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RELINKING GERMAN ECONOMICS TO THE MAIN STREAM: HEINRICH VON STACKELBERG

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Abstract

Between 1885 and 1930, German universities, then dominated by the Historical School, made hardly any significant contributions to modern economic theory. This began to change around 1930. Precisely at the time when the Nazis set out to purge the "liberalist" tradition from German universities, German theorists again began to make original contributions to mainstream economics. Except for Erich Schneider, nobody contributed as much to this development as Heinrich von Stackelberg, who began his (short) academic career as a "neoclassical" advocate of corporativism and ended it as a protagonist of the "social market economy". After a brief review of Stackelberg's life before the background of those troubled times, the paper provides a concise review of his many-facetted contributions to economic theory.

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LINKING GERMAN ECONOMICS TO THE MAIN STREAM: HEINRICH VON STACKELBERG

by Jürg Niehans

Between 1830 and 1860, Germany provided the world with some of the pioneers of marginalism. Thünen und Gossen are the shining examples. Then German theory declined. From 1880 to 1930, the German academic establishment, now dominated by the Historical School, was virtually cut off from the marginalism of mainstream economic theory. In 1885, Wilhelm Launhardt, an engineering professor in Hannover, had written a little book that made him one of the leading mathematical economists of his day (Launhardt 1885). It turned out to be the last contribution of this kind before the 1930s. German-language economic theory had its center in Vienna. Things began to change around 1930. Paradoxically, precisely at the time when the Nazis tried to purge the "liberal" tradition from German universites, German economists began again to make original contributions to mainstream economics. By the end of the war, they had provided Germany with a solid link to the main stream. Besides Erich Schneider, nobody contributed as much to this development as Heinrich von Stackelberg.

1. Economist in troubled times

The basic facts about Stackelberg's life, short as it was, are fairly simple and easy to document. Other aspects are more complex and difficult to clear up from published sources. The present section is mostly about the basic facts. For more extensive biographical material

the reader is referred to the forthcoming edition of Stackelberg's collected scientific papers, whose editors, Norbert Kloten and Hans Möller, are able to interpret the documentary sources in the light of their personal memories.[1]

Heinrich von Stackelberg was born on October 31, 1905, in Kudinovo, near Moskow. He belonged to the Estonian branch of a far-flung family of teutonic knights that can be traced back to the beginning of the 14th century.[2] His paternal grandfather had been a landowner, but his father, being a younger son, had gone to Dresden to study chemical engineering and become an industrialist. His mother, born Luisa de Vedia, was an Argentinian of Spanish descent, whom his father had met in Dresden. He grew up in the fiercly nationalistic, aristocratic and race-conscious spirit of those proud and defiant baltic junkers.[3] It became the spirit of his early life.

The First World War caused the Stackelbergs to move to Yalta, where the four boys, to the extent they were old enough, were taught by a private tutor. After the Bolshevist revolution the family moved to Silesia, and later to Cologne, where Heinrich obtained his high-school diploma in 1924. His many-sided ability, reaching from the classics to mathematics, from poetry to music (Eucken 1948, 134), was early recognized, and he graduated at the top of his class.

1 The basic biographical data are from Möller (1949, 1956, 1962), Krelle (1987), Kürschner (1941), Hochschullehrer (1938), and Müller (1965). Hans Möller graciously let me copy several documents relating to Stackelberg's life and he extended my background by numerous conversations. I am also indebted to Wilhelm Krelle for biographical information. Möller's and Krelle's writings on Stackelberg have also been helpful in interpreting his economic contributions.

2 For his family tree see Genealogisches Handbuch (1952, 414).

3 The nature of this spirit can be gleaned from the following sentence, written by a member of another branch of the Stackelberg familiy on the last page of his autobiographical recollections: "I shall hope, and have to believe, that this magic power of continuous proximity to the peasant's soil, of tenacious aristocratic self-preservation in this neogermanic democracy, does not languish forever; and since the days of the militia and the baltic regiment and its heroic warfare against the fivehundred-years-old enemy from the sarmatic plains I know that this magic power has indeed survived, beyond the collapse, on the soil of the last feudal society" (Stackelberg-Sutlem 1927, 181-2).

For his undergraduate work at the University of Cologne, Stackelberg chose economics as his main field, but he also trained himself to become an excellent mathematician. Among his teachers were Erwin von Beckerath, who had shifted his allegiance from historicism to (verbal) marginalism, and Eugen Schmalenbach, the czar of cost accounting. As a theorist, Stackelberg must have been largely self-taught. As he told it, it was a senior essay he had to write on Marshall's quasi-rent that aroused his interest in mainstream economic theory. He probably sensed how much some solid calculus could do under the given conditions. Having received his undergraduate degree at the end of 1927, he went on to write a dissertation on the theory of cost, on the basis of which he was awarded the degree of doctor rerum politicarum in 1930. Parts of it read almost like a response to Schmalenbach, whose classic "Cost Accounting and Pricing Policy" (Schmalenbach 1930) combined shrewd intuition with a complete lack of rigor.

By that time, Stackelberg had become active in what is called the "youth movement", that many-facetted and ever-fermenting German counterpart to the boy scouts. In a later photograph he still appears with that clean-shaven, somewhat bland face, with the eager eyes, and with the neatly trimmed and parted hair which we would expect in an actor playing a "wandervogel" of the '20s (Stigler and Friedland 1989). He belonged to the movement's nationalist and conservative wing that wanted to achieve "national dignity" and eastward expansion through soldierly virtues. In 1932 we find him as the editor of the rightist magazine "Jungnationale Stimmen", to which he contributed six articles. One of them develops a program in which national economic self-sufficiency ("apples for bananas") provides the basis for territorial expansion, and vice versa (Stackelberg 1932 b).[4] Another article on reparations and war debts does not go beyond a factual account (Stackelberg 1932 c). The remaining articles may be of interest to a historian of the youth movement, but not of economics.

4 Balanced judgment is helped by remembering that in the following year, in a famous article whose German translation was published in "Schmollers Jahrbuch" (Keynes 1933), Keynes advocated a higher degree of national self-sufficiency, though in the name of peace and not of aggrandizment.

In 1931 Stackelberg joined the Nazi party and he also became a member of the SS (Hochschullehrer 1938, 238). At his university he became the "führer" of the national-socialist lecturers[5]. This ideological development has sometimes been attributed to influences by some of his teachers. This is hardly fair to the latter. The Cologne faculty included hardly any nazis at that time (Heiber 1991). It is true that Beckerath in his scholarly analysis of fascism had shown some early sympathy for Mussolini's "neo-absolutism" (Beckerath 1927 (a) and 1927 (b), part. 154 f.). An aesthete rather than a fighter, with sensitive antennae for historical currents, he contemplated history as if it were a work of art and saw in Mussolini a "renaissance man". However, for the "vulgar" Nazis he never had anything but contempt. He did not join the party and even ceased his activities in cultural exchanges with Italy when Mussolini became Hitler's ally. Later he chaired the oppositional "Freiburg Group" that developed the concepts of a "social market economy" as a blueprint for the German post-war economy.[6] Schmalenbach, who had a jewish wife (Heiber 1991), even resigned his chair when (and because) Hitler came to power and later was prohibited from publishing (Hax 1965). Rather than by external influences, Stackelberg's political choice has to be explained in terms of his family background, his childhood experiences and also perhaps a certain immaturity of judgment.

