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CENTER FOR ECONOMIC STUDIES

# FOUR STUDIES IN THEORETICAL SPATIAL ECONOMICS

Jean H. P. Paelinck

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# FOUR STUDIES IN THEORETICAL SPATIAL ECONOMICS

## Abstract

In what follows, four research topics in theoretical spatial economics are presented, topics which have been worked on recently.
They start with two so-called "Tinbergen-Bos Systems" (TBS) analyses. The first is a check on the empirical presence of "Tinbergen hierarchies" in Dutch microregional data; the hypothesis is not rejected. The second develops two theoretical aspects: the computation of non-allowable TBS (supply of some goods and/or services being absent) and the solution via continuous linear programming (CLP) of Tinbergen-Bos metricised systems.
This CLP approach can be found back in an essay, the third one, on the maximal flow capturing problem, for which a solution in those terms is presented.
Coming back to TBS, it should be mentioned that spatial interdependence via transportation costs is a central idea in the analysis; this idea is applied again in an optimal control set-up with potentialised partial differential equations (PPDEs), for which a - once more potentialised - framework and an explicit solution are proposed.

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## 1. Empirical Evidence on Tinbergen-Bos Systems

TABLE OF CONTENTS

## *1. Introduction.*

Tinbergen-Bos Systems (TBS) go back to the sixties; since their emergence we have studied them closely (see references 6.1).

Attempts at observing TBS-structures are scarce (see Chin, 1964 and Meulemeester, 1969); the problem has been taken up again recently[1].

In what follows we will first expose the theoretical foundations of TBS, then study the linking problems with empirical evidence, introducing to a case study for the Netherlands.

## *2. Theoretical foundations.*

The starting point is the notion of an economic **activity** exercised by a certain number of plants of equal ("optimal") size. The set of basic activities can be ranked according to that number; the activity comprising the minimal number of plants is said to have the highest rank (say 1).

Activities concentrate in **centers**; with I activities $c_I = 2^I-1$ types of centers can obviously be composed.

A reference area is covered with certain types of centers, and a realisation thereof is called a **system**; for I activities the number of possible systems is equal to $s_I = 2^{c_I}-1-e_I$, where $e_I$ is the number of excluded systems, i.e. of systems not delivering all I goods and services[2].

Special cases of centers and systems can be pointed at. Two such cases are the weak Tinbergen hierarchy (WTH, applying to centers) and the strong Tinbergen hierarchy (STH, applying to systems)[3]. WTH implies that every center possesses **all** the activities of lower rank than the activity of highest rank present; STH implies that a system is built up of a **complete** set of centers with WTH. Postulating a STH solves of course the problem of selecting one of the $s_I$ possible systems.

1) Within the framework of an essay on Theoretical Spatial Economics at the Rotterdam School of Economics; we like to thank the two authors, Duncan Beeckman and Emiel Maasland, for having made available the original statistical material they collected, and Reinaud van Gastel for having performed the computations.

2) No analytical expression for $e_I$ is known to us; but we were able to derive an upper bound.

3) The terminology is ours.

*3. Statistical evidence.*

Let us start with the assumption that a system is built up from elementary activities and shows STH; table 1 then summarises the evidence.

TABLE 1

| Activities | | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | ............. | I |
| Number of centers | | | | | |
| $n^1 \equiv n_1$ | 1 | $n_{21}$ | $n_{31}$ | .............. | $n_{I1}$ |
| $n^2 = n_2 - n^1 n_{21}$ | 0 | 1 | $n_{32}$ | .............. | $n_{I2}$ |
| $n^3 = n_3 - n^1 n_{31} - n^2 n_{32}$ | 0 | 0 | 1 | .............. | $n_{I3}$ |
| ...................... | ...... | ...... | ...... | .............. | ...... |
| $n^I = n_I - n^1 n_{I1} - .. - n^{I-1} n_{I,I-1}$ | 0 | 0 | 0 | ............. | 1 |
| Number of activities | $n_1 <$ | $n_2 <$ | $n_3 <$ | ............. | $< n_I$ |

In practice, however, sectors and centers are aggregate entities, so aggregation over activities and elementary centers has to be considered. It is not difficult to prove that, for sector aggregates, if the aggregation is over $i<i'$ activities, zeros are kept down $i'$, and up $i'$ for $i>i'$; the reverse holds for center aggregation.

Let us now make two assumptions :

$A_1$ : sector aggregation is over neighbouring activities of table 1; this is not unreasonable in terms of homogeneous activity classification;

$A_2$ : center aggregation is over neighbouring centers; this again is not unreasonable in terms of common locational factors.

In the pure $A_1$-case zeros continue to appear below the still present ones, in the pure $A_2$-case also but the ones have disappeared. A special case is that of joint $A_1$-$A_2$ aggregation leading to a square matrix : the latter is then upper-triangular. The reason is obvious : table 1, being rectangular, is pseudo-triangular, and any "square" aggregation will keep zeros below the main diagonal. However one supposes implicitly that aggregate sector-ranking follows elementary activity ranking; exceptions to this assumption cannot be excluded.



*4. Case study.*

A square 43x43 matrix was produced for the Netherlands (1993), the regions being so-called "COROP-regions"; table 2 summarises the information (source : CBS, Statistiek van het Ondernemingenbestand 1993, table A.6). It should be mentioned that the sectors have been defined over enterprises and not over plants, which adds an additional perturbing element to the analysis to follow.

A way indeed to study the properties of the table is to compare it to the triangular structure which would have resulted from the $A_1$-$A_2$ aggregation on a pure STH. To that end an index t was constructed as :

$$t = ( u - l ) n^{-1} \qquad (1)$$

where u is the number of plants on and above the main diagonal, l the remaining number of plants, and n the total number of plants. t can be rewritten as :

$$t = 1 - 2 l n^{-1} \qquad (2)$$

It is easily seen that t = 1 for the pure triangular case; an extreme opposite case would be the equal spatial dispersion (ESD) of all aggregated sectors, in which case :

$$l = \Sigma_i n_i ( I - i ) I^{-1} \qquad (3)$$

A statistical test can be constructed supposing that all possible configurations between ESD and an aggregated STH are homogeneously distributed; in that case the probability of obtaining by chance a configuration between the observed one and a square matrix resulting from the aggregation of a pure STH is :

$$p = ( 1 - t' ) ( 1 - t_{ESD} )^{-1} \qquad (4)$$

$t'$ being the empirical value of t; for table 2 , $t_{ESD}$ has been computed as .698094. As however, given the structure of table 2, relatively high values of $t'$ are to be expected a priori (the case of $t_{ESD}$ illustrates this), a triangular distribution would be more in order; given the constraint on the probability integral and p(1)=0, one gets :

$$p = .045^{-1} - .045^{-1} t \qquad (5)$$

Another possible reference point - substitute for $t_{ESD}$ - is that of the biproportional adjustment of table 2, which respects row and column totals, followed by an adjustment along the lines of point (c) below; still another one is - still along that point (c) - a maximum-l objective. Results will also be reported.

