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VALUATION OF IRREVERSIBLE ENTRY OPTIONS UNDER UNCERTAINTY AND TAXATION

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Abstract

We analyze the tax effects on a potential firm with an irreversible entry option and subject to risky post-entry earnings. We formulate the problem in terms of optimal stopping and derive both the necessary conditions for optimal entry and the value of the optimal premium by relying on the classical theory of diffusions and the Greenian representation of the stochastic value functional. We show that to make the entry option invariant with respect to the tax policy when the government owns a call option on a fraction of firm's earnings, the tax allowance has to satisfy a first-order non-linear differential equation. We derive qualitive results for the neutrality of the tax policy. Using standard geometric Brownian motion to model price uncertainty, we provide examples of the requirements for tax invariance. The Johansson-Samuelson theorem is re-examined.

Keywords: Optimal entry premium, tax policy, Johansson-Samuelson theorem, regular diffusion process, Green-kernel, optimal stopping.

JEL Code: D21, D84, G33, C61

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1 Introduction

The market entry of a new firm is like a leap into the great unknown. The recent literature on the theory of entry including the contributions by Brennan and Schwartz (1985), McDonald and Siegel (1986) and Dixit (1989) has reached the fundamental conclusion, that because of the uncertain future, it is optimal for any potential entering firm to choose a rather high premium in terms of expected rents relative to the immediate cost of entry.[1] The stochastic environment of potential firms is, however, heavily dependent upon public policy, in particular the taxation of future rents. There is very little analysis of this problem. In the current paper, we raise the following questions: (i) how does taxation of future rents interact with the entry premium of potential new firms, and (ii) what are the requirements to be imposed on the tax system if the objective is to prevent taxation from creating an entry barrier?

The existing theory of profits taxation has provided a number of useful insights. However, these two questions have not been thoroughly addressed so far. Instead, most of the literature on profits taxation has focused on the interaction between tax rules and the marginal investment choice of an existing, mature firm which no longer faces the problem of entry. In sharp contrast, market entry can be regarded as a discrete choice characterized by irreversibility, rather substantial sunk costs, and market uncertainty. Such a discrete, once-and-for-all decision about whether or not to enter the market represents an option with high risk and should be distinguished from marginal investment decisions in the post-entry regime. It is thus possible that the analysis of the associated tax effects may lead to substantial revision of the way one is used to thinking of the interaction between profits taxation, business formation and capital choice.[2] We consider the problem of entry and business formation under tax policy. The government is assumed to hold a property right to a fraction of a firm's cash flow in terms of a call option. Regarding profits tax as a call option highlights the idea that firms which incur losses will not be subsidized through the tax system.

In interpreting the government's claim in terms of a call option, we would like to draw attention to an early paper by Ball and Bowers (1983) who evaluate a particular proposal for taxing the resources sector of Australian industry using a variation of the Brown tax.[3] Their analysis suggests that this particular variation, though allowing for an unlimited carry-forward of losses (with an adjustment for interest), does not actually provide full loss offset. Given the risk that there may be no positive revenue in the future against which to draw previous losses, the implied effective tax rate on the project may be very high, potentially creating an investment barrier. In the current paper, we take a step further. We show that even in the absence of carry-forward of losses, non-distorting taxation exists and we provide a characterization of its properties. Basically, to prevent taxation from creating an entry (or investment) barrier, the government should allow for sufficient public participation at the outset. Such a policy can be accomplished through various instruments, including an initial grant or early tax relief. The fact that such arrangements are widely used in many countries can therefore be justified by our analysis. However, the properties of such a tax structure have not been examined and are

[1] In our recent paper (1997), this view has been generalized to the case where the quality of information on future market conditions can be improved only by commitment to irreversible entry with a sunk cost.

[2] This claim is even more generally valid, extending to the case where an existing firm plans to undertake an expansion in its capacity and considers such an expansion as irreversible.

[3] Brown (1948) suggested taxing economic rents by excluding interest payments and receipts but allowing for immediate expensing of capital outlays and permitting full loss offset by remitting money to the enterprise when negative cash flows occur.

not well understood. We hope that our paper contributes towards better understanding of this problem.

When compared to the Ball and Bowers paper, a somewhat more optimistic view thus arises from our analysis, not forgetting that we share the concern that the basic threat to tax neutrality lies in the government's inability to commit itself to sharing risks.[4] Such an asymmetry has not been left unnoticed in the previous literature on taxation. Auerbach (1986) demonstrates both analytically and using simulations the importance of the asymmetric treatment of gains and losses under otherwise neutral tax systems. With discrete decisions with substantial sunk costs, such an asymmetry can be expected to have even more radical implications. From such a perspective, MacKie-Mason (1990) and Lund (1992) represent the earlier papers which study the tax effects on investment opportunties in a framework of real options as we do in the current paper. MacKie-Mason considers the effects on investment and asset values of the US percentage depletion allowance, calculated as a percentage of gross revenue from taxable income under full or no loss offset. He shows that such a subsidy may actually discourage investment in some projects, shutdown of marginal projects may be encouraged, and that effective marginal tax rates will vary with the riskiness of a project. Moreover, an increase in the corporate income tax rate may encourage some investment under the US depletion allowance - a form of taxation paradox, cf. Sinn (1987). The study undertaken by Lund (1992) reports numerical simulation results on the tax effects on projects representing real options.

One of the key results on non-distorting taxation on income from capital, called the Johansson-Samuelson theorem by Sinn (1987), points to a general efficiency condition relative to marginal investment and across the form of financing. The basic intuition behind this celebrated result goes as follows: when a tax falls on economic profits from a project uniformly across all types of capital income regardless of the source, the efficiency conditions are undisturbed. We will show below that the validity of the Johansson-Samuelson theorem is more limited than thought earlier. Its limited validity arises because the theorem abstracts from the forthcoming sunk costs faced by any, non-existing potential firm. Whenever a project represents an option with a positive value of waiting, any tax on the opportunity cost - even a uniform tax - raises the option value of the project. However, the Johansson-Samuelson tax also raises the entry threshold and thus postpones entry.

In the current paper, we derive the effects of profits tax on valuation of an entry option. In the absence of taxation, such a valuation is well-known from the literature starting with McDonald and Siegel (1986) and Dixit (1989). Introduction of taxation into such a framework provides a new angle on the issue of taxation of risky profits. The valuation of an entry option has previously been derived under the assumption that the firm expects its price to follow a simple geometric Brownian motion. We have chosen to work with a more general time-homogenous and regular stochastic diffusion process relying on the Greenian representation of Markovian functionals. There are several reasons for such a choice. First, though the Greenian representation has not been widely employed so far by the economic profession, it is highly efficient in providing a method of valuing options requiring hardly any technical calculations. This sharply contrasts with the approach based on stochastic dynamic programming and geometric Brownian motion.

