, $i^{fb} > i^{sb}$ because i^{fb} is the level of investment that solves the hold-up problem without inducing any sourcing diversion. Notice that the gap between i^{fb} and i_0^* is wider for lower ω. This reflects the fact that the hold-up problem becomes more severe as productivity increases (Remark 1). The two differences indicated in Lemma 3 decrease with ω because i^{sb} incorporates elements from both i^{fb} and i_0^*.

Recalling Figures 3a-b, the function $\hat{r}$ decreases in ω. This reflects substitutability in $\Delta\Psi$ between r and ω (or equivalently, complementarity between r and productivity). To understand that, note first that the marginal social gain (due to higher investment) when r increases is greater for high productivity Y-chains, because the hold-up problem is more severe for them (Remark 1). Given a binding ROO,

$$\frac{\partial^2 \Delta\Psi_{RS}}{\partial r \partial \omega} = \left(\frac{2c - b^2}{2c}\right) \alpha b \bar{Q} \frac{\partial i^{fb}}{\partial \omega} < 0.$$

In addition, the marginal social loss (due to sourcing diversion) when r increases is smaller for high productivity Y-chains, because they already use more within-FTA specialized inputs anyway, so the distortion due to raising q up to q_r is smaller. Formally,

$$\frac{\partial^2 \Delta\Psi_{SD}}{\partial r \partial \omega} = \left(\frac{2c - \alpha b^2}{4c}\right) \bar{Q} \frac{\partial q_0}{\partial \omega} < 0.$$

Thus, considering the Y-chain-level welfare effect, it is best to raise r as productivity increases. Every supplier with $\omega < p_w^z$ underinvests. The smaller ω is (that is, the higher the supplier productivity), the more severe is the hold-up problem and the less severe is sourcing diversion. Therefore, the rule of origin should be tighter, inducing a greater increase in investment, the smaller ω is.

5.2 Heterogeneous Suppliers

Now consider the effect of a ROO on a distribution of heterogeneous suppliers. Incorporating ROO-effectiveness, the aggregate welfare effect is

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,\omega) g(\omega) d\omega.$$

This integrates out over the density of Y-chains with suppliers $\omega \in [\omega_{UC}, \omega_{NC}(\tau)]$, for whom the ROO binds. The following proposition characterizes how the level of r affects $\Delta\Psi(r,\omega)$.

Proposition 3 *There exist values*

$$\begin{aligned} r^+ &\equiv \left[\tfrac{2c-\alpha^2 b^2}{\alpha(1-\alpha)b^2}\right] \sqrt{\tfrac{\tau}{\bar{Q}(2c-\alpha b^2)}} \text{ and} \\ \omega^0 &\equiv p_w^z - q_r \left[\tfrac{(2c-\alpha b^2)(2c-2\alpha b^2+\alpha^2 b^2)}{2(2c-\alpha^2 b^2)}\right] \end{aligned}$$

such that the Y-chain-level welfare effect, $\Delta\Psi(r,\omega)$, has the following properties:
(i) If $r \leq r^+$, then $\Delta\Psi(r,\omega)$ is an inverted-U function of ω, positive for $\omega \in [\omega_{UC}, \omega^0]$ and negative for $\omega \in [\omega^0, \omega_{NC}]$.

(ii) If $r \in (r^+, 2r^+)$, then $\Delta\Psi(r,\omega)$ is a positive inverted-U function of ω on $\omega \in [\omega_{UC}, \omega_{NC}]$.

(iii) If $r > 2r^+$, then $\Delta\Psi(r,\omega)$ is positive and strictly increasing in ω for all $\omega \in [\omega_{UC}, \omega_{NC}]$.

The intuition for this result is as follows. Ignoring ROO-effectiveness, $\Delta\Psi(r,\omega)$ is an inverted-U function of ω, with $\Delta\Psi(r,\omega_{UC}) = 0$ and $\Delta\Psi(r,\omega)$ increasing in ω at $\omega = \omega_{UC}$. Thus, if $\Delta\Psi(r,\omega_{NC}) \geq 0$ at $\omega = \omega_{NC}$, then $\Delta\Psi(r,\omega) \geq 0$ for all ω such that the ROO is binding. That condition is equivalent to $r \geq r^+$. Now, if $\Delta\Psi(r,\omega_{NC}) < 0$ at $\omega = \omega_{NC}$, then $\Delta\Psi(r,\omega)$ will be positive for ω relatively close to ω_{UC} but negative for ω relatively close to ω_{NC}. The cutoff ω^0 determines what "close" represents.

Hence, for any rule of origin, some Y-chains provide a positive contribution for welfare, and a sufficiently tight rule of origin (weakly) improves welfare for all Y-chains. To see the reasons behind the first claim, note that welfare is unchanged for Y-chain ω_{UC}. But for any rule of origin, no matter how "light," welfare must increase for a Y-chain with ω that is higher but sufficiently close to ω_{UC}. The reason is that the ROO induces a first-order relationship-strengthening effect, which is higher than the second-order loss from the sourcing diversion induced by the ROO.[23] For Y-chains with ω well above ω_{UC}, however, the sourcing-diversion effect may dominate.

Now, for a sufficiently high r, the welfare effect is positive for all $\omega \in [\omega_{UC}, \omega_{NC}]$. Intuitively, a "tight" rule of origin only affects the behavior of high-productivity suppliers (because low-productivity ones will choose to not comply). Since those suppliers' under-investment is more severe, the rule has a more beneficial role in mitigating that problem, relative to the sourcing distortion (for given investment) that it also engenders.

Corollary 2 *If $r > r^+$, then for any distribution of suppliers, the welfare effect of the rule of origin, $\Delta W(r)$, is positive.*

This result implies that, with a continuous density of productivity, a stricter ROO is more likely to generate a positive welfare effect for all affected suppliers. Figure 4 highlights this using an example where we consider different levels of r. Parameters satisfy Assumption 1 for $r = \{.35, ..., .65\}$, but not for $r = .25$ and $r = .75$, when, respectively, ω_{NC} and ω_{UC} are at a corner. Following Proposition 3, $\Delta\Psi(r, \omega)$ is increasing at $\omega = \omega_{UC}$ for all r and is concave on $\omega \in [\omega_{UC}, \omega_{NC}]$. For the particular τ chosen here, $\Delta\Psi(r, \omega)$ is an inverted-U on $\omega \in [\omega_{UC}, \omega_{NC}]$.

Using the results in Proposition 3, we can solve to find $r^+ \approx .53$ under the parametrization used in Figure 4. Hence, in the figure, $\Delta\Psi(r, \omega) > 0$ for all $\omega \in [\omega_{UC}, \omega_{NC}]$ for $r \geq 0.55$ and $r = 0.65$. By Corollary 2, if r is in this upper region, then the aggregate welfare effect of

[23] The relationship-strengthening effect can be rewritten as

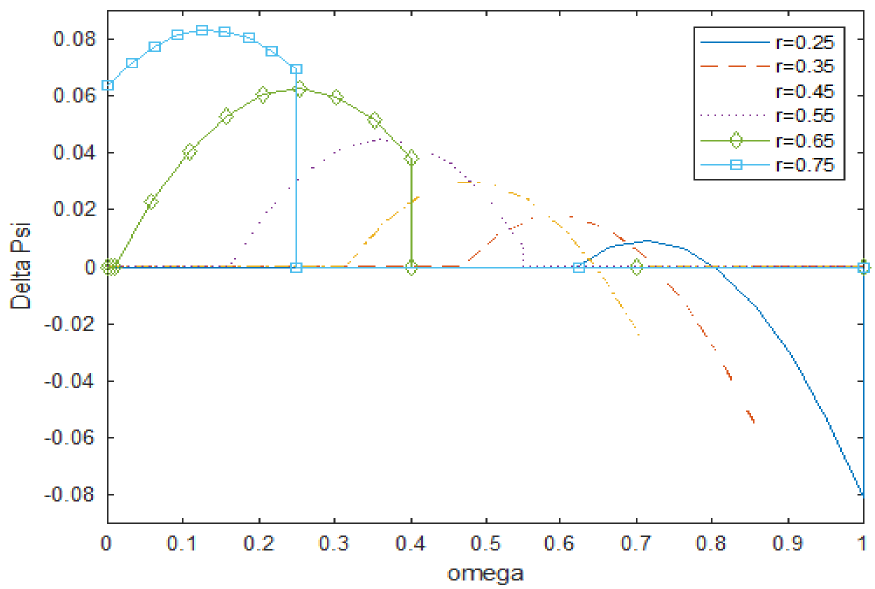
$$\Delta\Psi_{RS} = \left(\frac{\alpha b(2c - b^2)}{2c(2c - \alpha b^2)}\right)(\omega - \omega_{UC})\left(HUP_0 - EXC_r\right),$$

while the sourcing diversion effect can be rewritten as

$$\Delta\Psi_{SD} = -\frac{1}{2c}\left(\omega - \omega_{UC}\right)^2.$$

The former is first-order and positive for ω just above ω_{UC}, while the latter is negative but second-order for such ω.