TABLE 2

| | COROP | Coropnr. | 28 | 33 | 90 | 75 | 26 | 24 | 29,30 | 73 | 31 | 38 | 22 | 32 | 97 | 77 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Rijnmond | 29a | 32 | 9 | 12 | 23 | 10 | 17 | 83 | 614 | 62 | 124 | 85 | 82 | 157 | 118 |
| 2 | Amsterdam | 23a | 11 | 1 | 10 | 33 | 14 | 23 | 39 | 94 | 40 | 90 | 71 | 50 | 264 | 149 |
| 3 | W-N Brabant | 33 | 10 | 9 | 12 | 11 | 29 | 52 | 60 | 22 | 86 | 49 | 66 | 74 | 59 | 107 |
| 4 | Aggl Grav. | 26 | 10 | 4 | 12 | 6 | 8 | 11 | 39 | 21 | 16 | 81 | 56 | 37 | 301 | 160 |
| 5 | Alkmaar e.o. | 19 | 7 | 5 | 4 | 1 | 6 | 4 | 20 | 4 | 39 | 35 | 17 | 22 | 19 | 36 |
| 6 | Utrecht | 17 | 6 | 11 | 27 | 15 | 23 | 19 | 130 | 14 | 85 | 141 | 92 | 125 | 336 | 166 |
| 7 | Twente | 12 | 6 | 7 | 11 | 8 | 18 | 14 | 46 | 4 | 95 | 57 | 136 | 88 | 60 | 76 |
| 8 | Z-O N Brabant | 36 | 5 | 16 | 6 | 12 | 14 | 43 | 45 | 4 | 80 | 74 | 126 | 115 | 61 | 83 |
| 9 | Veluwe | 13 | 5 | 9 | 6 | 9 | 27 | 15 | 54 | 3 | 76 | 46 | 70 | 61 | 116 | 68 |
| 10 | Kop N-Holland | 18 | 5 | 3 | 1 | 10 | 4 | 3 | 19 | 40 | 31 | 33 | 47 | 25 | 31 | 78 |
| 11 | Achterhoek | 14 | 4 | 12 | 4 | 0 | 36 | 30 | 28 | 3 | 53 | 31 | 66 | 45 | 25 | 56 |
| 12 | Z-O Z Holland | 30 | 4 | 7 | 4 | 1 | 9 | 4 | 27 | 16 | 46 | 41 | 32 | 45 | 32 | 49 |
| 13 | Z-Limburg | 39 | 4 | 6 | 13 | 11 | 18 | 20 | 79 | 2 | 47 | 53 | 68 | 103 | 121 | 122 |
| 14 | N Overijssel | 10 | 4 | 5 | 5 | 0 | 1 | 3 | 22 | 14 | 50 | 27 | 88 | 44 | 30 | 43 |
| 15 | Zaanstreek | 22 | 4 | 1 | 5 | 2 | 23 | 2 | 22 | 10 | 17 | 20 | 20 | 9 | 7 | 24 |
| 16 | Aggl Arnhem | 15 | 3 | 10 | 10 | 5 | 20 | 33 | 35 | 8 | 59 | 74 | 63 | 104 | 103 | 123 |
| 17 | N Limburg | 37 | 3 | 6 | 6 | 1 | 17 | 6 | 13 | 0 | 49 | 23 | 38 | 102 | 32 | 38 |
| 18 | M Limburg | 38 | 3 | 6 | 4 | 1 | 3 | 13 | 17 | 1 | 22 | 19 | 26 | 71 | 21 | 32 |
| 19 | Ov Groningen | 3 | 3 | 2 | 6 | 2 | 6 | 7 | 17 | 72 | 28 | 38 | 16 | 43 | 78 | 104 |
| 20 | O Groningen | 1 | 3 | 2 | 2 | 0 | 14 | 2 | 19 | 1 | 12 | 9 | 9 | 24 | 7 | 15 |
| 21 | Z Vlaanderen | 31 | 3 | 0 | 2 | 0 | 2 | 5 | 13 | 46 | 3 | 10 | 29 | 11 | 9 | 39 |
| 22 | IJmond | 20 | 2 | 6 | 3 | 1 | 9 | 5 | 15 | 71 | 16 | 9 | 18 | 7 | 13 | 20 |
| 23 | Ov Zeeland | 32 | 2 | 4 | 8 | 7 | 2 | 8 | 13 | 37 | 17 | 15 | 30 | 35 | 32 | 80 |
| 24 | 's- Herthogenbosch | 35a | 2 | 4 | 7 | 3 | 1 | 16 | 31 | 1 | 34 | 21 | 24 | 40 | 35 | 31 |
| 25 | Z-W Gelderland | 16 | 2 | 4 | 1 | 1 | 5 | 11 | 23 | 4 | 32 | 21 | 27 | 56 | 17 | 49 |
| 26 | Gr A'dam | 23 | 2 | 2 | 8 | 57 | 17 | 10 | 33 | 7 | 27 | 52 | 41 | 41 | 50 | 76 |
| 27 | N Friesland | 4 | 2 | 2 | 8 | 0 | 4 | 3 | 14 | 33 | 26 | 18 | 23 | 37 | 32 | 76 |
| 28 | Gooi Vechtstreek | 24 | 2 | 2 | 2 | 11 | 7 | 5 | 31 | 2 | 21 | 33 | 27 | 25 | 29 | 39 |
| 29 | N Drenthe | 7 | 2 | 2 | 2 | 2 | 2 | 1 | 4 | 6 | 13 | 17 | 7 | 22 | 32 | 25 |
| 30 | O-Z Holland | 28 | 2 | 1 | 5 | 1 | 12 | 5 | 30 | 1 | 26 | 27 | 29 | 125 | 35 | 33 |
| 31 | Z-O Drenthe | 8 | 2 | 0 | 2 | 1 | 1 | 1 | 9 | 0 | 21 | 20 | 21 | 14 | 7 | 26 |
| 32 | Z-O Friesland | 6 | 2 | 0 | 2 | 0 | 0 | 3 | 5 | 13 | 17 | 20 | 17 | 20 | 13 | 28 |
| 33 | N-O N Brabant | 35 | 1 | 6 | 5 | 0 | 6 | 27 | 35 | 3 | 30 | 17 | 42 | 44 | 19 | 36 |
| 34 | Delfzijl e.o. | 2 | 1 | 2 | 0 | 0 | 1 | 0 | 12 | 107 | 3 | 2 | 1 | 5 | 6 | 10 |
| 35 | Flevoland | 40 | 1 | 1 | 2 | 13 | 11 | 5 | 21 | 7 | 28 | 21 | 21 | 19 | 35 | 25 |
| 36 | Aggl Haarlem | 21 | 1 | 0 | 5 | 3 | 3 | 5 | 23 | 10 | 12 | 32 | 27 | 21 | 30 | 42 |
| 37 | Z-W Friesland | 5 | 1 | 0 | 1 | 0 | 2 | 1 | 2 | 14 | 12 | 7 | 23 | 30 | 10 | 21 |
| 38 | M-N Brabant | 34 | 0 | 8 | 7 | 4 | 15 | 328 | 27 | 9 | 85 | 39 | 71 | 75 | 59 | 34 |
| 39 | Delft Westland | 27 | 0 | 3 | 2 | 3 | 9 | 4 | 24 | 2 | 19 | 26 | 13 | 37 | 41 | 20 |
| 40 | Aggl Leiden | 25 | 0 | 1 | 4 | 7 | 8 | 4 | 35 | 3 | 32 | 38 | 21 | 44 | 60 | 39 |
| 41 | Z-W Overijssel | 11 | 0 | 1 | 3 | 3 | 5 | 6 | 16 | 0 | 14 | 5 | 11 | 12 | 21 | 10 |
| 42 | Z-W Drenthe | 9 | 0 | 1 | 3 | 3 | 3 | 5 | 12 | 0 | 12 | 9 | 8 | 17 | 14 | 23 |
| 43 | Gr Rijnmond | 29 | 0 | 0 | 0 | 1 | 2 | 7 | 13 | 8 | 15 | 17 | 21 | 17 | 10 | 44 |
| | | | 162 | 181 | 242 | 272 | 426 | 786 | 1255 | 1331 | 1508 | 1541 | 1744 | 2023 | 2469 | 2473 |



