WHEN BACKWARD INTEGRATION BY A DOMINANT FIRM IMPROVES WELFARE

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Abstract

This paper studies the welfare consequences of a vertical merger that raises rivals' costs when downstream competition is *à la* Cournot between firms with constant asymmetric marginal costs. The main result is that such a vertical merger can nevertheless improve welfare if it involves a downstream firm whose cost is "low enough". This is because by raising the input price paid by the non-merging firms the merger thereby shifts production away from those relatively inefficient producers in favor of the more efficient firm. However there is a tradeoff between the gain in productive efficiency and the loss in consumers' surplus caused by a higher downstream price which follows a higher input price. It is also shown, through an example, that this result extends to price competition with differentiated products.

JEL Classification: L13, L4.

Keywords: foreclosure, raising rival costs, Cournot, welfare, asymmetric costs.

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1 Introduction

The effects of vertical integration are often called into question. Is a dominant input supplier able to extend its market power from the intermediate good market to the final good market by buying a downstream firm?[1] Conversely, can a final good producer raises its rivals' costs by integrating backward with an input firm?

These issues have been addressed by several authors. My purpose is to shed a new light on the welfare effects of backward vertical integration by taking into account asymmetries between firms. A key issue in merger analysis is the market shares of the firms involved.[2] Indeed, in most markets, market shares are not symmetric and the usual symmetric oligopolistic competition models can simply not capture this empirical fact. However, from a practical point of view, it would be most useful if the antitrust authorities could rely on an analysis of the relationship between the integrating firms' market shares and the welfare effects of the merger.

Adopting a Cournot (homogeneous good) competition framework, I present a model where firms in the final good market have asymmetric market shares due to the fact that they have heterogeneous marginal costs. It is then shown that (in a foreclosure environment) a vertical merger improves welfare if the integrating firm has a marginal cost that is low enough. Since in a Cournot model the lower the marginal cost the higher the market share, a striking result emerges: a vertical merger improves welfare if the market share of the integrating firm is large enough. On the other hand, welfare can be reduced if the merging firm only serves a small percentage of the demand. This result is in sharp contrast with the common belief that an action by a "dominant" firm should be more scrutinized than the same action taken by a small firm. For example, "It is inconceivable today", says Mr Pitofsky, "that mergers of companies with insignificant market shares would be challenged..."[3] The

[1] See Rey and Tirole (2002) for an extensive presentation and Kuhn and Vives (1999) for a recent analysis of this issue.

[2] Market shares play a crucial role in horizontal mergers. For vertical integration they are less important but they are not neglected.

[3] The Economist, October 5th 2000. Robert Pitofsky was at the time at the head of

present paper shows that the effects of a vertical merger are, in certain circumstances, easier to assess if the involved firm has a large market share rather than a small one. The "market share rule" exhibited in the paper can be viewed the other way around: a split of a vertically integrated structure can worsen welfare if the future market share of the disintegrated final good producer remains large. Even when such a split reduces the concentration in the final good market and lessens the price of the final good.

The controversial Brown Shoe case[4] can be used to illustrate the conclusions of my paper. In 1956 a merger between the Brown Shoe Co. and G. R. Kinney, a retail shoe chain, occurred. The acquisition was challenged by the Department of Justice (DOJ) and after several court decisions, Brown had to sell Kinney in 1963. Although Brown Shoe was one of the four largest manufacturers of shoes, it produced less than 5 percent of the total industry output.[5] Basically, the DOJ argued that the market share of the merged firm in the retail sector was large enough to affect both competition and reduce welfare.[6] This case is often mentioned as an example of an excessive antitrust policy. The analysis in the current paper does not contradict the view that the merger could have reduced welfare because, paradoxically, of the small market shares involved. That is, Brown Shoe and Kinney might not have been very efficient compared to the average firm in their sector and the efficiency gain due to the merger may not have compensated the reduction of competition.

In the special case of a linear demand function, the threshold value above which a merger improves welfare, depends only on two easily observable market conditions: the market share of the merging unit and the number of firms. The anticipated market price alone or the anticipated variation of the Herfindahl index of the market share distribution are poor indicators of the

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the Federal Trade Commission.

[4] See Peterman (1975) for a detailed analysis.

[5] While the figures vary from town to town, in 1954, Brown secured about 1.1 percent of national retail sales of shoes and Kinney stores had between 0.9 and 1.1 per cent.

[6] The argument was also a dynamic one: that is, if the merger was allowed it would accelerate the concentration trend in the shoe industry.

welfare effects of the merger.

The intuition behind this result is the following. Market share asymmetries reflect efficiency differences: the more efficient a firm (relatively to the average marginal cost),the larger its market share. Therefore, if a firm with a large market share merges and if the merger increases the marginal costs of its rivals, that means that after the merger the integrated firm produces more than before. This is good for welfare because the firm is more efficient than the average firm. This point is reminiscent of Farrell and Shapiro (1990) in which the authors show that a cost reducing investment improves welfare only if the market share of the investing firm is large enough.

The results are derived in a somewhat simple model and there are many features that might be relevant to a vertical merger that are not modeled. For example, a vertical merger may generate scope economies or may eliminate double marginalization within the merged firm. These, in turn, may decrease the downstream price, in which case the merger benefits both consumers and the merging firm. These features are not modeled because competition at the upstream market level is bang-bang: either Bertrand competition or a (constrained) monopoly. However, these omissions are an explicit modeling strategy. In this model, there is no reason for a vertical merger to be welfare improving except for the potential shift toward more efficient downstream production. If anything, the model is biased against finding benefits from vertical integration. Furthermore, the decisions to integrate or not, and to foreclose or not, are not analyzed. The conclusion relies on a second best analysis: when the effects of a merger are challenged, the comparison is one between a pre-merger and a post-merger suboptimal worlds. Before the merger the market is more competitive but less efficient than after the merger.

Formally, the results are valid in the special case of Cournot competition on a homogeneous good market when firms have constant marginal costs. The results also hold in a price competition duopoly model with product differentiation. More generally, the same kind of tradeoff as in the model should appear in any model where the most efficient firms have the largest market shares but these market shares are not large enough from a welfare point of view. If backward integration increases the marginal cost of the less efficient firms their market shares are reduced which could improve welfare if this efficiency gain is larger than the loss in consumers' surplus. It is worth to emphasize that if the asymmetry in the market shares are not due to efficiency reasons the results are invalidated.

My results are reminiscent of the analysis on "raising rivals' costs" initiated by Salop and Scheffman (1983) and Salop and Scheffman (1987). Riordan (1998) also develops a model with an asymmetric market structure and links the welfare effect of vertical acquisitions to ex ante market share properties. However, his results are qualitatively different and they rely on the existence of a dominant firm with a superior technology which competes with fringe firms that are price takers.[7] He shows that a small degree of vertical integration is socially desirable if the variable cost advantage of the dominant firm is not too large. He also shows that vertical integration is likely to reduce welfare at the margin if the dominant firm is already substantially vertically integrated, or if the dominant firm's output market share is substantially larger than its input market share.

Salinger (1988) initiated the analysis of a vertical merger when both upstream and downstream markets are oligopolistic. He showed that in a situation where some firms are already integrated and some are not, an additional merger can drive the final good price either up or down. Competition is *à la* Cournot in both markets[8] and firms are symmetric (integrated firms benefit from a lower marginal cost through a lower input price but otherwise the marginal costs are the same). In Ordover, Saloner, and Salop (1990) a more complete model of vertical integration is introduced but firms in the final good market are also symmetrical, therefore the net effect of a merger is a reduction in social welfare because the merger does not remedy any existing market imperfection. In Hart and Tirole (1990), the market structure is also a Bertrand duopoly upstream and a Cournot duopoly downstream. In their model, there is cost asymmetry among input producers. However,

[7]Moreover, the dominant firm has a first mover advantage in both upstream and downstream markets.