[4] The seminal analysis by Domar and Musgrave (1944) suggests that full risk-sharing and symmetric tax treatment of gains and losses make risk-averse investors increase their stake in risky asset as an optimal reponse to a rise in the tax rate. This result has since been qualified by Gordon (1985) in a general equilbrium, multi-period framework. For an analysis of tax neutrality under uncertainty in the case of a marginal investment project, see also Bond and Devereux (1995).

Indeed, the latter approach necessitates solving for a linear second-order differential equation with variable coefficients and subject to a set of boundary conditions which have to be satisfied at the unknown boundary while the techniques based on the classical theory of diffusions only require the use of simple non-linear programming and the Greenian representation of Markovian functionals. The payoff is that we are able to characterize explicitly both the form of the optimal entry premium and the value of the entry option; a property often absent in studies relying on dynamic programming techniques. Second, it is not the case that a geometric Brownian motion would be an appropriate model for most prices (cf. Dixit and Pindyck (1994) pp. 74-78). Our approach thus has the further advantage of allowing for introduction of more realistic ideas of how prices may evolve and has a wide range of applications in economic problems.

We report several new results. It turns out that what is relevant for the entry threshold in general is the value of unutilized production options, not the prices in the post-entry regime. This result is related to the "bad news principle" made famous by Bernanke (1983). We show that the project has to be profitable at the optimal threshold when adjusting for the tax effect. We derive a general neutrality condition for profits tax from the perspective of a potential, non-existent firm. It turns out that such a condition can be cast in terms of the required governmental risk sharing. Stated as initial participation, the condition is shown to be represented by a solution to a particular first-order differential equation. In a parametrized example it is also shown to be concave in the tax rate. What such a result implies is that the governmental participation in sharing risks should increase in a non-linear fashion as a function of the tax rate. This result once again points to the importance of unlimited loss offsets. The limits to neutrality of the cash flow tax are therefore also much more stringent than previously thought and neutrality will not be obtained in the form of tax structure suggested by the Meade Committee (1978) without due attention devoted to the problem of losses. A further implication is that such a result provides justification for accelerated depreciation instead of economic depreciation.[5]

To conclude, we would like to highlight that typically there is substantial asymmetry in the operations of potential new firms facing substantial sunk costs and high market risks compared to mature, established firms operating under more stable market conditions.[6] The latter group of firms often generates substantial positive cash flows facing a much lower risk of failure than a typical new and inexperienced firm. Most tax analyses have focused on defining tax rules in terms of neutrality from the perspective of mature firms with positive cash flows and tax reforms have often been conducted from such a perspective. However, given the higher failure risks of potential firms with the high probability that losses cannot be carried forward, a tax system which is non-distorting from the perspective of mature firms typically discriminates against new

[5]The idea of the efficiency of accelerated depreciation has already been introduced by Bulow and Summers (1983) for a stochastic tax base (due to stochastic economic depreciation). They consider the concept of economic depreciation in a risky environment and show that depreciation allowances, if set ex ante, should be adjusted to take account of future asset price risk. This result has since been largely neglected. Interpreting the case of failure of the firm in our model as "radical economic depreciation" makes it possible to build a bridge between the Bulow-Summers view and our model with the remaining difference that Bulow and Summers worked with marginal investment decisions while we model the discrete entry decision. The importance of both of these results derives from the fact that they are the only results justifying liberal depreciation schemes, thus providing a major challenge to the key principles of the tax reform wave of the 1980s which was designed to reduce write-off possibilities.

[6]The data reported by Eurostat (1995) suggests that the failure rate of new firms in the European Union is substantial in the early years of the business life-cycle. After a year, 20 % of new firms finish, and 35 % has disappeared within three first years. After five years, only 50 % remains in the market (Enterprises in Europe, Fourth Report, Eurostat (1995)).

firms. The problem is that a policy stance, which is neutral with respect to market entry is inherently time-inconsistent. Given that such a policy problem is rationally foreseen by potential firms, "neutral" taxation of existing firms may interact with the option value of potential new firms. As a result, policy-makers seem to face a trade-off between choosing one which is neutral with respect to new entry but subsidizes the marginal investment of existing firms and choosing a tax system which reduces the option value of entry but does not tax the return on marginal investment.

2 Irreversible Entry under Uncertainty and Taxation

Consider a firm operating under price uncertainty after potential entry. There are certain benefits in terms of generality of results in working with general diffusion processes. First, from the economic point of view, it is not the case that the assumption of geometric Brownian motion is the most natural one. In fact, prices in most markets do not behave in that way (Dixit and Pindyck (1994), section 3.B). Second, the formal analysis is more general and actually "easier" using the general diffusion process once one becomes familiar with the basic concepts. To capture the stochastic nature of the model, assume then that the post-entry price evolves according to a regular diffusion defined on a complete filtered probability space $(\Omega, P, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F})$, with state-space $(0, \infty)$, and described up to explosion times by the (Itô-) stochastic differential equation

$$dp(t) = \mu(p(t))dt + \sigma(p(t))dW(t), \tag{1}$$

where $p(0) = p$ is known and $W(t)$ denotes standard Brownian motion. For simplicity, we assume that both the drift coefficient $\mu : \mathbf{R}_+ \mapsto \mathbf{R}$, measuring the expected growth rate of demand, and the diffusion coefficient $\sigma : \mathbf{R}_+ \mapsto \mathbf{R}$, measuring the size of the stochastic fluctuations around the mean, are continuous. For simplicity, and in accordance with reality, we assume throughout this study that ∞ is an unattainable boundary for the diffusion $p(t)$. Thus, while demand can increase, it is never expected to become infinitely high in finite expected time.[7]

In accordance with standard microeconomic theory, we assume that in the absence of taxation the firm faces a continuously differentiable, increasing, and convex net cash flow $R : \mathbf{R}_+ \mapsto \mathbf{R}$. Moreover, we also assume that there is a threshold at which the cash flow vanishes. Put formally, we assume that there is a $p^* \in \mathbf{R}_+$ satisfying the equation $R(p^*) = 0$. It is clear that the monotonicity of R implies that the zero-revenue threshold p^* is unique. Therefore, $R(p) < 0$ for all $p < p^*$ and $R(p) \geq 0$ for all $p \geq p^*$.