TABLE 2 continued

| 39 | 37 | 36 | 23 | 35 | 85 | 76 | 74 | 92 | 25 | 63,64 | 34 | 27 | 20,21 | 94 | 52 | 82 | 71,72 | 93 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 165 | 296 | 175 | 153 | 313 | 350 | 1552 | 1552 | 498 | 372 | 584 | 765 | 530 | 589 | 640 | 750 | 1159 | 1393 | 989 |
| 265 | 110 | 178 | 553 | 100 | 264 | 668 | 296 | 295 | 372 | 785 | 217 | 1009 | 395 | 484 | 437 | 743 | 1410 | 817 |
| 92 | 103 | 131 | 106 | 180 | 167 | 222 | 253 | 229 | 260 | 381 | 428 | 291 | 448 | 357 | 412 | 471 | 626 | 566 |
| 176 | 33 | 129 | 103 | 105 | 133 | 240 | 25 | 350 | 256 | 393 | 186 | 492 | 338 | 579 | 540 | 903 | 684 | 776 |
| 47 | 43 | 60 | 64 | 42 | 68 | 64 | 20 | 105 | 130 | 153 | 117 | 208 | 147 | 134 | 169 | 211 | 149 | 241 |
| 212 | 141 | 277 | 182 | 250 | 287 | 301 | 166 | 451 | 490 | 698 | 550 | 746 | 603 | 815 | 740 | 1101 | 922 | 1294 |
| 83 | 46 | 118 | 219 | 258 | 121 | 164 | 16 | 251 | 303 | 281 | 358 | 252 | 368 | 329 | 346 | 403 | 412 | 649 |
| 175 | 67 | 241 | 190 | 229 | 214 | 132 | 20 | 302 | 501 | 458 | 556 | 387 | 440 | 360 | 446 | 471 | 533 | 675 |
| 99 | 115 | 100 | 65 | 182 | 150 | 110 | 32 | 240 | 253 | 348 | 347 | 291 | 436 | 346 | 395 | 451 | 512 | 635 |
| 46 | 107 | 77 | 60 | 106 | 106 | 72 | 83 | 163 | 171 | 150 | 178 | 173 | 205 | 229 | 293 | 268 | 321 | 355 |
| 54 | 53 | 48 | 56 | 155 | 109 | 102 | 12 | 131 | 257 | 163 | 296 | 212 | 319 | 245 | 279 | 280 | 310 | 399 |
| 47 | 140 | 88 | 40 | 123 | 136 | 177 | 758 | 178 | 208 | 174 | 285 | 190 | 221 | 224 | 268 | 265 | 535 | 408 |
| 96 | 47 | 117 | 159 | 108 | 168 | 126 | 52 | 348 | 272 | 189 | 311 | 250 | 393 | 393 | 448 | 495 | 618 | 683 |
| 44 | 98 | 42 | 33 | 99 | 80 | 58 | 286 | 109 | 165 | 112 | 157 | 154 | 198 | 219 | 200 | 258 | 288 | 347 |
| 29 | 65 | 54 | 40 | 77 | 37 | 46 | 59 | 78 | 101 | 84 | 147 | 134 | 100 | 74 | 97 | 118 | 109 | 136 |
| 112 | 69 | 120 | 132 | 134 | 140 | 163 | 263 | 286 | 238 | 319 | 321 | 352 | 356 | 534 | 376 | 509 | 497 | 899 |
| 48 | 23 | 51 | 54 | 138 | 80 | 138 | 13 | 98 | 139 | 114 | 237 | 112 | 192 | 205 | 191 | 202 | 336 | 313 |
| 40 | 34 | 43 | 70 | 73 | 76 | 54 | 361 | 83 | 140 | 87 | 195 | 128 | 171 | 149 | 159 | 220 | 259 | 237 |
| 60 | 70 | 47 | 44 | 64 | 85 | 98 | 156 | 142 | 189 | 108 | 132 | 241 | 179 | 318 | 243 | 292 | 268 | 403 |
| 23 | 17 | 19 | 9 | 32 | 30 | 26 | 16 | 55 | 60 | 33 | 65 | 48 | 98 | 100 | 85 | 115 | 167 | 139 |
| 10 | 27 | 13 | 6 | 15 | 30 | 68 | 231 | 44 | 40 | 38 | 66 | 34 | 93 | 72 | 101 | 98 | 137 | 132 |
| 15 | 34 | 25 | 30 | 53 | 52 | 61 | 35 | 74 | 71 | 87 | 112 | 71 | 94 | 79 | 135 | 113 | 142 | 186 |
| 40 | 63 | 34 | 43 | 67 | 74 | 93 | 157 | 120 | 67 | 99 | 136 | 91 | 185 | 230 | 196 | 165 | 301 | 354 |
| 64 | 26 | 52 | 50 | 78 | 106 | 56 | 29 | 119 | 174 | 174 | 168 | 150 | 174 | 185 | 204 | 290 | 215 | 339 |
| 24 | 66 | 44 | 27 | 62 | 66 | 45 | 122 | 91 | 311 | 153 | 192 | 137 | 141 | 103 | 147 | 138 | 360 | 211 |
| 66 | 88 | 82 | 86 | 106 | 142 | 389 | 73 | 196 | 194 | 358 | 186 | 292 | 218 | 231 | 337 | 512 | 528 | 430 |
| 38 | 144 | 34 | 25 | 70 | 117 | 44 | 158 | 155 | 138 | 96 | 130 | 164 | 242 | 241 | 213 | 249 | 329 | 303 |
| 66 | 35 | 72 | 58 | 69 | 83 | 62 | 32 | 128 | 119 | 352 | 110 | 286 | 126 | 197 | 227 | 269 | 193 | 317 |
| 29 | 16 | 19 | 12 | 31 | 40 | 26 | 6 | 53 | 64 | 59 | 48 | 75 | 67 | 148 | 98 | 112 | 114 | 152 |
| 80 | 62 | 55 | 34 | 87 | 85 | 74 | 58 | 105 | 172 | 196 | 151 | 156 | 204 | 160 | 261 | 269 | 352 | 319 |
| 26 | 14 | 21 | 15 | 33 | 48 | 28 | 12 | 62 | 42 | 36 | 109 | 61 | 81 | 94 | 89 | 102 | 165 | 151 |
| 34 | 57 | 22 | 20 | 31 | 49 | 23 | 42 | 70 | 110 | 63 | 99 | 100 | 114 | 125 | 102 | 147 | 192 | 193 |
| 54 | 40 | 57 | 97 | 102 | 123 | 56 | 26 | 132 | 231 | 123 | 211 | 140 | 244 | 160 | 240 | 288 | 328 | 342 |
| 7 | 12 | 5 | 1 | 16 | 14 | 14 | 46 | 12 | 18 | 10 | 16 | 16 | 23 | 35 | 34 | 26 | 56 | 62 |
| 38 | 58 | 48 | 63 | 75 | 60 | 67 | 45 | 128 | 86 | 116 | 102 | 129 | 170 | 105 | 99 | 124 | 161 | 189 |
| 68 | 25 | 71 | 50 | 39 | 57 | 71 | 19 | 79 | 98 | 198 | 79 | 214 | 113 | 200 | 205 | 223 | 132 | 324 |
| 26 | 138 | 14 | 8 | 51 | 20 | 15 | 128 | 44 | 56 | 44 | 75 | 56 | 86 | 61 | 72 | 66 | 115 | 102 |
| 92 | 87 | 84 | 122 | 173 | 117 | 97 | 216 | 228 | 291 | 414 | 316 | 207 | 323 | 284 | 328 | 372 | 447 | 473 |
| 34 | 30 | 76 | 20 | 46 | 63 | 66 | 19 | 88 | 61 | 69 | 294 | 101 | 113 | 137 | 269 | 151 | 280 | 209 |
| 68 | 71 | 65 | 37 | 84 | 89 | 104 | 40 | 159 | 132 | 188 | 116 | 212 | 226 | 248 | 208 | 302 | 254 | 373 |
| 14 | 10 | 12 | 14 | 31 | 28 | 33 | 16 | 49 | 41 | 67 | 47 | 80 | 83 | 87 | 95 | 101 | 88 | 135 |
| 16 | 24 | 11 | 16 | 32 | 36 | 18 | 51 | 57 | 59 | 64 | 74 | 91 | 92 | 107 | 84 | 100 | 143 | 146 |
| 18 | 63 | 38 | 21 | 78 | 53 | 91 | 173 | 72 | 96 | 103 | 151 | 74 | 139 | 98 | 179 | 192 | 359 | 203 |
| 2840 | 2867 | 3069 | 3187 | 4197 | 4353 | 6114 | 6173 | 6657 | 7548 | 8721 | 8836 | 9137 | 9547 | 10121 | 10797 | 13344 | 15740 | 16606 |