[8]See also Gaudet and Long (1996), Schrader and Martin (1998) and Avenel and Barlet (2000) for more on the integration and forclosure incentives in this framework.

they focus on merger incentives and on whether a merger leads to exclusion of competitors in an incomplete contract environment.

The remainder of the paper is organized in the following way. Section 2 establishes the Nash equilibrium properties of an asymmetric Cournot competition game with n firms. Section 3 studies the effect of a marginal increase of the marginal cost of the non merging firms for a general demand function. Section 4 extends the welfare analysis of a vertical integration to the case of a linear demand function. Specifically, section 4.1 considers the welfare effects of any price increase after the merger, section 4.2 analyzes the situation where ex post the input market is a monopoly, and section 4.3 determines the optimal input price increase which maximizes the welfare. Section 5 extends the analysis to price competition and section 6 concludes.

2 Hypothesis and Preliminary results

Consider the industry structure of figure 1: two upstream firms provide a homogeneous input to several downstream firms. The marginal costs of production of the final good firms are assumed to be constant. Yet, they are not symmetric: heterogeneity is allowed among the producers of the final good.[9] To elude any input substitution issues, it is assumed that downstream technology has fixed coefficients. So a given change in the upstream price simply changes downstream marginal costs by the same amount. The competition in the final good market is *à la* Cournot, while in the intermediate good market it is *à la* Bertrand. The number of firms in the downstream market is n, $n > 1$ and they face a demand function, $P(Q)$, with $P'(Q) < 0$ and where Q is the total production: $Q = \sum_{j=1}^{n} q_j$, where q_j is the individual production of firm j.[10]

Let c denote the ordered list of the constant marginal costs: $c = (c_1, c_2, \ldots,$

[9] Heterogeneity could also be allowed in the intermediate good market but it would not add much to the model.

[10] Throughout the paper the following convention is adopted: j denotes an arbitrary downstream firm, while i denotes the merging firm.

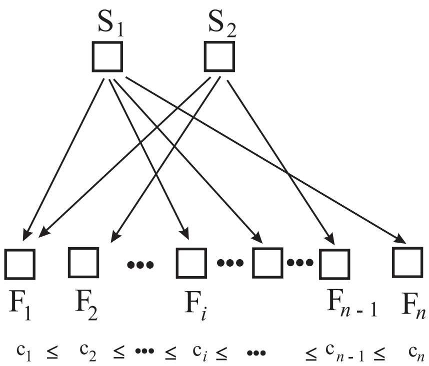

Figure 1: Market structure before a merger

c_n) with $c_1 \leq c_2 \leq \ldots \leq c_n$. The marginal cost of production of the upstream good producers is assumed to be identical and is denoted γ. For simplicity, I assume that in the ante-merger situation, $c_1, c_2, \ldots, c_n$ take into account the price of the intermediate good, so γ can be set to zero. Let $\overline{c} = \frac{1}{n}\sum_{j=1}^{n} c_j$ be the average marginal cost and $v = \frac{1}{n}\sum_{j=1}^{n}\left(c_j - \overline{c}\right)^2$ the variance of the marginal cost distribution of the final good industry. Finally, let Θ denote the elasticity of the slope of the inverse demand function: $\Theta\left(Q\right) = \frac{P''(Q)Q}{P'(Q)}$. Novshek (1985) showed that an equilibrium exists and (with constant marginal costs) is unique as long as $\Theta \geq -1$. That is why from now on this sufficient condition is assumed:

Assumption 1. $\Theta \geq -1$

In the following lemma the equilibrium properties of the Cournot-Nash equilibrium are given.

Lemma 1. *The Cournot-Nash industry output, Q^*, is the solution of:*

$$n\left(P\left(Q\right) - \overline{c}\right) = -P'\left(Q\right)Q,$$

the Cournot-Nash production of firm j and the market share of firm j are:

$$q_j^* = \frac{Q^*}{n} + \frac{\overline{c} - c_j}{-P'\left(Q^*\right)}, \ \ and \ \ s_j^* = \frac{q_j^*}{Q^*} = \frac{1}{n} + \frac{\overline{c} - c_j}{n\left(P\left(Q^*\right) - \overline{c}\right)},$$

the Cournot-Nash profit of competitor j is: $\Pi_j^* = (-P'(Q^*)) \left(q_j^*\right)^2$*, whenever* $P(Q^*) \geq c_j$ *and zero otherwise. Finally when all firms produce in equilibrium, the expression of welfare is:*

$$W^* = \int_0^{Q^*} P(u)\,du - \sum_{j=1}^{n} c_j q_j^* = \int_0^{Q^*} \left[P(u) - \overline{c}\right] du + \frac{n}{-P'(Q^*)} v. \qquad (1)$$

Proof. See appendix A. □

It is worth noting that the equilibrium welfare depends on both the mean and the variance of the marginal costs, while the equilibrium quantity, Q^*, and therefore the consumers' surplus depends only on the mean. That is, results based only on the variation of the price can be misleading in terms of welfare. In equilibrium, firms more efficient than the average firm ($c_j < \overline{c}$) have the largest market shares ($s_j^* > \frac{1}{n}$). Moreover, in equilibrium, the variance of the market share distribution is proportional to the variance of the marginal cost distribution. That means that if the average marginal cost $\overline{c}$ does not change, an increase of the Herfindahl index increases welfare. In a different context, Farrell and Shapiro (1990) showed a similar relationship between the Herfindahl index and welfare.

The consequences on the marginal cost distribution of a merger between a firm i of the downstream market and a producer of the intermediate good are twofold. On the one hand, the marginal cost of firm i may decrease from c_i to $c_i - \varepsilon$, with $\varepsilon \geq 0$, and on the other hand, the marginal costs of the remaining producers of the final good may increase or decrease, for all $j \neq i$, from c_j to $c_j + w$. The value of w depends on the structure of the upstream market after the removal of one producer and of the new partition of market power on the final good market after the changes in the marginal cost distribution on the final good market. The value of ε is the difference between the market price of the intermediary good before the merger and the marginal cost of production of the upstream firm that is integrated. The new market structure is represented in figure 2. Throughout the paper it is assumed that all firms remain active after the merger. That is, w is not too high to induce a firm to exit. It is worth noting that exit by some

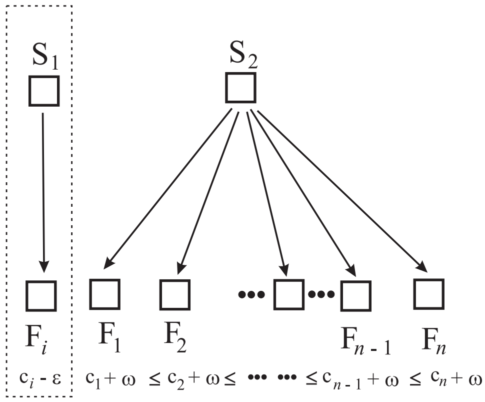

Figure 2: Market structure after the merger

non-merging firms could have either a positive or a negative welfare effect. However, allowing for exit would not qualitatively change the results. In particular the welfare function is continuous in w even when exit is taken into account.

