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SOME EVOLUTIONARY FOUNDATIONS FOR PRICE LEVEL RIGIDITY

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Abstract

This paper shows that price rigidity evolves in an economy populated by imperfectly rational agents who experiment with alternative rules of thumb. In the model, firms must set their prices in face of aggregate demand shocks. Their payoff depends on the level of aggregate demand, as well as on their own price and their "neighbor"'s price. The latter assumption captures local interactions. Despite the fact that the rational expectations equilibrium (REE) is characterized by a simple pricing rule that firms can easily adopt, the economy does not converge to the REE for highly autocorrelated aggregate demand shocks and a high level of local interaction. Instead, the aggregate price level exhibits rigidity, in that it does not fully react to contemporaneous aggregate demand shocks, and inertia, in that controlling to it positively depends on its past value. We show that local interactions and serial correlation of aggregate demand shocks play a key role in generating those results.

JEL Classification: E3, D83, D84.

Keywords: evolution, bounded rationality adaptive learning, experimentation, externalities, spillovers, local interaction, money, aggregate demand, price rigidity, rational expectations, monetary policy, macroeconomic fluctuations.

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1 Introduction and motivation

An important cornerstone of contemporary macroeconomic theory is the idea that the price level does not react one for one to nominal aggregate demand shocks. Otherwise, aggregate demand shocks would have a zero impact on real activity. It is therefore important to investigate the sources of such price rigidity. Historically, the literature has considered three options: assuming fixed and exogenous prices, imposing that prices can only be changed infrequently, and relying on some fixed "menu cost" of changing prices.[1]None of these options, however, is fully satisfactory.

In this paper, we investigate another route, namely we study the extent to which sticky price-setting behaviour may evolve as an equilibrium outcome in an economy populated by imperfectly rational agents. We assume that such agents are not able to compute their optimal price-setting rule, and instead have to make experiments with rules-of-thumb. They drop rules that yield a low payoff in favor of those which yield a high payoff. Another important ingredient of the model is *local interaction.* That is, an agent's payoff function depends on the price chosen by another "contiguous" agent, which may for example be interpreted as a supplier.

A crucial aspect of the model is that it is impossible to separate learning from action, and by experimenting, firms exert externalities on their neighbor, whose payoff depends on the firm's price. Consequently, experimentation by one agent affects the rules picked up by other agents.

We then ask the following questions: Does the economy converge to the REE, or do experiments and local interaction prevent this from happening? If so, do we observe any recognizable pattern in the behavior of aggregate prices and in the cross-section distribution of individual prices?

Despite that the rational expectations equilibrium (REE) price setting rule is feasible, i.e. belong to the set of rules that can be used by individual

[1]See Barro and Grossman (1971), Bénassy (1975,1976), Taylor (1980), Fischer (1977), Blanchard and Kiyotaki (1987), Rotemberg (1982), Caplin and Spulber (1987), Caballero and Engel (1993), Mankiw (1985), Akerlof and Yellen (1985)

agents, it turns out that for a range of parameters the economy does not converge to that equilibrium. Rather, it converges to an equilibrium where the aggregate price level does not react one for one to contemporary money shocks, contrary to what should happen in the REE where, in this model, it should precisely be equal to the money stock. The price level also exhibits inertia in that it depends on past money shocks, and also has the property of being less volatile than money. Furthermore, the response to contemporary shocks and the level of inertia are such that money is roughly neutral in the long run. To summarize, this economy exhibits the stylized macroeconomic properties of actual economies.

As we discuss below in greater details, sticky price behaviour arises from the combination of two factors: A high autocorrelation of monetary shocks, and a high degree of local interaction. If monetary shocks are not strongly autocorrelated, then the economy roughly converges to the REE. If local interaction is low, then inertia in the aggregate price level is entirely due to those firms that are temporarily experimenting with—typically suboptimal—alternative rules, and the aggregate price level is as volatile as money. But, when the two are combined, prices are sticky in a deep sense.

Experimentation by individual firms in order to try and find out a better pricing rule plays a key role in generating these results. Typically, when a firm abandons its best pricing rule for a while to experiment with a new one, this generates excess autocorrelation in its prices. If local interactions are strong, this inertia in turn spreads to those agents whose payoff depends on that firms' price. In general equilibrium, this leads the economy to a situation where the price level does not fully react to nominal aggregate demand shocks.

While our variables are interpreted in terms of "money" and "prices", the model is in fact quite general and provides evolutionary foundations for "rigid" behavior in a variety of settings.

The literature on evolution and adaptive learning is large, but has scarcely dealt with macroeconomic fluctuations[2]. A recent exception is Marcet and Nicolini (1997), who are able to obtain recurrent hyperinflations in a Sargent-Wallace (1987) style model where they impose an adaptive learning model. To my knowledge, the present paper is the first one to deal with price level stickiness.[3] Existing applications of adaptive learning to macroeconomics include Marimon et al. (1990), who deal with a problem of equilibrium selection based on Kiyotaki and Wright (1989). Sargent (1993) presents various applications of bounded rationality to macroeconomics, including an interesting one on the paradox of trade, but none of them is about deriving price stickiness and monetary policy effectiveness from evolutionary principles. Arifovic (1996, 2001), derives persistent fluctuations of the exchange rate in the context of a model where the rational expectations equilibrium is indeterminate. Applications to the time series properties of artifical stock markets have been studied by LeBaron et al (1999). Older applications to growth theory are found in Nelson and Winter (1985), who develop a model of individual firms which learn about their optimal capital/labor ratio by trial and error.

