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WHAT HAPPENS WHEN INFLATION TARGETS CHANGE?

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Abstract

This paper considers an open-loop Nash game between independent monetary and fiscal authorities which seek to achieve conflicting objectives. The monetary authority is concerned solely with achieving a desired rate of inflation. The fiscal authority has multiple objectives defined by specific preference parameters. Using a calibrated model, a dynamic game theory approach is employed to analyze the impact of a change in inflation targets on the dynamic paths of public debt, the real money supply, government expenditure, and real interest rates.

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1. INTRODUCTION

In recent years, the general approach of central banks has emphasized the reduction of inflation from the double-digit levels of the late 1970s and early 1980s. When a monetary authority makes an explicit commitment to a particular policy stance, such as in the adoption of an inflation target, there is potential for conflict between monetary policy and other independently determined arms of policy; in particular, conflicts can arise when fiscal and monetary authorities pursue different policy objectives.

There are at least two mechanisms through which differing monetary and fiscal policy objectives can lead to conflict. One source of conflict arises through unemployment/inflation trade-offs embodied in a Lucas supply function or a Phillips curve;[1] a second type of conflict can come about through the dynamics of asset accumulation, which defines a relationship between the size of the fiscal deficit, the stance of monetary policy and the level of public indebtedness.

This paper emphasizes both types of fiscal/monetary interactions. In earlier papers, Tabellini (1986) and Masciandaro and Tabellini (1988) used purely theoretical frameworks, so that the signs of policy effects but not their magnitudes were evaluated. These authors considered outcomes under a diverse range of objectives for monetary and fiscal authorities; they demonstrated that the equilibrium size of fiscal deficits and public indebtedness depend on the institutional features of the monetary regime.

In a largely descriptive paper, Nordhaus (1994) examines fiscal/monetary policy interactions using a framework that does not distinguish between trade-offs arising through the dynamics of asset accumulation and those arising through a Phillips curve. He considers outcomes when the monetary authority is more averse to inflation than the fiscal authority and when the fiscal authority is also concerned about the size of the fiscal deficit. Nordhaus argues that a lower fiscal deficit outcome is more likely to be achieved when fiscal and monetary policies are coordinated than is likely to be the case when these policies are implemented independently.

These earlier papers demonstrate that economic outcomes are sensitive to the specification of monetary and fiscal objectives. The emphasis of current policy-makers means that it is now more appropriate to assume an inflation target as the sole argument in the monetary authority's loss function.[2] Extrapolation of earlier results to a world where the monetary authority must adopt a particular inflation target, suggests that substantial changes in the inflation target may also be accompanied by substantial changes in fiscal policy and in the level of public indebtedness.

This paper uses a calibrated model to extend previous studies. In the chosen framework, where the monetary authority has an inflation target as its sole policy objective, the calibrated model is used to quantify the impact that changing inflation targets are likely to have on economic outcomes. The analysis shows that for changes in the inflation target, between 0% per annum and 10% per annum, there is some impact on the long-run equilibrium levels, but that empirically many of these effects are really quite small. However, the dynamic path between the equilibria can involve significant adjustments.

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[1] Alesina and Tabellini (1987) and Buckle and Stemp (1991) emphasize conflicts arising from unemployment/inflation trade-offs.

[2] For a discussion of whether the inflation target is an optimal central bank contract, see Walsh (1995a, 1995b).

2. THE MODEL

The analysis assumes a closed economy with a fixed level of capital stock and with all tradable wealth held either as money or government bonds. The dynamics of asset accumulation and price level dynamics have been chosen as the two sources of dynamics in the representative economy. The model abstracts from the longer-run dynamics of capital accumulation. The basic macro framework can be described by the following set of equations:

$$y = \alpha_0 + \alpha_1 g - \alpha_2 (r - \pi) \quad (1a)$$

$$m = \beta_0 + \beta_1 y - \beta_2 r \quad (1b)$$

$p = \gamma(y - \bar{y}) + \pi$, except at points where the price level jump (1c)

$$\dot{m} + \dot{b} = g - \tau + rb - p(m + b) \quad (1d)$$

All variables are functions of time, all model coefficients (the α's, β's and γ) are positive, a dot above a variable denotes the derivative with respect to time, and:

y = real output,
$\bar{y}$ = full employment level of output (assumed exogenous),
g = real government expenditure,
τ = real taxation revenues (assumed exogenous),
r = nominal interest rate,
p = actual rate of inflation,
π = expected rate of inflation,
m = real supply of base money,
b = real level of bonds (equal to public sector debt).

