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CONTESTS OVER PUBLIC GOODS: EVOLUTIONARY STABILITY AND THE FREE-RIDER PROBLEM

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Abstract

We analyze group contests for public goods by applying the solution concept of an evolutionary stable strategy (ESS). We show that a global ESS cannot exist, because a mutant free-rider can always invade group behavior successfully. There does exist, however, a unique local ESS, which we identify with evolutionary equilibrium. It coincides with Nash equilibrium, the hitherto dominant solution concept in contest theory, if and only if groups are symmetric. For asymmetric groups it always proposes a different and arguably more sensible solution than Nash equilibrium. We explore the properties of (local) ESS in detail.

JEL C79, D71, D72, H41.

Keywords: contests, public goods, evolutionary stability.

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1 Introduction

Collective action often takes the form of two or more groups contesting an issue. If this "issue" can be represented by a price V, groups formally compete with one another by expending effort or money in order to win the prize and a contest game results. Contest theory has entered many areas of economic theorizing; e.g. R&D-competition, rent-seeking or lobbying efforts, election campaigns, promotion races and legal battles have all been modelled and analyzed as contests. Most of these applications - even if competition between groups of agents is concerned - feature contests over a private good. The literature on contests over prizes, which exhibit features of a group-specific public good, is not well-developed, despite the fact that contests over those goods are frequent and stimulate, if not require collective action.

Katz et al. (1988) were the first to analyze a rent-seeking contest over a public good. In their model rent-seeking behavior among (homogenous) groups reduces (in equilibrium) to rent-seeking behavior among individuals, a feature that is reproduced by two variants of their model (for heterogenous groups) due to Baik (1993) and Baik et al. (2001). Those models only differ in the specification of the contest success functions. All of them use Nash equilibrium to determine solutions. The non-intuitive results derived from these models have been criticized by Riaz et al. (1995), who consider a more general model with preferences of contestants over public and private goods, which is not a proper contest anymore.

In this paper the basic modelling of the problem as a contest is fully maintained, but we apply a different solution concept. Instead of Nash equilibrium, which presupposes rational actors, we explore the implications of evolutionary stability of behavior. An evolutionarily stable strategy (ESS) represents the most basic solution concept from evolutionary game theory. It yields a refinement of Nash equilibrium, if applied to infinitely large populations of players. It need, however, not coincide with Nash equilibrium in finite population games. In fact, Leininger (2002) shows that ESS is always different from Nash equilibrium in the case of classic (private good) contests among individuals. We demonstrate in this paper that ESS can also fruitfully be applied to public good contests and argue its case vis-a-vis Nash equilibrium. ESS predicts intuitively more sensible behavior and sheds new light on the free-rider problem, which infests the competing groups.

The paper is organized as follows: section 2 reviews the reference model of Katz et al. (1988); section 3 introduces the notion of an ESS for finite asymmetric player games. In section 4 ESS is applied to the public good contest of section 2; the main result (Theorem 1) shows, that a (global) ESS does not exist, if each group has at least two members (and is hence susceptible to free-riding). Section 5 proves existence of a unique local ESS and explores its properties in detail. Section 6 concludes.

2 The reference model

Katz et al. (1988) examine the following model of a contetst for a pure public good, which directly builds on seminal work on individual rent-seeking by Tullock (1967,1980). Two locations contest a prize, which is valued at V by *each* individual in each of the two locations, e.g. V could represent public funds to be used for an environmental clean-up in either of the two locations. Other example might include the employment providing allocation of government agencies in one of the two locations, a classic example is provided by the contest of the two cities of Bonn and Berlin for capital status after reunification in Germany (see Leininger, 1993).
There are n individuals living at location 1 and m individuals at location 2. Hence the prize exhibits the property of a group-specific (resp. local) public good. It is assumed that the probability of location $i = 1,2$ winning the prize is given by the ratio of the *total* amount of expenditures (resp. effort) in the contest by individuals in location i to the total amount of expenditures by all individuals in both locations.
Hence the contest success function is given by

$$p_i(x_1,\dots,x_n,y_1,\dots,y_m) = \frac{\sum_{k=1}^n x_k}{\sum_{k=1}^n x_k + \sum_{j=1}^m y_j} \qquad i = 1,2$$

where $x_k,\ k = 1,\cdots,n$, denotes expenditure by individual k in location 1 and y_j denotes expenditure of individual in location 2.
Expected profit for player k in location 1 is therefore given by

$$\Pi_k(x_1,\dots,x_n,y_1,\dots,y_m) = \frac{\sum_{k=1}^n x_k}{\sum_{k=1}^n x_k + \sum_{j=1}^m y_j} \cdot V - x_k \qquad k = 1,\dots,n$$

while expected profit for j in location 2 is given by

$$\Pi_j(x_1,\dots,x_n,y_1,\dots,y_m) = \frac{\sum_{j=1}^m y_j}{\sum_{k=1}^n x_k + \sum_{j=1}^m y_j} \cdot V - y_j \qquad j = 1,\dots,m$$

Note that players simultaneously interact at an intra-group level and an inter-group level, the inter-group competition will affect intra-group behavior and vice versa.

