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THE OPTIMAL TAXATION OF COUPLES

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Abstract

The existing literature on optimal income taxation is limited in its ability to cast light on issues of family taxation, because the model of the household on which it is based fails to capture two important characteristics of real households: that they typically contain two adults and that domestic production is an important alternative use of time to market labour supply. This paper formulates a simple and tractable model of such a household and then uses it to evaluate alternative systems for taxing couples, in terms primarily of their equity properties.

Keywords: taxation, couples, equity

JEL Code: H21, H31, J22

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1. Introduction[1].

The point of departure of the theory of optimal income taxation, one of the cornerstones of modern public finance theory, is the proposition that ideally a tax should be levied on an individual's innate productivity endowment, but, since this is unobservable, it must instead be levied on money income. The underlying model of behaviour, whether in the general theory of non-linear taxation first developed by Mirrlees (1971), or in the theory of optimal linear taxation formulated by Sheshinski (1972), is that of a utility maximising individual who divides his time optimally between market labour supply and leisure, given an exogenously determined net wage. The gross wage measures his productivity. There is a given distribution of wage rates over the population, and the problem is to maximise some social welfare function defined on individual utilities subject, in the linear case, to the government's budget constraint, and, in the non-linear case, also to self-selection constraints. The theory permits an analysis of how concerns with the equity and efficiency effects of a tax system interact to determine the parameters of that system, and in particular its marginal tax rate and degree of progressivity.

When we come to consider debates on actual tax policy, it becomes clear that a central issue in this debate, that of how *couples* should be taxed, cannot be addressed by this literature. The debate is usually framed in terms of the choice of one of the following three systems:

1. *Joint taxation*, in which money incomes of household members are aggregated and a tax rate is applied to this aggregate;

2. *Anonymous individual taxation*, in which the same tax schedule is applied to the separate money incomes of the household members;

3. *Selective individual taxation*, in which different tax schedules are applied separately to the money incomes of male and female household members.

The standard models discussed above cannot be used to inform this debate, not only for the obvious reason that they are based on the labour supply decisions of a single individual, but also because they incorporate the simple dichotomisation of time into market labour supply and leisure. In actual fact individuals divide their time between market work and household production as well as "pure leisure" (the direct consumption of own time). This

[1]We are grateful to Bernd Huber, Pierre Pestieau, Robert Pollak and Agnar Sandmo for helpful comments on this paper. The usual disclaimer applies.

is a crucial aspect of the discussion of the taxation of couples because the empirical evidence shows that households with the same wage rates, demographic characteristics and nonwage incomes make widely different choices in their allocation of time between market and domestic work, especially in respect of the labour supply of the female partner. Given the assumption of identical preferences normally made in optimal tax analysis, this has to be modelled as resulting from varying productivities in household production. The purpose of this paper is to show that this productivity variation has important implications for the horizontal (in)equity of a tax system with money incomes as the tax base, which in turn influence the assessment of the relative merits of the three types of tax system just described.

The importance of household production for optimal tax analysis is also emphasised by Sandmo (1990), who derives formulae for optimal indirect and direct taxation in a model of single person households which produce a good that is a perfect substitute for a market good. In particular he shows how the nature of the correlation between productivities in domestic and market work (this latter measured by the gross market wage rate) will affect the marginal tax rate. This paper adopts a somewhat different model, in which domestic goods are traded only within the household, but the main departure is to have the household consist of two individuals, so that the issues of family taxation can be directly discussed. We extend Sandmo's results to deal with the issue of horizontal inequity inherent in tax systems based on observed money income in the presence of significant variations in market labour supply of at least one of the household members.

In Boskin and Sheshinski (1983), progress was made in the direction of generalising the optimal tax literature to take account of the two-person nature of households, by specifying a household utility function defined on a single consumption good and two types of leisure. This allows the following intuition to be explored. In an economy consisting of two-person households, it is possible to distinguish costlessly between the primary earner, usually male, and the secondary earner, usually female, so that in principle any of the three types of tax system defined above could be applied. Take also the stylised fact that female labour supply elasticities are significantly greater than those of males[2]. Then on standard Ramsey principles we would expect that optimally, at any given income, women should be subject to lower tax rates

[2] Although it should be noted that Heckman (1993) questions whether this stylised fact really is supported by a careful analysis and interpretation of the evidence.

than men. Subject to the usual qualification concerning cross-elasticities, this intuition is confirmed by the optimal linear tax analysis carried out by Boskin and Sheshinski. This argues therefore for selective individual taxation.

It is remarkable that neither of the characteristics of real households that are important - their two-person nature and the existence of household production with its concomitant possibilities of varying degrees of specialisation in household work - is taken into account in Boskin and Sheshinski's model. As a result, the distributional effects of taxation, *within* as well as *between* households, are inadequately identified and discussed. In the earlier public finance literature these issues were recognised as being of central importance, see for example Pechman (1971) and Munnell (1980). The purpose of this paper therefore is to set up models which allow us to explore more fully the implications of domestic production in two person households for the analysis of the equity issues arising from optimal linear income taxation. We shall focus primarily on the issue of the welfare distribution *between* as opposed to *within* households.