At the same time Stackelberg worked on his habilitation thesis, which in Germany is the door to an academic career. "Market Structure and Equilibrium" was submitted to the faculty in the fall of 1933 and published in 1934, but the lectureship was officially granted only in 1935. It seems the delay, perhaps surprising in the light of Stackelberg's credentials, both scientific and political, was caused by the fact that after Hitler's rise to power every habilitation was made

5 In a programmatic speech (1934 a), Stackelberg described "national-socialist science" as being characterized not only by its unconditional allegiance to Adolf Hitler and by its being based on specific "internal experiences", but also by the "incorruptibility and reliability of German intellectual work". It is clearly the latter requirement which he wanted to impress on his colleagues.

6 Beckerath's political views are admirably detailed and documented in Kloten (1966).

to require the approval of the Cultural Ministry in Berlin.[7] In his test lecture before the assembled faculty, Stackelberg used duopoly theory to demonstrate that liberal convictions (in the Jeffersonian sense), like other ideologies, have sometimes influenced the path of scientific research. He also took great pains to emphasize, however, that the validity of scientific results stands on its own, independent of changes in ideologies (1935 a).

In 1935 Stackelberg was transferred to the University of Berlin, first as a lecturer, then as an associate professor. In the following year he married countess Elisabeth von Kanitz, 19 years old, on her family's estate in East Prussia. His insistence on a church wedding earned him a reprimand from the SS.[8] Two daughters and a son arrived in due course, the latter in the year before his father's death. From 1938 to 1940, Stackelberg headed a research project on labor economics sponsored by the Deutsche Arbeitsfront. Hans Möller told me that Stackelberg, precisely because his ideological position was unassailable, felt he could afford to use such projects to shelter friends who were vulnerable. Among his colleagues and friends in Berlin was Jens Jessen, who had also been proud to call himself an "old fighter", but was executed as a participant in the officers putsch of July 20, 1944 (Heiber 1991, 197-208). During the war, Stackelberg served several tours of duty in the army (and not in the SS) on the Eastern front; a photograph shows him with the insignia of a "special officer" (Sonderführer), a subaltern rank used for functions like interpreters.

In 1941, Stackelberg was appointed to a professorship in Bonn. In his inaugural lecture, a bland description of German monetary policy since 1870, he was still looking forward to a continental payments system dominated by Germany (1942 b). During the following years, however, he began to participate in the deliberations of Beckerath's Freiburg Group, of which

7 Klaus Müller (1965, 18) seems to misinterpret the facts. The admonition he quotes was not addressed by the political authorities to the university, but by the responsible university official to the political authorities in Berlin.

8 For this telling detail I am indebted to Hans Möller.

Walter Eucken was one of the guiding spirits.[9] The group was in contact with Carl Goerdeler, the head of the officers conspiracy, and some of its members were incarcerated after the abortive coup, but survived (Hauenstein 1964). Both these activities and the teaching in Bonn came to an end in the fall of 1943, when Stackelberg was again sent to the front. For 1944 he had accepted the offer of a visiting professorship in Madrid, where he spent the last two and a half years of his life. Heinrich von Stackelberg died in Madrid on October 12, 1946, forty years old, from cancer.

2. Linking cost accounting to marginalism

In his first significant contribution to economic science, Stackelberg performed a marriage between the Cournot-Pareto tradition of economic analysis and German cost accounting. Schmalenbach's fundamental work on cost and accounting theory then dominated German management science. Under the guidance of Erwin von Beckerath, on the other hand, Stackelberg had convinced himself that economic theory, being essentially quantitative, required mathematical tools. His "Foundations of a Pure Theory of Cost" (1932 a) takes up Schmalenbach's problems with Paretian techniques.

The undertaking may be characterized as an effort to develop a theory of cost without an explicit theory of production. This is probably a Schmalenbach legacy. It is true that production functions make a formal appearance, but this happens only at the end, in the context of general equilibrium (1932 a, 108). There is yet no discussion of the quantitative relationships between inputs and outputs and no conceptual description of the state of technology. Subject to this qualification, which is serious, Stackelberg achieves a reasonably successful, if unelegant, restatement of cost theory.

9 On the (relative) intellectual independence in some circles of the SS see Ritter (1984, 427).

For one product, supplied either under pure competition or simple monopoly, the results do not go beyond Cournot. Like Edward Chamberlin in the United States (Chamberlin 1933) and Joan Robinson in England (Robinson 1933), Stackelberg played the game of "catching up with Cournot"[10]. This was a time when the equality of average and marginal cost could still be described as a "surprising property" of the average cost minimum (Stackelberg 1932 a, 29). For monopoly with free entry there is a clear, though brief, anticipation of the Chamberlin-Robinson case of firms producing under falling average costs (Stackelberg 1932, 90). One of the novel elements concerns the case of a firm with a parametric selling price, but with sales dependent on advertising costs. While it is remarkable as an early analysis of selling costs, the inconsistency between a parametric price and a limited demand robs it of much of its potential value. Stackelberg's later, and more successful, attack on the same problem is described below.

Stackelberg breaks new ground when he extends the analysis to joint products. His most original contribution is the use of polar coordinates to separate the structure (or composition) of output, measured by the slope of a ray through the origin, from its scale, measured by the length of this ray. This makes it possible to break up the firm's optimization problem into two subproblems. In the first stage, the firm selects an arbitrary structure of outputs and determines the optimal scale. This can be done on the basis of the same principles that are valid for a single output. The procedure is then repeated for other predetermined structures of output, yielding an optimal scale for each structure. The second stage consists in selecting the best of these sub-optima.

While this procedure was both novel and correct, it did not produce results that could not have been obtained more simply and intuitively without polar coordinates. What was lacking was the concept of homogeneity of the production function. For homogeneous functions it makes

10 The arriere-guard character of monopolistic competition was justly emphasized by Samuelson (1972).

indeed good sense to separate the choice of structure from that of scale, because the optimal structure is the same for every scale. For more general functions, the transformation of the original function in terms of several inputs and/or outputs into a function in terms of structure and scale offers no analytical advantage.

Potentially the most interesting of Stackelberg's problems concerns intra-firm transfer prices. He first considers the case in which the output of plant A becomes an input of plant B, which sells its final output in the market. The question is what transfer price A should charge B to make firm profit a maximum. It is considered by Stackelberg in several variations based on different assumptions, but the essential elements can best be brought out by concentrating on the simplest case in which (1) A does not sell its intermediate output in the market, (2) each unit of output B requires exactly one unit of output A, and (3) plant B sells in a competitive market.

