CESifo

AREA CONFERENCES 2022

Macro, Money, and International Finance
21 – 22 July 2022

Economic Stabilizers in Emerging Markets: The Case for Trade Credit

Bryan Hardy, Felipe E. Saffie, and Ina Simonovska

CESifo

Economic Stabilizers in Emerging Markets: The Case for Trade Credit*

Bryan Hardy†
Bank for International Settlements

Felipe E. Saffie‡
Darden School of Business, University of Virginia

Ina Simonovska §
University of California—Davis, NBER, CEPR

May 23, 2022

Abstract

We document that small and medium-sized enterprises (SMEs) trade off bank for trade credit, while large firms extend trade credit, especially during financial crises. We develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more (state-contingent) trade credit the more debt-constrained they are relative to their trading partner. We validate the model's predictions using detailed firm-level data from emerging economies. We find that the model with state-contingent trade credit generates lower GDP volatility and more sharply increasing share of trade credit in liabilities during crises than counterfactual economies without (state-contingent) trade credit. We conclude that the insurance channel of trade credit earns it a role of a macroeconomic stabilizer in emerging markets.

Keywords: trade credit, bank credit, insurance, supply chains
JEL Classification Numbers: E32, G21, G32

---

*We thank Gonzalo Basante Pereira and Nicole Fleskes for excellent research assistance. We thank Thanasis Geromichalos and participants at the HK Online Trade Seminar, VITM Seminar, Pontificia Universidad Catolica de Chile, Wisconsin, UC Davis, and NYU Abu Dhabi for comments and suggestions. The views are those of the authors and not necessarily those of the Bank for International Settlements.

†Email: bryan.hardy@bis.org. Address: Centralbahnplatz 2, 4051 Basel, Switzerland.

‡Email: SaffieF@darden.virginia.edu. Address: 100 Darden Blvd, Charlottesville, VA 22903, USA.

§Email: inasimonovska@ucdavis.edu. Address: One Shields Avenue, Davis, CA, 95616, USA.

1 Introduction

Firms in emerging markets rely extensively on non-traditional sources of financing in the absence of a well-developed financial sector. The most common source of alternative finance is trade credit, which is defined as non-financial firm-to-firm borrowing and lending. According to the World Bank Enterprise Survey conducted during the 2002-2010 period, across the 35 largest emerging markets, bank debt and alternative finance each comprise 20% of a firm's total source of funds, and within the latter, trade credit is the single most important category accounting for 25% of alternative finance (see Allen et al. (2013)). Furthermore, unlike large firms, the majority of small and medium-sized enterprises (SMEs), especially in less developed countries, lack access to a bank line of credit, which suggests that they utilize trade credit because they face binding financial constraints.

In this paper, we study the interplay between bank and trade credit. Using data for Mexico, we document several facts. First, for small and medium-sized enterprises (SMEs), bank and trade credit act as substitutes, as reliance on trade credit increases when bank credit declines. Second, large firms are more likely to extend trade credit, including at longer maturities, when SMEs receive less bank credit. Third, during the great financial crisis (GFC) of 2008, listed firms in Mexico increase the share of their financial resources which are used to support trade credit lending. To reconcile these facts, we develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more trade credit the more financially-constrained they are relative to their trading partner. Furthermore, net trade credit receipts are more state-contingent for more financially-constrained firms with more volatile sales, which confirms the role that trade credit plays in diversifying away idiosyncratic risks. We verify that these predictions are supported by firm-level data from eight emerging markets. When calibrated to match key firm-level moments in emerging markets' data, the model with state-contingent trade credit generates lower GDP volatility and a more sharply rising share of trade credit in liabilities during crises than counterfactual economies without (state-contingent) trade credit. Therefore, we conclude that trade credit acts as a macroeconomic stabilizer in emerging markets.

The theory that we develop features a continuum of intermediate- and final-good producers, respectively, all of whom are financially constrained and differentiated by their cost of borrowing from a bank. Intermediate-good producers have to finance their cost of labor prior to production, while final-good producers have to expense their purchases of intermediate goods prior to producing and delivering final goods to consumers. Final-good producers face demand uncertainty, which affects intermediate-good producers via the supply chain.

Producers potentially have access to two types of markets. With some positive probability, they may meet in a decentralized market where they exchange intermediate goods. In this setting, producers may opt to issue trade credit to each other in order to supplement bank loans needed to pay the upfront costs. Intermediate- (final-) good producers who refuse the terms of a bilateral meeting as well as those who do not get the opportunity to match can sell (buy) the intermediate good to (from) a wholesaler at a perfectly competitive price. However, the wholesaler requires that all purchases are paid for in advance and does not offer trade credit. Hence, producers have to rely entirely on bank borrowing in order to produce in this case.

The amount and the state-contingency of trade credit that firms extend to each other depends critically on the firms' abilities to borrow from a bank. Assuming that the provision of insurance is costly, firms extend state-contingent trade credit to each other only when they face binding borrowing constraints. A more financially-constrained firm raises less debt and produces less than a less-constrained one. However, it also receives more (state-contingent) trade credit from its partner, which relaxes its constraint and allows it to borrow more. In fact, debt levels are higher when trade credit is state contingent not only because recipients' constraints are relaxed, but also because providers need to borrow more from banks in order to supply more trade credit.

We derive several testable predictions of the model. First, the model predicts that more debt-constrained firms obtain more trade credit. The reasoning is simple: bank and trade credit are substitutes, so more debt-constrained producers need more trade credit. Additionally, the volatility of trade credit received, which reflects its degree of state-contingency, is higher for more debt-constrained producers with more volatile sales because they are in greater need of insurance. Put simply, less-constrained agents offer more insurance to more constrained ones via trade credit.

We verify these predictions using firm-level observations from the ORBIS database featuring eight emerging markets (Korea, Czech Republic, Croatia, Estonia, Poland, Slovenia, Hungary and Romania) during the 2009-2019 period. Following our theory as well as the empirical literature, we infer a firm's debt capacity from its profitability—most notably its EBITDA (see Lian and Ma (2021))—and we demonstrate that firms with lower EBITDA receive more net trade credit. Furthermore, the volatility of net trade credit received by firms is increasing in the volatility of their revenue and decreasing in their level of EBITDA. Viewed through the lens of our theory, these findings imply that more debt-constrained and more inherently volatile firms receive more insurance via trade credit from their partners.

Our theoretical and empirical results suggest that trade credit mitigates idiosyncratic risks. In order to evaluate whether these insurance properties of trade credit have aggregate effects, we calibrate the model's parameters to match moments from the distributions of firm sales, profitability, debt and trade credit, and we engage in a counterfactual exercise. In particular, we compare the volatility of GDP as well as the share of trade credit in liabilities in our model to alternative economies that are nested by our framework and feature (i) no trade credit and (ii) non-state-contingent trade credit only. In light of a negative aggregate financial shock, our model generates the lowest GDP volatility, the model without any trade credit generates the highest volatility, and the model that features only non-state-contingent trade credit lies in the middle. Moreover, while trade credit drops sharply relative to GDP in our model, its share in total liabilities rises considerably more than in the alternative economy with non-state-contingent trade credit. These findings demonstrate the macroeconomic stabilizing role of state-contingent trade credit.

The assumption that trade credit is state contingent is critical for our results, and it is supported by the empirical corporate finance literature. Trade credit is inherently flexible because, unlike financial institutions, non-financial firms face considerably fewer regulations regarding their balance sheet performance and their handling of accounts receivable that are past due. Firms exploit this flexibility and selectively enforce the trade-credit contract terms such as penalties for late payments. Late payment in trade credit is widespread. A 2018 survey of firms in Western Europe shows that 88% of them had frequent late payments, corresponding to 42% of trade credit volume.[1] Another survey estimates that trade credit globally in 2018 was 66 days overdue on average (Wu et al., 2020). In the US, a 2003 survey of small businesses revealed that half of firms reported that their main supplier did not impose late penalties on trade credit payment, while those that did charged 2% monthly.[2] These late payments help firms to manage liquidity (Wu et al., 2020).

There is a small corporate finance literature that examines the insurance properties of trade credit theoretically. Notably, Wilner (2000) demonstrates that, in the context of repeated interactions, trade creditor firms, desiring to maintain an enduring product market relationship, grant more concessions to a customer in financial distress than would be granted by lenders in a competitive credit market because the firms exercise market power in their bilateral relationships. Cuñat (2007) examines how, in a context of limited enforceability

[1] Atradius Payment Practices Barometer. The volume of trade credit paid late rose to 53% in 2021.

[2] 2003 U.S. Survey of Small Business Finances. 69% of firms surveyed used trade credit, while 28% used trade credit and made late payments. Of that number, 70% of the value was paid on time on average, with the other 30% paid late.

of contracts, suppliers act as liquidity providers insuring against liquidity shocks that could endanger the survival of their customer relationships. Finally, Yang and Birge (2018) model firms as sharing inventory risk with suppliers by basing repayment on portion of goods actually sold to customers, making trade credit effectively state contingent. Suppliers accept this arrangement and do not impose late penalties because it enables larger volumes to be purchased.

The above literature focuses on the decision of individual firms and ignores the general equilibrium aspects of trade credit. In contrast, the macroeconomic literature that examines trade credit models this source of financing similarly to bank debt, thus ignoring its insurance properties. Shao (2017) shows that trade credit helps channel funds from financially-unconstrained to constrained firms, and in the face of financial market distress, suppliers reduce trade credit lending, further tightening their customers' borrowing constraints. Similarly, Reischer (2019) argues that firms smooth shocks by substituting bank and trade credit, and an increase in the cost of trade credit amplifies financial shocks by tightening the financing condition of customers. In both models, trade credit propagates shocks through the economy. Alfaro et al. (2021) find that credit supply shocks can propagate downstream through production networks, both via price and the trade credit extended by suppliers. Kalemli-Ozcan et al. (2014) develop a model of production chains that predicts that more trade credit is supplied by firms that are more upstream, but these upstream firms are more sensitive to changes in the availability of credit. The model implies that shocks to bank credit can amplify its real impacts via production chains. Unlike these papers, in our model, state-contingent trade credit mutes aggregate shocks along the supply chain.

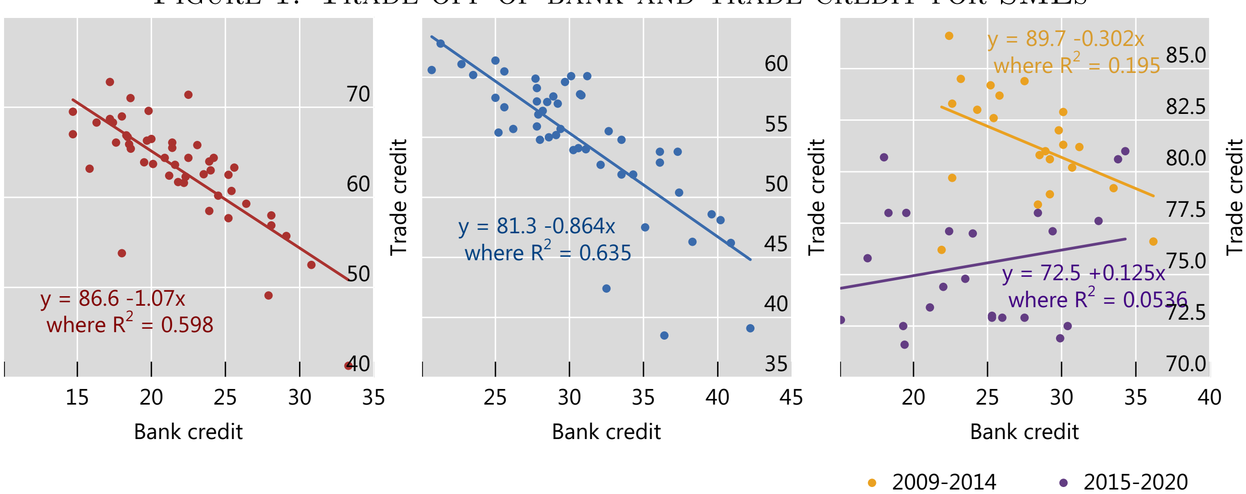
A large empirical literature documents the stabilizing role of trade credit, especially during crises episodes. Using a supplier–client matched sample, Garcia-Appendini and Montoriol-Garriga (2013) find that the negative shock to bank credit that firms experienced during the global financial crisis resulted in firms with high pre-crisis liquidity levels extending more trade credit to other corporations, and subsequently experiencing better performance as compared with ex-ante cash-poor firms. Additionally, trade credit taken by constrained firms increased during this period. Using data on liquidity shortfalls generated by the fraud and failure of Swedish cash-in-transit firm Panaxia, Amberg et al. (2021) demonstrate that firms manage liquidity shortages by increasing the amount of credit drawn from suppliers and decreasing the amount issued to customers. They find that the underlying mechanism in trade credit adjustments is in part due to shifts in overdue payments. In the context of natural disaster shocks, Ersahin et al. (2021) show that affected firms extend more trade credit, especially if their customers are difficult to replace. Love et al. (2007) examine emerging markets and find that trade credit lending by large publicly traded firms increases initially during the global financial crisis, but then contracts afterwards. They argue that firms with financial resources smooth out shocks by redistributing those resources via trade credit. Finally, Hardy and Saffie (2019) find that, during the peso depreciation that followed the Lehman Brothers' collapse, listed firms in Mexico which experienced a balance sheet shock contracted their investment more than other firms, but did not decrease their trade credit lending more than other firms. This finding suggests that large firms protected their value chains by absorbing most of the exchange rate shock.

The remainder of the paper proceeds as follows: In Section 2, we present basic facts on the trade off between bank and trade credit. In Section 3, we outline our novel theory of trade credit. In Section 4, we explore the theoretical predictions of the model. In Section 5, we present the empirical results. In Section 6, we report the results from the counterfactual exercises. We conclude in Section 7. We relegate all derivations and proofs to Appendix A.

2 Trade Credit Facts

In this section, we use data from Mexico to establish basic facts about the role that trade credit plays in a typical emerging market. We begin by studying the trade off between bank credit and trade credit for SMEs, which don't have access to a broad range of financial instruments. In order to examine this relationship for SMEs, we turn to the Credit Market Survey conducted quarterly by the Banco de Mexico. This survey asks at least 450 firms across Mexico about their access to different forms of credit, accessibility of bank credit, as well as their extension of trade credit to other firms. It then provides aggregated responses by firm size. There is a structural break in the survey around 2009, the time of the great financial crisis (GFC), so we examine the pre- and post-GFC data separately.

Figure 1: Trade-off of bank and trade credit for SMEs



(a) Small firms, 1998Q1-2008Q2

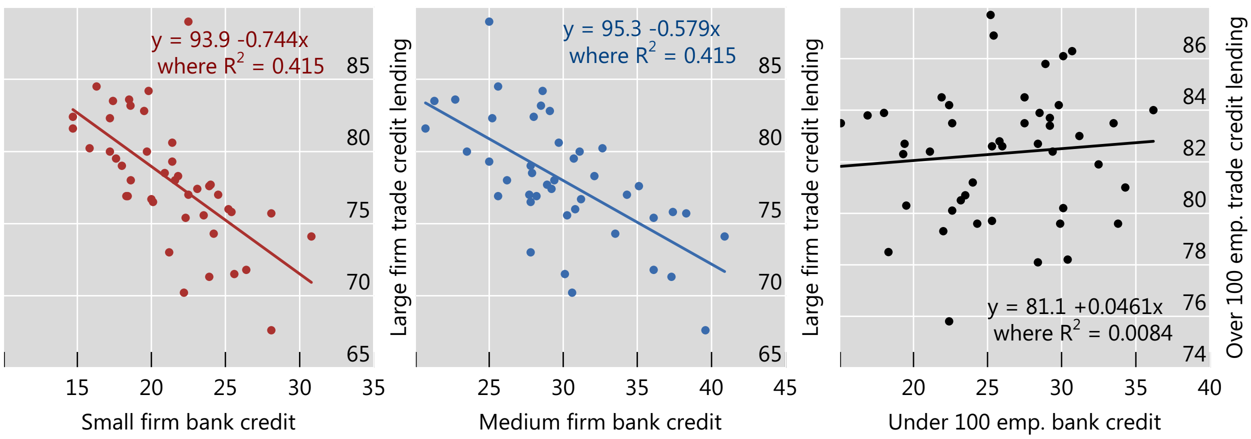
(b) Medium firms, 1998Q1-2008Q2

(c) Firms with <100 employees, 2009Q3-2020Q3

Percent of firms receiving bank credit vs. percent of firms receiving trade credit (or for pre-2009, whose main credit source is trade credit). Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Source: Credit market survey, Banco de Mexico.

Fact 1: SMEs trade off bank credit for trade credit. Figure 1 illustrates the trade-off between bank and trade credit for SMEs. The left and center panels show that, as the proportion of SMEs with any bank credit declines, the proportion listing trade credit as their most important source of credit rises. In the post-GFC survey in the right panel, we see that, for the first 5 years after the GFC, as the share of firms with any bank credit declines, the share of firms with any trade credit (not just those for whom it is most important) increases. The latter half of the sample (purple dots) shows that this relationship is not always a strict trade-off. Indeed, during periods of growth we may expect firms to increase their access to both bank credit and trade credit (although the purple slope is not statistically significant). Further, these surveys may miss the relationship between the volume of trade or bank credit that firms receive. Nevertheless, there appears to be a relevant trade off in use or importance of trade credit for SMEs.