One can show:

Proposition 1 (Katz et al.,1988) There exists a unique Nash equilibrium in pure stratagies, in which each player bids according to
$x^* = x_k^* = \frac{1}{4n} \cdot V$, if he belongs to location 1, and
$y^* = y_k^* = \frac{1}{4m} \cdot V$, if he belongs to location 2.
Aggregate expenditures amount to $\frac{1}{2} \cdot V$ and
$p_1(x^*, .., x^*, y^*, .., y^*) = p_2(x^*, .., x^*, y^*, .., y^*) = 1/2.$

Hence the total effort expended in the contest is *independent* of group size and equals one-half of the value of the rent. Moreover, total expenditures by each location are the same in equilibrium, thus they are equally likely to win the prize. This neutrality of group size is explained by the fact that whilst the free-rider problem increases with group size, the *aggregate* prize for the group also increases with group size. The negative incentive effect of another group member on effort by fellow group members, who partially free-ride on the new member's contribution, is exactly offset by the contribution of the new member. As a result, groups (of identical players) behave in a *public* good contest like single individuals in a private good context and reproduce Tullock's (1980) equilibrium results for rent-seeking between individuals for a privat good at an aggregate level. These non-intuitive features of the model have been critized by Riaz et al. (2001), who consider a more general model borrowed from the literature on public good provision, in which preferences depend on the public good as well as a private good. In this more general context they prove, that total effort expenditures *do* depend on groupsize(s) in a positive way. Their model, however, does not conform well to the theory of contests in general, or the literature on rent-seeking in particular. In the following we shall develop an analysis of the pure contest model of Katz et al.(1988) which exhibits a more intuitiv relationship between the intra-group incentives to free-ride and inter-group competitive forces.

3 Evolutionary stability of behavior

Ideas from evolutionary game theory find more and more applications outside the narrow confines of biological contexts. They have become increasingly popular in economics as a guide to the formulation of economic development in general and the evolution of economic processes in particular. One of the most basic concepts of evolutionary game theory is the notion of an evolutionarily stable strategy (ESS) as defined by Maynard Smith (Maynard Smith and Price (1973), Maynard Smith (1974,1982)). Its definition neatly shortcuts the full study of a detailed dynamic process by formalizing necessary requirerments for a *stable rest point* of such a process. It is this notion of a solution concept that shall be used to analyze contests for a pure local or group-specific public good.

Generally speaking, evolutionary game theory lends itself to a belief-free analysis of interactive decisions, it therefore requires less than "full rationality", if the latter is understood as utility maximization under optimal usage of all relevant information, which in interactive decisions includes preferences and beliefs of opponents. Strategy selection in evolutionary game theory is basically governed by - the economically meaningful - desire of *beating the average*. It is hence not surprising that quite often economic phenomena, which seem to contradict "traditional" rational theories, can be explained by resort to evolutionary arguments (the literature on experimental economics provides ample examples).

Hehenkamp, Leininger and Possajennikov (2001) have demonstrated the scope and usefulness of the solution concept ESS for contests by reanalyzing Tullock's (1980) classic rent-seeking model. The unique Nash equilibrium in that model is not evolutionarily stable. Still, an ESS exists precisely in those circumstances that do admit existence of a Nash equilibrium. The most prominent feature of the unique ESS is, that it implies underdissipation of the rent, if the rent-seeking technology exhibits decreasing returns to scale, full rent dissipation with constant returns to scale, and overdissipation (!) with increasing returns to scale (see Hehenkamp et al. (2001)).

In order to analyze the public good case of a contest from the point of view of evolutionary stability, we have to adapt the solution concept ESS to the *asymmetric* group context of the present contest.

Recall that a strategy is evolutionarily stable, if a whole population, which uses that strategy (or standard of behavior), cannot be invaded by a small group of mutants using a different ("mutant") strategy. The emphasis of the evolutionary approach is not on explaining actions as a result of choice, but on the diffusion of behavioral forms in groups (as a result of learning, imitation, reproduction or otherwise).

In a finite population of $r = n + m$ individuals the smallest meaningful number of mutants is one, hence we define:

Definition: *(Schaffer, 1988):*

i) *Let a strategy (standard of behavior)* x *be adapted by all players* i, $i = 1, \ldots, r$. *A mutant strategy* $\bar{x} \neq x$ *can invade* x, *if the pay-off for a single player using* $\bar{x}$ *(against* x *of the* $(r-1)$ *other players) is strictly higher than the pay-off of a player using* x *(against* $(n-2)$ *other players using* x *and the mutant player using* $\bar{x}$*).*

ii) *A strategy* x^{ESS} *is evolutionarily stable, if it cannot be invaded by any other strategy.*

Since Schaffer's reformulation of ESS for finite populations applies to symmetric games with identical roles for players, we have to use Selten's (1980) general technique to "symmetrize" an asymmetric game in order to make it analyzable by ESS.