Let $c' = (c_1 + w, \ldots, c_i - \varepsilon, \ldots, c_n + w)$ denote the list of the marginal costs after the merger.[11]

By definition, foreclosure means that $w \geq 0$. Salinger (1988) showed that if competition at the upstream level is *à la* Cournot, w can be either positive or negative. This result is still true with asymmetric downstream firms. In the present case, in order to make foreclosure a real issue, it is assumed that competition upstream is *à la* Bertrand before the merger. After the merger however, it is not clear that the integrated firm withdraws from the intermediate good market. Ordover, Saloner, and Salop (1990) state that it is rational but Hart and Tirole (1990) and Reiffen (1992) challenged this view

[11] The post-merger average and variance of the marginal cost distribution are

$$\overline{c'} = \overline{c} + \frac{n-1}{n} w - \frac{\varepsilon}{n},$$

and

$$v' = v + \frac{n-1}{n^2} (w + \varepsilon)^2 + \frac{2}{n} (w + \varepsilon) (\overline{c} - c_i) .$$

and Ordover, Saloner, and Salop (1992) provide the necessary clarifications. Here, as in Ordover, Saloner and Salop, it is assumed that the remaining input producer is either a monopoly or a constrained monopoly and that the integrated firm stops selling the intermediate good.[12] However, the results do obtain as long as the upstream price is increasing when the number of firms decreases, a property that occurs in many oligopoly models.

With Bertrand competition upstream, the input price is (ex ante) equal to the marginal cost of production of the intermediate good. Therefore, integration does not affect the input price for the merging firm, which means that $\varepsilon = 0$.

Assumption 2. *$\varepsilon = 0$, and $w > 0$.*

With $w > 0$ and $\varepsilon = 0$, the welfare formula (1) shows that two opposing forces are at work: the average marginal cost is increased which, in turn, reduces the quantity produced and therefore welfare but the variance of the marginal cost distribution is increased which improves welfare. Let $Q^*(w)$ denote the new equilibrium quantity. It is the solution of

$$P'(Q)\,Q + nP(Q) = n\overline{c} + (n-1)\,w.$$

Lemma 2. *After any vertical integration (with $\varepsilon = 0$ and $w > 0$) the market price of the final good increases and the surplus of the consumers decreases, while the profit of the integrated firm always increases.*

[12]To quote Ordover, Saloner, and Salop (1992) "The notion that vertically integrated firms behave differently from unintegrated ones in supplying inputs to downstream rivals would strike a businessperson, if not an economist, as common sense." Their idea is to model the competition between the integrated firm and the independent input supplier as a first price auction. As in a Dutch auction, a clock starts at a high price, say w, and shows a price declining with time until it reaches zero. At any time a firm can drop out. The firm who does not drop out wins the market at the current price (therefore the auction is more a declining English auction than a Dutch auction). As it is a weakly dominated strategy for the independent supplier to stay as long as the price is higher than zero. The unique Nash Equilibrium not involving weakly dominated strategy is for the integrated firm to drop out instantaneously.

Proof. After a vertical integration, the average marginal cost is $\overline{c} + \frac{n-1}{n} w$ which is strictly greater than $\overline{c}$ as $w > 0$. Now, the market price of the final good is strictly increasing with the average marginal cost and the consumers' surplus is strictly decreasing with the average marginal cost. To show formally that the profit of the integrated firm is increasing, first note that as the price increases, then $P^* - c_i$ increases. From the first order condition: $P + P' q_i^* = c_i$ it comes that $\frac{\partial q_i^*}{\partial w} = \left(-\frac{\partial Q^*}{\partial w}\right)(1 + \Theta s_i^*)$. Then as assumption 1 insures that $\Theta \geq -1$, it follows that $\frac{\partial q_i^*}{\partial w} > 0$. Therefore, the profit $(P^* - c_i)\, q_i^*$ increases. □

Consumers are worse off after such a vertical integration because it raises the market price of the final good. However, it cannot be concluded that the effect on welfare is negative because the profits of the firms may increase.

Let $W^*(w)$ denote the social welfare after a vertical integration. The expression of $W^*(w)$ can be derived from equation 1 of lemma 1 acknowledging that after the merger the marginal cost distribution is: $(c_1 + w, \ldots, c_{i-1} + w, c_i, c_{i+1} + w, \ldots, c_n + w)$ and noting that the input supplier makes a profit $w \sum_{j \neq i} q_j^*(w)$.[13]

$$W^*(w) = \int_0^{Q^*(w)} P(u)\, du - \sum_{j=1}^{n} c_j q_j^*(w).$$

The difference of welfare before vertical integration minus welfare after integration presents two parts: $\Delta W^*(w) = -\int_{Q^*(w)}^{Q^*(0)} (P(u))\, du - \sum_{j=1}^{n} c_j \Delta q_j^*$. That is the difference in consumers' surplus and the difference in production costs. However, this difference can be usefully rewritten in order to exhibit three effects.

[13] Of course, the monetary transfer from the non integrated producers to the remaining input producer does not (directly) affect the welfare. From the welfare point of view it is as if non-integrating firms still have the same marginal cost c_j as before the merger but they produce a different quantity $q_j^*(w)$ instead of $q_j^*(0)$.

Lemma 3. *The difference of welfare can be written as:*

$$\begin{aligned}\Delta W^*(w) = & -\int_{Q^*(w)}^{Q^*(0)} (P(u) - \overline{c})\, du \\ & + nv\left(\frac{1}{-P'(Q^*(w))} - \frac{1}{-P'(Q^*(0))}\right) \\ & + \frac{w}{-P'(Q^*(w))}(\overline{c} - c_i).\end{aligned}$$

Proof. See appendix B. □

That is, the effects of the integration on the welfare is composed of three effects. First an "average welfare effect", $-\int_{Q^*(w)}^{Q^*(0)} (P(u) - \overline{c})\, du$, which is the welfare loss due to the reduction of the quantity produced that would occur if all the firms would have the same marginal cost. Next a "scale selection effect", $nv\left(\frac{1}{-P'(Q^*(w))} - \frac{1}{-P'(Q^*(0))}\right)$. Indeed, under Cournot competition with heterogeneous firms there is competitive selection in the sense that the overall output is produced to a larger proportion by more efficient firms. When the demand function is nonlinear, an increase by w of the marginal cost of all firms decreases the total quantity but this reduction is not proportional for all firms. The way the production is distributed among firms is affected. The scale selection effect is positive if the inverse demand function is strictly concave (the output reduction helps the efficient firms to produce relatively more). On the contrary, it is negative if the inverse demand function is strictly convex. These first two effects are independent of the identity of the firm that vertically integrates. The average welfare effect does not even depend on the distribution of the marginal cost but only on the average marginal cost and on w, while the scale selection effect varies with both the mean and the variance of the marginal cost distribution and with w. Finally there is an "efficiency effect", $\frac{w}{-P'(Q^*(w))}(\overline{c} - c_i) = wQ^*(0)\left(s_i^* - \frac{1}{n}\right)\frac{-P'(Q^*(0))}{-P'(Q^*(w))}$, which is positive when the marginal cost of the integrating firm is lower than the average marginal cost. Of course, when all the marginal costs are equal, $v = 0$, and $\overline{c} = c_i$, the last two effects are null and a vertical integration always reduces welfare.

The tradeoff between consumers' surplus and production efficiency is now studied in detail. In section 3, it is done in the general case but only for a marginal marginal cost increase. In section 4 the tradeoff is studied in the special case of linear demand functions.

3 Marginal marginal cost distortion

The idea of this section is to determine the sign of $\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0}$ in order to assess if a marginal increase of the marginal costs of the non-merging firms is beneficial or not to welfare. Let θ denote the elasticity of the slope of the inverse demand function evaluated at the ex ante total quantity, that is: $\theta = \Theta\left(Q^*\left(0\right)\right)$.

Proposition 1. *For $n = 2$, if the ex ante market share of the merging firm is larger than a threshold value strictly larger than $1/2$ and lower than 1 a vertical integration that marginally increases rival costs is socially beneficial.*

For $n > 2$, there exists two threshold values $\widetilde{s}_0$ and $\widetilde{s}_1$ such that:

- *If the ex ante market share of the merging firm, s_i^*, is lower than $\widetilde{s}_0$ a vertical integration that marginally increases rival costs reduces welfare.*
- *If s_i^* is between $\widetilde{s}_0$ and $\widetilde{s}_1$, the merger reduces or improves welfare depending on the variance of the ex ante market shares and on the convexity of the demand function around the equilibrium.*
- *Finally, if s_i^* is larger than $\widetilde{s}_1$, welfare is improved regardless of the variance of the ex ante market shares.*

For any $n > 2$: if $\theta > 0$, $0 \leq \widetilde{s}_0 < \widetilde{s}_1$ and $\frac{1}{n} < \widetilde{s}_1 < \frac{2}{n+1}$ while if $\theta < 0$ then $\frac{2}{n+1} < \widetilde{s}_0 < \widetilde{s}_1 < 1$.