Figure 2: Trade credit lending by large firms and bank credit of SMEs



(a) Small firms, 1998q1-2008q2

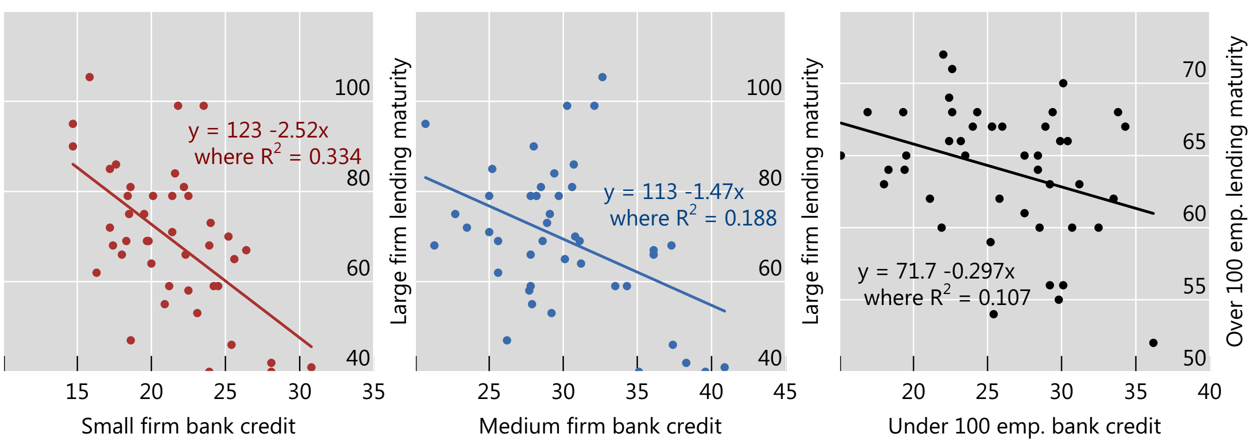
(b) Medium firms, 1998q1-2008q2

(c) Firms with <100 employees, 2009q3-2020q3

Percent of firms receiving bank credit vs. percent of large firms extending trade credit. Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Large firms have 1997 sales between 501-5000 million pesos. Firms with more than 5000 million pesos in sales make up less than 4% of the survey sample, so their responses are excluded. Source: Credit market survey, Banco de Mexico.

Fact 2: Large firms extend more trade credit when SMEs' bank loans decline. Figure 2 examines how trade credit lending by large firms evolves as SMEs report less use of bank credit. Figures 2a and 2b show that, as the share of SMEs with any bank credit declines, the share of large firms extending trade credit to other firms increases. The survey does not reveal whether the share of trade credit lending to SMEs increases, but the trade credit networks of large firms expand at the same time as SMEs are receiving less credit from banks. Post-GFC, this relationship is less clear in terms of the share of large firms extending trade credit. However, the volume of trade credit extended by such firms could increase. Consistent with this, we observe in Figure 3 that the maturity of trade credit lending by large firms lengthens as SMEs' use of bank credit declines. This holds both pre- and post-GFC. Effectively, giving longer repayment terms for trade credit could increase the volume of trade credit between firms in a given quarter, in addition to easing the financing burden on the recipient firms.

Figure 3: Average maturity of trade credit granted by large firms vs bank credit of SMEs



(a) Small firms, 1998q1-2008q2

(b) Medium firms, 1998q1-2008q2

(c) Firms with <100 employees, 2009q3-2020q3

Percent of SMEs receiving bank credit vs. average maturity in days of trade credit lent by large firms. Small firms have 1997 sales between 1-100 million pesos. Medium firms have 1997 sales between 101-500 million pesos. Large firms have 1997 sales between 501-5000 million pesos. Firms with more than 5000 million pesos in sales make up less than 4% of the survey sample, so their responses are excluded. Source: Credit market survey, Banco de Mexico.

Facts 1 and 2 are consistent with the empirical literature on trade credit. Petersen and Rajan (1997) document that small firms rely on trade credit when bank credit is unavailable, firms with access to (bank) credit offer more trade credit, and suppliers extend more trade credit to financially-constrained firms.

Fact 3: Trade credit rises during adverse economic conditions. To examine the trade credit lending of large firms in greater detail, we turn to detailed firm-level data for stock-market listed non-financial firms in Mexico. This dataset is derived from quarterly financial statements made by companies listed on the Mexican Stock Exchange (BMV) comprising 183 firms (unbalanced) over 2005q1-2015q2.[3] From this data, we have a detailed look at the sources of financing for these firms (i.e. the structure of their liabilities) as well as how these resources are used (i.e. the structure of their assets).

Large firms have much broader access to credit than SMEs, and are able to tap into sources of credit even when credit becomes tighter for the general economy. This easier access to credit enables large firms to serve as a type of financial intermediary for other firms, borrowing from traditional sources of credit and then increasing their extension of trade credit to other firms (Hardy and Saffie, 2019).

We next examine the propensity of firms to use each source of their borrowing to finance trade credit, and if that relationship changes during the GFC when credit conditions tighten. We run the following regression:

$$\frac{\Delta Accounts\ Receivable_{it}}{Assets_{it-1}} = \alpha_i + \alpha_t + \sum_{j \in FS} \frac{\Delta Funding\ Source^j_{it}}{Assets_{it-1}} (\beta^j_1 + \beta^j_2 Crisis_t) + \epsilon_{it}$$

where FS is the set of funding sources of the firm: net profits ("cash flow"), bonds, loans, trade credit, or other liabilities. $Crisis$ takes a value of 1 over 2008q3-2010q2.

The results are shown in Table 1. The interpretation of coefficients in this table is as

[3] See Hardy (2018) for more details of this dataset.

Table 1: Corporate funding and the supply of trade credit

	(1) Total	(2) Customers	(3) Others
Cash Flow	0.141***	0.0930***	0.0482*
	(0.0386)	(0.0220)	(0.0283)
Δ Bond	0.168***	0.0793**	0.0890**
	(0.0540)	(0.0366)	(0.0381)
Δ Loan	0.183***	0.127***	0.0554***
	(0.0301)	(0.0261)	(0.0167)
Δ Trade Credit	0.212***	0.187***	0.0249
	(0.0415)	(0.0346)	(0.0304)
Δ Other Liab	0.159***	0.0952***	0.0638**
	(0.0397)	(0.0226)	(0.0310)
Cash Flow × Crisis	0.229***	0.199***	0.0303
	(0.0654)	(0.0522)	(0.0455)
Δ Bond × Crisis	0.312***	-0.0800**	0.392***
	(0.0589)	(0.0391)	(0.0393)
Δ Loan × Crisis	0.106*	0.0196	0.0860***
	(0.0630)	(0.0450)	(0.0323)
Δ Trade Credit × Crisis	0.00193	-0.0338	0.0355
	(0.0731)	(0.0604)	(0.0453)
Δ Other Liab × Crisis	0.0600	0.0137	0.0466
	(0.0678)	(0.0513)	(0.0387)
Observations	4771	4779	4771
R^2	0.177	0.0633	0.222
Firms	183	183	183
FirmFE	Yes	Yes	Yes
TimeFE	Yes	Yes	Yes

Sample spans 2005q2-2015q2. Row labeled Firms reports the number of firms in each regression. Dependent variable is change in accounts receivable (either total, those to customers, or those to non-customers). Cash flow is net income over the previous quarter; Δ Bond is the change in bond debt over the previous quarter; Δ Loan is change in bank debt over the previous quarter; Δ Trade Credit is the change in trade credit liabilities (accounts payable) over the previous quarter. Δ Other is the change in all other liabilities (besides bank, trade, and bond credit) over the previous quarter. All variables are normalized by lagged assets. Crisis is a dummy taking a value of 1 over 2008q3-2010q2. Errors are clustered at the firm level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

follows: for every dollar increase that the firm receives via each source (e.g. \$1 more in loans), the coefficient indicates how much of this dollar is allocated towards trade credit lending (accounts receivables). For example, column (1) indicates that on average \$0.18 of a \$1 increase in bank loans finances the firms' extension of trade credit to other firms. This proportion during normal times (excluding the GFC) ranges from \$0.14 for internal cash flow from profits up to \$0.21 for credit received from other firms.

During the GFC, the propensity of firms to use these sources of credit to finance their own extension of trade credit increases. Specifically, an additional \$0.23 out of every dollar of profits goes to fund trade credit lending, as well as an additional \$0.31 from every dollar of new bond debt and \$0.11 of every new loan. Columns (2) and (3) break this relationship down by trade credit to the firm's customers vs. trade credit to other non-customer firms (suppliers, related firms, other firms). We see that these large firms utilize their profits in order to provide financing to their customers, whereas any additional bond or loan borrowing is actually used to support trade credit to non-customers. These findings suggest that firms extend trade credit when debt supply dries up, which makes trade credit assume the role of a macroeconomic stabilizer. In the next section, we develop a theory where trade credit assumes this very role.

3 Theory of Trade Credit

The economy consists of four types of agents: a unit measure of producers of an intermediate good, a unit measure of producers of a final good, who use the intermediate good to produce and deliver a final product to the consumers, a perfectly-competitive wholesaler whose role is to ensure that all markets for the intermediate good clear, and a perfectly-competitive bank that provides firms with credit. We label the intermediate-good producer (who sells goods to the final good producer) as "seller" and we label the final-good producer as "buyer".

The time horizon consists of two periods. In period 1, there is uncertainty regarding the realization of price (alternatively, demand) for the final good, which is sold in period 2.

In period 1, sellers use labor in order to produce the intermediate good according to a production function $X = \ln L$, where X denotes the quantity of intermediate good produced and L denotes the amount of labor units employed at wage rate w. Sellers begin the period with zero net worth, so in order to hire labor, they need to raise funds. Let s denote a given seller. They can borrow an amount D_s from a bank, which needs to be repaid in period 2 at interest rate r^*. Any amount saved between period 1 and 2 earns the same rate of interest, r^*. The seller also incurs a borrowing cost $\psi_s D_s^2$, where higher values of $\psi_s > 0$ are associated with relatively more debt-constrained sellers. Alternatively, a seller may obtain credit from a final-good producer to whom they sell the intermediate product, i.e. trade credit. Net trade credit can be negative; that is, a seller may opt to extend trade credit to a buyer. Buyers obtain intermediate goods from sellers in period 1 and transform them into final goods using a linear technology, where a unit of input yields a unit of final good. Like sellers, buyers begin period 1 with zero net worth and need to raise funds. Let b denote a given buyer characterized by their borrowing cost $\psi_b > 0$. They can raise debt D_b to be repaid in period 2 at interest rate r^* while incurring additional cost of borrowing $\psi_b D_b^2$. Alternatively, they can be a net recipient of trade credit, in which case they are a debtor.

Two variables in the model reflect the trade credit terms. First, A_b denotes the amount of funds that a buyer, b, pays to any seller in period 1. A positive amount of A_b represents a trade credit receipt for the seller or an advance payment for the buyer. In this case, the seller has accounts payable outstanding. Second, $T(z)$ denotes the amount of funds that the seller obtains from a buyer in period 2, where z denotes the realization of the price of the final good. A positive entry of $T(z)$ represents accounts receivable for the seller as it is a payment that the buyer makes after the delivery of the goods. For simplicity, assume there are two states of nature denoted by $\bar{z} > \underline{z} > 0$. The combination of A_b and $T(z)$ determines whether the seller is a net debtor or a net creditor vis-à-vis the buyer. The higher the value for $T(z)$, the more trade credit is awarded to the buyer, since they are more relieved from paying for inputs in advance (the lower is A_b) and therefore they are more likely to be a net debtor. Consider the extreme scenario in which $A_b = 0$. Then, the buyer does not pay anything in advance and settles all accounts in period 2. In this case, the buyer is a debtor and the seller is a creditor. Alternatively, suppose $T(z) = 0 \ \forall z$. In this case, the buyer pays the seller entirely in advance, which makes them the creditor and the seller the debtor. Notice that, unlike debt, trade credit is state-contingent. Hence, when the price of the final good is lower, the buyer may give a lower transfer $T(z)$ to the seller in period 2. Thus, the seller can effectively provide insurance to the buyer in bad states and demand compensation in good states via higher transfers $T(z)$. We assume that insurance provision is costly; that is, a seller incurs a cost of $\xi > 0$ units whenever $T(\bar{z}) \neq T(\underline{z})$.

Buyers (sellers) are differentiated by their cost of borrowing, $\psi_b > 0$ ($\psi_s > 0$). Each buyer (seller) draws ψ_b (ψ_s) from a beta distribution with shape parameters α and η at the beginning of period 1. Buyers and sellers randomly match in period 1 according to the following matching function $M(B, S) = \gamma \frac{B \cdot S}{B+S}$, where $B = 1$ is the measure of buyers, $S = 1$ is the measure of sellers and $\gamma \in (0, 2)$ is a parameter that measures the matching efficiency. Hence, the total number of exogenous matches per period is $\frac{\gamma}{2}$. Unmatched agents interact with a wholesaler. In particular, unmatched buyers buy the intermediate good from a wholesaler at a given price p_x. The wholesaler does not extend trade credit; hence, debt is the only source of funding for unmatched buyers. Similarly, unmatched sellers sell their (intermediate) product to the wholesaler at price p_x. The wholesaler does not extend them trade credit, so they need to finance all costs of production using debt. In contrast, buyers and sellers who have been matched bargain over the amount of intermediate good produced, X, the amount of debt raised in period 1, D_b and D_s, the amount of savings between the two periods, B_b and B_s, and the terms of trade credit, A_b and $T(z)$.

3.1 Dealings with Wholesaler

Figures 9a and 9b in Appendix A.7 summarize the timeline and the decisions of unmatched agents visually.

3.1.1 Seller's Problem

An unmatched seller has to raise debt from the bank in period 1 in order to produce. They get paid from the wholesaler in period 2 when they sell the product to them. They then pay off their debt. The seller's problem is summarized below[4]:

$$\max_{D_s \geq 0, L \geq 1} D_s - \psi_s D_s^2 - wL + \beta \left[p_x \ln L - D_s \left(1 + r^*\right)\right]$$

subject to:

$$\begin{aligned} D_s - \psi_s D_s^2 - wL &\geq 0 \\ p_x \ln L - D_s \left(1 + r^*\right) &\geq 0 \end{aligned}$$

The first constraint ensures that debt covers the cost of production and borrowing in the first period, while the second guarantees that the proceeds from a sale to the wholesaler are enough to cover debt repayment in the second period. The first constraint always holds with equality as it is sub-optimal to waste resources. Using the constraint to substitute out the expression for labor in the objective function and taking FOCs yields the following solution for the optimal amount of debt[5]:

[4] It is understood that all quantities are specific to a particular seller; for example $D_s(\psi_s)$, where ψ_s identifies a given seller. For ease of exposition, we suppress the notation in this section because we outline the problem for a given individual. We account for the individual's identity when we define equilibrium.

[5] See Appendix A for derivation.

$$D_s = \begin{cases} \frac{2\psi_s + \frac{1+r^*}{p_x} - \sqrt{4\psi_s^2 + \left(\frac{1+r^*}{p_x}\right)^2}}{2\psi_s \frac{1+r^*}{p_x}} & \text{if } p_x \geq (1+r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}} \\ 0 & \text{if } 0 < p_x < (1+r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}} \end{cases}$$

Substituting the optimal debt into the first (binding) constraint characterizes labor and production, where $X_s = \ln L$ denotes production by the seller. Substituting optimal debt and labor into the objective function yields the maximized value of a seller with cost draw ψ_s, which we denote by $\Gamma_s(\psi_s)$.[6]

3.1.2 Buyer's Problem

An unmatched buyer has to raise debt from the bank in the first period in order to buy the intermediate good (X_b) from the wholesaler. They sell the final good to the consumer in the second period and pay off their debt. The buyer's problem therefore is given by:

$$\max_{D_b, X_b \geq 0} D_b - p_x X_b - \psi_b D_b^2 + \beta E_z \left[z X_b - D_b \left(1 + r^*\right)\right]$$

subject to:

$$\begin{aligned} D_b - p_x X_b - \psi_b D_b^2 &\geq 0 \\ \underline{z} X_b - D_b \left(1 + r^*\right) &\geq 0 \end{aligned}$$

The two constraints have the same interpretation as in the case of the seller above, with the exception that the second constraint may only be binding in the low state of the world,

[6]It is understood that $\Gamma_s(\psi_s)$ also depends on p_x as well as parameters. We suppress the notation for ease of exposition until we define equilibrium.

making the constraint in the high state of the world redundant. The FOCs for the buyer's problem yield the following solution:

$$D_b = \begin{cases} \frac{1}{2\psi_b}\left[1 - \frac{p_x(1+r^*)}{\tilde{z}}\right] & \text{if } 0 < p_x \leq \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)} \\ \frac{1}{\psi_b}\left[1 - \frac{p_x(1+r^*)}{\underline{z}}\right] & \text{if } \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)} < p_x \leq \frac{\underline{z}}{1+r^*} \end{cases}$$

The optimal quantity of intermediate good purchased, X_b, follows from the constraints. Substituting optimal debt and intermediate good purchased into the objective function yields the maximized value of a buyer with cost ψ_b, denoted by $\Gamma_b(\psi_b)$. This value is decreasing in ψ_b, which can be verified by substituting the optimal debt in the objective function.

3.1.3 Summary of Unmatched Agents

The buyer and the seller put opposing pressures on the equilibrium price offered by the wholesaler. We assume that the wholesale market is perfectly competitive, and that wholesalers earn zero profits by ensuring that the amount of intermediate good purchased equals the amount of intermediate good sold among unmatched agents. We define equilibrium in Section 3.3. Before we do so, however, we turn to the problem that matched agents solve. In that decentralized marketplace, the maximized value functions of the buyer and of the seller from the centralized marketplace, $\Gamma_b(\psi_b)$ and $\Gamma_s(\psi_s)$, respectively, represent outside options. Since $\Gamma_b(\psi_b)$ is decreasing, more debt-constrained buyers have lower outside options and therefore lower bargaining power. Their outside option will play a critical role in the bilateral problem solution, which we describe next.

3.2 Bilateral Matches

Figures 10a and 10b in Appendix A.7 summarize the timeline and the decisions of matched agents visually. For simplicity, we assume that sellers make take-it-or-leave-it offers to buyers.

The seller solves the following problem:

$$\max_{D_s \geq 0, D_b \geq 0, L \geq 1, \underline{T}, \bar{T}, A_b, B_s, B_b} D_s + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} - \psi_s (D_s)^2 +$$
$$\beta\left[(B_s - D_s)(1 + r^*) + \tilde{T}\right] - \Gamma_s(\psi_s)$$

subject to:

$$D_b - \psi_b (D_b)^2 - A_b + \beta\left[(B_b - D_b)(1 + r^*) - \tilde{T} + \tilde{z}\ln L\right] - \Gamma_b(\psi_b) \geq 0 \tag{1}$$

$$B_b = D_b - A_b - \psi_b (D_b)^2 \tag{2}$$

$$B_s = D_s + A_b - wL - \xi \mathbf{1}\left\{\frac{\bar{T}}{\underline{T}} \neq 1\right\} - \psi_s (D_s)^2 \tag{3}$$

$$B_s \geq 0 \tag{4}$$

$$B_b \geq 0 \tag{5}$$

$$(B_s - D_s)(1 + r^*) + \underline{T} \geq 0 \tag{6}$$

$$(B_b - D_b)(1 + r^*) - \underline{T} + \underline{z}\ln L \geq 0 \tag{7}$$

$$(B_b - D_b)(1 + r^*) - \bar{T} + \bar{z}\ln L \geq 0 \tag{8}$$

In the above problem, $\bar{T} \equiv T(\bar{z}), \underline{T} \equiv T(\underline{z})$, $\tilde{T} \equiv p\underline{T} + (1-p)\bar{T}$, and $\tilde{z} \equiv p\underline{z} + (1-p)\bar{z}$, where $p \in (0, 1)$ is the probability that the final good price in period 2 is $\underline{z}$; i.e. the probability that the bad state of the world occurs. Furthermore, expression (2) represents the savings of the buyer between the two periods, while (3) denotes the savings of the seller.

