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ON THE TAXATION OF TRADE
WITHIN AND BETWEEN
HOUSEHOLDS

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Abstract

Tax reform proposals affect individual welfares in ways which strongly depend on the nature of specialisation in household production and the pattern of trade within households. Variation in the degree of specialisation in domestic production across households strongly influences the impacts on individual tax burdens of a given tax reform. The standard models of the economics literature cannot be used to analyse these issues because they ignore the two-person nature of households and the existence of household production and trade. This paper proposes a simple and tractable model to remedy this and uses it to analyse the impacts of the types of tax reform that have been the subject of recent policy debate.

Keywords: tax, reform, household, trade

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1 Introduction

Tax reform is high on the policy agenda in many developed countries, for example in Australia, Canada, Germany, the USA and UK. A frequent theme is the proposal to bring down the highest marginal tax rates as well as to simplify the rate structure. Interestingly, these countries have differing approaches to the taxation of couples. The USA and Germany have joint taxation, under which primary and secondary earners in a household face the same marginal tax rates¹. In the remaining countries there is individual taxation, with marginal rates varying with individual income, so that primary and secondary earners in the same household may face different marginal rates. A question that arises therefore is whether the contemplated reforms could be expected to have the same effects on incentives and equity under the different systems, and a further question is that of how the reforms will affect the comparison of the systems in terms of their equity and efficiency. It has been strongly argued for example that efficiency considerations support not just individual taxation, but individual taxation with a lower marginal rate schedule for secondary earners². Reducing higher marginal rates in an individual tax system would appear to move in the opposite direction to this, since it would bring the marginal rates faced by primary and secondary earners closer together.

In our view, the existing theoretical literature on optimal tax theory and tax reform³, despite the important insights it has contributed, is not of very much help in the debate over these issues, primarily because of the limitations of the models of household choice underlying the analysis. A rigorous treatment of the issues concerning the taxation of couples is beyond the scope of this literature, first, because such an analysis involves households consisting of *two individuals*, and secondly because, as we show below, it involves the concepts of *production and trade within the household*. In the standard analysis a single-person household consumes an untaxed good, "leisure", the time not allocated to market work, and the trades it makes with the rest

¹Though as Feldstein and Feenberg (1996) point out, working wives are in effect faced with higher marginal tax rates in the US system because they pay social security taxes far in excess of the actuarial value of the benefits they receive. The same is true of Germany.

²See in particular Boskin and Sheshinski (1983) and Feldstein and Feenberg. Piggott and Whalley (1996) challenge what they see as this "conventional wisdom", but for a critique of their argument see Apps and Rees (1997c).

³For an excellent recent exposition and extension of this theory, see Guesnerie (1995).

of the economy, essentially exchange of labour time for market goods, are taxed. Income taxation can be viewed as a tax on trade between the market and household sectors, with all trades between individuals mediated through the market place⁴. If, however, individuals form multi-person households, they may also allocate time for production for trade with other household members, and these trades are untaxed. The implicit wage incomes derived from production and trade with other household members cannot be taxed, and income taxation is then a *selective* tax on trade⁵.

This has important implications for the analysis of reforms to the taxation of couples. First, the empirical evidence shows that in many households, partners specialise either in market or in household production. A tax reform may therefore induce an opposite welfare change for each partner by changing the intra-household terms of trade of household for market goods. Since we are concerned with the welfares of *individuals*, these effects need to be captured in a formal model. Secondly, the evidence also shows that the degree of specialisation in domestic production of, typically, the female partner varies widely across households, even after controlling for demographic characteristics⁶ and wage rates. Households tend to divide into *traditional* households, where the female partner specialises almost entirely in domestic work, and *non-traditional* households, in which female market labour supply is significant, approaching that of the male partner in the upper half of the distribution. Since traditional households have higher levels of untaxed production and trade, a tax reform can have substantially different impacts on the different household types. This is reflected in the policy debate on the relative merits of joint vs. individual taxation. The difference between these systems only matters when there is significant variation in female labour supply across households.

In optimal tax theory a central element is the idea that individuals differ in their innate productivities in market work, where these are measured

⁴An early contribution in this area by Boskin (1975) analysed the problem as a distortionary tax on the inputs to production in the market sector, with no taxation of household sector inputs, and used a two-sector general equilibrium model to estimate the resulting welfare losses. These were shown to be substantial.

⁵For an earlier approach to models that capture this see Apps (1982).

