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COMPREHENSIVE INCOME TAXATION,  
INVESTMENTS IN HUMAN AND PHYSICAL  
CAPITAL, AND PRODUCTIVITY

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## COMPREHENSIVE INCOME TAXATION, INVESTMENTS IN HUMAN AND PHYSICAL CAPITAL, AND PRODUCTIVITY

### Abstract

This paper analyzes the implications of the comprehensive income tax for the accumulation of human and physical capital and for the overall productivity level of the economy. A comprehensive income tax, applying to both labour income and capital income, is shown to discriminate against investments in human capital relative to investments in physical capital. It has an adverse impact on human capital accumulation and lowers productivity.

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## I. Introduction

In a world with only unskilled labour and physical capital, a comprehensive income tax generates two distortionary effects on resource allocations. It distorts the intratemporal leisure-consumption tradeoffs and the intertemporal saving-consumption tradeoffs. The intratemporal distortion is created by the labour income component of the comprehensive income tax, while the intertemporal distortion is caused by the capital income component. In the presence of investments in human capital, the income tax generates additional distortions. First, since the capital income tax component does not apply to investments in human capital, it discriminates against investments in physical capital. On the other hand, since investments in human capital improve the earning capacity of labour, then the wage tax component reduces the net-of-tax rate of return on investment in human capital and thus discriminates against this kind of investment, relative to investment in physical capital. Therefore, it may look as if the two components of the comprehensive income tax (when applied with equal rates) offset each other with respect to the choice between the two forms of investment. However, this conclusion is valid only when human capital does not depreciate (or become obsolete). In particular, when people are *finitely* lived, human capital must depreciate to zero at retirement. Unlike the case with physical capital, this depreciation of human capital is not commonly tax-deductible. In other words, the wage tax applies to both the yield on, and the principal of, an investment in human capital, while the capital income tax applies only to the yield on the investment in physical capital.

In this paper we analyze the effects of these distortions on overall savings, the composition of savings between investments in human capital and physical capital, and productivity. Section II deals with the asymmetric treatment of human and physical capital by the income tax. Section III develops a stylized overlapping generations model which is used for analyzing the long-run effects of income taxation on investments in human and physical capital, productivity and welfare. We assume some productivity effect, external to the individual, typically associated with investments in human capital. Section IV provides some concluding remarks.

## II. The Income Tax: The Differential Treatment of Investments in Human and Physical Capital

Assume an individual who is considering investments in human or in physical capital. Suppose that by investing  $H$  ECUs in human capital (education, training, etc.), an individual increases her earning capacity in each period in the future by a factor  $g(H)$ ,  $g' > 0$ , and  $g'' < 0$  (namely, there is a positive, but diminishing, marginal return to investments in human capital). That is, a unit of her labour time is worth  $g(H)$  in units of a standard labour input, priced at a  $w$  ECUs per unit. This specification of investment in human capital follows the model of optimum income taxation developed by Sheshinski (1971) and Atkinson (1973). The specification focuses on the money outlays (e.g. tuition) necessary for investment in human capital rather than on foregone labor income. Alternatively, if the individual invests in physical capital, she earns a return of  $r$  ECUs per unit, at each period in the future. As she allocates her total investment between physical and human capital so as to maximize future returns, it must be the case that she earns at the margin the same return on both forms of investment.

Suppose people work for  $T$  periods. Then their human capital depreciates to zero at retirement. Since this depreciation is not tax-deductible, the income tax is levied essentially both on the investment (principal) and on its return (yield). A person who invests an additional unit in her human capital enjoys an additional net-of-tax return in present values of:

$$(1) \quad (1-\Theta)wg'(H)\sum_{t=1}^T \left[ \frac{(1-\mu)}{1+(1-\tau)r} \right]^t - \frac{(1-\Theta)w(1-\mu)}{(1-\tau)r+\mu} g'(H) \left[ 1 - \left( \frac{1-\mu}{1+(1-\tau)r} \right)^T \right]$$

where  $\Theta$  and  $\tau$  are the tax rates on labor income and capital income, respectively, and  $\mu$  is the rate of depreciation (or obsolescence) of human capital.<sup>1</sup> The present value of the returns on an additional unit of investment in physical capital is, by definition, one. Hence,