TABLE 2 continued

| 95,96 | 68 | 51 | 83 | 98 | 81 | 67 | 61,62 | 84 | 65,66 | Som | Hoofddiasom | | Bevolking |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 924 | 953 | 2246 | 2782 | 2709 | 4528 | 3080 | 6539 | 6619 | 8976 | 53609 | 53609 | | 1109755 |
| 1389 | 722 | 1061 | 4135 | 2581 | 5308 | 3998 | 5465 | 6461 | 8245 | 49652 | 49641 | | 722598 |
| 532 | 678 | 1394 | 1046 | 1494 | 1127 | 1744 | 3333 | 3415 | 4609 | 25741 | 25722 | | 574377 |
| 707 | 721 | 1460 | 2197 | 2011 | 2127 | 2425 | 3104 | 5878 | 6403 | 34236 | 34210 | | 703231 |
| 234 | 269 | 769 | 452 | 564 | 443 | 670 | 1031 | 1393 | 1893 | 10108 | 10091 | | 220918 |
| 1327 | 1134 | 2475 | 2845 | 2737 | 4002 | 2375 | 5907 | 9513 | 8613 | 52344 | 52262 | | 1053083 |
| 545 | 662 | 1240 | 1392 | 1262 | 1223 | 1436 | 2612 | 2578 | 4932 | 23485 | 23421 | | 583077 |
| 657 | 912 | 1543 | 1876 | 1644 | 1931 | 1966 | 3230 | 4571 | 5164 | 30575 | 30434 | | 669931 |
| 678 | 743 | 1377 | 1998 | 1248 | 1052 | 1679 | 2765 | 2805 | 4658 | 24675 | 24547 | | 609318 |
| 433 | 436 | 1299 | 472 | 809 | 531 | 1195 | 1248 | 1616 | 2668 | 14200 | 14084 | | 339179 |
| 454 | 510 | 973 | 924 | 738 | 627 | 1015 | 1688 | 1457 | 2872 | 15131 | 14930 | | 365129 |
| 295 | 457 | 1106 | 1402 | 824 | 636 | 717 | 2246 | 2126 | 2993 | 17584 | 17393 | | 397068 |
| 732 | 627 | 1233 | 884 | 1559 | 972 | 2846 | 1978 | 2660 | 5477 | 24908 | 24484 | | 644756 |
| 354 | 388 | 763 | 463 | 616 | 467 | 853 | 1354 | 1153 | 2711 | 12405 | 12112 | | 329097 |
| 144 | 151 | 439 | 218 | 367 | 448 | 269 | 583 | 777 | 1118 | 6265 | 6099 | | 146556 |
| 834 | 675 | 1230 | 1564 | 1470 | 1105 | 1765 | 2561 | 3140 | 4838 | 25652 | 24890 | | 665674 |
| 306 | 303 | 612 | 366 | 605 | 734 | 962 | 1182 | 917 | 2027 | 11032 | 10627 | | 265591 |
| 239 | 241 | 502 | 339 | 491 | 366 | 818 | 999 | 976 | 1911 | 9700 | 9344 | | 213535 |
| 390 | 402 | 626 | 629 | 708 | 471 | 986 | 1128 | 1721 | 2592 | 13214 | 12571 | | 350335 |
| 140 | 187 | 252 | 138 | 269 | 114 | 374 | 388 | 307 | 1165 | 4590 | 4371 | | 151204 |
| 94 | 112 | 274 | 176 | 244 | 207 | 530 | 472 | 350 | 1104 | 4990 | 4717 | | 107096 |
| 161 | 194 | 468 | 268 | 442 | 327 | 453 | 716 | 824 | 1811 | 7328 | 6863 | | 165725 |
| 303 | 308 | 574 | 416 | 624 | 787 | 1242 | 963 | 1025 | 2218 | 11265 | 10404 | | 255810 |
| 261 | 313 | 836 | 545 | 611 | 748 | 766 | 1500 | 1591 | 2369 | 12443 | 11613 | | 256442 |
| 203 | 275 | 684 | 427 | 536 | 384 | 515 | 1333 | 1276 | 1773 | 10099 | 8988 | | 207481 |
| 381 | 505 | 1134 | 1051 | 1123 | 1239 | 780 | 3026 | 3076 | 3252 | 20504 | 18301 | | 402180 |
| 358 | 358 | 745 | 326 | 583 | 424 | 1000 | 1030 | 916 | 2389 | 11297 | 9870 | | 317998 |
| 489 | 329 | 721 | 887 | 942 | 983 | 650 | 1748 | 2810 | 2426 | 15022 | 13314 | | 235159 |
| 186 | 188 | 306 | 430 | 295 | 172 | 476 | 533 | 631 | 1066 | 5589 | 4907 | | 160984 |
| 242 | 375 | 1033 | 734 | 753 | 997 | 588 | 1763 | 2313 | 2417 | 14427 | 12416 | | 313637 |
| 180 | 163 | 333 | 377 | 299 | 142 | 363 | 429 | 452 | 1166 | 5218 | 4322 | | 150235 |
| 208 | 269 | 456 | 212 | 396 | 248 | 467 | 774 | 592 | 1496 | 6851 | 5503 | | 191181 |
| 280 | 429 | 810 | 347 | 648 | 573 | 759 | 1569 | 1121 | 2391 | 12192 | 9269 | | 300443 |
| **45** | 54 | 89 | 33 | 114 | 51 | 129 | 108 | 113 | 330 | 1639 | 1066 | | 54526 |
| 234 | **223** | 435 | 244 | 515 | 503 | 407 | 1174 | 1393 | 1379 | 8580 | 6273 | | 249876 |
| 280 | 268 | **565** | 501 | 760 | 657 | 811 | 1091 | 1950 | 2148 | 11510 | 8483 | | 219575 |
| 157 | 116 | 273 | **122** | 198 | 141 | 354 | 318 | 297 | 782 | 4059 | 2212 | | 96719 |
| 479 | 542 | 1045 | 957 | **909** | 929 | 1172 | 2726 | 2149 | 3701 | 20041 | 11586 | | 453852 |
| 175 | 239 | 655 | 538 | 509 | **461** | 486 | 1120 | 1566 | 1434 | 9512 | 5067 | | 219155 |
| 377 | 386 | 991 | 711 | 825 | 1125 | **1023** | 2425 | 2320 | 3021 | 16476 | 8789 | | 373684 |
| 151 | 130 | 215 | 274 | 240 | 178 | 332 | **468** | 623 | 973 | 4722 | 2064 | | 130729 |
| 186 | 200 | 340 | 363 | 294 | 162 | 412 | 589 | **502** | 1079 | 5458 | 1581 | | 139116 |
| 142 | 262 | 590 | 608 | 414 | 478 | 409 | 1343 | 998 | **1459** | 9059 | 1459 | | 193116 |
| 16886 | 17409 | 36172 | 36669 | 36980 | 40128 | 45297 | 76559 | 88971 | 126049 | 687387 | 623909 | 81,53% | 15309131 |



