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**INTERNALIZING EXTERNALITIES
IN SECOND-BEST TAX SYSTEMS**

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Abstract

We analyze the analytical structure of optimal taxation of polluting and non-polluting goods in a second-best world where lump-sum taxes are infeasible. After deriving the environmental tax rate which exactly internalizes the external effect, we show how to separate the analysis of second-best optimal environmental taxes from the analysis of the tax structure which minimizes the excess burden. This separation reveals that standard results of optimal taxation essentially carry over to economies with externalities. Our approach clarifies the controversy regarding the relation between the first-best Pigovian tax rate on polluting goods and the optimal tax rate in a second-best world.

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I. Introduction

The double-dividend hypothesis has recently become a very popular argument both in the academic and in the political debate. Green taxes are expected to improve the quality of the environment *and* to reduce the distortions of the existing tax system. It is therefore widely accepted that the existence of a double dividend in a world with distortionary taxes makes green taxes superior to other environmental instruments which are considered to be efficient in a world without distortions.

A controversy, however, has emerged in the literature with respect to the question about the magnitude of the second-best optimal tax on a polluting good. As second-best theory suggests, first-best efficiency conditions will not hold if there are other distortions present in the economy. The early literature on the double dividend claims that, due to distortionary taxes, taxes on polluting goods should be higher than the so-called Pigovian tax which is equal to the marginal environmental damage (cf. Nichols 1984, Lee and Misiolek 1986). More recently, however, Bovenberg and de Mooij (1994) have argued that the opposite is true.

This ambiguity has caused considerable confusion within the profession. It seems to be unclear whether in second-best situations, characterized by distortionary taxes, optimal taxes on polluting goods should be higher or lower than the first-best Pigovian tax associated with the same allocation. Furthermore, the difference between first-best and second-best optimal taxes has mistakenly been interpreted as a measure of the "second dividend" in the sense that the second dividend is negative if the second-best optimal tax on a polluting good is lower than the corresponding Pigovian tax.

The difference in the results concerning the optimal tax rate on a polluting good is, as shown in Schöb (1994), due to different normalizations of tax rates which lead to different definitions of what the tax on a polluting good actually is. Hence, a comparison between the

second-best optimal tax on a polluting good and the first-best Pigovian tax associated with the same allocation turns out to be rather meaningless.

This paper tries to eliminate the confusion surrounding these issues by suggesting a measure of a tax which is imposed on the polluting good for environmental reasons only. Clarifying the question of what the concept of full internalization means in a second-best framework, we define an internalization tax on the polluting good which depends exclusively on the real allocation and consequently is independent of the normalization chosen. It is therefore a suitable reference standard for what may be considered the environmental tax.

The internalization tax or Pigovian component of the second-best optimal tax on a polluting good is always *lower* than the first-best Pigovian tax associated with the same allocation, provided the marginal cost of public funds exceeds unity, because a tax dollar is worth more than a private dollar. This does not imply, however, that the second-best optimal tax on the polluting good yields no "second dividend". The second dividend stems from the fact that the internalizing tax simultaneously helps to finance the public good and presumably reduces the excess burden. This fact has no immediate implication on the magnitude of the tax rate which fully internalizes the external effect. Thus, the magnitude of the internalizing tax rate allows no conclusions regarding the second dividend to be made.

Apart from the internalization tax, an additional tax on the polluting good may be imposed in order to minimize the excess burden of financing the (second-best) optimal quantity of the public good. This tax component, called the Ramsey component of the tax on a polluting good, can be analytically separated from the Pigovian component. Similar to Sandmo (1975), this definition implies that environmental tax components enter optimal tax formulas additively. Employing our definition of the environmental tax component, we can illuminate, by separating environmental considerations from minimizing the excess burden, the analytical structure of the optimal taxation of polluting and non-polluting goods in a second-best world. Furthermore, we can show that the standard results from the optimal

taxation literature carry over easily to the case with externalities. This holds for the case with one polluting good as well as for the case where multiple pollutants damage the environment.

The paper is organized as follows. Section II introduces the model. The concept of internalizing externalities within a second-best framework is discussed in Section III. Section IV then analyses the optimal tax structure for the three good case. Section V refers the results to the standard optimal taxation literature. Bovenberg and de Mooij's (1994) result is reconsidered in Section VI. In Section VII we generalize our results by considering the possibility of multiple externalities. Section VIII concludes.