<sup>1</sup>For simplicity, we abstract from the issue of the depreciation of physical capital because this depreciation is tax-deductible.

maximization of the present value of the net-of-tax returns from all kinds of investments requires:

$$(2) \quad \frac{(1-\Theta)w(1-\mu)}{(1-\tau)r+\mu} g'(H) \left[ 1 - \left( \frac{1-\mu}{1+(1-\tau)r} \right)^T \right] = 1.$$

If human capital does not depreciate ( $\mu = 0$ ) and if people work indefinitely ( $T \rightarrow \infty$ ), then (2) reduces to  $\frac{(1-\Theta)}{(1-\tau)} wg'(H)/r = 1$ , so that the distortion disappears when  $\Theta = \tau$ . However, when  $\mu > 0$  and  $T < \infty$ , then a comprehensive income tax with equal rates applied to labor income and capital income (i.e.,  $\Theta = \tau$ ) still *distorts* the trade-offs between the two kinds of investments. Total differentiation of (2) with respect to the income tax parameter ( $\tau = \Theta$ ) reveals that the income tax discriminates *against* investments in human capital, relative to investments in physical capital.

Here we focus on the out-of-pocket element in the cost of education (investment in human capital). If we were to introduce also time cost of investment in human capital (namely, foregone income during schooling), the distortion created by the comprehensive income tax is further magnified. This is because in a typical (progressive) income tax system the foregone income is taxable at a much lower rate than the extra income generated by the investment in human capital.<sup>2</sup>

## III. An Overlapping Generations Model of Investments in Human and Physical Capital

The long-run effects of income taxation on investments in human and physical capital are analyzed in a stylized overlapping generations model with production (see Diamond (1985)). The unique feature of the overlapping generations model is related to the fact that the newborn generation in each period lacks any human capital because human capital is not transferable from parents to children. Thus, every generation has to

<sup>2</sup>For this reason, we maintain that it is inappropriate to model investment in human capital in a framework in which all of the cost is in the form of foregone income and the tax rate is proportional. Obviously, in the later (unrealistic) case a proportional labor income tax has no effect on investment in human capital.

acquire its own human capital. However, investment in human capital has usually an external effect in that it adds to the general (economy-wide) state of knowledge or know-how, and the stock of knowledge is not fully perishable. Knowledge is transferable with some depreciation (or obsolescence) from one generation to another. A special case of the model is when the stock of knowledge is fully perishable, in which case no intergenerational transfer of human capital, directly or indirectly, is possible.

### III.1. The General Model

Suppose that each generation lives for two periods. Consider the generation which is born in period  $t$ . In order to simplify the notation, assume that there is just one individual in each generation (i.e., zero population growth). She consumes  $c_{1t}$  and  $c_{2t}$  in the first and the second periods of her life, respectively. Her preferences are represented by a utility function  $u(c_{1t}, c_{2t})$ . She is born with an initial endowment of  $N$  units of the consumption good. She does not work in the first period of her life. She spends her time acquiring human capital, which requires also out-of-pocket expenses. By investing  $H_t$  units of the consumption good in the first period, she augments her effective labour supply in the second period to  $g(H_t, B_t) = A_{t+1}$  units of a standard labour input, where  $B_t$  is the general state of knowledge. We assume that  $g$  is a Cobb-Douglas, constant-return to scale, function:<sup>3</sup>

$$(3) \quad A_{t+1} = g(H_t, B_t) = H_t^\gamma B_t^{1-\gamma}.$$

The function  $g$  is simply the production function of human capital. Since  $A_t$  is the effective labour, then  $B_t$  is the driving force behind a labour-augmenting technical change. Note that  $B_t$  is external (given) to the individual.