Related papers in microeconomics include Ellison (1993), and Ellison and Fudenberg (1993), who study the role of local interaction in determining outcomes in evolutionary games, and Harrington (1998, 1999), who, in particular, shows how rigid agents may be more successful in evolutionary hierarchical contests. None of this work is concerned with macroeconomic fluctuations.

Interaction and externalities in the context of the traditional sticky price literature (based on menu costs or exogenously imposed timing restrictions on price setting) have been studied by Ball and Romer (1991), Lindbeck and Snower (1999).[4]

[2] A substantial fraction is devoted to convergence of Bayesian or least square learning to rational expectations, see Bray (1982), Evans and Honkapohja (1995), Marcet and Sargent (1989) and Grandmont and Laroque (1990).

[3] In Marcet and Nicolini (1997), the learning process is totally different from this paper's, since experimentation plays no role, nor do local interactions.

[4] A more general analysis of the aggregate consequences of local interactions has been

The rest of the paper is organized as follows. First, we set up the model and derive the (trivial) rational expectations equilibrium. Then, we describe how pricing rules are set and how they evolve by selection. Main numerical results are reported in section 4, while section 5 discusses some intuition for these results.

2 The model

There are n firms indexed by $i \in Z/nZ$, located on a circle. Each firm has a left-neighbor $i-1$ and a right-neighbor $i+1$. At each date t firms set a choice variable p_{it}, called price, and an aggregate shock m_t, called money, is drawn. m_t is drawn from the [0,1] interval and p_{it} is also constrained to be chosen in that interval. When setting prices, firms observe their own past prices, their left-neighbor's past prices, and the current and past money stock. All firms set prices simultaneously without knowing other firms' contemporaneous prices. The payoff of firm i's at date t is then given by

$$U_{it} = -\gamma(p_{it} - m_t)^2 - (1-\gamma)(p_{it} - p_{i-1t})^2$$

This payoff function captures the existence of local interactions. The lower γ, the more the payoff depends on the neighbor's behavior and the greater the weight of the neighbor's price in an individual's optimal price rule. A firm's optimal price is a linear combination of money and its neighbor's expected price conditional on the firm's information set.

On may consider this as a circular flow model, with left-neighbors interpreted as suppliers and right-neighbors as customers.

It is straightforward to check that the unique rational expectations equilibrium is given by $p_{it} = m_t$, $\forall i$. In such an equilibrium all firms charge the same price and money is neutral. Past events do not affect current outcomes, since only contemporaneous variables appear in U_{it}.[5]

developed by Durlauf (1991,1996) and Brock and Durlauf (2001).

[5] Furthermore, the rational expectations equilibrium is unique, so that adaptive learning

3 Description of the procedure

We study what happens when, instead of perfectly rational agents, the economy is populated by boundedly rational artificial firms which gradually learn they payoff. These firms are experimenting with alternative price rules and select the best rules, i.e. those which yield the highest payoff.

The learning procedure is specified as follows.

At each date t, firm i's information set is given by $S_{it} = \{p_{it-1}, p_{i-1,t-1}, m_{t-1}, m_t\}$. They behave according to a rule which specifies their current price p_{it} as a function of that information set. This rule is specified as follows:[6]

$$\hat{p}_{it} = c_{0,it} + c_{1,it}\hat{m}_{t-1} + c_{2,it}\hat{m}_t + c_{3,it}\hat{p}_{it-1} + c_{4,it}\hat{p}_{i-1,t-1} \quad (1)$$

The pricing rule followed by firm i at date t is represented by the vector of parameters $(c_0, c_1, c_2, c_3, c_4)$. The hat denotes the logistic transformation:

$$\hat{x} = \ln \frac{x}{1-x}.$$

The use of this transformation guarantees that regardless of the values of $\{p_{it-1}, p_{i-1,t-1}, m_{t-1}, m_t\}$ and of the rule parameters (c_0, c_1, c_2, c_3), the price quoted by each firm remains in the $(0, 1)$ interval.

Two things should be noted regarding the price-setting behavior defined by Equation (1). First, agents do not conceptualize the notions of equilibrium, expectations, or parameters. Their mental ability does not go beyond mechanically applying rules such as (1) and learn through experience which ones are better. Hence, rational expectations, or even expectations, are meaningless for these agents. Similarly, Bayesian learning would be impossible, since a Bayesian must formulate a probability model with an underlying

will not play any role in equilibrium selection contrary to what happens in Grandmont and Laroque (1990), Marimon et al.(1990), and Arifovic (1996). In the latter case, there exists a continuum of rational expectations equilibria, and adaptive learning induces fluctuations.