II. The model

Consider a closed economy with N identical households, $C + D$ private consumption goods, a public good G and labour ℓ . The private goods $c \in \{1, \dots, C\}$ are clean (i.e. they have no external effect), whereas the private goods $d \in \{C + 1, \dots, C + D\}$ are dirty, i.e. their production or consumption create negative external effects which causes the environmental quality E to deteriorate.¹ In any equilibrium all households will consume identical consumption bundles $(x_1, \dots, x_C, \dots, x_{C+D})$ and supply an identical quantity x_ℓ of labour. The aggregate quantities are given by $(Nx_1, \dots, Nx_C, \dots, Nx_{C+D})$ and Nx_ℓ respectively. We assume that environmental quality is a decreasing function of the aggregate quantity Nx_d , produced and consumed, of each dirty good $d = C + 1, \dots, C + D$, i.e.

$$E = e(Nx_{C+1}, \dots, Nx_{C+D}), \quad e_i < 0, \quad (1)$$

with $e_i = de/d(Nx_i)$. When consuming a dirty good, the household does not take account of the negative effect on the environmental quality.

¹ Although we concentrate on *negative* external effects, it is also obvious that our analysis generalizes readily to a situation where both positive and negative external effects are present.

There is a linear technology for the production of the private goods and the public good, with labour being the only input. Assuming perfect competition, we can normalize the wage rate to unity and choose units for all goods such that all producer prices are equal to one. Then, production possibilities are described by

$$Nx_t = N \sum_c x_c + N \sum_d x_d + G. \quad (2)$$

The government provides the public good G . To finance it, the government raises either lump-sum taxes T from each household or taxes on labour and on commodities. The government's budget constraint is therefore given by:

$$G = N \sum_c t_c x_c + N \sum_d t_d x_d + N t_l x_l + NT, \quad (3)$$

where t_l denotes the tax rate on labour while t_c and t_d denote the commodity taxes on the clean and the dirty goods, respectively. T denotes a lump-sum tax. The benevolent government maximizes a utilitarian welfare function

$$W = Nu(x_1, \dots, x_C, \dots, x_{C+D}, x_v, G, E). \quad (4)$$

The preferences of a household with respect to both clean and dirty private goods, leisure x_v , the public good G , and the environmental quality E can be represented by a twice continuously differentiable, strictly quasi-concave utility function

$$U = u(x_1, \dots, x_C, \dots, x_{C+D}, x_v, G, E), \quad (5)$$

with $u_i > 0$, $i = 1, \dots, C + D, v, G, E$, denoting the marginal utility of good i . To simplify the analysis, we assume separability between private consumption and the environment E , and private consumption and the public good G , respectively. The time endowment is normalized to one, hence $x_v + x_l = 1$. The budget constraint of the household is given by

$$\sum_c (1+t_c)x_c + \sum_d (1+t_d)x_d = (1-t_l)(1-x_v) - T. \quad (6)$$

III. Optimal internalization of external effects

In the presence of externalities, Pareto efficiency requires the equality of *social* and *private* marginal welfare of consuming a dirty good. In a first-best environment this can be achieved by imposing a tax on a polluting good which equals the marginal environmental damage, i.e. the induced change in the environmental quality multiplied by the marginal rate of substitution between the environment and a numéraire. Such a Pigovian tax fully *internalizes* the external costs at the margin as the individual has to take account of all marginal costs resulting from her decision.

In a second-best framework we can apply the concept of *internalizing externalities* by looking for a tax rate for any given dirty good d , $t_d = t_d^E$, which would exactly internalize the external effect of this dirty good. Consider therefore the following thought experiment, which is a special – but for our purpose more intuitive case – of the Diamond (1975) approach to analyzing the welfare effect of marginal changes of private income, which we will consider below. One of the N households obtains an additional marginal unit of exogenous income Y . In the household optimum the household is indifferent to how to spend the additional income. Hence without loss of generality we assume that the household increases the consumption of d only, i.e. by $1/(1+t_d^E)$. The government uses the additional tax revenues to increase the supply of the public good by $t_d^E/(1+t_d^E)$. The effect of a marginal increase in income for one household on social welfare is therefore:

$$\frac{dW}{dY} = \frac{u_d + Nu_E e_d + Nu_G t_d^E}{1+t_d^E}. \quad (7)$$

The first term of the right-hand side denotes the increase in private utility while the second term denotes the external effect imposed on all households by the additional consumption of the dirty good d . The last term is the increase in all household's utility due to the additional provision of the public good G which is financed by the internalization tax imposed on the dirty good d .

Full internalization requires that the *private marginal utility* of consuming the dirty good, which is $du/dY = u_d/(1+t_d^E)$, is equal to the *social marginal welfare* of consuming the dirty good:

$$\frac{u_d}{1+t_d^E} = \frac{dW}{dY}. \quad (8)$$

The external effect is exactly internalized if and only if the tax on the dirty good is equal to $-u_E e_d / u_G$. Therefore, we define

$$t_d^E \equiv -\frac{u_E}{u_G} e_d \quad (9a)$$

as the component of the tax on the dirty good d the government has to impose in order to exactly internalize the external effect. It will be called the *Pigovian component* of t_d .

