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ON THE CALCULATION OF MARGINAL EFFECTS IN THE BIVARIATE PROBIT MODEL

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Abstract

We examine the effects of marginal changes in continuous variables on the joint, conditional and marginal probabilities involved in the bivariate probit model. The connection between effects in the univariate and bivariate probit models are also explored. We illustrate these effects using a bivariate model of welfare and labor force participation by lone mothers, using data from the 1989 Labour Market Activity Survey of Canada.

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INTRODUCTION

In this paper, we consider the effects associated with a change in an independent variable on probabilities of interest in the context of the bivariate probit model. We show that the effect on the marginal probabilities are identical to those presented in textbooks for the univariate probit model- e.g. Greene (1993). However, we also show that this equivalence holds only under symmetry of the conditional density functions and not in general. In the case of the bivariate probit model, the symmetry of the conditional distribution is guaranteed by the joint normality of the error structure. In the next section, we present some of the various effects that arise naturally in the context of the bivariate decision model. We then illustrate and discuss these effects using a bivariate probit model of the decision by lone mothers to enter the labor force and to participate in the welfare program. We use data from the Labour Market Activity Survey (LMAS) produced by Statistics Canada to illustrate this example. Finally we make some concluding remarks.

MARGINAL EFFECTS IN THE BIVARIATE PROBIT MODEL

In the context of a univariate binary choice model, it is standard practice when making policy evaluations to compute the marginal effect of a change in an independent variable on a desired probability. We write the univariate binary choice model as

$$y_i^* = x_i^T\beta + u \text{ , } y_i = 1 \text{ if } y_i^* > 0 \tag{1}$$
$$= 0, \text{ otherwise,}$$

where x_i is a px1 vector of explanatory variables referring to the ith observation and β is a px1 vector of unknown coefficients. In the case of a probit model u~N(0,1). In the above formulation, y^* is an unobserved latent variable, whereas y is the observed dummy variable with responses 1 and 0. The regression function is given by

$$E(y_i|x_i) = \Phi(x_i^T\beta),$$

where $\Phi(.)$ denotes the standard normal cumulative distribution function (cdf). The regression function also captures the probability of belonging to some group, such as the labor force, as well as the expected value of y itself. The effect of a unit change of variable x_{ik} on that probability of membership in a particular group is given by

$$\partial E(y_i|x_i)/\partial x_{ik} = \partial\Phi(x_i^T\beta)/\partial x_{ik}$$
$$= \phi(x_i^T\beta)\beta_k \tag{2}$$

In the above expression, $\phi(.)$ denotes the standard normal probability density function (pdf).

Similar considerations apply to the bivariate binary choice model, but these do not appear to have been explicitly discussed in the literature. In particular, the marginal effects of explanatory variables on specific probabilities have not been considered. These effects are especially useful in

policy applications. Below we derive the marginal effects for a bivariate probit model. Because of the special meaning of the term "marginal" in the bivariate context, we reserve the adjective for marginal distribution and simply speak of the effects of changes in explanatory variables on various probabilities. The bivariate probit model is written as

$$y_{1i}^* = x_{1i}^T\beta_1 + u_{1i} \text{ , } y_{1i} = 1 \text{ if } y_{1i}^* > 0$$
$$= 0 \text{ otherwise}$$

$$y_{2i}^* = x_{2i}^T\beta_2 + u_{2i} \text{ , } y_{2i} = 1 \text{ if } y_{2i}^* > 0$$
$$= 0 \text{ otherwise}$$