Now consider instead the post-entry net cash flow of the firm subject to taxation at a constant tax rate $\epsilon > 0$. It is assumed that the government holds an option on a fraction of the firm's profit implying that it will not subsidize losses. The net cash flow of the firm subject to taxation, denoted now by $\hat{R}(p)$, is

$$\hat{R}(p) = \begin{cases} (1-\epsilon)R(p), & p \geq p^* \\ R(p), & p < p^* \end{cases} \tag{2}$$

It is worth noticing that (2) can also be written in the alternative forms $\hat{R}(p) = (1-\epsilon)R^+(p) -$

[7] However, it is worth pointing out that we do not preclude the cases where $\lim_{t\to\infty} p(t) = +\infty$ almost surely. In the case of the lower boundary 0 we assume that it is either unattainable or a killing boundary for the diffusion $p(t)$.

$R^-(p) = R(p) - \epsilon R^+(p)$, showing that the intrinsic value of the tax authorities' option on the firm's profits is given by $\epsilon R^+(p)$.

Under these assumptions, the present value of the firm from a given entry date t up to an arbitrarily distant future[8], denoted as $Y(t)$, is given by the equation

$$Y(t) = \int_t^\infty e^{-\beta(s-t)} \hat{R}(p(s))ds,$$

where $\beta > 0$ is a measure of an opportunity cost. It can be thought to be expressed as net of tax, $\beta = (1-\hat{\epsilon})r$, where r denotes the nominal interest rate and $\hat{\epsilon}$ is the rate at which capital income is taxed. It is now clear from the definition above that the present expected value of the firm ("the underlying asset") is

$$E_p[e^{-\beta\tau}Y(\tau)] = E_p \int_\tau^\infty e^{-\beta s}\hat{R}(p(s))ds,$$

where τ is a Markov-time defined with respect to the standard filtration $\mathcal{F}_t$. The strong Markov property of diffusions and the time homogeneity of $p(t)$ then imply that

$$E_p \int_\tau^\infty e^{-\beta s}\hat{R}(p(s))ds = E_p[e^{-\beta\tau}J(p(\tau))],$$

where

$$J(p) = E_p \int_0^\infty e^{-\beta s}\hat{R}(p(s))ds \tag{3}$$

is the present expected value of the firm. Before proceeding in our analysis, we state the following auxiliary definition:

Definition: (Borodin and Salminen (1996), chapter II, and Itô and McKean (1965), section 4. 6, Mandl (1968), chapter II) The *Green-kernel* $G_\beta : (0,\infty) \times (0,\infty) \mapsto \mathbb{R}_+$ of the diffusion p is defined as

$$G_\beta(p,y) = \int_0^\infty e^{-\beta t}P(t;p,y)dt,$$

where $P(t;p,y)$ is the transition density of the diffusion p defined with respect to the speed measure of p. There are two linearly independent functions (the *fundamental solutions*) $\psi(p)$ and $\varphi(p)$, with $\psi(p)$ increasing and $\varphi(p)$ decreasing, spanning the set of solutions of the ordinary differential equation $((\mathcal{A}-\beta)u)(p) = 0$, where

$$\mathcal{A} = \frac{1}{2}\sigma^2(p)\frac{d^2}{dp^2} + \mu(p)\frac{d}{dp}$$

[8]It should be noticed that for a completely precise definition, the time horizon of the firm should be given from entry up to the first exit time $\zeta = \inf\{t \geq 0 : p(t) \notin (0,\infty)\}$ (the so-called *life time of the process*). Under our assumptions, this horizon may be finite only if 0 is a killing boundary. For the remaining cases, 0 is never attained in finite expected time. Therefore, in case of a lower killing boundary, the cash flow should read as $\hat{R}(p(t))1_{[0,\zeta)}(t)$.

is the differential operator representing the infinitesimal generator of p. The Green-kernel $G_\beta(p,y)$ can then be rewritten as (cf. Borodin and Salminen (1996), pp. 19, and Mandl (1968), pp. 31 for derivation)

$$G_\beta(p,y) = \begin{cases} B^{-1}\psi(p)\varphi(y), & p < y \\ B^{-1}\psi(y)\varphi(p), & p \geq y \end{cases}$$

where $B > 0$ is the constant *Wronskian* determinant of the fundamental solutions $\psi(p)$ and $\varphi(p)$.

Equation (3) can now be written alternatively in the form (Itô and McKean (1965), chapter 4, Mandl (1968), chapter II)

$$J(p) = \int_0^\infty G_\beta(p,y)\hat{R}(y)m'(y)dy \tag{4}$$

where

$$m'(p) = \frac{2}{\sigma^2(p)} \exp\left(\int^p \frac{2\mu(s)}{\sigma^2(s)} ds\right)$$

is the speed density of p.

Consider now a risk-neutral firm with an entry option and facing a stochastic cash flow (2) after potential entry. The role of government may take several forms. Government shares profits by taxing them and shares costs by providing initial grants or allowances when profits arise. These mechanisms are introduced explicitly. Government also provides general infrastructure which has an impact on the price process (1) but this need not be modeled explicitly. Moreover, it is assumed that there is a credible commitment to an announced tax policy. If entering is costly, then the objective of the firm is to determine

$$V(p) = \sup_\tau E_p[e^{-\beta\tau}(J(p(\tau)) - \alpha(\epsilon)c)^+], \tag{5}$$

where τ is an arbitrary Markov time defined with respect to the natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, c is the initial investment cost incurred during entry, and $\alpha : [0,1] \mapsto \mathbf{R}$ is a function depending on the corporate tax rate and satisfying the condition $\alpha(0) = 1$. Note that only in the case where entry cost is completely tax-deductible, we have that $\alpha(\epsilon) = (1-\epsilon)$.