Players of the present contest can assume one of two *roles*, either they become a member of the group in location 1 or they become a member of the group in location 2. Nature assigns these roles to each of the $r = n + m$ players with probabilities $\frac{n}{n+m}$ for group 1 resp. $\frac{m}{n+m}$ for group 2. After the role assignment the contest takes place. A strategy for a player in the symmetric version of the contest game reflects behavior *conditional* on roles, i.e. it specifies behavior for group 1-membership, x say, *and* group 2-membership, y say. Figure 1 illustrates the situation for player i.

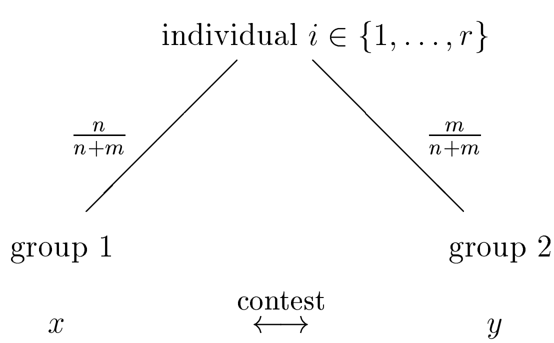


Fig.1 Role assignment and behavior

Since a player expects with probability $\frac{n}{n+m}$ (resp. $\frac{m}{n+m}$) to become assigned membership of group 1 and play x (resp. group 2 and play y) his expected pay-off, if viewed as an individual of the *monormorphic* population of $r = (n+m)$ players using the role-conditioned strategy (x, y), is

$$\Pi(x, \ldots, x, y, \ldots, y) = \frac{n}{n+m}\Big(\frac{n \cdot x}{n \cdot x + m \cdot y}V - x\Big) + \frac{m}{n+m}\Big(\frac{m \cdot y}{n \cdot x + m \cdot y}V - y\Big).$$

Such a *population game* (Selten, 1980) is necessarily symmetric and we can apply the solution concept ESS (as defined above) to it. We define:

Definition: (x^*, y^*) *is an evolutionarily stable strategy (ESS) if it cannot be invaded by an alternative strategy* $(x_M, y_M) \neq (x^*, y^*)$.

We now formalize ESS in the context of the pure public good contest of Katz et al. (1988).

4 Public Good Contest: an evolutionary analysis

Let (x, y) be a (role-conditioned) strategy, which is adopted by all players; (x, y) can be invaded, if there exists (x_M, y_M) such that

$$\Pi_M(x_M, x, \ldots, x, y_M, y, \ldots, y) > \Pi_l(x_M, x, \ldots, x, y_M, y, \ldots, y)$$

$l = 2, \ldots, m+n;$

we denote the pay-off of player 1, the mutant, by Π_M and the identical pay-off of all other players, $l = 2, \ldots, n$, by Π_{NM}.

As a consequence a strategy (x^*, y^*) is an ESS if and only if

$$(*) \qquad \Pi_M(x_M, x^*, \ldots, x^*, y_M, y^*, \ldots, y^*) \leq \Pi_{NM}(x_M, x^*, \ldots, x^*, y_M, y^*, \ldots, y^*)$$

for all $(x_M, y_M) \neq (x^*, y^*)$.

A useful and informative reformulation of condition (*) is, that (x^*, y^*) is an ESS if it solves

$$(**) \quad max_{(x_M, y_M)} \ \Pi_M(x_M, x^*, .., x^*, y_M, y^*, .., y^*) - \Pi_{NM}(x_M, x^*, .., x^*, y_M, y^*, .., y^*)$$

(**) shows that in an ESS, i.e. when all players use the strategy (x^*, y^*), a player behaves *as if* maximizing the pay-off *difference* between his and the *average* pay-off of other players. In particular, a player need not maximize his expected pay-off (as in Nash-equilibrium) in an ESS. His aim is to beat the *average* pay-off and this goal is not only furthered by high own pay-payoff, but also by a low(er) pay-off of rivals! Behavior in an ESS, that lowers rivals' pay-offs has been termed "spiteful" (Hamilton, 1971). It can occur in the present context via lowering the opponents' (group) probability of winning the contest. Hence the free-riding problem *within* a group becomes intertwined with *relative* competition *between* groups. For the latter relative group size should matter and one would expect that larger groups suffer more from the free-rider problem than smaller groups. As a consequence total expenditures should decline with relative size of m and n.

We look at these issues in detail now. We have that

$$\Pi_M^*(x_M, x^*, .., x^*, y_M, y^*, .., y^*) = \frac{n}{n+m}\Big[\frac{(n-1)\cdot x^* + x_M}{(n-1)\cdot x^* + x_M + m\cdot y^*}\cdot V - x_M\Big]$$
$$+\frac{m}{n+m}\Big[\frac{(m-1)\cdot y^* + y_M}{n\cdot x^* + (m-1)\cdot y^* + y_M}\cdot V - y_M\Big]$$

and

$$\begin{aligned}
\Pi_{NM^*} \quad & (x_M, x^*, \ldots, x^*, y_M, y^*, \ldots, y^*) \\
= \quad & \frac{n}{n+m}\Big[\frac{n}{n+m}\Big(\frac{(n-1)\cdot x^* + x_M}{(n-1)\cdot x^* + x_M + m\cdot y^*}\cdot V - x^*\Big) \\
+ \quad & \frac{m}{n+m}\Big(\frac{n\cdot x^*}{n\cdot x^* + (m-1)\cdot y^* + y_M}\cdot V - x^*\Big)\Big] \\
+ \quad & \frac{m}{n+m}\Big[\frac{n}{n+m}\Big(\frac{m\cdot y^*}{m\cdot y^* + (n-1)\cdot x^* + x_M}\cdot V - y^*\Big) \\
+ \quad & \frac{m}{n+m}\Big(\frac{(m-1)y^* + y_M}{n\cdot x^* + (m-1)y^* + y_M}\cdot V - y^*\Big)\Big]
\end{aligned}$$