For this case, a modern economist would immediately derive the rule that plant A should sell to B at marginal cost. Denote cost of plant A by $k = k(x)$ and cost of plant B by $K = K(x) + px$, where x is the quantity of output and p the transfer price. Under integrated optimization the firm maximizes the difference between revenue, at market price P, and cost, namely $Px - K(x) - k(x)$, for which the necessary condition is $P = K'(x) + k'(x)$. Under decentralized optimization, plant A would maximize $px - k(x)$ with necessary condition $p = k'(x)$ and plant B would maximize $Px - K(x) - px$ with necessary condition $P = K'(x) + p$. The two procedures clearly have the same outcome.

This is not Stackelberg's solution, however. He argues that plant A should charge average rather than marginal cost. The reason for the difference is not that one of the two solutions is false, but that they relate to different interpretations of the problem. A modern economist is inclined to search for a transfer price with the property that both plants can separately maximize their profits, given the transfer price, and still come out with the maximum joint profit. Stackelberg, on the other hand, regarded plant A as a captive operation without separate profit maximization. The question then concerned the transfer price that would correctly reflect A's costs in the cost accounting of B. Basically, Stackelberg's solution is trivial because it simply says that plant B should treat plant A as if it were a part of its own operations. The elaborate discussion and the mathematical formalism which Stackelberg bestowed on the many variants of the transfer price problem could not make up for the fact that he had missed the potentially most interesting aspects.

Stackelberg's original contribution in this book is limited to the theory of the firm. It is true that this is finally embedded in a general-equilibrium framework, but this is no more than a paraphrase of Pareto. It is worth noting, though, that in discussing the marginal productivity theory of distribution Stackelberg succeeded in avoiding the false issue of the linear homogeneous production (or transformation) functions. Instead he based his restatement on Wicksell's demonstration[11] that with all firms producing at minimum average cost total product is exhausted even without linear homogeneity (110-112). If by 1932 Stackelberg had perhaps not extended the frontiers of economic theory, he had at least reached them. For a German dissertation of that period this was no mean achievement. In fact, this dissertation was immediately recognized as an authoritative statement of cost theory[12] and thus became the basis for further progress.

3. Oligopolistic competition.

Stackelberg's main claim to a place in the pantheon of economic science is his second book, "Marktform und Gleichgewicht" (Market Structure and Equilibrium, 1934 b). The early 1930s have often (and perhaps falsely) been described as the time of a "revolution" in price

11 Stackelberg (1932 a, 79) refers to H. L. Moore (1929), 145.

12 Hans Möller reported in his obituary that Stackelberg's contribution appeared in examinations at the London School of Economics as early as 1933/34 (1949, 396).

theory. If Joan Robinson's imperfect competition represents the British wing, and Edward Chamberlin's monopolistic competition the American wing of this new attack on old problems, Stackelberg's theory of oligopolistic competition is their German counterpart.

The significance of Stackelberg's contribution must be seen in a wider historical context. In 1838, Antoine Augustin Cournot made two fundamental contributions to oligopoly theory (Cournot 1838). (1) For the case of homogeneous duopoly he developed the solution concept under which each duopolist optimizes his supply on the assumption that the other duopolist does not react. (2) For the case of heterogeneous duopoly he proposed an analogous solution for the limiting case in which the two products are strict complements. For almost a century, oligopoly theory, with the important exceptions of Francis Ysidro Edgeworth's "Mathematical Psychics" (1881) and John von Neumann's "Theory of Games of Chance" (1928), consisted in debating, extending, criticizing, and generalizing Cournot. Stackelberg's contribution still belongs to the Cournot century of oligopoly theory. While some of his answers were new, the questions he asked and the tools he used were essentially Cournot's.

In the course of the Cournot century, the pendulum was swinging back and forth between determinacy and indeterminacy. Cournot's determinate solutions remained unchallenged for about forty years, for the most part simply because they were ignored. After 1880 this began to change. Edgeworth (1881) argued that in the case of few traders the ultimate position on the contract curve is indeterminate within limits that get more narrow as numbers increase. Bertrand's criticism of Cournot (1883), though based on faulty economics, seemed to provide additional reasons for indeterminacy. Edgeworth later reiterated his argument in the analytical terms of the Cournot model, concluding that duopoly prices fluctuate indefinitely (Edgeworth 1925). By the end of the 19th century, the Cournot solution was largely abandoned. Oligopoly prices were regarded as indeterminate.

Then a reaction set in. It was foreshadowed by Launhardt (1885) who, apparently without knowing Cournot, provided the counterpart to the Cournot solution for heterogeneous duopoly. This important contribution was ignored, however, even (surprisingly) by Stackelberg. A similar model by Pareto (1909, App. 71) had the same fate, and only Hotelling (1929) succeeded in attracting the attention of the profession to this case. It was Wicksell who provided the impulse for the reaction. In his "Lectures" (1934-35) he presented the Cournot solution as valid, and he strongly reiterated his position in his review of Bowley's "Mathematical Groundwork of Economics" (Wicksell 1927). Schumpeter (1927) let himself be convinced by Wicksell, and Erich Schneider (1932) provided a lucid modern analysis of the Cournot solution. Around 1932, the determinate Cournot-Launhardt solution seemed to dominate the field. Against this historical background, Stackelberg undertook to demonstrate that Cournot equilibrium is unstable after all.

Stackelberg had first sketched his oligopoly theory at the end of his book on cost (1932 a), and it was presented concisely and elegantly in a paper published in Italian in the following year (1933 b), which helps to fix priorities relative to Robinson and Chamberlin. In "Market Structure and Equilibrium" the argument is extended and elaborated. It is based on a graphical apparatus in which Cournot's reaction lines (in Schneider's 1932 terminology) are supplemented by the two sets of isoprofit curves, which became known as "Stackelberg indifference curves". Stackelberg's comprehensive taxonomy covers both homogeneous and heterogeneous oligopoly and also bilateral monopoly. Since the differences and similarities between these cases are now well understood and are not the subject of Stackelberg's own contribution, the following exposition will be based on the case of homogeneous duopoly.

To evaluate the stability of market structures, Stackelberg developed what he called a new method, which he regarded as his main innovation. Price formation in a market, he postulated, should be considered as stable if both of the following requirements are satisfied (1934 b, 12): (1) "If this type of price formation is assumed and some individual is then given unlimited

freedom in pricing, then the behavior of this individual remains unchanged", (2) "The behavior of any one individual is not intended to change the type of price formation". These formulations are opaque. What, in particular, is meant by "type of price formation"? The answer must be derived from the context of Stackelberg's argument. It then appears that the "type of price formation" is meant to describe the expectations of one duopolist about the reaction of the other. The two stability requirements can thus be rephrased as follows: (1a) Each duopolist maximizes profits for given expectations about the reaction of the other. (2a) Neither duopolist has an incentive to change these expectations.