Within a first-best framework where lump-sum taxes are available and consequently $t_c = t_t = 0, \forall c \in \{1, \dots, C\}$ in the optimum, the budget constraint (3) becomes $G = \sum_d t_d^E x_d + NT$. Optimality requires that the marginal welfare of public expenditures, i.e. Nu_G is equal to the marginal welfare of private expenditures which for all clean goods $c \in \{1, \dots, C\}$ equals the marginal utility u_c (recall that all producer prices are normalized at unity). Hence, the first-best Pigovian tax component for good d becomes

$$t_d^E = -\frac{Nu_E}{u_c} e_d. \quad (9b)$$

This is the well-known Pigovian tax, which equals the marginal environmental damage measured in terms of private income (cf. e.g. Baumol and Oates 1988, p. 42).

If lump-sum taxes are not feasible, the dirty good might also be taxed for reasons other than internalizing the external effect. Denoting $t_d^R \equiv t_d - t_d^E$ as the *additional* tax on the dirty good on top of the Pigovian component (a justification of such a tax is given in section IV below), and allowing the household to allocate a marginal unit of additional income optimally

over all goods, we obtain the social marginal utility of private income as defined by Diamond [1975, his equation (6)]:

$$\begin{aligned} \frac{dW}{dY} &= Nu_G \left[\frac{u_Y}{Nu_G} + \frac{u_E e_d}{u_G} \frac{\partial x_d}{\partial Y} + t_c \frac{\partial x_c}{\partial Y} + (t_d^E + t_d^R) \frac{\partial x_d}{\partial Y} + t_t \frac{\partial x_t}{\partial Y} \right] \\ &= Nu_G \left[\frac{u_Y}{Nu_G} + t_c \frac{\partial x_c}{\partial Y} + t_d^R \frac{\partial x_d}{\partial Y} + t_t \frac{\partial x_t}{\partial Y} \right], \end{aligned} \quad (7)$$

where $u_Y \equiv du/dY$. For $t_d^R = 0$ and $\partial x_c / \partial Y = \partial x_t / \partial Y = 0$ this reduces to (7). The *social marginal utility of private income* now depends on the social evaluation of increased utility of the household made possible by higher income and the social evaluation of the additional tax revenues collected as a consequence of having more income, except for the additional tax revenues from the internalization tax component. As these revenues exactly compensate for the marginal environmental damage, this also shows that the definition (9a) gives the tax on the dirty good which exactly internalizes the external effect.

In a world where distortionary taxes exist, a dollar in the public purse is worth more (or less) than a dollar in the private purse. As the tax revenues from internalizing the external effect accrue at the governmental level, these tax revenues should be measured by the marginal welfare of public expenditures, i.e. the marginal welfare of the public good G . Consequently, the Pigovian component as defined in equation (9a) is measured in terms of public expenditures.²

To summarize this section: Measuring the social costs of pollution in this way allows for a more general definition of an optimal environmental tax which encompasses the first-best Pigovian tax as a special case. As already indicated above, in a second-best world this environmental tax is not necessarily the only tax on polluting goods. Therefore, we have to

² Notice that the equivalence is only true, if G is determined endogenously by the optimizing government. If G is exogenously fixed, the welfare change from additional public good provision and rebating additional tax revenues via reducing other taxes may differ.

analyze the optimal tax structure in a broader context where we will make use of the definition developed here.

IV. The optimal tax structure in a second-best framework

As a first step we focus on the three good case with one clean and one polluting consumption good and leisure. A generalisation of the results will be presented in Section VII. Assume that lump-sum taxes are not feasible. To derive the optimal tax structure, we make use of the indirect utility function $w(t_c, t_d, t_l, G, E)$, which already takes into account the utility maximizing behaviour of the household. With lump-sum taxes being infeasible, the government can only raise tax revenues by introducing taxes on the two private commodities or on labour. Hence, the government maximizes

$$W = Nw(t_c, t_d, t_l, G, E) = Nu[x_c(t_c, t_d, t_l), x_d(t_c, t_d, t_l), x_v(t_c, t_d, t_l), G, E], \quad (10)$$

with respect to t_c, t_d, t_l, G and subject to equations (2) and (3).³ Since (2) is implied by Walras' Law and since maximizing W/N is equivalent to maximizing W , we define the Lagrangian as

$$\mathcal{L}(t_c, t_d, t_l, G, \mu) = w(t_c, t_d, t_l, G, E) - \mu \left(\frac{G}{N} - t_c x_c - t_d x_d - t_l x_l \right). \quad (11)$$