The first term of Π_M^* refers to the possible role of the mutant as a member of group 1, the second refers to the possible role as a member of group

2. Π^*_{NM} consists of the analogue two terms for a non-mutant taking into account that the mutant can have *either* role.

If we substitute these two expressions into the maximization problem (**) we get, that an ESS must solve (after some tedious rearrangements)

$$(M) \qquad max_{(x_M,y_M)} \quad \Big\{ \frac{n \cdot m}{(n+m)^2} \cdot \frac{(n-1)x^* + x_M - m \cdot y^*}{(n-1)x^* + x_M + m \cdot y^*} \cdot V - \frac{n}{n+m}(x_M - x^*)$$

$$+ \frac{n \cdot m}{(n+m)^2} \cdot \frac{(m-1)y^* + y_M - n \cdot x^*}{(m-1)y^* + y_M + n \cdot x^*} \cdot V - \frac{m}{n+m}(y_M - y^*) \Big\}$$

The first-order conditions are (by the quotient rule)

i) $\frac{n \cdot m}{(n+m)^2} \cdot \frac{(n-1)x^* + x_M - m \cdot y^* - (n-1)x^* - x_M + m \cdot y^*}{((n-1)x^* + x_M + m \cdot y^*)^2} \cdot V - \frac{n}{n+m} = 0$

ii) $\frac{n \cdot m}{(n+m)^2} \cdot \frac{(m-1)y^* + y_M - n \cdot x^* - (m-1)y^* - y_M + n \cdot x^*}{((m-1)y^* + y_M + n \cdot x^*)^2} \cdot V - \frac{m}{n+m} = 0$

After some manipulations these yield

i') $\frac{m}{n+m} \cdot \frac{2m \cdot y^*}{((n-1)x^* + x_M + m \cdot y^*)^2} \cdot V = 1$

ii') $\frac{n}{n+m} \cdot \frac{2n \cdot x^*}{((m-1)y^* + y_M + n \cdot x^*)^2} \cdot V = 1$

Since the solution must be symmetric in roles, we impose

$y_M = y^*$ and $x_M = x^*$, which gives us

$$\frac{m}{n+m} \cdot \frac{2my^*}{(n \cdot x^* + m \cdot y^*)^2} \cdot V = 1 = \frac{n}{n+m} \cdot \frac{2nx^*}{(n \cdot x^* + m \cdot y^*)^2}$$

Consequently, $\qquad m^2 \cdot y^* = n^2 \cdot x^* \qquad$ (R)

If we substitute this into i') we get

$$\begin{aligned}\frac{2}{n+m}\cdot\frac{n^2\cdot x^*}{(n\cdot x^*+\frac{n^2}{m}x^*)^2}\cdot V &= 1 \qquad \text{or}\\ \frac{2n^2}{(n+m)(n+\frac{n^2}{m})^2\cdot x^*}\cdot V &= 1 \qquad \text{and therefore}\\ x^*=\frac{2n^2}{(n+m)}\cdot\frac{m^2}{n^2(m+n)^2}\cdot V &= 2\cdot\frac{m^2}{(n+m)^3}\cdot V\end{aligned}$$

and from this it follows that

$$y^*=2\cdot\frac{n^2}{(m+n)}\cdot V.$$

Note that this unique interior solution of (**) resp. (M) coincides with the Nash equilibrium of the previous section, if and only if $m=n$! Katz et al. (1988) *assume* a "regular, interior solution" (p. 51) for the first-order condition of the Nash equilibrium problem. Checking for boundary solutions (see Appendix) reveals, that – for all m and n – this *assumption* is indeed satisfied; i.e. the interior Nash solution is a global one for the model under consideration. This is not true for our unique interior ESS-candidate:

Theorem 1: *There is* no *evolutionarily stable strategy in a public good contest for V, if each group has at least two members.*