Figure 4: *The Y-Chain-Level Welfare Effect of a ROO*



Note: This diagram illustrates the Y-chain-level welfare effect $\Delta\Psi(r,\omega)$ using an example. We consider $r \in \{0.25, 0.35, 0.45, 0.55, 0.65, 0.75\}$. For each r, $\Delta\Psi(r,\omega)$ is shown for all ω. Other parameters are $\alpha = 0.5$, $b = 1.25$, $c = p_w^z = 1$, $\tau = 0.05$, $g(\omega) = 1$ and $\bar{Q} = 2.5$.

the ROO is positive for *any* distribution of ω. We also see in the figure that $\Delta\Psi(r,\omega) < 0$ at $\omega = \omega_{NC}$ for $r \leq 0.45$. If r is in this lower region, then the aggregate welfare effect of the ROO may be positive or negative depending upon the distribution of ω.

The ultimate magnitude of $\Delta W(r)$, and how it changes with r, depends on the distribution of ω. Although generally ambiguous, it is useful to sort out the various forces in play. To do so, let us write the general expression of how r affects $\Delta W(r)$, after using the Leibniz formula:

$$\frac{d\Delta W(r)}{dr} = \int_{\omega_{UC}(r)}^{\omega_{NC}(r)} \left(\frac{d\Delta\Psi(r,\omega)}{dr}\right) g(\omega)d\omega - \Delta\Psi(r,\omega_{UC})g(\omega_{UC})\omega'_{UC}(r) + \Delta\Psi(r,\omega_{NC})g(\omega_{NC})\omega'_{NC}(r). \tag{11}$$

There are three sets of effects. The first term captures the way welfare changes for chains that comply in a constrained way both with ROO r and with ROO $r + dr$. The second term captures the way welfare changes for chains that enter into constrained compliance as r increases to $r + dr$. The third term captures the way welfare changes for chains that exit constrained compliance (and enter non-compliance) as r increases to $r + dr$.

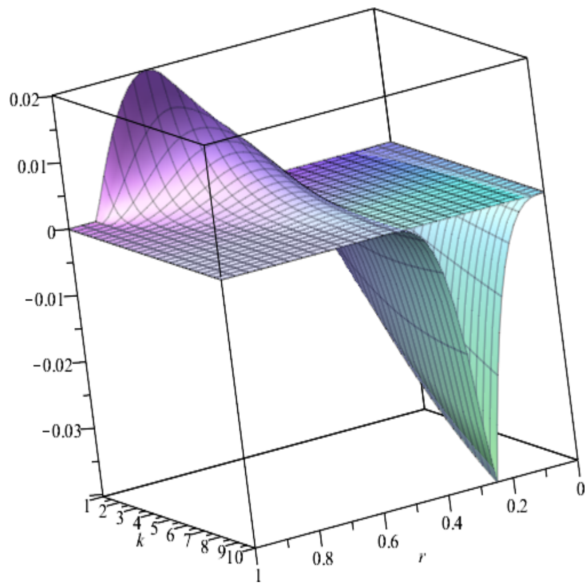
Consider the first term. For each $\omega \in (\omega_{UC}, \omega_{NC})$, the chain complies in a constrained way with ROO r and for ROO $r+dr$. As r increases, welfare changes according to $\frac{d\Delta\Psi(r,\omega_{UC})}{dr}$ for a given chain. This change is then multiplied by the density at that point, $g(\omega)$. For each $\omega \in (\omega_{UC}, \omega_{NC})$, these changes are added up, and we get the first term in equation (11). Let us analyze each ω. For ω just above ω_{UC}, at the status-quo ROO, the chain chooses investment i_r^* and inputs q_r^*, but these levels are only slightly above i_0^* and q_0^* respectively. Thus, investment is too low because of the HUP. As r increases, these chains increase their investments and inputs further, and welfare rises. From Proposition 3, we know that for any ω, there is an $\hat{r}(\omega)$ that optimizes $\Delta\Psi$ (assuming the ROO binds). For a given r, we can invert this expression to find $\hat{\omega}(r)$. For all $\omega < \hat{\omega}(r)$, welfare $\Delta\Psi(r,\omega)$ rises with a higher r. For all $\omega \geq \hat{\omega}(r)$ such that the chain complies both with status-quo ROO r and with the new ROO $r+dr$, the higher ROO exacerbates the over-investment problem. Hence, welfare falls for these chains. The entire welfare effect captured by the first term in equation (11) therefore depends on the distribution. If it has sufficient mass of high-productivity suppliers, welfare rises due to this force, but otherwise it falls.

Consider now the second term. At ω_{UC}, as r increases, the vertical chain moves from unconstrained compliance into constrained compliance, i.e., $q_0^* = q_r^*$. The welfare effect changes from zero to $\Delta\Psi(r,\omega_{UC})$, which is also zero because the unconstrained compliance input choice just meets the ROO constraint. Hence, there is no welfare change from this effect.

Finally, consider the third term of equation (11). At ω_{NC}, as r increases, the vertical chain opts out of constrained compliance and into non-compliance. The welfare effect changes to zero, so welfare falls by $\Delta\Psi(r,\omega_{NC})$. This loss/gain is weighted by the density of chains at that point, $g(\omega_{NC})$ and by the pace at which suppliers substitute out of constrained compliance into non-compliance, $\omega_{NC}^{'}(r)$. Naturally, this is the case when ω_{NC} is interior. If it is not, then this term vanishes.

Putting all this together, the two non-zero welfare effects are: (1) the changes in welfare to the chains that comply in a constrained way for both r and $r+dr$; and (2) the change in welfare from the ω_{NC} chains opting out of compliance as r increases to $r+dr$, when ω_{NC} is interior.

Figure 5: *The Welfare Effect of a ROO With Pareto Density*



Note: This diagram illustrates the welfare effect where $\frac{1}{\omega}$ is distributed according to a Pareto(k) with scale parameter $\frac{1}{p_w^z}$ and shape parameter $k \geq 1$.

To visualize the aggregate effects more clearly, let us consider that the familiar case where inverse productivity $\frac{1}{\omega}$ is distributed Pareto with scale parameter $\frac{1}{p_w^z}$ and shape parameter $k \geq 1$, so $G(\omega) = (\omega/p_w^z)^k$. In this case, we can solve explicitly for $\Delta W(r)$ as a function of primitives. Using the same parametrization of Figure 4, Figure 5 shows how $\Delta W(r)$ changes with r and k.

When $r = 0$, $\omega_{UC} = p_w^z < \omega_{NC}$, so there is no effect (trivially) because every chain complies unconstrained. As r starts to increase, $0 < w_{UC} < p_w^z < w_{NC}$, so the chains with ω smaller but very near p_w^z start to comply in a constrained way. But these are the chains for which i^{sb} is very small, so the welfare effect coming from them is surely negative. As r keeps increasing while $0 < \omega_{UC} < p_w^z < \omega_{NC}$, the ROO-induced overinvestment problem of the high-ω chains gets worse and some additional chains with lower-but-still-high ω start to overinvest, although by not as much. Thus, ΔW becomes more negative. Observe that, because $\Delta\Psi(\omega)$ is an inverted-U function and is positive for ω near ω_{UC} (Proposition 2), some chains do yield a positive welfare effect, but for low r this effect is too small to overturn the negative effect due to the least productive chains.

Once r increases enough so that ω_{NC} becomes slightly smaller than p_w^z, so $0 < \omega_{UC} < \omega_{NC} < p_w^z$ (that is, Assumption **??** holds), then the least productive chains stop complying (and overinvesting). This eliminates the most negative contributions to ΔW, which then starts to increase from a negative level. As r keeps rising while $0 < \omega_{UC} < \omega_{NC} < p_w^z$, the range of chains for which r generates a positive effect increases and the positive relationship-strengthening effect becomes greater, while at the same time other low-productivity chains stop complying and overinvesting. This pushes the welfare effect higher and eventually turns it positive.

Now, as r rises further so that $\omega_{UC} < 0 < \omega_{NC} < p_w^z$, the mass of high-productivity chains that are affected decreases, and this lowers ΔW. Finally, as r becomes high enough so that $\omega_{UC} < \omega_{NC} \leq 0 < p_w^z$, then no chain complies and $\Delta W = 0$ again.

Importantly, all of these effects must be weighted by the corresponding densities. Specifically, the negative effect from the high-ω chains overinvesting, as well as the gain when they stop complying, is more relevant when the distribution of productivity is shifted toward low-productivity suppliers. Similarly, the positive effect from the low-ω chains investing more is less relevant when the distribution of productivity is shifted toward low-productivity suppliers. Under a Pareto distribution, these happen when k is high.

