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TRANSPARENCY AND TACIT COLLUSION IN A DIFFERENTIATED MARKET

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Abstract

This paper investigates the effects on tacit collusion of increased market transparency on the consumer side of a market in a differentiated Hotelling duopoly. Increasing market transparency increases the benefits to a firm from underbutting the collusive price. It also decreases the punishment profit. The net effect is that collusion becomes harder to sustain. In the limiting homogeneous market, the effect vanishes. Here market transparency does not affect the possibilities for tacit collusion.

JEL Classification: L13, L40.

Keywords: transparency, tacit collusion, competition policy, internet.

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1 Introduction

The advent of the Internet has the potential of increasing market transparency in many markets. While consumers before had to spend considerable time searching many markets, there are now many well-established websites where price comparisons are available with a click on the mouse. Consumer agencies try to make markets more transparent in various ways, though magazines, press releases, and also through websites. Newspapers often have weekend sections, where a market is surveyed and prices compared. In these (and many other) cases market transparency is improved not by the firms selling in the market but by outside agents. The creation of the The European Internal Market was partly motivated by governments' belief (or hope?) that transaction costs would decrease and transparency would increase so competition intensified. This actualizes an old question in the competition policy debate: is improved market transparency for consumers good or bad? Does it promote competition and should a competition authority promote it or not? The purpose of this paper is to cast light on this question in a differentiated duopoly, by investigating whether improved transparency on the consumer side of the market will facilitate collusion or not. The answer turns out to hinge on the degree of product differentiation. In a differentiated market, improving transparency reduces the scope for collusion, while there are no effects in a homogeneous market.

In the competition policy debate, improved transparency is typically viewed as promoting competition if it affects the consumer side of the market. However, the arguments here usually refer to a static setting. On the other hand, improved transparency on the producer side is typically thought to be anti-competitive, see for instance Kuhn and Vives (1995). If firms are uncertain about their competitors' prices, tacit collusion is harder to maintain, as price undercutting is harder to detect. Taken alone this effect implies that increased transparency facilitates collusion. This effect is well understood and we will not concentrate on it. Improved transparency on the consumer side has different effects. In a static setting, the market becomes more competitive as the effective demand elasticity of a firm increases. However, the effects on collusion are ambiguous. The increased elasticity of demand makes it more tempting to undercut the other firm; this destabilizes collusion. On the other hand, a more severe punishment is possible in a transparent market. This facilitates collusion. The total effect on collusion is the net effect of these two forces. In the differentiated Hotelling market of the paper, the first effect dominates, so improving transparency on the consumer side makes tacit collusion more difficult to sustain.

We identify market transparency on the consumer side with consumers' information about the prices charged by the firms in the market. Some consumers are informed about prices and some are not. Informed consumers are supposed to know both firms' prices, as they would if they had access to a web site comparing prices. This follows the "Sales Model" of Varian (1980). The uninformed consumers have an expectation about the firm's price. In equilibrium, this expectation is correct. Nevertheless, the uninformed consumers affect the amount of demand a firm can gain by lowering its price - and therefore the effective demand elasticity - as they do not learn about the price change.

Tacit collusion requires that a potential deviation from the collusive path is discouraged by a sufficiently hard punishment. As is well known, the credibility of very hard punishments can be questioned. We therefore study both the case where the punishment consists of reversion to the static Nash-equilibrium in all future and the case of optimal (symmetric) punishments.

As shown by Varian, the Nash equilibrium is in mixed strategies in a homogeneous market, which is not completely transparent. This feature is present in our model, when product differentiation is low[1], but not when it is sufficiently high. In both cases we obtain qualitatively similar results: increasing transparency diminishes the scope for collusion. When Nash equilibria are mixed, however, we have to rely on simulations, as it has proven hard to find a closed form for the mixed strategy Nash equilibrium.

Increasing transparency makes collusion more difficult to sustain in a differentiated market. This shows up in two ways: first, the smallest discount factor necessary for sustaining collusion on the monopoly price is increasing in market transparency. Secondly, for lower discount factors, the best price the firms can collude on (if any) is decreasing in the level of market transparency.

In the limiting case of a homogeneous market, the effects of increased market transparency on the gains from a deviation and the punishment is exactly balanced and the crucial discount factor needed for collusion on the monopoly price is independent of market transparency. In this sense, the homogeneous and differentiated markets are qualitatively different, but it is also shown that the effect in the differentiated market tends to zero as product differentiation disappears. Furthermore, it is shown that in the homogeneous market collusion is impossible at all if the discount factor is too low for collusion at the monopoly price. The same feature is true when product differentiation is low. Also in this respect, the homogeneous market is the limit of the differentiated.

As is well known, if the discount factor is low, firms may do better by

[1] Actually, the precise condition for when the Nash equlibrium is in mixed strategies also depend non-linearly on the degree of transparency, so a low degree of product differentiation means low relatively to the level of market transparency.

using harder punishment phases than reversion to the one-shot Nash equilibrium. We therefore also briefly consider optimal symmetric stick and carrot equilibria. The result is the same: the smallest discount factor compatible with full collusion increases in the market transparency and for lower discount factors increasing transparency decreases the collusive price and profit.

Market transparency is viewed differently in different countries and by different competition authorities. While the previous Danish competition act actively tried to promote transparency in order to increase competitiveness (see Albæk, Møllgaard and Overgaard (1994)), the Danish government now acknowledges that, although transparency may be good in so far as it increases consumers' information, it may help firms collude if it increases firms' information, see e.g. Erhvervspolitiken (1999). The EU commission seems to have a mixed view. Kuhn and Vives (1995) conclude that the Commission mostly found increased transparency in the form of price-announcements by firms as anti-competitive. However, the internal European market as well as the single European currency has often been seen as adding to transparency and therefore competitiveness.

Market transparency has been analyzed from different angles in the literature. As discussed above Varian (1980) studied a homogeneous market, where market transparency is incomplete. He showed that the expected profit of the firms in the symmetric Nash equilibrium decreases in the level of market transparency. In this sense increasing market transparency intensifies competition. The search literature, see for instance Burdett and Judd (1993) or Stahl (1989) can be seen as a developing this theme. Lowering search costs increases search and this intensifies competition. Anderson and Renault (1999) study price competition when consumers have to search for prices and product characteristics. They show that market prices rise with search costs.

The literature on advertising, see e.g. Bester and Petrakis (1995), can also be seen as contributing to understanding market transparency. An important difference, however, is that in this case the firms in the market actively affect market transparency, while we consider the case where market transparency if affected by outside agents. Increased advertising is typically shown to lead to lower prices.

On the other hand, cartel theory stemming back to Stigler (1964), Green-Porter (1984) and others, see Tirole (1988) for an overview, has pointed to the anti-competitive effects of increased transparency on the producer side of the market for the reasons described above

The most closely related paper to the present paper is Nilsson (1999). He considers tacit collusion in a repeated search model of a *homogeneous* market a la Burdett and Judd (1993), where firms are perfectly informed of prices but consumers have to search in order to learn prices. Lowering search cost corresponds to increasing transparency in Varian's model, it lowers the price in a one shot equilibrium. However collusion is facilitated in the repeated game. In Nilsson's model, most of the consumers decide whether to search or not taking into account the expected benefits from searching, while a fraction of the consumers always search. The majority of consumers therefore stop searching if firms set the same prices. This happens if the firms collude on a high price. Undercutting the other firm in the collusive phase will therefore only give a relatively small increase in demand. This feature facilitates collusion. In the punishment phase of the collusive equilibrium, firms do not set the same price, (they play a mixed strategy as in Varian) therefore search occurs and increasing transparency through lowering search costs intensifies competition in this phase. Thus increasing transparency increases search in the punishment phase but not in the collusive phase, as a result increasing transparency facilitates collusion.

In the present paper, on the contrary, increasing transparency increases the information level of consumers in both phases of the equilibrium and the result therefore depends on the net effect on deviation profit and punishment profit. The other important difference is that we consider both a homogeneous and a *differentiated* market.