Proof: We will show that a mutant free-rider, who uses strategy $(x_M, y_M)=(0,0)$ always can invade (x^{ESS}, y^{ESS}), the only interior candidate for an ESS. Obviously, free-riding itself is not an ESS either.
So let player 1 use strategy $(x_M, y_M)=(0,0)$, while player $i, i=2,\ldots,r$, still uses $(x^*, y^*)=(x^{ESS}, y^{ESS})$. If we can show, that the relative pay-off for 1 is positive, the claim is proven. We have

$$\Pi_1(0,x^{ESS},..,x^{ESS},0,y^{ESS},..,y^{ESS})-\Pi_i(0,x^{ESS},..,x^{ESS},0,y^{ESS},..,y^{ESS})$$

$$=\frac{n}{n+m}\big(\frac{(n-1)\cdot x^*}{(n-1)\cdot x^*+m\cdot y^*}\cdot V\big)+\frac{m}{n+m}\big(\frac{(m-1)\cdot y^*}{n\cdot x^*+(m-1)\cdot y^*}\cdot V\big)$$

$$-\frac{n}{n+m}\Big[\frac{n}{n+m}\Big(\frac{(n-1)\cdot x^*}{(n-1)\cdot x^*+m\cdot y^*}\cdot V-x^*\Big)+\frac{m}{n+m}\Big(\frac{n\cdot x^*}{n\cdot x^*+(m-1)\cdot y*}\cdot V-x^*\Big)\Big]$$

$$-\frac{m}{n+m}\Big[\frac{n}{n+m}\Big(\frac{m\cdot y^*}{m\cdot y^*+(n-1)\cdot x^*}\cdot V - y^*\Big)+\frac{m}{n+m}\Big(\frac{(m-1)\cdot y^*}{n\cdot x^*+(m-1)\cdot y*}\cdot V - y^*\Big)\Big]$$

$$=\frac{n}{(n+m)}\Big(1-\frac{n}{n+m}\Big)\frac{(n-1)\cdot x^*}{(n-1)\cdot x^*+m\cdot y^*}\cdot V+\frac{m}{(n+m)}\Big(1-\frac{m}{n+m}\Big)\frac{(m-1)\cdot y^*}{n\cdot x^*+(m-1)\cdot y*}\cdot V$$

$$-\frac{n}{(n+m)}\cdot\frac{m}{(n+m)}\cdot\frac{n\cdot x^*}{n\cdot x^*+(m-1)\cdot y^*}\cdot V-\frac{m}{(n+m)}\frac{n}{(n+m)}\cdot\frac{m\cdot y^*}{m\cdot y^*+(n-1)\cdot x*}\cdot V$$

$$+\frac{n}{n+m}\cdot x^*+\frac{m}{n+m}\cdot y^*$$

$$=\frac{n}{(n+m)}\cdot\frac{m}{(n+m)}\cdot\frac{(n-1)\cdot x^*}{(n-1)\cdot x^*+m\cdot y^*}\cdot V+\frac{m}{(n+m)}\cdot\frac{n}{(n+m)}\cdot\frac{(m-1)\cdot y^*}{n\cdot x^*+(m-1)\cdot y*}\cdot V$$

$$-\frac{n}{(n+m)}\cdot\frac{m}{(n+m)}\frac{n\cdot x^*}{n\cdot x^*+(m-1)\cdot y^*}\cdot V-\frac{n}{(n+m)}\cdot\frac{m}{(n+m)}\frac{m\cdot y^*}{m\cdot y^*+(n-1)\cdot x^*}\cdot V$$

$$+\frac{n}{(n+m)}\cdot x^*+\frac{m}{n+m}\cdot y^*>0 \qquad \text{if and only if}$$

(divide by $\frac{n}{n+m}\cdot\frac{m}{n+m}$)

$$\Big(\frac{(n-1)\cdot x^*-m\cdot y^*}{(n-1)\cdot x^*+m\cdot y^*}+\frac{(m-1)\cdot y^*-n\cdot x^*}{n\cdot x^*+(m-1)\cdot y^*}\Big)\cdot V+\frac{n+m}{m}\cdot x^*+\frac{n+m}{n}\cdot y^*>0$$

We now substitute $x^*=\frac{2m^2}{(n+m)^3}\cdot V$ and $y^*=\frac{2n^2}{(n+m)^3}\cdot V$ and collect terms, which yields after tedious manipulations (see Appendix) the equivalent condition that

$$m\cdot n^3-2n^3+n\cdot m^3-2m^3+2n^2m^2+2n\cdot m-n^2m-nm^2>0;$$

Write this as

$$(m-2)n^3+(n-2)m^3+[n^2\cdot m(2m-1)-n\cdot m(m-2)]>0$$

and it is immediate, that all three terms of the sum are non-negative and the last one is strictly positive for $n>1$ and $m>1$; i.e. both groups need to be "groups" in the proper sense of the word. One contestant alone in a "group" cannot suffer from free-riding of group members. This completes the proof. q.e.d.