where $E(u_{1i}) = E(u_{2i}) = 0$, $var(u_{1i}) = var(u_{2i}) = 1$ and $cov(u_{1i},u_{2i}) = \rho$ for all i. Also u_{1i} and u_{2i} are jointly normally distributed. The dimensions of x_{1i} and x_{2i} and β_1 and β_2 are p_1x1 and p_2x1, respectively. Above, y_{1i}^* and y_{2i}^* denote observations on latent variables, whereas y_{1i} and y_{2i} denote dummy variables with observed responses 1 and 0, respectively. For illustrative purposes, let the subscripts 1 and 2 refer to welfare status and labor force participation status, such that $y_{Wi} = 1$ denotes the ith welfare participant and $y_{Li} = 1$ denotes the ith labor force participant. Table 1 contains some of the notation used as well as a summary of the relevant events and associated probabilities. The first two rows and columns of Table 1 indicate the joint distribution: For example, $\Phi_{W1,L1}$ is the probability of welfare and labor force participation, while $\Phi_{W1,L0}$ is the probability of welfare participation and labor force non-participation. Row 3 and column 3 in Table 1 report the marginal probabilities: Φ_{W1} is the marginal probability of welfare participation, while Φ_{L0} is the marginal probability of labor force non-participation. Rows 4 to 5 and columns 4 to 5, Table 1, indicate the relevant conditional probabilities: For example, $\Phi_{L1/W1}$ is the probability of being in the labor force given welfare participation.

In the context of the probit model and the assumed normality, the marginal probability Φ_{W1} is defined as

$$\Phi_{W1} \equiv \Pr(y_{W1} = 1) = \Pr(u_{W1} > -x_{W1}^T\beta_W) = \Phi(x_{W1}^T\beta_W)$$

while, for example, that for Φ_{L0} is defined as

$$\Phi_{L0} \equiv \Pr(y_{L1} = 0) = \Pr(u_{L1} \leq -x_{L1}^T\beta_L) = \Phi(-x_{L1}^T\beta_L).$$

Other elements of Table 1 include the joint probabilities

$$\Phi_{W1,L1} \equiv \Pr(y_{W1} = 1,\ y_{L1} = 1) = \Phi(x_{W1}^T\beta_W,\ x_{L1}^T\beta_L, \rho)$$

$$\Phi_{W1,L0} \equiv \Pr(y_{W1} = 1,\ y_{L1} = 0) = \Phi(x_{W1}^T\beta_W,\ -x_{L1}^T\beta_L, \rho)$$

and similarly for $\Phi_{W0,L1}$ and $\Phi_{W0,L0}$, as well as the following conditional probabilities:

$$\Phi_{L1/W1} \equiv P(y_{L1} = 1|\ y_{W1} = 1) = \Phi[(x_{L1}^T\beta_L - \rho x_{W1}^T\beta_W)/(1-\rho^2)^{1/2}], \text{ and}$$

$$\Phi_{L0/W1} \equiv P(y_{L1} = 0|\ y_{W1} = 1) = \Phi[(-x_{L1}^T\beta_L + \rho x_{W1}^T\beta_W)/(1-\rho^2)^{1/2}].$$

Finally, $\Phi_{L1/W0}$ and $\Phi_{L0/W0}$ are defined in a similar fashion.

In addition to the elements in Table 1, we will need corresponding concepts at the pdf level, i.e.

$$\phi_{W1} = \phi(x_{W1}^T\beta_W)$$

$$\phi_{L1/W1} = \phi[(x_{L1}^T\beta_L - \rho x_{W1}^T\beta_W)/(1-\rho^2)^{1/2}], \text{ and}$$

$$\phi_{L0/W1} = \phi[(-x_{L1}^T\beta_L + \rho x_{W1}^T\beta_W)/(1-\rho^2)^{1/2}].$$

Similar expressions hold for $\phi_{L1/W0}$ and $\phi_{L0/W0}$.

In order to simplify the notation, the discussion that follows dispenses with the observation subscript i. The effect of a change in a continuous variable α_k can be usefully evaluated at different levels; we initially assume that α_k appears in both x_W and x_L. The most obvious effect to consider is the impact of α_k on a marginal probability such as Φ_{W1}. Although α_k affects both the decision to go on welfare and enter the labor force, it can be shown that the effect of a marginal change in α_k on the probability of welfare participation Φ_{W1} is, as in the univariate model, given by equation (2) above. The effects of α_k on Φ_{L1} are similar.