Using Roy's identity the first-order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial t_c} = -\lambda x_c + u_E e_d N \frac{\partial x_d}{\partial t_c} + \mu \left(x_c + t_c \frac{\partial x_c}{\partial t_c} + t_d \frac{\partial x_d}{\partial t_c} + t_l \frac{\partial x_l}{\partial t_c} \right) = 0, \quad (12a)$$

$$\frac{\partial \mathcal{L}}{\partial t_d} = -\lambda x_d + u_E e_d N \frac{\partial x_d}{\partial t_d} + \mu \left(x_d + t_c \frac{\partial x_c}{\partial t_d} + t_d \frac{\partial x_d}{\partial t_d} + t_l \frac{\partial x_l}{\partial t_d} \right) = 0, \quad (12b)$$

³ Note that because of the separability between private consumption and the environment E , and private consumption and the public good G , respectively, the demand functions do not depend on E and G .

$$\frac{\partial \mathcal{L}}{\partial t_l} = -\lambda x_l + u_E e_d N \frac{\partial x_d}{\partial t_l} + \mu \left(x_l + t_c \frac{\partial x_c}{\partial t_l} + t_d \frac{\partial x_d}{\partial t_l} + t_l \frac{\partial x_l}{\partial t_l} \right) = 0, \quad (12c)$$

$$\frac{\partial \mathcal{L}}{\partial G} = u_G - \mu \frac{1}{N} = 0. \quad (12d)$$

Since x_c, x_d and x_l are each homogenous of degree zero in consumer prices, the equations (12a) – (12c) are linearly dependent.⁴ Hence, we can arbitrarily normalize the system of tax rates⁵ or, equivalently, the consumer prices. In particular, we can set any of the three tax rates equal to zero. As we will make use of different normalizations, we present all first order conditions here. The shadow price λ denotes the *private marginal utility of private income*. The shadow price μ equals the *marginal utility of public expenditures* [cf. equation (12d)]. Defining the *Ramsey component* t_d^R implicitly by

$$t_d = t_d^R + t_d^E, \quad (13)$$

we can rewrite the first order conditions:

$$-\lambda x_c + \mu \left(x_c + t_c \frac{\partial x_c}{\partial t_c} + t_d^R \frac{\partial x_d}{\partial t_c} + t_l \frac{\partial x_l}{\partial t_c} \right) = 0, \quad (14a)$$

$$-\lambda x_d + \mu \left(x_d + t_c \frac{\partial x_c}{\partial t_d} + t_d^R \frac{\partial x_d}{\partial t_d} + t_l \frac{\partial x_l}{\partial t_d} \right) = 0, \quad (14b)$$

⁴ Let $q_c = 1 + t_c$, $q_d = 1 + t_d$, and $q_l = 1 - t_l$ denote the consumer prices. Homogeneity implies

$$0 = q_c \frac{\partial x_i}{\partial q_c} + q_d \frac{\partial x_i}{\partial q_d} + q_l \frac{\partial x_i}{\partial q_l} = q_c \frac{\partial x_i}{\partial t_c} + q_d \frac{\partial x_i}{\partial t_d} - q_l \frac{\partial x_i}{\partial t_l}, \quad i = c, d, l.$$

Note that the household's budget constraint (6) with $T = 0$ implies: $\frac{q_c}{q_l} x_c + \frac{q_d}{q_l} x_d = x_l$. Multiplying (12a) by q_c/q_l , (12b) by q_d/q_l , making use of the homogeneity property and adding up gives (12c).

⁵ Given any tax vector $t = (t_c, t_d, t_l)$, the tax vector $t' = (t_c', t_d', t_l')$, where

$$t_c' = \frac{1+t_c}{\gamma} - 1, \quad t_d' = \frac{1+t_d}{\gamma} - 1, \quad t_l' = 1 - \frac{1-t_l}{\gamma}$$

and $\gamma > 0$, gives identical budget equations for all households and the government for any parameter $\gamma > 0$. Consequently, t' leads to the same allocation as t . With $\gamma = 1 + t_c$, we get $t_c' = 0$, with $\gamma = 1 - t_l$, we get $t_l' = 0$.

$$-\lambda x_t + \mu(x_t + t_c \frac{\partial x_c}{\partial t_t} + t_d^R \frac{\partial x_d}{\partial t_t} + t_t \frac{\partial x_t}{\partial t_t}) = 0, \quad (14c)$$

$$Nu_G = \mu, \quad (14d)$$

$$t_d^E = -\frac{u_E}{u_G} e_d, \quad (14e)$$

where (14e) restates (9a). These first-order conditions (14a) – (14d) are very similar to the first-order conditions we would obtain for an optimization problem without considering externalities. The presence of externalities only implies one modification. Instead of the total tax on the dirty good we have to consider the Ramsey component of the total tax. The Ramsey component is determined by *non-environmental* welfare considerations only, i.e. by minimizing the excess burden of the tax system.