Theorem 1 is remarkable, it nicely illustrates the different stability conditions embodied by Nash equilibrium and ESS. Take the case when $m=n$ and hence both solution candidates coincide. While $(x^*,y^*)=(\frac{1}{4n}V,\frac{1}{4n}V)$

is stable against all Nash deviations, it is not stable against all mutants. When considering a deviation from $x^* = \frac{1}{4n}V$ a Nash player compares *his* pay-off after the deviation with *his* pay-off before the deviation; whereas an ESS-player – according to the *as if*-interpretation of a relative pay-off maximizer – compares *his* pay-off after the deviation with the *other* players pay-off after the deviation. In a *private* good contest for V (see Hehenkamp, Leininger and Possajenikov, 2001, and Leininger, 2002) the latter consideration always leads to a higher expenditure than in Nash equilibrium, since Nash equilibrium always occurs at a solution, that yields a positive value of the first derivative of the *relative* pay-off function. A marginal increase of expenditures therefore increases relative pay-off (Leininger, 2002, Proposition 1). Heuristically, a marginal increase of expenditures beyond the Nash equilibrim level has second-order negative effect on a player's own pay-off (as the first derivative of his pay-off function is zero at the Nash equilibrium level) and a first-order negative effect on other players' pay-off (as cross derivatives in Nash equilibrium are always negative). As a consequence the *difference* between own and others pay-offs increases. In a *public* good contest this incentive to increase one's expenditure is counteracted by the free-rider problem: although an increase above the Nash level still advances one's position in relation to member's of the *other* group, it puts oneself at a disadvantage in relation to fellow members of the own group; they free-ride on additional "spiteful" effort. An extreme free-rider, who puts in zero effort regardless of his role, can even invade the only candidate ESS (x^*, y^*) as Theorem 1 shows. (The "same" extreme free-rider cannot upset Nash equilibrium). Free-riding is – from a point of view of ESS – sensible spiteful behavior against (ex post) members of the *own* group. It is costless in terms of expenditures and, somewhat perversely, increases (ex post) the pay-off of members of the other group (which works towards a *decrease* in one's own relative pay-off), but it increases one's own pay-off by even more. An important reason for the latter is, that in the *ex ante*-calculus of role conditioned behavior a player has a reduced incentive to guard himself against such a deviant, since he might benefit from it (in relative terms), if he happens to become a member of the *other* group. Free-riding is thus even more destabilising in evolutionary equilibrium than in Nash equilibrium, which is governed by absolute (own) pay-off considerations. I.e. "rationality" *helps* to stabilize behavior and bring about predictable patterns of expenditures.

5 Local evolutionary equilibrium

The damaging message of Theorem 1 is now put into perspective with the help of our unique interior candidate solution (x^*, y^*) of the last section. We shall show, that this candidate solution *is* an ESS-solution, if we introduce a restriction due to Alós-Ferrer and Ania (2001), who define the notion of a local evolutionarily stable strategy (local ESS) for finite populations of players.

Definition: *A strategy $\bar{x}^{ESS}$ is local evolutionarily stable, if it cannot be invaded by any strategy from a neighbourhood of $\bar{x}^{ESS}$.*

Clearly, any ESS as defined before qualifies as a local ESS, since non-invadability there refers to all other strategies, not just local ones. Since our strategy sets are connected continua in Euclidian space, neighbourhood is straightforwardly defined with respect to Euclidian norm $d(x, y)$, i.e.

$$B(\bar{x}, \delta) = \{x \in \ R | d(\bar{x}, x) < \delta\}$$

Local evolutionary stability requires that mutants enter with behavior similar to the one used before mutation. In an evolutionary context this requirement has almost natural appeal. Although we have not modelled an adjustment dynamics to ESS, we note that the most popular dynamics used in evolutionary game theory (like e.g. replicator dynamics) portray agents as boundedly rational agents, who either learn, imitate or experiment with limited information about the environment. In our particular context of a contest between two *groups* of agents for a public good, those behavioral restrictions, which forbid behavior too deviant from group behavior, may stem from group norms or homogeneity enforcing sanction mechanism (those may have developed in response to the content of Theorem 1).

We can now state

Theorem 2: There is a unique local evolutionarily stable strategy in a public good contest for V. It is given by $(x^*, y^*) = \left(\frac{2m^2}{(n+m)^3}V, \frac{2n^2}{(n+m)^3}V\right)$.

Proof: Consider the maximization problem (M) again and restrict it to choices from $(x_M, y_M) \in \ B(x^*, y^*, \delta)$. Then any local maximizer, that is symmetric in roles, is seen to be a local ESS. In particular, (x^*, y^*) is a local ESS. Moreover, the derivation preceding Theorem 1 shows, that $(x^*, y^*) = \left(\frac{2m^2}{(n+m)^3}V, \frac{2n^2}{(n+m)^3}V\right)$ is the only interior candidate for a local ESS. Note, that the free-riding option $(0, 0)$ is not a local ESS.

q.e.d.

Because of Theorem 2 we identify (x^*, y^*) with evolutionary equilibrium and discuss some properties of evolutionary equilibrium behavior. We first relate it to Nash equilibrium behavior.

Corollary: Nash equilibrium in a public good contest for V is (local) evolutionarily stable, if and only if the groups are of the same size (i.e. if $m = n$).

Furthermore we have

Proposition 2:

In a public good contest for V total expenditure in evolutionary equilibrium is always less or equal than in Nash equilibrium.