She can also invest  $K_{t+1}$  units of the consumption good in the first period and receive  $K_{t+1} [1 + (1 - \tau_{t+1}) r_{t+1}]$  in the second period, where  $r_{t+1}$  is the return to

<sup>3</sup>Since some kind of homotheticity is needed for the steady-state analysis that follows, we have simply assumed Cobb-Douglas functional forms throughout.

physical capital (i.e., the real rate of interest) and  $\tau_{t+1}$  is the tax rate on capital income in period  $t+1$ . Thus, the budget constraints in the first and second periods are, respectively:

$$(4) \quad c_{1t} + H_t + K_{t+1} = N$$

and

$$(5) \quad c_{2t} = (1 - \Theta_{t+1}) w_{t+1} g(H_t, B_t) + K_{t+1} [1 + (1 - \tau_{t+1}) r_{t+1}] + T_{2t},$$

where:

$w_t$  = wage per efficiency unit of labour in period  $t$ ,

$\Theta_t$  = tax rate on labour income in period  $t$ , and

$T_{2t}$  = lump-sum transfer to generation  $t$  in the second period of its life.

Naturally,  $K_{t+1}$  is the capital stock in period  $t+1$ . Constraints (4) and (5) can be consolidated into a single, present value budget constraint:

$$(6) \quad c_{1t} + c_{2t} [1 + (1 - \tau_{t+1}) r_{t+1}]^{-1} = N - H_t + (1 - \Theta_{t+1}) w_{t+1} \cdot [1 + (1 - \tau_{t+1}) r_{t+1}]^{-1} g(H_t, B_t) + T_{2t} [1 + (1 - \tau_{t+1}) r_{t+1}]^{-1} = M_t.$$

A Maximisation of  $M_t$  with respect to  $H_t$  yields the rule for an optimal human capital investment:

$$(7) \quad h_t = \frac{H_t}{B_t} = \left[ \frac{(1 - \Theta_t) w_{t+1} \gamma}{1 + (1 - \tau_{t+1}) r_{t+1}} \right]^{\frac{1}{1-\gamma}}$$

Assuming Cobb-Douglas preferences ( $u = \alpha \log c_1 + (1 - \alpha) \log c_2$ ), we also find the consumption demand functions:

$$(8) \quad \begin{aligned} (a) \quad c_{1t} &= \alpha M_t \\ (b) \quad c_{2t} &= (1 - \alpha) M_t [1 + (1 - \tau_{t+1}) r_{t+1}] \end{aligned}$$

Turning to the production side of the economy, we assume a Cobb-Douglas, constant-returns-to scale production function:

$$(9) \quad Y_t = F(K_t, A_t) = K_t^\varepsilon A_t^{1-\varepsilon},$$

where  $Y_t$  is output in period  $t$ .

Denoting the capital-effective labour ratio by  $k_t = K_t / A_t$  and the output per efficiency unit of labour by  $y_t = Y_t / A_t$ , we have:

$$(10) \quad y_t = k_t^\varepsilon.$$

The marginal productivity (profit maximization) conditions are:

$$(11) \quad r_t = \varepsilon k_t^{\varepsilon-1}$$

and

$$(12) \quad \omega_t = (1-\varepsilon) k_t^\varepsilon.$$

To focus on the distortionary effects of income taxation, we make the conventional assumption (in the literature on excess burden of taxation) that the government does not redistribute income across generations but rather returns in a lumpsum fashion to each person the taxes paid by her, i.e.:

$$(13) \quad T_{2t} = \Theta_{t+1} \omega_{t+1} g(H_t, B_t) + \tau_{t+1} r_{t+1} K_{t+1}.$$

Substituting (13) into the definition of wealth (namely,  $M_t$  in equation (6)), we obtain:

$$(14) \quad M_t = N - h_t B_t + \omega_{t+1} h_t^\gamma B_t [1 + (1 - \tau_{t+1}) r_{t+1}]^{-1} + \tau_{t+1} r_{t+1} K_{t+1} [1 + (1 - \tau_{t+1}) r_{t+1}]^{-1},$$

where use was made also of equation (3).

The economy-wide resource constraint at period  $t$  is given by:

$$(15) \quad c_{1t} + c_{2,t-1} + H_t + K_{t+1} - K_t = Y_t + N.$$

This constraint simply states that total consumption (of old and young), plus total investments (in human and physical capital) must equal total output, plus initial endowment.<sup>4</sup>

Finally, the evolution rule for the stock of knowledge is specified by:

$$(16) \quad B_{t+1} = B_t (1 - \delta) + H_{t+1}.$$

That is, the stock of knowledge depreciates at the rate of  $\delta$  and accumulates through the investment in human capital.