It should be made clear how the aggregated centers ("COROP"-regions) should be ranked in order to compute t, the (aggregated) sectors having of course a fixed ranking (according to the number of units observed in each sector; see table 2, last row). Three possible criteria could be proposed (amongst others, for sure) :

(a) a ranking according to the maximal number of units in the ranked sector (specialisation);

(b) the average rank of the aggregated centers in terms of sector ranks;

(c) minimal l; as the sector ranking is fixed, this leads to a linear assignment problem (LAP), of which the solution can be computed by means of linear programming[4].

Table 3 shows the obtained rankings of the COROP-regions, the corresponding t-values and the probabilities p. The correspondence between (a) and (b) on the one hand, (c) on the other, is not perfect, but the (b)-rule comes closer to (c) than (a), according to the t-values which play the role of e.g. a Kendall-τ rank correlation coefficient. If one opts for a triangular distribution, a chance factor of 15% is present in the (c)-ranking, but given the nature of the aggregate data described above, they do not contradict a fundamental STH-tendency.

4) Garfinkel and Nemhauser, 1972, p. 73-74; the problem can be written indeed as :

$$\max \Sigma_{i=1-43}\Sigma_{j=1-43}x_{ij}\Sigma_{k\geq j}m_{ik} \qquad (5)$$

s.t.

$$Jx = i \qquad (6)$$

$$\dot{x}x = x \qquad (7)$$

(6) being the so-called assignment-conditions and (7) the 0-1 constraints, which can be replaced by :

$$0 \leq x \leq i \qquad (8)$$

$x_{ij}$ assigns a spatial unit i to rank j, the $m_{ik}$'s summed being the (shrinking) sum of row elements of table 2 as i occupies a lower rank.

J is the well-known assignment matrix, x the vector of the $x_{ij}$'s, i the unit column vector.

TABLE 3 : results for criteria (a), (b) and (c)

| Criteria | | (a) | (b) | (c) |
|---|---|---|---|---|
| Ranked regions | | 29a | 17 | 29a |
| | | 36 | 29a | 17 |
| | | 17 | 23a | 33 |
| | | 23 | 36 | 23a |
| | | 14 | 33 | 34 |
| | | 34 | 15 | 36 |
| | | 39 | 39 | 39 |
| | | 2 | 13 | 15 |
| | | 12 | 26 | 12 |
| | | 23a | 12 | 26 |
| | | 10 | 34 | 13 |
| | | 28 | 23 | 3 |
| | | 26 | 30 | 23 |
| | | 15 | 14 | 18 |
| | | 13 | 18 | 25 |
| | | 4 | 25 | 30 |
| | | 33 | 3 | 35 |
| | | 35 | 28 | 14 |
| | | 37 | 35 | 10 |
| | | 30 | 24 | 4 |
| | | 25 | 10 | 38 |
| | | 38 | 35a | 16 |
| | | 18 | 37 | 35a |
| | | 16 | 4 | 28 |
| | | 24 | 32 | 24 |
| | | 27 | 16 | 37 |
| | | 3 | 19 | 19 |
| | | 32 | 21 | 32 |
| | | 21 | 38 | 21 |
| | | 35a | 27 | 27 |
| | | 19 | 40 | 29 |
| | | 29 | 29 | 6 |
| | | 40 | 22 | 40 |
| | | 6 | 20 | 9 |
| | | 9 | 6 | 20 |
| | | 20 | 9 | 8 |
| | | 11 | 7 | 7 |
| | | 22 | 8 | 22 |
| | | 31 | 1 | 31 |
| | | 7 | 31 | 1 |
| | | 8 | 11 | 11 |
| | | 1 | 5 | 5 |
| | | 5 | 2 | 2 |
| t | | .8661 | .8762 | .8814 |
| p | rectangular | .4455 | .4101 | .3912 |
| | triangular | .1989 | .1711 | .1544 |



The biproportional adjustment, followed by the (c)-programme, resulted in a t-value of .8814, very near to the (c)-value itself (.8819). The minimising LAP however lead to a t-value of .3848; recomputing the rectangular and triangular probabilities for the (c)-case gave values of .1920 and .0365 respectively. which only reinforced the statement made above.

## *5. Conclusions.*

Another type of data base, starting from elementary technical units, would be a much better entry to WTH and STH checking; some census data provide them, and they could be exploited along the lines exposed and implemented above.

Other lines of investigation would be to check on the size of the plants concerned (employment, production) to analyse the nature of joint plant location; again more refined data would be necessary.

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## 2. On Two Problems in the Analysis of Tinbergen-Bos Systems

TABLE OF CONTENTS

## 1. Introduction.

Tinbergen-Bos Systems (TBS) continue to pose interesting problems; we recently encountered two of them, on the solution of which we report hereafter.

The first problem concerns inadmissible systems, i.e. systems not delivering all goods and services demanded by the market.

The second derives from the computation of metricised TBS, a problem in mixed integer-continuous linear programming.

## 2. Inadmissible systems.

In Paelinck, 1995, an overview was given of TBS theory; briefly it considers systems - i.e. sets - of centers, the latter being defined as (spatial) clusters of plants belonging to different activity sectors. The number of possible systems for I activity sectors was given as :

$$s_I = 2^{c_I} - 1 - e_I \tag{1}$$

with :

$$c_I = 2^I - 1 \tag{2}$$

$e_I$ being the number of excluded or inadmissible systems. No analytical expression of it is known to us, but hereafter we derive an upper bound.

Consider the following example for I = 4; we first construct matrix **A**, linking the number of centers ($c_I$ = 15) to the total number of possible systems ($2^{c_I} - 1 = 32,767$) :

TABLE 1 : Center-System Matrix

| $S_i \backslash C_i$ | 1 | 2 | 3 | . | . | . | 15 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | . | . | . | 0 |
| 2 | 0 | 1 | 0 | . | . | . | 0 |
| 3 | 0 | 0 | 1 | . | . | . | 0 |
| . | | | | | | | |
| . | | | | | | | |
| . | | | | | | | |
| 15 | 0 | 0 | 0 | . | . | . | 1 |
| 16 | 1 | 1 | 0 | . | . | . | 0 |
| . | | | | | | | |
| . | | | | | | | |
| . | | | | | | | |
| $S_I$ | 1 | 1 | 1 | . | . | . | 1 |

We then construct matrix **B**, linking the number of centers to the number of activities :



TABLE 2 : Center-Activity Matrix

| $C_i \backslash I_i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 |
| 4 | 0 | 0 | 0 | 1 |
| 5 | 1 | 1 | 0 | 0 |
| 6 | 1 | 0 | 1 | 0 |
| 7 | 1 | 0 | 0 | 1 |
| 8 | 0 | 1 | 1 | 0 |
| 9 | 0 | 1 | 0 | 1 |
| 10 | 0 | 0 | 1 | 1 |
| 11 | 1 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 1 |
| 13 | 1 | 0 | 1 | 1 |
| 14 | 0 | 1 | 1 | 1 |
| 15 | 1 | 1 | 1 | 1 |
| Number of zeros | 7 | 7 | 7 | 7 |

The number of zeros in each column is equal to 7, i.e. $2^3 - 1 = c^{I-1}$; this result is derived from the fact that zeros and ones are distributed in an anti-symmetrical way inside the distribution of center types, there being one more one due to the absence of empty centers.