[6]Note that the problem is specified in 'analog', rather than digital terms, as in Nelson and Winter, but contrary to the more recent literature which uses genetic algorithms, classifiers, or neural networks. See the Santa Fe Institute volumes (Anderson et al. (1988); Arthur et al. (1997)) for discussions. A previous attempt to formulate the problem in terms of classifier-style rules was unsuccessful, essentially because mutation or generalization of crucial bits yielded rules that made little sense.

parameter to be learned. These concepts are out of reach for our agents. Second, the REE is a special case where all firms follow the rule given by $c_2 = 1, c_1 = c_3 = c_0 = c_4 = 0$. Therefore, it is perfectly attainable. Failure to converge to the REE, if it occurs, cannot come from the agents' inability to adopt the correct behavior.

The rule coefficients can have any value. However, to prevent the economy from having unstable dynamics, we will only consider rules which satisfy the following restriction:

$$|c_3| + |c_4| < 1. \tag{2}$$

Experimentation and the evolution of rules.

The economy starts from an arbitrary distribution of prices. At the beginning of time, firms select a rule randomly and apply it to set their price. After a rule has been used for at least T periods, firms decide to experiment with another rule. There are two modes of experimentation, denoted by $e = 1, 2$. At each date t there is a probability q_{iet} that firm i abandons its rule and experiments with another one under mode e instead. The initial value of q_{iet} is exogenously set within the interval $[q_{\min}, q_{\max}]$. The two modes are:

–Local mutation ($e = 1$). In such a case the new rule is defined by

$$c_{it+1} = c_{it} + \mu z_{it},$$

where μ is a parameter capturing the size of a local mutation, and z_{it} is an i.i.d random variable distributed over $(-1, 1)$.

–Global randomization ($e = 2$). In such a case the new rule is drawn randomly in the same way as the initial one.

In both cases, if the new rule does not satisfy restriction (2), a new rule is drawn until (2) is satisfied.

During the experiment the new rule is used but the firm remembers the preceding rule as well as the average payoff experienced with it so far. Experimentation lasts for at least T periods, after which the average payoff per period is compared with that of the previous rule (note that there is no discounting). If it is inferior, then the firm returns to the previous rule, and the probability of experimenting according to mode e is adjusted downwards according to the following formula:

$$q_{iet+1} = (1-\theta)q_{iet} + \theta q_{\min}.$$

If it is better, then the experiment continues as long as the average payoff is higher than that of the previous rule, up to the point where the total duration of the experiment equals $\bar{T}$, where $\bar{T} = \min(100, T'/2)$, and T' is the total length of time during which the previous rule has been used. At this point the new rule is definitely adopted, and the probability of experimenting according to mode m is adjusted upwards using the following formula:

$$q_{iet+1} = (1-\theta)q_{iet} + \theta q_{\max}.$$

If the average payoff of the experimented rule falls below that of the previous rule at any point between T and $\bar{T}$, then the experiment is immediately abandoned and one reverts to the previous rule.

This rather complex procedure is used to prevent a rule which has proved quite good for a very long time to be abandoned in favor of a rule which has improved on it for a much shorter time, say because of an unlikely sequence of shocks. Thus we impose a probation period set to half the duration of the previous rule before an experiment is adopted. But, given that, as time proceeds, firms will find good rules that will be used for a very long time, in order to avoid too long experiments we put a cap of 100 periods on this probation period.

4 Results

We now describe how the model was solved numerically and report our main quantitative results.

The total number of firms was set to 10. Initial prices were drawn from a uniform distribution over $(0, 1)$. Initial values of c_i, $i = 0, ..., 3$, were drawn using a uniform distribution over [-1,1]. The minimum length of time a rule must be used before another can be tried is set to $T = 15$. The initial probabilities of experimenting were set at their maximum $q_1 = q_2 = q_{\max} = 0.03$. We have set $q_{\min} = 0.001$ and $\theta = 0.1$.

The money process to which this economy is subjected is an AR1:

$$m_{t+1} = \rho m_t + \varepsilon_t, \tag{3}$$

where ε is distributed uniformly over $((2\bar{m} - 1)/(1 - \rho), 1/(1 - \rho))$, so that the resulting distribution of m has a mean $\bar{m} \geq 1/2$, a minimum $2\bar{m} - 1$, and a maximum equal to 1. The mean was chosen to be equal to 0.6.

We are interested in how the average behaviour of the economy depends on γ and ρ. Therefore, we tried 4 values of ρ, $\rho = 0, 0.5, 0.9$, and 0.95, and 7 values of γ, $\gamma = 1, 0.8, 0.6, 0.4, 0.2, 0.1, 0.05$. We let the economy evolve for 500,000 periods, and then for another 3,000 periods during which we collect statistics on the aggregate price level and the aggregate money stock. This was repeated 10 times for each pair (ρ, γ), thus generating 10 simulations for each set of parameter. For each of these simulations we characterize the behaviour of the economy by running the following regression over the final 3,000 periods when statistics were collected:

$$\bar{p}_{t+1} = a_0 + a_1 m_{t+1} + a_2 \bar{p}_t + \eta_t,$$

where $\bar{p}_t$ is the aggregate price level defined as the average across firms of individual prices:

$$\bar{p}_t = \frac{1}{n} \sum_{i=1}^{n} p_{it}.$$

Under the REE one should have $a_0 = a_2 = 0$, $a_1 = 1$, and the regression should yield an R^2 equal to 1. Even if individual firms end up adopting such a rule, one should however expect small deviations from this ideal point because a fraction of firms are experimenting. In the long run, this fraction should not exceed $q_{\min}$ as experimentation should yield lower payoffs than the optimal rule.