The broader lessons from the Pareto distribution are general. First, when the ROO is very lenient, there is ample room for negative welfare effects. A low r does not affect the high-productivity chains that should be affected, and does affect the low-productivity ones that should not (from a social standpoint). It is especially harmful when the distribution of productivity is skewed toward low-productivity suppliers. The opposite happens when the ROO is relatively strict, as it induces more investment precisely by the chains that are underinvesting more. It is especially beneficial when the distribution of productivity is not too skewed toward low-productivity suppliers. Put in a different way, when there is plenty of low-productivity suppliers, there is little hope for a welfare-improving ROO, but when there is a sufficiently large share of high-productivity suppliers, a relatively strict ROO is likely to generate the highest possible welfare gain.[24]

[24] The case of a uniform distribution ($k = 1$) is worth noting. Under Assumption **??**, $[\omega_{UC}, \omega_{NC}]$ shifts to the left as r increases while its range $\rho(\tau)$ does not change. In general, the mass of affected chains changes

5.3 Special Cases

Several prior papers have studied rules of origin in competitive environments. Generally, their view is that rules of origin reduce welfare. This is consistent with our analysis. Intuitively, when firms act as price takers, there are no hold-up problems. Accordingly, there is no scope for welfare-enhancing relationship strengthening and ROOs produce only distortions in the inputs market.

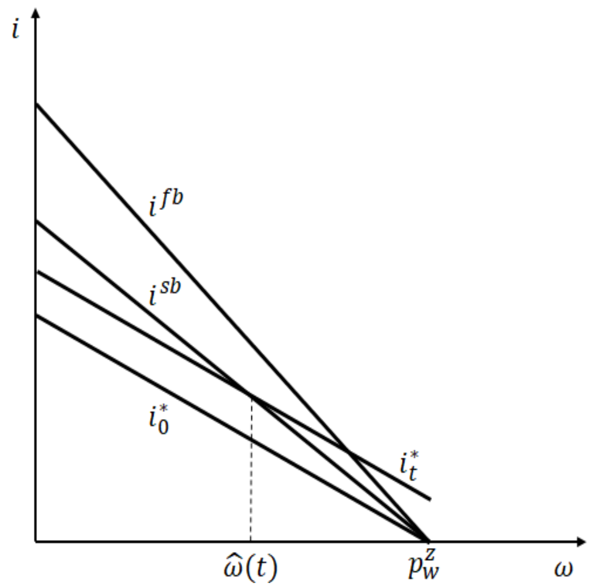
Indeed, welfare also falls for several special limiting cases in our model. Consider first the possibility that either investment is useless ($b = 0$) or the hold-up problem cannot be solved ($\alpha = 0$). Then there is effectively no investment decision, because equilibrium investments are zero and do not change with a rule of origin (recall equation (6)). Hence, the relationship-strengthening effect in equation (9) is zero. Meanwhile, the firms still have some incentive to comply with the ROO, inducing sourcing diversion. Welfare cannot improve, and it strictly decreases whenever the ROO is binding.

Welfare also falls for sure when the hold-up problem is non-existent, $\alpha = 1$. In this case, investment equals the first-best with no ROO. Hence, $HUP_0 = 0$ and $EXC_r \geq 0$, so the relationship-strengthening effect cannot be positive and is strictly negative if the ROO is binding. Again, welfare cannot improve, and it strictly decreases whenever the ROO is binding.

Proposition 4 *In the limiting cases of ineffective investment (*$b = 0$*) or extreme supplier bargaining ability (*$\alpha = 0$ *or* $\alpha = 1$*), any binding ROO strictly decreases welfare.*

as the interval $[\omega_{UC}, \omega_{NC}]$ shifts, despite the constant range, but that does not happen in the uniform case, since density is constant across all ω. Thus, in that case the mass of affected chains does not vary with r. It can then be shown that, under Assumption **??**, $\Delta W(r)$ strictly increases with r when $g(\omega)$ is uniform on $[0, p_w^z]$.

Figure 6: *Benchmark Investment Levels*



Note: This diagram illustrates the relationship between inverse productivity ω and first-best, second-best and equilibrium investment, with and without an input external tariff.

6 Positive Input Tariff

Now consider the case where the input external tariff is strictly positive, $t > 0$. Absent a binding ROO, equilibrium investment and output become

$$i^* = i^*_t = \frac{\alpha b(p^z_w + t - \omega)}{2c - \alpha b^2},$$
$$q^*_i = q^*_t = \frac{2(p^z_w + t - \omega)}{2c - \alpha b^2}.$$

Note that $i^*_t > i^*_0$ and $q^*_t > q^*_0$. The input tariff increases the privately optimal number of inputs in the F-S bargain; in response, S invests more.

This can have either a positive or negative effect on welfare. Figure 6 shows benchmark investment levels from subsection 5.1, together with i^*_t. Clearly, investment absent a ROO may now be either below or above i^{sb}. Indeed, $i^*_t \geq i^{sb}$ if and only if $\omega \geq \hat{\omega}(t)$ in the diagram.

The input tariff alters both the effectiveness of a ROO and its ability to increase welfare. Regarding effectiveness, we return to the homogeneous supplier case of subsection 5.1 and redefine the Y-chain-level welfare effect as $\Delta\Psi(r, t, \omega)$.[25] The cutoffs $\underline{r}(\omega)$ and $\bar{r}(\omega, \tau)$ become

[25]Observe that the aggregate welfare effect of a ROO is still given by the integral over the joint profit of

now functions of t :

$$\underline{r}(\omega, t) \equiv \tfrac{2(p_w^z+t-\omega)}{\bar{Q}(2c-\alpha b^2)} \text{ and}$$

$$\bar{r}(\omega, \tau, t) \equiv \tfrac{2(p_w^z+t-\omega)}{\bar{Q}(2c-\alpha b^2)} + 2\sqrt{\tfrac{\tau}{\bar{Q}(2c-\alpha b^2)}}.$$

Because specialized sourcing is increasing in t, it becomes more likely that vertical chains will meet a given r unconstrained. Thus, to be binding, the ROO needs to be tighter. The upshot is that the input tariff shifts $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ higher.

The shift in $[\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$ may improve welfare, provided that τ is relatively low. In particular, if $\bar{r}(\omega, \tau, 0) < \hat{r}(\omega) < \bar{r}(\omega, \tau, t)$, then input tariff t enables the use of ROO $\hat{r}(\omega)$ to yield investment i^{sb}. On the other hand, if τ is high enough so that $\hat{r}(\omega) < \bar{r}(\omega, \tau, 0)$, then $\hat{r}(\omega)$ is binding with no input tariff but there is no additional welfare improvement to be had. But if t is sufficiently high, then $\hat{r}(\omega) \leq \underline{r}(\omega, t)$, so that the optimal ROO at the Y-chain level is infeasible. In this case, $i_t^* \geq i^{sb}$, as in Figure 6 for $\omega \geq \hat{\omega}(t)$.[26] In such cases, it is optimal to choose an innocuous ROO. We have the following proposition.

Proposition 5 *Let the distribution of inverse productivity be degenerate and centered on any $\omega \in (0, p_w^z)$. There exists*

$$\hat{t}(\omega) = \frac{\alpha(1-\alpha)b^2(p_w^z - \omega)}{(2c - 2\alpha b^2 + \alpha^2 b^2)}$$

such that:

(i) If $t < \hat{t}(\omega) - \sqrt{(2c-\alpha b^2)\tau\bar{Q}}$, then $\Delta\Psi(r, t, \omega)$ is maximized with rule of origin $r^*(\omega, t) = \bar{r}(\omega, \tau, t)$;

(ii) If $t \in [\hat{t}(\omega) - \sqrt{(2c-\alpha b^2)\tau\bar{Q}}, \hat{t}(\omega)]$, then $\Delta\Psi(r, t, \omega)$ is maximized with rule of origin $r^*(\omega, t) = \hat{r}(\omega)$;

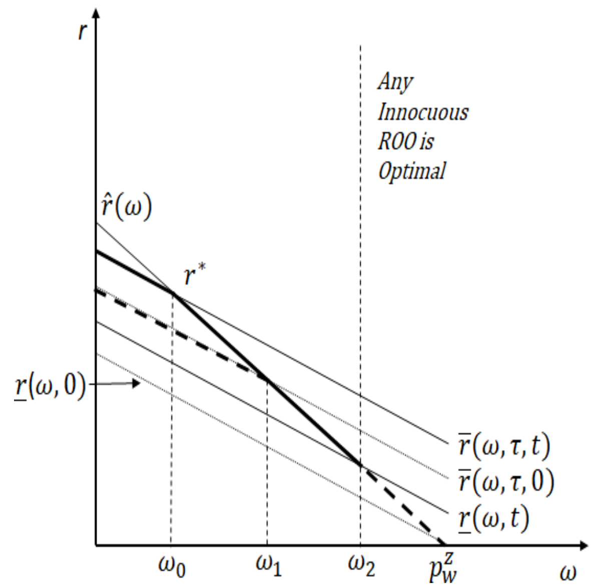
(iii) If $t > \hat{t}(\omega)$, then welfare cannot be improved with a rule of origin.

The lower cutoff, $\hat{t}(\omega) - \sqrt{(2c-\alpha b^2)\tau\bar{Q}}$, is found by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau, t)$ and solving for t. The higher cutoff, $\hat{t}(\omega)$, is found by setting $\hat{r}(\omega) = \underline{r}(\omega, t)$ and solving for t. It maximizes

all Y-chains, gross of tariff payments. The reason is that the tariff revenue due to imports of z is a transfer from the Y-chains to the *Home* government, and therefore neutral from society's perspective.