This is a standard IS/LM model augmented by a Phillips curve (which defines price dynamics) and extended to incorporate the dynamics of asset accumulation. To close the model it is necessary to define time-paths for the real supply of money and for government expenditure. Before these variables can be chosen, however, the optimization decisions of fiscal and monetary authorities are discussed; it is also necessary to define a time-path for private agents' expectations about inflation (given by the variable, π).

The fiscal authority

The fiscal authority is assumed to desire a target level of output, y^T. Consistent with Barro and Gordon (1983a, 1983b) this target level of output is chosen to be "too large", so that $y^T > \bar{y}$. The fiscal authority is also assumed to desire a zero level of public debt.

Specifically, the fiscal authority chooses government expenditure, g, so as to minimize a loss function of the form:

$$L_g = \tfrac{1}{2}\int_0^{\infty} \exp(-\delta t)[(y - y^T)^2 + \phi . b^2]dt \qquad (2)$$

The time discount rate of the fiscal authority is given by δ. The parameter ϕ determines the relative weight to be given to the alternative objective criteria. When $\phi = 0$, all weight is given to achieving the target level of output, y^T. As ϕ becomes larger, increasing weight is given to stabilizing public debt; when ϕ approaches ∞ all weight is given to stabilizing debt irrespective of the outcome for real output.

The fiscal authority has incomplete information about the workings of the economy. In particular, the fiscal authority does not know how the monetary authority will react to a change in government expenditure. Accordingly, it determines fiscal policy by assuming that its choice of government expenditure will have no impact on the real money supply. At any point in time, it takes the real money supply as exogenous. Also, the fiscal authority does not know how private agents form their expectations about future inflation outcomes. At any point in time it takes inflationary expectations, π, as exogenous.

The optimization decision of the fiscal authority can then be summarized as follows: to choose g so as to minimize equation 2 subject to equations 1a-1d where the variables, m and π, are exogenous to the fiscal authority's decision. The outcome of the fiscal authority's optimization problem is a time path for g that depends on the exogenously determined time-paths for m and π. In other words, at the beginning of the optimization period the fiscal authority chooses a path for government expenditure conditional on paths for m and π. When, contemporaneously, the realized values for m and π are observed, then the fiscal authority chooses a value for g consistent with its optimization decision.[3]

The Hamiltonian for the Euler-Lagrange problem associated with the fiscal authority's optimization decision is given by:

$$H = \tfrac{1}{2}\exp(-\delta t)[(y - y^T)^2 + \phi . b^2]$$

$$+ \exp(-\delta t)\lambda[\dot{m} + \dot{b} - g + \tau - rb + p(m + b)] \qquad (3)$$

Euler's Theorem gives the necessary conditions for minimizing L_g by the following equations:

$$\frac{\partial H}{\partial g} = 0 \qquad (4a)$$

$$\frac{\partial H}{\partial b} = \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{b}}\right) \qquad (4b)$$

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[3] In the real world inflationary expectations can be measured only approximately using surveys and with a time lag. This analysis abstracts from these complications by assuming that π can be observed instantaneously.

Thus, noting that the values of m and π are exogenous to the fiscal authority's decision, the necessary conditions for minimizing the fiscal authority's loss function are given by:

$$y = \alpha_0 + \alpha_1 g - \alpha_2 (r - \pi) \tag{5a}$$

$$m = \beta_0 + \beta_1 y - \beta_2 r \tag{5b}$$

$$p = \gamma(y - \bar{y}) + \pi \text{, except at points where the price level jumps} \tag{5c}$$

$$y - y^T = \frac{\lambda[1 + r_g b - p_g (m + b)]}{y_g} \tag{5d}$$

$$\dot{\lambda} = \lambda[\delta - r + \pi] + \phi b \tag{5e}$$

$$\dot{m} + \dot{b} = g - \tau + rb - p(m + b) \tag{5f}$$

where r_g and p_g are the short run multipliers derived from equations 1a-1c, so that:

$$r_g = \left(\frac{dr}{dg}\right)_{SR} = \frac{\alpha_1 \beta_1}{\beta_2 + \alpha_2 \beta_1} > 0$$

and

$$p_g = \left(\frac{dp}{dg}\right)_{SR} = \gamma\left(\frac{dy}{dg}\right)_{SR} = \frac{\gamma \alpha_1 \beta_2}{\beta_2 + \alpha_2 \beta_1} > 0$$

This determines a time-path for g that is a function of m and π.