The significant requirement is clearly the second. It recognizes that expectations about reactive behavior are not implied in profit maximization and that the model, as Bowley had clearly recognized (1966, 38), remains incomplete as long as they are not specified. Edgeworth's indeterminacy thus appears as the consequence of incomplete model specification. In addition, however, expectations must be consistent in the sense that experience provides no reason to change them. This analysis, though it represented a step forward, was "in the air" at that time. Almost simultaneously, Ragnar Frisch put forth his concept of the "conjectural variation" in the other duopolist's decision variable (Frisch 1933). While his exposition was more lucid and forceful than Stackelberg's, his analysis stopped short of the consistency requirement, which was later emphasized, though not made analytically fruitful, by Wassily Leontief (1936) and R. F. Kahn (1937).

Presumably, the expected reaction of one duopolist to the decision by the other may vary over a wide range. Stackelberg had many suggestive things to say about this interaction, but his formal analysis is limited to just two possibilities, namely what he called "dependent" and "independent" behavior. Under dependent behavior, a duopolist chooses his optimal supply on the assumption that his opponent's supply is independent of his own. Under independent behavior, a duopolist expects his opponent to adopt a dependent strategy. A "follower", characterized by dependent behavior, expects his opponent to be a leader; a "leader", characterized by independent behavior, expects his opponent to be a follower. All duopolists are either followers or leaders; there are no intermediate possibilities[13]. Though Stackelberg does not use the leader/follower terminology, the correspondence between his oligopoly theory and his political philosophy is intriguing.

Stackelberg's central question concerns the extent to which oligopolistic markets can be expected to satisfy his second stability requirement. On the basis of the two behavior types he distinguished between two classes of duopolies. Under symmetrical duopoly, both duopolists have the same type of expectation. In what Stackelberg called the Edgeworth-Bowley case both act as leaders, whereas in the Cournot-Launhardt case both act as followers. In the first case the inconsistency of expectations is obvious and had long been recognized. A mathematical proof of it had been provided by Pareto (1909, App. 71).[14] It is evident that at least one duopolist is bound to find out that his opponent does not, in fact, act as a follower, thus forcing him to change his expectations. Stackelberg's second requirement is violated; there is no stable equilibrium.

In the other case of symmetric duopoly, with both opponents acting as followers, instability is not evident. Each duopolist finds that his opponent indeed supplies the expected quantity. Once equilibrium is reached, neither duopolist has a reason to revise his expectations. It is also easy to specify a tâtonnement process that leads to this equilibrium, and Cournot had already analyzed its convergence. Symmetric duopoly with dependent strategies seems to be stable.

Stackelberg demurred. He was not the first to do so. Edgeworth had noticed that in a Cournot duopoly one of the opponents will generally find it advantageous to change his (or her)

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13 Leontief (1936) criticized Stackelberg for neglecting an intermediate case in which each duopolist expects the other to react to some finite extent and each maximizes profits for the given reaction of the other.

14 Stackelberg (1934 b, 135-8; 1938 b, 114-6) justly criticized Harrod (1934) for implying this inconsistency. Hicks's (1935) and Kaldor's (1936) defense of Harrod, to Stackelberg's surprise, completely missed the point.

strategy. If the opponent is known to be a follower, profits can usually be increased by taking the leadership position (1925). Stackelberg said of Edgeworth's criticism of Cournot that "its clarity, brevity and rigor leaves nothing to be desired" (1934 b, 72-3). He also referred to a similar analyis by Kurt Sting (1931). Stackelberg concluded that the Cournot equilibrium, too, violates his second requirement and is, therefore, unstable. The same applies to the Launhardt-Pareto-Hotelling solution for heterogeneous products. True, these solutions are stable as long as expectations, implying dependence, are unchanged. They cease to be stable, however, once expectations, as postulated by Stackelberg's second requirement, are subject to modification.

This leaves the class of asymmetrical duopoly. It is clearly able to satisfy both requirements: the leader finds that the opponent actually acts as a follower, and vice versa. Asymmetrical duopoly thus emerged as the only stable market structure. Not every asymmetrical duopoly is necessarily stable, however. It is possible that either or both of the two opponents would find another strategy more profitable, thus initiating changes in expectations. Overall, therefore, unstable market structures predominate; stable duopoly is an exception.

From his theoretical analysis Stackelberg derived far-reaching policy implications. Up into the 19th century, he believed, stable markets predominated, but then technological progress gave large firms an increasing advantage over small firms. The point of minimum average cost moved upward. As a consequence, the number of firms declined. Competitors became fewer and fewer. Markets began to be dominated by oligopoly and bilateral monopoly. In addition, the behavior of firms became increasingly rational; traditional, ethical, and political constraints to profit maximization were progressively eliminated. As a consequence, the frictions that had once counteracted the instability of oligopoly and bilateral monopoly were gradually reduced. In the end, the conflicts that characterize oligopoly and bilateral monopoly are bound to become

so acute that the state has to step in. Government-controlled syndicates, cartels, trade associations and labor unions will maintain the stability which competition no longer provides. The unstability of oligopoly and bilateral monopoly thus leads to the corporative state.

These were Stackelberg's views at the time of the Great Depression. In his case, as in some others, they were associated with national-socialism. In themselves, however, they were not necessarily fascist. They were shared, though with different conclusions about private enterprise, by most marxists. Non-marxist epigones of Karl Marx like Werner Sombart and Joseph Schumpeter held similar views, in the latter's case even after World War II. Stackelberg's anti-nazi teacher Schmalenbach still argued in 1949 that free competition will be distroyed by the increasing weight of fixed costs (1949). In the United States, many New Dealers shared similar ideas; the "decline of competition" was widely regarded as a truism.

The profession received Stackelberg's analysis with great respect, but also with reservations concerning its main point (Hicks 1935; Lange 1935; Kaldor 1936; Leontief 1936. Stackelberg's reply is 1938 b). Wilhelm Krelle, who went further than anybody else in developing duopoly theory along Stackelbergian lines (1961), was led to the conclusion that there is not only one stable equilibrium but a whole region of them. In Nash's version (1950), the Cournot solution became one of the fundamental concepts of modern game theory and general equilibrium theory.

Indeed, Stackelberg's analysis is flawed. Its basic limitation is the assumption that duopolists can be nothing but leaders or followers. This excludes the possibility that each expects the other to reason in the same way he does himself. If this possibility is allowed for, it is at least conceivable that, even without collusion, both act symmetrically, maximizing their profits for given expectations, and that these expectations are compatible with each other. One variant of this case is illustrated in Figure 1, in which the decision variables, say the quantities offered, are measured along the two axes. The solid lines relate to one duopolist and the broken lines

relate to the other. The U-shaped curves are the respective iso-profit contours. The thin expectation lines represent, in Frisch's terminology, the "conjectural variations" with which one duopolist expects the other to react to his own action. The reaction lines connect those points where the expectation lines are tangent to the iso-profit contours. There is no particular reason why the expectation curves, at the point where the two reaction lines intersect, should not be tangent to each other, thus expressing consistent expectations. Stackelberg might have objected that this case is contrived and unlikely, and he would have been right. The construction nevertheless shows that consistent expectations are logically conceivable.