[26]Note that $\hat{\omega}(t)$ is the level of inverse productivity such that the tariff is exactly $t = \hat{t}(\hat{\omega}(t))$, where $\hat{t}$ is derived in Proposition 5.

Figure 7: *Optimal ROO, Homogeneous Suppliers, $t > 0$*



Note: This diagram highlights the optimal ROO for the case where all suppliers have the same level of inverse productivity, for the case of a positive input tariff ($t > 0$). The x-axis is the level of inverse productivity ω that all suppliers have, while the bold-face function is the optimal ROO $r^*(\omega)$ for that set of suppliers. Note that the optimal ROO r^* under $t = 0$ (Figure 3a) is shown by the dashed line for $\omega < \omega_1$ and $\omega > \omega_2$, and is the same as the optimal ROO with $t > 0$ for $\omega \in [\omega_1, \omega_2]$.

the Y-chain-level welfare effect when an input tariff is used but no ROO is used. If t is higher than that, then investment already exceeds i^{sb} with no ROO. Because the ROO can only increase investment, it is best not to have a binding ROO.

Figure 7 shows how $r^*(\omega)$ changes with a positive t. The input external tariff shifts the range $[\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$ up for all ω. Generally, the input tariff is more likely to be helpful when productivity is higher; it makes $r^*(\omega)$ strictly higher for $\omega \leq \omega_1$. For $\omega \leq \omega_0$, $r^*(\omega) = \bar{r}(\omega, \tau, t)$. For $\omega \in (\omega_0, \omega_1]$, $r^*(\omega) = \hat{r}(\omega)$. Welfare rises in each of these cases. For $\omega \in (\omega_1, \omega_2)$, $r^*(\omega)$ is the same as in Figure 3a (and note the optimal ROO in the $t = 0$ case is otherwise represented by the bold dashed line). For $\omega > \omega_2$, the input tariff eliminates the usefulness of a ROO. To the right of the vertical dashed line, any effective ROO will worsen welfare. An innocuous ROO is therefore optimal.

Note that τ may be both a complement and a substitute for t. In Figure 7, τ and t are complements for some $\omega < \omega_0$. If τ rises, then an increase in t enables higher welfare for additional levels of ω. But if $\tau = \tau'$ is as large as in Figure 3b, then with no input tariff, $r^*(\omega) = \hat{r}(\omega)$ for all ω. With an input tariff, $\underline{r}(\omega, t)$ shifts up as in Figure 7, and for $\omega > \omega_2$, a ROO now cannot help. But because τ' is so large, there are no ω where the input tariff enables an increase in welfare. The final-goods tariff only substitutes for the input tariff.

Now consider the effect of a ROO for the case of heterogeneous suppliers. Incorporating ROO-effectiveness, the aggregate welfare effect can now be written as

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r, t, \omega) g(\omega) d\omega.$$

The following result characterizes how the level of r affects $\Delta\Psi(r, t, \omega)$.

Proposition 6 *There exist values*

$$\begin{aligned} r_t^- &\equiv t\left(\tfrac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right), \\ r_t^+ &\equiv t\left(\tfrac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right)\left(\tfrac{(2c-\alpha^2b^2)(4c-3\alpha b^2+\alpha b^2)}{2(2c-\alpha b^2)^2}\right) + \left(\tfrac{(2c-\alpha^2b^2)\sqrt{\tau\bar{Q}}}{\sqrt{2c-\alpha b^2}\alpha(1-\alpha)b^2}\right) \end{aligned}$$

and

$$\omega_t^0 \equiv p_w^z - t\left(\frac{2c - 2\alpha b^2 + \alpha^2 b^2}{2c - \alpha^2 b^2}\right) - q_r\left[\frac{(2c - \alpha b^2)(2c - 2\alpha b^2 + \alpha^2 b^2)}{2(2c - \alpha^2 b^2)}\right]$$

such that the Y-chain-level welfare effect of the rule of origin, $\Delta\Psi(\omega, r)$, has the following properties:

(i) If $r < r_t^-$, then $\Delta\Psi(r, t, \omega) < 0$ for all $\omega \in (\omega_{UC}, \omega_{NC}]$.

(ii) If $r \in [r_t^-, r_t^+]$, then $\Delta\Psi(r, t, \omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega_t^0]$ and $\Delta\Psi(r, t, \omega) \leq 0$ for all $\omega \in [\omega_t^0, \omega_{NC}]$.

(iii) If $r > r_t^+$, then $\Delta\Psi(r, t, \omega) > 0$ for all $\omega \in [\omega_{UC}, \omega_{NC}]$.

As with $t = 0$, a ROO is more likely to be beneficial if it is tighter, and for the same reasons: a tight rule affects the behavior of high-productivity suppliers. The key difference when $t > 0$ is that the aggregate welfare effect can be strictly *negative* for any distribution of ω, because a ROO may aggravate overinvestment by *all* affected suppliers.

Corollary 3 *If $r < r_t^-$, the welfare effect of the rule of origin is strictly negative for any continuous distribution of suppliers. If $r > r_t^+$, the welfare effect of the rule of origin is strictly positive for any continuous distribution of suppliers.*

Note that both r_t^- and r_t^+ increase with t. With a higher external tariff on intermediate inputs, more suppliers overinvest and fewer underinvest, so a higher r is needed to affect only those that underinvest. For a sufficiently high t, all suppliers overinvest and welfare improvements are impossible.

Corollary 4 *If* $t > \frac{\alpha(1-\alpha)b^2\bar{Q}}{2}$, *then the introduction of any rule of origin strictly decreases welfare.*

7 Extensions

In this section, we discuss how our results would change under some important alternative specifications to our benchmark model. We also describe some testable implications of the model.

7.1 Location of Inputs

Since a large fraction of global value chains are actually regional, and are often circumscribed to members of FTAs, we conduct our analysis assuming that the key economic relationship between firms takes place within a trading bloc. Now, this is of course not always the case, and specialized and generic suppliers may operate in geographic regions that are different from our baseline model.

Consider first the situation where both types of suppliers are in ROW. In that case, the ROO would be irrelevant because there would be no within-bloc sources of input supply. The ROO would be equally mute if both types of inputs are fully available within the trading bloc: regardless of the input mix, compliance would be assured.

The more interesting alternative obtains when specialized suppliers are in ROW while generic suppliers are in either $Home$ or $Foreign$. Essentially, this reverses the location considered in our analysis. In that case, constrained compliance with a ROO would induce more sourcing of generic within-bloc inputs, crowding out the supply of specialized inputs. Thus, the ROO would yield inefficient sourcing while also reducing relationship-specific investments, thereby worsening hold-up problems. This is clearly bad for welfare. Extending the model in this direction would therefore reinforce a central insight from our analysis, made clear in section 5.3: if ROOs do not stimulate relationship-specific investments and attenuate holdup problems, then they cannot improve welfare.

7.2 Adjustments in Q

In our benchmark model, we impose assumptions that make it optimal for vertical chains to always choose $Q = \bar{Q}$. This is very helpful to highlight the implications of a ROO for investment, the key variable for the welfare analysis, but is of course an artificial assumption. More generally, vertical chains have two margins of adjustment when complying with a ROO: (1) the mix of inputs for given level of production; and (2) the level of production for a given mix of inputs. Our analysis shuts down the second margin, but in general firms will find optimal to alter both margins.

In particular, firms might reduce the total number of inputs, Q, to comply with the ROO. Intuitively, lowering Q reduces the need for higher q, and this could be a less costly way to comply with the ROO in some settings. When this occurs, the welfare effect of the ROO changes both because fewer final goods are produced within the bloc and because this reduces the need to increase investments. Importantly, whether lower production of the final good within the bloc increases or decreases welfare by itself is, in principle, ambiguous. In the case of enhanced protection considered here, it is welfare-improving because fewer final goods mitigate trade diversion, but the opposite would happen if there were trade creation in final goods.

Handling this possibility is relatively easy in our setting, but cases where $Q < \bar{Q}$ obtains in equilibrium require a number of conditions to hold simultaneously.[27] To see the intuition, note that to preclude this outcome, it suffices to assume that $(p_w^x - p_w^z)$ is sufficiently large. By reducing Q, a vertical chain costs itself the margin $(p_w^x - p_w^z)$ on every unit not produced in the equilibrium bargain that would be produced absent that bargain. When this margin is large, the chain prefers to set $Q = \bar{Q}$, and instead just adjusts the mix of inputs to comply with the ROO, as in our benchmark analysis. A high final-good tariff also pushes the chain in this direction, because reducing Q also sacrifices τ of surplus per unit of foregone final-

[27]See Appendix B for the basic analysis.

good production. A lenient ROO and high supplier productivity similarly push the chain to keep Q high, because both changes make it easier to comply by increasing specialized inputs. Thus, the most likely situation where a chain would adjust Q downwards would be if r is both large and binding for relatively unproductive suppliers, but where the final-good tariff is relatively low. Recalling that a high r tends to bind for high-productivity suppliers and a low τ decreases the range in which the ROO binds, it follows that the conditions that make $Q < \bar{Q}$ optimal are relatively difficult to satisfy and, when they are satisfied, the ROO is not particularly relevant (i.e., τ is low and the rule has little bite).