To close the model, it is necessary to include additional equations which determine the time path of the real money supply, m, and of inflationary expectations, π. These are determined by the behaviour patterns of the monetary authority and of private agents.

The monetary authority

The monetary authority is assumed to adopt a policy objective devoted solely to achieving a given inflation target. Specifically, the monetary authority chooses the time path of the nominal money supply to minimize a loss function of the form

$$L_m = \tfrac{1}{2}\int_0^\infty \exp(-\rho t)[(p - p^T)^2]dt \qquad (6)$$

where the symbol p^T defines an exogenously specified target value for the inflation rate. All agents, including the monetary authority, are assumed to be able to instantaneously observe the contemporaneous price level. Hence, the monetary authority's choice of the nominal money supply can be modeled by allowing the monetary authority to choose the real money supply, *m*.

The parameter, ρ, is the time discount rate of the monetary authority. This discount rate is allowed to differ from the discount rate of the fiscal authority, δ, reflecting the possibility that the two authorities may have different time-frames in which they need to achieve their objectives. Typically, the fiscal authority might hope to achieve its objectives before the next election, so that the value of δ can be significantly larger than ρ. For the loss function given by equation 6 this issue is irrelevant since the optimal path for inflation is independent of ρ.

Since any jumps in the price level will only occur at the beginning of the optimization period and since such jumps occur on a set of measure zero the loss function of the monetary authority, L_m, is minimized when

$$p = p^T, \text{ except at points where the price level jumps.} \qquad (7)$$

The monetary authority chooses m to achieve this outcome.

Private agents

Private agents are assumed to form inflationary expectations as if the inflation target is achieved. Hence, inflationary expectations satisfy the following equation:

$$\pi = p^T \tag{8a}$$

Alternatively, private agents can be assumed to have perfect foresight so that:

$\pi = \dot{p}$, except at points where the price level jumps. (8b)

Equations 8a and 8b lead to identical results. Without loss of generality we henceforth assume that private agents form their expectations about inflationary outcomes according to equation 8a.

Conflicting objectives

Combining equations 1c, 7 and 8a (or 8b), it can be observed that the monetary authority's objective is consistent with an output target given by $y = \bar{y}$. This conflicts with the objective of the fiscal authority which (as long as the parameter, ϕ, is finite) is spending at least some of its resources trying to achieve an output target given by $y = y^T$. Thus, as long as $y^T \neq \bar{y}$, there is a direct conflict between the objectives of fiscal and monetary authorities. Such is the case in this paper where it is assumed that $y^T > \bar{y}$.

3. SOLVING THE MODEL

We can combine the outcome from the fiscal authority's optimization decision (given by equations 5a-5f), together with the outcome from the monetary authority's decision (given by equation 7) and private agents' formation of expectations (given by equation 8a) to yield a Nash game outcome for the interaction between fiscal and monetary authorities as follows:[4]

$$y = \alpha_0 + \alpha_1 g - \alpha_2 (r - \pi) \quad (9a)$$

$$m = \beta_0 + \beta_1 y - \beta_2 r \quad (9b)$$

$$p = \gamma(y - \bar{y}) + \pi \text{, except at points where the price level jumps} \quad (9c)$$

$$p = p^T \text{, except at points where the price level jumps} \quad (9d)$$

$$\pi = p^T \quad (9e)$$

$$y - y^T = \frac{\lambda[1 + r_g b - p_g (m + b)]}{y_g} \quad (9f)$$

$$\dot{\lambda} = \lambda[\delta - r + \pi] + \phi b \quad (9g)$$

$$\dot{m} + \dot{b} = g - \tau + rb - p(m + b) \quad (9h)$$

The remainder of this paper focuses on the outcome from this Nash game under a variety of assumptions.

4. CALIBRATING THE MODEL

For illustrative purposes the model of the Nash game, given by equations 9a-9h, is calibrated by assuming a plausible set of parameter values. These are detailed in Table 1.[5]

(Table 1 about here)

This paper employs the calibrated model to examine the steady-state and dynamic properties of fiscal and monetary responses. The solutions are independent

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[4] This is the solution to an open loop Nash game. For a discussion of related issues, see Basar and Olsder (1982).