If $0 \leq a_1 < 1$ then prices are "rigid" in the sense that they react less than one-for-one to monetary shocks. If $0 < a_2 \leq 1$ then there is "price inertia" in the sense that a price change lasts longer than the money shock which has triggered it (which should not be the case under the REE).[7]

Finally, if $a_1 + a_2 \approx 1$, then money is "neutral" in the long run, in that prices would eventually adjust fully to a permanent change in m.[8]

Tables 1 to 5 report the means, maximum, minimum, and standard deviations of the regression coefficients of interest across the 10 simulations for each pair of values of ρ and γ.

The following pattern emerges:

1. If monetary shocks are not very correlated, then the economy follows on average the REE. This is true for both $\rho = 0$ and $\rho = 0.5$, for virtually all levels of local interaction among producers. However, at strong levels of local interaction, there tends to be some price rigidity, but the average price response is never more than one standard deviation away from its predicted value of 1 in the REE. Also, there is no significant price inertia at these low values of ρ.

2. At high levels of ρ, things are quite different. First, there is significant price rigidity as well as price inertia. Rigidity and inertia are greater, the more money shocks are autocorrelated. Furthermore, an increase in the intensity of local interactions increases rigidity and inertia. For example, the contemporaneous price response a_1 steadily falls from 0.9 to 0.3 when γ falls

[7] Formally, we have $p_t = (1 - a_2 L)^{-1}(1 - \rho L)^{-1} a_1 \varepsilon_t = P(L)\varepsilon_t$. The mean lag is $P'(1)/P(1) = \frac{a_2}{1-a_2} + \frac{\rho}{1-\rho}$, while the mean lag for m is just $\frac{\rho}{1-\rho}$.

[8] Note however that such a change is a zero probability event. As long as p is stationary, it tends to return to its mean, as does m, so that any rejection of the neutrality hypothesis should be interpreted with caution. Things would be different if the driving process for m were I(1).

from 1 to 0.05 at $\rho = 0.9$. And at $\rho = 0.95$ it can be as low as 0.1. Conversely, a_2 rises with γ.

3. Overall, aggregate prices behave in accordance with the long-run neutrality hypothesis (See Table 3). This is especially true at high values of γ. At low values of γ monetary-shocks tend to have long-lasting effects, but $a_1 + a_2$ is less than one standard deviation away from 1.

4. Finally, an increase in the intensity of local interactions makes the long-run behaviour of the economy more arbitrary. Hence, the standard deviation of coefficients tends to increase as γ falls, although this is not true at high values of ρ and very low values of γ. For example, at $\rho = 0.5$ and $\gamma = 1$, the range of responses to a monetary shock is 0.971–0.992, and the range of estimated price inertia coefficients across the 10 simulations is 0.0006–0.019. At $\gamma = 0.05$, however, we get much wider ranges of 0.773–1.198 and -0.166–0.085.

Intuitively, the lower γ, the less determinate the outcome, since the optimal price response of a given firm is more dependent on its neighbour's pricing rule and less on the aggregate shock m. In the limit, when $\gamma = 0$, any pricing rule is an equilibrium provided it is followed by all agents.

$\gamma \backslash \rho$	0	0.5	0.9	0.95
1	0.984 (0.007)	0.983 (0.006)	0.902 (0.036)	0.675 (0.079)
0.8	0.998 (0.009)	0.993 (0.014)	0.898 (0.048)	0.586 (0.12)
0.6	0.973 (0.026)	0.978 (0.013)	0.785 (0.108)	0.495 (0.133)
0.4	0.986 (0.027)	0.981 (0.019)	0.743 (0.099)	0.308 (0.133)
0.2	1.006 (0.035)	0.98 (0.033)	0.574 (0.166)	0.207 (0.146)
0.1	0.956 (0.056)	0.962 (0.094)	0.375 (0.151)	0.096 (0.062)
0.05	0.929 (0.081)	0.905 (0.119)	0.29 (0.185)	0.114 (0.06)

Table 1 – Mean and standard deviation of coefficient on money

$\gamma\backslash\rho$	0	0.5	0.9	0.95
1	0.002 (0.006)	0.008 (0.007)	0.102 (0.027)	0.341 (0.0726)
0.8	-0.001 (0.007)	0.005 (0.014)	0.111 (0.057)	0.406 (0.116)
0.6	-0.013 (0.021)	0.012 (0.015)	0.169 (0.072)	0.506 (0.154)
0.4	0.005 (0.026)	0.017 (0.026)	0.228 (0.076)	0.624 (0.117)
0.2	0.008 (0.037)	0.004 (0.029)	0.359 (0.168)	0.721 (0.147)
0.1	-0.0163 (0.043)	0.026 (0.043)	0.494 (0.156)	0.835 (0.058)
0.05	-0.001 (0.067)	-0.016 (0.071)	0.609 (0.163)	0.842 (0.085)