⁶It is worth noting that in countries such as those mentioned earlier, although it is true that female labour force participation in households with young children is significantly lower than in households without, it still does not fall below 40% or so in the former households.

by their market wages. It seems then only consistent to allow their productivities in domestic production also to differ⁷. Moreover, this has the advantage of allowing us to *explain* the varying degrees of specialisation in domestic production as between traditional and non-traditional households in an analytically tractable way, without having to depart from the assumption of identical utilities, also a standard element of optimal tax models. Indeed, variation in domestic productivities is the *raison d'être* for introducing household production into the model. If productivities were identical across households then it would suffice for most purposes to regard household members as simply consuming "time".

In this paper we present a simple formal model to analyse the effects of tax reform on individual welfares, taking account of the multi-person nature of households and the existence of the two household types⁸. There are two tax rates, one paid by primary earners, the other by secondary earners, and these rates may or may not be equal. Thus the initial situation could be one of either *individual taxation* or *joint taxation*. In the case of individual taxation, we might either have a *progressive anonymous* system, with the same progressive rate schedule applying to each individual's income; or a *selective* system with separate rate schedules and lower rates for secondary earners.

We first consider the case of a revenue neutral reform that increases the rate paid by secondary and reduces that paid by primary earners. This could be thought of as a move toward a flat (or flatter) rate tax in a progressive anonymous system, such as that prevailing in Australia, Canada and the UK, or to a change in the rates in the selective individual tax system favoured by the economics literature. We assume for simplicity only two household types and no wage dispersion among men on the one hand and women on the other, which is a substantial abstraction, but it allows us to focus on the central issues that have been neglected in the existing literature. We then go on to consider the case of a revenue neutral move away from a system of joint taxation toward selective individual taxation with a lower rate for secondary earners. Here we allow for wage dispersion across households but

⁷This point is emphasised in the important contribution of Sandmo (1990), who considers both optimal income and commodity taxation in a model of one-person households with domestic production and productivity variation across households.

⁸Curiously enough, issues of this kind were recognised as central in the longstanding debate in the earlier public finance literature on the choice between individual vs. joint taxation of married couples.

not as between primary and secondary earners. The main purpose is to show how the welfare evaluation of the tax reforms in each case depends crucially on the way in which labour supplies vary with domestic productivity.

2 A Model of the Household

In this section we present a model of a two-person household with domestic production. We assume that each household member produces a domestic good with a constant returns technology. The assumption that output of a domestic good is proportional to time input allows considerable simplification of the comparative statics analysis of the effects of tax rate changes without, in the end, sacrificing too much generality. Because it is the variation in the *female's* allocation of time to household production across households that is important, we allow female domestic productivities to vary across households but assume those of men are all the same. This variation could be due to either or both of differences in human capital among women⁹ and differences in household physical capital, which, since it is assumed exogenous, does not appear explicitly. Since preferences of all individuals are assumed identical, and women all receive the same market wage, variations in female labour supply are due to this productivity variation.

The gross market wage represents an individual's productivity in market production, as is usual in tax models. There is a single market good, x , produced by both women and men. All men are equally productive, as are all women, but male and female productivities can differ. Since labour is the only input and there are constant returns to scale we can write the production function simply as

$$x = \sum_h n_h (l_m^h + w l_f^h) \quad (1)$$

where w is the female productivity parameter, l_f^h is the labour input of a woman in household of type $h = 1, 2$ and l_m^h is that of a man, while n_h is the number of households of type h . We assume $l_i^h > 0$, $\forall h, i$, though of course household types may differ in the precise amounts of labour they supply. The households differ in respect of f 's productivity in domestic production, as we shall shortly specify. We let x be the numeraire with price 1. The

⁹For a paper which explores the implications of this for the specification of empirical models of labour supply, see Apps, Killingsworth and Rees (1996).

wage rates are w_f and w_m respectively. However the assumed production functions imply

$$w_m = 1, w_f = w \quad (2)$$

given profit maximisation. As usual in optimal tax models, wage rates are exogenous.

In a household, each partner produces a good which is partly consumed by her or himself and partly traded for the other goods. The production of domestic goods is also characterised by constant returns to scale. Since it is also assumed that men in all households have the same productivity, their production functions can be written simply as

$$y_m^h = t_m^h, h = 1, 2 \quad (3)$$

while those of women are

$$y_f^h = k_h t_f^h, h = 1, 2 \quad (4)$$

where t_j^h is the time spent in producing the household good y_j^h by individual $j = f, m$ in household of type $h = 1, 2$. We assume $k_1 > k_2$, so type 1 (2) households¹⁰ will be referred to as the high (low) productivity households. The individuals have the strictly quasi-concave and increasing utility functions

$$u(x_i^h, y_{if}^h, y_{im}^h), h = 1, 2, i = f, m \quad (5)$$

where

$$\sum_{i=f,m} y_{ij}^h = y_j^h, j = f, m \quad (6)$$

and y_{ij}^h is the amount of the domestic good produced by individual j that is consumed by i . Note that all these utility functions are assumed identical, the usual assumption in optimal tax models.