As is obvious from Table 1, other effects of a change in α_k can be evaluated. For example, noting that $\Phi_{W1,L1} = \Phi_{W1}\Phi_{L1/W1}$, the effect on the probability $\Phi_{W1,L1}$ of being on welfare and in the labor force is given by

$$\partial\Phi_{W1,L1}/\partial\alpha_k = \partial(\Phi_{W1}\Phi_{L1/W1})/\partial\alpha_{ik}$$
$$= \Phi_{L1/W1}\phi_{W1}\beta_{Wk} + \Phi_{W1}\phi_{L1/W1}\beta_{Lk} \quad (3)$$

The first term in equation (3) is the effect of an increase in α_k on the probability of being on welfare weighted by the probability of labor force participation given welfare participation. It should be noted that a non-zero effect on $\Phi_{L1,W1}$ may obtain even when α_k does not enter x_W and hence no direct effect on the marginal probability of welfare participation exists; an indirect effect may arise if α_k enters x_L and hence $\phi_{L1/W1}$, the second term in equation (3). A somewhat similar decomposition in the context of the Tobit model was noted by McDonald and Moffitt (1980).

Since $\Phi_{W1,L0} = \Phi_{L0,W1}$, the analogous effect of a change in α_k on $\Phi_{W1,L0}$ is given by the expression

$$\partial\Phi_{W1,L0}/\partial\alpha_k = \partial(\Phi_{W1}\Phi_{L0/W1})/\partial\alpha_{ik}$$
$$= \Phi_{L0/W1}\phi_{W1}\beta_{Wk} + \Phi_{W1}\phi_{L0/W1}(-\beta_{Lk}) \quad (4)$$

and comments concerning the direct and indirect effects made in the above paragraph hold here as well. We note that the effects of changes of α_k on other joint probabilities in Table 1 are similar to those presented above in equations (3) and (4). Equations (3) and (4) elucidate why the effect of a change in α_k on, for example, the marginal probability Φ_{W1} is given by the univariate result in equation (2). Since

$$\Phi_{W1} = \Phi_{W1,L1} + \Phi_{W1,L0} \quad (5)$$

the derivative of Φ_{W1} with respect to α_k consists of the four terms in equations (3) and (4). The direct effects, that is the first terms in equations (3) and (4), add up to

$$\phi_{W1}\beta_{Wk}(\Phi_{L1/W1} + \Phi_{L0/W1}) = \phi_{W1}\beta_{Wk},$$

the univariate result in equation (2). The indirect effects, that is the second terms in equations (3) and (4), add up to zero because, by the symmetry of the standard normal pdf,

$$\phi_{L1/W1} = \phi_{L0/W1}, \text{ and}$$

$$\Phi_{W1}\beta_{Wk}(\phi_{L1/W1} - \phi_{L0/W1}) = 0.$$

Thus, the result that the effect of a change in α_k on the probability of (say) welfare participation Φ_{W1} is equal to the univariate effect relies on the symmetry of the conditional density functions which is satisfied by the normality assumption. This result will also generalize to a multivariate probit model where the symmetry of the conditional density functions also holds because of the joint normality of the decision process. However, in a more general environment, where symmetry of conditional densities is not implied by the joint error distribution, this equivalence will not hold. In that case, the indirect effects will not cancel each other out and the marginal effects of the univariate and bivariate effects will differ.

It should be noted that, under independence, equation (3) and (4) become

$$\partial\Phi_{W1,L1}/\partial\alpha_k = \Phi_{L1}\phi_{W1}\beta_{Wk} + \Phi_{W1}\phi_{L1}\beta_{Lk} \tag{6}$$

and

$$\partial\Phi_{W1,LO}/\partial\alpha_k = \Phi_{LO}\phi_{W1}\beta_{Wk} + \Phi_{W1}\phi_{LO}(-\beta_{Lk}) \tag{7}$$