The structure of the first-order conditions shows that we can separate the Pigovian component and the Ramsey component of the second-best optimal tax t_d . Given the second-best allocation, the equation system (14a) – (14c) yields the optimal tax rates (subject to normalization) for goods which do not generate an external effect plus the optimal Ramsey component for the tax on the dirty good. Substituting equation (14e) into equation (13) then yields the optimal tax on the dirty good. Notice, however, that the households' decision concerning x_c , x_d and x_t depend on $t_d = t_d^R + t_d^E$ and that therefore all equations (14a) – (14e) have to be solved simultaneously. The separation of the two tax components is hence a conceptual and not a computational separation.

The concept of internalization suggested above turns out to be consistent with Sandmo's (1975) additivity property. By solving explicitly for the optimal tax structure, Sandmo (1975, p. 92) has shown that the marginal environmental damage of the polluting good enters the tax formula for the polluting good as an additional term only. It does not enter

the tax formulas for other goods and factors directly, regardless of the pattern of complementarity and substitutability.

V. Optimal Taxation Rules in the Presence of Externalities: A Reconsideration

The separation of the internalization problem from the optimal taxation problem makes it easy to carry over the results from the optimal taxation literature into a more general framework where externalities occur in the economy. This advantage will be exemplified for two well-known optimal taxation rules. In the next section we show how the Ramsey rule and the Corlett-Hague rule derived for the case without externalities apply to the case where externalities are present.⁶ Again we confine the analysis to the three good case.

The Ramsey rule

Recall that many tax rate normalizations lead to the same allocation (see n. 5 above). Thus we are free to normalize the tax rate on labour to zero, $t_t = 0$. Defining $t_c^R \equiv t_c$ and using the Slutsky decomposition

$$\frac{\partial x_i}{\partial(1+t_j)} = s_{ij} - x_j \frac{\partial x_i}{\partial Y}, \quad (15)$$

with s_{ij} denoting the compensated (cross-)price effect. From this and (14a), or (14b) respectively, we obtain after some re-arrangement

$$\frac{\sum_{i=c,d} t_i^R s_{ic}}{x_c} = \left(\frac{\lambda - \mu}{\mu} + \sum_{i=c,d} t_i^R \frac{\partial x_i}{\partial Y} \right) \equiv -(1-b), \quad (16a)$$

⁶ For a modification of a third well-known optimal taxation rule, the inverse elasticity rule, see Sandmo (1975).

$$\frac{\sum_{i=c,d} t_i^R s_{id}}{x_d} = \left(\frac{\lambda - \mu}{\mu} + \sum_{i=c,d} t_i^R \frac{\partial x_i}{\partial Y} \right) \equiv -(1-b), \quad (16b)$$

where $b \equiv \left(\frac{\lambda}{\mu} + \sum_{i=c,d} t_i^R \frac{\partial x_i}{\partial Y} \right)$ is to be interpreted as the social marginal utility of private income

(cf. Diamond 1975, p.338).⁷ However, in the presence of externalities, the definition of b is conditional on the full internalization of the external effect as suggested in Section III.

Though the equations (16a) and (16b) are similar to the Ramsey rule (cf. Atkinson and Stiglitz 1980, p. 372), the interpretation has to be slightly modified, as in the presence of externalities we have to consider the Ramsey components instead of the total tax rate.

Ramsey rule in the presence of externalities: If commodity taxes are set optimally, a small equiproportional increase in all Ramsey components, i.e. in all non-environmental tax components, will cause all compensated commodity demands to fall by the same proportion.

A formal derivation of this result is given in Appendix A.

Corlett-Hague rule

The Corlett-Hague rule in the presence of externalities can easily be derived from the equation system (16a) – (16b). Multiplying (16a) by x_c and (16b) by x_d respectively, and using Cramer's rule, we can solve for the optimal tax rates:

$$t_c = t_c^R = \frac{(1-b)(x_d s_{dc} - x_c s_{dd})}{s_{dd} s_{cc} - s_{dc} s_{cd}}, \quad (17a)$$

$$t_d = t_d^R = \frac{(1-b)(x_c s_{cd} - x_d s_{cc})}{s_{dd} s_{cc} - s_{dc} s_{cd}}. \quad (17b)$$

⁷ Note that b as well as λ depend on the chosen normalization since the normalization influences consumer prices and thus the real value of nominal income.

Dividing (17a) and (17b) by the consumer prices q_j (cf. n. 4) and defining the elasticity of compensated demand as $\varepsilon_{ij} = q_j s_{ij} / x_i$; $i, j = c, d, v$, we can derive the following expressions:

$$\frac{t_c^R}{q_c} = \frac{(1-b)}{(s_{dd} s_{cc} - s_{dc} s_{cd})} \frac{x_c x_d}{q_c q_d} (\varepsilon_{cd} - \varepsilon_{dd}), \quad (18a)$$

$$\frac{t_d^R}{q_d} = \frac{(1-b)}{(s_{dd} s_{cc} - s_{dc} s_{cd})} \frac{x_c x_d}{q_c q_d} (\varepsilon_{dc} - \varepsilon_{cc}), \quad (18b)$$