2. The model of the household.

Households are in fact microcosms of the whole economy. Individuals within them specialise, produce and exchange, and moreover this production and exchange activity is untaxed. By means of outside markets individuals in different households also exchange labour for goods, and these transactions external to the household are taxed. In evaluating the effects of this taxation on the welfare of *individuals*, we have to show how these are mediated through the household decision process.

To solve the household's decision problem we need some principle of rationality, and the most obvious one to take is that of Pareto efficiency: in choosing their resource allocation the household members exhaust all possibilities of costless welfare improvement. This of course leaves open the question of income distribution within the household. In the most general approach we assume that each household member possesses a reservation utility which must be (weakly) exceeded if he or she is to remain in the household. For simplicity we assume the achieved utilities strictly exceed these reservation levels. The household equilibrium is then found by maximising one member's utility subject to a given level of utility for the other, where this constraint level expresses the distributional outcome in the household and is in general a function of net wage rates, individual non-wage incomes, and other exogenous variables left unspecified. More precision can of course be obtained if we assume the household distributes its income to maximise some specific "household welfare function" to generate a specific sharing rule, but we shall first see how far we can go with the more general approach. As is usual in optimal tax models, we assume all individuals have identical preferences, so that they differ only in their productivities.

In a household, individual $i = f, m$ divides total time available, T, between market labour supply l_i and time t_i spent in producing a domestic good[3] y_i. The production functions for this are simply

$$y_i = k_i t_i, \ k_i > 0, \quad i = f, m \tag{1}$$

where k_i is an exogenous productivity parameter[4]. The utility functions, assumed strictly concave and increasing, are

$$u_i = u(x_i, y_{if}, y_{im}), \ i = f, m$$

where y_{ij} is i's consumption of the domestic good produced by j and x_i is i's consumption of the bought in market good. The household's budget constraint is

$$\sum_i x_i \leq \alpha + \beta \sum_i w_i l_i \tag{2}$$

where α is the lump sum payment under the linear tax system and $(1 - \beta)$ is the marginal tax rate. For simplicity we assume there is no non-wage income. Note that the lump sum is assumed to be paid to a household rather than to an individual. The wage rate w_i is, as usual in optimal tax models, taken to be the measure of the individual's innate productivity in market work and is exogenously given and unobservable. Note finally that the consumption good is the numeraire.

Using the time constraint to eliminate t_i, we can write the household's problem as

[3] Note that we exclude household public goods, the presence of children in the household, and the possibility of pure leisure- direct consumption of own time as a good in the utility function. For a model which includes all of these see Apps and Rees (1995). We exclude them here not because they are unimportant but because they would complicate unduly the analysis of optimal taxation.

[4] This productivity parameter will be determined by the human capital of the individual. For a study which develops this approach empirically, se Apps, Killingsworth and Rees (1996).

AP (allocation problem):

$$\max\ u(x_f, y_{ff}, y_{fm}) \tag{3}$$

$$s.t.\ u_m \leq u(x_m, y_{mf}, y_{mm}) \tag{4}$$

$$\sum_i x_i \leq \alpha + \beta \sum_i w_i l_i \tag{5}$$

$$\sum_i y_{ij} = k_j(T - l_j), \quad j = f, m \tag{6}$$

$$T \geq l_f, l_m \geq 0 \tag{7}$$

We assume that, for every household, at the optimal solution to this problem $T > l_i \geq 0$ and $l_m > 0$, but that for some households it may be the case that $l_f = 0$. The constrained level of m's utility u_m is assumed to be a differentiable function of α and the net of tax wage rates and, as already suggested, is such as to yield both individuals more than the minimum utilities they need to stay in the household. The first order conditions for this problem can be written:

$$\frac{u_{y_{ff}}}{u_{x_f}} = \frac{u_{y_{mf}}}{u_{x_m}} = \frac{\mu_f}{\lambda} \equiv p_f \tag{8}$$

$$\frac{u_{y_{fm}}}{u_{x_f}} = \frac{u_{y_{mm}}}{u_{x_m}} = \frac{\mu_m}{\lambda} \equiv p_m \tag{9}$$

$$c_f \equiv \frac{\beta w_f}{k_f} \leq \frac{\mu_f}{\lambda} \equiv p_f \tag{10}$$

$$0 = (p_f - c_f) l_f \tag{11}$$

$$c_m \equiv \frac{\beta w_m}{k_m} = \frac{\mu_m}{\lambda} \equiv p_m \tag{12}$$

In these conditions the μ_i are the Lagrange multipliers associated with the production constraints (7), and λ that attached to the budget constraint (6). We can think of λ as the marginal utility of household income (in f-utils) and the first order conditions imply

$$u_{x_f} = \lambda = \rho u_{x_m} \tag{13}$$

where ρ is the Lagrange multiplier attached to the constraint on m's utility. Then the ratios μ_i/λ give the implicit prices of the domestically produced goods p_i, $i = f, m$. c_i denotes the marginal opportunity cost of y_i. and where i supplies labour to the market, price and marginal cost of the domestic good he or she produces must be equal. However if in a household f specialises in domestic production and supplies no time to the market, then the price of the good she produces may be higher than its marginal cost.