Proof. See appendix C. □

Proposition 1 provides sufficient conditions for a vertical integration to improve welfare. First, when s_i^* is larger than $\widetilde{s}_1$, the efficiency effect is so important that it leads to a welfare gain. This happens when the firm that buys an input supplier is "dominant" and despite the reduction of the consumers' surplus. The merging firm is sufficiently more efficient than the average firm and the merger induces a beneficial reallocation of the (reduced) production.

Next, depending on the sign of the scale selection effect around the equilibrium, and on the magnitude of the variance of the marginal cost distribution, the market share of the merging firm can be more or less large to induce a welfare gain. More precisely, when θ is strictly positive, the scale selection effect is positive which means that a marginal reduction of the production improves welfare because it makes the most efficient firms produce more. The larger this effect (which increases with the variance of the marginal cost distribution), the lower the efficiency effect has to be to compensate the average welfare effect. When θ is strictly positive, a slightly surprising result is obtained. It is not a necessary condition that the merging firm is more efficient than the average firm for welfare to improve. Indeed, if the variance of the ex ante market share distribution is large enough the scale selection effect can be large enough to compensate for both a negative average welfare effect and a negative efficiency effect.[14] That means that while a merger of a large firm improves welfare, it would be wrong to think that a merger of a small firm reduces welfare. If there exists a large enough non merging firm, the merger of a small firm can improve welfare because the production is reallocated from the inefficient non merging firms to some of the efficient non merging firms.

Finally, when the scale selection effect is negative ($\theta < 0$) only a large efficiency effect can induce a welfare gain. This is why $\widetilde{s}_0$ and $\widetilde{s}_1$ are larger when $\theta < 0$.

[14]It is easy to show that $\left.\frac{\partial q_j^*(w)}{\partial w}\right|_{w=0} > 0$ if and only if θ is strictly positive and $\frac{2+\theta}{\theta}\frac{1}{n-1} < s_j^*$. That is, if the ex ante market share of a non merging firm is large enough its production increases after the merger.

4 Linear demand

In order to solve the model in closed form, it is assumed in this section that the demand function is linear. This assumption leads to a tradeoff between a negative average welfare effect and a possibly positive efficiency effect, while the scale selection effect is null. Let $P(Q) = a - bQ$ where a and b are strictly positive.

The following lemma states the equilibrium properties of the Cournot-Nash equilibrium when n firms compete with different marginal costs with a linear demand function.

Lemma 4. *The Cournot-Nash industry output and the Cournot-Nash price of the final good are:*

$$Q^* = \frac{n(a-\overline{c})}{b(n+1)}, \ P^* = \frac{a+n\overline{c}}{n+1},$$

the Cournot-Nash production of firm j and the market share of firm j are:

$$q_j^* = \frac{a-\overline{c}}{b(n+1)} + \frac{\overline{c}-c_j}{b}, \text{ and } s_j^* = \frac{q_j^*}{Q^*} = \frac{1}{n} + \frac{n+1}{n}\frac{\overline{c}-c_j}{a-\overline{c}},$$

the Cournot-Nash profit of competitor j is: $\Pi_j^* = b\left(q_j^*\right)^2$, finally the Cournot-Nash welfare is decreasing with the mean of the marginal cost distribution and is increasing with its variance:

$$W^* = \frac{n(n+2)}{2b}\left(\frac{a-\overline{c}}{n+1}\right)^2 + \frac{n}{b}v. \tag{2}$$

Proof. Apply lemma 1 with $P(Q) = a - bQ$. □

4.1 Ex post constrained monopoly upstream

In this section, it is supposed that the remaining input supplier is not free to rise its price to the monopoly level after the withdrawal of its competitor. This can happen for several reasons,[15] the simplest being to assume that an

[15] See Ordover, Saloner, and Salop (1990) and Ordover, Saloner, and Salop (1992).

alternative (but slightly less efficient) input is available at a fixed price. Let w denote the post merger input price. As it has been assumed that throughout the paper all firms remain active after the merger, w must not be too large.[16]

Proposition 2. *Vertical integration is socially beneficial if and only if the ex ante market share of the final good firm which merges is larger than*

$$\widetilde{s} + \beta w$$

where

$$\widetilde{s} = \frac{2}{n+1}, \text{ and } \beta = \frac{(n-1)^2}{2n(n+1)(a-\overline{c})}.$$

Proof. See appendix D □

The intuition behind proposition 2 is fairly simple: when a firm is more efficient than its competitors, there are efficiency gains if it produces more and its competitors produce less. Therefore on the one hand, the monopolization of the input market is beneficial because it leads to a rise of the marginal cost of the inefficient firms which in turn leads them to produce less. Unfortunately, this way to improve the efficiency of the production is harmful for the consumers. When the ex ante market share of the integrating firm is relatively small (meaning that the firm is not very efficient compared to the average firm), then the surplus loss is larger than the efficiency gain and vertical integration is socially costly. However, when the marginal cost gap between the merging firm and the other firms is large, then its ex ante market is large and the vertical integration is socially beneficial despite the increase of the price on both the input and final good market. Note also that if s_i^* is lower than $\widetilde{s}$, then w should be negative to obtain a positive variation of welfare. Note that for a given marginal cost avantage of the integrating firm : $(\overline{c} - c_i)$ and for a given number of firms, if the size of the market a increases, then s_i^* goes to $1/n < \widetilde{s}$. That is, it could be misleading to look only at the magnitude of the difference between the marginal cost of

[16]More formally, it is assumed here that $w \leq w^{\max}$, with $w^{\max} = \frac{n+1}{2}(P^* - c_n)$ (where P^* is the ex-merger market price). The value of $w^{\max}$ is easily derived from the expression $s_n^*(w) \geq 0$.

the integrating and the average marginal cost. One has also to look at this difference relatively to the size of the market.

4.2 Ex post monopoly upstream

When the remaining input producer can behave like a monopoly, the marginal costs of the unintegrated firms are increased by $w^m > 0$. The monopoly price on the input market is the solution of $\max_w wQ^*_{-i}(w)$, where $Q^*_{-i}(w) = \sum_{j \neq i} q^*_j(w)$ is the sum of the Cournot-Nash demands of input by the non-integrated firms. Two interesting features are at stake here. First, does w^m induce a large enough efficiency effect? Second, as w^m is a declining function of s^*_i, is it always true that the integration of the largest firm[17] is the most beneficial from a welfare point of view?

Proposition 3. *When the input producer sets a monopoly price, vertical integration is socially beneficial if and only if the ex ante market share of the final good firm which merges is larger than a threshold value* $\underline{s}$ *where*

$$\underline{s} = \frac{n+15}{9n+7}.$$

Proof. See appendix E. □

That is, the merger is socially beneficial only if the market share of the merging firm is large enough. Again, to increase welfare, the integrating firm has to be more efficient than the average firm on the final good market. If the linearity assumptions give a good approximation of a particular market, then from a practical point of view this result is striking as it only relies on the market share s^*_i and on the number of firms, but not on the values of the marginal costs or the slope and intercept of the demand function. So very little information is needed to determine if a merger benefits or harms society.

[17] The larger the market share of the integrating firm, the lower the increase of rivals' costs.