Applying the property of the Slutsky term for the compensated elasticities, $\varepsilon_{ic} + \varepsilon_{id} + \varepsilon_{iv} = 0$, $i = c, d$ (cf. e.g. Deaton and Muellbauer 1980, p. 62), we obtain $\varepsilon_{cd} - \varepsilon_{dd} = -\varepsilon_{cc} - \varepsilon_{dd} - \varepsilon_{cv}$ and $\varepsilon_{dc} - \varepsilon_{cc} = -\varepsilon_{cc} - \varepsilon_{dd} - \varepsilon_{dv}$. Together with equations (18a) and (18b) and $(1-b)/(s_{dd} s_{cc} - s_{dc} s_{cd}) > 0$ this implies

$$\varepsilon_{cv} > \varepsilon_{dv} \Leftrightarrow \frac{t_d^R}{q_d} > \frac{t_c^R}{q_c}. \quad (19)$$

The left-hand side of condition (19) considers the case where the dirty good is a relatively stronger complement (weaker substitute) to leisure than the clean good. This implies that the optimal Ramsey component of the tax on the dirty good relative to its consumer price should be higher than the optimal Ramsey component of the tax on the clean good relative to its consumer price. This is a result which is close to the Corlett-Hague (1953) rule.

Corlett-Hague rule in the presence of externalities: The Ramsey component, i.e. the non-environmental tax component, relative to the consumer price should be higher for goods which are complementary to leisure than for goods which are substitutes for leisure.

VI. The result from Bovenberg and de Mooij (1994) reconsidered

Bovenberg and de Mooij (1994) have recently derived a somewhat puzzling result. For the case where a combination only of labour taxes and green taxes is optimal, which is equivalent

to the normalization $t_c = 0$, they show "that, in the presence of pre-existing distortionary taxes, the optimal pollution tax typically lies below the Pigovian tax, which fully internalizes the marginal social damage from pollution". Furthermore, from this they infer that "environmental taxes typically exacerbate, rather than alleviate, pre-existing tax distortions" and that "marginal costs of environmental policy rise with the marginal costs of public funds" (Bovenberg and de Mooij, 1994, p.1085).

In our view this interpretation is misleading for two reasons. First, with distortionary taxes it is the tax rate given by (9a), not the (first-best) Pigovian tax rate, given by (9b), which achieves full internalization of the marginal social damage from pollution, and this tax rate *decreases* *cet. par.* with increasing marginal costs of public funds. Second, the optimal tax on the polluting good depends on the chosen normalization, whereas the Pigovian tax (9b) does not. Therefore, the chosen normalization may - and typically will - determine whether the difference between these two tax rates is positive or negative (cf. Schöb 1994). Consequently, the sign of this difference is, to some extent at least, arbitrary and its interpretation, therefore, not very meaningful.

The formal results of Bovenberg and de Mooij (1994) can easily be reproduced in the context of our model. Given the normalization $t_c = 0$, the optimal tax rates follow from the first order conditions (14b) and (14c), as (17a) and (17b) followed from (14a) and (14b) for the normalization $t_c = 0$. This gives:

$$t_t^R = t_t = \frac{(1-b')(x_t s_{dt} + x_d s_{dt})}{s_{dd} s_{tt} - s_{dt} s_{td}}, \quad (20a)$$

$$t_d^R = \frac{-(1-b')(x_d s_{tt} + x_t s_{td})}{s_{dd} s_{tt} - s_{dt} s_{td}}, \quad (20b)$$

where b' is the analogue to b of (16a) and (16b) of Section V for the different normalization chosen in this section.

In addition to the assumptions made in Section II, Bovenberg and de Mooij (1994) assume that the utility function is separable between leisure and consumption goods and homothetic in consumption goods. Under these assumptions (20b) implies $t_d^R = 0$ (see Appendix B). This result is closely related to the literature on uniform commodity taxation. For the case without externalities, Sandmo (1974) shows that uniform commodity taxation is optimal for a utility function which satisfies the assumptions made above. Uniform commodity taxation is, in turn, equivalent to levying a tax on labour only. This explains why the normalization $t_c = 0$ implies that the Ramsey component for the dirty good is zero in the model of Bovenberg and de Mooij (1994).

The optimal tax on the dirty good consists of the Ramsey component and the Pigovian component [equation (13)]. As the Ramsey component is zero, the optimal tax is given by

$$t_d^* = t_d^E. \quad (21)$$

This confirms the results derived by Bovenberg and de Mooij (1994, p. 1087). In the presence of distortionary taxes we can expect that $Nu_G > u_c$ (for a discussion of this issue, see Atkinson and Stern 1974). Hence, a comparison of equations (9b) and (14e) shows that the second-best optimal tax on the dirty good has to be below the (hypothetical) first-best Pigovian tax rate associated with the second-best allocation.