[5] These parameter values are derived from DeLong and Summers (1986) and Mankiw and Summers (1986).

of the monetary authority's discount rate, ρ, but can change with the parameters, δ, y^T and ϕ (in the fiscal authority's objective criterion) and with target inflation, p^T.[6]

Throughout these simulations, the parameter, δ, is chosen to be 0.15 above the steady-state real interest rate. This is consistent with a situation where the fiscal policy-maker is more myopic than an underlying representative consumer. The target level of output, y^T, is chosen to be "too high", at 105% of the full employment level of output. Also, the parameter ϕ has been chosen from a broad range of values: from values of ϕ close to zero (representing the case when the fiscal authority's objective emphasizes achieving big government) to values of ϕ close to ∞ (when all weight in the fiscal authority's objective criterion is given to eliminating public debt).

In this analysis, the focus of attention is on economic outcomes under two differing targets for the inflation rate. The variable p^T is allowed to take the values of 0.0 and 0.1, consistent with inflation outcomes of 0% per annum and 10% per annum.

Analysis of the dynamics of the game interaction proceeds in three phases: firstly, it is necessary to determine values taken by economic variables at steady-state equilibria; secondly, dynamic properties in the neighborhood of each steady-state must be determined; thirdly, it is appropriate to examine properties of the optimal dynamic path. Results derived in these three phases uniquely define local properties of the optimal solution.

[6] The solutions described in this paper have been derived using Mathematica 3.0 (Wolfram, 1996).

5. STEADY-STATE EQUILIBRIA

Equations 9a-9h can be solved to give the dynamic path of the economic variables: y, r, π, m, p, b and g, as well as the co-state variable, λ. Henceforth, an asterisk, *, is used to denote steady-state equilibria. With the chosen parameter values, there are two distinct steady-state solutions to this model. However only one of the steady-states is associated with a positive steady-state real interest rate ($r^* - \pi^*$) and a positive steady-state level of public sector debt (b^*). Table 2 describes the (unique) steady-state associated with a positive b^* given alternative values for the fiscal preference parameter, ϕ, and the inflation target, p^T.

(Table 2 about here)

Ceteris paribus, changes in the inflation target, p^T, have a minimal impact on all steady-state outcomes except the equilibrium real money supply, m. To the contrary, changes in fiscal authority preferences, as measured by the parameter, ϕ, have a significant impact on the steady-state.

Ceteris paribus, as the weight given by the fiscal authority to stabilizing public debt increases (that is, as ϕ increases), the value of steady-state public sector debt is reduced, while steady-state government expenditure increases. This seemingly perverse result for government expenditure is driven by the steady-state budget constraint. For example, when $p^T = 0$, this constraint is given by:

$$g^* - \tau + r^* b^* = 0 \qquad (10)$$

As a consequence, with a positive interest rate, public sector debt and government expenditure are negatively related in the steady-state. The relationship between government expenditure and public sector debt is more easily understood in the short-run, where, as is to be expected, high levels of public debt are associated with high levels of government expenditure.

6. DYNAMIC PROPERTIES

The linearized dynamic system associated with equations 9a-9h has two eigenvalues. Table 3 shows the value of eigenvalues in the neighborhood of the chosen steady-state over a range of values for the parameters, p^T and ϕ.

(Table 3 about here)

Once again, preferences of the fiscal authority are much more important in determining the rate of convergence of the optimal solution than are the preferences of the monetary authority concerning the target inflation rate.

The results in Table 3 demonstrate that, at least over the range of parameters considered here, the system will be characterized by one stable and one unstable eigenvalue (a saddlepoint instability). This will require an appropriate adjustment in one jump variable to move the economy towards a stable path. Focusing on the stable (negative) eigenvalue, it can be observed that, ceteris paribus, changes in p^T have little impact on the stability properties of the optimal solution. To the contrary, changes in fiscal authority preferences are much more important, with increasingly rapid convergence toward the steady-state as ϕ becomes larger.

7. PROPERTIES OF THE OPTIMAL PATH

As shown above, the linearized model has exactly one stable eigenvalue so that the optimal solution follows a saddle-path. At the beginning of the optimization period, an adjustment in an appropriate jump variable will move the economy onto the stable saddle-path. There are two candidates for jump variable: an open market operation (a policy response) and a price level adjustment (a market response).

Table 4A describes key points on the dynamic adjustment path of the economy following an increase in the inflation target from 0% per annum to 10% per annum. These key points are the initial equilibrium (which assumes that the central bank has been targeting 0% inflation for a considerable period of time), the short-run equilibrium (reached after the appropriate policy response by the jump variable) and then the long-run equilibrium (to which the economy converges following a gradual dynamic adjustment from the short-run equilibrium). Table 4B summarizes the economic adjustment following a decrease in the inflation target from 10% per annum to 0% per annum. Tables 4A and 4B consider the outcomes following both an open market operation and a price level adjustment.