Møllgaard and Overgaard (2000) also study the consumer side. They identify transparency with the ability of consumers to compare the characteristics of goods and services and study a repeated version of Singh and Vives' (1984) model of a differentiated duopoly. They interpret the substitutability of the goods in the consumers' utility functions as a measure of transparency. Increased transparency therefore makes consumers switch producer more easily. In collusive trigger-strategy equilibria, this increases the temptation to undercut the other firm - since more demand will be gained. However, it may also make the punishment phase of the equilibrium more severe. An "optimal degree" of transparency therefore exists. Their paper - as well as the present - is related to the literature on stability of collusion in differentiated markets. See, for example, Deneckere (1983), Chang (1991), Ross (1992) and Häckner (1995). These authors show that as products become more substitutable, a deviation becomes increasingly attractive. Although the punishment phase also becomes more severe, the first effect dominates and collusion is harder to maintain. Compared with the present paper, the difference is that this literature compares the effect of changing preferences. We let preferences and product characteristics remain fixed and investigate the effect of price transparency.

The organization of the paper is the following. Section 2 introduces a simple Hotelling market. The one shot Nash equilibrium is characterized in

Section 3 for the different cases of product differentiation. Section 4 studies the effect of transparency on tacit collusion. Section 5 briefly handles optimal symmetric punishment phases, while Section 6 concludes.

2 A Hotelling market

Consider a Hotelling market with a continuum of consumers. Consumer x is located at $x \in [0,1]$. Each consumer either buys one unit of the (differentiated) good or does not buy. There are 2 firms, located at 0 and 1 respectively. If firms charge prices p_0 and p_1, consumer x gets utility $u - p_0 - tx$ from buying one unit from firm 0 and $u - p_1 - t(1 - x)$ from buying a unit from firm 1. The parameter $t > 0$ is the transportation cost. We will assume that all consumers are potential costumers at each firm: $u \geq t$. A consumer, who knows the prices of the firms, is therefore indifferent between buying from 0 and 1 if she is located at

$$x = x(p_0, p_1) \equiv \frac{1}{2} + \frac{p_1 - p_0}{2t} \tag{1}$$

There are two different *information types* of consumers: a fraction ϕ are informed about both firms' prices, while a fraction $(1 - \phi)$ are uninformed. The variable ϕ is our measure of market transparency, the higher is ϕ, the more transparent is the market. We conceive of the informed consumers as having easy access to the pricing information, perhaps through an internet site. Both information types of consumers are supposed to be uniformly distributed on locations. In principle one could of course also imagine that some consumers are only informed about one of the firms' price. We will however leave these complications aside.

All consumers know the locations of the firms, regardless of whether they are informed about the firms' prices or not. A consumer, who is uninformed about firm $i's$ price has an *expectation* p_i^e of this price. For this consumer, the *expected* utility from buying form firm i is $u - p_i^e - tx$. An uninformed consumer is indifferent between buying form the two firms, if she is located at $x(p_0^e, p_1^e)$. A consumer can only visit one firm in a period. Hence, it is not possible for uninformed consumers to visit the firms in a sequence. The time line is as follows. First consumers form expectations, second firms set prices, which are observed by some consumers only. Based either on their knowledge or expectations about prices consumers decide on which firm to go to - if any. If an uninformed consumer arrives at a firm and finds that the price is higher than expected, the consumer may decline to buy if she wishes, this occurs, for instance, if she visits firm 0 and $u - tx - p_0 < 0$. In equilibrium this will not happen, as consumers have correct - rational - expectations. Finally, transactions take place.

If $p_0 - p_1 > t$, all informed consumers buy from firm 1, if $p_1 - p_0 > t$, they all buy from firm 0. In the sequel we will only consider symmetric equilibria where the expected prices are the same for the two firms, i.e., where $p_0^e = p_1^e = p^e$. As will become clear, the equilibrium price will be so low (less than or equal to $u - \frac{t}{2}$) that the market is covered and each firm faces $(1-\phi)/2$ uninformed consumers. The demand facing firm 0 can then be written without explicit reference to the expected prices as

$$D(p_0, p_1, \phi) = \begin{cases} \phi + \frac{1-\phi}{2} \text{ if } p_0 < p_1 - t \\ \phi\left(\frac{1}{2} + \frac{p_1 - p_0}{2t}\right) + \frac{1-\phi}{2} \text{ if } p_1 - t \leq p_0 \leq p_1 + t \\ \frac{1-\phi}{2} \text{ if } p_1 + t \leq p_0 \leq u - \frac{t}{2} \\ \frac{1-\phi}{2}\left(\frac{u-p}{t}\right) \text{ if } p_1 = u - \frac{t}{2} \leq p_0 \leq u \end{cases} \tag{2}$$

For simplicity, we assume that marginal costs are constant. We normalize marginal costs to zero, so firm $0's$ profit in a period is $\pi_0 = p_0 D(p_0, p_1 \phi)$.

3 Static Nash equilibrium

We first concentrate on the one period Nash equilibrium. Depending on the degree of product differentiation - as given by $\frac{t}{u}-$ and the transparency of the market, as given by ϕ, the equilibrium will either be in pure or in mixed strategies.

3.1 Pure strategy Nash equilibrium

In a symmetric equilibrium, firm 0's problem is

$$\text{Given } p_1, \phi \ \max_{p_0} \ p_0 D(p_0, p_1, \phi).$$

In a symmetric pure strategy equilibrium, the firms set the same price, and the relevant part of the demand function is given by the second line in (2). Using this, we get the best reply

$$p = \frac{1}{2}\left(p_1 + \frac{t}{\phi}\right)$$

The Nash equilibrium price $p^N(\phi)$ and profit, $\pi^N(\phi)$ are therefore given by

$$p^N(\phi) = \frac{t}{\phi}, \ \pi^N(\phi) = \frac{t}{2\phi}.$$

Both are clearly decreasing in ϕ. Thus more transparency increases competition and lowers the Nash-equilibrium price. When a firm decreases its price, only informed consumers notice it and increase demand. The more informed consumers, ϕ, the more demand is gained by lowering price and the more intense competition is among the firms. This also shows that in the one shot game the firms - jointly − have no interest in promoting market transparency, they have the opposite interest. It is straightforward to check that the second order condition for maximum is fulfilled.

In deriving the equilibrium we assumed that the second line of (2) is relevant, hence it should not be advantageous to undercut the other firm by t and gain the whole informed market. This takes that $\left(\frac{t}{\phi} - t\right)\left(\phi + \frac{1-\phi}{2}\right) < \frac{t}{2\phi}$, which is fulfilled for all positive ϕ and t.

Furthermore, the equilibrium is derived under the assumption that the market is covered at the Nash equilibrium price $\frac{t}{\phi}$, i.e. that $u - t/2 \geq \frac{t}{\phi}$ or

$$\frac{t}{u} < \frac{2\phi}{2+\phi} \tag{3}$$

If this condition is *not* fulfilled, there is effectively no competition in the market. Even in a Nash equilibrium, the firms are not competing about the consumers, as they have strong incentives to exploit the uninformed consumers.

3.2 Nash equilibrium with monopoly pricing,

If (3) is not fulfilled, each firm is effectively a monopolist in its own half of the line. Consider firm 0 again, its marginal informed consumer has an x given by $u - p_0 - tx = 0$, i.e. $x = (u - p_0)/t$. The marginal uninformed consumer, who turns up at the firm in equilibrium is located at $x^e = (u - p_0^e)/t$. An uniformed consumer, x, who turns up at the firm will only buy if the *actual* price p_0 is such that $u - tx - p_0 \geq 0$. The firm can decrease the demand from the uninformed consumers by raising p_0, but it can not increase their demand by lowering p_0. For $p^e \geq u - \frac{t}{2}$, the demand of the uninformed consumers is therefore

$$x^u = \begin{cases} (1-\phi)(u - p_0^e)/t \text{ for } p_0 \leq p_0^e \\ (1-\phi)(u - p_0)/t \text{ for } p_0 > p_0^e \end{cases} \tag{4}$$

Given p_0^e, the firm's (monopoly) profit as a function of p_0 is therefore

$$\pi^m(p_0, p_0^e, \phi) = \begin{cases} \left(\phi\frac{u-p_0}{t} + (1-\phi)\frac{u-p_0^e}{t}\right) p_0 \text{ for } p_0 \leq p_0^e \\ \left(\phi\frac{u-p_0}{t} + (1-\phi)\frac{u-p_0}{t}\right) p_0 = \frac{u-p_0}{t} p_0 \text{ for } p_0 \geq p_0^e \end{cases} \tag{5}$$