The seller maximizes their surplus from being matched over not being matched subject to the constraint in expression (1) that the buyer's surplus within a match does not fall short of their outside option, the constraints in expressions (4) and (5) that savings are non-negative, and the debt repayment constraints for the seller and the buyer, respectively, in expressions (6), (7) and (8). Clearly, if constraint (6) binds when $z = \underline{z}$, it cannot hold with equality when $z = \bar{z}$. Hence, it is sufficient to only consider the former case for the seller. Furthermore, substituting expression (2) into constraint (1) yields

$$\beta \left[(B_b - D_b)(1 + r^*) - \tilde{T} + \tilde{z} \ln L \right] - \Gamma_b (\psi_b) \geq 0 \tag{9}$$

From expression (9) it follows that constraints (7) and (8) cannot jointly bind for as long as the buyer's outside option is strictly positive. This observation will play an important role in arriving at the solution to the problem. In particular, we will examine various combinations of binding constraints, subject to the restrictions discussed above.

Solving the problem involves characterizing the solutions to four distinct cases. In the first case, the debt repayment constraints (6)-(8) are not binding. This is the unconstrained solution. In the second case, the debt repayment constraint for the seller is binding and debt repayment constraint (7) for the buyer is also binding, while the second-period transfers are equalized across the states of nature in order to avoid paying the insurance cost. In these two cases, trade credit does not provide agents with insurance. In the third (fourth) case, the debt repayment constraint for the seller is binding and debt repayment constraint 7 (8) for the buyer is also binding, but the second-period transfers are not equalized across the states of nature, so the seller incurs the cost $\xi > 0$. These two are the more interesting cases since agents provide each other with insurance via trade credit. We characterize each of these cases in turn below.

Finally, we assume that $\beta(1 + r^*) = 1$. Consequently, agents have no incentives to save, which implies that $B_s = B_b = 0$ and expressions (2)-(5) hold with equality throughout. For tractability, to each allocation below, we assign a numerical subscript that corresponds to the respective case, ex. $D_{b,1}$ is the debt allocation for a given buyer in case 1.

3.2.1 Case 1: Unconstrained agents

We assume that the debt repayment constraints (6)-(8) do not bind in any state of the world. This implies that $\bar{T}_1 = \underline{T_1}$ is feasible. Since $\xi > 0$, it is optimal to set $T_1 = \bar{T}_1 = \underline{T_1}$. That is, since insurance is costly to provide, in the equilibrium where agents are not credit constrained and therefore not in need of insurance, trade credit is equalized across states of the world. In addition, the seller always has the incentive to extract all the surplus from the buyer, which implies that constraint (1) is binding. In order to understand how debt levels and production behave in this equilibrium, denote by $\lambda_{i,1}$ the multiplier for the constraint in expression (i) above for case 1, take FOCs, and simplify to obtain:

$$L_1 = \frac{\tilde{z}\beta}{(1-\tilde{\lambda}_1)w} \tag{10}$$

$$D_{s,1} = \frac{\tilde{\lambda}_1}{2\psi_s\left(\tilde{\lambda}_1 - 1\right)} \tag{11}$$

$$D_{b,1} = \frac{\tilde{\lambda}_1}{2\psi_b\left(\tilde{\lambda}_1 - 1\right)}, \tag{12}$$

where $\tilde{\lambda}_1 \equiv \lambda_{3,1}$. Expressions (11) and (12) imply that, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner: $D_s/D_b = \psi_b/\psi_s$. Using this equality, together with expression (10) into constraints (2) and (3) yields a unique solution for the multiplier $\tilde{\lambda}_1 < 1$ given by:

$$\tilde{\lambda}_1 = 1 - \frac{\psi_s\psi_b}{\psi_s+\psi_b}\left[2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2}\right] \quad \text{if} \tag{13}$$

$$\frac{2\underline{z}\beta(\psi_b\psi_s)^2}{(\psi_s+\psi_b)^2}\ln\left(\frac{\frac{\tilde{z}\beta(\psi_s+\psi_b)}{w\psi_s\psi_b}}{\left[2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2}\right]}\right) \geq \frac{\psi_s\psi_b}{\psi_s+\psi_b} - \frac{1}{\left[2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2}\right]}$$

Having found the optimal $\tilde{\lambda}_1$, we can substitute expression (13) into expressions (10), (11), (12) and constraint (1) which holds with equality, in order to obtain the optimal levels of $L_1, D_{s,1}, D_{b,1}$ and T_1, respectively.

3.2.2 Case 2: Constrained agents and no insurance

When debt repayment constraints begin to bind, the optimal level of trade credit T_1 above is no longer feasible. Since insurance is costly ($\xi > 0$), debt constrained sellers may still find it optimal to not provide insurance to buyers. In this case, $T_2 = \bar{T}_2 = \underline{T_2}$, where T_2 is the transfer under the constrained scenario. Further, as discussed above, constraints (7) and (8) cannot be jointly binding. We consider the case where constraint (7) is the binding one.[7] Substituting expression (3) into the objective function for the seller obtains

$$\beta\left[\left(B_{s,2} - D_{s,2}\right)\left(1 + r^*\right) + T_2\right] - \Gamma_s\left(\psi_s\right)$$

Clearly, when $T_2 = \bar{T}_2 = \underline{T_2}$, constraint (6) also cannot bind for as long as the seller's outside option is strictly positive. Similarly, when $T_2 = \bar{T}_2 = \underline{T_2}$ and constraint (7) is binding, constraint (1) cannot be binding as long as the buyer's outside option is strictly positive, which is apparent from expression (9).

If $\lambda_{i,2}$ is the multiplier for the constraint in expression (i) above, the above discussion

[7] In Appendix A, we demonstrate that a case in which the debt repayment constraint for the seller is binding and debt repayment constraint (8) for the buyer is also binding, but the second-period transfers are equalized across the states of nature in order to avoid paying the insurance cost, is not feasible.

implies that $\lambda_{8,2} = \lambda_{6,2} = \lambda_{1,2} = 0$. Taking FOCs and simplifying yields:

$$L_2 = \frac{\underline{z}\beta}{(1-\tilde{\lambda}_2)w} \tag{14}$$

$$D_{s,2} = \frac{\tilde{\lambda}_2}{2\psi_s\left(\tilde{\lambda}_2 - 1\right)} \tag{15}$$

$$D_{b,2} = \frac{\tilde{\lambda}_2}{2\psi_b\left(\tilde{\lambda}_2 - 1\right)}, \tag{16}$$

where $\tilde{\lambda}_2 \equiv \lambda_{3,2}$.

Even when agents are constrained, in order to maximize production, each agent will borrow according to her borrowing capacity relative to her partner, which can be seen from expressions (15) and (16): $D_{s,2}/D_{b,2} = \psi_b/\psi_s$. Using this equality, together with expression (14) into constraints (2) and (3) allows to obtain a unique solution for the multiplier $\tilde{\lambda}_2 < 1$ given by:

$$\tilde{\lambda}_2 = 1 - \frac{\psi_s\psi_b}{\psi_s+\psi_b}\left[2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2}\right] \quad \text{if} \tag{17}$$

$$\underline{z}\beta \geq \frac{w\psi_s\psi_b}{\psi_s+\psi_b}\left[2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2}\right]$$

Having found the optimal $\tilde{\lambda}_2$, we can substitute expression (17) into expressions (14), (15), (16) and constraint (7) which holds with equality (imposing that $T_2 = \bar{T}_2 = \underline{T_2}$), in order to obtain the optimal levels of $L_2, D_{s,2}, D_{b,2}$ and T_2, respectively.

Notice that, since $\tilde{z} > \underline{z}$, it must be that $\tilde{\lambda}_2 > \tilde{\lambda}$. Since debt is decreasing in the multiplier, $D_{s,2} < D_{s,1}$ and $D_{b,2} < D_{b,1}$. Substituting expression (17) into (14) and taking the derivative with respect to $\underline{z}$ demonstrates that labor is increasing in the price of the final good, so $L_2 < L_1$. Thus, when the agents are constrained, production as well as debt are lower.

3.2.3 Case 3: Constrained agents and insurance; buyer is constrained in bad state

The next two cases are more interesting from the point of view of this paper because they describe situations in which transfers are state contingent, and therefore, trade credit provides agents with insurance. Denote by $\lambda_{i,3}$ the multiplier for the constraint in expression (i) above and combine the FOCs for $\bar{T}_3$ and $\underline{T_3}$ to obtain:

$$\lambda_{8,3}\frac{p}{1-p} = \lambda_{7,3} - \lambda_{6,3}$$

As discussed above, constraints (7) and (8) cannot be jointly binding. We consider the case where constraint (7) is the binding one. Then, $\lambda_{8,3} = 0$ implies that $\lambda_{6,3} = \lambda_{7,3}$.

After combining the FOCs with the constraints and simplifying, we have the following system of eight equations and eight unknowns ($\lambda_{2,3}, \lambda_{4,3}, \lambda_{6,3}, L_3, D_{s,3}, D_{b,3}, \bar{T}_3$ and $\underline{T_3}$):

$$L_3 = \frac{\beta\tilde{z} + \underline{z}\lambda_{6,3}}{(1-\lambda_{2,3})w}$$

$$0 = 1 + \lambda_{2,3} + \lambda_{4,3} + \frac{\lambda_{6,3}}{\beta}$$

$$D_{s,3} = \frac{1+\lambda_{4,3}}{2\psi_{s,3}\left(1-\lambda_{2,3}\right)}$$

$$D_{b,3} = \frac{1+\lambda_{4,3}}{2\psi_{b,3}\left(1-\lambda_{2,3}\right)}$$

$$\frac{\Gamma_b(\psi_b)}{\beta} = \tilde{z}\ln L_3 - p\underline{T_3} - (1-p)\bar{T}_3 - D_{b,3}(1+r^*) \tag{18}$$

$$0 = -\left(D_{s,3} + D_{b,3}\right) + wL_3 + \psi_s\left(D_{s,3}\right)^2 + \psi_b\left(D_{b,3}\right)^2 + \xi \tag{19}$$

$$\underline{T_3} = D_{s,3}(1+r^*) \tag{20}$$

$$D_{b,3} + D_{s,3} = \frac{\underline{z}\ln L_3}{1+r^*} \tag{21}$$

Similarly to the cases above, each agent will borrow according to her borrowing capacity relative to her partner: $D_{s,3}/D_{b,3} = \psi_b/\psi_s$. Using this equality in expression (19) allows us to characterize the optimal level of debt for the seller, $D_{s,3}$. Having found that, $D_{b,3}$ follows trivially from the proportionality result, and $\bar{T}_3, \underline{T_3}$ and L_3 follow from expressions (18), (20) and (21), respectively.

From expression (19), the optimal level of the seller's debt solves the following implicit function:

$$\begin{aligned} (1-\psi_s D_{s,3})\left(1+\frac{\psi_s}{\psi_b}\right) D_{s,3} &= we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,3}}{\underline{z}}} + \xi \\ \text{if} \quad \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) &\geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)} + \xi \end{aligned} \tag{22}$$

3.2.4 Case 4: Constrained agents and insurance; buyer is constrained in good state

As discussed above, constraints (7) and (8) cannot be jointly binding. Now, we consider the case where constraint (8) is the binding constraint for the buyer, while constraint (6) continues to bind for the seller. Taking FOCs and simplifying yields the following two expressions for debt levels:

$$D_{s,4} = \frac{p\,(1-\lambda_{1,4}) - \lambda_{3,4}}{2\psi_s\,(1-\lambda_{3,4})} \tag{23}$$

$$D_{b,4} = \frac{p\,(1-\lambda_{1,4}) - \lambda_{3,4}}{2\psi_b\,(1-\lambda_{3,4})} \tag{24}$$

Constraints (6) and (8) yield the values for $\underline{T_4}$ and $\bar{T}_4$, respectively, which together yield:

$$\tilde{T}_4 = D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s} - 1\right) + (1-p)\,\bar{z}\ln L_4 \tag{25}$$

Note that constraint (1) is binding in this case. In fact, because constraint (7) is not binding, we can increase $\underline{T_4}$. This would relax constraint (6) and not affect constraint (8). This change clearly benefits the seller, so the only reason not to do this is if it violates another constraint. That would be constraint (1). Combining constraints (1) and (2) and substituting expression (25) yields an expression for the wage bill:

$$wL_4 = we^{\frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}}} \tag{26}$$

Similarly to previous cases, each agent will borrow according to her borrowing capacity relative to her partner: $D_{s,4}/D_{b,4} = \psi_b/\psi_s$. Using this equality together with expression (26) in constraints (2) and (3) allows us to characterize the optimal level of debt for the seller, $D_{s,4}$. Having found that, $D_{b,4}$ follows trivially from the proportionality result, and $\underline{T}_4, \bar{T}_4$ and L_4 follow from constraints (6) and (8) and expression (26), respectively. The optimal level of debt $D_{s,4}$ solves the following implicit function:

$$\begin{aligned} (1-\psi_s D_{s,4})\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4} &= we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4}}{\underline{z}}+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi \\ \text{if} \quad \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) &\geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi \end{aligned} \tag{27}$$

The solutions to cases 3 and 4 closely resemble each other. They both yield state contingent transfers. The level of the transfers, as well as the level of debt and production varies across the cases. In Appendix A, we show that case 4 yields lower levels of debt and production than case 3.

3.3 Equilibrium

Let $\Omega \equiv \{w, \beta, \eta, \alpha, \gamma, r^*, p, \underline{z}, \bar{z}, \xi\}$ be a vector of parameters in the model. Let $V_{s,j}(\psi_s, \psi_b; p_x, \Omega)$ denote the value of a seller with cost draw ψ_s who is matched to a buyer with cost draw ψ_b in case $j \in \{1, 2, 3, 4\}$.[8] Similarly, let $V_{b,j}(\psi_s, \psi_b; p_x, \Omega)$ denote the value of a buyer with

[8]The exact expression for each case can be found in Appendix A.

cost draw ψ_b who is matched to a seller with cost draw ψ_s in case $j \in \{1,2,3,4\}$. Define the indicator $I_s(\psi_s, \psi_b; p_x, \Omega)$ to be 1 when $\forall j \in \{1,2,3,4\}$, $\Gamma_s(\psi_s; p_x, \Omega) \geq V_{s,j}(\psi_s, \psi_b; p_x, \Omega)$. Recall that the probability of a match (unconditional on bargaining outcomes) is $\frac{\alpha}{2}$, which implies that $1 - \frac{\alpha}{2}$ is the probability that agents will not be given the opportunity to match. Then, aggregate supply of the intermediate good is:

$$S(p_x, \Omega) = \left(1 - \frac{\alpha}{2}\right) \int_{\underline{\psi_s}}^{\bar{\psi_s}} X_s\left(\psi_s; p_x, \Omega\right) d\psi_s + \frac{\alpha}{2} \int_{\underline{\psi_s}}^{\bar{\psi_s}} \int_{\underline{\psi_b}}^{\bar{\psi_b}} I_s(\psi_s, \psi_b; p_x, \Omega) X_s\left(\psi_s; p_x, \Omega\right) d\psi_b d\psi_s \tag{28}$$

Define the indicator $I_b(\psi_s, \psi_b, ; p_x, \Omega)$ to be 1 when $\forall j \in \{1,2,3,4\}$, $\Gamma_b(\psi_b; p_x, \Omega) \geq V_{b,j}(\psi_s, \psi_b; p_x, \Omega)$. Then, aggregate demand for the intermediate good is:

$$D(p_x, \Omega) = \left(1 - \frac{\alpha}{2}\right) \int_{\underline{\psi_b}}^{\bar{\psi_b}} X_b\left(\psi_b; p_x, \Omega\right) d\psi_d + \frac{\alpha}{2} \int_{\underline{\psi_s}}^{\bar{\psi_s}} \int_{\underline{\psi_b}}^{\bar{\psi_b}} I_b(\psi_s, \psi_b; p_x, \Omega) X_b\left(\psi_b; p_x, \Omega\right) d\psi_b d\psi_s \tag{29}$$

Market clearing in the centralized market implies that

$$S(p_x, \Omega) = D(p_x, \Omega) \tag{30}$$

Definition 1 *For given parameter set Ω, equilibrium is a price $p_x \in (0, \infty)$, allocations for the centralized market $\{D_s(\psi_s; p_x, \Omega), D_b(\psi_b; p_x, \Omega), X_s(\psi_s; p_x, \Omega), X_b(\psi_b; p_x, \Omega)\}$, Lagrange multipliers for the decentralized market $\{\tilde{\lambda}_1(\psi_s, \psi_b; p_x, \Omega), \tilde{\lambda}_2(\psi_s, \psi_b; p_x, \Omega)\}$, and debt allocations for the decentralized market $\{D_{s,3}(\psi_s, \psi_b; p_x, \Omega), D_{b,3}(\psi_s, \psi_b; p_x, \Omega), D_{s,4}(\psi_s, \psi_b; p_x, \Omega),$ $D_{b,4}(\psi_s, \psi_b; p_x, \Omega)\}$ that satisfy: (i) the buyer's problem's solution in the centralized market, (ii) the seller's problem's solution in the centralized market, (iii) market clearing in the centralized market given by expression (30), (iv) shadow prices in expressions (13) and (17); (v) the optimal levels of seller debt in the decentralized markets given by expressions (22) and (27); and (vi) the optimal levels of buyer debt in the decentralized markets that satisfy $D_{b,j}(\psi_s, \psi_b; p_x, \Omega) = D_{s,j}(\psi_s, \psi_b; p_x, \Omega)\frac{\psi_s}{\psi_b}$ for $j = \{3, 4\}$.*

4 Numerical Analysis: Model Predictions

To better highlight the insights from the model we calibrate its parameters to match moments from the distributions of firm sales, profitability, trade credit, and debt in eight emerging markets, and we generate numerical solutions to a number of variables of interest.