Alternatively, conditions (10) to (12) can be interpreted as saying that at an interior solution the optimal allocation of time between household and market equalises the net wage with the marginal value product of the domestically produced good, valued at the imputed domestic price, but at a corner solution for f, the marginal value product may exceed her net wage or outside opportunity cost. Note that unlike the standard model, preferences do not enter directly into the determination of the time allocation, but only indirectly through the implicit prices of the domestic goods. There is no pure leisure in this model, so time not spent on market work does not yield utility directly, but only indirectly through the goods it is used to produce.

For purposes of the analysis of optimal income taxation in the next section it is useful to analyse the comparative statics of the household equilibrium. To simplify this we can exploit some properties of the model. First we note that the household has the *imputed income*, equal to its total expenditure on both bought in and domestically produced goods, of

$$Y = x + \sum p_i y_i = \alpha + \sum \beta w_i l_i + \sum p_i y_i \tag{14}$$

Adding and subtracting $\sum c_i y_i$ on the right hand side and recalling the definition of c_i gives

$$Y = \alpha + \sum \beta w_i T + \sum (p_i - c_i) y_i \tag{15}$$

The second term on the right hand side is *net household full income*, while the third is the sum of imputed profits from domestic production. If both f and m supply labour to the market then this third term is zero, while if f specialises entirely in domestic production then $p_f - c_f > 0$. In any case we see that net money income $\alpha + \sum \beta w_i l_i$ understates the value of consumption a household can enjoy. It is as if the national income of a country that produces significant amounts of non-traded goods were set equal to the value of its exports plus transfers from abroad, or equivalently its imports.

We can then apply the Second Theorem of Welfare Economics to argue that the household equilibrium determined by conditions (9)-(13) can be generated as follows. The household first shares its income Y between the two members by giving them lump sums s_i such that $\sum s_i = Y$ identically. Given the equilibrium prices, each $i = f, m$ then solves the problem:

CA (consumption allocation):

$$\max\ u(x_i, y_{if}, y_{im}) \quad s.t.\ s_i = x_i + p_f y_{if} + p_m y_{im} \tag{16}$$

In problem **CA** each household member chooses his or her optimal consumption given the budget constraint defined by the income share and the equilibrium prices. This is just a standard consumer problem, and so will yield the usual demand functions $x_i(\ p_f, p_m, s_i)$, $y_{ij}(\ p_f, p_m, s_i)$ and indirect utility functions $v_i(p_f, p_m, s_i)$ with marginal utilities of income $\lambda_i(\ p_f, p_m, s_i)$. The derivatives of these functions will be important in the optimal tax analysis, and in taking these it must be remembered that the prices are not truly exogenous, but in fact we have the following. There is a set N of individuals for which

$$l_i \geq 0\ and\ p_i = \frac{\beta w_i}{k_i} \tag{17}$$

while for $f \notin N$

$$l_f = 0\ and\ p_f > \frac{\beta w_f}{k_f} \tag{18}$$

For $f \notin N$ it follows that there are no direct effects on labour supply of varying the marginal tax rate. In the optimal tax analysis we will need expressions for the derivatives of labour supplies with respect to the tax parameters. As opposed to the standard model, where l_i appears directly in the utility function, here it is derived from the demand for the domestic outputs. Thus we can write the labour supply functions (using (7)) as

$$l_j = T - \kappa_j \sum_i y_{ij}(p_f, p_m, s_i) \quad for\ j \in N \tag{19}$$

$$= 0 \quad for\ j \notin N \tag{20}$$

where $\kappa_j = 1/k_j$. For individuals not in N (females in some households), the partials of l_i are always zero. For $i \in N$ we *assume* (as is usual in optimal tax analysis)[5]

[5]For f such that $l_f = 0$ and $p_f = \beta w_f / k_f$, that is a household in which f is just on

$$\frac{\partial l_i}{\partial \alpha} \leq 0 \tag{21}$$

$$w_i \frac{\partial l_i}{\partial \beta w_i} = \frac{\partial l_i}{\partial \beta} > 0 \tag{22}$$

We can use (20) to evaluate these derivatives:

$$\frac{\partial l_j}{\partial \alpha} = -\kappa_j \sum_i \frac{\partial y_{ij}}{\partial s_i} \frac{\partial s_i}{\partial \alpha} \quad j = f, m. \tag{23}$$

$$\frac{\partial l_j}{\partial \beta} = -\kappa_j \sum_i [\sum_{n=f,m} \frac{\partial y_{ij}}{\partial p_n} \frac{\partial p_n}{\partial \beta} + \frac{\partial y_{ij}}{\partial s_i} \frac{\partial s_i}{\partial \beta}] \quad j = f, m \tag{24}$$

We see from (24) that it is sufficient for (22) that the domestic goods are normal goods and that both household members receive a share of an increase in lump sum payment α. It seems reasonable to assume that both these conditions hold. Recalling (15) we have in fact

$$\sum_i \frac{\partial s_i}{\partial \alpha} = \frac{\partial Y}{\partial \alpha} = 1 \tag{25}$$

From (25) we see that (23) will hold if demands for the domestic goods vary negatively with their prices (since $\partial p_i / \partial \beta = w_i \kappa_i > 0$) *and* these price effects outweigh the income effects resulting from the fact that a reduction in the marginal tax rate increases net household income and therefore, given that they are normal, the demands for the domestic goods.This assumption is of course restrictive and rules out backward-bending labour supply curves. Note that we have

$$\sum \frac{\partial s_i}{\partial \beta} = \frac{\partial Y}{\partial \beta} = T \sum w_i \tag{26}$$

for $i \in N$.