Our principal result on optimal entry under taxation is summarized in

Theorem 1. *If $\tilde{p} \in \{p \in \mathrm{R}_+ : J(p) \geq \alpha(\epsilon)c\}$ is the optimal entry threshold, then*

$$J'(\tilde{p})\psi(\tilde{p}) = (J(\tilde{p}) - \alpha(\epsilon)c)\psi'(\tilde{p}) \tag{6}$$

and the value of the optimal entry strategy is

$$V(p) = \begin{cases} J(p) - \alpha(\epsilon)c, & p \geq \tilde{p} \\ (J(\tilde{p}) - \alpha(\epsilon)c)\frac{\psi(p)}{\psi(\tilde{p})}, & p < \tilde{p}. \end{cases} \tag{7}$$

Proof. If $\tilde{p} \in \{p \in \mathrm{R}_+ : J(p) \geq \alpha(\epsilon)c\}$ is the optimal entry threshold, then the optimal stopping (exercise) date is, by assumption, $\tau = \tau(\tilde{p}) = \inf\{t \geq 0 : p(t) = \tilde{p}\}$, implying that (Borodin and Salminen (1996), chapter II and Itô and McKean (1965), section 4.6)

$$V(p) = (J(\tilde{p}) - \alpha(\epsilon)c)E_p[e^{-\beta\tau(\tilde{p})}; \tau(\tilde{p}) < \infty] = (J(\tilde{p}) - \alpha(\epsilon)c)\frac{\psi(p)}{\psi(\tilde{p})}, \tag{8}$$

proving (7). However, since the stopping strategy has to be such that it maximizes the value conditional on the stopping boundary (otherwise there would be another stopping strategy rendering a higher value and thus contradicting maximality), we see that $\tilde{p} \in (0, \infty)$ can be optimal (see Alvarez (1995), Proposition 3.1) only if for any $p < \tilde{p}$

$$\frac{\partial V(p)}{\partial \tilde{p}} = \frac{\psi(p)}{\psi(\tilde{p})^2}[J'(\tilde{p})\psi(\tilde{p}) - (J(\tilde{p}) - \alpha(\epsilon)c)\psi'(\tilde{p})] = 0,$$

proving the necessary condition (6). □

Theorem 1 shows that the optimal threshold $\tilde{p}$ depends both on the tax rate and fraction $\alpha(\epsilon)$ and satisfies

$$J(\tilde{p}) - \alpha(\epsilon)c = J'(\tilde{p})\frac{\psi(\tilde{p})}{\psi'(\tilde{p})} \tag{9}$$

where the right-hand side of equation (9) is the ***explicit opportunity cost*** of irreversibly using the entry opportunity. That is, the returns accrued from initiating production must exceed the entry costs by a nonnegative amount measuring the required entry premium. It is also worth pointing out that $\tilde{p}$ can be a (local) maximum of (8) only if the local concavity condition

$$J''(\tilde{p})\psi(\tilde{p}) - (J(\tilde{p}) - \alpha(\epsilon)c)\psi''(\tilde{p}) < 0 \tag{10}$$

is satisfied. Moreover, the value of the firm following the optimal entry strategy satisfies the familiar smoothness requirements stated in

Corollary 1. *If* $\tilde{p} \in \{p \in \mathbf{R}_+ : J(p) \geq \alpha(\epsilon)c\}$ *is the unique optimal entry boundary, then*

$$\lim_{p \to \tilde{p}} V(p) = J(\tilde{p}) - \alpha(\epsilon)c \qquad \textit{(value-matching)} \tag{11}$$

and

$$\lim_{p \to \tilde{p}} V'(p) = J'(\tilde{p}) \qquad \textit{(smooth-fit)}. \tag{12}$$

Proof. It is clear from Theorem 1 that $\lim_{p \downarrow \tilde{p}} V(p) = J(\tilde{p}) - \alpha(\epsilon)c$ and $\lim_{p \downarrow \tilde{p}} V'(p) = J'(\tilde{p})$. Similarly, by letting p approach the optimal entry threshold $\tilde{p}$ from below in (7) we obtain $\lim_{p \uparrow \tilde{p}} V(p) = J(\tilde{p}) - \alpha(\epsilon)c$, proving the value-matching condition.[9] Consider now $V'(p)$ on the set $(0, \tilde{p})$. By letting p approach $\tilde{p}$ from below and invoking necessary condition (6), we obtain $\lim_{p \uparrow \tilde{p}} V'(p) = J'(\tilde{p})$, which is the required result. This completes our proof. □

While (6) presents us the necessary condition for the optimal entry threshold in a compact form, it does not show us how the *Greenian representation* (4) of the value function can be applied to characterizing the optimal entry strategy. According to (4),

$$J(p) = B^{-1}\varphi(p) \int_0^p \psi(y)\hat{R}(y)m'(y)dy + B^{-1}\psi(p) \int_p^\infty \varphi(y)\hat{R}(y)m'(y)dy. \tag{13}$$

[9]It is worth noticing that the value-matching condition is a more or less trivial property in the case of a linear diffusion since all β-excessive functions for such a process have to be continuous (see Dynkin (1965 b), Theorem 12.4, page 7, and Dynkin (1965 a), definition 3.23, page 104).

Thus, by ordinary differentiation we obtain

$$J'(p) = B^{-1}\varphi'(p)\int_0^p \psi(y)\hat{R}(y)m'(y)dy + B^{-1}\psi'(p)\int_p^\infty \varphi(y)\hat{R}(y)m'(y)dy. \tag{14}$$

It is now a direct consequence of equations (13) and (14) that the necessary condition (6) can be rewritten in the form

$$\int_0^{\tilde{p}} \psi(y)\hat{R}(y)m'(y)dy = \alpha(\epsilon)c\frac{\psi'(\tilde{p})}{S'(\tilde{p})}, \tag{15}$$

where $S'(p) = \exp\left(-\int^p \frac{2\mu(s)}{\sigma^2(s)}ds\right)$ is the scale density of p. Interestingly, (15) shows that the optimal entry threshold is determined only by the value of the lost entry opportunities in the sense that it is not affected by prices above $\tilde{p}$. Thus, the determination of the optimal boundary will always be dependent only on the prices at which the entry option is left unexercised. An interesting property is summarized in

Lemma 1. *If $\alpha(\epsilon) > 0$ then the project has to be profitable even in the post-tax sense at the optimal entry threshold.*[10] *Put formally, if $\alpha(\epsilon) > 0$, then $\tilde{p} > p^*$ and $\hat{R}(\tilde{p}) > 0$.*

Proof. The required result is a direct consequence of (15), since $\psi(p)$ and $m'(p)$ are nonnegative functions, and $\hat{R}(p)$ is monotonically increasing. □

Moreover, it is now clear that (15) can be rewritten in the form

$$\int_0^{\tilde{p}} \psi(y)R(y)m'(y)dy - \epsilon\int_{p^*}^{\tilde{p}} \psi(y)R(y)m'(y)dy = \alpha(\epsilon)c\frac{\psi'(\tilde{p})}{S'(\tilde{p})}. \tag{16}$$