Table 2 —Mean and standard deviation of coefficient on lagged aggregate price level

$\gamma\backslash\rho$	0	0.5	0.9	0.95
1	0.986 (0.01)	0.991 (0.005)	1.004 (0.013)	1.016 (0.021)
0.8	0.997 (0.009)	0.998 (0.014)	1.009 (0.018)	0.993 (0.035)
0.6	0.96 (0.037)	0.991 (0.016)	0.954 (0.058)	1.001 (0.035)
0.4	0.992 (0.034)	0.998 (0.016)	0.971 (0.05)	0.932 (0.05)
0.2	1.014 (0.034)	0.984 (0.039)	0.933 (0.1)	0.928 (0.042)
0.1	0.94 (0.058)	0.988 (0.103)	0.869 (0.07)	0.931 (0.045)
0.05	0.921 (0.1)	0.889 (0.13)	0.899 (0.13)	0.956 (0.046)

Table 3 — Mean and standard deviation of sum of coefficients (testing for long-run neutrality))

$\gamma\backslash\rho$	0	0.5	0.9	0.95
1	0.968–0.994	0.971–0.992	0.829–0.958	0.585–0.849
0.8	0.982–1.011	0.972–1.023	0.806–0.96	0.437–0.796
0.6	0.927–1.014	0.958–0.996	0.593–0.893	0.302–0.699
0.4	0.943–1.04	0.955–1.014	0.615–0.938	0.162–0.59
0.2	0.935–1.046	0.924–1.039	0.3–0.82	0.02–0.419
0.1	0.856–1.043	0.714–1.063	0.214–0.622	0.024–0.227
0.05	0.786–1.088	0.773–1.198	0.066–0.657	0.021–0.21

Table 4 — Minimum and maximum values of coefficient on money

$\gamma\backslash\rho$	0	0.5	0.9	0.95
1	-0.0068–0.0184	0.0006–0.019	0.056–0.164	0.208–0.447
0.8	-0.013–0.011	-0.0135–0.041	0.024–0.213	0.222–0.536
0.6	-0.037–0.035	-0.003–0.042	0.042–0.277	0.262–0.721
0.4	-0.037–0.049	-0.025–0.054	0.111–0.367	0.442–0.807
0.2	-0.05–0.074	-0.054–0.046	0.004–0.626	0.455–0.914
0.1	-0.115–0.057	-0.035–0.101	0.176–0.7	0.74–0.906
0.05	-0.084–0.133	-0.166–0.085	0.366–0.886	0.636–0.953

Table 5 — Minimum and maximum values of coefficient on lagged aggregate price level

To summarize, there is both price inertia and price rigidity—prices react less than one for one to money. This phenomenon is more salient, the more the money process is autocorrelated, and the stronger the local interactions. This does not necessarily imply, however, that prices are less volatile than money. There could be more noise in prices, in particular because of experimentation periods. In order to see whether the aggregate price level is more or less volatile than money, we have computed its relative volatility, as measured by the ratio between its standard deviation and that of money. This is reported in Table 6. As we can see, prices are equally volatile as money for low levels of money autocorrelation and local interaction. But for higher values of these parameters, i.e. in the zone of interest where price inertia prevails, prices are significantly less volatile than money.

$\gamma\backslash\rho$	0	0.5	0.9	0.95
1	0.98	0.99	1.0	1.02
0.8	1.00	1.00	1.01	0.99
0.6	0.98	0.99	0.94	1.0
0.4	0.99	0.99	0.95	0.8
0.2	1.01	0.99	0.87	0.67
0.1	0.96	0.98	0.71	0.60
0.05	0.93	0.91	0.69	0.71

Table 6 – relative standard deviation of the aggregate price level.

Do the above numbers bear any resemblance to the data? To have a rough comparison, we take U.S. quarterly log nominal GDP as our measure of nominal aggregate demand (m in the model), and U.S. quarterly log GDP deflator as a measure of the aggregate price level $\bar{p}$.[9] Both series were detrended by taking the residual of a regression on a linear trend and the price of oil. The latter variable was included as a crude way to filter out supply shocks, which are ignored in the model. The first order autocorrelation in our resulting measure of m is 0.968, implying a value of ρ close to 0.95. Regressing our measure of p on m and $p(-1)$ yields coefficients equal to 0.21 and 0.78 respectively. These coefficients are comparable to their corresponding values of 0.21 and 0.72 for $\gamma = 0.2$ and $\rho = 0.95$ (Tables 1 and 2). Finally the relative standard deviation of our measure of p is equal to 0.91, which is higher than the predicted value of 0.67 (Table 6) for $\gamma = 0.2$ and $\rho = 0.95$, but not widely off the mark.

5 Elements of explanation

In this section we provide some intuition about the results of the preceding section, as well as about some features of the equilibrium rules.