Returning now to **CA**, we see that this is a standard consumer problem so that all the usual duality results can be applied. Denote i's marginal utility of income by λ_i. Then we have

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the margin of participation. the derivative in (22) is the left hand derivative (the right hand derivative is always zero) and that in (23) is the right hand derivative (the left hand derivative is always zero).

$$\frac{\partial v_i}{\partial \alpha} = \lambda_i \frac{\partial s_i}{\partial \alpha} \tag{27}$$

$$\frac{\partial v_i}{\partial \beta} = -\lambda_i y_{if} \frac{\partial p_f}{\partial \beta} - \lambda_i y_{im} \frac{\partial p_m}{\partial \beta} + \lambda_i \frac{\partial s_i}{\partial \beta} \tag{28}$$

$$= -\lambda_i (w_f t_{if} + w_m t_{im} - \frac{\partial s_i}{\partial \beta}) \tag{29}$$

where(29) follows from Roy's identity and (30) from defining $y_{ij} = k_j t_{ij}$, that is, t_{ij} is the amount of time j spends producing the amount of the domestic good j consumed by i.

Since **CA** is a standard problem, we can in the usual way obtain Slutsky equations:

$$\frac{\partial y_{ij}}{\partial p_n} = \sigma_{ijn} - y_{in} \frac{\partial y_{ij}}{\partial s_i} \qquad i, j, n = f, m. \tag{30}$$

where the σ_{ijn} are compensated substitution terms, strictly negative for $n = j$. Inserting this into (25) and rearranging gives a Slutsky equation for labour supplies:

$$\frac{\partial l_j}{\partial \beta} = -\kappa_j \sum_i [\sum_{n=f,m} \sigma_{ijn} \frac{\partial p_n}{\partial \beta} + (\frac{\partial s_i}{\partial \beta} - \sum_{n=f,m} y_{in} \frac{\partial p_n}{\partial \beta}) \frac{\partial y_{ij}}{\partial s_i}] \qquad j = f, m \tag{31}$$

The first term in the square bracket is a substitution effect: it shows the change in labour supply resulting from *compensated* changes in demands for domestic goods. The second term is an income effect and has an interesting interpretation: the term in the round bracket is the difference between the change in i's income share and the compensation required to buy the initial quantities. If this is positive the individual is better off and this income effect has the usual sign. If negative, then even though i's demand for the domestic good produced by j is normal, this income effect will tend to offset the substitution effect.. However if, as we would expect, the household's distributional preferences are non-decreasing in the utilities of both members, that is, the Pareto property holds, then this latter case could not arise.

The next step in the analysis is to determine how labour supplies vary with the productivity parameters for *given* market wage rates. Equation (20) can be used to derive the very simple relationship

$$e_{k_i} = \frac{t_i}{l_i}(1 - e_i) \tag{32}$$

where e_{k_i} is the elasticity of i's market labour supply with respect to productivity k_i and e_i is the price elasticity of total household demand for i's domestic output. Thus if this demand is relatively elastic i's market labour supply will fall as k_i increases and conversely if demand is relatively inelastic. Note that this elasticity also depends on how significant the supply of time to domestic production is relative to market labour supply. The intuition underlying this elasticity is quite simple. A productivity increase on the one hand reduces the implicit price of the domestic good and therefore increases its demand and so would tend to reduce i's market labour supply, but on the other hand i can now produce a given output with a smaller input of domestic time t_i and so this would tend to increase market labour supply. Only if the domestic demand for i's output is sufficiently price elastic will the net result then be a reduction in market labour supply.

Since domestic productivities, by the assortative mating assumption, vary positively with each other, we also have to consider the cross effects. Again using (20) we have quite simply

$$e^i_{k_j} = \frac{t_i}{l_i} e_{ij} \; i, j = f, m, i \neq j \tag{33}$$

where $e^i_{k_j}$ is the elasticity of i's market labour supply with respect to k_j and e_{ij} is the elasticity of demand for i's output with respect to p_j. Thus if the domestic goods are complements this cross-effect of increasing household productivity is to reduce i's market labour supply and conversely if they are substitutes. The intuition for this is that an increase in j's productivity reduces the implicit price of the corresponding domestic good, this increases (decreases) the demand for i's domestic good if they are complements (substitutes), and so increases (decreases) the amount of time i must spend in household production.