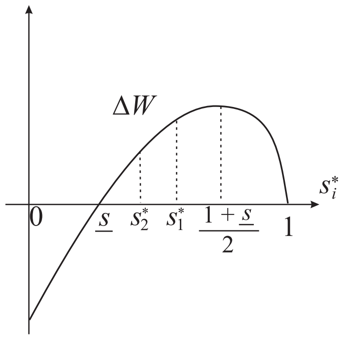

Figure 3: Variation of welfare with s_i^*

Appendix E shows that the welfare variation $\Delta W = K\left(1 - s_i^*\right)\left(s_i^* - \underline{s}\right)$.[18] As long as s_i^* is larger than $\underline{s}$, ΔW is positive but its amplitude is larger or lower depending on s_i^* as shown in figure 3. Of course, the variation of welfare is null if $s_i^* = \underline{s}$ and if $s_i^* = 1$. It is at its maximum for $s_i^* = \frac{1+\underline{s}}{2} = \frac{5n+11}{9n+7}$. Moreover, the variation of welfare is always larger when the merging firm is the most efficient one. First, if s_1^* is lower than $\frac{1+\underline{s}}{2}$, then the variation of welfare is the largest when firm 1 vertically integrates (like in figure 3).[19] Next, if s_1^* is larger than $\frac{1+\underline{s}}{2}$, then[20] as s_2^* must be lower than $1 - s_1^*$ and it is easy to show, as in figure 4, that $\Delta W\left(s_2^*\right) \leq \Delta W\left(s_1^*\right)$.[21]

4.3 The optimal input price increase

The final step is, for a given ex ante market share, s_i^*, of the merging firm, to find the value of w such that the welfare gain is at its maximum.

[18] Where $K = \frac{(9n+7)n^2(a-\overline{c})^2}{32b(n-1)(n+1)^2}$.

[19] In figures 3 and 4, as s_i^* increases along the horizontal axes, the underlying change in model parameters is a mean-preserving change in the distribution of marginal costs that involves a higher/lower s_i^*. If not the function ΔW would have a different shape because K would change.

[20] As $\frac{1+\underline{s}}{2}$ is larger than $\frac{1}{2}$, only one firm can have a market share that exceeds this optimal value.

[21] Indeed, $\Delta W\left(s_2^*\right) \leq \Delta W\left(1 - s_1^*\right)$ and $\Delta W\left(1 - s_1^*\right) \leq \Delta W\left(s_1^*\right)$ if and only if $s_1^* \geq 1/2$ which is true as $s_1^* \geq \underline{s} \geq 1/2$.

Proposition 4. *If firm i is the merging firm, the welfare gain is at its maximum if $w = w_i^*$ such that*

$$w_i^* = \frac{s_i^* - \widetilde{s}}{2\beta}.$$

Moreover,

$$w^m - w_i^* = \frac{n\,(5n+3)\,(a-\overline{c})}{4\,(n-1)^2}\,(\widehat{s} - s_i^*)\,, \;\; \textit{where } \widehat{s} = \frac{n+7}{5n+3}.$$

Proof. Straightforward calculations from equation 3 of appendix D and from the definition of w^m in appendix E. □

That is, for the small market share firms, $0 \leq s_i^* \leq \widetilde{s}$, the welfare gain is negative except if w is negative: as the firm is not sufficiently efficient, it is better to reduce the marginal costs of all its rivals rather than raising them. For an intermediate market share firm that vertically integrates, $\widetilde{s} < s_i^* < \widehat{s}$, welfare (for some value of w) can be improved and this gain is at its maximum for w_i^* strictly positive but not larger than w^m. Finally, for a large market share firm, $\widehat{s} \leq s_i^*$, the value of w that maximizes the welfare gain, w_i^*, is greater than the monopoly price w^m.[22] Indeed, when s_i^* is large, w^m is small and the marginal cost of the less efficient firms is not raised by

[22]In this last case, w could be so large that the least efficient firm, n, is driven out of the market. However if the ex ante market share s_n^* is larger than $\frac{2}{5n+3}$ it never happens. In particular, assume that $s_2^* = s_3^* = \ldots = s_n^* = \frac{2}{5n+3}$, then $s_1^* = \frac{3n+5}{5n+3} > \frac{n+7}{5n+3}$.

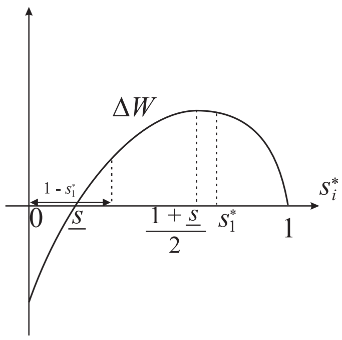

Figure 4: Variation of welfare when s_1^* is larger than $\frac{1+s}{2}$.

much, which leads to an insufficient reallocation of the production. Figure 5 shows the welfare difference $\Delta W = Rw\left(s_i^* - \widetilde{s} - \beta w\right)$ as a function of w (see appendix D) for different integrating firms. Note that as the market share s_n^* of the less efficient firm is always lower or equal to $1/n$, then $s_n^* - \widetilde{s}$ is always negative. On the other hand, $s_1^* - \widetilde{s}$ is not always positive. Indeed if firms are very similar, all market share are close to $1/n < \widetilde{s}$.

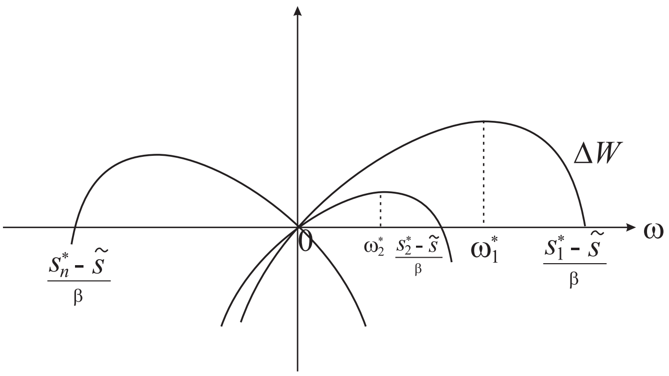

Figure 5: Variation of welfare with w for different values of s_i^*.

5 Price competition

So far, the results rely on the Cournot competition assumption. Do they also hold in a differentiated good market with price competition? The purpose of this section[23] is to show that the results obtained for Cournot competition do extend to price competition with differentiated products. In a differentiated good market, a firm can have the highest market share thanks to a lower cost of production or simply because it has a better product. This last effect is absent form a Cournot model with a homogeneous product. Therefore in this section, first the tradeoff between productive efficiency and loss in consumers' surplus is studied in presence of price competition. Next, it is shown that "product quality" can play the same role as "productive efficiency".

[23]This section has been added thanks to the suggestions of the co-editor and of an anonymous referee.

Unfortunately, industrial organization does not benefit from the existence of a price competition model as tractable as the Cournot one. For this reason, only two illustrative examples are studied in this section. First, the standard duopoly model with a representative consumer who has a quadratic utility function, second, the Hotelling linear city model.

5.1 Product differentiation and linear demand

Two competing downstream firms (1 and 2) sell differentiated products at prices p_1 and p_2. It is assumed that the demand side of the market is described by a consumer whose utility is:

$$U(q_1, q_2) = a\,(q_1 + q_2) - \sigma\, q_1\, q_2 - \frac{1}{2}\left({q_1}^2 + {q_2}^2\right) + R - p_1\, q_1 - p_2\, q_2 \ ,$$

where R is the revenue of the consumer, q_i the consumption level of good i ($i = 1, 2$), a is the "size of demand" and $\sigma \in\ ]-1, 1[$ measures the degree of differentiation between the products. The monopoly case is $\sigma = 0$, if $\sigma \to 1$ the products become homogeneous, while if $\sigma < 0$ the products are complements instead of substitutes. The demand function of firm i ($i = 1, 2$) is then given by

$$q_i = \frac{a}{1+\sigma} + \frac{1}{1-\sigma^2}\left(-p_i + \sigma p_j\right).$$

For simplicity, it is assumed that firms have constant marginal costs and the lowest marginal cost is normalized to zero: $c_1 = 0 < c_2 = c$. Exactly as in the Cournot model, if firm 1 vertically integrates, its own marginal cost remains unchanged, while its competitor's marginal cost increases: $c_1 = 0$ and $c_2 = c + \omega$.