5.1 From experimentation to inertia

We first explore the reasons behind the observed patterns of price inertia and rigidity at high values of ρ and low values of γ.

Let us first rule out local interaction among agents, i.e. to assume $\gamma = 1$. Then, the above results imply full response and no inertia of the price level for low values of ρ, but inertia and less than full response at high values of ρ. Further inspection of simulation results suggests that this is entirely due to periods of experimentation. That is, if one defines the aggregate price level as the average among firms which are not currently experimenting, then the regression coefficient of the aggregate price level on money is again close to one, while its coefficient on itself lagged once is again close to zero.

[9] Source: *OECD Main Economic Indicators.*

Hence, experimentation tends to generate price inertia for highly correlated shocks. Why is that so? Typically, during periods of experimentations firms pursue a price rule different from their usual one, which tends to generate more autocorrelation in prices than implied by the individual rules themselves. To get a grasp of this phenomenon and how it relates to autocorrelation in the driving shocks (ρ), consider the following simple model:

$$y_t = a_t x_t,$$

where x_t is the driving exogenous variable (m_t in our model), and y_t is the endogenous variable (p_t in our model). a_t is a time-dependent coefficient representing the current rule being used in order to set y_t as a function of x_t. Hence movements in a capture experimentations with the rule.

Clearly, agents follow rules that are contemporaneous and do not depend on past variables. For simplicity we shall assume that a_t and x_t follow independent processes, and that the following holds:

$$\begin{aligned}
Ex_t &= 0 \\
Ea_t &= 1 \\
Ex_t^2 &= 1 \\
Ea_t^2 &= 1 + \sigma_a^2 \\
Ex_t x_{t-1} &= \rho_x \\
Ea_t a_{t-1} &= 1 + \rho_a \sigma_a^2
\end{aligned}$$

Then, an econometrician who wants to estimate the response of y to x using the following regression:

$$y_t = c_0 + c_1 x_t + c_2 y_{t-1},$$

would, by least square theory, get the following coefficients for a large sample:

$$\begin{aligned}
c_0 &= 0 \\
c_1 &= \frac{1 + \sigma_a^2 - \rho_x^2(1 + \rho_a \sigma_a^2)}{1 + \sigma_a^2 - \rho_x^2} \\
c_2 &= \frac{\rho_x \rho_a \sigma_a^2}{1 + \sigma_a^2 - \rho_x^2}
\end{aligned}$$

These formulas make it clear that at $\rho_x = 0$ the regression would yield $y = x$, while as ρ_x goes up, c_1 falls and c_2 rises. This roughly correspond to the pattern observed in tables 1 and 2. On the other hand, if experimentation did not take place ($\sigma_a = 0$), one would always get $c_1 = 1$ and $c_2 = 0$ regardless of ρ_x. As already pointed above, this is indeed what we get when excluding firms that are experimenting from the aggregate price level.

Therefore, the inertia and rigidity properties do not seem so interesting, as they do not affect "normal" firms. However, when local interactions are reintroduced into the picture, inertia spreads. In order to get their prices more in line with that of their neighbour, which is autocorrelated because of experimentation, firms tend to index their prices on their neighbor's previous price. That is, they select rules with a positive value of c_4 in (1). This is indeed what is observed, as Table 7 shows. By doing so, they reinforce inertia in the aggregate price level. At the same time, since it is profitable for them to maintain long-run neutrality, this is partially offset by a reduction in their price response to contemporaneous money shocks. This in turn reduces aggregate price responsiveness.

$\gamma \backslash \rho$	0	0.5	0.9	0.95
1	-0.03	-0.00	-0.02	-0.02
0.8	0.03	0.11	0.02	0.11
0.6	-0.00	-0.02	0.10	0.14
0.4	0.02	0.01	0.11	0.14
0.2	-0.02	0.11	0.21	0.27
0.1	0.01	0.09	0.25	0.19
0.05	0.03	0.13	0.33	0.39

Table 7 – Average weight (c_4) on lagged left-neighbor's price, non experimenting firms only.

This mechanism only takes place if the neighbor's price is significantly autocorrelated, i.e. if ρ is large enough. If not, then the average rule absent local interactions involves no autocorrelation, so that when local interaction is introduced, it does not pay to use one's neighbor's lagged price.

Finally, having more price inertia and less response to money tends to reduce the volatility of aggregate prices if ρ is large enough, which explains the results in Table 6.[10]

To summarize: While absent local interactions, inertia and rigidity seem a statistical artifact due to those firms that are experimenting, when local interactions are strong, rigidity is embodied in individual rules through indexation on the left-neighbor's lagged price; this is the way firms adjust to the externality exerted upon them when their left-neighbor experiments with an alternative rule.

5.2 On error-correcting behavior

In this subsection, we explain one aspect of the rules followed by individual firms, namely that they tend to have a positive value of c_1 and a negative value of c_3. This feature is basically unrelated to the inertia and response properties analyzed so far. We discuss it for completeness and because it is interesting on its own.