We can now draw some conclusions on how market labour supplies of household members and therefore household wage income will vary with domestic productivity, at *given* wage rates. If demands for both goods are relatively elastic and they are complements then market labour supplies and household wage income will certainly fall as productivity increases. On the other hand if demands for the domestic goods are relatively price inelastic and the outputs are substitutes, then market labour supplies and wage income will rise with domestic productivities.

The empirically most relevant case appears to be that in which the main variation occurs in the market labour supply of the female partner, male hours worked remaining fairly constant as we move through the distribution of households. In everything that follows we shall focus on this case. We consider here two main cases in relation to female labour supply.

Case I: f's labour supply elasticity with respect to the productivity parameter is strongly negative, primarily because the price elasticity of demand for her output is high. In this case, given a subset of households with the same wage rates, those households in which women tend to specialise in domestic production - which we call here *traditional households* - are also those with the higher domestic productivities. Those women who specialise in household production do so because they are relatively more productive at it.

Case II: f's labour supply elasticity with respect to the productivity parameter is strongly positive, because demand for her output is relatively price-inelastic. The effect of increasing productivity in reducing the amount of time a woman needs to spend in household production dominates. In that case traditional households are at the lower end of the productivity spectrum.

In interpreting the results of the optimal tax analysis in the following sections we shall concentrate on these focal cases.

Finally since the solutions to **AP** and **CA** are identical we have, using (14):

$$u_{x_f} = \lambda_f = \lambda = \rho u_{x_m} = \rho \lambda_m \tag{34}$$

These relationships between the marginal utilities of income of the individuals in the household will be useful in what follows.

3. Optimal linear taxation.

We assume a continuum of households, which could in principle be distributed along five dimensions: the individual wage rates w_i, the productivities in domestic production k_i, and their distributional preferences. To keep things tractable we first make the assumption:

Assortative Mating: individual wage rates vary non-negatively and monotonically with each other across households, as do productivities in household production. Thus we write:

$$\begin{aligned} w_m &= \phi(w_f),\ \phi' \geq 0 && (35) \\ k_m &= \psi(k_f),\ \psi' \geq 0 && (36) \end{aligned}$$

This implies that each household can be characterised by a pair of values (w_f, k_f). Let female wage rates be defined on the compact interval W and productivities on the compact interval K. Then $g(w_f, k_f)$ is the joint density function of wages and productivities across households, normalised so that

$$\int_W \int_K g(w_f, k_f) dw_f dk_f = 1 \qquad (37)$$

We assume that the "social planner" is utilitarian and so wants to maximise the social welfare function

$$S = \int_W \int_K [v_f + v_m] g(w_f, k_f) dw_f dk_f \qquad (38)$$

subject to the revenue constraint

$$(1-\beta) \int_W \int_K [w_f l_f + w_m l_m] g(w_f, k_f) dw_f dk_f - \alpha \geq R \qquad (39)$$

where $R \geq 0$ is exogenous net revenue. With γ the Lagrange multiplier attached to (38) the first order conditions are

$$\begin{aligned} \int_W \int_K [\sum_{i=f,m} (\lambda_i \frac{\partial s_i}{\partial \alpha} + \gamma(1-\beta) w_i \frac{\partial l_i}{\partial \alpha}] g(w_f, k_f) dw_f dk_f - \gamma &= 0 && (40) \\ \int_W \int_K [\sum_{i=f,m} (\frac{\partial v_i}{\partial \beta} + \gamma(1-\beta) w_i \frac{\partial l_i}{\partial \beta} - \gamma w_i l_i)] g(w_f, k_f) dw_f dk_f &= 0 && (41) \end{aligned}$$

These conditions show clearly that the optimal tax parameters will depend on the precise nature of the household sharing rules, since these determine the way in which *individual utilities*, which are the arguments of the social welfare function, are affected by variations in the tax parameters. The key question is whether there is *dissonance* between the distributional preferences of the household and of the social planner, in the sense that the implicit weights applied to the individual utilities by the household differ from those of the planner, here unity. Since we have discussed this issue at some length in earlier papers[6] we eliminate it by making the assumption

Non-dissonance: the implicit weights attached by the household to individual utilities at the household equilibrium are identical to those of the planner, in this case unity.

Thus the household is also utilitarian. In terms of the results in the previous section this implies that $\rho = 1$ and also that each member's marginal utility of income is equal to that of the household, λ. It is then straightforward to show that the first order conditions become

$$\int_W \int_K [(\lambda + \gamma(1-\beta) \sum_{i=f,m} w_i \frac{\partial l_i}{\partial \alpha}] g(w_f, k_f) dw_f dk_f - \gamma = 0 \quad (42)$$

$$\int_W \int_K [\sum_{i=f,m} (\lambda w_i l_i + \gamma(1-\beta) w_i \frac{\partial l_i}{\partial \beta} - \gamma w_i l_i)] g(w_f, k_f) dw_f dk_f = 0 \quad (43)$$

4. Alternative tax systems and the welfare distribution.

We can now compare the results with those of the standard analysis[7]. The issue of income distribution within the household has been finessed by the non-dissonance assumption. In a sense, the planner can delegate to households the problem of achieving the appropriate welfare distribution between *individuals within households,* since they have the same distributional preferences as he has. He has to choose the tax parameters in the light of the welfare distribution across individuals in different households, as indicated by the household marginal utility of (*full*) income λ.