4.1 Data Description

We use firm-level data spanning 2009-2019 from eight emerging markets: Korea, Czech Republic, Croatia, Estonia, Poland, Slovenia, Hungary and Romania. We utilize the ORBIS database, which has annual firm-level observations with balance-sheet and other information.[9]

We clean the sample, dropping unusual observations (ex. negative assets or employees).[10] The main restriction on our sample is coverage for variables of bank debt, trade credit borrowing, and trade credit lending. For these variables, it is difficult to distinguish between a true 0 and a missing observation marked with a 0. We restrict our sample to firms that had non-missing data during 2009-2019, and had a positive value for one of these three variables at least once over that period.

4.2 Calibration

We set the exogenous wage to 0.1 and the discount factor β to 0.95, which fixes the interest rate r^*. We solve the model with 100 grid points for each buyer and seller type, which gives 100 x 100 potential pairs of matches. We simulate 7500 pairs, or 15000 firms in total. With this panel of firms, we calculate model moments and we match the corresponding moments

[9] We choose to work with these countries because they are emerging economies with solid coverage in the ORBIS database. Our findings are robust to including in the analysis Mexico and Brazil, which have very limited coverage. Bajgar et al. (2020) describe the degree of coverage for different countries in the ORBIS database.

[10] We largely follow the construction/cleaning process outlined in Kalemli-Ozcan et al. (2015). We drop financial and public administration firms from our sample.

in the data. There are seven moments and seven parameters. Every moment is related to all the parameters due to the general-equilibrium nature of the problem, but some moments are more informative about some parameters. Table 2 summarizes the parameter estimates and the moments that are most informative about each parameter. The calibration minimizes the average of the absolute percent deviation between model and data. The overall loss is 7.68%.

Externally Calibrated Parameters		
Parameter	*Description*	*Value*
ω	Wage	0.1
β	Discount factor	0.95

Table 2: Internally Calibrated Parameters

	Calibrated Parameters				
Parameter	*Description*	*Value*	*Moment*	*Data*	*Model*
$\underline{z}$	Low demand	2.73	Acc. Pay. (P50 to P25)	4.8	4.84
$\bar{z}$	High demand	6.11	Acc. Pay. (P75 to P50)	3.6	3.56
ξ	Cost of insurance	0.28	Within firm stdev NTC to Sales (P75)	0.2	0.22
α	Shape parameter of Beta(α,η)	1.08	EBITDA (P50 to P25)	4.6	5.32
η	Shape parameter of Beta(α,η)	1.09	Total Debt (P50 to P25)	4.1	3.43
p_L	Low state probability	0.15	EBITDA to Sales (P75 to P25)	9.5	9.01
$\frac{\gamma}{2}$	Match probability	0.94	Fraction firms $AP > 0$	89%	93.01%

Since unmatched firms in the model pay for all purchases in advance, not all firms in the model have positive levels of trade credit. The same is true in the data. Thus, the first moment that we target is the fraction of firms with positive levels of accounts payable, which is informative regarding the value of the matching efficiency parameter, γ. Further, in the model, firms vary according to their cost of borrowing, ψ_j, which is drawn from the Beta distribution. This cost has a direct implication regarding the firm's level of debt, sales, and profitability. While the three variables are highly correlated with each other, the correlation is by no means perfect because the variables are affected by the firm's trade partner's characteristics. Furthermore, economy-wide variables such as aggregate demand and the cost of insurance affect the endogeneous choice that firms make whether to match as well as how much debt to raise, trade credit to extend, and output to produce. Therefore, we calibrate economy-wide parameters as well as parameters of the Beta distribution in order to match moments from the distributions of firms' sales, profitability, and debt.

In both the model and in the data, we normalize debt, trade credit, EBITDA and sales by their respective means, and we proceed to generate percentiles from these unit-free distributions. We target the ratio of the 50th to the 25th percentile as well as the ratio of the 75th to the 50th percentile from the distribution of accounts payable. Since accounts payable are state contingent, we use the expected accounts payable computed over the two states.

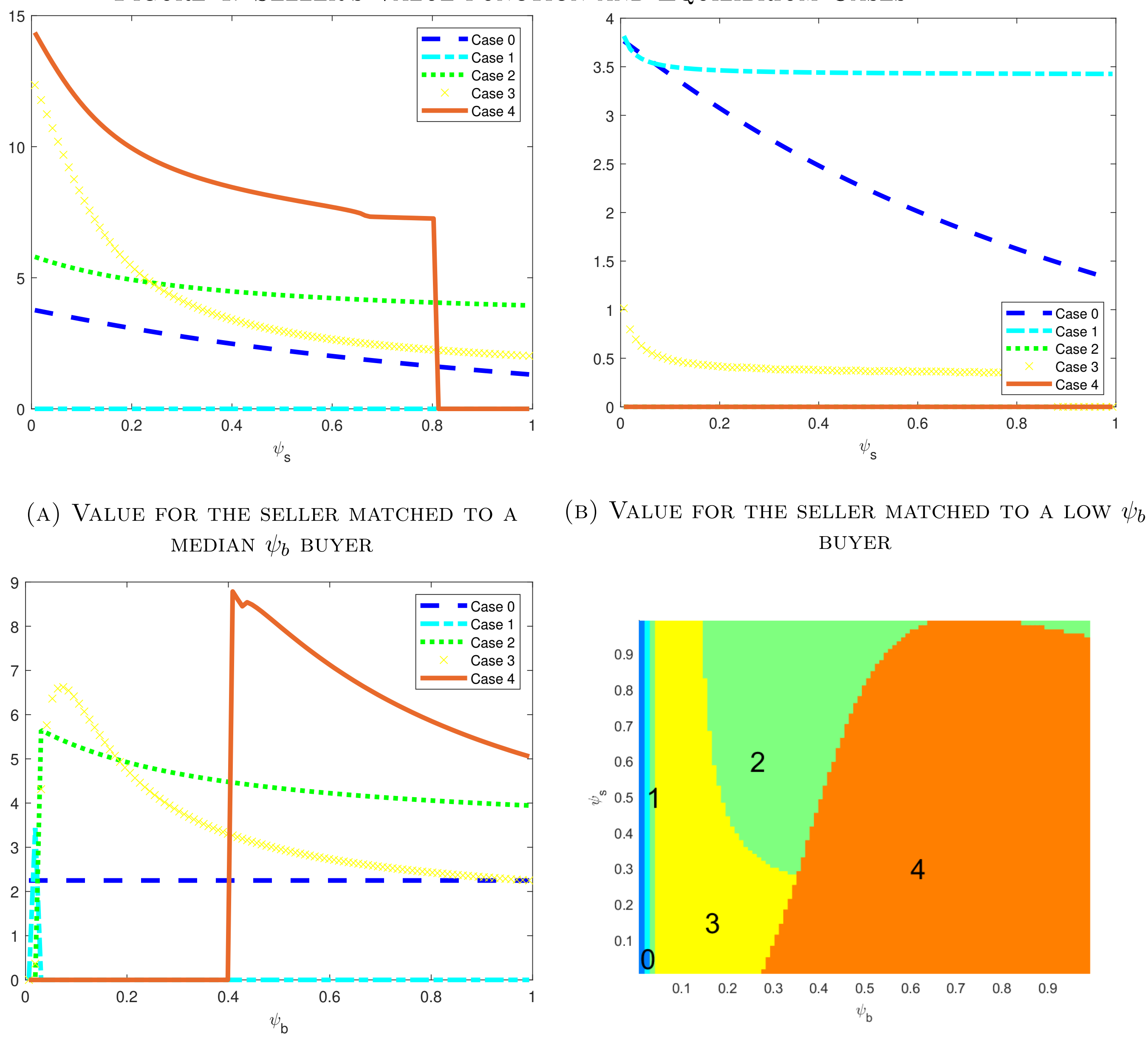
To measure profitability, we use a standard measure in the literature–EBITDA (see Lian and Ma (2021). We target the ratio of the 50th to the 25th percentile from this distribution. We target the same ratio from the debt distribution. In the data, we define total debt to be the sum of trade credit debt (Accounts Payable) and short-term debt. In the model, bank debt is given by the variable D_j.

We also target the ratio of the 75th to the 50th percentile from the distribution of EBITDA relative to sales across firms. Finally, we target the 75th percentile of the distribution of firm-level standard deviation of the fraction of net trade credit (defined as Accounts Payable minus Accounts Receivable) in sales. We compute first the ratio in the high and in the low state, respectively, then take the standard deviation of this ratio, and finally generate moments from the distribution of this standard deviation variable. We chose a moment from the upper tail of the distribution since only cases 3 and 4 in the model generate state-contingent trade credit, and therefore variation in this variable.

The Figures in Appendix A.7 plot the model's predicted distributions of firm borrowing costs, sales, profitability, and debt. The Beta distribution of firm types resembled the uniform distribution, but the distributions of firm sales, profitability and debt feature tails that resemble the commonly-reported empirical distributions.

4.3 Model Predictions

Figure 4: Seller's Value function and Equilibrium Cases



(a) Value for the seller matched to a median ψ_b buyer

(b) Value for the seller matched to a low ψ_b buyer

(c) Value for the median ψ_s Seller

(d) Case Policy

Figure 4 illustrates the equilibrium choices of a seller offered the opportunity to match with a buyer. The first three panels show for every case (0 to 4, where 0 denotes unmatched agent) the value $V_{s,j}(\psi_s, \psi_b)$ of a seller who was offered the possibility to match with a buyer. Note that, in case 0, the seller chooses to exercise their outside option. When a given case is not feasible, we set the value of that case to zero. Panel 4a displays the value for the seller when matched to the median buyer as a function of the type of the seller. Note that, for every case, the value of the seller decreases in ψ_s as borrowing becomes more expensive for the seller. When facing the median buyer, accepting a match strictly dominates dealing with the broker (case 0 is never preferred). Also, in this type of a match, the unconstrained allocation is not feasible, which is why case 1 always appears to be at value of 0. When the seller has access to relatively cheap bank loans (low ψ_s), the best option is to provide insurance to the buyer using contingent trade credit (cases 3 and 4). As borrowing becomes more expensive for the seller the dominating strategy is to provide non-contingent trade credit (case 2). Panel 4b shows the value for the seller when matched with a buyer who has a low value of ψ_b. In this case, the buyer does not need insurance from the seller since debt is relatively cheap for them, and the outside option for the buyer (dealing with the broker) is relatively high. Therefore, the seller either decides to refuse the match (case 0) or to select a non-contingent strategy (the unconstrained allocation in this match since it is feasible).

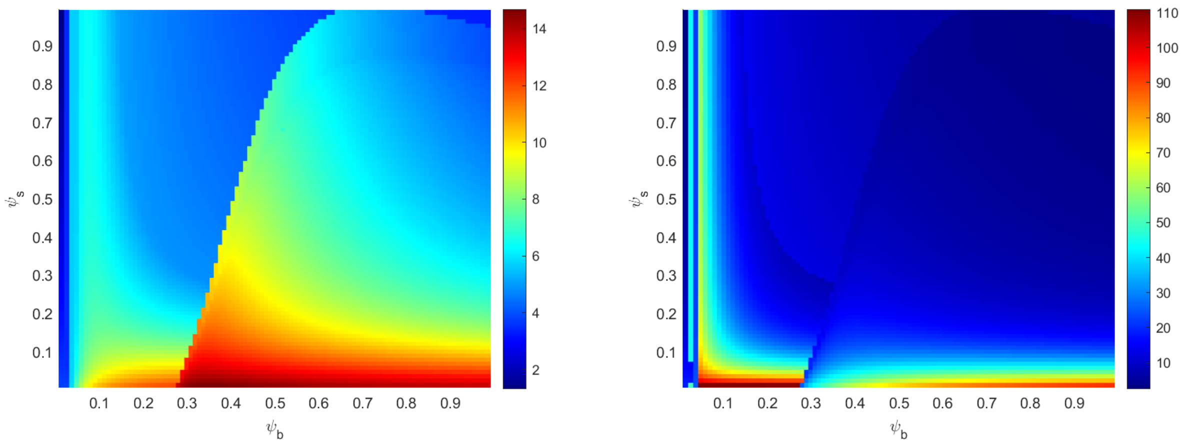
Panel 4c takes a different approach focusing on the median seller and seeing how their value changes as a function of the type of the buyer. Naturally, the value of the seller when interacting with the broker (case 0) is independent of the type of the buyer. Consistent with our former analysis, when the median seller faces a buyer with access to cheap credit, they decide to refuse the match as compensating the buyer for the outside option is very expensive. As the borrowing cost of the buyer increases, the seller decides to match without providing insurance (cases 1 and 2), only when the borrowing cost of the buyer becomes high enough, the seller decides to provide insurance (cases 3 and 4). Note that the value function of the seller is not always monotonic on the type of the buyer. This non-monotonicity is particularly clear for case 3. On the one hand, as the borrowing cost of the buyer increases, their outside option decreases allowing the seller to capture more surplus. On the other hand, when the borrowing cost of the buyer is too expensive, the joint borrowing capacity of the match is compromised and the production scale decreases, implying a lower surplus for the pair.

Panel 4d maps the complete equilibrium space. In a nutshell, sellers matched with buyers who have very low ψ_b prefer to deal with the wholesaler. When the buyer can borrow at low cost, or when borrowing is very expensive for both the buyer and the seller, the deal features non-contingent transfers. For buyers with higher cost ψ_b, the seller provides at least some insurance. In this equilibrium, 7% of agents that interact with wholesaler, of which 6% cannot match exogenously. Hence, very few sellers refuse the match offered (case 0) and prefer to interact with the wholesaler. Only 0.63% of the sellers end up in the unconstrained case (case 1), 22.33% are in case 2 and offer non-contingent trade credit, while case 3 and case 4 occur in 70.05% of the cases (or 74.86% of all matches). Therefore, a majority of the buyers are allowed to deliver contingent transfers to pay for their inputs.

Figure 5 shows how the value of the seller and their hiring decision are determined by the cost of borrowing of the match. Panel 5a show that sellers with low borrowing cost who are matched to buyers with high cost benefit the most from being matched (red region). Interestingly, these pairs are not the ones with the highest production. In fact, Panel 5b shows that low-ψ_s sellers matched with low- to medium-ψ_b buyers are the ones hiring more workers and therefore producing more intermediate goods. These sellers cannot benefit as much from the borrowing needs of the buyer and instead focus on maximizing the total surplus increasing production. Moreover, even relatively high-ψ_s sellers are able to scale production significantly when matched with low ψ_b buyers under a contingent transfer scheme (left border between cases 2 and 3 in Figure 4d).

The contrast between value and production in Figure 5 highlights the duality of trade credit. First, trade credit allows a high-ψ_s seller (ψ_b buyer) to scale up production by taking

Figure 5: Value and Labor

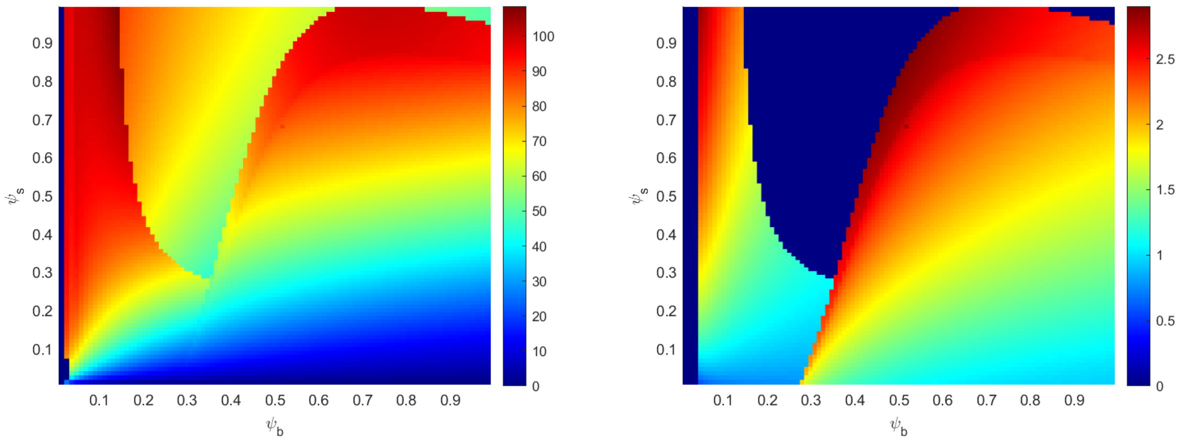


(a) Value for the Seller (b) Labor Hired by Seller

advantage of matching with a low-ψ_b buyer (ψ_s seller). The goal of trade credit in this case is to find cheap finance for intermediate good production. A second goal for trade credit is to provide repayment insurance to the buyer. This role is particularly important when buyers and sellers face moderate spreads. In that range, the buyer incurs non-trivial borrowing costs and, therefore, the second period repayment constraint is likely to bind in some state. In this situation, contingent trade credit allows the pair to achieve higher production by allowing the buyer to repay in every state.