### III.2. Steady State

Given the fixed amount of the first-period endowment (per capita), it follows that the only possible steady state is with zero per-capita growth of output, consumption, investment, etc. Rewriting (16) in proportional rate of change terms we conclude that at the steady state:

$$(16') \quad h_t = h - \delta.$$

The other steady-state equations then follow by rewriting (7), (11), (12), (14) and (15) in steady-state forms:

$$(7') \quad h = \left[ \frac{(1 - \Theta) \omega \gamma}{1 + (1 - \tau) \pi} \right]^{\frac{1}{1 - \gamma}}$$

$$(11') \quad r = \varepsilon h^{\varepsilon-1}$$

<sup>4</sup>The reader can check that the resource constraint follows from the individual and the government budget constraints (Walras law).

$$(12') \quad w = (1-\epsilon) k^\epsilon$$

$$(14') \quad m = n - \lambda + \omega h^\gamma [1+(1-\tau)r]^{-1} + \tau k h^\gamma (1-\delta+h)[1+(1-\tau)r]^{-1}$$

$$(15') \quad m[\alpha + (1-\alpha)[1+(1-\tau)r(1-\delta+h)^{-1}] + \lambda + k h^\gamma (-\delta+h) - k^\epsilon h^\gamma - n - 0,$$

where  $m \equiv M/B$  and  $n \equiv N/B$ , and where use is made of (8) and (10).

#### IV. Effects of Income Taxation

We now resort to numerical simulation methods in order to analyze the effects of the income tax parameters on the productivity level ( $B$ ), the physical to human capital ratio ( $k$ ), and welfare.<sup>1</sup> Note that the (no-tax) steady state equilibrium does not necessarily follow the Golden Rule (e.g. Diamond (1970)). Indeed, in our case it does not follow the Golden Rule, because population is stationary. Hence, taxes have two effects, possibly conflicting, on steady state welfare. On the one hand, they may distort the margins of choice between present and future consumption and between human and physical capital investments. On the other hand, they may lower or increase the rate interest, thereby bringing the economy closer to, or farther out of, the Golden Rule.

For our parameter values, we have chosen  $\alpha = 0.727$ , which corresponds to a subjective rate of time preference of 4% per year under the assumption that the two periods are separated apart on average by 25 years. Following the calculations performed by Lucas (1990) for the external effect of human capital accumulation, we assumed that  $\gamma = 0.48$  (corresponding to a Hicks-neutral technological change coefficient of 0.36 in his calculations). The share of capital in output,  $\epsilon$ , is assumed to be 0.25. The rate of depreciation of the stock of knowledge,  $\delta$ , is assumed to be 0.533, which corresponds to an annual rate of depreciation of 3%

<sup>1</sup>In a similar model (see Nerlove, Razin, Sadka and von Weizsäcker (1990)) we were able to provide also analytical results on the effect of income taxation on the physical to human capital ratio.

Table 1 describes the effects of the income tax on the physical to human capital ratio ( $k$ ). As expected, moving in this table along any line from left to right (that is, raising the labor income tax rate, while keeping constant the capital income tax rate) results in an increase in the physical to human capital ratio. That is, the wage tax discourages investment in human capital, relative to physical capital. The opposite is true with respect to the capital income tax: it encourages investment in human capital, relative to physical capital. Moving along the diagonal ( $\tau = \theta$ ) of the table (that is, along an increase in the comprehensive income tax rate), we confirm our intuition that the comprehensive income tax discriminates against investment in human capital, relative to physical capital. The effect of the income tax on  $k$  is significant: A 50% comprehensive income tax raises  $k$  by 1340% over its no-tax level! The pronounced effect on  $k$  is due to the magnifying effect of the endogenous productivity ( $B$ ) on human capital.

Table 2 demonstrates the effects of income taxation on productivity. The wage tax lowers productivity, while the capital income tax raises it. Here too, the effects of the two taxes are not symmetric and do not offset each other. The table confirms our intuition that all in all the comprehensive income tax ( $\tau = \theta$ ) lowers productivity. A 50% comprehensive income tax decreases productivity by over 90%!