In the general case where the production function of the final good is smooth in the number of inputs, Q, constrained compliance will typically imply both lower Q and more specialized inputs, q.[28] Thus, our current analysis of how a ROO induces more sourcing of within-bloc specialized inputs and its novel welfare implications would carry over that general case. The difference is that the conventional – and ambiguous – welfare changes due to the final-goods market would need to be taken into account as well.

7.3 Administrative Cost of Compliance

We assume that, if a vertical chain chooses quantities so that $q \geq rQ$, then it automatically satisfies the ROO. In reality, there are additional costs of compliance related to the administrative costs of proving to the customs authority that, indeed, the firm's choice of inputs satisfies $q \geq rQ$. These include, for example, the cost of keeping additional records of transactions and of filling out additional border documents. Such costs are often considered to be equivalent to an increase in marginal costs (or to a reduction of the preferential margin), although the magnitudes of the estimates vary with the study and with the trading bloc in analysis (e.g., in the analyses of Cadot et al., 2006, they vary from 2 to 7 percent). They could also include a fixed cost component. Numerous authors argue that such costs are significant enough to induce non-compliance by many firms.

It is relatively straightforward to incorporate such costs into our setup. Recall that the gain from compliance for a vertical chain is $\tau\bar{Q}$, the total tariff savings when it exports $\bar{Q}$

[28]No compliance, in turn, will be associated with lower Q but no changes in q, because under dual sourcing p_w^z determines q when the ROO does not bind.

units of the final good to the FTA partner. Following the literature, the documentation costs of compliance could be represented as $\delta\bar{Q}$, thus reducing the gain from compliance to $(\tau-\delta)\bar{Q}$. Naturally, if the per-unit documentation cost were larger than the final-good tariff, $\delta \geq \tau$, then no vertical chain complies and the ROO does not affect the inputs market. Otherwise, the analysis of the private decisions of the vertical chains carries over just as before, but for an "adjusted" final-good tariff of $\tau' \equiv \tau - \delta$.[29]

Now, the welfare analysis does change with the introduction of the fixed documentation/administrative costs, because such costs are a real burden to the economy, and not just a transfer (as in the case of the tariff proceeds, $\tau\bar{Q}$). Therefore, whenever there is rule compliance, we would need to subtract $\delta\bar{Q}$ from the welfare calculation. The upshot is that, the higher the documentation and administrative costs, the less attractive are ROOs for the society, all else equal.

7.4 Endogenous Matching

In this paper we consider that producers of final goods and suppliers of customized inputs have already matched. By doing so we are simplifying on two grounds. First, and most obviously, we do not study how a ROO would disturb those matches. While potentially important, many of the insights from the analysis of how a PTA affects matching in OTB would carry over for a ROO. In particular, while the prospect of free trade in final goods increases the attractiveness of matching inside the bloc, the ROO requirement tends to decrease the extent of "matching diversion" – i.e., the matching with low-productivity suppliers inside the FTA at the expense of matching with high-productivity suppliers outside the bloc.

Second, even if we left aside how a ROO would affect the matching equilibrium, we know from OTB that, by endogenizing the matching process, an inevitable consequence is that low-productivity suppliers will not match, being relegated in favor of specialized suppliers outside the FTA. The implication is that the potential for a ROO to cause harm is reduced. Consider, for example, Figure 4. The only source of welfare loss stems from high-ω suppliers (when r is relatively low). If those suppliers are not matched into Y-chains and revert to producing the numéraire good, then it becomes more likely that a ROO will induce a welfare

[29]Observe that, in our setting, we can think of $\delta\bar{Q}$ also as a fixed cost.

gain, even when r is relatively low.

7.5 Positive Implications

An empirical assessment of the welfare predictions of our model would be challenging, and would probably require a quantitative model. On the other hand, testing the observable implications of the model would be more straightforward. For that, one would need the introduction of ROOs in a new FTA, or changes in existing ROOS, in addition to firm-level information.

First, our model is precise about what type of firm is likely to comply with ROOs: high-productivity ones. Moreover, as the rules become stricter, the range of firms complying shortens, and the ones that stop complying are those with intermediate levels of productivity, for whom compliance was barely profitable initially.

Second, our model is also clear about which firms, among the compliers, will change behavior because of ROOs. Our results show that, although high-productivity firms will generally satisfy the rules, they will not change their behavior – that is, for them the rule is not binding. On the other hand, medium-productivity compliers increase within-FTA sourcing and investment. Accordingly, their **observed** productivity (which considers both their "fundamental" productivity parameter ω and their investment) should increase as a result of introducing (or tightening) ROOs.

This heterogeneous behavior, with greater investment reaction from mid-range productivity firms, is similar to what Lileeva and Trefler (2010) find when studying the U.S.-Canada FTA.[30] As data on utilization rates becomes more available (see Kniahin and Melo, 2022), testing these implications will become more feasible, and falsifying the building blocs of the model will become easier.

[30] The model also has other, more straightforward, testable implications. For example, a higher tariff on final goods induces compliance by a wider range of firms. This has been documented in several FTAs.

8 Conclusion

We study the welfare effects of rules of origin in free trade agreements with a property-rights model. Given the nature of modern global value chains and their prevalence within FTAs, this approach seems natural. We design the details of the model so that we can derive clear-cut analytical solutions, but the two key assumptions on which the analysis rests are probably much more general. First, hold-up problems matter. Second, trade policies affect investment incentives. This makes clear when our conclusion do and do not hold. A prominent case when they do not is a competitive market.

Indeed, in addition to being novel, the implications of ROOs from a property-rights perspective are vastly different from those under a competitive setting. We show that welfare may rise or fall with the imposition of such a rule, but that the effects tend to be more positive when the rule requires a higher fraction of within-bloc inputs. Moreover, a sufficiently strict rule can ensure a positive welfare impact, at least if input external tariffs are not exceedingly high.

Some analysts (e.g., Crivelli et al., 2021) suggest that more lenient rules should be used to induce higher levels of preference utilization. This is also a common message of several papers that find that ROOs induce changes in sourcing patterns, which implicitly assume that sourcing and investment are efficient in the absence of ROOs. Our analysis recommends caution in such policy proposals. In our setting, making rules of origin more lenient would induce some low-productivity suppliers to comply and would make the rule redundant for some high-productivity suppliers. While the former effect is unlikely to raise welfare by much (and could actually decrease it), the latter may imply forgoing gains from mitigating hold-up problems precisely when they matter more. More generally, our analysis shows that understanding the organization of the firms affected by the rules is critical for their normative assessment.

Clearly, the design of ROOs has several practical dimensions that we bypass in our analysis. The lack of transparency and clarity in actual ROOs, their multiplicity across products and agreements, and the distinct ways of defining origin are important dimensions from a practical perspective. Attempts at defining "best practices" and at "multilateralizing" the rules under the auspices of the World Trade Organization have so far failed, but developments in those directions would surely be helpful.[31] Indeed, the insights from our analysis (as well as other theoretical analyses) would be more useful following those developments.

[31] See, for example, the discussion and suggestions of Hoekman and Inama (2018) and Kniahin and Melo (2022).

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Appendix A: Proofs

Proof of Lemma 1. Ignoring the ROO constraint, note that both Σ^{RC} and Σ^{NC} are strictly concave in q and are maximized by $q_0 = (p_w^z - \omega + bi)/c$. Then, because $\Sigma^{RC}(q_0, i, \omega) > \Sigma^{NC}(q_0, i, \omega)$, if q_0 is ROO-compliant, then q_0 is optimal. The expression for $i_{UC}(\omega) \equiv$

$\frac{\omega-(p_w^z+t)+cq_r}{b}$ is found by setting $q_0 = q_r$ and solving for ω, and it follows that $q_0 > q_r$ precisely when $i > i_{UC}$. This shows that $q_i = q_0$ if $i > i_{UC}$.

To find the expression for i_{NC}, note first that Σ^{RC} and Σ^{NC} are both increasing functions of i, with $\frac{d\Sigma^{RC}(q_r,i,\omega)}{di} = bq_r$ and $\frac{d\Sigma^{NC}(q_0,i,\omega)}{di} = bq_0$. As i falls below i_{UC}, we know that $q_r > q_0$; hence $\Sigma^{RC}(q_r, i, \omega)$ falls faster than $\Sigma^{NC}(q_0, i, \omega)$, and $\Sigma^{RC}(q_r, i, \omega) = \Sigma^{NC}(q_0, i, \omega)$ for a unique value of i. The cutoff $i_{NC}(\omega)$ is the level of investment that solves this. It follows that for $\Sigma^{RC}(q_r, i, \omega) < \Sigma^{NC}(q_0, i, \omega)$ for $i < i_{NC}$, so that $q_i = q_0$ if $i < i_{NC}$. It also follows that $\Sigma^{RC}(q_r, i, \omega) > \Sigma^{NC}(q_0, i, \omega)$ for $i \in (i_{NC}, i_{UC})$, so that $q_i = q_r$ if $i \in (i_{NC}, i_{UC})$. Note that, if $i = i_{UC}$, then $q_r = q_0$ and $\Sigma^{RC}(q_r, i_{UC}, \omega) = \Sigma^{RC}(q_0, i_{UC}, \omega)$. ■

Proof of Lemma ??. Because q_0 and q_r are continuous functions, it is obvious that $\Sigma^{NC}(i, q_0, \omega)$ is continuous on $[0, i_{NC}]$, that $\Sigma^{RC}(q_r, i, \omega)$ is continuous on $[i_{NC}, i_{UC}]$, and that $\Sigma^{RC}(q_0, i, \omega)$ is continuous for $i \in [i_{UC}, \frac{bp_w^z}{2c-b^2}]$, where the upper bound of this interval corresponds to S's choice of investment when $\omega = 0$ and $\alpha = 1$. It exceeds i_{UC} by the assumption $C_q(\bar{Q}, b(p_w^z)\,(2c-b^2)^{-1}, 0) > p_w^z$.