(Tables 4A and 4B about here)

(Figure 1 about here)

To illustrate the interpretation of Tables 4A and 4B it is appropriate to focus on one particular outcome. Figure 1 presents a typical dynamic adjustment path of *m* and *b* following an increase in the inflation target. Once jumps in *m* and *b* have taken place, thus determining the time-paths of the real money supply and real public sector debt, it is a straightforward matter to determine the dynamic time-paths of government expenditure and the real interest rate. The complete methodology for calculating the magnitude of the respective adjustments is summarized in the Appendix.

Consider the case of a 10% increase in the inflation target, summarized in Table 4A, when $\phi = 0.01$. The initial equilibrium is given by E_1 (b = 0.76278, m = 0.04189, g = 0.226314, $r - \pi$ = 0.031052). Following an open-market operation at the beginning of the optimization period the economy would move instantaneously to E_{2A} (b = 0.77107, m = 0.03361, g = 0.204904, $r-\pi$ = 0.013923) and then follow a slow adjustment to E_3 (b = 0.75964, m = 0.03173, g = 0.228349, $r-\pi$ = 0.032679). Alternatively, if the jump to the stable arm of the saddle-path were to occur through an instantaneous jump in the price level then the economy would move from E_1 to E_{2B} (b = 0.85112, m = 0.04675, g = 0.040661, $r-\pi$ = -0.117471) and then follow a slow adjustment to E_3.

There are two significant observations to be made from this example. Firstly, even when the changes in equilibria are very small, the dynamic adjustment path can involve significant movements in economic variables along the dynamic adjustment path; economic variables may actually move a large distance away from the new equilibrium before eventually returning Secondly, the adjustment paths can differ significantly depending on the nature of the initial jump that brings the economy onto the stable path. These two observations also apply to the results for other values of ϕ presented in Table 4A and to the results presented in Table 4B.

The results allow the examination of another interesting question: If the economy is going to jump to the stable arm of the saddle-path, then what is the mechanism by which such a stable path is to be reached? Specifically, is the appropriate jump likely to involve a policy response (in this case, an open market operation) or a market response (here, a price level adjustment)?[7]

When ϕ equals 1.0 or 100.0, the outcomes immediately following a jump in the price level are totally implausible, with negative values for the real money supply, levels of real government expenditure that are excessively high or excessively low and with real interest rates that are unrealistic. Even for lower values of ϕ, a jump in the price level leads to much more significant swings in the values taken by economic variables. Generally, it is unlikely that the economy would be able to sustain adjustments consistent with those required by a jump in the price level. On the contrary, the optimal adjustments following an open-market operation fall well within the range of plausibility. However, even in the case of an open-market operation, the short run movements in economic variables can be quite substantial irrespective that the magnitudes of steady-state effects are minimal.

9. CONCLUSION

This paper has considered an open-loop Nash game between independent monetary and fiscal authorities. The monetary authority is concerned solely with achieving a desired rate of inflation. The fiscal authority has multiple objectives defined by specific preference parameters. More precisely, the fiscal authority is concerned with minimizing the present discounted value in a loss function which gives weight both to achieving a desired output target and also to reducing the level of public indebtedness. A calibrated model, which emphasizes fiscal/monetary policy trade-offs through a Phillips curve and the dynamics of asset accumulation, has been used to assess the impact of changes in the inflation target on the dynamic path of the economy. As a by-product of the analysis, the impact of changes in fiscal authority preferences on economic outcomes has been examined.

Not surprisingly, the results show that changes in fiscal authority preferences can have a substantial impact on the time-path of the economy. When the fiscal

[7] I am grateful to Christopher Allsopp and David Vines for most helpful discussions on this issue.

authority gives emphasis to achieving its output objective, the public debt level can be exceedingly large (even more than ten times GDP levels); when emphasis is placed on reducing public debt, the level of public indebtedness can be substantially reduced (to less than 1% of GDP).

The paper focused on changes in the monetary authority's inflation target between 0% per annum and 10 % per annum. The central result of the paper is that a change in the inflation target has minimal impact in the long-run, but that economic impacts can be quite substantial during the adjustment phase. As a by-product, two short-run adjustment mechanisms were considered - a policy response (involving an open market operation) and a market response (involving a jump in the price level). This paper has argued that the policy response was by far the more plausible adjustment mechanism. The clear message is that, if the central bank is to change its target for the inflation rate, then any such change must be accompanied by an appropriate open market operation. Furthermore, even when the appropriate open market operation is implemented, economic variables are likely to undergo substantial adjustments before the new equilibrium is reached.