In what follows it will be convenient to use the expectations operator $E[.]$ in place of the integral notation $\int_W \int_K (.) g(w_f, k_f) dw_f dk_f$. Now multiply through condition (41) by $E[\sum w_i l_i]$ and subtract the result from (42) to obtain

$$Cov(\lambda, \sum w_i l_i) + \gamma(1-\beta)\Delta = 0 \quad (44)$$

where

[6] See Apps and Rees (1988), (1997).

[7] For a good, concise account of this see Dixit and Sandmo (1977).

$$Cov(\lambda, \sum w_i l_i) = E[\lambda(\sum w_i l_i - E[\sum w_i l_i])] \tag{45}$$

and

$$\Delta \equiv E[\sum w_i \frac{\partial l_i}{\partial \beta}] - E[\sum w_i \frac{\partial l_i}{\partial \alpha}] E[\sum w_i l_i] \tag{46}$$

The covariance in (43) is the distributional term in the optimal tax expression and Δ represents the efficiency term. Given the assumptions in (22) and (23), we have $\Delta > 0$. Solving for the optimal marginal tax rate in (43) gives

$$(1-\beta) = -\frac{1}{\gamma}\frac{Cov(\lambda, \sum w_i l_i)}{\Delta} \tag{47}$$

We have $\gamma > 0$ and so the sign of the covariance in the numerator will determine the sign of the marginal tax rate. In what follows we take Δ as given and focus on the covariance term.

Consider the entire population of households, across which wage rates as well as domestic productivities vary[8]. It seems most plausible to assume that there would be a positive correlation between wage rates and domestic productivities across the population of households, since increasing human capital should increase productivity in all spheres of activity. Clearly the higher the wage rates and domestic productivities the greater the utility levels enjoyed by the individuals in the household. It is also then reasonable to assume that the marginal utility of household income, λ, falls as we move through the distribution. A possible consequence of this for the marginal tax rate has been well described by Sandmo[9]:

> Assume [.....] that individuals who are highly productive in the market tend also to have high productivity in domestic work. If $\text{Cov}(\lambda, \sum w_i l_i)$ is negative initially [*ie without household production*] a positive correlation between w_f and k_f should make it

[8]We can regard the kind of model used by Boskin and Sheshinski, in which domestic production is ignored, as implicitly assuming that the productivity parameters are identical across households. In that case units can be chosen so that these parameters are unity, and the respective wage rates become the prices of the domestic goods, which might as well be called "leisure". Thus our general model can be specialised to that of Boskin and Sheshinski by making the non-dissonance and identical-productivity assumptions.

[9]Sandmo (1990) p 89.

more negative, thus increasing the marginal tax rate. The economic intuition clearly is that the tax on labour income indirectly also manages to capture the "untaxable" benefits from household production, which strengthens its effectiveness as an instrument of redistribution.[*our notation*]

However, this is unambiguously the case only if something like Case II defined in the previous section is assumed. It requires that increasing wages and domestic productivity also increases total household money income. Suppose instead that Case I holds, and the effect of increasing domestic productivity in reducing female market labour supply is strong enough to more than offset the effect of the increasing wage rate and indeed so strong that it leads to a falling household money income overall. Then this covariance term will be positive, implying that the marginal tax rate will optimally be negative. The intuition for this apparently strange result is clear: since the utilities of household members are higher the lower is their combined market income, the tax system should redistribute incomes to those with higher money incomes, which are actually a negative indicator of utilities in this case.

We may regard as more plausible the case in which household money income increases with wage rates and domestic productivities even if there is some reduction in female labour supply, so that the optimal tax rate will be positive.The distinction between cases I and II is however still useful in alerting us to the possibility of the significant degree of horizontal inequity that may arise under a linear tax system with household money income as the base. Take any subset of households with the same market wage rates w_f^0, w_m^0, and consider what happens as the productivity parameters vary. The utilities of household members certainly increase with domestic productivity since the implicit prices of the household goods fall (household full income remains unchanged). In the case in which female labour supply *falls* with increasing productivity, while male labour supply stays roughly constant, household money income also *falls*. Thus there is an inverse relationship between household money income and the productivity parameter. This is illustrated in Figure 1. $K = [k_f^0, k_f^1]$ is the interval of possible productivity values. Along DB we have a subset of households with the same wage rates w_f^1, w_m^1. Household income $\sum w_i^1 l_i$ falls with k_f, while household utility increases with k_f. Since in the (optimal) linear tax system tax payments increase with money income, there is an inverse relationship between taxes and utilities. Let AC show the relationship between household income and domestic productivity for a subset of households with the wage rates $w_i^0 < w_i^1, i = f, m$. Then a household at A pays the same tax as a household at B even though B enjoys both higher wage rates and higher domestic productivities and so higher utilities.