It remains to establish that $U_S(i)$ is continuous at $i = i_{NC}$ and at $i = i_{UC}$. For the first condition, the proof of Lemma 1 shows that i_{NC} is defined as the unique i such that $\Sigma^{RC}(q_r, i, \omega) = \Sigma^{NC}(q_0, i, \omega)$. Hence $\lim_{i \to i_{NC}} U_S(i) = \Sigma^{RC}(q_r, i_{NC}, \omega) = \Sigma^{NC}(q_0, i_{NC}, \omega)$, and $U_S(i)$ is continuous at $i = i_{NC}(\omega)$.

For the second condition, note that $i_{UC}(\omega)$ is defined so that $q_0 = q_r$. Hence, $\Sigma^{RC}(q_0, i_{UC}, \omega) = \Sigma^{RC}(q_r, i_{UC}, \omega)$ and $U_S(i)$ is continuous at $i = i_{UC}$. ■

Proof of Proposition 1. We first show that $U_S(i)$ is continuous on $\left[0, \frac{bp_w^z}{2c-b^2}\right]$. Because q_0 and q_r are continuous functions, it is obvious that $\Sigma^{NC}(i, q_0, \omega)$ is continuous on $[0, i_{NC}]$, that $\Sigma^{RC}(q_r, i, \omega)$ is continuous on $[i_{NC}, i_{UC}]$, and that $\Sigma^{RC}(q_0, i, \omega)$ is continuous for $i \in [i_{UC}, \frac{bp_w^z}{2c-b^2}]$, where the upper bound of this interval corresponds to S's choice of investment when $\omega = 0$ and $\alpha = 1$. It exceeds i_{UC} by the assumption $C_q(\bar{Q}, b(p_w^z)\,(2c-b^2)^{-1}, 0) > p_w^z$.

It remains to establish that $U_S(i)$ is continuous at $i = i_{NC}$ and at $i = i_{UC}$. For the first condition, the proof of Lemma 1 shows that i_{NC} is defined as the unique i such that $\Sigma^{RC}(q_r, i, \omega) = \Sigma^{NC}(q_0, i, \omega)$. Hence $\lim_{i \to i_{NC}} U_S(i) = \Sigma^{RC}(q_r, i_{NC}, \omega) = \Sigma^{NC}(q_0, i_{NC}, \omega)$, and $U_S(i)$ is continuous at $i = i_{NC}(\omega)$.

For the second condition, note that $i_{UC}(\omega)$ is defined so that $q_0 = q_r$. Hence, $\Sigma^{RC}(q_0, i_{UC}, \omega) = \Sigma^{RC}(q_r, i_{UC}, \omega)$ and $U_S(i)$ is continuous at $i = i_{UC}$.

Given that $U_S(i)$ is continuous on $\left[0, \frac{bp_w^z}{2c-b^2}\right]$, it attains a maximum. It is straightforward to see that maximizing U_S when either $\Sigma = \Sigma^{RC}(q_i, i, \omega)$ or $\Sigma = \Sigma^{NC}(q_i, i, \omega)$ yields the condition $I'(i^*) = -\alpha C_i(q_{i^*}, i^*, \omega)$. If $q_{i^*} = q_0$, then with our functional forms we can solve to find

$$\begin{array}{l} q_0^* = \frac{2(p_w^z - \omega)}{2c - \alpha b^2}, \\ i_0^* = \frac{\alpha b (p_w^z - \omega)}{2c - \alpha b^2}. \end{array} \tag{12}$$

If $q_{i^*} = q_r$, then $i^* = i_r^* = \frac{\alpha b q_r}{2}$. From Lemma 1, we know that q_0 maximizes Σ when i is such that $q_0 \geq q_r$. Hence, i_0^* maximizes $U_S(i)$ if q_0^* in (12) exceeds $q_r^* = r\bar{Q}$. The cutoff $\omega_{UC} = p_w^z - \frac{r\bar{Q}(2c - \alpha b^2)}{2}$ is found by setting $q_0^* = q_r^*$ and solving for ω. Because q_0^* is decreasing in ω, we have shown that $i^* = i_0^*$ if $\omega < \omega_{UC}$.

To find the expression for ω_{NC}, note that $\Sigma^{RC}(q_0^*, i_0^*, \omega)$ equals $\Sigma^{NC}(q_0^*, i_0^*, \omega)$ plus a constant term that reflects the additional producer surplus coming from ROO compliance. Thus, the envelope theorem implies that the rate of change of supplier profit is

$$\frac{dU_S}{d\omega} = -\alpha C_\omega(q_0^*, i_0^*, \omega) = -\alpha q_0^*.$$

Because q_0^* is itself a decreasing function of ω, these profit functions are decreasing and strictly convex functions of ω. The envelope theorem also applies for the rate of change for profit under constrained compliance:

$$\frac{dU_S(q_r^*, i_r^*, \omega)}{d\omega} = -\alpha C_\omega(q_r^*, i_r^*, \omega) = -\alpha q_r^*.$$

If $\omega > \omega_{UC}$, then $q_r^* > q_0^*$. Hence, $U_S(q_r^*, i_r^*, \omega)$ has a steeper slope than $U_S(q_0^*.i_0^*, \omega)$. This implies that there is a unique value ω_{NC} satisfying $U_S(q_r^*, i_r^*, \omega_{NC}) = \Sigma^{NC}(q_0^*, i_0^*, \omega_{NC})$. Solving this equation yields $\omega_{NC} = p_w^z - \frac{r\bar{Q}(2c - \alpha b^2)}{2} + \sqrt{(2c - \alpha b^2)\tau\bar{Q}}$.

Moreover, it follows that $\Sigma^{RC}(q_r^*, i_r^*, \omega) < \Sigma^{NC}(q_0^*, i_0^*, \omega)$ for $\omega > \omega_{NC}$, so that $i^* = i_0^*$ if $\omega > \omega_{NC}$. It also follows that $\Sigma^{RC}(q_r^*, i_r^*, \omega) > \Sigma^{NC}(q_0^*, i_0^*, \omega)$ for $i \in (\omega_{UC}, \omega_{NC})$, so that $i^* = i_r^*$ if $\omega \in [\omega_{UC}, \omega_{NC}]$. Note that if $\omega = \omega_{UC}$, then $i_r^* = i_0^*$ and $\Sigma^{RC}(q_r^*, i_r^*, \omega_{UC}) = \Sigma^{RC}(q_0^*, i_0^*, \omega_{UC})$. ■

Proof of Lemma 2. By definition, $\Delta\Psi(\omega, r) = \Psi(\omega, q_{i^*}, i^*) - \Psi(\omega, q_0^*, i_0^*)$. For ω outside $[\omega_{UC}, \omega_{NC}]$, $i^* = i_0^*$ and $q_{i^*} = q_0^*$, so $\Delta\Psi(r, \omega) = 0$.