Suppose consumers expect a price $p_0^e \geq u - \frac{t}{2}$. If the firm considers raising the price above the expected price, the relevant part of the profit function is the second line in (5). Maximizing this yields is $p_0 = u/2 \leq\ u - \frac{t}{2}$ as $u \geq t$. It therefore follows that if $p_0^e \geq u - t/2$, the firm never wishes to raise the price above p_0^e. Then consider the option of lowering the price. The relevant part of the profit function is the first line in (5). The marginal profit from a price change is

$$\phi\frac{u-2p_0}{t} + (1-\phi)\frac{u-p_0^e}{t}$$

If this is positive evaluated at $p_0 = p_0^e$, then the firm does not wish to lower the price. Hence *all* $p_0 \geq u - \frac{t}{2}$ for which

$$\phi\frac{u-2p_0}{t} + (1-\phi)\frac{u-p_0}{t} \geq 0$$

are compatible with rational expectations equilibria. This yields

$$u - \frac{t}{2} \leq p_0 \leq \frac{u}{1+\phi}$$

The left hand side is (weakly) less than the right hand side if

$$\frac{2\phi}{1+\phi} \leq \frac{t}{u} \tag{6}$$

If $\frac{2\phi}{2+\phi} < \frac{t}{u} < \frac{2\phi}{1+\phi}$ the firm would like to lower the price if $\left(\phi\frac{u-p_0}{t} + (1-\phi)\frac{u-p_0^e}{t}\right) p_0$ were the relevant profit function and $p_0^e = u - \frac{t}{2}$. However, as we saw above, for $\frac{2\phi}{2+\phi} < \frac{t}{u}$, there are no Nash equilibria with lower price than $u - \frac{t}{2}$. The equilibrium in this case has both firms setting $p = u - \frac{t}{2}$. The relevant profit function is derived using the second line in equation (2). As we saw in the last section, the firm does then not wish to lower the price. Above we saw that it did not wish to raise the price as well. The equilibrium in this case is therefore $p_0 = p_1 = u - \frac{t}{2}$.

Summarizing the results of this section: if $\frac{2\phi}{2+\phi} \leq \frac{t}{u} \leq, \frac{2\phi}{1+\phi}$, the Nash equilibrium is unique and involves $p_0 = p_1 = p^m = u - \frac{t}{2}$. For $\frac{2\phi}{1+\phi} < \frac{t}{u}$, there is a continuum of symmetric Nash equilibria, corresponding to all prices $p \in [u - \frac{t}{2}, \frac{u}{1+\phi}]$.

3.3 Mixed strategy Nash equilibrium

Finally, consider the case where the goods are close substitutes, and t/u is low. In this case, the pure strategy Nash equilibrium price derived above $p = t/\phi$ becomes very low, and it may be better for a firm to raise its price and only sell to the uninformed consumers.[2] If only uninformed consumers are served, the best price maximizes $(1-\phi) \min [\frac{u-p_0}{t}, \frac{1}{2}] p_0$. For small t, the solution is $p_0 = u - \frac{t}{2}$. This is gives higher profit than $p = \frac{t}{\phi}$ if

$$\frac{t}{u} < \frac{2(1-\phi)\phi}{(1+\phi)(2-\phi)} \tag{7}$$

In Figure 1 below the conditions (3), and (7) are depicted for with ϕ along the first axis and $\frac{t}{u}$ along the second.

[2] One could object, that the ability of a firm to exploit the uninformed consumers rely on the fact that we have made it impossible for an uninformed consumer to visit the other firm if she is surprised by a high price. If one assumed that the uninformed consumer could visit the other firm at a very low cost if her expectations were proven wrong, then she could not be exploited as much, and the pure strategy Nash equlibrium would continue to exist (except for very low degrees of product differentiation). In this case the analysis of the pure strategy Nash equilibrium would be valid for an even larger part of the parameter space. However, there would still be a small part of the parameter space, where a pure strategy Nash equilibrium would not exist.

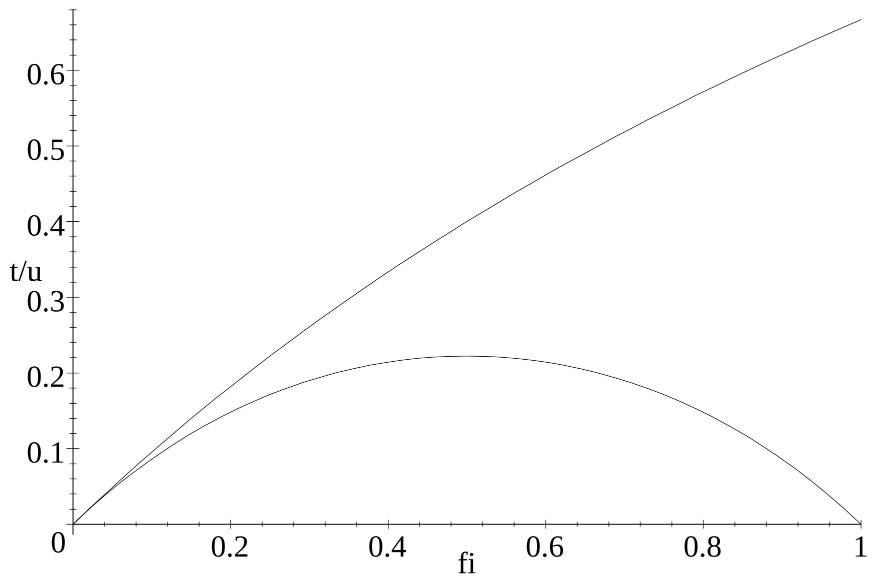

Figure 1:

When condition (7) is fulfilled, the pure strategy Nash equilibrium derived above does not exist. Given the results of Varian (1980), this is not surprising. Varian showed that in a homogeneous market where a fraction ϕ of the consumers is uninformed, there are no pure strategy Nash equilibria, but a symmetric mixed strategy equilibrium exists. The same happens in our model, when the goods become close substitutes.[3] Notice, that the crucial degree of substitutability depends on the degree of transparency. For high or low degrees of transparency, the Nash equilibrium will be in pure strategies even though product differentiation is low. This non-linearity stems from the fact that an increase in transparency both decrease the pure strategy Nash

[3] As readers familiar with the literature will realize, the following is an adaptation of the analysis of Varian to our differentiated market.

equilibrium profit and the profit the firm can obtain by raising the price and only serve uninformed consumers.

In the symmetric mixed strategy equilibrium both firms choose prices according to the distribution function F.[4] Let f be the corresponding density function. Since the equilibrium is symmetric, the uninformed consumers again divide evenly between the two firms. As is clear from Section 3.2 a firm will never set a price above $u-\frac{t}{2}$, all uninformed consumers therefore buy regardless of the realization of the randomization. If firm 0 chooses the price $p\leq u-\frac{t}{2}$ and firm 1 chooses prices according to F, firm $0's$ expected profit equals

$$E\pi_0\equiv\left(\phi\left((1-F(p+t))+\int_{p-t}^{p+t}\left(\frac{1}{2}+\frac{p_1-p}{2t}\right)f(p_1)dp_1\right)+\frac{1-\phi}{2}\right)p \tag{8}$$

If firm 1 sets a price, p_1, above $p+t$, firm 0 will sweep the whole demand from the informed consumers. This happens with probability $1-F\left(p+t\right)$. If firm 1 sets a price in-between $p-t$ and $p+t$, demand from the informed consumers will be shared according to (1) and for prices p_1 below $p-t$, firm 0 receives no demand from the informed consumers. Furthermore, half of the uninformed buy from firm 0.