Consider now $\mathbf{C} = \mathbf{A}.\mathbf{B}$, an $s_I x I$ matrix, which shows the activities present in each type of system. The rows of the the first (identity) submatrix of **A** hit 7 times the zeros of a column of **B**; the rows of the second submatrix hit them $\binom{7}{2}$ times. It is obvious that once a row of **A** has 8 or more ones, a one in **B** will always be hit.

This allows of drawing the following table of upper bounds for inadmissible systems, as double counting is present between columns of **B**; the number in the parenthesis of e(.) relates to the number of ones in the rows of the submatrices of **A**.

TABLE 3 : Inadmissible Systems, Upper Bounds

| **e** | Formula | I = 4 | Exact Number |
|---|---|---|---|
| e(1) | $I\binom{c_{I-1}}{1}$ | 28 | 14 |
| e(2) | $I\binom{c_{I-1}}{2}$ | 84 | 66 |
| e(3) | $I\binom{c_{I-1}}{3}$ | 135 | ? |
| ........ | ........ | ........ | ........ |
| $e_I$ | $I(2^{c_{I-1}} - 1)$ | 508 | ? |

One has in general :

$$e_I < I.s_{I-1} \qquad (3)$$

hence the following proposition :

<u>Proposition 1</u> : $e_I/s_I$ tends towards zero for I large.

Proof : for large I, $e_I/s_I$ tends towards $I/2^{c_I - c_{I-1}} = I/2^{2^{I-1}}$, from which convergence to zero is obvious.*



3. Computing metricised TBS-equilibria.

Kuiper, Kuiper and Paelinck, 1993, presented the equations for solving a TBS metricised system; in condensed form, they can be presented as follows :

$$\min_{\mathbf{x},\mathbf{d}} f = \mathbf{w}'\mathbf{x} \qquad (4)$$

s.t.

$$\mathbf{J}\mathbf{x}^{\#} = \mathbf{c} \qquad (5)$$

where :

$$\mathbf{x}^{\#} = [\ \mathbf{x}'\ ;\ \mathbf{d}'\ ]' \qquad (6)$$

and:

$$\mathbf{i}'\mathbf{d}_k = n_k\ ,\ \forall\ k \qquad (7)$$

Here **x** is a column vector of intra- and intercenter flows, **d** a vector of possible locations for the producers of each product k (numbering $n_k$) and **c** a vector of constants; the elements of **d** are integers.

Now from (5) a number of variables can be substituted via :

$$\mathbf{x}_2^{\#} \in \mathbf{J}_1^{-1}(-\mathbf{J}_2\mathbf{x}_1^{\#} + \mathbf{c}) \qquad (8)$$

Let $\mathbf{x}_2^{\#}$ contain **d** and the vector of $x_{iik}$s, the deliveries inside an elementary area, at transport cost zero; substituting (8) into (4) one obtains :

$$\min_{\mathbf{d},\mathbf{x}_{ii},\mathbf{x}_{ij}} f = \alpha'\mathbf{d} + \alpha_1'\mathbf{x}_{ii} + \alpha_2'\mathbf{x}_{ij} \qquad (9)$$

subject to the constraints mentioned above. Consider now :

$$\alpha_2'\mathbf{x}_{ij} = (\mathbf{w}_2 - \mathbf{w}_1\mathbf{J}^{-1}\mathbf{J}_2)\mathbf{x}_2 \qquad (10)$$

and in (4) the total differential, for a fixed value of f :

$$d\mathbf{w}'\mathbf{x} = \mathbf{w}_1 d\mathbf{x}_{ij1} + \mathbf{w}_2 d\mathbf{x}_{ij2} = 0 \qquad (11)$$

From (10) and (8) follows immediately that :

$$\alpha_2' d\mathbf{x}_{ij} = 0 \qquad (12)$$

which holds for $\alpha_2'\mathbf{x}_{ij}$, given linearity.

This leads to the following proposition :

Proposition 2 : problem (4) trough (7) can be solved by solving only a linear problem in **d** and $\mathbf{x}_{ii}$ under the constraints (7), integrality for the elements of **d**, and binarity for the elements of $\mathbf{x}_{ii}$; moreover $d_{ik} > 0 \Rightarrow x_{ii} = 1$ (Kuiper, Mares and Paelinck, 1995).

The new system, comprising only the above-mentioned constraints (the last one being substituted by $d_{ik} \geqslant x_{i,ik}$), can be solved by linear programming, which increases the possibilities of computing solutions to metricised TBS for large k and a large number of possible locations.

The following example illustrates the foregoing; take three possible locations equally spaced on a line, with unit transport costs equal to the line-lenghts to be bridged (1 and 2), in which case :

$$f = x_{21} + 2x_{31} + x_{12} + x_{32} + 2x_{13} + x_{23} \qquad (13)$$

together with the demand and supply constraints (5).

Substituting along the lines exposed above gives :

$$f = -1.5d_2 - 2x_{11} - 2x_{33} + 5 \qquad (14)$$

which for $n_1 = 2$ has a minimum equal to 1 for $x_{11} = x_{33} = 1$ (1/2 being transported from locations 1 and 3 to location 2). If we put the 1-3 unit transport costs equal to 3 instead of 2, the objective function becomes

$$f = -3d_2 + x_{22} - 3x_{11} - 3x_{33} + 7 \qquad (15)$$

with the same optimal solutions as above.



4. References.

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## 3. On Solving the Maximal Flow Capturing Problem by Means of Continuous Linear Programming

TABLE OF CONTENTS

## 1. Introduction.

The maximal flow capturing problem (MFCP) is known to be a binary linear program (see e.g. Hodgson, 1990; Hodgson et al. 1995 and forthcoming; Berman et al., 1992).

In what follows a respecification will be presented, which should allow of using continuous linear programming for its solution.

## 2. Specifications.

The main structure of the MFCP can be presented as follows (Table 1) :

Table 1 : Fundamental Matrix for the MFCP

| $\mathbf{f}^*$ | $\mathbf{f/n'}$ | 1 | 2 | 3 | . . . . . . | $j^*$ | $\mathbf{m}^*$ |
|---|---|---|---|---|---|---|---|
| $f^*_1$ | 1 | | | | | | $m^*_1$ |
| $f^*_2$ | 2 | | | | | | $m^*_2$ |
| $f^*_3$ | 3 | | | | | | $m^*_3$ |
| . | . | | | | $\mathbf{S}^*$ | | . |
| . | . | | | | | | . |
| . | . | | | | | | . |
| $f^*_{i*}$ | $i^*$ | | | | | | $m^*_{i*}$ |
| | $\mathbf{n}^{*\prime}$ | $n^*_1$ | $n^*_2$ | $n^*_3$ | | $n^*_{j*}$ | $\mathbf{i'n}^* = \mathbf{i'm}^*$ |

Each row shows in a binary way whether a flow passes through certain nodes ($n_j$), each column showing the flows ($f_i$) which pass through a given node. The respective row and column totals are $m^*_i$ and $n^*_j$. A column vector $\mathbf{f}^*$ shows the flow intensities.