For $\omega \in [\omega_{UC}, \omega_{NC}]$, $i^* = i_r^*$ and $q_{i^*} = q_r^*$. We can then write

$$\begin{aligned} \Psi(q_{i^*}, i^*, \omega) &= (p_w^x - p_w^z)\bar{Q} + p_w^z q_r^* - C(q_r^*, i_r^*, \omega) - I(i_r^*) \\ &= (p_w^x - p_w^z)\bar{Q} + p_w^z q_r^* - (\omega - b i_r^*) q_r^* - \tfrac{c}{2} q_r^{*2} - i_r^{*2} \\ &= (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega) q_r^* + b\left(\tfrac{\alpha b q_r^*}{2}\right) q_r - \tfrac{c}{2} q_r^{*2} - \left(\tfrac{\alpha b q_r^*}{2}\right)^2 \\ &= (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega) q_r^* - \tfrac{(2c - 2\alpha b^2 + \alpha^2 b^2) q_r^{*2}}{4}. \end{aligned}$$

Similarly, we can derive

$$\Psi\left(\omega, q_0, i_0^*\right) = (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega)q_0^* - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)q_0^{*2}}{4}.$$

Collecting terms, we then have

$$\Delta\Psi(\omega, r) = \left(p_w^z - \omega\right)\left(q_r^* - q_0^*\right) - \frac{\left(2c - 2\alpha b^2 + \alpha^2 b^2\right)\left(q_r^{*2} - q_0^{*2}\right)}{4},$$

concluding the proof. ■

Proof of Proposition 2. Ignoring the effectiveness constraints,

$$\Delta\Psi(r, \omega) = (p_w^z - \omega)(q_r^* - q_0^*) - \frac{\left(2c - 2\alpha b^2 + \alpha^2 b^2\right)\left(q_r^{*2} - q_0^{*2}\right)}{4}.$$

This is a strictly concave function of r that equals 0 at $r = \underline{r}(\omega)$ (where $q_0^* = q_r^*$). The first and second derivatives are

$$\frac{d\Delta\Psi(\omega,r)}{dr} = (p_w^z - \omega)\bar{Q} - \frac{(2c-2\alpha b^2+\alpha^2 b^2)\left(2q_r^*\bar{Q}\right)}{4}$$
$$\frac{d^2\Delta\Psi(\omega,r)}{dr^2} = -\frac{(2c-2\alpha b^2+\alpha^2 b^2)\bar{Q}^2}{2}.$$

The second-derivative is negative, so the function is strictly concave. Evaluating the first derivative at $r = \underline{r}(\omega)$ and rearranging, we have

$$\frac{d\Delta\Psi(\omega, r)}{dr}|_{r=\underline{r}(\omega)} = \frac{(p_w^z - \omega)\alpha(1-\alpha)b^2\bar{Q}}{2c - \alpha b^2} > 0.$$

Hence $\Delta\Psi(\omega, r)$ is increasing at $r = \underline{r}(\omega)$. Setting the first derivative equal to zero and solving, we find that $\Delta\Psi(\omega, r)$ is maximized for

$$\hat{r}(\omega) = \frac{2(p_w^z - \omega)}{\bar{Q}(2c - 2\alpha b^2 + \alpha^2 b^2)} > \underline{r}(\omega).$$

Hence, if $\hat{r}(\omega) \leq \bar{r}(\omega, \tau)$, then $\hat{r}(\omega)$ is binding and $r^* = \hat{r}(\omega)$. We find $\underline{\tau}(\omega)$ by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau)$ and solving for τ. Because $\bar{r}(\omega, \tau)$ is increasing in τ, it follows that for any $\tau > \underline{\tau}(\omega)$, the unconstrained optimum $\hat{r}(\omega) < \bar{r}(\omega, \tau)$, so $\Delta\Psi(\omega, r)$ is an inverted-U function of r on $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ and is maximized at $r^* = \hat{r}(\omega)$ (part (i)). If $\tau \leq \underline{\tau}(\omega)$, then $\hat{r}(\omega) \geq \bar{r}(\omega, \tau)$ and $\Delta\Psi(\omega, r)$ is strictly increasing on $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ and is maximized at $r^* = \bar{r}(\omega, \tau)$ (part (ii)). If $\tau = \underline{\tau}(\omega)$, then $r^* = \bar{r}(\omega, \tau) = \hat{r}(\omega)$. ■

Proof of Lemma 3. We have $i^{fb} = \frac{b(p_w^z - \omega)}{2c - b^2}$, $i^{sb} = \frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2}$, and $i_0^* = \frac{\alpha b(p_w^z - \omega)}{2c - \alpha b^2}$. $i_0^* \leq i^{sb}$ follows from $2c - 2\alpha b^2 + \alpha^2 b^2 \leq 2c - \alpha b^2$ (which holds because $\alpha \leq 1$). Comparing i^{sb} and i^{fb}, we need to show

$$\frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2} \leq \frac{b(p_w^z - \omega)}{2c - b^2}.$$

Manipulating this expression leads to $2c \geq \alpha b^2$, which holds by assumption. ■

Proof of Proposition 3. Relaxing the ROO constraint, we can write

$$\Delta\Psi(r, \omega) = (q_r^* - q_0^*)\left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r^* + q_0^*)}{4}\right]. \tag{13}$$

At $\omega = \omega_{UC}$, $q_r^* = q_0^*$, so $\Delta\Psi(r, \omega_{UC}) = 0$. Recalling that q_0^* is a function of ω, the first derivative is

$$\begin{aligned} \frac{d\Delta\Psi(r,\omega)}{d\omega} &= (q_r^* - q_0^*)\left[-1 - \left(\frac{2c-2\alpha b^2+\alpha^2 b^2}{4}\right)\left(\frac{-2}{2c-\alpha b^2}\right)\right] \\ &+ \left(\frac{2}{2c-\alpha b^2}\right)\left[(p_w^z - \omega) - \frac{(2c-2\alpha b^2+\alpha^2 b^2)(q_r+q_0)}{4}\right], \end{aligned}$$

or equivalently,

$$\frac{d\Delta\Psi(r,\omega)}{d\omega} = -(q_r^* - q_0^*)\frac{(2c - \alpha^2 b^2)}{2(2c - \alpha b^2)} + \left(\frac{2}{2c - \alpha b^2}\right)\left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r + q_0)}{4}\right]. \tag{14}$$

At $\omega = \omega_{UC}$, the terms on the first line vanish. We can then solve to find

$$\frac{d\Delta\Psi(r,\omega)}{d\omega}|_{\omega=\omega_{UC}} = \frac{r\bar{Q}\alpha(1-\alpha)b^2}{2c - \alpha b^2} > 0.$$

Hence, $\Delta\Psi(r, \omega)$ is increasing in ω at $\omega = \omega_{UC}$. The second derivative is, after some rearranging,

$$\frac{d^2\Delta\Psi(r,\omega)}{d\omega^2} = \frac{-2(2c - \alpha^2 b^2)}{2c - \alpha b^2} < 0.$$

Hence, $\Delta\Psi(r, \omega)$ is concave.

With these characteristics, $\Delta\Psi(r, \omega)$ on $\omega \in (\omega_{UC}, \omega_{NC}]$ can cross zero at most once and only from above. The cutoff r^+ is then found by inserting $\omega = \omega_{NC}$ into (13), setting it to zero, and solving for r. This yields

$$r^+ = \left(\frac{2(2c - \alpha^2 b^2)}{\alpha(1-\alpha)b^2}\right)\sqrt{\frac{\tau}{\bar{Q}(2c - \alpha b^2)}}.$$

Because ω_{NC} is strictly decreasing in r plus the aforementioned concavity and single-crossing characteristics of $\Delta\Psi(r, \omega)$, it follows that for any $r > r^+$ and for any $\omega \in (\omega_{UC}, \omega_{NC})$, $\Delta\Psi(r, \omega) > \Delta\Psi(r, \omega_{NC}) > 0$. This proves part (ii).

The cutoff ω^0 is found by setting the term in brackets in (13) equal to zero and solving for ω. It is easily shown that $\omega^0 \in [\omega_{UC}, \omega_{NC}]$ if and only if $r \leq r^+$. Then, assuming $r \leq r^+$ and noting again the concavity and single-crossing characteristics of $\Delta\Psi(r,\omega)$, it follows that $\Delta\Psi(r,\omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega^0]$ and $\Delta\Psi(r,\omega) < 0$ for all $\omega \in (\omega^0, \omega_{NC}]$. This proves part (i). ∎

Proof of Corollary 2. Suppose $r \geq r^+$ and let $g(\omega)$ be arbitrary. From the Proof of Proposition 4, we know that $\Delta\Psi(r,\omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega_{NC}]$. Further, because $\Delta\Psi(r,\omega)$ is strictly decreasing in ω, we know that $\Delta\Psi(r,\omega) > 0$ for all $\omega \in (\omega_{UC}, \omega_{NC})$. Then $\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,\omega) g(\omega) d\omega > 0$. ∎

Proof of Proposition 4. Suppose either $b = 0, \alpha = 0$ or $\alpha = 1$. Then from the Proof of Proposition 3, we know that

$$\frac{d\Delta\Psi(r,\omega)}{dr}|_{r=\underline{r}(\omega)} = \frac{(p_w^z - \omega)\alpha(1-\alpha)b^2 r\bar{Q}^2}{2c - \alpha b^2} = 0,$$

and that $\frac{d^2\Delta\Psi(r,\omega)}{dr^2} < 0$. Summarizing, $\Delta\Psi(r,\omega_{UC}) = 0$ and $\Delta\Psi(r,\omega)$ is a decreasing and strictly concave function on $r \in (underliner(\omega), \bar{r}(\omega,\tau)]$. Hence any binding ROO strictly decreases welfare. ∎

Proof of Proposition 5. Ignoring the effectiveness constraints,

$$\Delta\Psi(r,t,\omega) = (p_w^z - \omega)(q_r^* - q_t^*) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)\left(q_r^{*2} - q_t^{*2}\right)}{4}.$$

This is a strictly concave function of r that equals 0 at $r = \underline{r}(\omega,t)$ (where $q_t^* = q_r^*$). The first and second derivatives are

$$\begin{aligned} \frac{d\Delta\Psi(\omega,t,r)}{dr} &= (p_w^z - \omega)\bar{Q} - \frac{(2c-2\alpha b^2+\alpha^2 b^2)\left(2q_r^*\bar{Q}\right)}{4} \\ \frac{d^2\Delta\Psi(\omega,t,r)}{dr^2} &= -\frac{(2c-2\alpha b^2+\alpha^2 b^2)\bar{Q}^2}{2}. \end{aligned}$$

The second-derivative is negative, so the function is strictly concave. Evaluating the first derivative at $r = \underline{r}(\omega,t)$ and rearranging, we have

$$\frac{d\Delta\Psi(\omega,t,r)}{dr}|_{r=\underline{r}(\omega,t)} = \bar{Q}\left[\frac{(p_w^z - \omega)\alpha(1-\alpha)b^2}{2c - \alpha b^2} - \left(\frac{2c - 2\alpha b^2 + \alpha^2 b^2}{2c - \alpha b^2}\right)t\right].$$

This is strictly negative if $t > \hat{t}(\omega)$. Because $\Delta\Psi(\omega,t,r)$ is strictly concave, if $t > \hat{t}(\omega)$, then $\Delta\Psi(\omega,t,r) < 0$ for $r \in (\underline{r}(\omega,t), \bar{r}(\omega,\tau,t)]$. This proves part (iii).