In a symmetric mixed strategy equilibrium, all prices in the support of f give the same profit to the firm, $E\pi_0$. Integrating by parts yields

$$E\pi_0=\left(\phi\left((1-F(p+t))+\left[\left(\frac{1}{2}+\frac{q-p}{2t}\right)F(q)\right]_{p-t}^{p+t}-\frac{1}{2t}\int_{p-t}^{p+t}F(q)dq\right)+\frac{1-\phi}{2}\right)p$$

[4] As we are amending Varian's (1980) analysis we just characterize the distribution function F directly and do not repeat some straightforward replications of Varian's lemmata (for instance that there are no atoms).

which reduces to

$$E\pi_0 = \left(\phi\left(1 - \frac{1}{2t}\int_{p-t}^{p+t} F(q)dq\right) + \frac{1-\phi}{2}\right)p. \tag{9}$$

Accordingly, we get

$$\frac{1}{2t}\int_{p-t}^{p+t} F(q)dq = 1 - \frac{\frac{E\pi_0}{p} - \frac{1-\phi}{2}}{\phi} \tag{10}$$

Above we saw that a firm will not set a higher price than $u - \frac{t}{2}$. In the appendix, we show that there is an equilibrium where $F\left(u - \frac{t}{2} - \varepsilon\right) < 1$ for all $\varepsilon > 0$, hence prices arbitrarily close to $u - \frac{t}{2}$ is charged with positive density. Firm $0's$ expected profit therefore equals

$$E\pi_0 = \left(u - \frac{t}{2}\right)\left(\frac{1-\phi}{2} + \phi\left(1 - \frac{1}{2t}\int_{u-\frac{3t}{2}}^{u+\frac{t}{2}} F(q)dq\right)\right)$$

Which we rewrite as

$$E\pi_0 = \left(u - \frac{t}{2}\right)\left(\frac{1-\phi}{2} + \phi\Delta(t)\right) \tag{11}$$

Where

$$\begin{aligned} \Delta(t) &\equiv 1 - \frac{1}{2t}\int_{u-\frac{3t}{2}}^{u+\frac{t}{2}} F(q)dq && (12) \\ &= 1 - \frac{1}{2t}\left(\int_{u-\frac{3t}{2}}^{u-\frac{t}{2}} F(q)dq + t\cdot 1\right) && (13) \\ &= \frac{1}{2} - \frac{1}{2t}\int_{u-\frac{3t}{2}}^{u-\frac{t}{2}} F(q)dq \rightarrow 0 \text{ as } t \rightarrow 0. && (14) \end{aligned}$$

where the second line follows from the fact that $F(q) = 1$ for all $q \geq u - \frac{t}{2}$. Notice that for all relevant $t > 0$, $\Delta(t) < \frac{1}{2}$.

In the homogeneous market, $t = 0$ and $\Delta(t) = 0$. The expected profit of the firm in the mixed strategy Nash equilibrium then equals

$$E\pi_0|_{t=0} = u\frac{1-\phi}{2}$$

which just is a restatement of Varian's result.

4 Tacit Collusion

Now we consider the repeated game. There are infinitely many periods, $\tau = 0, ..., \infty$. In each period the market is as described above. Firms seek to maximize the discounted sum of profits and both have the discount factor δ, which fulfills $0 < \delta < 1$. We will assume that a consumer's information type (as well as her location) is the same in all periods.

We focus on a trigger-strategy equilibrium, where in the collusive phase, firms collude on the best possible price, either the monopoly price, p^m or some lower price. Deviations from collusion are punished with reversion to the one-shot Nash equilibrium for the rest of the game as suggested by Friedman (1971). We only consider the case, where (3) is fulfilled, i.e. where the products are reasonably close substitutes, such that the firms cannot sustain the monopoly price even in a one shot game.

If the firms collude on the price p, each firm earns a profit $\pi(p) = \frac{p}{2}$ in all periods. The best deviation price maximizes the one period profit, $p'D(p', p, \phi)$. Notice, that only a fraction ϕ will learn that the firm has lowered its price before they decide to visit the firm. The rest $1 - \phi$ expects the firm to set p. Half of these consumers will visit the firm and get a nice surprise and will not decline to buy from the firm. The other half will not observe the deviation, as they buy from the other firm. The optimal deviation price is given by

$$p^d = \begin{cases} \frac{1}{2}\left(p + \frac{t}{\phi}\right) \text{ if } p \leq 2t + \frac{t}{\phi} \\ p - t \text{ if } p > 2t + \frac{t}{\phi} \end{cases}. \tag{15}$$

Notice that $\frac{1}{2}\left(p + \frac{t}{\phi}\right)$ is decreasing in ϕ and less than the collusive price p, when $p > t/\phi = p^N$. The more transparent the market is, the more demand is captured by a price decrease, and the lower is the optimal price. The associated profit is

$$\pi^d(p) = \begin{cases} \frac{1}{8}\frac{(\phi p+t)^2}{\phi t} \text{ if } p \leq 2t + \frac{t}{\phi} \\ (p-t)\frac{1+\phi}{2} \text{ if } p > 2t + \frac{t}{\phi} \end{cases} \tag{16}$$

both expressions are increasing in ϕ when $p > t/\phi$. Hence, the more transparent the market, the more can potentially be gained from deviating from collusive play. Clearly, this effect by itself makes collusion harder to sustain.

A deviation is punished by reversion to the Nash equilibrium in all future. Clearly, the results depend on whether this equilibrium is in pure or mixed strategies. We consider the two possibilities in turn.

4.1 Relatively high product differentiation

We first consider the case where product differentiation is relatively high such that (7) is not fulfilled and the Nash equilibrium is in pure strategies.

Suppose now that the firms seek to collude on the monopoly price $p^m = u - \frac{t}{2}$. In this case they each earn the monopoly profit per firm $\pi^m = p^m/2$ in each period. If $p^m = u - \frac{t}{2} < 2t + \frac{t}{\phi}$, or

$$\frac{t}{u} > \frac{2\phi}{5\phi + 2} \tag{17}$$

then the optimal deviation is given by the first expression in (16). In Figure 2 below this condition has been inserted in our old Figure 1.

Collusion can be sustained if the present value of monopoly profits exceeds the deviation profit plus the present value of the punishment profits, i.e., if

$$\frac{1}{1-\delta}\pi^m \geq \pi^d(p^m) + \frac{\delta}{1-\delta}\pi^N \tag{18}$$

Inserting the relevant expressions, this condition becomes

$$\frac{1}{1-\delta}\frac{u-\frac{t}{2}}{2} \geq \frac{1}{8}\frac{\left(\phi\left(u-\frac{t}{2}\right)+t\right)^2}{\phi t} + \frac{\delta}{1-\delta}\frac{t}{2\phi}.$$

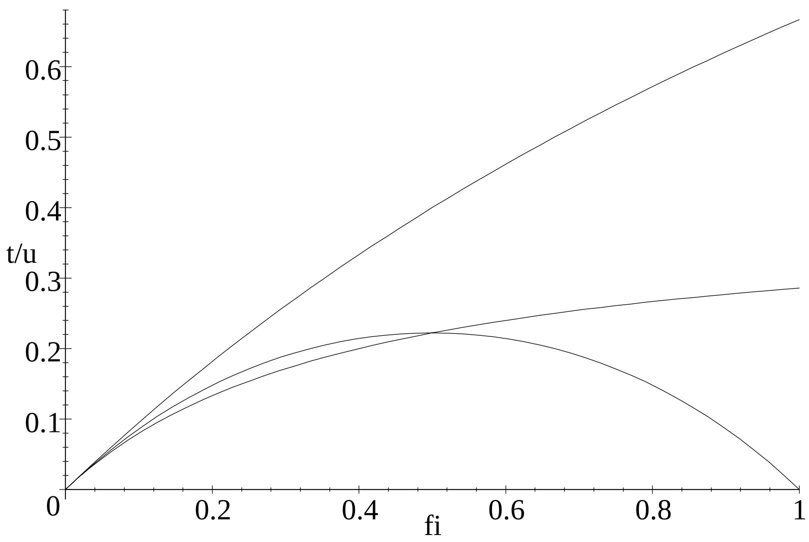

Figure 2:

It is fulfilled when firms are sufficiently patient, namely when

$$\delta \geq \delta_1 \equiv 1 - \frac{8t}{\phi\,(2u-t)+6t}.$$

Clearly, $0 < \delta_1 < 1$ and δ_1 is increasing in the level of market transparency, ϕ. In this sense full collusion on the monopoly price is more difficult to sustain when the market is more transparent. For $t/u = \frac{2\phi}{\phi+2}$, the right hand side equals zero, and full collusion can be sustained even with $\delta = 0$. This, of course conforms with analysis above, in this case the one shot Nash equilibrium price equals the monopoly price.

Then consider, the case where (17) is not fulfilled. The optimal deviation price and profit is then given by the second expression in (15) and (16) respectively. Inserting in the no-deviation constraint (18) then yields that the monopoly price can only be sustained if the discount factor at least equals

$$\delta_2 = \phi \frac{\phi\,(2u - 3t) - 2t}{(2u - 3t)\,\phi\,(1 + \phi) - 2t}$$

Differentiating one finds that the sign of $\frac{\partial \delta_2}{\partial \phi}$ equals the sign of

$$(\phi u - 2t\,(1 + \phi))\,4u\phi + \left(3\phi^2 + 12\phi + 4\right) t^2 \tag{19}$$

For $t = 0$, this expression is positive. The roots in t are

$$t = \frac{\phi}{4 + 3\phi^2 + 12\phi} \left(4\phi + 4 \pm 2\sqrt{(\phi - 4)\,\phi}\right) u$$

In the relevant interval $\phi \in [0, 1]$, these roots are imaginary. Therefore, the expression (19) does not change sign in the relevant interval and is positive for all relevant ϕ. We conclude that $\frac{\partial \delta_2}{\partial \phi} > 0$.