Another special case holds if α_k enters only (say) x_W. Then

$$\partial\Phi_{W1,L1}/\partial\alpha_k = \Phi_{L1/W1}\phi_{W1}\beta_{Wk} \tag{8}$$

and

$$\partial\Phi_{W1,LO}/\partial\alpha_k = \Phi_{LO/W1}\phi_{W1}\beta_{Wk} \tag{9}$$

and only <u>direct</u> effects obtain. Of course, only <u>indirect</u> effects on joint probabilities are also possible if α_k enters x_L but not x_W. We stress that the terms <u>direct</u> and <u>indirect</u> hold for a given conditioning in $\Phi_{LO/W1}\Phi_{W1}$; that is, we could have written $\Phi_{W1,LO} = \Phi_{W1/LO}\Phi_{LO}$. In that case,

$$\partial\Phi_{W1,LO}/\partial\alpha_k = \Phi_{W1/LO}\phi_{LO}\beta_{Lk} + \Phi_{LO}\phi_{W1/LO}(-\beta_{Wk}) \tag{10}$$

While the total effect in equation (4) must clearly equal that in equation (10), what appears as a <u>direct</u> effect in equation (4) will appear as an <u>indirect</u> effect in equation (10).

We conclude the discussion by noting that the above derivatives are estimable in practice at specific points of the sample space such as the sample means of x_W and x_L using the maximum likelihood parameter estimates $\hat{\beta}_W$ and $\hat{\beta}_L$ in place of the true parameters. In the next section, we illustrate these effects by estimating a bivariate probit model of the joint decision by lone mothers to enter the labor force and to participate in the welfare

program.

AN EMPIRICAL EXAMPLE

Data for this application are derived from 1989 information contained in the 1988/89 longtitudinal LMAS produced by Statistics Canada. Use of the longtitudinal rather than cross-sectional data base allows us to include in the analysis information on each respondent's marital and job training status and the extent of her welfare and labor force participation in the preceding year (1988). The working sample includes females under the age of 65 who have children and who, in 1989, resided in households in which there were children present but no spouse. Observations in which the lone mother was a welfare recipient and a labor force participant in 1989, but the two states did not overlap in terms of the actual 1989 months involved, were removed from the sample. Without this exclusion, an individual would be classified as working while on welfare even though the two states did not overlap. Only 44 observations were discarded from the entire sample because of this restriction. Lone mothers who were full-time students or were not resident in a known province were also excluded. The resulting sample contains 2,643 lone mothers: 641 (24.3 percent) were neither on welfare nor in the labor force; 243 (9.2 percent) were on welfare and in the labor force; 471 (17.8 percent) were on welfare but not in the labor force while 1288 (48.7) were in the labor force but not on welfare.

The explanatory variables in the bivariate probit equations control for the personal characteristics of the lone mother, including age, education, disability status, head of family status, and the age and number of children.

Additional regressors pertinent to the welfare and labor force participation decisions include measures of current-year non-labor income, other household members' earned income and job training and welfare status in the previous year. Three welfare program regressors are of particular interest. The relative basic allowance, the imputed program tax rate (or benefit-reduction rate) on earned income and provincial dummy variables. The relative basic allowance variable, defined as the basic allowance to which the lone mother is entitled divided by her potential full-time earnings captures the generosity (or replacement value) of welfare benefits relative to potential labor market earnings. As in other studies of welfare effects, the wage rate pertinent to individuals who were not employed during 1989 was generated using the standard Heckman (1979) two step procedure. In the wage equation, the logarithm of the wage rate was regressed on region, age, education, visible minority status, disability status, immigrant status, marital status, job training and a sample selection correction variable. The underlying probit model is a labor force participation equation where participation is defined in terms of paid employment in 1989. The regressors include, in addition to the variables in the wage equation, the age and the total number of children and measures of non-labor income. The program tax rate is a composite rate based on the proportion of earned income (assuming a 20 hour work-week) lost because of decreased welfare benefits. Thus the tax rate on earned income varies according to program design and the market wage rate imputed for each individual. The provincial dummies capture, among other things, parameters of welfare programs such as liquid asset exemption levels and other administrative features that vary by province. Unlike in the United States, welfare programs in Canada fall under provincial jurisdiction.

The full results for the bivariate probit equations are, in the interest of brevity, not reported here; the general fit of this model is satisfactory-see Christofides, Stengos and Swidinsky (1995) for further details. Instead, in Table 2, we concentrate on results pertinent to two variables which best illustrate the arguments made in the previous section. The variable Other Income reports earned income by members of the household other than the lone mother. It is of interest in the current context because, as Table 2 indicates, it is negative and significant at the 5% level in the welfare equation, indicating that Other Income discourages welfare participation, but it is not relevant to the labor force decision. That is, it enters x_W but not x_L. The relative Basic Allowance variable enters the welfare equation positively and the labor force equation negatively; both coefficients are significantly different from zero at the 1% level.