The first term on the left hand side of (16) captures the value of the lost entry opportunities in the absence of taxation valued at prices which do not justify entry. The second term is interpreted as *the value of the lost option of the tax authorities on the profits of the firm for the pre-entry prices.*

Let us now demonstrate that (15) proves that if an exit threshold exists, then it is unique. The β-harmonicity of the function $\psi(p)$ implies that (15) can also be written in the form (Borodin and Salminen (1996), chapter II)

$$\int_0^{\tilde{p}} \psi(y)[\hat{R}(y) - \beta\alpha(\epsilon)c]m'(y)dy = \alpha(\epsilon)c\frac{\psi'(0)}{S'(0)}. \tag{17}$$

Therefore, $\hat{R}(\tilde{p}) \geq \beta\alpha(\epsilon)c$. By now defining the continuously differentiable function $\Delta : \mathbf{R}_+ \mapsto \mathbf{R}$ as

$$\Delta(p) = \int_0^p \psi(y)[\hat{R}(y) - \beta\alpha(\epsilon)c]m'(y)dy - \alpha(\epsilon)c\frac{\psi'(0)}{S'(0)}$$

we notice by ordinary differentiation that

$$\Delta'(p) = \psi(p)[\hat{R}(p) - \beta\alpha(\epsilon)c]m'(p).$$

[10] It is of interest to point out that in some cases, it is necessary to have $\alpha(\epsilon) < 0$ to make entry worthwhile, see below.

The monotonicity of $\hat{R}(p)$ now implies that $\Delta(p)$ is also monotonically increasing as long as $\hat{R}(p) > \beta\alpha(\epsilon)c$. Combining this result with (17) proves the alledged result.

Let us now develop the value function slightly further. By recalling that $BS'(p) = \psi'(p)\varphi(p) - \psi(p)\varphi'(p)$, we notice from (7) that if $p < \bar{p}$, then

$$\begin{aligned} V(p) &= \frac{\psi(p)}{\psi(\bar{p})}\Big[B^{-1}\varphi(\bar{p})\int_0^{\bar{p}} \psi(y)\hat{R}(y)m'(y)dy \\ &+ B^{-1}\psi(\bar{p})\int_{\bar{p}}^{\infty} \varphi(y)\hat{R}(y)m'(y)dy - \alpha(\epsilon)c\Big] \end{aligned}$$

By now invoking the necessary condition (15) and the results stated above we see that before entry has occurred, the value of the option is

$$V(p) = \frac{\psi(p)}{B}\int_{\bar{p}}^{\infty} \varphi(y)[(1-\epsilon)R(y) - \beta\alpha(\epsilon)c]m'(y)dy.$$

That is, the value of the opportunity measures the expected discounted returns (per unit of time) from $\bar{p}$ up to infinity. The value of the entry option therefore depends on the prices at which the opportunity is unexercised only through $\bar{p}$. It is also worth noticing that the value of the option of the tax authorities on the profits of the firm, denoted $T(\epsilon, p)$, is

$$T(\epsilon, p) = \epsilon B^{-1}\varphi(p)\int_{p^*}^{p} \psi(y)R(y)m'(y)dy + \epsilon B^{-1}\psi(p)\int_{p}^{\infty} \varphi(y)R(y)m'(y)dy - (1-\alpha(\epsilon))c.$$

Therefore, at entry the value of this option is

$$T(\epsilon, \bar{p}) = \epsilon B^{-1}\varphi(\bar{p})\int_{p^*}^{\bar{p}} \psi(y)R(y)m'(y)dy + \epsilon B^{-1}\psi(\bar{p})\int_{\bar{p}}^{\infty} \varphi(y)R(y)m'(y)dy - (1-\alpha(\epsilon))c.$$

It is now obvious that under asymmetric taxation, the optimal entry threshold is not generally invariant to taxation, nor is the option value linear in ϵ. It is worth noticing that this negative result holds even under full tax-deductability of entry costs, that is, when $\alpha(\epsilon) = (1-\epsilon)$. To illustrate this argument more rigorously, differentiate (6) implicitly with respect to the corporate tax rate ϵ and invoke (15) to obtain

$$[J''(\bar{p})\psi(\bar{p}) - (J(\bar{p}) - \alpha(\epsilon)c)\psi''(\bar{p})]\frac{\partial\bar{p}}{\partial\epsilon} = -\alpha'(\epsilon)c\psi'(\bar{p}) - S'(\bar{p})\int_{p^*}^{\bar{p}} \psi(y)R(y)m'(y)dy.$$

The local concavity condition implies that the multiplier of $\frac{\partial\bar{p}}{\partial\epsilon}$ is negative. Therefore, since $R(p) \geq 0$ on $[p^*, \infty)$ we see that if $\alpha'(\epsilon)c\psi'(\bar{p}) + S'(\bar{p})\int_{p^*}^{\bar{p}} \psi(y)R(y)m'(y)dy \geq 0$, then *taxation increases the investment threshold and thus prolongs waiting and leads to the postponement of entry*. This result casts serious doubt on the celebrated proposition of the Meade Committee (1978) on taxation in the UK suggesting tax neutrality of cash flow taxation. Such a view applies to a mature profitable firm but does not hold for a potential firm with an entry option under uncertainty. Tax distortion of a mature existing firm subject to asymmetric tax treatment of profits and losses has previously been analyzed by Auerbach (1986). From the above condition we note that something analogous to the so-called taxation paradox in the theory of corporation taxation (cf. Sinn 1987) may also arise in the current model; i.e. a rise in the tax rate may reduce the entry threshold. Such a case arises if the condition $\alpha'(\epsilon)c\psi'(\tilde{p})+S'(\tilde{p})\int_{p^*}^{\tilde{p}}\psi(y)R(y)m'(y)dy \leq 0$ holds. Intuitively, this amounts to stating that division of the cost of entry, c, between the firms and the tax authority is sufficiently favorable to the firm. This happens when the rise in the stake of the tax authority is sufficient to compensate for the lost revenue of the firm. As we have explained, the firm evaluates the tax option of the authorities at pre-entry prices, entering the market only when the value of lost production options becomes high enough. Both of these offsetting mechanisms are captured by this condition stated as a response to a marginal change in the tax rate. It is worth mentioning that if $\alpha(\epsilon)$ is decreasing, then a sufficient condition for a positive relationship between taxation and entry is that $R(\tilde{p}) \leq -\alpha'(\epsilon)\beta c$. Summarizing,