The comparison of the total tax on the dirty good with the first-best Pigovian tax, however, is somehow misleading. The freedom to normalize one tax rate to zero makes the Ramsey component, and therefore the total tax rate, dependent on the normalization chosen. Consequently, there is no unambiguous answer to the question whether the second-best optimal tax on the dirty good is larger or smaller than the first-best Pigovian tax (cf. Schöb 1994).

Defining the Pigovian component in the way we suggested in Section III offers a tax component which is independent of the normalization chosen. As this component is smaller than the Pigovian tax would be for the same allocation, we can make the clarifying statement that the *tax component necessary to internalize external effects* is smaller in a world with distortionary taxes than it is in a world without - whatever the Ramsey-optimal taxes on the dirty good might be.

To give an intuition for this result, notice that tax revenues from internalizing external costs are measured in terms of public expenditures. Therefore, it is also necessary to measure the marginal external costs in terms of public expenditures. This is exactly what the Pigovian tax component does. As the value of public expenditures Nu_G is independent of the normalization chosen, the Pigovian tax component as defined here is also independent.

In a world with distortionary taxes a dollar in the public purse is worth more than a dollar in the private purse. Hence, it needs less than one dollar of *tax revenues* to compensate the households for one dollar of additional environmental damage measured in terms of private income. "This implies that each unit of pollution does not have to yield as much public revenue to offset the environmental damage if this revenue becomes more valuable..." (cf. Bovenberg and van der Ploeg 1994, p. 361).

It is worth emphasizing, however, that the fact that the Pigovian tax component is smaller than the first-best Pigovian tax does not contradict the essence of the double dividend hypothesis. The second-best tax on the dirty good will, in general, yield two dividends, one resulting from the internalization of the external effect, the other from reducing tax distortions. However, contrary to a seemingly wide-spread view, the difference between the second-best tax rate on the dirty good and the (hypothetical) Pigovian tax rate is not an adequate measure of either of these two dividends.

VII. The case of multiple externalities

Consider now the general case where we have C clean goods and D dirty goods. With lump-sum taxes not feasible, the government can raise taxes only on private commodities and on labour. Hence the government maximizes

$$W = Nw(t_1, \dots, t_C, t_{C+1}, \dots, t_{C+D}, t_t, G, E) \quad (10')$$

subject to equation (3), as in Section IV. Assuming again separability between private consumption and the environment E , and private consumption and the public good G , respectively, the first-order conditions for the general case become:

$$-\lambda x_c + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_c} + \mu (x_c + \sum_c t_c \frac{\partial x_c}{\partial t_c} + \sum_d t_d \frac{\partial x_d}{\partial t_c} + t_t \frac{\partial x_t}{\partial t_c}) = 0, \quad (12a)$$

$$\forall c \in \{1, \dots, C\}$$

$$-\lambda x_d + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_d} + \mu (x_d + \sum_c t_c \frac{\partial x_c}{\partial t_d} + \sum_d t_d \frac{\partial x_d}{\partial t_d} + t_t \frac{\partial x_t}{\partial t_d}) = 0, \quad (12b)$$

$$\forall d \in \{C+1, \dots, C+D\}$$

$$-\lambda x_t + \sum_d u_E e_d N \frac{\partial x_d}{\partial t_t} + \mu (x_t + \sum_c t_c \frac{\partial x_c}{\partial t_t} + \sum_d t_d \frac{\partial x_d}{\partial t_t} + t_t \frac{\partial x_t}{\partial t_t}) = 0, \quad (12c')$$

$$u_G - \mu \frac{1}{N} = 0. \quad (12d')$$

Defining the Pigovian tax components for all dirty goods according to definition (9a) and using (12d'), we obtain:

$$\mu \sum_d t_d^E \frac{\partial x_d}{\partial t_j} = - \sum_d u_E e_d N \frac{\partial x_d}{\partial t_j}, \quad j = 1, \dots, C, \dots, C+D, \ell. \quad (22)$$

Analogously to (13), the Ramsey components are implicitly defined by

$$t_d = t_d^R + t_d^E, \quad \forall d \in \{C+1, \dots, C+D\}. \quad (13')$$

Substituting the equations (13') and (22) into (12a') - (12c'), we can rewrite the first order conditions as:

$$-\lambda x_c + \mu(x_c + \sum_c t_c \frac{\partial x_c}{\partial t_c} + \sum_d t_d^R \frac{\partial x_d}{\partial t_c} + t_t \frac{\partial x_t}{\partial t_c}) = 0, \quad (14a')$$

$$\forall c \in \{1, \dots, C\}$$

$$-\lambda x_d + \mu(x_d + \sum_c t_c \frac{\partial x_c}{\partial t_d} + \sum_d t_d^R \frac{\partial x_d}{\partial t_d} + t_t \frac{\partial x_t}{\partial t_d}) = 0, \quad (14b')$$

$$\forall d \in \{C+1, \dots, C+D\}$$

$$-\lambda x_t + \mu(x_t + \sum_c t_c \frac{\partial x_c}{\partial t_t} + \sum_d t_d^R \frac{\partial x_d}{\partial t_t} + t_t \frac{\partial x_t}{\partial t_t}) = 0. \quad (14c')$$