These results have extended previous studies by focusing, in particular, on the inflation target as an objective of monetary policy and by using a calibrated model which makes it possible to estimate the relative magnitude of different effects. In particular, the major conclusions concerning the relative importance of short-run and long-run adjustments and the greater plausibility of an open market operation over a price level adjustment would not have been possible in a purely theoretical framework without the benefit of calibration.

APPENDIX

CALCULATING SHORT-RUN EQUILIBRIA

The linearized model has exactly one stable eigenvalue so that the optimal solution follows a saddle-path. At the beginning of the optimization period, an adjustment in an appropriate jump variable will move the economy onto the stable saddlepath. Two types of jump have been considered in this paper: an open market operation and a price level jump.

Let μ_1 (<0) and μ_2 (>0) denote the stable and unstable eigenvalues, respectively, of the optimal solution. Let $b_1^*, m_1^*, g_1^*, r_1^* - \pi_1^*$ denote the steady-state equilibria of relevant variables. Then the general solution for public sector debt, b(t), is given by:

$$b(t) - b_1^* = C_1 \exp(\mu_1 t) + C_2 \exp(\mu_2 t) \qquad \text{(A.1)}$$

where C_1 and C_2 are constants determined by the stability properties and initial conditions of the optimal solution. In order to ensure stability, it is necessary to set C_2 equal to zero. This will require an appropriate adjustment (either an open market operation or a jump in the price level), at the beginning of the optimization period, leading to a corresponding adjustment in public sector debt. Let $b(0)$ denote the level of public sector debt after the initial adjustment has occurred. Then the optimal solution reduces to the form:

$$b(t) - b_1^* = (b(0) - b_1^*).\exp(\mu_1 t) \qquad \text{(A.2a)}$$

The optimal solutions for the real money supply, *m(t)*, real government expenditure, *g(t)*, and for real interest rates, $r(t) - \pi(t)$, then satisfy equations of the form:

$$m(t) - m_1^* = \left(\frac{dm}{db}\right)_{stable\ path} .(b(0) - b_1^*).\exp(\mu_1 t) \qquad \text{(A.2b)}$$

$$g(t) - g_1^* = \left(\frac{dg}{db}\right)_{stable\ path} .(b(0) - b_1^*).\exp(\mu_1 t) \qquad \text{(A.2c)}$$

$$(r(t) - \pi(t)) - (r_1^* - \pi_1^*) = \left(\frac{dr}{db}\right)_{stable\ path} .(b(0) - b_1^*).\exp(\mu_1 t) \qquad \text{(A.2d)}$$

The values of $\left(\frac{dm}{db}\right)_{stable\ path}$, $\left(\frac{dg}{db}\right)_{stable\ path}$ and $\left(\frac{dr}{db}\right)_{stable\ path}$ which define the slope of optimal stable solutions in m-b space, g-b space and r-b space, respectively, are summarized for a range of parameter values in Table A1.

(Table A1 about here)

The slope of the stable paths given in Table A1 can be used to calculate optimal initial jumps to: *b(0), m(0), g(0)* and $r(0) - \pi(0)$ as follows. Assume that initially the economy is in an equilibrium given by: $b_0^*, m_0^*, g_0^*, r_0^* - \pi_0^*$. Then assume that there is an exogenous shock to the system leading to a new steady state given by: $b_1^*, m_1^*, g_1^*, r_1^* - \pi_1^*$. This will lead to instantaneous initial jumps in all variables to: $b(0), m(0), g(0), r(0) - \pi(0)$.

Jump occurs through open market operation

When the initial jump occurs through an open market operation then b and m will jump at the beginning of the optimisation period so as to preserve the initial wealth constraint. The initial solution satisfies:

$$m(0) - m_1^* = \left(\frac{dm}{db}\right)_{stable\ path} .(b(0) - b_1^*) \qquad \text{(A.3a)}$$

Hence,

$$[m(0) - m_1^*] + [b(0) - b_1^*] = \left[1 + \left(\frac{dm}{db}\right)_{stable\ path}\right].(b(0) - b_1^*) \qquad \text{(A.3b)}$$