Proposition 1. *If the firm is not almost surely profitable, then the value of an entry option is not invariant to a cash flow tax where the firm is allowed an initial grant $\alpha(\epsilon) = 1-\epsilon$ and where the earnings of the firm are subject to taxation at a rate of $\epsilon > 0$.*

3 A Condition for Tax-neutrality

It is of interest to study if there is a function $\alpha(\epsilon)$ which leads to tax neutrality with respect to the entry decision in the absence of loss offset. It is clear from the analysis of the previous section that $\frac{\partial\tilde{p}}{\partial\epsilon} = 0$ only if

$$\alpha'(\epsilon) = -\frac{S'(\tilde{p})}{c\psi'(\tilde{p})}\int_{p^*}^{\tilde{p}}\psi(y)R(y)m'(y)dy. \tag{18}$$

On the other hand, (16) implies that

$$\int_{p^*}^{\tilde{p}}\psi(y)R(y)m'(y)dy = \frac{\alpha(\epsilon)}{(1-\epsilon)}c\frac{\psi'(\tilde{p})}{S'(\tilde{p})} - \frac{1}{(1-\epsilon)}\int_0^{p^*}\psi(y)R(y)m'(y)dy \tag{19}$$

By combining the results of (18) and (19), we finally obtain that if a neutral $\alpha(\epsilon)$ exists, then it must satisfy the ordinary differential equation[11]

$$(1-\epsilon)\alpha'(\epsilon)+\alpha(\epsilon) = \frac{S'(\tilde{p})}{c\psi'(\tilde{p})}\int_0^{p^*}\psi(y)R(y)m'(y)dy, \tag{20}$$

which implies that

$$\frac{d}{d\epsilon}\Big[\frac{\alpha(\epsilon)}{(1-\epsilon)}\Big] = \frac{S'(\tilde{p})}{(1-\epsilon)^2c\psi'(\tilde{p})}\int_0^{p^*}\psi(y)R(y)m'(y)dy.$$

Since the right hand side of this equation is negative for any corporate tax rate $\epsilon \in [0,1)$ and $\alpha(0) = 1$, we see that if there is an α which leads to neutrality, then $\alpha(\epsilon) < (1-\epsilon)$. Summarizing,

Theorem 2. *(a) If $\alpha(\epsilon)$ satisfies (20) then the optimal entry threshold is invariant to taxation.*
(b) If $\{p \in \mathbf{R}_+ : R(p) < 0\} \neq \emptyset$, then the tax invariance of the entry option requires that $\alpha(\epsilon) < 1-\epsilon$.
(c) If the firm does not face potential losses, that is, if $\{p \in \mathbf{R}_+ : R(p) < 0\} = \emptyset$, then the tax invariance of the entry option requires that $\alpha(\epsilon) = 1-\epsilon$.

[11] It is worth noticing that according to the necessary condition for optimal entry, the optimal entry threshold $\tilde{p}$ is dependent on the function $\alpha(\epsilon)$. Therefore, the ordinary differential equation describing the function $\alpha(\epsilon)$ leading to neutrality is generally highly nonlinear.

We should emphasize that Theorem 2 is valid rather generally; in deriving it we have not assumed harmonized tax rates on capital income regardless of the source. The significance of this is its implication that one should allow for accelerated depreciation for efficiency reasons. The effect of tax asymmetry is so strong that this result holds regardless of the extent to which the tax system may discriminate, say, against the alternative return standing for the opportunity cost. Most countries have experimented with accelerated depreciation, though many tax reforms of the 1980s aimed at eliminating it. Such an elimination may be acceptable with established, mature firms generating positive free cash flow in the state of no growth opportunities. However, for potential non-existing firms, restricted fiscal depreciation generates a tax threat which reduces the option value of costly entry. Interpreting initial grants as a substitute for accelerated depreciation provides justification for such grants, which are widely used in industrial policies. Below we show that in an explicit model with prices following a geometric Brownian motion, the α-function satisfying condition (20) is decreasing and strictly concave in the tax rate.

4 Illustration of Results

In the preceding sections uncertainty was modeled through the general diffusion (1) so as to obtain general results. However, to illustrate our results we now invoke a familiar diffusion process, standard geometric Brownian motion. That is, in this section we assume that the price evolves according to the stochastic differential equation

$$dp(t) = \mu p(t)dt + \sigma p(t)dW(t) \qquad p(0) = p,$$

where μ and σ are constants. For simplicity, assume now that the cash flow of the firm is of the form

$$\hat{R}(p) = (1-\epsilon)\delta p^{\theta},$$

where $\delta \in \mathbf{R}_+$ and $\theta > 1$ are constants (in this case $p^* = 0$). If the assumption $\beta > q(\theta) := \theta\mu + \sigma^2\theta(\theta-1)/2$ (*absence of speculative bubbles*) holds, ordinary integration yields

$$J(p) = \frac{(1-\epsilon)\delta p^{\theta}}{\beta - q(\theta)}.$$

In this case, the optimal entry problem becomes

$$V(p) = \sup_{\tau} E_p\Big[e^{-\beta\tau}(\frac{(1-\epsilon)\delta p^{\theta}}{\beta - q(\theta)} - \alpha(\epsilon)c)^+\Big].$$

By now invoking the necessary condition (6) we find that the optimal entry boundary has to satisfy the equation

$$\psi((1-\epsilon)M\tilde{p}^{\theta} - \alpha(\epsilon)c) = \theta(1-\epsilon)M\tilde{p}^{\theta}, \tag{21}$$

where $M := \frac{\delta}{\beta - q(\theta)}$. It is a straightforward consequence of (21) that (provided that $\psi > \theta$)

$$\tilde{p} = \Big(\frac{\psi c}{M(\psi - \theta)}\Big)^{\frac{1}{\theta}}\Big(\frac{\alpha(\epsilon)}{(1-\epsilon)}\Big)^{\frac{1}{\theta}}. \tag{22}$$