The household's budget constraint is given by

$$\sum_{i=f,m} x_i^h = \alpha + \sum_{i=f,m} \beta_i w_i l_i^h, h = 1, 2 \quad (7)$$

where α is the lump sum payment made to each household and $1 - \beta_i$ is the marginal tax rate paid by $i = f, m$ under the given tax system.

¹⁰The fact that there are only two types of households is inessential. For a model with a continuum of households see Apps and Rees (1997b).

Notice that, given the individual time constraints

$$l_i^h + t_i^h = T, i = f, m \quad (8)$$

where T is total available time, if we define the implicit prices of the domestic goods as

$$p_f^h = \frac{\beta_f w_f}{k_h}, h = 1, 2, \quad (9)$$

$$p_m = \beta_m w_m \quad (10)$$

we can replace the separate constraints (3), (4), (7) and (8) with the single *full-income budget constraint*

$$\sum_{i=f,m} (x_i^h + p_f^h y_{if}^h + p_m y_{im}^h) = \alpha + T \sum_i \beta_i w_i \equiv Y, h = 1, 2 \quad (11)$$

Note also that only the price of the domestic good produced by f varies across household types, with of course more productive households having the lower price. These prices have a straightforward explanation: they are the marginal opportunity costs of the domestic outputs. All households have the same full income Y , but high productivity households are clearly better off because they face a lower implicit price of the female domestic good, and so can enjoy higher total consumption at any given full income.

The substantive assumption we make to solve for the equilibrium household resource allocation is that this is Pareto efficient. This could be regarded as a characterisation of household as opposed to individual rationality. It means that household members behave sufficiently cooperatively that all possibilities of costless welfare improvement are exhausted. This of course leaves open the question of welfare distribution within the household. This is handled as follows. Each member has a reservation utility level that has to be weakly exceeded for him or her to remain within the household. We assume that these constraints are non-binding at the equilibrium household resource allocation. The actual distributional outcome is defined by a constraint giving a level of m 's utility. This is assumed to be a differentiable function of the components of the household's full income, the lump sum α and the net of tax wage rates $\beta_i w_i$, as well as other variables not specified. An equivalent, and, as it turns out, more useful approach is to apply the Second Theorem of Welfare Economics to model the household *as if* it first

distributed its full income between the household members, who then choose their consumptions to maximise their individual utilities at the given prices p_m, p_f^h . For appropriate choices of income shares this results in the identical resource allocation¹¹. We now set out the formal analysis corresponding to these two approaches.

By solving the problem

$$\max u(x_f^h, y_{ff}^h, y_{fm}^h) \text{ s.t. } u(x_m^h, y_{mf}^h, y_{mm}^h) \geq u_m^0 \quad (12)$$

and subject also to the budget constraint (11) we obtain individual demand functions $x_i^h(p_f^h, p_m, Y, u_m^0)$, $y_{ij}^h(p_f^h, p_m, Y, u_m^0)$, together with the marginal utility of household full income $\lambda^h(p_f^h, p_m, Y, u_m^0)$, the shadow price of the full income budget constraint.

It turns out however that the second approach to characterising the household equilibrium is more useful for the comparative statics analysis. In this alternative approach we can regard the household as generating its consumption allocation by first choosing income shares s_i^h , such that

$$\begin{aligned} s_f^h &= \alpha_f^h + p_f^h y_f^h + \beta_f w_f l_f^h = \alpha_f^h + \beta_f w_f T \\ s_m^h &= \alpha_m^h + p_m y_m^h + \beta_m w_m l_m^h = \alpha_m^h + \beta_m w_m T \\ \sum_{i=f,m} s_i^h &= Y \quad h = 1, 2 \end{aligned} \quad (13)$$

where $\alpha_i^h \geq 0$ is a lump sum transfer to i , with $\sum_{i=f,m} \alpha_i^h = \alpha$. Thus each partner has a full income given by the income earned (implicitly) from household production and from market labour supply, plus a transfer. The sum of these transfers must equal the lump sum transfer from the tax system, α , but the intra-household transfer itself could be negative for one household member. This simply reflects the distributional choices of the household members. Then each solves the problem

$$\max u(x_i^h, y_{if}^h, y_{im}^h) \text{ s.t. } x_i^h + p_f^h y_{if}^h + p_m y_{im}^h = s_i^h, \quad i = f, m \quad (14)$$

to yield demand functions $x_i^h(p_f^h, p_m, s_i^h)$, $y_{ij}^h(p_f^h, p_m, s_i^h)$, together with the marginal utilities of individual income $\lambda_i^h(p_f^h, p_m, s_i^h)$, the shadow prices of the individual budget constraints.