Setting the first derivative equal to zero and solving, we find that (relaxing the effectiveness constraint) $\Delta\Psi(\omega, t, r)$ is maximized for

$$\hat{r}(\omega) = \frac{2(p_w^z - \omega)}{\bar{Q}(2c - 2\alpha b^2 + \alpha^2 b^2)}.$$

If $\hat{r}(\omega) \in [\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$, then $\hat{r}(\omega)$ is binding and $r^* = \hat{r}(\omega)$. If $\hat{r}(\omega) > \bar{r}(\omega, \tau, t)$, then $\hat{r}(\omega)$ is innocuous. Then by the concavity of $\Delta\Psi(\omega, t, r)$, $r^* = \bar{r}(\omega, \tau, t)$. We thus find the upper bound on t in part (ii) of the proof by setting $\hat{r}(\omega) = \underline{r}(\omega, t)$ and solving for t. We find the lower bound on t in part (ii) of the proof and the bound in part (i) by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau, t)$ and solving for t. ■

Proof of Proposition 6. Relaxing the ROO constraint, we can write

$$\Delta\Psi(r, t, \omega) = (q_r^* - q_t^*)\left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r^* + q_t^*)}{4}\right]. \tag{15}$$

At $\omega = \omega_{UC}$, $q_r^* = q_t^*$, so $\Delta\Psi(r, t, \omega_{UC}) = 0$. Recalling that q_t^* is a function of ω, the first derivative is

$$\begin{aligned} \frac{d\Delta\Psi(r,t,\omega)}{d\omega} &= (q_r^* - q_t^*)\left[-1 - \left(\frac{2c-2\alpha b^2+\alpha^2 b^2}{4}\right)\left(\frac{-2}{2c-\alpha b^2}\right)\right] \\ &+ \left(\frac{2}{2c-\alpha b^2}\right)\left[(p_w^z - \omega) - \frac{(2c-2\alpha b^2+\alpha^2 b^2)\left(q_r^*+q_t^*\right)}{4}\right], \end{aligned}$$

At $\omega = \omega_{UC}$, the terms on the first line vanish. We can then solve to find

$$\frac{d\Delta\Psi(r, t, \omega)}{d\omega}|_{\omega=\omega_{UC}} = \left(\frac{2}{2c - \alpha b^2}\right)\left(\frac{r\bar{Q}\alpha(1-\alpha)b^2}{2} - t\right).$$

Hence, $\Delta\Psi(r, t, \omega)$ is increasing in ω at $\omega = \omega_{UC}$ if and only if $r \geq t\left(\frac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right) \equiv r_t^-$. The second derivative is, after some rearranging,

$$\frac{d^2\Delta\Psi(r, t, \omega)}{d\omega^2} = \frac{-2(2c - \alpha^2 b^2)}{2c - \alpha b^2} < 0.$$

Hence, $\Delta\Psi(r, t, \omega)$ is concave. Because it is concave and $\Delta\Psi(r, \omega_{UC}) = 0$, it follows that if $r < r_t^-$, then $\Delta\Psi(r, t, \omega_{UC}) < 0$ for all $\omega \in (\omega_{UC}, \omega_{NC}]$. This proves part (i).

If $r > r_t^-$, then $\frac{d\Delta\Psi(r,t,\omega)}{d\omega}|_{\omega=\omega_{UC}} > 0$. Given the concavity of the function, $\Delta\Psi(r, t, \omega)$ on $\omega \in (\omega_{UC}, \omega_{NC}]$ can cross zero at most once and only from above. The cutoff r_t^+ is then found by inserting $\omega = \omega_{NC}$ into (15), setting it to zero, and solving for r. Because ω_{NC} is strictly decreasing in r plus the aforementioned concavity and single-crossing characteristics of $\Delta\Psi(r,t,\omega)$, it follows that for any $r > r_t^+$ and for any $\omega \in (\omega_{UC}, \omega_{NC})$, $\Delta\Psi(r,t,\omega) > \Delta\Psi(r,t,\omega_{NC}) > 0$. This proves part (iii).

The cutoff ω_t^0 is found by setting the term in brackets in (15) equal to zero and solving for ω. It is easily shown that $\omega_t^0 \in [\omega_{UC}, \omega_{NC}]$ if and only if $r \leq r_t^+$. Then, assuming $r \leq r_t^+$ and noting again the concavity and single-crossing characteristics of $\Delta\Psi(r,t,\omega)$, it follows that $\Delta\Psi(r,t,\omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega_t^0]$ and $\Delta\Psi(r,t,\omega) < 0$ for all $\omega \in (\omega_t^0, \omega_{NC}]$. This proves part (ii). ■

Proof of Corollary 3. The welfare effect is

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,t,\omega) g(\omega) d\omega.$$

If $r < r_t^-$ then by Proposition 6 the integrand is strictly negative for all $\omega > \omega_{UC}$. Hence $\Delta W(r) < 0$. If $r > r_t^-$ then by Proposition 6 the integrand is strictly positive for all $\omega > \omega_{UC}$. Hence $\Delta W(r) > 0$. ■

Proof of Corollary 4. If $t > \frac{\alpha(1-\alpha)b^2\bar{Q}}{2}$, then $r_t^- > 1$. Then any ROO satisfies $r < r_t^-$ so by Proposition 6 welfare is negative for any ω such that the ROO is binding. ■

Appendix B: Endogenous Q

Relax the assumption that $Q = \bar{Q}$. In the default of no bargain and no compliance, production of $\bar{Q}$ units is always optimal. So we can write

$$U_F^T - U_F^0 = [(p_w^x + \tau)Q - p_w^z z - p^s q] - \left[(p_w^x - p_w^z)\bar{Q}\right].$$

With this expression, the bargaining surplus becomes

$$\Sigma^{RC} \equiv \tau Q + p_w^z q - C(q,i,\omega) - (p_w^x - p_w^z)(\bar{Q} - Q).$$

Obviously, if $Q = \bar{Q}$, this collapses back to the case considered in the main analysis. But if $Q < \bar{Q}$, the bargaining surplus is reduced because there is a margin of $p_w^x - p_w^z$ that is earned on $(\bar{Q} - Q)$ extra units in the default (versus the equilibrium).

Incorporating this, we can then write the maximization problem as

$$\max_{\{q,Q\}} \qquad \Sigma^{RC} \equiv \tau Q + p_w^z q - C(q,i,\omega) - (p_w^x - p_w^z)(\bar{Q} - Q)$$

such that

$$q \geq rQ,$$
$$Q \leq \bar{Q}.$$

Imposing the first constraint, relaxing the second and optimizing over Q, we find

$$Q^*(i) = \frac{\tau + (p_w^x - p_w^z) + (p_w^z - (\omega - bi))\,r}{r^2 c}.$$

Substituting for $i^* = \frac{\alpha b r \bar{Q}}{2}$, we obtain

$$Q^* = \frac{2\,(\tau + (p_w^x - p_w^z) + r(p_w^z - \omega))}{r^2(2c - \alpha b^2)}.$$

It is easy to show that Q^* is decreasing in r and ω. Hence, the condition $Q^* \geq \bar{Q}$ holds more easily if r is low or if ω is low.

To guarantee that $Q^* \geq \bar{Q}$ for any r, we need it to hold for $r = 1$. This implies

$$\tau + p_w^x - \omega \geq \frac{(2c - \alpha b^2)\bar{Q}}{2}.$$

This condition is easiest to meet with low ω, i.e., high-productivity suppliers.[32] To guarantee further that $Q^* \geq \bar{Q}$ for any ω we need it to hold for $\omega = p_w^z$. This implies

$$\tau + (p_w^x - p_w^z) \geq \frac{(2c - \alpha b^2)\bar{Q}}{2}.$$

This condition holds more easily if either τ or p_w^x are higher, and can hold for negligible τ as long as p_w^x is sufficiently high.

[32] In principle, τ and/or p_w^x could be functions of ω. If they increase with ω, the condition would more readily hold.