Figure 6 explores the dual role of trade credit in the model. First, Panel 6a reflects the fraction of the labor cost of the seller that is covered by trade credit from the buyer $\left(\frac{A_b}{wL}\right)$ in percentage terms. Red colors indicate that the labor cost of the seller is intensively financed by trade credit from the buyer. It is clear that low-ψ_b buyers can also help high-ψ_s sellers achieve a higher production scale. In fact, up to 90% of the production cost of the seller in Panel 6a can be covered with trade credit from the buyer. Panel 6b plots the ratio between the contingent payments agreed upon by the seller and the buyer, in log terms, $\log\left(\frac{\bar{T}}{\underline{T}} - 1\right)$. This figure emphasizes the insurance role of trade credit. Red colors signal payments that are highly contingent on the demand realization. Although, with the exception of case 0,

Figure 6: Trade Credit: Scale and Insurance



(a) Labor cost covered by Buyer

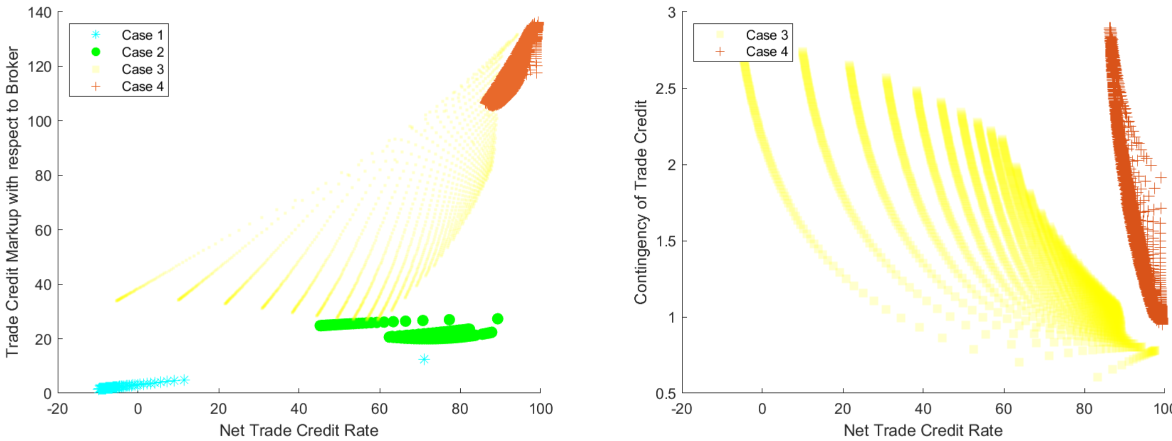
(b) State Contingency

every case exhibits some trade credit, only cases 3 and 4 feature contingent trade credit. For a given ψ_b, when ψ_s increases, the buyer receives more contingent trade credit. The reason is simple, the more costly borrowing becomes for the seller, the more borrowing is channeled through the buyer, therefore, the more insurance the buyer needs in order to be able to repay in both states of nature. Similarly, for a given ψ_s, contingency decreases with ψ_b as the seller borrows more and needs to repay unconditionally to the bank.

Typically trade credit carries a high implicit cost for the borrower (see Klapper et al. (2011)). Figure 7 studies how the cost, size, and contingency of trade credit are related. We infer the cost of trade credit for the buyer by comparing the average unit price of an intermediate good within the match to the market price when interacting with the wholesaler; since a market price is not directly observed in a bilateral meeting, we plot the total values of each exchange, in percentage terms, $\left(\frac{A_b+\beta\tilde{T}}{p_x X} - 1\right)$. This is an effective markup over the centralized market price and reflects how costly trade credit is.

Panel 7a plots the mark-up against the expected net trade credit rate received by the buyer defined as $\frac{\beta\tilde{T}-A_b}{A_b+\beta\tilde{T}}$ and plotted in percentage terms. The numerator reflects the difference between the discounted expected transfers that the buyer will make after receiving the

Figure 7: The Cost of Trade Credit

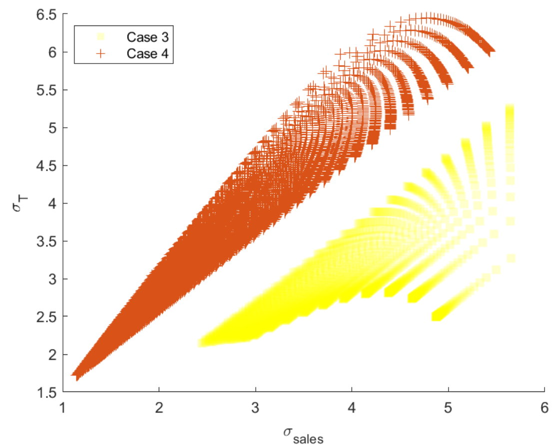


(a) Net trade credit to the Buyer (b) Labor cost covered by Buyer

intermediate inputs and the payment that they make before the seller delivers the intermediate goods. The denominator is the total size of the expected payout that the buyer makes to the seller. Note that, for all cases, the more credit the seller extends to the buyer, the more expensive trade credit becomes. Interestingly, under case 1, the buyer can be a net lender to the seller, while in all other cases the buyer is always a net debtor. Moreover, contingent trade credit (cases 3 and 4) is associated with higher trade credit markups. Panel 8 plots the contingency measure described in Panel 6b against the expected net trade credit rate received by the buyer for the cases that exhibit contingent trade credit. Note that case 4 is characterized by large, highly contingent and extremely expensive trade credit for the buyer, while trade credit under case 3 is less contingent, smaller, and cheaper. Finally, note that, within each case, the more trade credit the seller provides, the less contingent it becomes.

The above discussion suggests that the dispersion of trade credit is an appropriate statistic to measure the degree of state contingency. We have established that more debt-constrained firms receive more state-contingent trade credit. Moreover, Figure 8 shows that trade credit volatility is also increasing in the volatility of firms' sales as it is the same set of shocks that governs both variables. Therefore, it is important to control for sales volatility in our

Figure 8: Volatility of Trade Credit



empirical analysis to which we turn below.

5 Empirical Analysis: Test of Model's Predictions

In this section, we examine some predictions of the model using the ORBIS dataset described earlier.

Testable Prediction 1: Trade credit provision is driven by financial constraints of revenue-maximizing firms. We first examine how a firm's ability to raise debt as well as its scale of production affect its level of trade credit borrowing. In the model, a producer's cost of borrowing determines the amount of trade credit they receive: more (less) debt-constrained agents are net recipients (providers) of trade credit. These costs are not observable in the data; however, a firm's profitability is informative about the firm's financial capacity. Empirically, firm profitability is commonly measured by EBITDA, where firms with higher levels of EBITDA are considered to be less financially-constrained (see Lian and Ma (2021)).

To test the model's predictions regarding trade credit provision, we implement the following empirical specification:

$$NetTC_{icst} = \alpha_i + \alpha_{cst} + \beta_e EBITDA_{icst} + \zeta X_{icst} + \epsilon_{icst}$$

where i denotes a firm, c is a country, s is the 2-digit ISIC sector that firm i is in, and t denotes year.[11] $NetTC_{icst}$ is the firm's net trade credit received (Accounts Payable minus Accounts Receivable), normalized by the firm's total assets in order to remove scale effects and $EBITDA_{icst}$ is the firm's EBITDA normalized by assets. The theory predicts that more financially-constrained firms obtain more trade credit from their partners. Hence, it should be the case that the coefficient estimate for β_e is negative.

In addition, we control for firm and sector-country-year fixed effects to account for time invariant differences across firms and shocks to specific industries in each country. Implicitly, we assume that firms in same country, sector and year have similar trade partners, conditional on other firm observable characteristics. These characteristics include EBITDA as described above, as well as a range of variables that comprise the X_{icst} vector including the firm's sales normalized by assets, short-term debt normalized by assets, capital normalized by assets, the log of the firm's employment, and the log of the firm's assets in each period. All variables are in millions USD and are winsorized at 1%.

We opt to use short-term debt as a measure of the firm's indebtedness since, in our model, firms use debt and trade credit to finance daily operations; i.e. working capital needs. This interpretation is standard in the existing literature on financial frictions (see Dinlersoz et al. (2019)). We also show that our results are robust to using total (short- and long-term) debt.

The first column in Table 3 reports the regression results using short-term debt, while the second uses total debt. In both cases, as predicted by our theory, the coefficient estimate on EBITDA is negative. All coefficient estimates are statistically significant at the 1% level.

[11] The firms in the sample span 48 2-digit ISIC sectors.

Table 3: Net Trade Credit and Bank Debt

	(1) ST Debt	(2) Total Debt
EBITDA_{it}	-0.0987***	-0.103***
	(0.00227)	(0.00211)
Debt_{it}	-0.0505***	-0.0311***
	(0.00386)	(0.00328)
Sales_{it}	0.00368***	0.00497***
	(0.000441)	(0.000498)
Capital_{it}	0.0233***	0.0232***
	(0.00180)	(0.00191)
Log Emp_{it}	-0.00734***	-0.00638***
	(0.000679)	(0.000698)
$\log(\text{Assets}_{it})$	0.0215***	0.0203***
	(0.00149)	(0.00147)
Observations	2941910	2318117
R^2	0.0219	0.0224
FirmFE	Yes	Yes
CountrySectorYearFE	Yes	Yes

Dependent variable is net trade credit (accounts payable - accounts receivable) relative to total assets. Debt is total debt liabilities, ST debt is short term debt liabilities. Sales is the net sales revenues. Each of these is normalized by total assets. Other firm controls include EBITDA over assets, capital over assets, log employment, and log assets. All independent variables (except those in logs) are winsorized at 1%. R^2 is within R^2. Errors are clustered at the industry-year level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Other firm controls also affect the amount of trade credit that firms receive. In particular, firms with larger scale of operation receive more trade credit, which suggests that firms use trade credit to reach optimal production scale. Additionally, firms with more debt receive less trade credit, which suggests that the two sources of financing are substitutable. A large empirical literature has documented this fact. Furthermore, firms with more capital and assets receive more trade credit, while larger firms, as measured by employment size, receive less.

Testable Prediction 2: Trade credit provides insurance along supply chains. In order to test the model's predictions regarding insurance, we examine the volatility of trade credit received. Since trade credit is state contingent, the higher is the volatility of trade credit, the more insurance an agent receives because they have to pay less in a bad state of the world. According to the theory, more debt-constrained firms need more and therefore receive more state-contingent trade credit. Additionally, firms whose sales are more volatile by construction have more volatile trade credit levels because the same shock governs both variables.

With these predictions in mind, we run the following regression in the model:

$$Vol[NetTC_{ics}] = \alpha_{cs} + \beta_e EBITDA_{ics} + \beta_v Vol[Sales_{ics}] + \zeta X_{ics} + \epsilon_{ics}$$

where the regressand is given by $Vol\{\beta T - A_b\}$, which represents the standard deviation of net trade credit for the buyer computed over the two states of nature in the model corresponding to $\bar{T}$ and $\underline{T}$. The volatility of sales is computed following the same convention. Since we are computing one variable—the volatility–per firm, we no longer include firm fixed effects in the regression. The vector of additional controls includes the mean sales, capital, log of employment, log of assets over the same time period used to compute the volatility. In addition, in includes the volatility of debt because of the inherent trade off between debt and trade credit. Note that in the model, debt is non state-contingent, so the volatility of debt is by construction zero. Finally, sales, debt and capital are once again normalized by total assets.

Table 4: Net Trade Credit Volatility

	(1) ST Debt	(2) Total Debt
EBITDA_i	-0.128***	-0.121***
	(0.00897)	(0.00846)
Debt_i	-0.114***	-0.0907***
	(0.0161)	(0.0119)
Sales_i	0.00345**	0.00390***
	(0.00140)	(0.00139)
Capital_i	0.0374***	0.0350***
	(0.00313)	(0.00312)
Log Emp._i	0.00719***	0.00653***
	(0.00184)	(0.00186)
Log Assets_i	-0.00686***	-0.00580***
	(0.00147)	(0.00148)
Vol Sales_i	0.0267***	0.0253***
	(0.00215)	(0.00215)
Vol Debt_i	0.00239***	0.225***
	(0.000233)	(0.0187)
Observations	75934	75934
R^2	0.200	0.213
CountrySectorFE	Yes	Yes

Regression is run at the firm level (one observation per firm). Dependent variable is the standard deviation over the sample for each firm of accounts payable (columns (1) and (4)), accounts receivable (columns (2) and (5)), and net trade credit (accounts payable - accounts receivable) (columns (3) and (6)), all normalized by total assets. Debt is sample average of total debt liabilities for each firm relative to total assets. Sales is sample average of the net sales revenues to total assets. Other firm controls include the sample average for each firm of: EBITDA over assets, capital over assets, log employment, and log assets (the latter two are averaged first, and then logged), as well as the standard deviation for each firm of their sales to assets ratio and the debt (or ST debt) to assets. All variables (except those in logs) are winsorized at 1%. R^2 is within R^2. Errors are clustered at the industry level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The theory predicts that the coefficient estimate of β_e should be negative and the estimate of β_v should be positive. The first column in Table 4 reports the regression results using short-term debt, while the second uses total debt. In both cases, as predicted by our theory, the coefficient estimate on sales volatility is positive, while the coefficient estimate on EBITDA is negative. The coefficient estimates are statistically significant at the 1% level. Other firm controls also affect the state contingency of trade credit. As in the specification above, firms with larger scale of operation receive more state contingent trade credit, while firms with more debt receive less.

6 Trade Credit as Macroeconomic Stabilizer

6.1 Counterfactual Analysis

In this section, we illustrate the macroeconomic stabilizing role of trade credit via counterfactual analysis. In particular, we consider the implications of a negative aggregate financial shock to our model economy with state-contingent trade credit and we compare the outcomes to counterfactual economies without trade credit as well as with non-state-contingent trade credit only.

To proceed with the analysis notice that our model nests the two alternative economies. First, if the matching efficiency in the model is set to zero, then no firm is given the opportunity to access the bilateral market, and all firms instead interact with the wholesaler. Since the wholesaler only buys and sells the intermediate good and does not provide any trade credit, this economy has zero trade credit. This economy effectively collapses to case 0 in our model. Second, if the cost of providing insurance, ξ, is prohibitively high, firms that interact with each other bilaterally will never choose to provide state-contingent trade credit. Instead they will choose the optimal amount $\tilde{T} = \underline{T} = \bar{T}$. Hence, this economy collapses to cases 0, 1 and 2 of our model, where firms either interact with the wholesaler, or when they are interact wit each other, they are either unconstrained or constrained and provide non-state contingent trade credit to each other.

Table 5: Response to Negative Financial Shock

	GDP (%)	$\frac{D_b+D_s}{GDP}$ (pp)	$\frac{AP+AR}{GDP}$ (pp)	$\frac{AP+AR}{AP+AR+D_b+D_s}$ (pp)
Baseline	-2.94	-2.22	-7.14	0.39
No Trade Credit, $\gamma = 0$	-5.47	-0.97	na	na
No state-contingent TC, $\xi = \infty$	-3.01	-0.19	0.01	0.16

Table 5 reports statistics for the three models in the counterfactual scenario where all firms' costs of borrowing, ψ_j, have increased by 10%. This scenario can be interpreted as a financial crisis, since debt becomes more expensive, and therefore shrinks, for all agents in the economy. The parameters of the models have been fixed at the levels calibrated in previous sections with the exception of two parameters: (i) in the model without trade credit, the matching efficiency is set to zero, $\gamma = 0$, and (ii) in the model without state-contingent trade credit, the cost of insurance is prohibitively high, $\xi = \infty$.

The second column of the table shows that debt shrinks relative to GDP by the largest amount in the baseline economy (2.22 percentage points). However, GDP shrinks the least (2.94 percentage points). What explains this pattern? The answer is the adjustment of trade credit. The fourth column of the table shows that the share of trade credit, which is the sum of accounts payable and accounts receivable, in all assets flows (debt and trade credit) expands by 39 percentage points. Thus, even though trade credit, relative to GDP shrinks by 7.14 percentage points, it expands relative to debt. This means that, while all sources of finance—both debt and trade credit—fall during the crisis, state-contingent trade credit absorbs the shock.

In fact, even non-state-contingent trade credit acts a stabilizer. To see this, compare the fall in GDP in the economy without trade credit of 3.01 percentage points to the economy without trade credit of 5.47. However, non-state-contingent trade credit expands as a fraction of output during a financial crisis, which is a counterfactual observation. These findings lead us to conclude that trade credit, especially when state-contingent, acts as a macroeconomic stabilizer.

6.2 Empirical Support

The analysis above suggests that economies with trade credit should experience lower aggregate volatility. We test this prediction using the ORBIS dataset described above. Ideally, one would want to verify the prediction at the country level. However, country-level volatility can be attributed to a wide variety of factors. Instead, we test the prediction at the sector level. In particular, we aggregate firm-level data up to the country-sector-year level, then compute the standard deviation and the average over time, and we derive the ratio of the standard deviation of sales to assets, as well as the ratio of trade credit by sales. The prediction of the model is that sectors with more trade credit have lower volatility of sales.

Table 6: Trade Credit and Aggregate Volatility

	(1)	(2)	(3)	(4)	(5)	(6)
$(AP_{cs}+AR_{cs})/Sales_{cs}$	-0.117**	-0.126***				
	(0.0504)	(0.0210)				
$AP_{cs}/Sales_{cs}$			-0.0670**	-0.127***		
			(0.0317)	(0.0157)		
$AR_{cs}/Sales_{cs}$					-0.454***	-0.293***
					(0.0995)	(0.0774)
Observations	399	399	399	399	399	399
R^2	0.0341	0.0544	0.00758	0.0393	0.0809	0.0363
SectorFE	No	Yes	No	Yes	No	Yes
CountryFE	Yes	Yes	Yes	Yes	Yes	Yes

Observations are at the country-sector level. Dependent variable is standard deviation of sales divided by assets. This is computed by first aggregating firm sales and assets to the country-sector-year level, and then computing the standard deviation of sales within each country-sector observation, and the average assets within each country-sector observation. The independent variable is the given measure of trade credit (AP = accounts payable; AR = accounts receivable) divided by sales. These are first aggregated to the country-sector-year level from the firm data, and then averaged over time, and then the ratio computed. R^2 is within R^2. Errors are clustered at the industry level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6 reports the results from a regression of the volatility of sales on trade credit. Notice that both accounts payable and accounts receivable have the same effect: the higher they are, relative to sales, the less volatile is the sector. All regressions include country fixed effects, while the second, fourth and sixth column also include sector fixed effects. Therefore, sectors that have a greater proportion of trade credit, relative to their sales, have less volatile sales, even when comparing industries in the same country and controlling for industry specific factors.

7 Conclusion

We document that small and medium-sized enterprises (SMEs) trade off bank for trade credit, while large firms extend trade credit, especially during financial crises. We develop a model of heterogeneous firms that extend state-contingent credit to each other along supply chains for the purpose of providing insurance in the case of adverse economic shocks. The model predicts that firms obtain more (state-contingent) trade credit the more debt-constrained they are relative to their trading partner. We validate the model's predictions using detailed firm-level data from emerging economies. We find that the model with state-contingent trade credit generates lower GDP volatility and more sharply increasing share of trade credit in liabilities during crises than counterfactual economies without (state-contingent) trade credit. We conclude that the insurance channel of trade credit earns it a role of a macroeconomic stabilizer in emerging markets.