The separability of the Pigovian tax component from the non-environmental tax component is therefore also applicable in the case of multiple externalities. Hence, all optimal taxation rules derived for the case of many goods in a world without externalities can be modified for the case of multiple externalities in the same way as has been demonstrated for the case with three goods and one externality in Section IV.

VIII. Conclusions

We have analysed the analytical structure of the optimal taxation of polluting and non-polluting goods in a second-best world where lump-sum taxes are not feasible. In this area there is some controversy and confusion, which is due to the use of concepts which are not independent of the arbitrary normalization of tax rates.

By clarifying what is meant by internalization of external effects in a world with distortionary taxes we offer a definition of an environmental tax (component) which is independent of the normalization of tax rates. The application of this definition enables us to separate the analysis of second-best optimal environmental taxes from the analysis of the tax structure which minimizes the excess burden. This separation illuminates the basic structure of the optimal taxation in the presence of external effects and distortionary taxes and is, therefore, helpful for a better understanding of this area.

By simplifying the analysis in this way, we can show that the standard results of the optimal taxation literature easily carry over to economies with externalities. It also allows for a generalization of the results to the case of multiple externalities. Furthermore, our approach can help to clarify the controversy about the relation between optimal environmental taxation (i) in a first-best world, where lump-sum taxes are feasible, on the one hand and (ii) in a second-best world, where taxes are distortionary, on the other.

Appendix

A. Interpretation of the Ramsey rule

Consider the compensated demand function for e.g. the clean good $x_c^{com}(t_c, t_d, u)$. The effect of a change in the tax rates on compensated demand can be written as

$$dx_c^{com} = s_{cc} dt_c + s_{cd} dt_d. \quad (A1)$$

Now consider a proportional change of all Ramsey components, i.e. $dt_c = \alpha t_c = \alpha t_c^R$ and $dt_d = \alpha t_d^R$. Making use of equation (16a) and of the symmetry property of the compensated cross-price effects, i.e. $s_{cd} = s_{dc}$, we can reformulate (A1):

$$\frac{dx_c^{com}}{x_c} = \alpha \frac{s_{cc} t_c^R + s_{dc} t_d^R}{x_c} = -\alpha(1-b). \quad (A2)$$

As this result applies analogously for all taxed goods, the interpretation for the Ramsey rule in the presence of externalities follows.

B. Derivation of a zero Ramsey component for the dirty good

We can reformulate the second term in brackets of the nominator of equation (20b) by (note that $s_{td} = -s_{dt}$)

$$(x_d s_{tt} + x_t s_{td}) = x_t x_d (((1-t_t) s_{tt}/x_t - (1-t_t) s_{dt}/x_d) / (1-t_t)) = x_t x_d (\epsilon_{tt} - \epsilon_{dt}) / (1-t_t), \quad (B1)$$

whereby ϵ_{ij} denotes the compensated (cross-)price elasticity. Using the property of the Slutsky term for the compensated elasticities $\epsilon_{ic} + \epsilon_{id} + \epsilon_{iv} = 0$, $i = c, d$ together with the condition $(-x_v/x_t) \epsilon_{vv} = \epsilon_{tt}$ for the compensated labour supply elasticity and substituting into (B1) we obtain:

$$(x_d s_{tt} + x_t s_{td}) = -x_t x_d \left(\frac{x_v}{x_t} \epsilon_{vv} + \epsilon_{dt} \right) / (1-t_t) = x_v x_d (\epsilon_{vc} + \epsilon_{vd} - \frac{x_t}{x_v} \epsilon_{dt}) / (1-t_t). \quad (B2)$$

Sandmo (1974, p. 705) shows that separability between leisure and consumption goods and homotheticity of the preferences with respect to the private consumption goods implies $\epsilon_{dv} = \epsilon_{dt} = \alpha x_v (1-t_t)$, $\epsilon_{vi} = \alpha x_i (1+t_i)$ (whereby $\alpha = \beta/x_v$ in his notation, with β being a constant term). Substituting in yields

$$(x_d s_{tt} + x_t s_{td}) = \frac{-x_v x_d}{(1-t_t)} \alpha [(x_c(1+t_c) + x_d(1+t_d) - x_t(1-t_t))] \quad (B3)$$

Comparing the term in square brackets with the household's budget constraint [cf. equation (6)] shows that this term is identically zero. Hence, $t_d^R = 0$.

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