Suppose then that the discount factor is lower than the relevant crucial discount factor, δ_1 or δ_2. In this case, the most favorable equilibrium from the point of view of the firms involves a collusive price which exactly makes the non-deviation constraint fulfilled, i.e. the price solves

$$\frac{1}{1 - \delta} \pi(p) = \pi^d(p) + \frac{\delta}{1 - \delta} \pi^N \tag{20}$$

Assuming that the collusive price, p, is such that $p \leq 2t + \frac{t}{\phi}$, such that the first expression in (16) is relevant, we get two solutions by inserting the relevant expressions, the one shot Nash equilibrium price $p^N(\phi) = t/\phi$ and

$$p_c^1 = \frac{1 + 3\delta}{(1 - \delta)} \frac{t}{\phi}, \tag{21}$$

which yields the highest profit possible given the constraint (20) should be fulfilled. Clearly, p_c^1 is decreasing in the market transparency and so is the profit (which equals $p_c^1/2$).

The price p_c^1 is derived under the assumption that $p \leq 2t + \frac{t}{\phi}$, it is therefore only valid if

$$\delta \leq \frac{\phi}{2+\phi}$$

Consider then the case where the relevant deviation profit is given by the second expression in (16) . In this case, the maximal sustainable price is

$$p_c^2 = \frac{t}{\phi} + t \frac{1-\delta}{\left(1-\delta-\frac{\delta}{\phi}\right)} \tag{22}$$

Differentiating (22) it is readily seen that p is decreasing in ϕ. The formula is only valid for $p > 2t + \frac{t}{\phi}$, or $\delta \geq \frac{\phi}{2+\phi}$.

In conclusion, whenever the Nash equilibrium is in pure strategies, increasing transparency makes tacit collusion more difficult, both in the sense that in increases the minimal discount factor compatible with full collusion on the monopoly price and in the sense that it lowers the highest sustainable price when the discount factor is lower than the crucial discount factor. This result is, however, a net result of two opposing forces. First, the deviation profit is higher in a more transparent market (equation (16)), this makes collusion harder to sustain. On the other hand, the Nash equilibrium profit is smaller in a more transparent market, which makes the punishment harder. On balance, the effect on the deviation profit is larger, and therefore the net result is that increasing market transparency makes collusion harder to sustain.

4.2 Relatively low product differentiation

We now consider the case where product differentiation is relatively low and (7) is fulfilled such that the Nash equilibrium is in mixed strategies. Consider the case where the optimal deviation price is given by the second part of the expression (15). As is clear from Figure 2, this is fulfilled for low t and is true for most of the relevant parameter space. We will only focus on this case.

Assume that the firms are able to collude on the monopoly price. Inserting the second part of (16) and (11) into the non-deviation constraint (18) and solving for δ gives the lowest discount factor compatible with full collusion,

$$\delta_3 = \frac{1}{2}\frac{2u - 3t - 2\frac{t}{\phi}}{2u\left(1-\Delta\right) - t\left(2-\Delta\right) - \frac{t}{\phi}} \tag{23}$$

which is less than one as $t \leq 2u$.

Consider first a homogeneous product market, where $t = 0$ and $\Delta = 0$. Then δ_3 reduces to

$$\delta_3|_{t=0} = \frac{1}{2}.$$

This is *independent* of ϕ. In the homogeneous market, an increase in ϕ leads to an increase in the deviation profit which exactly balances the decrease in the profit of the punishment, such that the incentive constraint is unchanged.

In the differentiated market, we find the effect of increasing ϕ by differentiating δ_3 wrt ϕ. We get

$$\frac{\partial \delta_3}{\partial \phi} = \frac{1}{2}\frac{\left(2u-t\right)\left(\left(1-2\Delta\right)t + \left(2u\phi^2 - 3t\phi^2 - 2t\phi\right)\frac{\partial \Delta}{\partial \phi}\right)}{\left(2u\phi - 2u\phi\Delta - 2t\phi + t\phi\Delta - t\right)^2}. \tag{24}$$

Unfortunately, the sign of $\frac{\partial \delta_3}{\partial \phi}$ cannot directly be assessed. The problem is that we do not have a closed form solution for the mixed strategy F and therefore not for Δ. As can be seen, the sign depends on the term $\left(\left(1-2\Delta\right)t + \left(2u\phi^2 - 3t\phi^2 - 2t\phi\right)\frac{\partial \Delta}{\partial \phi}\right)$. Here the first term is positive (as $\Delta < 1/2$), while the sign of the second term depends on $\frac{\partial \Delta}{\partial \phi}$. I have carried out simulations, described the appendix. The simulations show that for small t, $\frac{\partial \delta_3}{\partial \phi}$ is small in absolute value but positive[5]. Hence, we again get the result

[5] I get this result for all t, I have tried, but the approximations used are better for small t.

that increasing transparency makes collusion on the monopoly price more demanding. While the effect of transparency is qualitatively different in the homogeneous and the differentiated market, the simulations in the appendix reveal that $\Delta \to 0$ and $\frac{\partial \Delta}{\partial \phi} \to 0$ for $t \to 0$ and therefore also that $\frac{\partial \delta_3}{\partial \phi} \to 0$ for $t \to 0$. So, the homogeneous market is the limiting case of the differentiated market, also with respect to the size of $\frac{\partial \delta_3}{\partial \phi}$.

Then, consider the case where the discount factor is so low, that the firms cannot collude on the monopoly price. We consider the possibilities for collusion on prices $p \in [2t + \frac{t}{\phi}, u - \frac{t}{2}]$ so the second part of (16) is relevant. Inserting this and (11) in the non-deviation constraint (18) and solving for p yields

$$p_c^3 = \frac{\frac{\delta}{1-\delta}\left(u - \frac{t}{2}\right)\left(\frac{1-\phi}{2} + \phi\Delta\right) - t\frac{1+\phi}{2}}{\frac{1}{2(1-\delta)} - \frac{1+\phi}{2}} \tag{25}$$

Again, consider first the homogeneous market (where $t = 0$, and $\Delta = 0$). Here, p_c^3 reduces to

$$p_c^3\big|_{t=0} = \frac{(1-\phi)\,\delta}{(1+\phi)\,\delta - \phi} u \tag{26}$$

Recall that in the homogeneous market all consumers have the same reservation price, u. Since $\delta < \delta_3|_{t=0} = \frac{1}{2}$, we have that $p_c^3|_{t=0} > u$ if $(1+\phi)\,\delta - \phi > 0$, and $p_c^3|_{t=0} < 0$ otherwise. Hence, the solution $p_c^3|_{t=0}$ is economically irrelevant and tacit collusion is impossible for all prices in $p \in [2t + \frac{t}{\phi}, u\ ]$ if $\delta < \frac{1}{2}$.

In the appendix, we show that exactly the same happens in the differentiated market if t is not too large.

Concluding this section, transparency has no effect on the scope for collusion in a homogeneous market. Furthermore, in such a market collusion is either possible on the monopoly price or not at all. In a slightly differentiated market the latter is also true, but transparency has an effect on collusion albeit small in the simulations performed. Nevertheless, the effect is qualitatively the same as in the more differentiated market, raising transparency makes collusion more demanding in the sense that a higher discount factor is required.

5 Optimal symmetric punishments

In this section we briefly consider equilibria with optimal symmetric punishment phases. When the discount factor is sufficiently high, reversal to the one shot Nash equilibrium is sufficient for sustaining the monopoly price. For lower discount factors this is not possible, and as is well known, the firms may realize higher profits in the normal phase of the equilibrium by using harder punishment phases.

In this section, we restrict attention to the case where $p^m = u - \frac{t}{2} < 2t + \frac{t}{\phi}$, i.e. where (17) is fulfilled.

In principle, there are no bounds below the prices the firms charge. They could even start giving money to customers by charging a negative price. Since we disregard unit cost, the price in the model should be considered as a net-of-cost price. A negative price does therefore not necessarily imply a negative gross price. The specification of consumers also allows for negative prices, a consumer finds a firm's product more attractive the more negative the price.