Proof:

Total expenditure in Nash equilibrium is $\frac{1}{2} \cdot V$, whereas in evolutionary equilibrium it is given by

$$n \cdot \frac{2m^2}{(n+m)^2} \cdot V + m \cdot \frac{2n^2}{(n+m)^3} \cdot V = \frac{2mn}{(n+m)^2} \cdot V$$

$$\frac{2m \cdot n}{(n+m)^2} \cdot V \geq \frac{1}{2} \cdot V \Leftrightarrow 2m \cdot n > n^2 + m^2$$

$$\Leftrightarrow (n-m)^2 \geq 0$$

The latter always holds. q.e.d.

Proposition 2 is in contrast to private good contests, for which the opposite relationship holds (Leininger, 2002). We have already seen that – albeit in some unexpected way – the free-rider option in the evolutionary game is far more effective than in the Nash game. This not only accounts for Proposition 2, but also for the following properties, which are in strong contrast to the neutrality properties of Nash equilibrium in public good contests.

Proposition 3:

In evolutionary equilibrium the following holds true:

i) If relative group size $\frac{n}{m}$ increases, then total contest expenditures decrease.

ii) A larger group always spends less than a smaller group.

iii) A group's probability of winning the contest is decreasing in own group size and increasing in rival group size:

$$p_{x^*}(x^*, \ldots, x^*, y^*, \ldots, y^*) = \frac{m}{n+m} \qquad ,$$

$$p_{y^*}(x^*, \ldots, x^*, y^*, \ldots, y^*) = \frac{n}{n+m}$$

Proof:

i) We have that total expenditures equal

$$\frac{2m \cdot n}{(n+m)^2} \cdot V = \frac{2 \cdot \frac{n}{m}}{(1+\frac{n}{m})^2} \cdot V$$

Consequently,

$$\frac{d}{d(\frac{n}{m})}\Big(\frac{2 \cdot \frac{n}{m}}{(1+\frac{n}{m})^2} \cdot V\Big) < 0 \quad ,$$

if we assume - without loss of generality - that $n > m$.

ii) We have from condition (R) that in equilibrium $m^2 \cdot y^* = n^2 \cdot x^*$ holds. Setting $m \cdot y^* = Y^*$ and $n \cdot x^* = X^*$, this reads as $m \cdot Y^* = n \cdot X^*$ which implies that the larger group spends less tha the smaller group on aggregate. (Note that in Nash equilibrium (section 2) $X^* = Y^*$ holds).

iii) Obvious from

$$p_{x^*}(x^*, \ldots, x^*, y^*, \ldots, y^*) = \frac{n \cdot x^*}{n \cdot x^* + m \cdot y^*} = \frac{m}{n+m}$$

$$p_{y^*}(x^*, \ldots, x^*, y^*, \ldots, y^*) = \frac{m \cdot y^*}{n \cdot x^* + m \cdot y^*} = \frac{n}{n+m}$$

q.e.d.

We now turn to individual rent-seeking in each role:

Proposition 4:
In evolutionary equilibrium it holds true that

i) individual expenditures decrease with increasing size of a contestant's own group; and

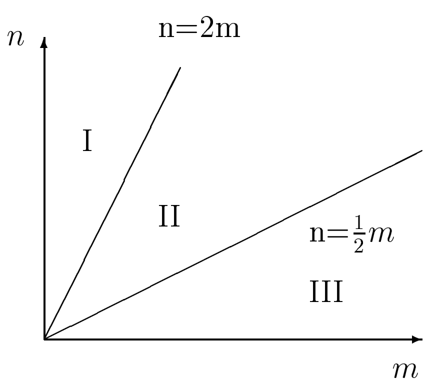
ii) individual expenditures decrease (increase) with increasing rival group size, if the rival group is at least twice (at most half) the size of a contestant's own group.

Proof:

i) $x^* = \frac{2m^2}{(n+m)^3} \cdot V$ is decreasing in n and
$y^* = \frac{2n^2}{(n+m)^3} \cdot V$ is decreasing in m.

ii) We have $\frac{dx^*}{dm} = \frac{4m(n+m)^3 - 2m^2 \cdot 3(n+m)^2}{(n+m)^6} \cdot V \quad \gtrless \quad 0$
if and only if $m \lessgtr 2n$ resp. $\frac{n}{m} \gtrless \frac{1}{2}$.
The reasoning for $\frac{dy^*}{dn}$ is completely analogous. q.e.d.

The interesting part of Proposition 4 is illustrated in Figure 2:



area I: $\frac{dx^*}{dm} > 0$ and $\frac{dy^*}{dn} < 0$

area II: $\frac{dx^*}{dm} > 0$ and $\frac{dy^*}{dn} > 0$

area III: $\frac{dx^*}{dm} < 0$ and $\frac{dy^*}{dn} > 0$

Fig.2: Critical relative group size

Areas I and III represent "lop-sided" contests: one group is more than twice as large as the other one and stands no chance, because of endemic free-riding: a further increase in the number of its members even allows members of the rival group to *reduce* their efforts! Only, if groups are similar enough in size; i.e. none is at least twice the size of the other, do members of both groups respond with an increase of expenditures in the number of their rivals.