References

Alfaro, Laura, Manuel García-Santana, and Enrique Moral-Benito, "On the direct and indirect real effects of credit supply shocks," *Journal of Financial Economics*, 2021, *139* (3), 895–921.

Allen, Franklin, Elena Carletti, Jun "QJ" Qianc, and Patricio Valenzuela, "Chapter 11 - Financial Intermediation, Markets, and Alternative Financial Sectors," in

George M. Constantinides, Milton Harris, and Rene M. Stulz, eds., *George M. Constantinides, Milton Harris, and Rene M. Stulz, eds.*, Vol. 2 of *Handbook of the Economics of Finance*, Elsevier, 2013, pp. 759–798.

Amberg, Niklas, Tor Jacobson, Erik von Schedvin, and Robert Townsend, "Curbing Shocks to Corporate Liquidity: The Role of Trade Credit," *Journal of Political Economy*, 2021, *129* (1), 182–242.

Bajgar, Matej, Giuseppe Berlingieri, Sara Calligaris, Chiara Criscuolo, and Jonathan Timmis, "Coverage and representativeness of Orbis data," 2020.

Cuñat, Vicente, "Trade Credit: Suppliers as Debt Collectors and Insurance Providers," *The Review of Financial Studies*, 2007, *20* (2), 491–527.

Dinlersoz, Emin, Sebnem Kalemli-Özcan, Henry Hyatt, and Veronika Penciakova, "Leverage over the firm life cycle, firm growth, and aggregate fluctuations," 2019.

Ersahin, Nuri, Mariassunta Giannetti, and Ruidi Huang, "Trade Credit and the Stability of Supply Chains," Technical Report, Swedish House of Finance Research Paper No. 21-13 2021.

Garcia-Appendini, Emilia and Judit Montoriol-Garriga, "Firms as liquidity providers: Evidence from the 2007–2008 financial crisis," *Journal of Financial Economics*, 2013, *109* (1), 272–291.

Hardy, Bryan, "Foreign currency borrowing, balance sheet shocks, and real outcomes," *BIS Working Paper*, 2018, *No 758.*

_ **and Felipe Saffie**, "From carry trades to trade credit: financial intermediation by nonfinancial corporations," *BIS Working Paper*, 2019, *No 773.*

Kalemli-Ozcan, Sebnem, Bent Sorensen, Carolina Villegas-Sanchez, Vadym Volosovych, and Sevcan Yesiltas, "How to Construct Nationally Representative Firm Level Data from the Orbis Global Database: New Facts and Aggregate Implications," Working Paper 21558, National Bureau of Economic Research September 2015.

_ , **Se-Jik Kim, Hyun Song Shin, Bent E Sorensen, and Sevcan Yesiltas**, "Financial shocks in production chains," in "American Economic Association meetings, January" 2014.

Klapper, Leora, Luc Laeven, and Raghuram Rajan, "Trade Credit Contracts," *The Review of Financial Studies*, 12 2011, *25* (3), 838–867.

Lian, Chen and Yueran Ma, "Anatomy of corporate borrowing constraints," *The Quarterly Journal of Economics*, 2021, *136* (1), 229–291.

Love, Inessa, Lorenzo A. Preve, and Virginia Sarria-Allende, "Trade credit and bank credit: Evidence from recent financial crises," *Journal of Financial Economics*, 2007, *83* (2), 453–469.

Petersen, Mitchell A. and Raghuram G. Rajan, "Trade Credit: Theories and Evidence," *The Review of Financial Studies*, 04 1997, *10* (3), 661–691.

Reischer, Margit, "Finance-thy-Neighbor. Trade Credit Origins of Aggregate Fluctuations," Technical Report 2019.

Shao, Lin, "Aggregate Fluctuations and the Role of Trade Credit," 2017.

Wilner, Benjamin S., "The Exploitation of Relationships in Financial Distress: The Case of Trade Credit," *The Journal of Finance*, 2000, *55* (1), 153–178.

Wu, Jing, Hsiao-Hui Lee, and John R Birge, "Trade credit late payment and industry structure," *mimeo*, 2020.

Yang, S. Alex and John R. Birge, "Trade Credit, Risk Sharing, and Inventory Financing Portfolios," *Management Science*, 2018, *64* (8), 3667–3689.

A Proofs and Derivations

A.1 Dealings with Intermediary Algebra

A.1.1 Seller

The FOC's of the seller's problem yield:

$$D_s = \frac{2\psi_s + \frac{1+r^*}{p_x} \pm \sqrt{4\psi_s^2 + \left(\frac{1+r^*}{p_x}\right)^2}}{2\psi_s \frac{1+r^*}{p_x}}$$

The larger of the two roots violates the restriction imposed by the log production function. To see this, substitute L into the production function, $\ln L$, to obtain the following expression: $\ln(D_s - \psi_s D_s^2) - \ln(w)$. Production is well defined when $D_s - \psi_s D_s^2 \geq w$. The larger of the two roots violates this inequality. The smaller of the two roots satisfies the inequality if the equilibrium price, p_x, is high enough:

$$p_x \geq (1 + r^*)\sqrt{\frac{2\psi_s}{w(2\psi_2 w - 1)}}$$

Note that when the second constraint is binding, the seller's value function is exactly zero. Hence, in this case, optimal debt and production are also zero. This solution is summarized in the main body of the paper.

A.1.2 Buyer

Since the first constraint must hold with equality, we can substitute it into the objective function to obtain:

$$\max_{D_b \geq 0} \beta E_z \left[\frac{z}{p_x} \left(D_b - \psi_b D_b^2 \right) - D_b \left(1 + r^* \right) \right]$$

subject to:

$$\frac{\underline{z}}{p_x} \left(D_b - \psi_b D_b^2 \right) - D_b \left(1 + r^* \right) \geq 0$$

The FOCs yield the following optimal solution, as a function of the Lagrange multiplier λ:

$$D_b = \frac{1}{2\psi_b} \left[1 - \frac{p_x \left(1 + \lambda \left(1 + r^* \right) \right)}{\beta \tilde{z} + \lambda \underline{z}} \right]$$

There are two cases; one where the constraint does not bind and one where it binds. In the first case, the Lagrange multiplier must be zero, so

$$D_b = \frac{1}{2\psi_b} \left[1 - \frac{p_x \left(1 + r^* \right)}{\tilde{z}} \right] \tag{31}$$

which is non-negative whenever the equilibrium price is low enough $p_x \leq \frac{\tilde{z}}{1+r^*}$. In the second case, we obtain D_b directly from the second constraint, which holds with equality. The solution is given by:

$$D_b = \frac{1}{\psi_b} \left[1 - \frac{p_x \left(1 + r^* \right)}{\underline{z}} \right] \tag{32}$$

which is non-negative whenever the equilibrium price is even lower, $p_x \leq \frac{\underline{z}}{1+r^*}$.

Which of the two expressions characterizes the optimal amount of debt depends on feasibility. The objective function of the buyer is a quadratic equation which has an inverted U shape. The limits on the positive and negative spectrum are both negative infinity. It attains a maximum when D_b is given by the unconstrained solution (31), which is non-negative under the aforementioned parameter restrictions. The solution to the constrained problem in (32) may be higher or lower than the solution to the unconstrained problem. However, when it is higher, the constrained solution is sub-optimal, since the max is achieved at the unconstrained solution. Thus, the solution to the buyer's problem is given by the unconstrained solution, unless it is infeasible, in which case it is given by the constrained solution. Substituting the unconstrained solution (31) into the constraint shows that the unconstrained solution is infeasible when $p_x > \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$. Hence, the solution to the buyer's problem is given by expression (32) whenever $\frac{\underline{z}}{1+r^*} \geq p_x > \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$ and by expression (31) whenever $p_x \leq \frac{\underline{z}}{(2-\frac{\underline{z}}{\tilde{z}})(1+r^*)}$. This solution is summarized in the main body of the paper.

Next we show that $\Gamma_b(\psi_b)$ is decreasing in ψ_b. In the case where the buyer is unconstrained, their maximized value is:

$$\Gamma_b(\psi_b) = \frac{1}{4\psi_b}\left[1 - \frac{p_x}{\beta\tilde{z}}\right]\left[\frac{\tilde{z}}{p_x} - \frac{1}{\beta}\right] \tag{33}$$

In the case where the buyer is constrained, their maximized value is

$$\Gamma_b(\psi_b) = \frac{1}{\psi_b}\left[1 - \frac{p_x}{\beta\underline{z}}\right]\left[\frac{\tilde{z}}{\underline{z}} - 1\right] \tag{34}$$

These values are positive as long as $\beta\tilde{z} > p_x$ ($\beta\underline{z} > p_x$ when the buyer is constrained) which ensures that debt levels are non-negative. Taking the derivative of each expression with respect to ψ_b yields the result.

A.2 Case 1 Algebra

The FOCs to the seller's problem are given by:

$$
\begin{aligned}
&[\bar{T}_1]: && (1-\lambda_1)\,\beta\,(1-p)=0 \quad \Rightarrow \quad \lambda_{1,1}=1\\
&[\underline{T_1}]: && (1-\lambda_{1,1})\,\beta p=0 \quad \Rightarrow \quad \lambda_{1,1}=1\\
&[L_1]: && (\lambda_{3,1}-1)w+\beta\lambda_{1,1}\frac{\tilde{z}}{L_1}=0 \quad \Rightarrow L_1=\frac{\tilde{z}\beta}{(1-\lambda_{3,1})w}\\
&[A_{b,1}]: && 1-\lambda_{1,1}+\lambda_{2,1}-\lambda_{3,1}=0 \quad \Rightarrow \lambda_{2,1}=\lambda_{3,1}\\
&[D_{s,1}]: && -2\psi_s D_{s,1}-\lambda_{3,1}\,(1-2\psi_s D_{s,1})=0 \quad \Rightarrow D_{s,1}=\frac{\lambda_{3,1}}{2\psi_s\,(\lambda_{3,1}-1)}\\
&[D_{b,1}]: && 2\psi_s D_{b,1}\,(\lambda_{2,1}-\lambda_{1,1})-\lambda_{2,1}=0 \quad \Rightarrow D_{b,1}=\frac{\lambda_{2,1}}{2\psi_b\,(\lambda_{2,1}-\lambda_{1,1})}=\frac{\lambda_{3,1}}{2\psi_b\,(\lambda_{3,1}-1)}\\
&[B_{s,1}]: && 1+\lambda_{3,1}+\lambda_{4,1}=0\\
&[B_{b,1}]: && 1+\lambda_{2,1}+\lambda_{5,1}=0 \quad \Rightarrow \lambda_{4,1}=\lambda_{5,1}
\end{aligned}
$$

Let $\tilde{\lambda}_1=\lambda_{3,1}$. Sum constraints (2) and (3) to get:

$$-D_{b,1}+\psi_b\,(D_{b,1})^2-D_{s,1}+wL_1+\psi_s\,(D_{s,1})^2 \;=\; 0$$

Substitute out the debt levels and labor to obtain:

$$\frac{1}{4}\left(\frac{\psi_b+\psi_s}{\psi_b\psi_s}\right)\tilde{\lambda}_1^2-\left(\frac{1}{2}\left(\frac{\psi_s+\psi_b}{\psi_b\psi_s}\right)-\tilde{z}\beta\right)\tilde{\lambda}_1-\tilde{z}\beta \;=\; 0$$

This is a quadratic equation in $\tilde{\lambda}_1$ whose only root that satisfies $\tilde{\lambda}_1<1$ is given in the main text of the paper.

In order for this case to be feasible, the solution needs to satisfy constraints (6)-(8). If constraint (7) is satisfied, then (8) holds trivially. If constraint (6) is satisfied, then the maximized value for the seller is non-negative, which is another necessary condition.

The optimal unconstrained allocation satisfies constraint (6) only if

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right) - \frac{\psi_s+\psi_b}{2\beta\psi_b\psi_s}\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1-1} \geq \Gamma_b(\psi_b)/\beta \tag{35}$$

The optimal unconstrained allocation satisfies constraint (7) only if

$$\Gamma_b(\psi_b)/\beta \geq (\tilde{z}-\underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right) \tag{36}$$

The two restrictions are jointly satisfied only if

$$\underline{z}\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right) \geq \frac{\psi_s+\psi_b}{2\beta\psi_b\psi_s}\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1-1} \tag{37}$$

Substituting the solution for $\tilde{\lambda}_1$ into (37) yields the parameter restriction in the main text.

Finally, due to the functional form for the production function, it must be that $L_1 \geq 1$, which requires that $\tilde{z}\beta \geq w(1-\tilde{\lambda}_1)$. Restriction (37), however, is more strict than this restriction because $\tilde{\lambda}_1/(\tilde{\lambda}_1-1) \geq 0$, so it is trivially satisfied.

A.3 Case 2 Algebra

The FOCs to the seller's problem are given by:

$$
\begin{aligned}
[L_2]: &\qquad (\lambda_{3,2}-1)w+\beta\lambda_{1,2}\frac{\tilde{z}}{L_2}+\frac{\underline{z}}{L_2}\lambda_{7,2}+\frac{\underline{z}}{L_2}\lambda_{8,2}=0 \quad \Rightarrow L_2=\frac{\beta\lambda_{1,2}\tilde{z}+\underline{z}\lambda_{7,2}+\lambda_{8,2}\bar{z}}{(1-\lambda_{3,2})w}\\
[B_{s,2}]: &\qquad 1+\lambda_{3,2}+\lambda_{4,2}+\frac{\lambda_{6,2}}{\beta}=0\\
[B_{b,2}]: &\qquad \lambda_{1,2}+\lambda_{2,2}+\lambda_{5,2}+\frac{\lambda_{7,2}}{\beta}=0\\
[A_{b,2}]: &\qquad 1-\lambda_{1,2}+\lambda_{2,2}-\lambda_{3,2}=0\\
[D_{s,2}]: &\qquad -2\psi_s D_{s,2}-\lambda_{3,2}\left(1-2\psi_s D_{s,2}\right)-\frac{\lambda_{6,2}}{\beta}=0 \quad \Rightarrow D_{s,2}=\frac{\lambda_{3,2}+\frac{\lambda_{6,2}}{\beta}}{2\psi_s\left(\lambda_{3,2}-1\right)}=\frac{1+\lambda_{4,2}}{2\psi_s\left(1-\lambda_{3,2}\right)}\\
[D_{b,2}]: &\qquad 2\psi_b D_{b,2}\left(\lambda_{2,2}-\lambda_{1,2}\right)-\lambda_{2,2}-\frac{\lambda_{7,2}}{\beta}=0 \quad \Rightarrow D_{b,2}=\frac{\lambda_{2,2}+\frac{\lambda_{7,2}}{\beta}}{2\psi_b\left(\lambda_{2,2}-\lambda_{1,2}\right)}=\frac{\lambda_{1,2}+\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)}\\
[T_2]: &\qquad \beta\left(1-\lambda_{1,2}\right)+\lambda_{6,2}-\lambda_{7,2}=0
\end{aligned}
$$

Let $\lambda_{8,2} = \lambda_{6,2} = \lambda_{1,2} = 0$. Then we need to solve the following system:

$$\begin{aligned}
L_2 &= \frac{\underline{z}\lambda_{7,2}}{(1-\lambda_{3,2})w} \\
0 &= 1 + \lambda_{3,2} + \lambda_{4,2} \\
0 &= \lambda_{2,2} + \lambda_{5,2} + \frac{\lambda_{7,2}}{\beta} \\
0 &= 1 + \lambda_{2,2} - \lambda_{3,2} \\
D_{s,2} &= \frac{\lambda_{3,2}}{2\psi_s\left(\lambda_{3,2}-1\right)} \\
D_{b,2} &= \frac{\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)} \\
\lambda_{7,2} &= \beta \\
0 &= -D_{b,2} + A_{b,2} + \psi_b\left(D_{b,2}\right)^2 \\
0 &= -D_{s,2} - A_{b,2} + wL_2 + \psi_s\left(D_{s,2}\right)^2 \\
0 &= -D_{b,2}(1+r^*) - T_2 + \underline{z}\ln L_2
\end{aligned}$$

The system simplifies to:

$$\begin{aligned}
L_2 &= \frac{\underline{z}\beta}{(1-\lambda_{3,2})w} \\
0 &= 1 + \lambda_{3,2} + \lambda_{4,2} \quad \Rightarrow \quad \lambda_{4,2} = -\left(1+\lambda_{3,2}\right) \\
0 &= 1 + \lambda_{2,2} + \lambda_{5,2} \quad \Rightarrow \quad \lambda_{5,2} = -\left(1+\lambda_{2,2}\right) = -\lambda_{3,2} \\
0 &= 1 + \lambda_{2,2} - \lambda_{3,2} \quad \Rightarrow \quad \lambda_{2,2} = \left(\lambda_{3,2}-1\right) \\
D_{s,2} &= \frac{\lambda_{3,2}}{2\psi_s\left(\lambda_{3,2}-1\right)} \\
D_{b,2} &= \frac{\lambda_{5,2}}{2\psi_b\left(1-\lambda_{3,2}\right)} \\
0 &= -D_{b,2} + A_{b,2} + \psi_b\left(D_{b,2}\right)^2 \\
0 &= -D_{s,2} - A_{b,2} + wL_2 + \psi_s\left(D_{s,2}\right)^2 \\
0 &= -D_{b,2}(1+r^*) - T_2 + \underline{z}\ln L_2
\end{aligned}$$

Let $\tilde{\lambda}_2 = \lambda_{3,2}$. To solve for this object, substitute out $A_{b,2}$ in the second to the last equation:

$$0 = -D_{b,2} - D_{s,2} + \psi_b (D_{b,2})^2 + \psi_s (D_{s,2})^2 + wL_2$$

Substitute out the debt levels and labor to obtain:

$$0 = \frac{1}{4}\left(\frac{\psi_s + \psi_b}{\psi_s \psi_b}\right)\tilde{\lambda}^2 + \left[\underline{z}\beta - \frac{1}{2}\left(\frac{\psi_s + \psi_b}{\psi_s \psi_b}\right)\right]\tilde{\lambda}_2 - \underline{z}\beta$$

This is a quadratic equation in $\tilde{\lambda}_2$ whose only root that satisfies $\tilde{\lambda}_2 < 1$ is given in the main text of the paper.