Fig 1 about here

Where female market labour supply increases with domestic productivity there may still be horizontal inequities since households whose members enjoy the same utility levels may pay different amounts of tax, but these will be less marked than in the case shown in Figure 1 because at least market wage income is increasing with utility in this case.

Clearly the essential problem in the case where horizontal inequity is large lies in the fact that there is no tax instrument in this simple linear tax system that can capture the covariance, conditioned on given wage rates, between the utilities of household members and household productivities. The non-observability of domestic production rules this out. This means that both of the first two tax systems listed in the introduction - joint taxation and anonymous individual taxation - when they are based on a constant marginal tax rate. could be associated with a high degree of horizontal inequity, the former because it is precisely the type of tax system so far considered, the latter because, in a linear tax system, it is fully equivalent to joint taxation. Therefore we now turn to an analysis of selective individual taxation to examine whether this can offer an improvement in this regard

5. Selective individual taxation.

We remain within the confines of a linear tax system, but now consider the possibility of differing marginal tax rates $(1 - \beta_i)$. $i = f, m$. Because of the non-dissonance assumption we can without loss of generality continue to regard the lump sum α as being paid to the household and not to an individual[10]. Thus the planner's problem is to choose α, β_f, β_m so as to maximise

$$S = E[v_f + v_m] \tag{48}$$

[10]This would certainly not be true in the absence of this assumption since we would expect separate lump sums to affect the household distribution of income in different ways in general. This kind of perception underlay for example the change from child tax allowances to child benefit in the UK.

subject to

$$E[\sum_{i=f,m}(1-\beta_i)w_i l_i]-\alpha \geq R \tag{49}$$

The first order conditions are

$$E[\lambda+\gamma\sum_{i=f,m}(1-\beta_i)w_i\frac{\partial l_i}{\partial\alpha}]-\gamma=0 \tag{50}$$

$$E[\lambda w_i l_i+\gamma\sum_{j=f,m}(1-\beta_j)w_j\frac{\partial l_j}{\partial\beta_i}-\gamma w_i l_i]=0 \; i=f,m \tag{51}$$

Multiplying (51) through by $E[w_f l_f]$ and subtracting it from (52) with $i=f$, and then by $E[w_m l_m]$ and subtracting it from (52) with $i=m$, allows the conditions to be expressed as

$$(1-\beta_f)\delta_{ff}+(1-\beta_m)\delta_{fm} = -\frac{1}{\gamma}Cov(\lambda, w_f l_f) \tag{52}$$

$$(1-\beta_f)\delta_{mf}+(1-\beta_m)\delta_{mm} = -\frac{1}{\gamma}Cov(\lambda, w_m l_m) \tag{53}$$

where

$$\delta_{ii} \equiv E[w_i\frac{\partial l_i}{\partial\beta_i}-w_i\frac{\partial l_i}{\partial\alpha}E[w_i l_i]] \;\; i=f,m \tag{54}$$

$$\delta_{ij} = E[w_j\frac{\partial l_j}{\partial\beta_i}-w_j\frac{\partial l_j}{\partial\alpha}E[w_i l_i]] \;\; i,j=f,m,i\neq j \tag{55}$$

We shall not discuss the interpretation of the "efficiency terms", the δ's, since here we are primarily interested in the equity implications of these conditions. Suppose that the optimal values of all variables are inserted into the conditions, so that they are identities. Then treating them as a linear system in the marginal tax rates we can write

$$1-\beta_i=-\frac{1}{\gamma\Delta}[Cov(\lambda, w_i l_i)\delta_{jj}-Cov(\lambda, w_j l_j)\delta_{ji}] \;\; i,j=f,m,i\neq j \tag{56}$$

where Δ is now the determinant of the matrix $[\delta_{ij}]$. It is reasonable to assume this is positive. The reason both covariances influence the value of a marginal tax rate is of course that other things equal, the higher is one tax rate the lower is the other given the revenue constraint. An obvious point is that there is no reason in general to expect these tax rates to be equal, and so since the assumption is that it is costless to set two tax rates, selective individual taxation is in general optimal. Here we wish to focus on the covariance terms as determinants of the difference between the two marginal tax rates. Thus we have from (56)

$$\beta_f - \beta_m = \frac{1}{\gamma\Delta}\{Cov(\lambda, w_f l_f)[\delta_{mm} + \delta_{fm}] - Cov(\lambda, w_m l_m)[\delta_{ff} + \delta_{mf}]\} \quad (57)$$

We assume that all the efficiency terms, the δ's, are positive. Consider what happens as we move through the distribution of households. Wage rates and domestic productivities of both household members are increasing, and so their utilities must also be increasing. There are conflicting effects on the implicit prices of the domestic goods. The increasing wage rates will imply that these prices tend to increase while the increasing domestic productivities on the other hand imply that the prices of the domestic goods will tend to decrease. The net changes in the implicit prices will then affect the individual labour supplies in ways we have analysed in the previous sections. There is no reason for these effects to be identical for both household members. There will also be positive income effects on the demands for these goods.