From Abreu (1988) it is known that any equilibrium payoff can be realized in an equilibrium with simple strategies, consisting of a normal phase and a punishment phase. We will study equilibria where the punishment only lasts one period - stick and carrot equilibria (cf Abreu (1986)). As the game is symmetric, we look at symmetric equilibria.

Suppose both firms set p, the best deviation is to p^d as given in the first line of (15) (since (17) is fulfilled). Notice, that if $p < p^N = t/\phi$, the deviation price is *higher* than the original price. In this case the uninformed consumers get an unpleasant surprise and they would have preferred to go to the other firm if they had been aware of the higher price. However, the deviation price is still below the Nash equilibrium price, and all consumers prefer to buy at this price rather than not buying. The associated deviation profit is given by the first line in (16).

Consider the following prescription for "stick and carrot "strategies. Let p be a (high) normal price and q a (low) punishment price. Both firms set p in the first period and set p in each period where p - or q - were set by both firms in the previous period. If a firm deviates from this prescription, both firms set q for one period and then return to setting p. If a firm deviates from this, they both set q for one period and then return to setting p. The equilibrium is characterized by the following incentive equations

$$\frac{1}{1-\delta}\pi(p) \geq \pi^d(p) + \delta\left(\pi(q) + \frac{\delta}{1-\delta}\pi(p)\right) \tag{27}$$

The present value of choosing the normal phase price p should at least equal the deviation profit plus the present value of being punished for a period and then returning to the normal phase. A firm may deviate from the punishment, the punishment phase will then be restarted. Such a deviation is not attractive if

$$\pi(q) + \frac{\delta}{1-\delta}\pi(p) \geq \pi^d(q) + \delta\left(\pi(q) + \frac{\delta}{1-\delta}\pi(p)\right) \tag{28}$$

which can be rewritten

$$(1-\delta)\,\pi\,(q) + \delta\pi\,(p) \geq \pi^d\,(q) \tag{29}$$

In the optimal equilibrium (27) and (29) are both fulfilled with equality. Inserting the deviation profit from (16) gives two equations in two unknowns

$$\begin{aligned} \frac{1}{1-\delta}\frac{p}{2} &= \frac{1}{8}\frac{(\phi p+t)^2}{\phi t}+\delta\left(\frac{q}{2}+\frac{\delta}{1-\delta}\frac{p}{2}\right) \qquad (30) \\ (1-\delta)\frac{q}{2}+\delta\frac{p}{2} &= \frac{1}{8}\frac{(\phi q+t)^2}{\phi t} \end{aligned}$$

With two pairs of solutions, $p = q = t/\phi = p^N$ and

$$p = (1+8\delta)\frac{t}{\phi} \text{ and } q = (1-8\delta)\frac{t}{\phi} \qquad (31)$$

It is clear that the collusive price, p, is decreasing in ϕ. Equations (30) make it clear that transparency affects the equilibrium, through the incentives to deviate in both phases of the equilibrium. Notice that while increasing ϕ increases the deviation profit in the collusive phase, $\frac{\partial}{\partial\phi}\left(\frac{1}{8}\frac{(\phi p+t)^2}{\phi t}\right) > 0$, the effect in the punishment phase, depends on the level of q. For positive $q < \frac{t}{\phi}$, the effect is negative

$$\frac{\partial\left(\frac{1}{8}\frac{(\phi q+t)^2}{\phi t}\right)}{\partial\phi} = \frac{1}{8}(\phi q+t)\frac{\phi q-t}{t\phi^2} < 0 \text{ for } 0 \le q < \frac{t}{\phi} = p^N.$$

However, for sufficiently negative q, the effect changes. Nevertheless, the effect in the collusive phase dominates.

Clearly, q is only positive if $\delta < 1/8$. Notice, however that the total discounted profit in the punishment phase (given in the second line of (30)) is positive regardless of the value of q, (the reason being that the continuation profit is positive)[6]. The collusive price $p < u - \frac{t}{2}$ if

$$\delta < \delta_{op} \equiv \frac{(2u-t)\phi - 2t}{16t}$$

[6] Again, the firm has the option to raise the price to $u - \frac{t}{2}$ in the punishment phase and only serve the uninformed consumers. Straightforward calculations show that this is not attractive if $\delta < \frac{1}{4} + \frac{\sqrt{2(2u-t)(1-\phi)\phi t}}{8t}$, which is fulfilled for $\delta < \frac{\phi}{4}$.

Furthermore, we used the first line in (16), so the solution is only valid if $(1+8\delta)\frac{t}{\phi} \leq \ t+\frac{t}{\phi}$ or $\delta \leq \frac{\phi}{4}$, i.e. for small discount factors. The crucial discount factor $\delta_{op} \leq \frac{\phi}{4}$ if $\frac{t}{u} \geq \frac{2\phi}{5\phi+2}$, which is condition (17).

If the discount factor $\delta \geq \delta_{op}$ then the optimal symmetric punishment suffices for sustaining the monopoly price, $u - \frac{t}{2}$. Again we see that this required discount factor is increasing in ϕ.

It is easy to check that p is higher than the collusive price which can be sustained by a punishment phase consisting of reversion to the one shot Nash equilibrium given by (21) when $0 < \delta < 1/2$.

Concluding, this section, when optimal symmetrical punishments are used, we find the same result as with Nash punishments. Increasing transparency increases the smallest discount factor compatible with sustaining collusion on the monopoly price. Furthermore, when the discount factor is lower than the crucial value, the profit and price of the most collusive equilibrium decreases as transparency increases.

6 Concluding remarks

This paper has analyzed the effect on tacit collusion from an increase in market transparency on the consumer side of the market. There are two opposing effects from this: the temptation to undercut the other firm increases but so does the toughness of the punishment (whether this consists of Nash-reversion or the optimal symmetric punishment). In the differentiated Hotelling market, the first effect dominates, increasing transparency makes collusion more difficult and is pro-competitive. This is a qualitatively different result from the result obtained in the corresponding homogeneous market, where the effects on deviation profit and punishment profit exactly balances each other. In case of a almost homogeneous market, we had to rely on simulations, and the results obtained approached the results of the homogeneous market as product differentiation is decreased.

7 Appendix

In the symmetric mixed strategy equilibrium both firms choose prices according to the distribution function F. First we show that there is a mixed strategy equilibrium where $F\left(u-\frac{t}{2}\right)=1$ and $F\left(u-\frac{t}{2}-\varepsilon\right)<1$ for all $\varepsilon>0$. I.e. prices abitraily close to $u-\frac{t}{2}$ is chosen with positive density.

In the text it was shown that F fulfills

$$\frac{1}{2t}\int_{p-t}^{p+t}F(q)dq=1-\frac{\frac{E\pi_0}{p}-\frac{1-\phi}{2}}{\phi} \tag{32}$$

Differentiating both sides

$$\frac{\partial\left(\frac{1}{2t}\int_{p-t}^{p+t}F(q)dq\right)}{\partial p}=\frac{1}{2t}\left(F\left(p+t\right)-F\left(p-t\right)\right)=\frac{E\pi_0}{2\phi p^2} \tag{33}$$

Assume that firm 1's mixed strategy is given by F and $F\left(u-\frac{t}{2}\right)=1$ and $F\left(u-\frac{t}{2}-\varepsilon\right)<1$ for all $\varepsilon>0$. We then show, that firm 0 is indifferent between all prices $p\leq u-\frac{t}{2}$ such that $f(p)>0$, . thus chosing prices according to F is a best response to the other firm choosing prices according to F. Consider any such p. Differentiating (9) of the text wrt p yields

$$\begin{aligned}\frac{\partial E\pi_0}{\partial p} &= \phi\left(1-\frac{1}{2t}\int_{p-t}^{p+t}F(q)dq\right)+\frac{1-\phi}{2}-\frac{\phi}{2t}\left(F\left(p+t\right)-F\left(p-t\right)\right)p \\ &= \frac{E\pi_0}{p}-\frac{1-\phi}{2}+\frac{1-\phi}{2}-\frac{E\pi_0}{p}=0\end{aligned}$$

here we have used (32) and (33). At $p=u-\frac{t}{2}$, the above derivative is the left derivative (and $F\left(u-\frac{t}{2}+t\right)=F\left(u-\frac{t}{2}\right)=1$). Raising p above $u-\frac{t}{2}$

reduces the demand of the uninformed, this is not optimal as shown in section 3.2. We conclude that firm 0 is indifferent between all prices in the support of F of the other firm 1. Accordingly, there is a symmetric mixed strategy equilibrium where $F(u-\frac{t}{2})=1$ and $F\left(u-\frac{t}{2}-\varepsilon\right)<1$ for all $\varepsilon>0$.