The problem can be stated in the following way :

$$\max_{\mathbf{y}} g = \mathbf{f}^{*\prime}\mathbf{y} \qquad (1)$$

s.t.

$$\mathbf{y} \leq \mathbf{S}^{*}\mathbf{x} \qquad (2)$$

$$\mathbf{i}'\mathbf{x} = p^{*} \qquad (3)$$

$$\mathbf{y},\mathbf{x} \in \{ 0 , 1 \} \qquad (4)$$

One wants to maximise the number of flows meeting fixed objects located in a given number $p^*$ of nodes, those flows being dependent on the fact that they pass through the (optimally) selected nodes, **x**.

It is known that continuous linear programming (CLP), { 0 , 1 } being replaced by [ 0 , 1 ], can produce fractional solutions in **x**; a simple 5x5 example will show this.

Consider the following 5x5 $\mathbf{S}^*$-matrix (Table 2) :

Table 2 : An $\mathbf{S}^*$-matrix

| $f^*$ | f\n' | 1 | 2 | 3 | 4 | 5 | $m^*$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 2 |
| 2 | 2 | 0 | 1 | 1 | 0 | 0 | 2 |
| 3 | 3 | 0 | 0 | 1 | 1 | 0 | 2 |
| 4 | 4 | 0 | 0 | 0 | 1 | 1 | 2 |
| 5 | 5 | 0 | 0 | 1 | 0 | 1 | 2 |
| | $n^{*\prime}$ | 1 | 2 | 3 | 2 | 2 | 8 |

For $p^* = 2$, an optimal binary solution is the $\mathbf{x}'$-vector [ 0 0 1 1 0 ] giving an objective function value of g = 14. However the vector $\mathbf{x}' = 1/2$ [ 0 1 1 1 1 ] satisfies the constraints and allows of obtaining g = 14.5.



Consider now the matrix :

$$\mathbf{Y} = \mathbf{S}^{*}\hat{\mathbf{x}} \qquad (5)$$

It could lead to the following specification of the problem :

$$\max g = \mathbf{f}^{*\prime}\mathbf{y} \qquad (6)$$

s.t.

$$\mathbf{y} \leq \mathbf{Y}\mathbf{i} \qquad (7)$$

$$\mathbf{i}'\mathbf{Y}\hat{\mathbf{n}}^{*-1}\mathbf{i} = p^{*} \qquad (8)$$

$$\mathbf{y},\ \mathbf{Y},\ \in \{ 0 , 1 \} \qquad (9)$$

Instead of producing weighted solutions as specification (1) through (4) does, CLP now "samples" flows from various nodes, as the following example shows (Table 3) :

Table 3 : A **Y**-matrix

| $f^*$ | f\n | 1 | 2 | 3 |
|---|---|---|---|---|
| 1 | 1 | $y_{11}$ | $y_{12}$ | |
| 2 | 2 | $y_{21}$ | | $y_{23}$ |
| 3 | 3 | $y_{31}$ | $y_{32}$ | $y_{33}$ |
| 4 | 4 | $y_{41}$ | $y_{42}$ | |
| 5 | 5 | | $y_{52}$ | $y_{53}$ |
| 6 | 6 | $y_{61}$ | $y_{62}$ | $y_{63}$ |
| 7 | 7 | $y_{71}$ | | $y_{73}$ |
| 8 | 8 | | $y_{82}$ | $y_{83}$ |
| 9 | 9 | $y_{91}$ | $y_{92}$ | $y_{93}$ |

For $p^* = 1$, the optimal one-node solution is 1 for node 3, with $g = 40$; CLP, however, drops flow $y_{23}$ and replaces it by $y_{42}$, producing $g = 42$, all active $y_{ij}$s being equal to one.

It is known also that a "naive" method, picking succesively the nodes ranked downwards according to their total flow intensity, does not necessarily produce an optimum. Consider the following example, given only for illustrative purposes, as some flows are duplicated and others "singletons" (Table 4) :

Table 4 : Again an $\mathbf{S}^*$-matrix

| $\mathbf{f}^*$ | $\mathbf{f\backslash n}$ | 1 | 2 | 3 |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 |
| 2 | 2 | 1 | 0 | 1 |
| 3 | 3 | 1 | 1 | 0 |
| 4 | 4 | 1 | 0 | 1 |
| 5 | 5 | 1 | 1 | 0 |
| 6 | 6 | 0 | 1 | 0 |
| 7 | 7 | 0 | 0 | 1 |
| Total flows | | 15 | 14 | 13 |

For $p^* = 2$, the naive method would select nodes 1 and 2, and so would a "semi-naive" method, which would eliminate the flows already encountered; in this case it would start with node 1, eliminate flows 1 through 5, and then pick up node 3. The optimal solution obviously comprises nodes 2 and 3; table 5 compares the values of the objective function.

Table 5 : Values of an Objective Function

| Method | Value of g |
|---|---|
| Naive | 21 |
| Semi-naive | 22 |
| Optimal | 27 |



What triggers off some thinking about using CLP for the solution of the MFCP is the set of inequalities (2); in fact often their right hand side exceeds the left hand side, which is contrary to the logic of the problem, according to which at least (but also at most) one flow should be captured once by an optimal node choice. This invites to reconsider specification (1) through (4) and replace it by the following one :

$$\max_{\mathbf{y}} g = \mathbf{f}^{*\prime}\mathbf{y} \tag{10}$$

s.t.

$$\mathbf{y} \leqslant \mathbf{max}\ \mathbf{S}^*\hat{\mathbf{x}} \tag{11}$$

$$\mathbf{i}'\mathbf{x} = p^* \tag{12}$$

$$\mathbf{0} \leqslant \mathbf{x} \leqslant \mathbf{i} \tag{13}$$

Given the binary nature of $\mathbf{S}^*$, obviously $\mathbf{0} < \mathbf{y} < \mathbf{i}$. The logic is that, (10) having to be maximised, $\mathbf{y}$ will always tend to hit an upper ceiling, obliging the $x_{ij}$s to produce 0-1 solutions.

In continuous practice, (11) can be realised by introducing a vector of slack variables, $\mathbf{e}$, and extending the objective function (10) by a penalty term $+\alpha\mathbf{i}'\mathbf{e}$ ($\alpha$ > $\mathbf{min}\ \mathbf{f}^*$); two additional conditions have to be introduced :

$$\mathbf{0} \leqslant \mathbf{y} \leqslant \mathbf{i} \tag{14}$$

$$\mathbf{0} \leqslant \mathbf{e} \leqslant (p^* - 1)\mathbf{i} \tag{15}$$

and (11) replaced by :

$$\mathbf{y} \leqslant \mathbf{S}^*\mathbf{x} - \mathbf{e} \tag{16}$$

the logic of (15) being that at most $p^*$ redundancies can occur.

3. Applications.

Taking up again Table 2, one gets for $p^* = 2$ the following programme :

$$\max_{\mathbf{y}} g = y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5 \quad (17)$$

subject to the conditions mentioned above. Recall that if conditions (14) through (16) are ignored, the optimal solution is fractional ($\mathbf{x}' = 1/2$ [ 0 1 1 1 1]), producing g = 14.5, and that the optimal binary solution is $\mathbf{x}'$ = [ 0 0 1 1 0 ], generating g = 14. Fixing $\alpha$ at 1.1 produces the latter solution with $e_3 = 1$ and g = 15.1, splitting up into 14 + 1.1 > 14.

Table 4 did not need to be **e**-corrected, as CLP immediately produced the desired binary solution.

4. Conclusions.

More experience is needed with the method, especially in fixing appropriate values for $\alpha$. It could then be applied to empirical - i.e. large - $\mathbf{S}^*$-matrices.