But, from the initial wealth constraint,

$$m(0) + b(0) = m_0^* + b_0^* \qquad \text{(A.3c)}$$

Thus,

$$b(0) = b_1^* + \left\{\frac{1}{1 + \left(\frac{dm}{db}\right)_{stable\ path}}\right\}[(m_0^* + b_0^*) - (m_1^* + b_1^*)] \qquad \text{(A.3d)}$$

Jump occurs through instantaneous price level adjustment

When the initial jump occurs through an instantaneous price level adjustment then b and m will jump at the beginning of the optimisation period so as to preserve the following relationship:

$$\frac{b(0)}{m(0)} = \frac{b_0^*}{m_0^*} \qquad \text{(A.4a)}$$

But,

$$\frac{b(0)}{m(0)} = \frac{b(0)}{m_1^* + \left(\frac{dm}{db}\right)_{stable\ path}.(b(0) - b_1^*)} \qquad \text{(A.4b)}$$

Hence, combining equations A.3a and A.3b,

$$b(0).m_0^* = b_0^*.m_1^* + \left(\frac{dm}{db}\right)_{stable\ path}.(b_0^*.b(0) - b_0^*.b_1^*) \qquad \text{(A.4c)}$$

So that,

$$b(0) = \frac{b_0^* \left[m_1^* - b_1^* \cdot \left(\frac{dm}{db} \right)_{stable\ path} \right]}{m_0^* - b_0^* \cdot \left(\frac{dm}{db} \right)_{stable\ path}} \qquad \text{(A.4d)}$$

Determining short-run jumps in other variables

Once *b(0)* has been determined it is a straightforward matter to derive initial conditions for other relevant variables by setting t equal to zero in equations A.2b-A.2d.

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FIGURE 1
Dynamic Adjustment Following An Increase in Target Inflation
(not drawn to scale)

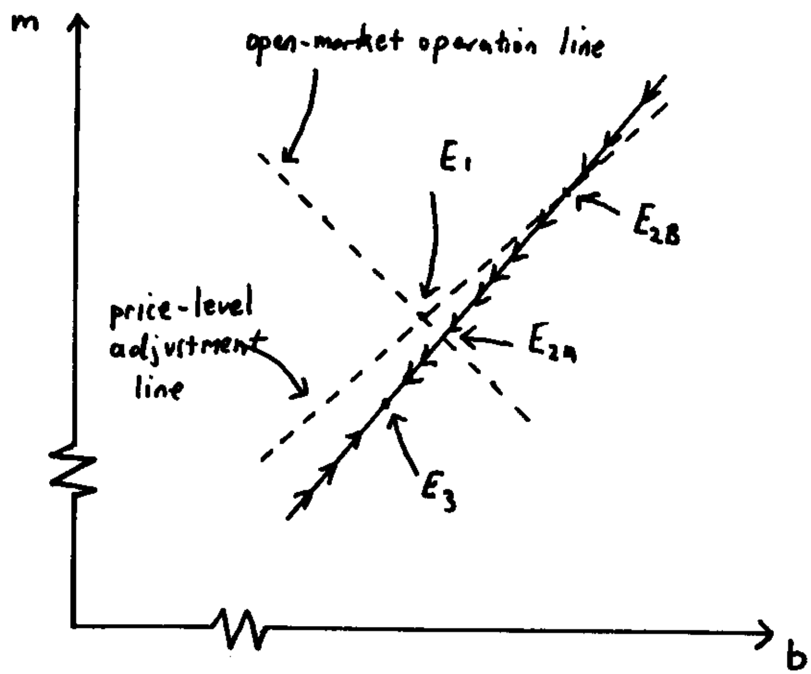

TABLE 1
Parameter Values for Calibrated Model

parameter	value
α_0	0.625
α_1	2.000
α_2	2.500
β_0	0.005
β_1	0.040
β_2	0.100
γ	0.400
$\bar{y}$	1.000
y^T	1.050
τ	0.250

TABLE 2
Optimal Steady-State Values

ϕ	p^T	b^*	m^*	g^*	$r^* - \pi^*$
0.0001	0.00	76.3722	0.0449195	0.188506	0.000805
	0.10	76.0623	0.0349146	0.188567	0.000854
0.01	0.00	0.76278	0.0418948	0.226314	0.031052
	0.10	0.75964	0.0317321	0.228349	0.032679
1.0	0.00	0.00762	0.0400303	0.249621	0.049697
	0.10	0.00759	0.0297933	0.252584	0.052067
100.0	0.00	0.00008	0.0400000	0.249996	0.049997
	0.10	0.00008	0.0297622	0.252972	0.052378