Differentiating both sides of (33) yields

$$\frac{\partial^2\left(\frac{1}{2t}\int_{p-t}^{p+t}F(q)dq\right)}{\partial p^2}=\frac{1}{2t}\left(F'\left(p+t\right)-F'\left(p-t\right)\right)=-\frac{E\pi_0}{4\phi p^3}<0$$

so

$$F'\left(p+t\right)<F'\left(p-t\right)$$

for all p. It follows that $F''\left(p\right)<0$, so F is strictly concave.

We know that $F\left(u-\frac{t}{2}\right)=1$. Using (33), we therefore get

$$F\left(u-\frac{t}{2}-2t\right)=1-t\frac{E\pi_0}{\phi\left(u-\frac{t}{2}-t\right)^2}$$

Iterating, we find

$$F\left(u-\frac{t}{2}-2nt\right)=1-\frac{t}{\phi}E\pi_0\sum_{i=1}^{n}\left(\frac{1}{\left(u-\frac{t}{2}-\left(2i-1\right)t\right)^2}\right) \tag{34}$$

Given $E\pi_0$, this gives F for all $p\left(n\right)\equiv u-\frac{t}{2}-2nt$, where $n=1,2,......\bar{n}$, and $\bar{n}$ is the smallest integer such that $F\left(p\left(\bar{n}+1\right)\right)<0$.

The expected profit equals

$$E\pi_0=\left(u-\frac{t}{2}\right)\left(\frac{1-\phi}{2}+\phi\Delta(t)\right) \tag{35}$$

Where

$$\Delta(t)\equiv1-\frac{1}{2t}\int_{u-\frac{3t}{2}}^{u+\frac{t}{2}}F(q)dq$$

The last term is the average of F over the small interval $[u - \frac{t}{2} - t, u - \frac{t}{2} + t]$. If F is close to one, then Δ is small. We are interested in evaluating the sign of

$$\frac{\partial\delta}{\partial\phi} = \frac{1}{2}\frac{(2u-t)\left((1-2\Delta)t - \left(2t\phi - 3t\phi^2 + 2u\phi^2\right)\frac{\partial\Delta(t)}{\partial\phi}\right)}{(2u\phi - t - 2t\phi - 2u\phi\Delta + t\phi\Delta)^2}$$

This involves evaluating Δ and $\frac{\partial\Delta(t)}{\partial\phi}$, where

$$\frac{\partial\Delta(t)}{\partial\phi} = -\frac{1}{2t}\frac{\partial\left(\int_{u-\frac{3t}{2}}^{u+\frac{t}{2}} F(q)dq\right)}{\partial\phi} \tag{36}$$

As I have no closed form solution for F, I have been unable to solve this analytically. I have therefore resorted to Mathematica[7] using the value $u = 3$, and different small values of t. The following is based on $t = .0005$. I tried for many different values of t and got similar results. However, it should be remembered that the larger t the approximations used below is.

I have used (34) to calculate F for all $p(n)$. Unfortunately, a recursion problem is involved. The formula in (34) includes $E\pi_0$ on the right hand side, and $E\pi_0$ on the other hand depends on F, as is clear from equation (35). In the simulations I have first approximated $E\pi_0$ by $\frac{1-\phi}{2}\left(u - \frac{t}{2}\right)$, . for small t and consequently small $\Delta(t)$ this is a reasonable approximation. I have then calculated F by means (34). With this F, I have calculated $E\pi_0$ using (35) including a term approximating Δ. Using this $E\pi_0$, I have recalculated F using (34). I have iterated like this three times. The difference between the two last approximations of F is vanishingly small, (of the order 10^{-15}), I have therefore made no further iterations here.

[7] The Mathematica notebook with the simulations is avaliable on request or can be downloaded from my homepage at www.econ.ku/cschultz/???????

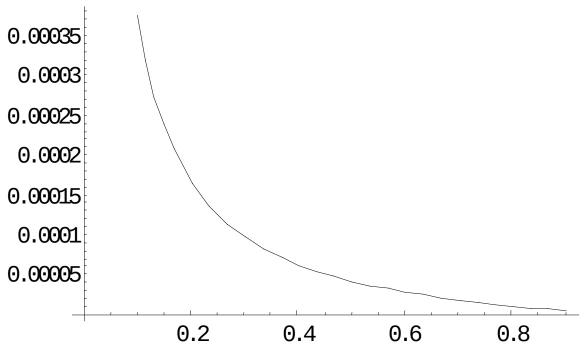

Figure 3: Approx DELA as fctn of fi

In the calculations I have approximated Δ with

$$\Delta \approx APP\Delta \equiv 1 - \left(\frac{1 + F\left(u - \frac{5t}{2}\right)}{2} \right)$$

Rather than taking the average over the interval $[u - \frac{t}{2} - t, u - \frac{t}{2} + t]$, I have used the average of 1 (which is the value of F for all $p \geq u - t/2$) and $F\left(u - \frac{5t}{2}\right)$ which is the value of F in the closest point to $u - \frac{t}{2} - t$, I have a formula for. This is not perfect, but the best which can be done. As F is concave, we underestimate the average F and the approximated value of Δ is therefore smaller than the true Δ. For small t, the approximation is better than for larger t. The approximated value of Δ for different ϕ is depicted in Figure 3. As it should be it is positive and small. As expected it is decreasing in ϕ.

Figure 4 plots the simulated F function for five values of ϕ , $\{.1, .2, .3, .4, 5\}, .$ where F corresponding to $\phi = .1$ is lowest and higher F correspond to higher ϕ. The true F is zero whenever the simulated F takes on negative values. E.g. F for $\phi = .1$ is zero for $p \lessapprox 2.1$. Figure 5 depicts the same, but for values of p very close to $u - \frac{t}{2}$.

In the following we write $F(p, \phi)$ for $F(p)$ given ϕ, so for $\phi_1 < \phi_2$, we

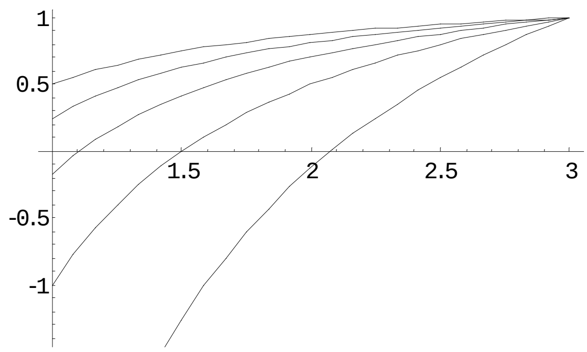

Figure 4: Simulated F for different values of fi

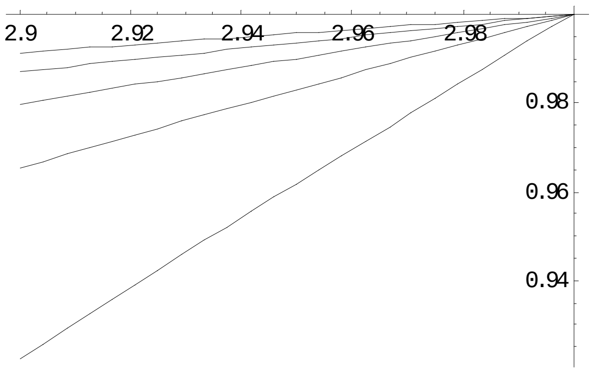

Figure 5: Simulated F for different values of fi

have $F(p, \phi_1) < F(p, \phi_2)$, $F(\bullet, \phi_1)$ stochastically dominates $F(\cdot, \phi_2)$. This imply that higher ϕ leads to lower expected prices and profits, which confirms the intuition gained from equation (35) when we remember that Δ is small.

Since F is strictly concave, and we also have $F(1, \phi_1) = F(1, \phi_2)$ as well as $F(p, \phi_1) < F(p, \phi_2)$ (for all p and $\phi_1 < \phi_2$) it follows that $\frac{\partial F(p,\phi_2)}{\partial p} < \frac{\partial F(p,\phi_1)}{\partial p}$, (see Figures 1 and 2). Furthermore, it is clear from the figures that $F(p, \phi) - F(p, \phi - .1)$ is smaller the higher is ϕ. The same is true for small differences in ϕ. According to the simulations, the derivative $\frac{\partial F}{\partial \phi}$ is positive and decreasing in F.