It is readily confirmed that the Nash equilibrium prices are:

$$p_1^* = \frac{a\,(2-\sigma-\sigma^2) + \sigma c + \sigma\omega}{4-\sigma^2} \text{ and } p_2^* = \frac{a\,(2-\sigma-\sigma^2) + 2c + 2\omega}{4-\sigma^2}$$

From which it is clear that both prices increase with ω and therefore the consumer is worse off. However, the quantity produced by firm 1 (resp. 2) increases (resp. decreases) which is an efficient reallocation of the production. That is, in this example, price competition in a differentiated product environment leads to the same tradeoff as in the Cournot model.

Lemma 5. *If c is large enough, backward integration by the dominant firm increases welfare.*

Proof. See appendix F. □

Of course, the larger c, the larger the market share of firm 1. Therefore, as in the Cournot model, vertical backward integration reduces welfare if the integrating firm has a small market share but increases welfare when its market share is large enough.

5.2 Hotelling line

In the representative consumer model of section 5.1, the consumer does not prefer *a priori* one good over the other. The Hotelling model allows for this kind of distinction. Let us consider quickly a standard Hotelling linear city model in which one firm is located in the middle and the other is at one end of the city. That is, firm 1 is located at $x_1 = 1/2$ and firm 2 is located at $x_2 = 1$. Locations are fixed. The utility of a consumer located at $x \in [0, 1]$ who patronizes firm i ($i = 1, 2$) is assumed to be $v - p_i - t\,(x_i - x)^2$. The parameter v is large enough so in equilibrium all consumers buy. Firms simultaneously choose their prices and consumers buy where they enjoy the highest utility. It is assumed that firms have constant returns to scale and no fixed costs. That is, before any vertical integration the cost structure is $c_1 = c_2 = c$ and after vertical integration by firm 1, it is $c_1 = c$ and $c_2 = c+\omega$. In equilibrium, it is easy to show that the indifferent consumer is located at:

$$\widetilde{x} = 7/12 + \omega/3t.$$

A consumer located between 0 and $\widetilde{x}$ patronizes firm 1, while the others buy at firm 2. From a welfare point of view, welfare is maximum when the transportation costs

$$t\int_0^{\widetilde{x}} (1/2 - x)^2\, dx + t\int_{\widetilde{x}}^1 (1 - x)^2\, dx$$

are minimized that is when $\widetilde{x} = 3/4$ (exactly at the middle of firms' locations). Therefore, welfare increases with ω between 0 and $t/2$. Of course, there is no welfare loss in this model because all consumers have an inelastic demand of one unit and it has been assumed that they all buy. A trivial way to extend this idea to models of vertical differentiation is to assume that firms have the same marginal cost of production initially.[24] The high quality firm has the largest market share, but not large enough form the welfare point of view (it should sell to all consumers).

6 Conclusion

The next logical step would be to specify a particular oligopolistic competition model with more than two firms in the upstream market so that vertical integration does not lead to a monopoly situation. Moreover, in the spirit of Ordover, Saloner and Salop, a bidding game could be introduced to establish explicitly which firms merge as well as the incentives to merge after a first merger. However, it could introduce some confusion in the analysis. In particular with Cournot competition at both levels, Salinger shows that vertical integration could lead to a decrease or an increase of the market price. That is, foreclosure is not certain to happen in this context. The integrating firm benefits from a lower marginal cost and therefore increases its production which may lower the final good price even if the marginal cost of its competitors is increased. Finally the profit of the merging unit may decrease so the incentives to merge are ambivalent.

This paper suggests that in the presence of a strong link between market share and cost efficiency, antitrust authorities should be more lenient towards a large market share firm than to a small one. However, the paper does not suggest that competition authorities should involve themselves in merger cases in which there is little concentration. This for two reasons: first, when the demand function is strictly concave, vertical integration by an inefficient firm can also create a beneficial reallocation of the production when a non merging firm has a large market share. Second, if one considers the uncertainties involved in any merger analysis and the issues not modeled

[24] See Tirole (1988), pages 296-97, for example.

in the paper, the welfare effects of a merger by a small firm are certainly difficult to assess.

The paper proves that in some circumstances, a price increase and a market power increase (through a higher Herfindahl index) are not contradictory with a welfare gain. However if the link between market share and cost efficiency is weak (for example due to consumers' switching costs, an old inefficient firm could enjoy a large market share), the results of this paper should be amended.[25]

More generally, as pointed out by a referee, market prices in asymmetric models are such that given the output level, customers are inefficiently distributed across firms. Any cause that leads to a more efficient redistribution of the customers, will (all else equal) improve welfare (i.e. distribute consumers from the high cost to the low cost firm, or from the less to the more preferred differentiated product). Unfortunately, the redistribution can be costly (i.e. induce a higher price for the consumers). The vertical integration studied in this paper is one way to get such a redistribution but other ways can be imagined. An open question is to find a more general statement of when such redistributions can outweigh the deadweight loss incurred due to the redistribution process itself.

[25]Another example is the following: assume that $c_i = \overline{c} + \varepsilon_i(t)$ with $\varepsilon_i(t)$ a random variable centered around zero. That is at each period of time, the marginal costs are randomly increased or decreased. In such a context, efficiency today does not mean efficiency tomorrow, and a vertical integration creates an efficient reallocation of the current production but an inefficient reallocation of the future production.

Appendix

A Cournot-Nash equilibrium with asymmetric firms

The profit function of firm j can be written $\Pi_j = (P(Q) - c_j) q_j$. Therefore the best reply function of firm j, BR_j is the solution of:

$$P'(Q) q_j + P(Q) = c_j.$$

Then the usual trick is to sum over all j these first order conditions which leads to:

$$P'(Q) Q + nP(Q) = n\overline{c},$$

which defines the total quantity Q^* produced at the Cournot equilibrium. Next, the quantity produced by firm j is:

$$q_j^* = \frac{\overline{c} - c_j}{-P'(Q^*)} + \frac{Q^*}{n},$$

and the market share of firm j is:

$$s_j^* = \frac{1}{n} + \frac{\overline{c} - c_j}{n(P(Q^*) - \overline{c})}.$$

By definition the marshallian equilibrium welfare is:

$$W^* = \int_0^{Q^*} P(u)\, du - \sum_{j=1}^{n} c_j q_j^*,$$

substituting q_j^* by $\frac{\overline{c}-c_j}{-P'(Q^*)} + \frac{Q^*}{n}$ it follows that

$$W^* = \int_0^{Q^*} [P(u) - \overline{c}]\, du + \frac{n}{-P'(Q^*)} v.$$

B Difference of welfare

The difference of welfare is

$$\Delta W = W(w) - W(0) = -\int_{Q^*(w)}^{Q^*(0)} P(u)\,du - \sum_{j=1}^{n} c_j \left(q_j^*(w) - q_j^*(0)\right).$$

From the equilibrium formulae, it follows that:

For $j \neq i$,

$$\begin{aligned} q_j^*(w) - q_j^*(0) = \quad & \tfrac{Q^*(w)-Q^*(0)}{n} + (\overline{c} - c_j)\left[\tfrac{1}{-P'(Q^*(w))} - \tfrac{1}{-P'(Q^*(0))}\right] \\ & - \tfrac{w}{-nP'(Q^*(w))} \end{aligned}$$

and for i

$$\begin{aligned} q_i^*(w) - q_i^*(0) = \quad & \tfrac{Q^*(w)-Q^*(0)}{n} + (\overline{c} - c_i)\left[\tfrac{1}{-P'(Q^*(w))} - \tfrac{1}{-P'(Q^*(0))}\right] \\ & - \tfrac{w}{-nP'(Q^*(w))} + \tfrac{w}{-P'(Q^*(w))}. \end{aligned}$$