Basically, if shocks are correlated, the rules do not converge to the optimal

[10] Consider the simple aggregate rule:

$$p_t = \alpha m_t + (1-\alpha)p_{t-1} + \varepsilon_t,$$

where m_t follows

$$m_t = \rho m_{t-1} + \eta_t$$

Assume that ε_t, which may be interpreted as experimentation, and η_t are uncorrelated. Then the variance of p_t is given by

$$Var(p) = \frac{\sigma_\varepsilon^2}{1-(1-\alpha)^2} +$$

$$\sigma_\eta^2 \left[\frac{\alpha^2\rho^2}{(1-\alpha-\rho)^2(1-\rho^2)} + \frac{\alpha^2(1-\alpha)^2}{(1-\alpha-\rho)^2(1-(1-\alpha)^2)} \right]$$

For ρ large enough, the term in $\frac{\alpha^2\rho^2}{(1-\alpha-\rho)^2(1-\rho^2)}$, which is increasing in α, dominates.

one $p_{it} = m_t$, but, rather, to an *error-correcting* rule of the type:

$$p_{it} = \alpha m_t + \beta(m_{t-1} - p_{it-1}) \tag{4}$$

This is confirmed if one looks at the average values of the rule coefficients $(c_0, c_1, c_2, c_3, c_4)$. At the end of the first simulation with $\gamma = 1$ and $\rho = 0$, for example, the average across firms of these coefficients is given by

c_0	-0.0015
c_1	0.105
c_2	1.02
c_3	0.036
c_4	-0.123

Table 8 – Average rule followed in one simulation with $\gamma = 1$ and $\rho = 0$.

Clearly, this is quite close to the optimal rule $p = m$. On the other hand, at the end of the first simulation with $\gamma = 1$ and $\rho = 0.95$, the average values of these coefficients is given by

c_0	-0.041
c_1	.412
c_2	1.07
c_3	-0.368
c_4	-0.026

Table 9 – Average rule followed in one simulation with $\gamma = 1$ and $\rho = 0.95$.

The coefficient on m is again close to one, but there is now a strong error correction component.

If $\alpha = 1$ and no error is initially made, any error-correcting rule is as good as the optimal rule. But, if α is close to but different from one, errors will be made, and an error-correcting rule will have a better performance by mechanically offsetting the errors due to the inappropriate value of α in subsequent prices.

In fact, the optimal value of the error-correction coefficient β is independent of α, so that we should expect firms to converge to error-correcting rules with this value of β and $\alpha = 1$, since, along the whole of their convergence path where $\alpha \neq 1$, they will be better-off by using such an error-correction term. To put it otherwise, the error-correcting rule is robust to model uncertainty, while the optimal rule is not.

To see this, note that if $\gamma = 1$, and assuming the constant term c_0 tracks the mean of m, the objective function of any firm is

$$-Var(m_t - p_{it})$$

If an error-correcting rule such as (4) is followed, then we have

$$m_t - p_{it} = \frac{(1-\alpha)}{1+\beta L} m_t$$

Substituting in the monetary process, we get

$$m_t - p_{it} = \frac{(1-\alpha)}{1+\beta L} \frac{1}{1-\rho L} \varepsilon_t.$$

Factorizing, and computing the variance, we get

$$Var(m_t - p_{it}) = \frac{(1-\alpha)^2 \sigma_\varepsilon^2}{(\rho+\beta)^2} \left[\frac{\beta^2}{(1-\beta^2)} + \frac{\rho^2}{(1-\rho^2)} + \frac{2\beta\rho}{1+\beta\rho} \right]$$

There exists an optimal value of β which minimizes the RHS, and it only depends on ρ.

It is easy to solve this equation numerically. The following table gives the solution as a function of ρ, where ρ takes the values that we have used:

ρ	β
0	0
0.5	0.43
0.9	0.74
0.95	0.81

Table 10 – Theoretical values of the error-correction term under $\gamma = 1$.

Surprisingly, if one compares the last line with table 9, which implies a value of β at around 0.4, this is substantially above the error-correction coefficients found in the simulations. But the point remains that both the theoretical and actual values increase with ρ and are equal to zero for $\rho = 0$.

Note that error correction is only valuable if shocks are correlated, that is if the unknown target price at t is related to the unknown target price at $t-1$. Only in this case does correcting past errors bring the firm on average closer to the optimum in subsequent periods. Otherwise, the error correction term only adds noise to the price process.

6 Conclusion

This paper has provided evolutionary foundations for aggregate price level stickiness in the face of shocks to nominal aggregate demand. Stickiness evolves when local interactions are strong and shocks are highly correlated across time, as an outcome of individual agents' tendency to experiment with alternative pricing rules.

A natural extension of the model would be to introduce idiosyncratic shocks, say shocks to production costs. The model would then have the potential to capture the evolution of the distribution of prices.[11]

As we already pointed out, the model could be applied to a variety of settings where agents are imperfectly rational, local interactions prevail, and there are aggregate shocks.

[11] See Lach and Tsiddon (1992) for an empirical analysis.