Symmetry can also be combined with consistency in a more general way. Stackelberg never made clear how the choice between the different market structures is assumed to be made. He gave extensive suggestive descriptions of the dynamic game in the course of which the duopolists might learn about each other's reactions, but what James Konow (1990) calls his "general theory" is not analytically worked out.[15] While his words are about market dynamics, his analysis remained static. Suppose the choice between market structures is modelled in the simplest possible way as follows. Both duopolists are fully informed about both iso-profit contour sets and about market demand. In the two asymmetric cases the duopolists announce their quantities sequentially, so that the second knows the first's offer but not vice versa. The result will be one of the stable equilibria of Stackelberg's asymmetric cases. In the symmetric case both duopolists put their decisions into an envelope, each without knowing the other's decision, and the envelopes are then opened simultaneously. In this case a rational duopolist will probably proceed on the assumption that his opponent will reason equally rationally, which brings on the Cournot solution.

In an extension of the game, the duopolists may also decide which of the three variants of the game they wish to play. In a first stage of the game, say in the morning, each player has

15 The extensive discussion in (1934 b) was later supplemented by (1938 b).

the choice between depositing his quantity decision in an envelope ("move") and postponing his decision ("wait"). At noon, the available envelopes are opened. If only one player has moved, the other announces his decision in the afternoon. If both turn out to have waited, both outputs will be zero. The pay-off matrix may look as in Table 1, where the first number relates to A's pay-off and the second number relates to B's. With the chosen numbers, rational players would both move, because they would understand that this is the only strategy that neither will have a reason to regret. The symmetric Cournot solution would thus be chosen over asymmetric Stackelberg duopoly. This simple game-theoretic reasoning is far from doing justice to the richness of Stackelberg's thought, but is shows that Cournot equilibrium may indeed be stable. Today there seems to be a near-consensus that it is. Indeed, in the form of Nash equilibrium it is widely regarded in game theory as the prototpe of stable equilibrium with rational players[16]. Stackelberg won the battle for asymmetric duopoly, but he lost the war against Cournot stability.

Table 1

A's strategies	B's strategies	
	move	wait
move	3, 3	4, 1
wait	1, 4	0, 0

4. Adding analytical dimensions

Stackelberg first became known for his theories of cost and duopoly. In the course of the following ten years, his work acquired additional dimensions, including, in particular, the theory of the firm, general equilibrium, spatial economics, international economics, and capi-

16 Binmore and Dasgupta (1986, 1-10) provide an example (I owe this refereuce to Gerhard Illing).

tal theory.

(a) Theory of the firm

An early contribution to the theory of the firm concerned demand and supply curves. Around the middle of the 1930s, it was well known that household demand curves might slope upwards (Slutsky 1915, Hicks and Allen 1934) and that supply curves might be bending backwards (Launhardt 1885)[17]. Erich Schneider had argued (1932) that the firm's demand for factors may be upward-sloping, too. It seems that Stackelberg was the first to prove that for the demand and supply of factors by firms there can be no Giffen paradoxes (Stackelberg 1938 c). The source of those paradoxes, he pointed out, is the utility function; with profit maximization there are no paradoxes. This is true, it is shown, under both pure competition and monopoly. The mathematical analysis underlying this proposition is a model of lucidity and stringency. The proposition itself soon became generally known through Hicks's "Value and Capital", first published in the following year (Hicks 1946, 90-96).

Another extension of the theory of the firm related to its marketing costs and product quality. Chamberlin had drawn attention to selling costs as an important decision variable, and discussions of oligopoly were replete with references to non-price competition (Kaldor 1936, p. 230, offers an example), but theoretical analysis had not progressed to the joint optimization of price and advertising expenditure. Stackelberg undertook to fill this gap (1939 a). The price a monopolistic firm obtains for its product is written as a function of the quantity it wishes to sell and its advertising expenditures, p = f(x,a). Costs depend positively not only on output but

17 In a lengthy (and uninspiring) paper, written for the German Labor Front in 1938 (Stackelberg 1942 a), Stackelberg included a discussion of labor supply.

also on price, K = k(x, p), because at a higher price, as the demand function states, more advertising is needed to sell a given quantity. Profit maximization then yields two marginal conditions:

(1) $p = k_x$ (p. 60, eq. 14). This means that price equals marginal cost even for a monopolist, which Stackelberg regarded as a highly remarkable result.

(2) $x = k_p$ (p. 60, eq. 15). This was the additional condition introduced by selling costs, but Stackelberg found it difficult to give it an intuitive interpretation.

To a modern reader it is evident that Stackelberg's expositional problems were due to the peculiar way he wrote his cost function. Suppose one writes the cost function as K = K(x,a) and the demand function as x = F(p, a). Profit maximization then yields the following conditions:

(1a) $p = K_x + \frac{K_a}{F_a}$. Price equals the sum of marginal production costs and marginal selling costs.

(2a) $p + \frac{F}{F_p} = K_x$. Marginal revenue equals marginal production costs.

It is clear that the two pairs of marginal conditions are equivalent, but Stackelberg had a hard time making explicit what his functions implied. Actually, his first condition is somewhat misleading inasmuch as it obscures the fact that his marginal cost differs from the usual concept by including selling costs. Nevertheless, Stackelberg's paper was a brilliant little piece of analysis. It took 15 years for the results to be rediscovered by Robert Dorfman and Peter Steiner (1954).

Stackelberg also advanced the analysis of price discrimination (1939 b). Theodore Yntema (1928, 688) had established that a monopolist faced with segmented markets will differentiate

prices in such a way that marginal revenues are the same in all markets and equal to marginal cost. In this problem, the demand curves for the several markets are assumed to be given. Stackelberg considered the more difficult problem of a monopolist who decides not only on prices in given submarkets, but also on the segmentation of the market into submarkets according to the buyers' willingness to pay. Buyers willing to pay at least p_1 thus form segment 1, the remaining buyers willing to pay at least $p_2 < p_1$ form segment 2, and so on. Stackelberg then established the theorem that "marginal revenue in each market equals the price of the following market", and "marginal revenue in the last market equals marginal cost" (p.6).

For two markets, the proof is straightforward. Let $p_1 = f(x_1)$ and $p_2 = f(x_1 + x_2)$. Denoting cost by K, profit is

$$G = x_1 f(x_1) + x_2 f(x_1 + x_2) - K(x_1 + x_2).$$

Taking the partial derivatives with respect to x_1 and x_2, equating these to zero and rearranging, one obtains the optimum conditions

$$R_1' = f(x_1) + x_1 f'(x_1) = -x_2 f'(x_1 + x_2) + K'(x_1 + x_2) = p_2,$$

$$R'_2 = p_2 + x_2 f'(x_1 + x_2) = K'(x_1 + x_2),$$

where R'_1 and R'_2 are the marginal revenues in the two submarkets. While the theorem is neat as far as it goes, it does not go very far. Few monopolists will be able to segment their markets in this way, even approximately, at no cost.