Columns 1 and 2, Table 2, report results pertinent to the bivariate probit model, while columns 3 and 4 contain results derived from separate, univariate probit equations of the decision to participate in welfare and in the labor force. Rows 1 to 4, Table 2, contain the estimated coefficients and the absolute values of the t statistics for the welfare and labor-force participation decisions. The remaining rows in Table 2 show some of the possible effects that can be examined. These effects are evaluated at the overall means of the explanatory variables using the appropriate coefficients. We concentrate on the effects spelled out in equations (3) and (4). Row 9, Table 2, indicates the overall effect on $\Phi_{W1,L1}$ in equation (3), resulting from a change in α_k, where k stands either for Other Income (columns 1 and 3, Table 2) or Relative Benefit Allowance (columns 2 and 4, Table 2). In row 10, Table 2, the effects calculated in row 9 are scaled by the standard deviation

σ_k of the relevant variables (104.4 for Other Income and 0.192 for Relative Basic Allowance) in order to evaluate the impact on the relevant probabilities of an increase in the variables by a meaningful magnitude. The remaining rows in Table 2 indicate the components of equation (4) as well as the total effect on $\Phi_{W1,L0}$, $\Phi_{W1,L0}\sigma_k$ and on the marginal probability $\Phi_{W1}\sigma_k$. Thus row 16, Table 2, shows the effect of an increase in the relevant variable by one standard deviation on the probability of being on welfare but not in the labor-force. The sum of the effects in rows 10 and 16, Table 2, produces the effect on Φ_{W1} in equation (5), i.e. the effect on the probability of being on welfare. This effect can, of course, be obtained directly as $\phi_{W1}\beta_k$ and the numbers obtained in row 17, Table 2, can be verified, see note "e" in Table 2.

The results in Table 2 indicate that the effects of Other Income and Relative Benefits Allowance are, in general, more muted in the bivariate model where independence is not assumed. Given the significance of $\hat{\rho}$, this is the appropriate model; the results under independence are reported for illustrative purposes only. Table 2, illustrates the direct and indirect results within $\Phi_{W1,L1}$ and $\Phi_{W1,L0}$ as well as the symmetry $\Phi_{W1}\phi_{L1/W1} = \Phi_{W1}\phi_{L0/W1} = 0.0004$. The latter makes the results in the last row row of Table 2 the appropriate marginal effects on the probability of being on welfare Φ_{W1}. Effects on other probabilities can also be evaluated, but these would follow along the lines detailed in Table 2 for $\Phi_{W1,L1}$, $\Phi_{W1,L0}$ and Φ_{W1} and are, therefore, not reported. It is noteworthy, that even though the coefficient estimate of Other Income is zero in the labor force participation equation, implying a zero marginal effect on the probability of labor-force participation Φ_{L1}, this need not be true for the joint probabilities $\Phi_{L1,W1}$ and $\Phi_{L1,W0}$. The effect on $\Phi_{L1,W1}$ (which is shown in row 9, columns 1 and 3,

Table 2) is clearly non-zero albeit small. The intuition is that, although Other Income does not affect the probability of labor-force participation, it does affect the probability of being on welfare and hence the joint probabilities of labor-force and welfare participation.

CONCLUSION

We have explored the effects of changes in continuous variables on several probabilities in the bivariate probit model. The connection between effects in the univariate and bivariate probit models have been explored and the correspondence between the effect of a change in a variable on the univariate and bivariate probabilities elucidated. We have illustrated our remarks using data from the 1989 LMAS.