To derive a neutrality condition, differentiate (22) implicitly with respect to the tax rate ϵ. Completing this task gives

$$\frac{\partial \tilde{p}}{\partial \epsilon} = \left(\frac{\psi c}{M(\psi-\theta)}\right)^{\frac{1}{\theta}} \frac{1}{\theta} \left(\frac{\alpha(\epsilon)}{(1-\epsilon)}\right)^{\frac{1}{\theta}-1} \frac{((1-\epsilon)\alpha'(\epsilon)+\alpha(\epsilon))}{(1-\epsilon)^2},$$

proving that tax neutrality can be achieved provided that the ordinary differential equation

$$(1-\epsilon)\alpha'(\epsilon) + \alpha(\epsilon) = 0 \tag{23}$$

is satisfied. Remembering that $\alpha(0) = 1$ we see that the solution of (23) is $\alpha(\epsilon) = (1-\epsilon)$. That is, in the absence of potential losses, revenue taxation leads to neutrality with respect to the entry decision, a result which is in accordance with the standard results of taxation and decisions on marginal investment. However, it is clear that this result arises only because of the fact that the firm in the current example does not face the risk of ever becoming unprofitable. To show that this is indeed the case, consider now instead a firm facing a net cash flow

$$\hat{R}(p) = (p-d) - \epsilon(p-d)^+, \tag{24}$$

where $d > 0$ is a constant operating cost. Ordinary integration shows that if $\beta > \mu$ (*absence of speculative bubbles*)

$$J(p) = \begin{cases} (1-\epsilon)\left(\frac{p}{\beta-\mu} - \frac{d}{\beta}\right) + \frac{2\epsilon d^{1-\varphi}p^{\varphi}}{\sigma^2(\psi-\varphi)\varphi(1-\varphi)}, & p \geq d \\ \frac{p}{\beta-\mu} - \frac{d}{\beta} - \frac{2\epsilon d^{1-\psi}p^{\psi}}{\sigma^2(\psi-\varphi)\psi(\psi-1)}, & p < d \end{cases} \tag{25}$$

where $\psi = (\frac{1}{2} - \frac{\mu}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\beta}{\sigma^2}}$ and $\varphi = (\frac{1}{2} - \frac{\mu}{\sigma^2}) - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\beta}{\sigma^2}}$ are the positive and negative roots of the quadratic equation $\frac{1}{2}\sigma^2 x^2 - (\frac{1}{2}\sigma^2 - \mu)x - \beta = 0$. The optimal entry threshold, denoted by $\tilde{p}$, is now determined from the necessary condition

$$(1-\epsilon)\frac{(\psi-1)\tilde{p}}{\beta-\mu} - (1-\epsilon)\frac{d\psi}{\beta} - \psi\alpha(\epsilon)c + \frac{2\epsilon d^{1-\varphi}\tilde{p}^{\varphi}}{\sigma^2\varphi(1-\varphi)} = 0. \tag{26}$$

Now define the function

$$\Lambda(p) = (1-\epsilon)\frac{(\psi-1)p}{\beta-\mu} - (1-\epsilon)\frac{d\psi}{\beta} - \psi\alpha(\epsilon)c + \frac{2\epsilon d^{1-\varphi}p^{\varphi}}{\sigma^2\varphi(1-\varphi)}.$$

It is then clear that Λ is strictly increasing, and explodes to $+\infty$ as p tends to infinity. Moreover, since Λ is continuous and

$$\Lambda(d) = \frac{2d}{\sigma^2\varphi(1-\varphi)} - \psi\alpha(\epsilon)c < 0,$$

we notice that a root $\tilde{p}$ satisfying the optimality condition exists on the set (d, ∞). Now consider the determination of the proper α leading to tax neutrality again. Ordinary differentiation shows that tax invariance requires that the ordinary differential equation

$$\alpha'(\epsilon) + \frac{\alpha(\epsilon)}{1-\epsilon} = -\frac{d^{1-\varphi}\tilde{p}^{\varphi}}{\beta(1-\varphi)c(1-\epsilon)} \tag{27}$$

has to be satisfied. (27) is a nonlinear differential equation which has to be solved simultaneously with (26) to obtain the correct α leading to neutrality. In Figure 1, we illustrate the neutral α-function under the assumptions that $d = 0.2$, $c = 1$, $\sigma = 0.25$, $\mu = 0.02$, $\beta = 0.0425$.

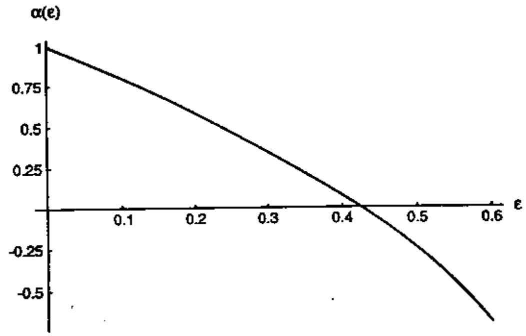

Figure 1: Neutral $\alpha(\epsilon)$

5 Stochastic Cash Flows: The Johansson-Samuelson Theorem Revisited

In their well-known contribution to the theory of tax-neutrality, Johansson (1961) and Samuelson (1964) proposed the general result to which we drew attention at the outset. It is our purpose in this section to re-examine their proposal from the perspective of costly entry. To this end, consider a competitive firm facing the stochastically evolving cash flow described in (2). In the absence of taxation, the value of the firm is then given as

$$F(p) = E_p \int_0^\infty e^{-\beta s} R(p(s))ds. \tag{28}$$

It is well-known that the functional (28) satisfies the ordinary differential equation

$$((\mathcal{A} - \beta)F)(p) = -R(p),$$

where

$$\mathcal{A} = \frac{1}{2}\sigma^2(p)\frac{d^2}{dp^2} + \mu(p)\frac{d}{dp}$$

is the differential operator representing the infinitesimal generator of p. Now consider the firm in the presence of asymmetric taxation subject to a depreciation allowance $D(p)$ instead. The value of the firm under harmonized taxation now reads

$$\hat{F}(p) = E_p \int_0^\infty e^{-(1-\epsilon)\beta s}[R(p(s)) - \epsilon(R^+(p(s)) - D(p(s)))]ds. \tag{29}$$

The effect of taxation is both to reduce the expected post-tax earnings of the firm and shift part of the market risk onto the tax authorities, reducing the post-tax variance rate faced by the firm. The market value of an existing firm may therefore rise or decline depending upon the fiscal depreciation. We want to find out whether the well-known Johansson-Samuelson condition survives and if it does under what conditions. Furthermore, we want to find out what difference taxation makes to the option value of potential projects under the Johansson-Samuelson condition. Value (29) satisfies the ordinary differential equation

$$((\mathcal{A}-(1-\epsilon)\beta)\hat{F})(p) = -(1-\epsilon)R(p)+\epsilon[R^-(p)-D(p)].$$