REFERENCES

Akerlof, George and Janet Yellen (1985), "A near-rational model of the business cycle, with wage and price inertia", *Quarterly Journal of Economics* 100(5), 823-38

Anderson, Philip W., Kenneth J. Arrow and David Pines (1988), *The Economy as an evolving complex system.* Santa Fe: Santa Fe Institute.

Arthur, W. Brian, Steven N. Durlauf, and David Lane (1997), *The Economy as an evolving complex system II.* Santa Fe: Santa Fe Institute.

Arifovic, Jasmina (1996), "The behavior of the exchange rate in the genetic algorithm and experimental economies" *Journal of Political Economy* 104(3) 510-41

——————— (2001), "Evolutionary algorithms in macroeconomic models" *Macroeconomic Dynamics*

Ball, Laurence and David Romer (1991), "Sticky prices as coordination failure", *American Economic Review* 81(3), 539-52

Barro, Robert and Herschel Grossman (1971), "A general disequilibrium model of income and employment" *American Economic Review* 61(1), 82-93

Bénassy, Jean-Pascal (1975), "Neo-Keynesian disequilibrium theory in a monetary economy", *Review of Economic Studies* 42(4), 1975, 503-23

——————— (1976), "The disequilibrium approach to monopolistic price setting and general monopolistic disequilibrium", *Review of Economic Studies* 43(1), 1976, 69-81

Blanchard, Olivier and Nobuhiro Kiyotaki (1987), "Monopolistic competition and the effects of aggregate demand", *American Economic Review* 77(4), 647-666

Bray, Margaret M. (1982) "Learning, estimation, and the stability of rational expectations" *Journal of Economic Theory 26,* 318-339

Brock, William and Steven N. Durlauf (2001) "Discrete choice with social interactions" *Review of Economic Studies* 68(2), 235-60

Caballero, Ricardo and Eduardo Engel (1993) "Heterogeneity and output fluctuations in a dynamic menu cost economy" *Review of Economic Studies*

60(1), 95-119

Caplin, Andrew and Daniel F. Spulber (1987) "Menu costs and the neutrality of money" *Quarterly Journal of Economics* 102(4), 703-25

Durlauf, Steven N. (1991) "Multiple equilibria and persistence in aggregate fluctuations", *American Economic Review,* May, 81(2), 70-74

——————— (1996) "Statistical mechanics approaches to socio-economic behavior", NBER technical paper #203

Ellison, Glenn (1993), "Learning, local interaction, and coordination" *Econometrica* 61(5), 1047-71

Ellison, Glenn and Drew Fudenberg (1993), "Rules of thumb for social learning", *Journal of Political Economy 101(4),* 612-43

Evans, G.W. and Seppo Honkapohja (1995) "Local convergence of recursive learning to steady states and cycles in stochastic nonlinear models", *Econometrica 63, 195-296*

Fischer, Stanley (1977) "Long-term contracts, rational expectations, and the optimal money supply rule", *Journal of Political Economy,* 85(1), 191-205

Grandmont, Jean-Michel and Guy Laroque (1990), "Stability, expectations, and predetermined variables" in Paul Champsaur et al, eds. *Essays in Honor of Edmond Malinvaud,* vol. 1. Cambridge MA: MIT Press, 71-92

Harrington, Joseph E. (1998), "The social selection of flexible and rigid agents", *American Economic Review* 88(1), 63-82

———————— (1999), "The rigidity of social systems", *Journal of Political Economy* 107(1), 40-64

Kiyotaki, Nobuhiro and Randall Wright (1989), "On money as a medium of exchange", *Journal of Political Economy* 97(4), 927-54

Lach, Saul and Daniel Tsiddon (1992) "The behavior of prices and inflation: An empirical analysis using disaggregated price data", *Journal of Political Economy* 100(2), 349-89

LeBaron, Blake, W. Brian Arthur, and Richard Palmer (1999) "Time series properties of an artificial stock market", *Journal of Economic Dynamics* *and Control* 23(9-10) 1487-1516

Lindbeck, Assar and Snower, Dennis (1999), "Price dynamics and production lags", *American Economic Review 89(2),* May, 81-88

Mankiw, N. Gregory (1985), "Small menun costs and large business cycles", *Quarterly Journal of Economics* 10(2) 529-38

Marcet, Albert and Thomas J. Sargent (1989) "Convergence of least squares learning in environments with hidden state variables and private information" *Journal of Political Economy* 97(6), 1306-22

Marcet, Albert, and Juan Pablo Nicolili (1997) "Recurrent hyperinflations and learning", Universitat Pompeu Fabra working paper, November 1997

Marimon, Ramon, Ellen McGrattan and Thomas J. Sargent "Money as a medium of exchange in an economy with artificially intelligent agents", *Journal of Economic Dynamics and Control* 14(2), 329-73

Nelson, Richard and Sidney Winter (1985) *An Evolutionary Theory of Economic Change*

Rotemberg, Julio (1982) "Sticky prices in the United States", *Journal of Political Economy* 90(6), 1187-1211

Sargent, Thomas J. (1993), *Bounded rationality in macroeconomics.* Oxford: Oxford University Press

Taylor, John B. (1980) "Aggregate dynamics and staggered contracts" *Journal of Political Economy* 88(1), 1-23