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Table 1

Event Probabilities in the Bivariate Probit Model

Welfare Participation		Labor Force Participation: Joint Distribution		Marginal W	Conditional on W	
		L1	L0		L1	L0
W1		$\Phi_{W1,L1}$	$\Phi_{W1,L0}$	Φ_{W1}	$\Phi_{L1/W1}$	$\Phi_{L0/W1}$
W0		$\Phi_{W0,L1}$	$\Phi_{W0,L0}$	Φ_{W0}	$\Phi_{L1/W0}$	$\Phi_{L0/W0}$
Marginal L		Φ_{L1}	Φ_{L0}	1		
Conditional on L	W1	$\Phi_{W1/L1}$	$\Phi_{W1/L0}$			
	W0	$\Phi_{W0/L1}$	$\Phi_{W0/L0}$			

Table 2

Coefficients and Components of Effects of Continuous Variables

Row	Coefficients	$\hat{\rho} = -0.35$ k = Other Income[b]	k = RBA[c]	Independence[a] k = Other Income	k = RBA
1	$\hat{\beta}_{Wk}$	-0.0010	5.2600	-0.0020	7.290
2	(|t| statistics)	(2.2650)	(8.8000)	(5.0400)	(8.470)
3	$\hat{\beta}_{Lk}$	0.0000	-10.4001	0.0000	-12.657
4	(|t| statistics)	(0.6601)	(22.0801)	(0.4700)	(16.450)
	Computed Components of Effects on $\Phi_{W1,L1}$, $\Phi_{W1,L0}$ and Φ_{W1}:				
5	$\Phi_{L1/W1}\phi_{W1}$	0.0897	0.0897	0.1276	0.1276
6	$\Phi_{L1/W1}\phi_{W1}\hat{\beta}_{Wk}$	-0.0001	0.4718	-0.0003	0.9302
7	$\Phi_{W1}\phi_{L1/W1}$	0.0004	0.0004	0.0456	0.0456
8	$\Phi_{W1}\phi_{L1/W1}\hat{\beta}_{Lk}$	0.0000	-0.0040	0.0000	-0.5772
9	$\Phi_{W1,L1}$	-0.0001	0.4678	-0.0003	0.3530
10	$\Phi_{W1,L1}\sigma_k$	-0.0094	0.0903	-0.0271	0.0681
11	$\Phi_{L0/W1}\phi_{W1}$	0.1283	0.1283	0.0891	0.0891
12	$\Phi_{L0/W1}\phi_{W1}\hat{\beta}_{Wk}$	-0.0001	0.6748	-0.0002	0.6495
13	$\Phi_{W1}\phi_{L0/W1}$	0.0004	0.0004	0.0456	0.0456
14	$\Phi_{W1}\phi_{L0/W1}\hat{\beta}_{Lk}$	(0.0000)[d]	(-0.0041)	(0.0000)	(-0.5772)
15	$\Phi_{W1,L0}$	-0.0001	0.6789	-0.0002	1.2267
16	$\Phi_{W1,L0}\sigma_k$	-0.0136	0.1310	-0.0188	0.2368
17	$\Phi_{W1}\sigma_k$	-0.0230[e]	0.2213	-0.0459	0.3049

Table 2 (continued)

Notes:

a : Under independence, the expressions in the leftmost column (which are appropriate for the dependence case of columns 1 and 2) get appropriately modified. Thus $\Phi_{L1/W1}\phi_{W1}$ becomes $\Phi_{L1}\phi_{W1}$ and so on with marginal distribution functions replacing the conditional ones. the coefficients used under independence derive from two separate univariate probit equations on welfare and on labor force participation. The coefficients on Other Income and on the Relative Benefit Allowance (RBA) variable in columns 1 and 2 Table 2, derive from a bivariate probit equation in which the estimated correlation coefficient is $\hat{\rho} = -0.35$ with a |t| statistic of 8.47.

b : This variable has a mean of $52.58 (in 100 $) and a standard deviation of 104.4.

c : This variable has a mean of 0.618 and a standard deviation of 0.193.

d: The minus sign in equation (4) is represented, in Table 2, by the brackets in this row.

e: These effects can be also calculated directly as $\phi_{W1}\hat{\beta}_{Wk}\sigma_k$, where ϕ_{W1} = 0.2188. Apart from rounding errors the direct calculation approach yields the same results as the ones reported.

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