Using the fact that $\sum_{j=1}^{n} c_j (\overline{c} - c_j) = -nv$, it follows that:

$$\begin{aligned} \Delta W(w) = \quad & -\int_{Q^*(w)}^{Q^*(0)} (P(u) - \overline{c})\,du \\ & + nv\left(\tfrac{1}{-P'(Q^*(w))} - \tfrac{1}{-P'(Q^*(0))}\right) \\ & + \tfrac{w}{-P'(Q^*(w))}(\overline{c} - c_i). \end{aligned}$$

C Marginal marginal cost distortion

The objective is to determine the sign of

$$\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0},$$

Of course,

$$W^*(w) = \int_0^{Q^*(w)} P(u)\,du - \sum_{j=1}^{n} c_j q_j^*(w).$$

However, it is more convenient to use the expression of welfare from equation 1 of lemma 1 (substituting $\overline{c}$ and v by the after merger average marginal cost and variance, and adding the profit of the input supplier) because the average cost and the variance of the marginal cost distribution are more apparent. Let $Q^*_{-i}(w)$ be equal to $\sum_{j\neq i} q_j^*(w)$. The starting point is then $W^*(w) =$

$$\int_0^{Q^*(w)} \left[P(u) - \overline{c} - \frac{n-1}{n}w\right] du + \frac{n}{-P'(Q^*(w))}\left(v + \frac{n-1}{n^2}w^2 + \frac{2w}{n}(\overline{c} - c_i)\right) + wQ^*_{-i}(w),$$

it follows that after some simplifications:

$$\begin{aligned}\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0} &= \left.\frac{\partial Q^*(w)}{\partial w}\right|_{w=0}\left[P(Q^*(0)) - \overline{c}\right] - \frac{n-1}{n}Q^*(0) \\ &+ \left.\frac{\partial Q^*(w)}{\partial w}\right|_{w=0}\left(\frac{P''(Q^*(0))}{(P'(Q^*(0)))^2}\right)nv \\ &+ \frac{2}{-P'(Q^*(0))}\left((\overline{c} - c_i)\right) + Q^*_{-i}(0).\end{aligned}$$

The introduction of $\theta = \frac{P''(Q^*(0))Q^*(0)}{P'(Q^*(0))}$, the elasticity of the slope of the inverse demand function (evaluated at $Q^*(0)$), of s_i^* the ex ante market share of the merging firm and of the variance of the ex ante market share distribution $v(s) = \frac{v}{n^2(P(Q^*(0))-\overline{c})^2}$ allows some additional simplifications:

From

$$P'(Q)Q + nP(Q) = n\overline{c} + (n-1)w,$$

it follows that

$$\left.\frac{\partial Q^*(w)}{\partial w}\right|_{w=0} = \frac{n-1}{P'(Q^*(0))}\frac{1}{n+1+\theta}.$$

Then, it is easy to show that

$$\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0} = Q^*(0)\left(s_i^* - \widetilde{\widetilde{s}}\right),$$

where

$$\widetilde{\widetilde{s}} = \frac{1}{n}\left[1 + \frac{n-1}{n+1+\theta}\left(1 - \theta n^2 v(s)\right)\right].$$

Therefore, the sign of $s_i^* - \widetilde{\widetilde{s}}$ has to be determined. However, for a given s_i^*, $\widetilde{\widetilde{s}}$ can take several values depending on the variance of the market shares distribution (that is the variance of the marginal costs distribution). For a given s_i^*, $v(s)$ has a minimal and a maximal value. These lower and upper bounds are now introduced:

For a given s_i^*, let $\underline{v_i}$ denote the minimal value of $v(s)$:

$$\underline{v_i} = \frac{\left(\frac{1}{n} - s_i^*\right)^2}{n-1},$$

and let $\overline{v_i}$ denote the maximal value of $v(s)$:

$$\overline{v_i} = \frac{n-1}{n^2} - \frac{2s_i^*(1-s_i^*)}{n}.$$

Therefore, if $\theta > 0$, it comes that

$$\delta_i \leq s_i^* - \widetilde{\widetilde{s}} \leq \Delta_i,$$

while if $\theta < 0$

$$\Delta_i \leq s_i^* - \widetilde{\widetilde{s}} \leq \delta_i,$$

where

$$\delta_i = s_i^* - \frac{1}{n} - \frac{n-1}{n(n+1+\theta)} + \frac{(n-1)\,n\theta\underline{v_i}}{n+1+\theta},$$

and

$$\Delta_i = s_i^* - \frac{1}{n} - \frac{n-1}{n(n+1+\theta)} + \frac{(n-1)\,n\theta\overline{v_i}}{n+1+\theta}.$$

The expressions of δ_i and Δ_i can finally be rewritten as follows

$$\delta_i = \frac{\theta n (s_i^*)^2 + (n+1+\theta)\,s_i^* - 2}{n+1+\theta},$$

and

$$\Delta_i = \delta_i + \frac{\theta}{n+1+\theta}(n-2)(1-s_i^*)^2,$$

from which it is apparent that both δ_i and Δ_i are second degree polynomial expressions of s_i^*. Moreover, $\delta_i = \Delta_i$ when $n = 2$. Basically, the remaining of the proof is to show that figure 6 is correct.

First note that if θ is positive they are convex, while concave if θ is negative. For any $n \geq 2$ and any $\theta \geq -1$ (assumption 1) it is easy to show that:

$$\delta_i(0) < 0 \quad \text{and } \delta_i(1) = \Delta_i(1) > 0.$$

Therefore, there exists a unique $\sigma \in [0,1]$ such that $\delta_i(\sigma) = 0$. Moreover, let Σ be the largest solution of $\Delta_i(\Sigma) = 0$ that is lower than 1. As

$$\Delta_i(0) < 0 \Leftrightarrow -1 \leq \theta \leq \frac{2}{n-2},$$

the root Σ is in $[0,1]$ only if $-1 \leq \theta \leq \frac{2}{n-2}$. Therefore, let $\widetilde{s}_0$ and $\widetilde{s}_1$ be such that

$$\widetilde{s}_0 = \begin{cases} \sigma & \text{if } \theta < 0 \\ \max\{0;\Sigma\} & \text{if } \theta > 0 \end{cases}$$

and

$$\widetilde{s}_1 = \begin{cases} \Sigma & \text{if } \theta < 0 \\ \sigma & \text{if } \theta > 0 \end{cases}$$

The three following properties,

$$\delta_i\left(\frac{1}{n}\right) = -\frac{n-1}{n(n+1+\theta)} < 0,$$

$$\delta_i\left(\frac{2}{n+1}\right) = \frac{2\theta(n-1)}{(n+1)^2(n+1+\theta)},$$

which sign is the sign of θ, and

$$\Delta_i\left(\frac{2}{n+1}\right) = \frac{\theta}{(n+1)^2(n+1+\theta)}\left[n^3 - 4n^2 + 7n - 4\right],$$

the fact that $[n^3 - 4n^2 + 7n - 4] > 0$ shows that when $\theta > 0$, then $0 \leq \widetilde{s}_0 < \frac{2}{n+1}$ and $\frac{1}{n} < \widetilde{s}_1 < \frac{2}{n+1}$ while if $\theta < 0$ then $\frac{2}{n+1} < \widetilde{s}_0 < \widetilde{s}_1 < 1$.