If male labour supplies vary little as we move through the distribution, male wage income will increase and $Cov(\lambda, w_m l_m)$ will be negative. This will contribute positively to the difference in (57). Ignoring efficiency aspects (which would almost certainly reinforce the point) the male tax rate will be positive. Male wage income and utilities of the household members will be positively related and so the problem of horizontal inequity across males is unlikely to be acute, since the tax paid by the male member of the household increases with household utilities.

In Case I, $Cov(\lambda, w_f l_f)$ will be positive, since higher-utility households have lower female wage income. This then also contributes positively to the difference in (57). Moreover we see from (56) that the marginal tax rate on female income will be negative, reflecting the fact that female wage income is a negative indicator of the utility of household members. This means also that the kind of horizontal inequity illustrated in Figure 1 earlier is reduced. Across a subset of women with the same wage rate, since increasing productivity reduces wage income, two women with the same wage income and (negative) tax bill could have widely differing utilities. However the negative female tax rate, coupled with the fact that the male in the better-off household will be paying more tax, means that the extent of the horizontal inequity is smaller than under the joint taxation system. Clearly a selective individual tax system cannot entirely solve the horizontal inequity problem because wage income is still the tax base. We still lack an instrument to capture the effects on utilities of variation in domestic productivities. However, moving to such a system from one of joint taxation can reduce horizontal inequity.

In Case II $Cov(\lambda, w_f l_f)$ will be negative, the female marginal tax rate will be positive and the difference in male and female tax rates could take either sign. Since female labour supplies and wage incomes increase with the productivity parameter, tax payments and utilities are positively related, and so the horizontal inequity of the tax system will be less acute.

Thus we can conclude that selective individual taxation will be superior to the other two systems, not only for the obvious reason that generically an equal tax rate for males amd females would not be given by the solution to the optimal tax problem, given that it is costless to set different tax rates[11], but also because the differentiation between male and female tax rates can better reflect the different effects on their labour supply behaviour of variations in their domestic productivities, and thus reduce the degree of horizontal inequity in the tax system in cases where this could be expected to be high.

6. Conclusions.

In this paper we have proposed and applied a simple and tractable model of two-person households whose alternative use of time to market labour supply is household production. We have used the model to discuss the choice between alternative ways of taxing couples. We showed that joint taxation of household wage income can be associated with considerable horizontal inequity - households with the same utility level pay very different taxes, and there is considerable utility variation across households paying the same taxes. This is essentially due to the variation in productivity in domestic work, particularly that of women, and its influence on the allocation of time

[11] This of course applies equally in an analysis concerned solely with the efficiency effects of the tax system, as Boskin and Sheshinski point out.

between market and household labour supply. A system of selective individual taxation can reduce this horizontal inequity. However, any linear tax system based on market wage income has a fundamental difficulty in this respect, because it cannot take account of the household production dimension. We will in further work examine whether introducing more progressivity into the tax system in the form of increasing marginal tax rates can be expected to improve the second best solution.

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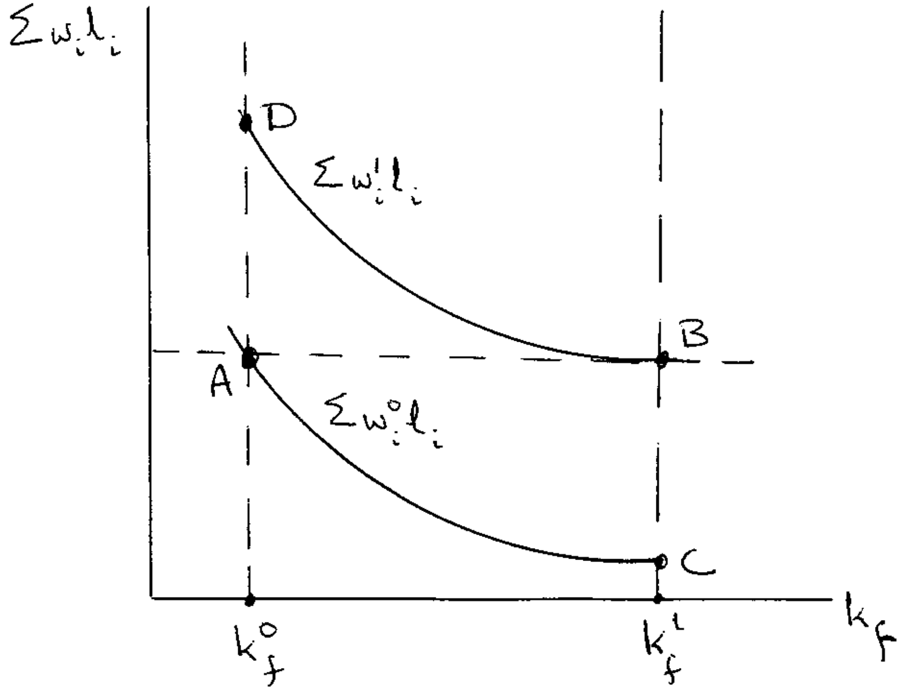

Figure 1. Apps and Rees: The Optimal Taxation of Couples.

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