(b) The existence of general equilibrium

Though Stackelberg is mainly known as a partial-equilibrium theorist, some of his earliest contributions were about general equilibrium. In German-speaking countries, Walrasian general equilibrium became known mainly in the simplified version of Gustav Cassel. This version elicited from Stackelberg two critical comments (1933 a).

The first point concerned what was later called the "integrability problem". Cassel had denounced utility theory as useless; nothing is lost, he maintained, by starting directly with demand functions. Stackelberg argued that a paretian utility function and a casselian set of demand functions are analytically equivalent. Just as a utility function can be used to derive demand functions, so can demand functions be used to construct a system of indifference curves. Each point on a demand curve can be mapped into commodity space as a quantity point combined with an (infinitesimally) short line segment indicating relative prices. Eventually, the line segments can be connected to yield indifference curves. That integrability may pose problems is not noted. In fact, Stackelberg's argument did hardly go beyond Pareto. Much was still left to do for Paul Samuelson, Hendrik Houthakker, Nicholas Georgescu-Roegen and others far into the second half of the century.

Stackelberg's second comment concerned the existence of a Cassel equilibrium. Cassel had assumed a technology with fixed coefficients; factor substitution was excluded. In this case, Stackelberg showed, the system is inconsistent whenever the number of factors exceeds the number of products. Suppose there are two products, measured along the axes of Fig. 2. For each of three factors there is a linear production possibility curve, labeled $F_1, F_2, \ldots$, indicating the maximum of one good that can be produced for each given quantity of the other. In general, the three lines have no point in common; there is no output combination that makes full use of all factors. If some factors are not fully utilized, they become free goods. Stackelberg concludes that a meaningful system of general equilibrium requires substitutability of factors. At about the same time, the question of the existence of economic equilibrium was also raised, in a somewhat different form, by Hans Neisser (1932) and Friedrich Zeuthen (1933). Together the three ushered in the era of existence proofs, culminating in the contributions by John von Neumann, Kenneth Arrow and Gérard Debreu.

(c) Spatial equilibrium

Stackelberg, always interested in location theory, gave to his own research a spatial dimension by proposing some sort of turnpike theorem (1938 a). It was inspired by physics, more particularly by optics, and Stackelberg described it as a striking illustration of isomorphism between sciences. If a ray of light hits the dividing plane between two media at an oblique angle, it changes its direction in such a way that (the sines of) the deviations from the perpendicular to the plane are in the same ratio as the speeds of light in the two media. This means that light, by deviating from the shortest trajectory, minimizes the traveling time between two points.

Stackelberg transformed this physical model into an economic model by considering a border between two modes of transportation, say land and water, with different transportation costs per ton-mile. If goods have to be shipped from a point inside one area to a point inside the other area at an oblique angle to the border, the minimization of transportation costs prescribes a broken route, the consequent increase in the overall distance being more than outweighed by the shortening of the distance in the more expensive mode. This led to his "refraction law of transport economics": "If two transport media are homogeneous from the point of view of transport economics and if they are separated by a linear border, then the shipping route from a point in one medium to a point in the other medium ... will be refracted at the border in such a way that the sines of the two angles of incidence are in inverse proportion to the freight rates" (684).

This theorem is then applied to a variety of spatial problems, including transportation between a coastal city and a landlocked city, the relationship between feeder lines and trunk lines, the maps of Thünen's "isolated state" with different transportation modes and with two cities connected by a low-cost railroad, and the provisioning of three cities, connected by rail, with agricultural products from the country-side.

(d) International economics

Toward the end of the 1930s, Stackelberg also added to his economics an international dimension. As a young man, it will be remembered, he had been a protagonist of national autarky. While expressing great respect for Gottfried Haberler's masterly "International Trade" of 1933, he still criticized it in 1936 for disregarding those political considerations that may require deviations from free trade (1936, 62). Two articles written for an international business journal just before the war show him as a competent, scholarly, but so far unoriginal expositor of international monetary problems (1939 c, d). With the paper "The theory of exchange rates under pure competition" (1949 b), ready for publication in 1944, Stackelberg became one of the originators of the elasticity approach in balance of payments theory. He explicitly acknowledged the similarity of his approach to that of Fritz Machlup (1939-40), whose paper he said he did not know when he developed his own analysis.

Stackelberg showed, in particular, how the reaction of the trade balance to an exchange rate change depends on the elasticities of exports and imports. The result is not expressed in a comprehensive formula as earlier by C. F. Bickerdike (1920) and Joan Robinson (1937, 194), but such a formula can easily be put together from the components listed in the paper (p. 29). It turns out that Stackelberg's result is identical, term by term, to Robinson's.

From this analysis Stackelberg emerges as an "elasticity pessimist". While normal reactions of the trade balance are surely possible, abnormal reactions and instability, he argues, cannot be excluded and may actually be widespread. This, in turn, may require government intervention into foreign trade and the foreign exchange market, including exchange controls.

(e) Time, capital, and interest

During his years in Berlin, Stackelberg must have devoted a large, and increasing, part of his analytical efforts to problems of time, capital and interest. Faithful to his scholarly eclecti-

cism, he developed his original contributions by building on the foundations laid by men like Thünen, Jevons, Böhm-Bawerk, Wicksell, and Fisher. In particular, he defended the "real" approach to capital and interest against the "monetary" approach proposed, as Stackelberg saw it, by Schumpeter and championed by Keynes (Stackelberg 1947). He did not believe in a "crisis in economics", he wrote shortly before his death; "true insight and confirmed results can only be obtained if we proceed along the traditional path shown by the great masters of the real approach" (314).

The fruit of Stackelberg's labors was a set of three papers, which eventually might have become a book. The first step concerned the supply of capital. Its result was a theory of saving and consumption derived from the optimizing behavior of individuals (1938/39). It was essentially the theory of Irving Fisher, expressed in terms of a Fisher-Pareto-Hicks/Allen indifference curve framework. The multiperiod problem was effectively reduced to two dimensions by limiting the individual to a choice between present goods at time zero and a constant stream of future goods beginning at time one. The main results are well known from later mainstream economics. With a rising interest rate, negative savings will be reduced and positive savings will be increased up to a point at which the supply of savings begins to bend backward. With rising income, savings will usually rise, though exceptions are theoretically conceivable. The average propensity to save, however, is likely to decline. In addition to these standard problems, Stackelberg also analyzed the reaction of savings to a lengthening of the constant future stream. It was the first time such results were mathematically derived from intertemporal utility maximization.

The second of Stackelberg's capital papers related to the demand for capital in a stationary state (1941 a). Malte Faber (1979, p. 21) described it as a culmination point of the Austrian tradition. In view of his detailed restatement the following summary can be brief. Wicksell's reformulation of Böhm-Bawerk's model for the point-input/point-output case, exemplified by

growing trees or maturing wine, is taken as a basis. This model is then modified to allow for the "recycling" of some outputs as intermediate inputs and for the continuous-input/point-output case.