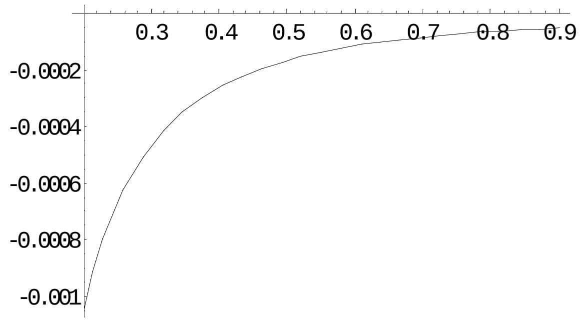

Figure 6: Approx DELTA-prime as fctn of fi

We wish to assess the value of $\frac{\partial\Delta(t)}{\partial\phi}$, i.e. the derivative with respect to ϕ of the average F over the small interval $[\ u-\frac{3t}{2}, u+\frac{t}{2}]$. We approximate this by

$$\frac{\partial\Delta(t)}{\partial\phi} \approx APP\Delta' \equiv \frac{1-\left(\frac{1+F\left(u-\frac{5t}{2},\phi\right)}{2}\right)-\left(1-\left(\frac{1+F\left(u-\frac{5t}{2},\phi-x\right)}{2}\right)\right)}{x}$$

evaluated at very small x. Figure 6 below depicts $APP\Delta'$ for different values of ϕ and $x = 10^{-8}$. As can be seen $APP\Delta'$ is negative and of the order $10^{-3} - 10^{-4}$. The negative sign corresponds to the fact that F is increasing in ϕ. We approximate the average derivative of F wrt ϕ over the interval $[u-\frac{3t}{3}, u-\frac{t}{2}]$ with an approximated value of the derivative of F at $u-\frac{5t}{2}$. The derivative of F wrt ϕ decreases in p for all $p\,(n)$ where we know the value of F. (see figure $A.3$). Assuming this is true for all p, we overestimate the value of the derivative of F wrt ϕ. Consequently our approximation of $\frac{\partial\Delta(t)}{\partial\phi}$ is more negative than the true $\frac{\partial\Delta(t)}{\partial\phi}$. We wish to assess the size and sign of

$$\frac{\partial\delta}{\partial\phi} = \frac{1}{2}\frac{(2u-t)\left((1-2\Delta)\,t-\left(2t\phi-3t\phi^2+2u\phi^2\right)\frac{\partial\Delta(t)}{\partial\phi}\right)}{\left(2u\phi-t-2t\phi-2u\phi\Delta+t\phi\Delta\right)^2}$$

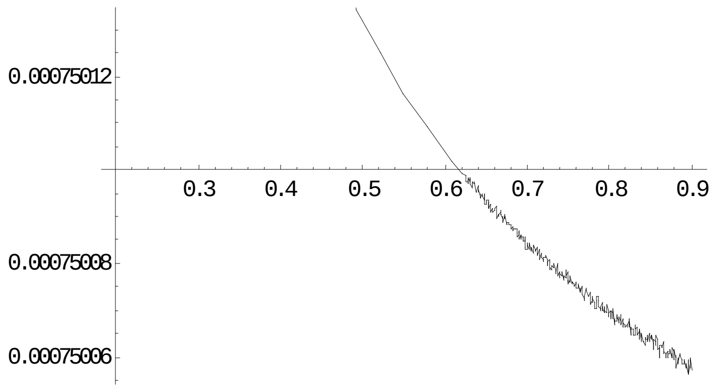

Figure 7:

The sign equals the sign of the term $(1-2\Delta)\,t-\left(2t\phi-3t\phi^2+2u\phi^2\right)\frac{\partial\Delta(t)}{\partial\phi}$. The approximated value of this term for different values of ϕ is given in Figure 7

We see that it is positive and conclude that the approximation leads to a positive $\frac{\partial\delta}{\partial\phi}$. Remembering that our estimate of Δ is too small and the estimate of $\frac{\partial\Delta(t)}{\partial\phi}$ too negative, we conclude that the true $\frac{\partial\delta}{\partial\phi}$ is positive.

Finally, Figure 8 gives the approximated value of $\frac{\partial\delta}{\partial\phi}$. We see that it is positive and small. Reiterating the calculations for smaller t, yields smaller values for $\frac{\partial\delta}{\partial\phi}$.

Finally, we plot the approximated discount factor as a function of ϕ

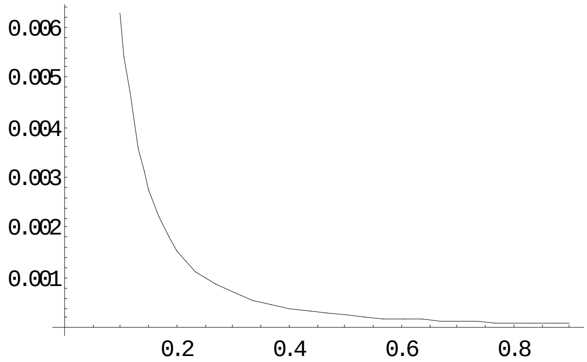

Figure 8: Approx ddelta/dfi as fctn of fi

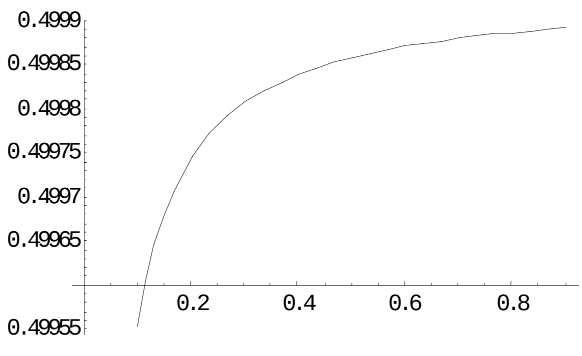

Figure 9: Approx discount-factor as fctn of fi

Proof that collusion is impossible if the discount factor is below δ_3 when t is small.

The price p_c^3 is negative, if the numerator and denominator of (25) have different signs. Consider first the case where $\delta < \frac{\phi}{1+\phi}$, then the denominator is negative. If

$$t < \frac{(1-\phi(1-2\Delta))\,2\delta}{2(1+\phi)-(1+2\phi+(1-2\Delta)\phi)\,\delta}u \tag{37}$$

then the numerator of (25) is positive, and p is then negative. As, $\Delta < \frac{1}{2}$, the right hand side of the last inequality is positive and we see that if t is sufficiently small, the solution to (25) is negative.

For $\delta = \frac{\phi}{1+\phi}$, there are no solutions to (25) except in the degenerate case where the numerator is also zero, (where infinitely many solutions occur).

Consider then $\delta > \frac{\phi}{1+\phi}$, such that the denominator in (25) is positive. If (37) is not fulfilled, the numerator of (25) is negative and p_c^3 is negative. If (37) is fulfilled p_c^3 is positive. We now consider this case. Recall, that we are considering discount factors such that collusion on $u - \frac{t}{2}$ is impossible, and the expressions used in the derivation of p_c^3 are only valid if $p_c^3 < u - \frac{t}{2}$, i.e. if

$$\frac{\frac{\delta}{1-\delta}\left(u-\frac{t}{2}\right)\left(\frac{1-\phi}{2}+\phi\Delta\right)-t\frac{1+\phi}{2}}{\frac{1}{2(1-\delta)}-\frac{1+\phi}{2}} < u - \frac{t}{2}$$

At δ^3, the left hand side equals the right hand side. Differentiating the left hand side we find,

$$\frac{\partial lhs}{\partial \delta} = \frac{1}{2}\frac{-(1-(1-2\Delta)\phi)\,2\phi u + \left(2-(1-2\Delta)\phi^2+3\phi\right)t}{(\delta-\phi+\phi\delta)^2},$$

which is negative for

$$t < 2\phi\frac{1-\phi(1-2\Delta)}{2-\phi^2+2\phi^2\Delta+3\phi}u \tag{38}$$

For t fulfilling (38) therefore, the solution $p_c^3 > u - \frac{t}{2}$. The right hand side of (38) is increasing in Δ. Therefore if

$$t < 2\phi \frac{1-\phi}{2-\phi^2+3\phi} u \tag{39}$$

then surely (38) is fulfilled.

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