6 Conclusion

We have studied a model of collective rent-seeking over a public good due to Katz et al. (1988) from an evolutionary point of view by applying the solution concept of an evolutionary stable strategy (ESS). It is known, that for rent-seeking over a private good ESS always differs from Nash equilibrium (Leininger, 2002). It turns out that the free-rider problem is even more severe in evolutionary equilibrium than for Nash players: a (global) ESS does not exist in our model as the only candidate solution can be invaded by a free-rider strategy (Theorem 1). However, the candidate solution is the unique local ESS of the model (Theorem 2). This evolutionary solution is not neutral with respect to group size(s), which is in contrast to an artificial property of Nash equilibrium. Rent-seekers collectively always spend less in evolutionary equilibrium than rational Nash players would do (Proposition 2). Their behavior is exclusively determined by relative group size $\frac{n}{m}$: collective expenditures decrease with an increase of $\frac{n}{m}$ and the larger group always spends less than the smaller group (Proposition 3). Individual expenditures, too, vary with relative group size in an intiuitive, but non-monotone way (Proposition 4).

Other than these formal results, we like to conclude that evolutionary analysis based on the notion of evolutionary stability yields qualitatively more sensible predictions for behavior of contestants than rational analysis based on Nash stability. A further message of the paper is, that collective rent-seeking efforts for public goods will be small, a provider of a public good, who wishes to encourage rent-seeking contributions needs to resort to additional measures.

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Appendix

A) Interiority of Nash solution:

Nash equilibrium pay-off for a group 1 member is given by

$$U_1(x^*, y^*) = \frac{1}{2}V - \frac{1}{4n} \cdot V.$$

Free-riding by choosing $x_1 = 0$ instead of $x_1 = x^*$ gives 1 pay-off

$$\frac{(n-1) \cdot x^*}{(n-1)x^* + m \cdot y^*} \cdot V = \frac{\frac{n-1}{4n}}{\frac{n-1}{4n} + 1/4} \cdot V.$$

Now

$$\frac{1}{2}V - \frac{1}{4n} \cdot V \geq \frac{\frac{n-1}{4n}}{\frac{n-1}{4n} + 1/4} \cdot V$$

$$\Leftrightarrow \quad 1/2 - \frac{1}{4n} \geq \frac{1}{1 + \frac{n}{n-1}} = \frac{n-1}{2n-1}$$

$$\Leftrightarrow n - 1/2 - 1/2 + \frac{1}{4n} \geq n - 1$$

$$\Leftrightarrow n - 1 + \frac{1}{4n} \geq n - 1$$

The interior solution dominates free-riding for a member of group 1. The proof for a member of group 2 is completely analogous.

B) Proof of Theorem 2:

Substitution of $x^* = \frac{2m^2}{(n+m)^3} \cdot V$ *and* $y^* = \frac{2n^2}{(n+m)^3} \cdot V$ yields the condition

$$\left[\frac{(n-1) \cdot \frac{2m^2}{(n+m)^3} - m \cdot \frac{2n^2}{(n+m)^3}}{(n-1) \cdot \frac{2m^2}{(n+m)^3} + m \cdot \frac{2n^2}{(n+m)^3}} + \frac{(m-1) \cdot \frac{2n^2}{(n+m)^3} - n \cdot \frac{2m^2}{(n+m)^3}}{n \cdot \frac{2m^2}{(n+m)^3} + (n-1) \cdot \frac{2n^2}{(n+m)^3}} \right] \cdot V$$

$$+ \frac{(n+m)}{m} \cdot \frac{2m^2}{(n+m)^3} \cdot V + \frac{(n+m)}{n} \cdot \frac{2n^2}{(n+m)^3} \cdot V \geq 0$$

Now multiply both sides with $\frac{(n+m)^3}{V}$ and get - equivalently -

$$\frac{(n-1)m^2 - m \cdot n^2}{(n-1)m^2 + m \cdot n^2} + \frac{(m-1)n^2 - n \cdot m^2}{n \cdot m^2 + (n-1) \cdot m^2} + \frac{2m}{(n+m)^2} + \frac{2n}{(n+m)^2} \geq 0$$

or - tediously -

$$\frac{((n-1)m^2 - m\cdot n^2)(n\cdot m^2 + (n-1)\cdot m^2) + ((m-1)n^2 - n\cdot m^2)((n-1)m^2 + m\cdot n^2)}{((n-1)m^2 + m\cdot n^2)(n\cdot m^2 + (n-1)m^2)}$$

$$+\frac{2}{n+m} \geq 0$$

which is equivalent to

$$\frac{(1-n-m)m^2\cdot n^2}{((n-1)m^2 + m\cdot n^2)(n\cdot m^2 + (n-1)\cdot m^2)} + \frac{2}{n+m} \geq 0$$

or

$$\frac{2}{n+m} \geq \frac{(n+m-1)m^2\cdot n^2}{((n-1)m^2 + m\cdot n^2)(n\cdot m^2 + (n-1)\cdot m^2)} \quad .$$

After multiplication and collection of terms we get

$$mn^3 - 2n^3 + nm^3 - 2m^2 + 2n^2m^2 + 2nm - n^2m - nm^2 \geq 0$$

which is the equation used in the proof.