Of special interest is the effect of the income tax on welfare (the utility of the representative generation in the steady state). We measure this effect by the proportional change in the pre-tax ( $\tau = \theta = 0$ ) levels of  $c_1$  and  $c_2$  that will bring the pre-tax utility level up to the post-tax utility level. Since utility is homothetic, this is a consistent welfare measure. Even though taxes are distortionary, they are not necessarily steady state welfare-reducing, because they may bring the economy closer to the Golden Rule. Indeed, table 3 suggests that the dominant factor determining the effect of taxation on steady-state welfare is the proximity to the Golden Rule. When a tax brings the economy closer to the Golden Rule (by increasing  $k$  and lowering  $r$ ), then it improves welfare in spite of its distortionary effects on the margins of choice between present and future consumption and between investments in human and physical capital. Since the comprehensive income tax is biased against investment in human capital relative to physical capital (that is, it

raises  $k$  and lowers  $r$ ), it follows that it improves welfare quite significantly. The effect of the income tax on  $k$  and hence on welfare is magnified by the external economy associated with productivity.

#### V. Conclusion

This paper analyzes the implications of tax policy for the accumulation of human capital (as specified by Usawa (1985), Razin (1972) and Adams (1990)), the accumulation of physical capital and the overall productivity level of the economy. A comprehensive income tax, applying both to labor and capital income, discriminates against investment in human capital, relative to investment in physical capital. Taking into account a positive external effect of investments in human capital on overall productivity, the adverse effect of income taxation on human capital investments is significantly magnified.

The paper assumed that there is no public subsidy to investment in human capital (an education subsidy). Of course, if the government uses the proceeds from the wage-income tax in order to subsidize education, the distortion of this tax could be mitigated. It is noteworthy that a subsidy may be called for on Pigouvian grounds as well, because of the positive externality of investment in human capital on productivity.

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Table 1: *The Effects of Income Taxes on the Ratio of Physical-to-Human Capital (percentage deviations from the zero-tax level)*

Rates of capital income tax	Rates of Labour Income Tax					
	0	0.1	0.2	0.3	0.4	0.5
0	0.0	48.1	131.7	287.7	607.6	1351.4
0.1	-1.0	47.0	130.0	286.2	605.8	1349.2
0.2	-2.0	45.8	129.0	284.6	604.0	1347.0
0.3	-3.0	44.6	127.7	283.1	602.0	1345.0
0.4	-4.0	43.4	126.3	281.5	600.0	1342.8
0.5	-5.0	42.2	125.0	279.0	598.0	1340.4

Table 2: *The Effects of Income Taxes on Productivity (percentage deviations from the zero-tax-level)*

Rates of capital income tax	Rates of Labour Income Tax					
	0	0.1	0.2	0.3	0.4	0.5
0	0.0	-31.3	-55.4	-73.0	-85.0	-92.7
0.1	0.8	-30.0	-55.2	-72.9	-85.0	-92.6
0.2	1.6	-30.5	-55.0	-72.8	-85.0	-92.6
0.3	2.4	-30.0	-54.8	-72.7	-85.0	-92.6
0.4	3.2	-29.6	-54.6	-72.7	-84.9	-92.6
0.5	4.1	-29.1	-54.4	-72.6	-84.8	-92.6

Table 3: *The Effects of Income Taxes on Steady State Welfare (percentage deviations from the zero-tax Welfare Level)*

Rates of capital income tax	Rates of Labour Income Tax					
	0	0.1	0.2	0.3	0.4	0.5
0	0.0	45.0	122.7	287.0	582.0	1248.0
0.1	-0.8	44.1	121.7	285.9	580.7	1244.5
0.2	-1.5	43.3	120.7	284.8	559.4	1242.8
0.3	-2.3	42.4	119.7	283.6	558.0	1241.2
0.4	-3.1	41.5	118.7	282.4	556.6	1239.0
0.5	-3.8	40.6	117.7	281.0	555.3	1238.0

Note: The change in welfare is measured by the factor that should multiply the zero-tax consumption vector ( $c_1$  and  $c_2$ ) so as to bring the zero-tax steady-state utility up (+) or down (-) to the post-tax utility level.

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