5. References.

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## 4. On Optimal Control of Potentialised Partial Differential Equations

TABLE OF CONTENTS

## 1. Introduction.

In Kaashoek and Paelinck (references in Section 6) results were presented on (systems of) potentialised partial differential equations (PPDEs). Potentialising (global spatial discounting, say) was used to adapt the use of PDEs (recent references also in Section 6) to (theoretical) spatial economics, where generalised spatial interaction plays an important part.

In this paper we will try to introduce optimal control (see recent references in Section 6) to steer the spatial (one- or more-dimensional) process to a given target (for example the null-solution, possibly but not necessarily supposing mass conservation, that constant mass being zero in certain applications). We first present a discrete example, then generalise it to the continuous case, and finally adapt the solution to a second order PPDE.

Conclusions follow.

## 2. Discrete example.

Consider the following quadratic programming problem :

$$\min \phi \equiv \tfrac{1}{2} [u_1^2(0) + u_2^2(0) + u_1^2(1) + u_2^2(1)] \tag{1}$$

the figures between parentheses referring to time periods, the subindices to sites or regions, the **u**-vector being one of control variables; (1) could be interpreted as cost minimisation, perfect cost symmetry having been introduced for reasons of simplicity.

We now suppose region 1 to be at level 2, region 2 at level -1, and we want to control them to levels 0 in period 3; this leads to :

$$u_1(0) + u_1(1) + \tfrac{1}{2} [u_2(0) + u_2(1)] = -2 \tag{2a}$$

$$\tfrac{1}{2} [u_1(0) + u_1(1)] + u_2(0) + u_2(1) = 1 \tag{2b}$$

The special <u>spatial</u> feature here is the fact that control on site i influences the dynamics on site j, and vice versa, be it with a spatial discounting factor (here ½).

Lagrangean optimisation leads to the system :

$$\begin{bmatrix} I & -A \\ A' & 0 \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ a \end{bmatrix} \tag{3}$$

from which :

$$u° = A\lambda \qquad (4)$$

a linear combination of the lagrangean multipliers, and :

$$\lambda = (A'A)^{-1}a \qquad (5)$$

For the above mentioned problem, the outcomes are **λ'** = [-3.11,2.89] and **u'** = [-1.66,1.34,-1,66,1.34], which in this special case (no time discount) gives a constant vector of the local control variables. Here and further down it can be checked that the second order conditions are satisfied.

## 3. Continuous version.

Consider now the continuous analog of problem (1)-(2) :

$$\min \phi \triangleq \tfrac{1}{2} \int_0^T \int_a^b u^2(x,t)\,dx\,dt \qquad (6)$$

s.t. :

$$\dot f(x,t) = \int_a^b w(x,\xi)u(\xi,t)\,d\xi \qquad (7)$$

The Hamiltonian is :

$$H \triangleq \int_a^b \tfrac{1}{2}u^2(x,t)\,dx + \int_a^b \lambda(\eta,t) \int_a^b w(x,\xi)u(\xi,t)\ d\xi\ d\eta \qquad (8)$$

whence, for given x :

$$\delta H/\delta u(x,t) = u(x,t) + \int_a^b w(x,\xi)\lambda(\xi,t)\,d\xi = 0 \qquad (9)$$

and :

$$u°(x,t) = -\dot\lambda(x,t) \qquad (10)$$

which shows that, like in the discrete case, the optimal control u°(x,t) is a linear combination of the costate variables.

Furthermore :

$$-\delta H/\delta f(x,t) = \dot\lambda(x,t) = 0 \qquad (11)$$



hence λ(x,t) is a constant for given x, implying :

$$\dot\lambda(x,t) = \dot\lambda(x) \qquad (12)$$

The transversality conditions finally impose :

$$\lambda(x,T) = 0,\ \forall x \qquad (13)$$

hence all u°(x,T) = 0.

From (10) and (12) one derives that :

$$u°(x,t) = -\dot\lambda(x) \qquad (14)$$

so, putting :

$$\int_a^b w(x,\xi)u°(\xi)\,d\xi \triangleq \bar u°(x) \qquad (15)$$

one obtains, integrating (7) with respect to time :

$$\bar u°(x) = -f(x,0)/T = \dot f(x) \qquad (16)$$

Equations (10) and (16) should allow of computing the local λ(x) and u°(x) (compare equations (4) and (5)).

## 4. Second order PPDE.

One can now start from a potentialised partial differential equation, e.g. the one studied in Kaashoek and Paelinck (o.c.) :

$$\ddot f(x,t) = \alpha \int_a^b m(x,\xi)f''(\xi,t)\,d\xi \qquad (17)$$

for which explicit solutions were derived (Kaashoek and Paelinck, 1994a) for different specifications of the weighting function m(x,ξ), so, omitting unnecessary details :

$$\ddot f(x,t) = \alpha\ f(x,t)\ g(x) \qquad (18)$$

Substituting $\dot f \equiv h$, and adding a potentialised control to (18), gives the system :

$$\dot{h}(x,t) = \alpha\, f(x,t)\, g(x) + \beta \int_a^b w(x,\xi) u(\xi,t) d\xi \qquad (19a)$$

$$\dot{f}(x,t) = h(x,t) \qquad (19b)$$

and adding now a time-discount function to (6) gives the Hamiltonian :

$$H \equiv \tfrac{1}{2} \int_a^b u^2(x,t) \exp(-\gamma t) + \int_a^b \lambda(\eta,t) \left[\alpha\, f(x,t)\, g(x) + \beta \int_a^b w(\xi,t) u(\xi,t) d\xi\right] d\eta + \mu(x,t)\, h(x,t) \qquad (20)$$

giving :

$$\delta H/du(x,t) = u(x,t) \exp(-\gamma t) + \beta \int_a^b w(\xi,t) \lambda(\xi,t) d\xi = 0 \qquad (21)$$

whence :

$$u^{\circ}(x,t) = -\beta \exp(\gamma t)\, \dot{\lambda}(x,t) \qquad (22)$$

in line with (10), but with the addition of a relative efficiency factor $\beta$ and the (inverse) time-discounting function (as discounted costs decrease with time, controls should logically increase over time).

Furthermore one can derive :

$$-\delta H/\delta h(x,t) = \dot{\lambda}(x,t) = -\mu(x,t) \qquad (23)$$

and :

$$-\delta H/\delta f(x,t) = \dot{\mu}(x,t) = -\alpha\, g(x,t)\, \lambda(x,t) \qquad (24)$$

the transversality conditions giving again for $\forall x$ that :

$$\lambda(x,T) = 0 \qquad (25a)$$

$$\mu(x,T) = 0 \qquad (25b)$$

so controls are put to zero in T.



Solving (23) and (24) gives :

$$\lambda(x,t) = \pi_1(x) \exp[\epsilon_1(x)t] + \pi_2(x) \exp[\epsilon_2(x)t] \qquad (26)$$

and :

$$\mu(x,t) = \rho_1(x) \exp[\epsilon_1(x)t] + \rho_2(x) \exp[\epsilon_2(x)t] \qquad (27)$$

where $\epsilon_1$ and $\epsilon_2$ are the eigenvalues of system (23)-(24).

Combining (22) and (26) one sees that this time the controls are combinations, additive and multiplicative, of exponential functions of time.

## 5. Conclusions.

We have shown that classical optimal control techniques can be applied to PPDEs; generalisations to two spatial dimensions and to systems of PPDEs is immediate, and in this way models in spatial variables (regional models, urban models) can be treated in order to select optimal development and physical planning policies. The important point, we recall, is the spatial interdependence of state and control variables.

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