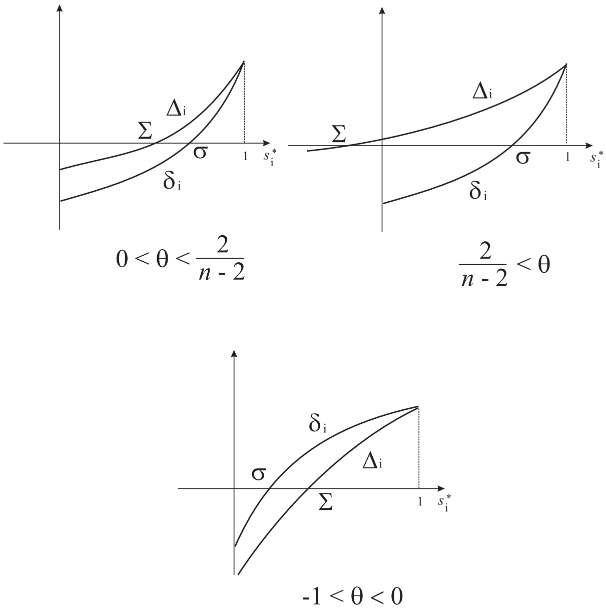

Figure 6: The value of $s_i^* - \widetilde{\widetilde{s}}$ is between δ_i and Δ_i.

D Proof of proposition 2

From lemma 3 it comes that:

$$\Delta W = -\int_{Q^*(w)}^{Q^*(0)} \left(P\left(u\right) - \overline{c}\right) du + wQ^*\left(0\right)\left(s_i^* - \frac{1}{n}\right)$$

using the fact that $Q^*\left(0\right) = \frac{n}{n+1}\frac{a-\overline{c}}{b}$, and that $Q^*\left(w\right) = Q^*\left(0\right) - \frac{n-1}{n+1}\frac{w}{b}$ it is easy to show that

$$\Delta W = \frac{n\left(a-\overline{c}\right)}{b\left(n+1\right)}\left(s_i^* - \frac{2}{n+1} - \frac{\left(n-1\right)^2}{2n\left(n+1\right)\left(a-\overline{c}\right)}w\right)w \tag{3}$$

and therefore,

$$\Delta W > 0 \Leftrightarrow \left(w > 0 \text{ and } s_i^* > \widetilde{s} + \beta w\right),$$

where

$$\widetilde{s} = \frac{2}{1+n}, \text{ and where } \beta = \frac{\left(n-1\right)^2}{2n\left(n+1\right)\left(a-\overline{c}\right)}.$$

E Proof of proposition 3

The first step is to calculate w^m. The upstream demand, given w, is:

$$Q^*_{-i} = \sum_{\substack{j=1\\ j\neq i}}^{n} q_j^*(w) = Q^*(w) - q_i^*(w),$$

from $Q^*(w) = Q^*(0) - \frac{n-1}{n+1}\frac{w}{b}$ and $q_i^*(w) = q_i^*(0) + \frac{n-1}{n+1}\frac{w}{b}$ it follows that

$$Q^*_{-i} = (1 - s_i^*) Q^*(0) - \frac{n-1}{n+1}\frac{2w}{b},$$

as the monopoly profit is wQ_{-i} it follows that

$$w^m = \frac{1}{4}\frac{n}{n-1}(a - \overline{c})(1 - s_i^*).$$

The difference of welfare between after vertical integration and before is given by equation 3 of appendix D where w is replaced by w^m and which simplifies in:

$$\Delta W = \frac{(9n+7)\, n^2 (a - \overline{c})^2}{32b(n-1)(n+1)^2}(1 - s_i^*)\left(s_i^* - \frac{n+15}{9n+7}\right).$$

F Proof of lemma 5

Let $W(w)$ denote the welfare function.[26]

$$W(w) = U^* + p_1^*(w)\, q_1^*(w) + (p_2^*(w) - c - w)\, q_2^*(w) + wq_2^*(w)$$

Tedious (but straightforward) computations lead to the following expression of the equilibrium welfare as a function of w.

$$W(w) = R + \frac{2a(1-\sigma)(3-2\sigma)(2+\sigma)^2\left(a - c - \frac{1-\sigma}{3-2\sigma}w\right)}{2(4-\sigma^2)(1-\sigma^2)} + \frac{(c+w)\left((12 - 9\sigma^2 + 2\sigma^4)c - (4 - 3\sigma^2)w\right)}{2(4-\sigma^2)(1-\sigma^2)}$$

[26] That is the sum of the profits of the firms (including the input supplier) and of the utility of the consumer.

Therefore $\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0} > 0$ if and only if

$$c > \widetilde{c} = a\frac{\left(2-\sigma+\sigma^2\right)^2}{4-3\sigma^2+\sigma^4}$$

Moreover, it is readily confirmed that firm 2 enjoys a positive market share if

$$c < \widetilde{\widetilde{c}} = a\frac{(1-\sigma)(2+\sigma)}{2-\sigma^2}$$

and it is easy to verify that

$$a\frac{(1-\sigma)(2+\sigma)}{2-\sigma^2} > a\frac{\left(2-\sigma+\sigma^2\right)^2}{4-3\sigma^2+\sigma^4}$$

which proves that for c larger than $\widetilde{c}$ and lower than $\widetilde{\widetilde{c}}$, then $\left.\frac{\partial W^*(w)}{\partial w}\right|_{w=0}$.

References

Eric Avenel and Corinne Barlet. Vertical foreclosure, technological choice, and entry on the intermediate market. *Journal of Economics & Management Strategy*, 9(2):211–30, Summer 2000.

Joseph Farrell and Carl Shapiro. Asset ownership and market structure in oligopoly. *RAND Journal of Economics*, 21(2):275–92, Summer 1990.

Gerard Gaudet and Ngo Van Long. Vertical integration, foreclosure, and profits in the presence of double marginalization. *Journal of Economics & Management Strategy*, 5(3):409–32, Fall 1996.

Oliver Hart and Jean Tirole. Vertical integration and market foreclosure. *Brookings Papers on Economic Activity*, 0(0):205–276, Special Issue 1990.

Kai Uwe Kuhn and Xavier Vives. Excess entry, vertical integration, and welfare. *RAND Journal of Economics*, 30(4):575–603, Winter 1999.

William Novshek. On the existence of cournot equilibrium. *Review of Economic Studies*, 52(1):85–98, January 1985.

Janusz A. Ordover, Garth Saloner, and Steven C. Salop. Equilibrium vertical foreclosure. *American Economic Review*, 80(1):127–42, March 1990.

Janusz A. Ordover, Garth Saloner, and Steven C. Salop. Equilibrium vertical foreclosure: Reply. *American Economic Review*, 82(3):698–703, June 1992.

John Peterman. The brown shoe case. *Journal of Law and Economics*, 18 (1):81–146, 1975.

David Reiffen. Equilibrium vertical foreclosure: Comment. *American Economic Review*, 82(3):694–97, June 1992.

Patrick Rey and Jean Tirole. *A Primer on Foreclosure, forthcoming in Handbook of Industrial Organization.* Elsevier, North Holland, Amsterdam, 2002.

Michael H. Riordan. Anticompetitive vertical integration by a dominant firm. *American Economic Review*, 88(5):1232–48, December 1998.

Michael A. Salinger. Vertical mergers and market foreclosure. *Quarterly Journal of Economics*, 103(2):345–56, May 1988.

Steven C. Salop and David T. Scheffman. Raising rivals' costs. *American Economic Review*, 73(2):267–71, May 1983.

Steven C. Salop and David T. Scheffman. Cost-raising strategies. *Journal of Industrial Economics*, 36(1):19–34, September 1987.

Alexander Schrader and Stephen Martin. Vertical market participation. *Review of Industrial Organization*, 13(3):321–31, June 1998.

Jean Tirole. *The Theory of Industrial Organization.* MIT Press, Cambridge, Massachusetts, 1988.

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