TABLE 3
Eigenvalues of Optimal Solution

ϕ	p^T	μ_1	μ_2
0.0001	0.00	-1.85838	2.01400
	0.10	-1.86246	2.01817
0.01	0.00	-2.80389	3.46832
	0.10	-2.81946	3.48953
1.0	0.00	-10.3295	58.9408
	0.10	-10.4119	59.4293
100.0	0.00	-12.4681	4853.95
	0.10	-12.5679	4894.43

TABLE 4A
Key Points on Dynamic Path Following an Increase in Target Inflation from 0% Per Annum to 10% Per Annum

ϕ	Key Points on Dynamic Path	b	m	g	$r-\pi$
0.0001	**Initial Equilibrium**	76.3722	0.04492	0.188506	0.000805
	Short-run Equilibria following:				
	• open-market operation	76.3814	0.03569	0.178929	-0.006856
	• jump in price level	81.4245	0.04789	0.026629	-0.128697
	Final Equilibrium	76.0623	0.03491	0.188567	0.000854
0.01	**Initial Equilibrium**	0.76278	0.04189	0.226314	0.031052
	Short-run Equilibria following:				
	• open-market operation	0.77107	0.03361	0.204904	0.013923
	• jump in price level	0.85112	0.04675	0.040661	-0.117471
	Final Equilibrium	0.75964	0.03173	0.228349	0.032679
1.0	**Initial Equilibrium**	0.00762	0.04003	0.249621	0.049697
	Short-run Equilibria following:				
	• open-market operation	0.00942	0.03823	0.147075	-0.032340
	• jump in price level	-0.00837	-0.04398	1.174730	0.789784
	Final Equilibrium	0.00759	0.02979	0.252584	0.052067
100.0	**Initial Equilibrium**	0.00008	0.040000	0.025000	0.049997
	Short-run Equilibria following:				
	• open-market operation	0.00011	0.039973	0.125332	-0.049734
	• jump in price level	-0.00001	-0.004068	0.675855	0.390684
	Final Equilibrium	0.00008	0.029762	0.025297	0.052378

TABLE 4B
Key Points on Dynamic Path Following a Decrease in Target Inflation

from 10% Per Annum to 0% Per Annum

ϕ	Key Points on Dynamic Path	b	m	g	$r-\pi$
0.0001	**Initial Equilibrium**	76.0623	0.03491	0.188567	0.000854
	Short-run Equilibria following:				
	• open-market operation	76.0531	0.04415	0.198083	0.008467
	• jump in price level	71.2909	0.03272	0.340995	0.122807
	Final Equilibrium	76.3722	0.04492	0.188506	0.000805
0.01	**Initial Equilibrium**	0.75964	0.03173	0.228349	0.032679
	Short-run Equilibria following:				
	• open-market operation	0.75135	0.04002	0.249699	0.049760
	• jump in price level	0.68046	0.02842	0.394693	0.165755
	Final Equilibrium	0.76278	0.04189	0.226314	0.031052
1.0	**Initial Equilibrium**	0.00759	0.02979	0.252584	0.052067
	Short-run Equilibria following:				
	• open-market operation	0.00579	0.03159	0.355121	0.134097
	• jump in price level	-0.00695	-0.02730	1.091220	0.722977
	Final Equilibrium	0.00762	0.04003	0.249621	0.049697
100.0	**Initial Equilibrium**	0.00008	0.029762	0.025297	0.052378
	Short-run Equilibria following:				
	• open-market operation	0.00005	0.029789	0.377636	0.152109
	• jump in price level	-0.00080	-0.297211	4.465129	3.422103
	Final Equilibrium	0.00008	0.040000	0.025000	0.049997

TABLE A1
Slope of Stable Path in the Neighborhood of the Steady-State

ϕ	p^T	$\left(\frac{dm}{db}\right)_{stable\ path}$	$\left(\frac{dg}{db}\right)_{stable\ path}$	$\left(\frac{dr}{db}\right)_{stable\ path}$
0.0001	0.0	0.00240	-0.03001	-0.02401
	0.1	0.00242	-0.03020	-0.02416
0.01	0.0	0.16364	-2.04554	-1.63643
	0.1	0.16413	-2.05167	-1.64133
1.0	0.0	4.61966	-57.7457	-46.1966
	0.1	4.62184	-57.7730	-46.2184
100.0	0.0	383.676	-4795.95	-3836.76
	0.1	383.842	-4798.03	-3838.42

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