By construction, the solution satisfies all the constraints, and the seller's maximized value is non-negative due to constraint (6). An additional restriction is that production is non-negative, which requires that $\underline{z}\beta \geq (1 - \lambda_2)w$. Substituting out $\tilde{\lambda}_2$ in this inequality yields the parameter restriction in the main text.

A.3.1 Ruling out Case 5

Consider the problem in Case 2 and the FOCs in the above section, before imposing values for the multipliers. Just as in Case 2, constraint (8) can never bind given that constraint (7) with a flat T is more restrictive than constraint (8). In Case 2, we assumed that constraints (1), (6) and (8) did not bind, so only (7) was binding. Suppose that we assume that constraint (6) binds instead and that $\lambda_{1,2} = \lambda_{8,2} = \lambda_{7,2} = 0$. Replacing these restrictions in the first FOC we get:

$$[L]: \qquad L_2 = \frac{0}{(1 - \lambda_{3,2})w} = 0$$

In fact, if constraint (6) binds, the value of the seller is 0 because T is flat. Therefore, this case cannot exist. Even if constraint (1) is not binding, the seller would not be willing to participate in this match.

A.4 Case 3 algebra

Taking FOCs of the seller's problem yields:

$$
\begin{aligned}
[L_3]: &\qquad (\lambda_{3,3}-1)w+\beta\lambda_{1,3}\frac{\tilde{z}}{L_3}+\frac{\underline{z}}{L_3}\lambda_{7,3}+\frac{\bar{z}}{L}\lambda_{8,3}=0 \quad \Rightarrow L_3=\frac{\beta\lambda_{1,3}\tilde{z}+\underline{z}\lambda_{7,3}+\bar{z}\lambda_{8,3}}{(1-\lambda_{3,3})w}\\
[B_{s,3}]: &\qquad 1+\lambda_{3,3}+\lambda_{4,3}+\frac{\lambda_{6,3}}{\beta}=0\\
[B_{b,3}]: &\qquad \lambda_{1,3}+\lambda_{2,3}+\lambda_{5,3}+\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}=0\\
[A_{b,3}]: &\qquad 1-\lambda_{1,3}+\lambda_{2,3}-\lambda_{3,3}=0\\
[D_{s,3}]: &\qquad -2\psi_s D_{s,3}-\lambda_{3,3}\left(1-2\psi_s D_{s,3}\right)-\frac{\lambda_{6,3}}{\beta}=0 \quad \Rightarrow D_{s,3}=\frac{\lambda_{3,3}+\frac{\lambda_{6,3}}{\beta}}{2\psi_s\left(\lambda_{3,3}-1\right)}=\frac{1+\lambda_{4,3}}{2\psi_s\left(1-\lambda_{3,3}\right)}\\
[D_{b,3}]: &\qquad 2\psi_b D_{b,3}\left(\lambda_{2,3}-\lambda_{1,3}\right)-\lambda_{2,3}-\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}=0 \quad \Rightarrow D_{b,3}=\frac{\lambda_{2,3}+\frac{\lambda_{7,3}+\lambda_{8,3}}{\beta}}{2\psi_b\left(\lambda_{2,3}-\lambda_{1,3}\right)}=\frac{\lambda_{1,3}+\lambda_{5,3}}{2\psi_b\left(1-\lambda_{3,3}\right)}\\
\left[\bar{T}_3\right]: &\qquad \beta\left(1-p\right)\left(1-\lambda_{1,3}\right)-\lambda_{8,3}=0\\
\left[\underline{T}_3\right]: &\qquad \beta p\left(1-\lambda_{1,3}\right)+\lambda_{6,3}-\lambda_{7,3}=0
\end{aligned}
$$

Combining FOCs with the constraints yields a system of 13 equations and 13 unknowns:

$$
\begin{aligned}
L_3 &= \frac{\beta\lambda_{1,3}\tilde{z} + \underline{z}\lambda_{7,3} + \bar{z}\lambda_{8,3}}{(1-\lambda_{3,3})w} \\
0 &= 1 + \lambda_{3,3} + \lambda_{4,3} + \frac{\lambda_{6,3}}{\beta} \quad \text{then} \quad (4) \quad \Rightarrow \quad \lambda_{4,3} = \lambda_{5,3} \\
0 &= \lambda_{1,3} + \lambda_{2,3} + \lambda_{5,3} + \frac{\lambda_{7,3}}{\beta} \quad \text{then} \quad (4) \quad \Rightarrow \quad \lambda_{4,3} = \lambda_{5,3} \\
0 &= 1 - \lambda_{1,3} + \lambda_{2,3} - \lambda_{3,3} \quad \text{then} \quad (3) \quad \Rightarrow \quad \lambda_{2,3} = \lambda_{3,3} \\
D_{s,3} &= \frac{1+\lambda_{4,3}}{2\psi_s\left(1-\lambda_{3,3}\right)} \\
D_{b,3} &= \frac{\lambda_{1,3}+\lambda_{5,3}}{2\psi_b\left(1-\lambda_{3,3}\right)} \\
0 &= \beta\left(1-p\right)\left(1-\lambda_{1,3}\right) \quad \text{then} \quad (1) \quad \Rightarrow \quad \lambda_{1,3} = 1 \\
0 &= \beta p\left(1-\lambda_{1,3}\right) + \lambda_{6,3} - \lambda_{7,3} \quad \text{then} \quad (2) \quad \Rightarrow \quad \lambda_{7,3} = \lambda_{6,3} \\
\frac{\Gamma_b}{\beta} &= \tilde{z}\ln L_3 - \tilde{T}_3 - D_{b,3}(1+r^*) \\
A_{b,3} &= D_{b,3} - \psi_b\left(D_{b,3}\right)^2 \\
0 &= -\left(D_{s,3}+D_{b,3}\right) + wL_3 + \psi_s\left(D_{s,3}\right)^2 + \psi_b\left(D_{b,3}\right)^2 + \xi \\
\underline{T_3} &= D_{s,3}(1+r^*) \\
D_{b,3} + D_{s,3} &= \frac{\underline{z}\ln L_3}{1+r^*}
\end{aligned}
$$

The system reduces to the 8 equations and 8 unknowns in the main text. The solution method involves first characterizing $D_{s,3}$ via the implicit function in expression (22). The quadratic equation in expression (22) has either no solution or two solutions (with a knife edge case of a unique solution). The max of the LHS is $D_{s,3} = \frac{1}{2\psi_s}$. Evaluating the LHS and the RHS at this value obtains:

$$\begin{aligned} LHS &= \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \\ RHS &= we^{\frac{\frac{1+r^*}{2}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}{\underline{z}}}+\xi \end{aligned}$$

To ensure the existence of a pair of solutions, the following parameter restriction is necessary:

$$\frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}+\xi \tag{38}$$

Given the two roots, the optimal level of $D_{s,3}$ is the one that is associated with a higher value function for the seller. To see when that occurs, substitute expressions (18), (20) and (21) into the seller's objective function to obtain

$$V_{s,3} = \ln L_3(1-p)(\bar{z}-\underline{z}) - \Gamma(\psi_b)/\beta$$

Clearly $V_{s,3}$ is maximized when L_3 is maximized. Since, according to expression (21), labor is increasing in the amount borrowed, it must be that the higher value of debt is the optimal solution to this case. Substituting expression (21) into $V_{s,3}$ yields

$$V_{s,3} = D_{b,3}(1+r^*)\frac{\psi_s+\psi_b}{\psi_s}(1-p)\left(\frac{\bar{z}}{\underline{z}}-1\right) - \Gamma(\psi_b)/\beta$$

We will compare this value to the value of the case 4 below.

A.5 Case 4 Algebra

Taking FOCs of the seller's problem yields:

$$
\begin{aligned}
[L_4]: &\qquad L_4 = \frac{\beta\lambda_{1,4}\tilde{z} + \bar{z}\lambda_{8,4}}{(1-\lambda_{3,4})w} \\
[B_{s,4}]: &\qquad 1 + \lambda_{3,4} + \lambda_{4,4} + \frac{\lambda_{6,4}}{\beta} = 0 \\
[B_{b,4}]: &\qquad \lambda_{1,4} + \lambda_{2,4} + \lambda_{5,4} + \frac{\lambda_{8,4}}{\beta} = 0 \\
[A_{b,4}]: &\qquad 1 - \lambda_{1,4} + \lambda_{2,4} - \lambda_{3,4} = 0 \\
[D_{s,4}]: &\qquad D_{s,4} = \frac{1 + \lambda_{4,4}}{2\psi_s\left(1 - \lambda_{3,4}\right)} \\
[D_{b,4}]: &\qquad D_{b,4} = \frac{\lambda_{1,4} + \lambda_{5,4}}{2\psi_b\left(1 - \lambda_{3,4}\right)} \\
\left[\bar{T}_4\right]: &\qquad \lambda_{8,4} = \beta\left(1-p\right)\left(1-\lambda_{1,4}\right) \\
\left[\underline{T_4}\right]: &\qquad \lambda_{6,4} = -\beta p\left(1-\lambda_{1,4}\right)
\end{aligned}
$$

Simplifying the FOCs and adding the constraints yields the following system:

$$\begin{aligned}
[L_4]: &\quad L_4 = \frac{\beta\lambda_{1,4}\tilde{z} + \bar{z}\beta\left(1-p\right)\left(1-\lambda_{1,4}\right)}{(1-\lambda_{3,4})w} \\
[B_{s,4}]: &\quad 1+\lambda_{3,4}+\lambda_{4,4}-p\left(1-\lambda_{1,4}\right)=0 \\
[B_{b,4}]: &\quad 1+\lambda_{2,4}+\lambda_{5,4}-p\left(1-\lambda_{1,4}\right)=0 \\
[A_{b,4}]: &\quad \lambda_{3,4}-\lambda_{2,4}=1-\lambda_{1,4}=\lambda_{5,4}-\lambda_{4,4} \\
[D_{s,4}]: &\quad D_{s,4}=\frac{1+\lambda_{4,4}}{2\psi_s\left(1-\lambda_{3,4}\right)}=\frac{p\left(1-\lambda_{1,4}\right)-\lambda_{3,4}}{2\psi_s\left(1-\lambda_{3,4}\right)} \\
[D_{b,4}]: &\quad D_{b,4}=\frac{\lambda_{1,4}+\lambda_{5,4}}{2\psi_b\left(1-\lambda_{3,4}\right)}=\frac{1+\lambda_{4,4}}{2\psi_b\left(1-\lambda_{3,4}\right)}=\frac{p\left(1-\lambda_{1,4}\right)-\lambda_{3,4}}{2\psi_b\left(1-\lambda_{3,4}\right)} \\
\left[\bar{T}_4\right]: &\quad \lambda_{8,4}=\beta\left(1-p\right)\left(1-\lambda_{1,4}\right) \\
\left[\underline{T_4}\right]: &\quad \lambda_{6,4}=-\beta p\left(1-\lambda_{1,4}\right) \\
&\quad \tilde{z}\ln L_4 - D_{b,4}(1+r^*) - \tilde{T}_4 = \frac{\Gamma_b\left(\psi_b\right)}{\beta} \\
&\quad D_{b,4}\left(1+\frac{\psi_b}{\psi_s}\right)-\psi_b\left(D_{b,4}\right)^2-\frac{1}{\psi_s}\left(\psi_b D_{b,4}\right)^2-wL_4-\xi=0 \\
&\quad \underline{T_4}=D_{b,4}(1+r^*)\frac{\psi_b}{\psi_s} \\
&\quad \bar{z}\ln L_4-\bar{T}_4=D_{b,4}(1+r^*)
\end{aligned}$$

Note that:

$$\begin{aligned}
\tilde{T}_4 &= pD_{b,4}(1+r^*)\frac{\psi_b}{\psi_s}+\left(1-p\right)\bar{z}\ln L_4-\left(1-p\right)D_{b,4}(1+r^*) \\
\tilde{T}_4 &= D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s}-1\right)+\left(1-p\right)\bar{z}\ln L_4
\end{aligned}$$

Then using the first constraint we get:

$$\begin{aligned}
p\underline{z}\ln L_4 - D_{b,4}(1+r^*)\left(p\frac{\psi_s+\psi_b}{\psi_s}\right) &= \frac{\Gamma_b\left(\psi_b\right)}{\beta} \\
\Rightarrow \quad \ln L_4 &= \frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}} \\
\Rightarrow \quad wL_4 &= we^{\frac{D_{b,4}(1+r^*)p\left(1+\frac{\psi_b}{\psi_s}\right)+\frac{\Gamma_b(\psi_b)}{\beta}}{p\underline{z}}}
\end{aligned}$$

The above derivations arrive at the solution to this problem as described in the main text. $D_{s,4}$ is characterized by the implicit equation in expression (27). Expression (27) resembles closely the equilibrium expression (22) for Case 3 above. Once again, the quadratic equation in expression (27) has either no solution or two solutions (with a knife edge case of a unique solution). The max of the LHS is still $D_s = \frac{1}{2\psi_s}$. However the RHS of expression (27) is clearly larger than the RHS in expression (22) as long as the buyer's outside option is strictly positive. Evaluating the LHS and the RHS at the maximum value obtains:

$$\begin{aligned}
LHS &= \frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \\
RHS &= we^{\frac{\frac{1+r^*}{2}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)}{\underline{z}}+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi
\end{aligned}$$

To ensure the existence of a pair of solutions, the following parameter restriction is necessary:

$$\frac{1}{4}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p\underline{z}}}+\xi \tag{39}$$

Given the two roots, the optimal level of $D_{s,4}$ is the one that is associated with a higher value function for the seller. To see when that occurs, substitute constraints (1), (2) and (3) into the seller's objective function to obtain

$$V_{s,4} = -(D_{s,4} + D_{b,4})(1 + r^*) + \tilde{z} \ln L_4 - \Gamma(\psi_b)/\beta$$

Substituting out L_4 using expression (26) in the above and using the proportionality result between the debt levels shows that the seller's value function is maximized whenever $D_{s,4}$ is maximized because $\bar{z} > \underline{z}$. Hence, the higher value of debt is once again the optimal solution to this case. Furthermore, the shapes of the RHS and the LHS allow us to derive some useful comparative statics. In particular, the LHS is a parabola with an inverted U shape as a function of $D_{s,4}$ while the RHS is an increasing exponential. Since the solution is the larger of the two roots, any parameter that shifts the RHS up results in a lower optimal debt level. In particular, suppose a parameter raises the buyer's outside option, $\Gamma_b(\psi_b)$. This necessarily lowers the optimal debt level.

Given the discussion above, the maximized value of the seller is[12]:

$$V_{s,4} = D_{b,4}(1 + r^*)\frac{\psi_s + \psi_b}{\psi_s}(1 - p)\left(\frac{\bar{z}}{\underline{z}} - 1\right) + \frac{(1 - p)\bar{z}}{p\underline{z}}\Gamma(\psi_b)/\beta$$

Comparing cases 3 and 4, clearly case 4 yields a lower value of debt because the RHS in this case is strictly higher. Since labor is increasing in debt, production is also lower in this case.

[12] The expression suggests that the seller's value function is increasing in the outside option of the buyer, $\Gamma_b(\psi_b)$, which is counter-intuitive. But that is not the case. There are two countervailing forces of $\Gamma_b(\psi_b)$ on $V_{s,4}$. The second part of the expression is clearly increasing in $\Gamma_b(\psi_b)$. However, the first part of the expression is increasing in $D_{b,4}$, which is a decreasing function of $\Gamma_b(\psi_b)$ since $D_{b,4}$ is proportional to $D_{s,4}$. Taking the derivative of $V_{s,4}$ with respect to $\Gamma_b(\psi_b)$ and applying the Implicit Function Theorem to expression (27) yields the following condition to ensure that $\frac{\partial D_{s,4}}{\partial \Gamma_b(\psi_b)} < 0$: $we^{\frac{(1+r^*)\left(1+\frac{\psi_s}{\psi_b}\right)D_{s,4}}{\underline{z}} + \frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}} < \beta\bar{z}(1 - 2\psi_s D_{s,4})$.

A.6 Existence of Different Cases

A.6.1 Cases Without Insurance

Whether the equilibrium is as in Case 1 or as in Case 2, depends on whether the seller's value function dominates in the first or the second case. The difference in optimized value functions, $V_{s,1} - V_{s,2}$, is positive only if

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right) - \underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right) + \frac{\psi_b + \psi_s}{2\beta\psi_s\psi_b}\left[\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - 1} - \frac{\tilde{\lambda}_2}{\tilde{\lambda}_2 - 1}\right] - (\tilde{z}\ln(1-\tilde{\lambda}_1) - \underline{z}\ln(1-\tilde{\lambda}_2)) > \Gamma_b(\psi_b)/\beta \tag{40}$$

Recall that case 1 is feasible when restrictions (35)-(37) hold. These restrictions can coexist with restriction (40). First, note that:

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right) - \underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right) + \frac{\psi_b + \psi_s}{2\beta\psi_s\psi_b}\left[\frac{\tilde{\lambda}_1}{\tilde{\lambda}_1 - 1} - \frac{\tilde{\lambda}_2}{\tilde{\lambda}_2 - 1}\right] - (\tilde{z}\ln(1-\tilde{\lambda}_1) - \underline{z}\ln(1-\tilde{\lambda}_2)) >$$
$$(\tilde{z} - \underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right)$$

Also, restriction (35) constitutes the seller's maximized value under case 1, while restriction (40) is the difference in maximized values between cases 1 and 2. By construction, it must be that restriction (40) is more binding. But when

$$\Gamma_b(\psi_b)/\beta < (\tilde{z} - \underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\tilde{\lambda}_1)}\right) \tag{41}$$

case 1 is ruled out because it is not feasible. So case 2 dominates case 1 when (41) holds. But case 1 dominates case 2 when

$$\tilde{z}\ln\left(\frac{\tilde{z}\beta}{w}\right) - \underline{z}\ln\left(\frac{\underline{z}\beta}{w}\right) + \frac{\psi_b + \psi_s}{2\beta\psi_s\psi_b}\left[\frac{\lambda}{\lambda - 1} - \frac{\lambda_2}{\lambda_2 - 1}\right] - \left(\tilde{z}\ln(1-\lambda) - \underline{z}\ln(1-\lambda_2)\right) >$$
$$\Gamma_b(\psi_b)/\beta \geq (\tilde{z} - \underline{z})\ln\left(\frac{\tilde{z}\beta}{w(1-\lambda)}\right)$$

and (37) hold, excluding the range in (41).