The main innovation concerns the period of production. In the Böhm-Wicksell model, output can be written simply as a function of time. In more complicated models this is not possible, but Stackelberg proposed an ingenious trick to save the concept of an average period of production. Every stationary model, no matter how complicated and "realistic", ends up by determining output, wages and an interest rate. In the Böhm-Wicksell model, wages are equal to output as discounted over the production period at the equilibrium rate of interest. By analogy, the production period in more complicated models is *defined* as the period over which output has to be discounted, at the current interest rate, to be equal to wage incomes. In symbols, if output, factor income, the rate of interest and the period of production are denoted, respectively, by x, w, r, and t, then the average period of production can be determined by solving for t the expression $w(1+r)^t = x$. In the special case of capital satiation, with a zero interest rate, the Stackelberg period of production is simply the quotient of the capital stock and the income flow (p. 51). This result is formally similar, though not substantially identical, to Robert Dorfman's "bathtub theorem" (Dorfman 1959).

Stackelberg realized, of course, that his procedure does not validate Böhm-Bawerk's use of the production period for less simple cases. Stackelberg's production periods clearly depend on the market rate of interest and thus cannot be used to explain the latter. Output cannot generally be written simply as a function of waiting time. Stackelberg undertook to demonstrate, however, that his average production period is nevertheless an illuminating descriptive statistic inasmuch as it helps to trace the consequences of an increased capital stock through the various components of the model.

In the third stage, the capital model is made dynamic. This raised the famous problem of the transition or "traverse" of the capital stock from one steady state to another, which had just defeated the analytical powers of Friedrich Hayek and was going to defeat those of Joan Robinson and John Hicks. Stackelberg's contribution, consisting in the first two parts of an unfinished article (1941 b), remained incomplete, too. Nevertheless it contains suggestive ideas and results. It is worth noting that in this context Stackelberg uses a "Keynesian" savings function in which saving is a rising function of aggregate income and the marginal propensity to save exceeds the average propensity (1941 b, part 2, p. 74). The general question is how the development of the economy depends on savings.

The answer is shown to depend, in part, on the technology. Under a "plastic" technology, the production period of existing capital goods can be changed at any time. Trees can be cut or left standing regardless of what was intended at the time they were planted. Under a "rigid" technology, the production period is fixed at the time the capital goods are built. In later jargon, one would probably talk about "putty-clay" or "vintage" capital goods. Stackelberg had a hard time finding a suitable example, but different qualities of wine that reach maturity after fixed periods of 2, 4, or 6 years depending on the choice of grape may help to illustrate his point.

With a plastic capital stock there is shown to be a different equilibrium growth path of the economy for every initial capital intensity (or production period), where equilibrium means that saving is continuously equal to investment. These equilibrium trajectories are not, in general, paths of balanced growth, but characterized by either a continuous rise or a continuous decline in capital intensity, output, consumption, saving and the capital stock, while the rate of interest moves in the opposite direction. There is one particular initial value of capital intensity for which the economy is stationary and saving is zero. For higher capital intensities there is continuous growth; for lower capital intensities the economy is in continuous contraction. Since declines in interest rates are not assumed to reduce saving, there is no self-braking of the growth process

as in classical economics. In contrast to Robert Solow's famous growth model (Solow 1956), the Stackelberg path is unstable. Perhaps it might inspire modern growth theorists to construct yet another version of "take-off into self-sustaining growth".

With a rigid capital stock the problem becomes much more complicated, and Stackelberg's analysis remains incomplete. The main result is that the lags inherent in the adjustment of vintage capital create the possibility, and indeed the likelihood, of endogenous oscillations, which Stackelberg, in the spirit of Frisch's propagation-impulse paper, interprets as business cycles. This seems to be the closest "Austrian" capital theory ever got to a mathematically articulated theory of business cyles.

5. Competitive equilibrium

In 1933, Stackelberg had followed the flag of the corporative state. With his "Principles of Theoretical Economics", published in 1943, he made himself a protagonist of competitive equilibrium. The slender book was intended as an undergraduate microeconomics text, undoubtedly an outgrowth and accompaniment of his own teaching. In purpose and scope it is comparable to, say, Erich Schneider's later "Prices and Equilibrium" (1949) or, in a different generation, Jack Hirshleifer's "Price Theory and Applications" (1976). There are some original developments, particularly about interrelationships between markets, but on the whole the book is meant to be an introduction into existing mainstream economics, including Stackelberg's own earlier contributions. A planned second part, mostly on macroeconomics, trade and location, was never written.

A second edition, with the title changed from "Principles" to "Foundations", appeared posthumously in 1948. In the editor's preface, Valentin Wagner reports that a large part of the first edition had been destroyed in a bombing raid and that not many copies had reached their

readers. The text of the second edition, finished in 1945 in Madrid, had been prepared by Stackelberg for his lectures in Madrid, and a Spanish edition had appeared in 1946. An English translation, entitled "The Theory of the Market Economy", came out in 1952.

The changes from the first to the second edition consist almost entirely of additions. Stackelberg is right in pointing out in his preface that the content of the first edition had remained substantially unchanged. Some specific references to German conditions were omitted and some sections were reorganized. The additions, however, are extensive. They make the second edition a different, and much richer, book. In particular, substitution, complementarity, joint products and cross price effects are analyzed in more detail, there is a half-hearted introduction of the concept of a production function, and there is a more extensive discussion of oligopoly and bilateral monopoly.

The most remarkable qualities of Stackelberg's "Principles", written when World War II was approaching its most ferocious stage, are its cosmopolitian scholarship and its scientific up-to-dateness. As his intellectual ancestors, Stackelberg mentions Menger, Jevons, Walras and Cournot. Important impulses are credited to Böhm-Bawerk, Pareto, Wicksell, and Marshall and also to Eucken, Allen and Hicks, and Amoroso. Hick's "Value and Capital", published in 1939, is fully absorbed. It might have been difficult in 1943 to find a more catholic and up-to-date text at this level anywhere in the world. Up to the appearance of the microeconomic part of Schneider's famous text (Schneider 1949), Stackelberg provided German-language students (and their teachers) with the only competent introduction into mainstream economic theory.

Stackelberg's expository talents and literary skills were limited, however. The style of the book is turgid and discursive. The emphasis on verbal explanation often leaves in the dark what some additional mathematics and graphs might have made clear. Professorial taxonomies abound, and the organization leaves room for improvement. On the other hand, a Stackelberg reader is constantly stimulated to do his own thinking. One gets the impression that Stackelberg

must have been an outstanding teacher of graduate students.

From the point of view of Stackelberg's intellectual development, the most remarkable feature of "Foundations" concerns the policy conclusions. In "Market Structure and Equilibrium" the future had been said to belong to corporativism and state planning. In the following year he provided the planners with a thumbnail sketch of welfare economics (1935 b). For a given distribution of endowments, so he pointed out, utility maximization by households and profit maximization by firms in a system of competitive prices result in an efficient allocation of resources. However, the corresponding income distribution may not be socially desirable; political and social needs may not be adequately reflected in private demand; and markets may be far from the competitive ideal. As a consequence, there is a large role for government control and planning. Social welfare requires, however, that the regulators and planners learn to treat the profit motive as their ally rather than their ennemy. On the horizon, we see the rightist counterpart to Abba Lerner's "economics of control".