The value of an existing firm is thus invariant to taxation only if the depreciation allowance $D(p)$ satisfies

$$D(p) = R^-(p) - (\mathcal{A}\hat{F})(p).$$

In that case, (29) satisfies the ordinary differential equation

$$((\mathcal{A}-\beta)\hat{F})(p) = -R(p),$$

that is, the valuation is invariant to taxation. It is of interest to notice that since

$$\mathcal{A}\hat{F}(p) = \lim_{t\downarrow 0} \frac{E_p[\hat{F}(p(t))]-\hat{F}(p)}{t}$$

corresponds to the expected rate of increase in $\hat{F}(p)$, the tax neutral depreciation allowance of the firm facing asymmetric taxation is greater than under symmetricity. That is, under asymmetric taxation the tax authorities should allow for accelerated depreciation. To relate the results above to those obtained in Samuelson (1964), notice that according to Itô's theorem, if $\hat{F} \in C^2(\mathbf{R}_+)$, then

$$\frac{1}{dt}E[d\hat{F}(p(t))] = \mathcal{A}\hat{F}(p(t)).$$

We thus have an answer to the valuation problem discussed by Samuelson (1964) and an extension of his results to the case with (i) asymmetric taxation and (ii) a stochastic tax base, as follows. The losses $R^-(p)$ have to be tax-deductible. In other words, there must be complete loss offset. The uncertainty arising from the stochasticity of the tax base has to be fully shared with the tax authorities, i.e., the volatility of the tax base has to be taken into account. Under these conditions the above results show that there is a tax system which is neutral with respect to the post-entry value of the firm. Note that it was essential to assume a harmonized tax system in the proof. However, this finding leads to the question of whether this same tax can be neutral with respect to the option value of entry. The answer must be negative. The reason for such a conclusion is that the option will typically be exercised at some later date subject to discounting at a tax-dependent opportunity cost. Even more important, writing the option value as

$$V(p) = \begin{cases} \hat{F}(p) - c, & p \geq \tilde{p} \\ (\hat{F}(\tilde{p}) - c)\frac{\psi(p,\epsilon)}{\psi(\tilde{p},\epsilon)}, & p < \tilde{p} \end{cases}$$

one can see an important conclusion. While valuation of existing projects in the economy is tax invariant under the Johansson-Samuelson tax, it actually raises the option values of all the potential projects which have so far not been undertaken. The intuition behind such a conclusion is that the Johansson-Samuelson tax involves "levelling the playing field" to the extent that the values and attractiveness of alternative assets are reduced. This finding suggests that the Johansson-Samuelson tax interferes with the entry threshold, too. In technical terms, solving for the optimal entry threshold from above, we obtain the necessary condition

$$(\hat{F}(\tilde{p}) - c)\frac{\partial\psi(\tilde{p},\epsilon)}{\partial p} = \hat{F}'(\tilde{p})\psi(\tilde{p},\epsilon).$$

Carrying out the comparative static analysis yields that the sign of the relationship between the entry threshold $\tilde{p}$ and the tax rate ϵ is determined by the sign of

$$X(\tilde{p},\epsilon) := \frac{\partial^2\psi(\tilde{p},\epsilon)}{\partial p\partial\epsilon} - \frac{\partial\psi(\tilde{p},\epsilon)}{\partial p}\frac{\partial\psi(\tilde{p},\epsilon)}{\partial\epsilon}\frac{1}{\psi(\tilde{p},\epsilon)}.$$

Because generally $X(\tilde{p},\epsilon) \neq 0$, we know that the entry threshold will not be tax-invariant. It is not possible to establish the sign of $X(\tilde{p},\epsilon)$ in the general case. However, our calculations show that when the price follows geometric Brownian motion, the Johansson-Samuelson tax unambiguously raises the entry threshold discouraging new entry.

Theorem 3. *The Johansson-Samuelson tax which makes the value of an existing firm invariant to taxation raises the option value of entry and makes the optimal entry threshold tax-dependent.*

This result stands in sharp contrast to the view associated with the Johansson-Samuelson theorem which abstracts from the possibility that the firm may face sunk costs in the future. This theorem is informative of the tax effects on an existing firm. The problem faced by a potential new firm is a different one. It has to value all future options using an opportunity cost which is not invariant to taxation.

As an illustration of the results above, consider a firm facing a linear net cash flow $(p(t)-\nu)$, where ν is a constant and $p(t)$ evolves according to the mean reverting process (*Ornstein - Uhlenbeck process*)

$$dp(t) = \mu(\hat{p} - p(t))dt + \sigma dW(t),$$

where μ is now a positive constant measuring the rate of adjustment and $\hat{p}$ denotes the expected long run equilibrium. By straightforward integration, we obtain

$$F(p) = \frac{(\hat{p}-\nu)}{\beta} + \frac{(p-\hat{p})}{\beta+\mu}.$$

On the other hand, under taxation the value of the firm is given by

$$\hat{F}(p) = \frac{(\hat{p}-\nu)}{(1-\epsilon)\beta} + \frac{(p-\hat{p})}{(1-\epsilon)\beta+\mu} - \epsilon E_p \int_0^\infty e^{-(1-\epsilon)\beta s}((p(s)-\nu)^+ - D(p(s)))ds.$$

By choosing the depreciation allowance satisfying the required neutrality condition we see that

$$D(p) = (p-\nu)^- + \frac{\mu(p-\hat{p})}{\mu+\beta}$$

represents the depreciation allowance under the tax-invariance requirement of the Johansson-Samuelson proposal.

6 Conclusions

The interaction between tax policy and firm behavior has been subject to an extensive research effort. In guiding tax reforms towards improved efficiency, one of the lessons on which much of intellectual thinking and policy suggestions have been built has been the Johansson-Samuelson theorem. Unfortunately, this has directed the focus towards tax treatment of existing mature firms with positive cash flows. The prospects of potential firms facing substantial sunk costs and uncertainty are rather different. The risk of not being able to write off the full sunk cost and the property right of the government on the firm's cash flow as an option tend to interact with the option value of potential new entry. This paper has derived results on tax invariance from such a perspective, departing from the premises of the Johansson-Samuelson model. These results have been derived without being restricted to uniform tax rates on capital income, though the case studied by Samuelson (1964) with harmonized tax rates is also reconsidered. Generally, the results justify the accelerated depreciation approach, and differentiated tax treatment of established mature firms and potential firms facing irreversible sunk costs with substantial uncertainty.

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