There is also a range of parameters in which case 2 is not feasible but case 1 is, so case 1 dominates case 2 in that range. To find that range, note that, in order for there to exist an equilibrium, the debt levels and hours worked must be non-negative. The joint restriction for these variables implies that the Lagrange multiplier $\tilde{\lambda}_1$ (or $\tilde{\lambda}_2$) must be non-positive. Since $\tilde{\lambda}_2 > \tilde{\lambda}_1$, it must be that $\tilde{\lambda}_2$ attains the zero upper bound first. At that point, case 2 becomes infeasible. This occurs when $\tilde{\lambda}_2 \geq 0$. Using the expression for $\tilde{\lambda}_2$, the range must be

$$\frac{\psi_s + \psi_b}{\psi_s\psi_b} \geq 2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s + \psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2} \tag{42}$$

In order for case 1 to remain feasible, it must be that $\tilde{\lambda}_1 \leq 0$. Using the expression for $\tilde{\lambda}_1$, the range must be

$$\frac{\psi_s + \psi_b}{\psi_s\psi_b} \leq 2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s + \psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \tag{43}$$

Combining expressions (42) and (43) yields

$$2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s + \psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \geq \frac{\psi_s + \psi_b}{\psi_s\psi_b} \geq 2\left(\underline{z}\beta\right) + \sqrt{\left(\frac{\psi_s + \psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\underline{z}\beta\right)^2} \tag{44}$$

For as long as parameters satisfy the restriction in expression (44), case 2 is not feasible, case 1 is feasible (subject to the restriction in expression (41) which can co-exist), and case 1 dominates case 2.

A.6.2 Cases With Insurance

In this section, we show that equilibria with insurance can arise. Recall that, cases 3 and 4 correspond to situations where agents use trade credit for insurance purposes. First, we show that cases 3 and 4 can dominate case 2 under certain conditions. Second, we show that cases 3 and 4 can dominate case 1 under certain conditions. Finally, we derive conditions that guarantee that case 3 dominates case 4 and vise verse.

Cases 3 and 4 vs. Case 2. We want to show that there exist an equilibrium in which the transfers are state contingent. To do that, we show that, starting with an equilibrium as in case 2 where the transfers are equalized, there exists a deviation that yields higher utility for the seller and is still feasible. Feasibility will be determined by a set of parameters.

In case 2, we know that constraint (7) binds, but constraints (1), (6) and (8) do not. The seller would like to extract all surplus from the buyer if possible, i.e. they would like to make constraint (1) bind. They can do so for given $D_{b,2}$ and L_2 by increasing the average transfer. However, in an equilibrium where the transfers are state-contingent, increasing the mean transfer (by either increasing the transfer in the good state or lowering it in the bad state) implies that they would have to pay the fixed cost ξ. This cost is financed with debt, which mean that the seller would have to increase the amount that they borrow. Since their surplus is increasing in the mean transfer and falling in the amount of debt, the deviation will only be profitable under some parameter values.

Suppose they want to expropriate all of the buyer's surplus. Then the mean transfer that they receive would have to increase by:

$$\epsilon = (\tilde{z} - \underline{z}) \ln L_2 - \Gamma(\psi_b)/\beta, \tag{45}$$

which corresponds to the value such that constraint (1) binds for given scale of production L_2. The seller is better off if this gain surpasses the loss due to higher debt:

$$\epsilon > x_d \tag{46}$$

where x_d is the extra debt, in addition to $D_{s,2}$, needed in order to cover all costs, keeping $A_{b,2}$ and L_2 fixed, and satisfies

$$D_{s,2} + x_d = \xi + \psi_s[(D_{s,2} + x_d)^2 - d_s^2] \tag{47}$$

This quadratic equation has two roots. We take the smaller of the two roots because we want to minimize the amount of debt raised. The solution is:

$$x_d = \frac{1 - 2\psi_s D_{s,2} - \sqrt{(2\psi_s D_{s,2} - 1)^2 - 4\psi_s(\xi - D_{s,2})}}{2\psi_s} \tag{48}$$

Notice that x_d is increasing in ξ. We can find a sufficient condition to ensure that $x_d > 0$. First, note that the upper bound for $D_{s,2} = \frac{1}{2\psi_s}$ which occurs as $\tilde{\lambda}_2 \to -\infty$. At the upper bound, $x_d < 0$. The lower bound for $D_{s,2} = 0$ which occurs as $\tilde{\lambda}_2 \to 0$. At this bound,

$$x_d = \frac{1 - \sqrt{1 - 4\psi_s \xi}}{2\psi_s}, \tag{49}$$

which is positive if and only if

$$0 < \xi < \frac{1}{4\psi_s}. \tag{50}$$

Hence, there exists small enough $\xi > 0$ such that $x_d > 0$. Since x_d is decreasing in ξ and ϵ is independent of ξ, there exists a small enough $\xi > 0$ such that $\epsilon > x_d$. Hence, cases 3 and 4 exist against case 2.

Cases 3 and 4 vs. Case 1 A sufficient condition to ensure that case 3 dominates case 1 is that case 1, which is the unconstrained equilibrium, is not feasible. Case 1 is not feasible whenever constraint (7) is violated. Substituting the expression for T_1 and L_1 into this constraint yields the following parameter restriction which ensures that case 3 dominates case 1:

$$\frac{\Gamma_b(\psi_b)}{\beta} < (\tilde{z} - \underline{z}) \ln \left(\frac{\tilde{z}\beta(\psi_s + \psi_b)}{w\psi_s\psi_b \left[2\left(\tilde{z}\beta\right) + \sqrt{\left(\frac{\psi_s+\psi_b}{\psi_s\psi_b}\right)^2 + 4\left(\tilde{z}\beta\right)^2} \right]} \right) \tag{51}$$

The same restriction ensures that case 4 is preferred to case 1 because constraint (7) is slack in case 4.

Case 4 vs. Case 3 Comparing the value functions in the two cases, $V_{s,4} > V_{s,3}$ if and only if:

$$\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}} > \frac{\psi_b + \psi_s}{\psi_b} \frac{(1-p)}{\beta \tilde{z}} \left(\frac{\bar{z}}{\underline{z}} - 1 \right) (D_{s,3} - D_{s,4}) > 0 \tag{52}$$

The difference $D_{b,3} - D_{b,4}$ is increasing in $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$. However, there exist parameter restrictions that ensure that this increase is less than linear such that the inequality in (52) holds. Consider the limiting case of $\xi = \epsilon > 0$, where ϵ is a small positive number. Taking logs of expression (27) yields the following log approximation of the implicit function that defines $D_{s,4}$:

$$\log \left((1 - \psi_s D_{s,4}) \left(1 + \frac{\psi_s}{\psi_b} \right) D_{s,4} \right) \approx \log w + \frac{(1 + r^*) \left(1 + \frac{\psi_s}{\psi_b} \right)}{\underline{z}} D_{s,4} + \frac{\Gamma_b(\psi_b)}{\beta p \underline{z}} \tag{53}$$

The LHS of expression (53) is a parabola that has an inverted U shape, while the RHS is linear in $D_{s,4}$. Since $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$ enters the intercept of the RHS, an increase shifts the RHS up by that amount. However, since the LHS has curvature, the decline in $D_{s,4}$, $\Delta D_{s,4}$, is strictly less than $\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}$. From expressions (22) and (27), $\Delta D_{s,4} = D_{s,3} - D_{s,4}$. All that remains is to ensure that the proportionality factor that scales this difference in expression (53) is not very large. A sufficient condition is that the following parameter restriction holds:

$$\frac{\psi_b + \psi_s}{\psi_b} \frac{(1-p)}{\beta \tilde{z}} \left(\frac{\bar{z}}{\underline{z}} - 1 \right) \leq 1 \tag{54}$$

Hence, case 4 dominates whenever restriction (54) holds for small $\xi > 0$.

Comparing expressions (38) and (39), clearly the conditions for existence of case 3 are less strict than those for case 4. Hence, case 3 dominates at the minimum whenever case 4 is not feasible, which occurs when

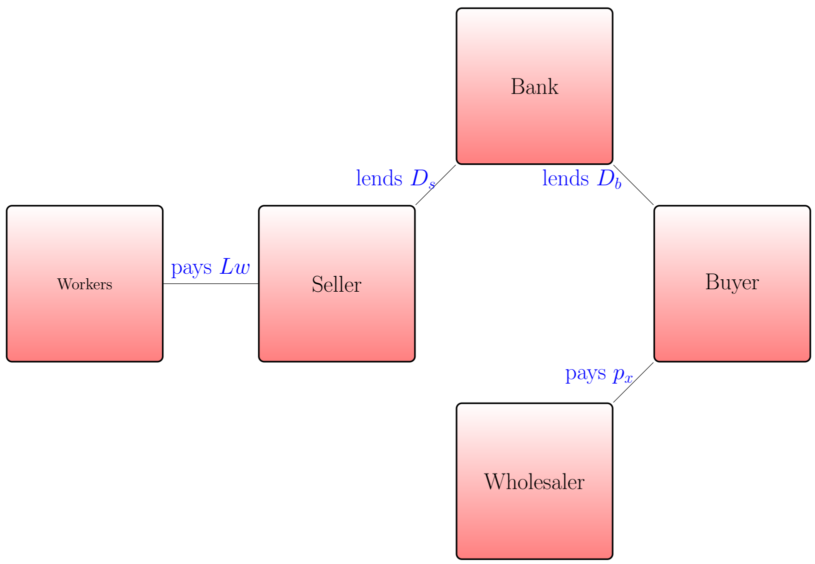
$$we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)+\frac{\Gamma_b(\psi_b)}{\beta p \underline{z}}} + \xi > \frac{1}{4}\left(\frac{1}{\psi_s} + \frac{1}{\psi_b}\right) \geq we^{\frac{1+r^*}{2\underline{z}}\left(\frac{1}{\psi_s}+\frac{1}{\psi_b}\right)} + \xi \tag{55}$$

To understand why debt is lower in case 4 than in case 3, start from a scenario where constraint (6) is binding. In order to relax it, you need to decrease D_s. But because D_s/D_b is fixed, this implies that D_b must also fall. Constraints (2) and (3) then imply that if D_s, D_b are declining, L is declining. Meanwhile, this allows you to raise $\bar{T}, \underline{T}$ exactly by $\delta D_b(1+r^*)$ to keep (1) unchanged where you are subtracting . Because $\underline{z} < \bar{z}$, the derivative of D_b with respect to L is bigger in (8) than in (7). This means that it is more costly to decrease debt using constraint (8). So, the same reduction in L will generate a bigger reduction in D_b in (8) than in (7), so the level of debt is lower in (8). Trade credit is more dispersed because

In order not to violate constraint (1) (keeping the buyer from dissolving the match), $\tilde{T}$ has to remain unchanged, which means that $\underline{T}$ must decrease. This means (8) will start to bind while (7) is being relaxed.

A.7 Other Figures

(a) Period 1



(b) Period 2

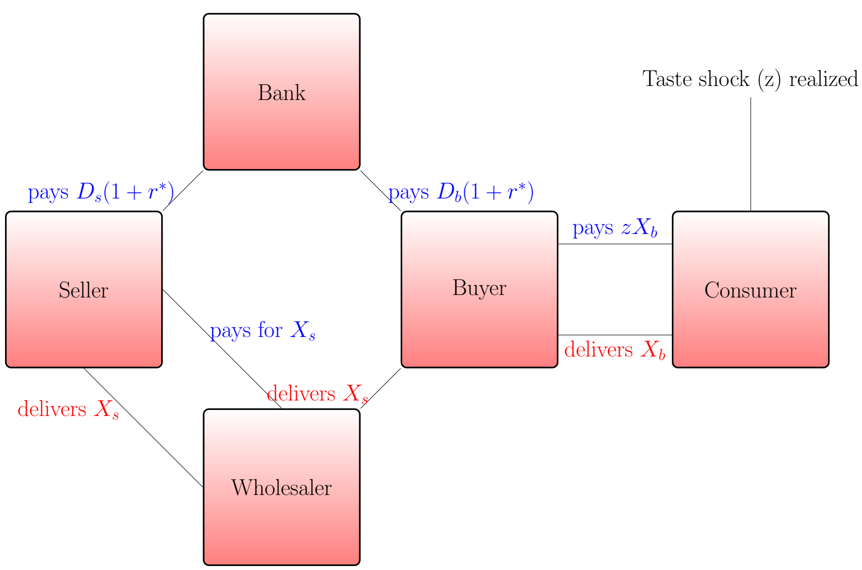
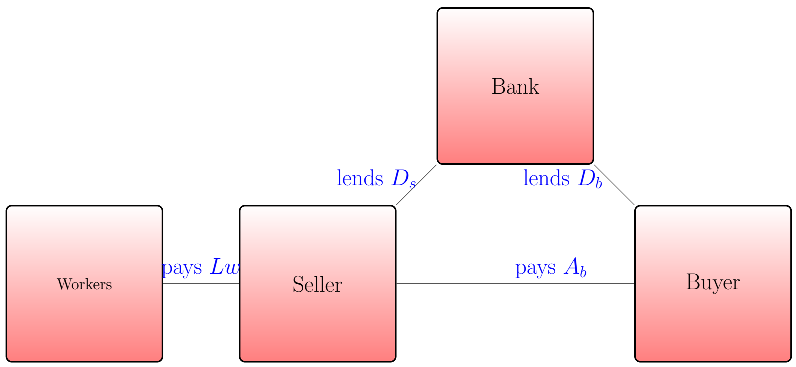


Figure 9: Model Description: Unmatched Agents

(a) Period 1



(b) Period 2

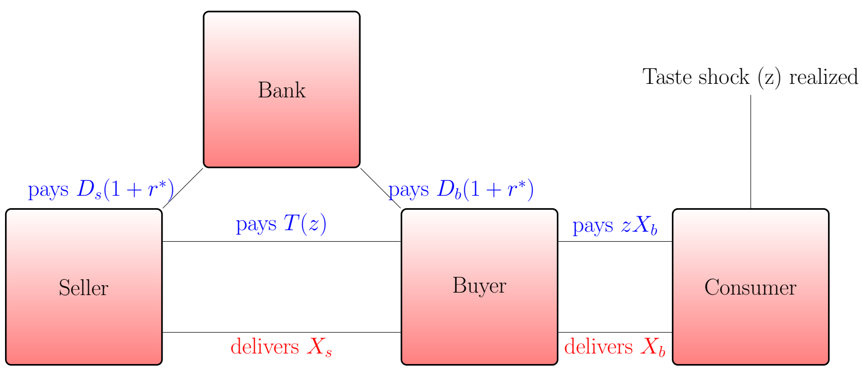
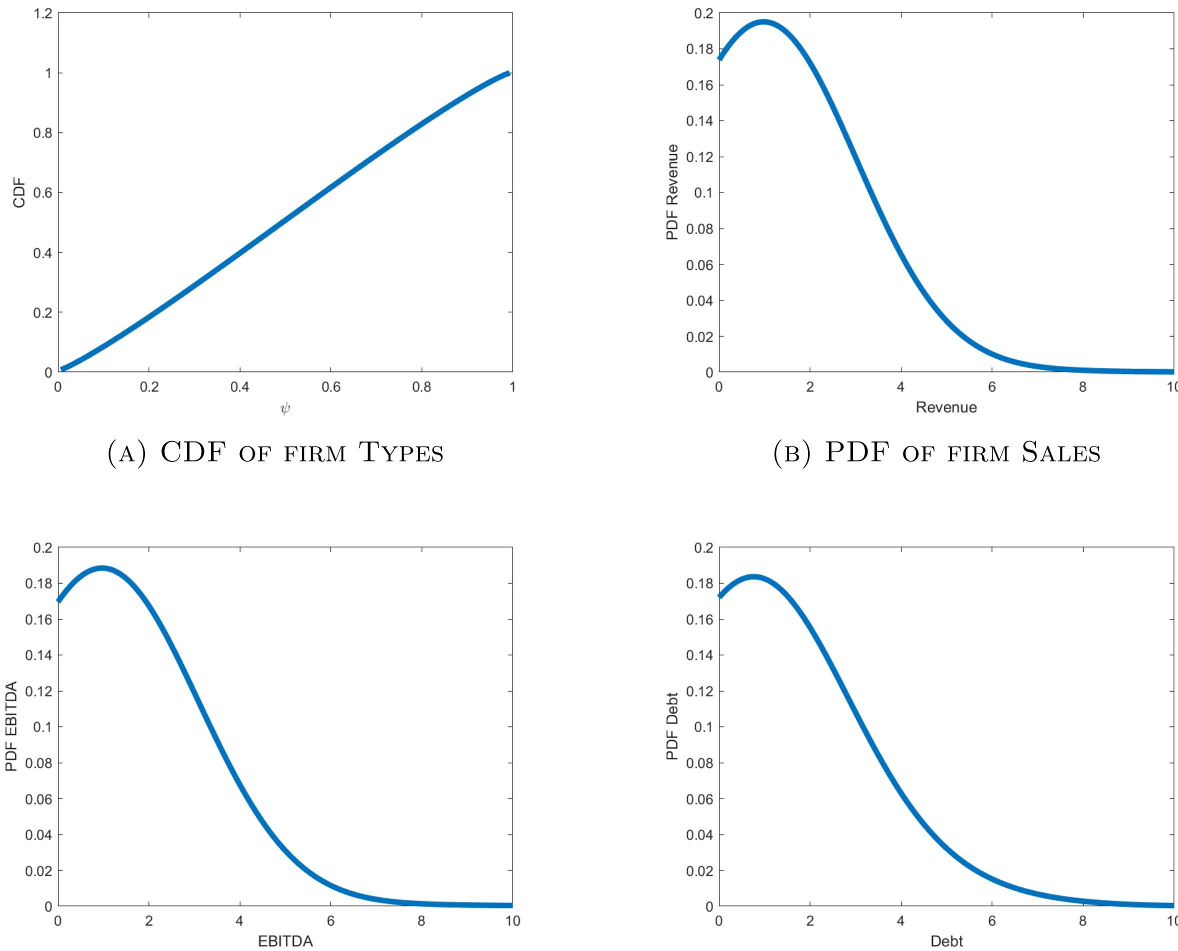


Figure 10: Matched Agents



(a) CDF of firm Types

(b) PDF of firm Sales

(c) PDF of firm EBITDA

(d) PDF of firm Total Debt

Figure 11: Model Distribution