A few years later, the "invisible hand" had become all-important and government planning was regarded with skepticism. To the work of the Freiburg Group Stackelberg contributed a paper, published posthumously, on the impossibility of economic planning (1949 a). The paper developed the thesis, familiar from libertarian economists like Mises and Hayek, that it is utterly impossible for planners to calculate those myriads of marginal costs and products that would be required for efficient planning. Free competition is compared to a computer that solves the problems which planners fail to solve. With almost prophetic insight, Stackelberg argues that the Soviet economy may temporarily furnish the means for gigantic government expenditures, but this is at the expense of private privations, and in 30 or 40 years the losses through inefficiency will demonstrate the superiority of a market economy. Stackelberg insists that this does not imply laissez faire, but any government measures would have to be carefully selected and designed to make them compatible with the working of a market economy.

This is the type of economy which Stackelberg envisaged in his "Principles". Perfect competition is described as, "from the production point of view, the most efficient form of economic organization inasmuch as it offers the most favorable condition for the development of productive resources" (1943, 199; 1948, 337). At the same time, "the personal distribution of income can, within certain limits, be freely determined by government economic policy without impeding the essential functions of perfect competition as described above" (1943, 209; 1948, 349-50). This is a description of a competitive economy with a social "safety net".

This leaves the government with two main economic functions. First, where free markets deviate particularly far from perfect competition, corrective measures may be able to push reality closer to the ideal. Externalities, adjustment lags and imperfections of competition are cases in point (1943, 206 f.; 1948, 344 f.). Second, the government, by its taxes, expenditures, and transfers, has to modify endowments in accordance with the prevailing views about equity and justice (1943, 208 f.; 1948, 349 f.). These were essentially the views of a large number of democratic economists of diverse political persuasions, both bourgeois and socialist, that turned out to dominate Western economic policies in the post-war period.

Stackelberg, like other economists, still found it difficult to account for the persistence of profits in a competitive economy. Characteristically, he quotes Schumpeter to the effect that profit is a reward for "leadership" (1943, 188; 1948, 324). Leadership ability, he explains, is distributed unequally between individuals. The marginal entrepreneur earns no profit, but the intramarginal entrepreneur earns profit even in full equilibrium. Profits are thus made to appear as the same sort of "static" income as differential land rent (1943, 189; 1948, 325). This argument remained fairly nebulous, though. It did not occur to Stackelberg that pure profits (and losses) may rather be the result of changes in the valuation of scarce factors due to shifts in equilibrium.

6. Concluding remark

Some economists pictured themselves as revolutionaries against the orthodoxy of their contemporaries, claiming to lay new foundations for their science. Thus Gossen saw himself as the Copernicus of the social universe, Jevons rebelled against the authority of Mill, and Keynesians proclaimed their "revolution". Others, not necessarily less original, based their work on the structure of science left by their predecessors, trying to add new floors or wings. Ricardo, Marshall, Wicksell and Samuelson may serve as examples. Stackelberg clearly belongs to the latter group. He was an eclectic in the sense of being a true scholar.

He expressed his creed in many places, but most clearly in his review of Walter Eucken's "Foundations of Economics" (1940). Eucken had declared all previous economics a failure; efforts to build a better science, he maintained, had to start from scratch. Stackelberg objected. "The science of economics", he wrote, "is now several hundred years old. It stands before our eyes as a web of many strands complexly intertwined and diverging. Experience and theory, induction and deduction, conceptual analysis and verification, observation and abstraction, true insight and errors of all sorts have joined hands, combined forces, and succeeded each other. All this heterogeneous stock of knowledge is today the foundation for the education of every economist. And, in spite of everything, it is a good foundation" (p. 257).

At the same time Stackelberg was original inasmuch as he added something new to every subject he took up. His early contributions got immediate international attention, making him one of the respected mathematical economists of the 1930s. In view of political developments it is perhaps natural that his later works became less widely known. What Stackelberg regarded as his most important result, namely the instability of Cournot duopoly, did not become part of mainstream theory; it was later overshadowed by game theory. Today he is remembered rather for asymmetric duopoly, for "Stackelberg leaders" and "Stackelberg followers".

Through his creative eclecticism, Stackelberg, together with Erich Schneider and others, became instrumental in relinking German economics, after a break of about half a century, to the main stream of marginalism. This was undoubtedly his most important contribution to the history of economic science.

Early life, youthful idealism, and some immaturity of judgment led Stackelberg to national socialism. He never let his political views corrupt his research and teaching, though. He rather seems to have been happy that his impeccable ideological credentials seemed to gain him a measure of academic freedom and independence, and he had the confidence of antinazis among his students and collaborators. In his last years he must have been increasingly disillusioned, and he became an active participant in the opposition group around Eucken and Beckerath. He is a striking example of the ideological pluralism of mainstream economics, where a atheist radical like Wicksell, a Christian moralist like John Bates Clark and an aristocratic antiliberal like Pareto join forces in developing the marginal productivity theory, and where a leftist Robinson, a bourgeois Chamberlin and a nazi Stackelberg initiate the decade of imperfect, monopolistic and oligopolistic competition.

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Figure 1

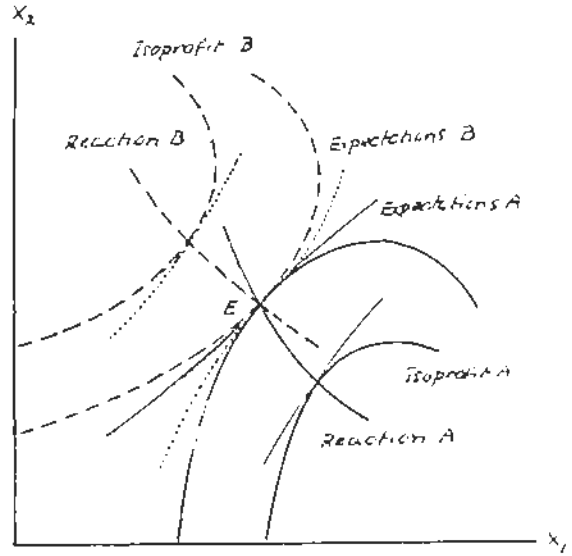
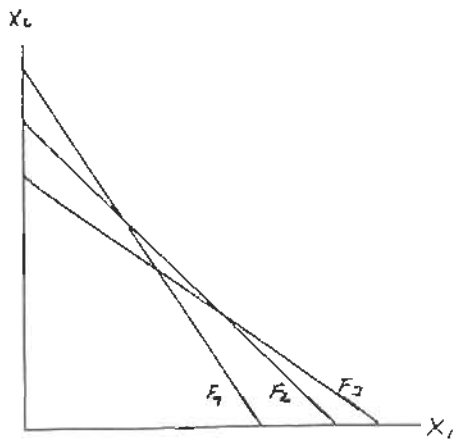


Figure 2



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