¹¹For further discussion of this approach see Apps and Rees (1988), (1997a). The hypothesis of Pareto efficiency is of course non-trivial. It was suggested as an approach to household decision taking by Samuelson (1956). For evidence that it may not hold see Udry (1996).

The problem in (14) is a standard consumer problem and we can in the usual way define the indirect utility functions

$$v_i^h = v(p_f^h, p_m, s_i^h) \quad h = 1, 2, \quad i = f, m. \quad (15)$$

Then applying Roy's Identity gives the partial derivatives

$$\frac{\partial v_f^h}{\partial p_f^h} = -\lambda_f^h(y_{ff}^h - y_f^h), \quad \frac{\partial v_m^h}{\partial p_f^h} = -\lambda_m^h y_{mf}^h \quad (16)$$

$$\frac{\partial v_f^h}{\partial p_m} = -\lambda_f^h y_{fm}^h, \quad \frac{\partial v_m^h}{\partial p_m} = -\lambda_m^h(y_{mm}^h - y_m^h) \quad (17)$$

$$\frac{\partial v_i^h}{\partial s_i^h} = \lambda_i^h \quad h = 1, 2, \quad i = f, m. \quad (18)$$

The derivatives in (16) and (17) reflect the significance of trade within the household. In general we expect an individual to consume less than the output of the good he or she produces, the resulting surplus being the amount traded with the other partner, at terms of trade given by the prices of the domestic goods. What is important here is that where partners specialise differentially in domestic production, changes in the terms of trade have greatly differing effects on the welfares of the individual household members. An increase (decrease) in the price of a good of which one is a net seller has a positive (negative) effect on one's utility that is proportionate to the amount sold.

From the solution to (14) we can also derive the market labour supplies as

$$l_f^h(p_f^h, p_m, s_f^h) = T - \frac{1}{k_h} \sum_{i=f,m} y_{if}^h(p_f^h, p_m, s_i^h) \quad (19)$$

$$l_m^h(p_f^h, p_m, s_m^h) = T - \sum_{i=f,m} y_{im}^h(p_f^h, p_m, s_i^h) \quad (20)$$

Thus an individual's labour supply is the difference between total time available and the amount of time required to meet total household demand for his or her domestic output.

An alternative way of expressing the welfare gain of the high productivity households is to say that for the same level of consumption of domestic goods they can supply more labour to the market and so earn a higher wage income,

though of course they may not *choose* to do this. Thus differentiating (19) with respect to k_h gives

$$\frac{\partial l_f^h}{\partial k_h} = \frac{t_f^h}{k_h} (1 - e_f^h) \quad (21)$$

where e_f^h is the price elasticity of demand for f 's domestic output (defined to be positive). Increasing domestic productivity increases f 's market labour supply if this demand elasticity is less than one and reduces it in the converse case. The intuitive reason for this ambiguity is that a change in the productivity has both a demand side and a supply side effect. The former results from the demand-increasing effect of a reduction in the implicit price of the domestic good following an increase in productivity, and is expressed by the demand elasticity. The latter results from the fact that with a higher productivity a given amount of domestic output can be supplied in less time. Whether females in high productivity households supply more or less time to the market therefore depends, inter alia, on the relative strengths of these effects. This is important because, as we shall see in the tax reform analysis, whether the households with higher female market labour supplies are also those with higher productivity and consequently higher welfare is significant for the interpretation of the degree of progressivity of a given tax reform. Note also that in deriving (21) we assume that changing the female's domestic productivity does not affect the distributional outcome in the household, as expressed by s_i^h .

It is also useful to express (21) in elasticity form. Thus we have

$$\kappa_f^h \equiv \frac{k_f^h}{l_f^h} \frac{\partial l_f^h}{\partial k_f^h} = \frac{t_f^h}{l_f^h} (1 - e_f^h) \quad (22)$$

We see that at relatively low levels of market labour supply small proportionate productivity differences can produce large proportionate changes in female labour supply, if the demand elasticity for the female's domestic good diverges significantly from unity in either direction.

Finally, we can note a perverse result of increasing the tax rate on women: such an increase causes a larger shift in time allocation from the market to the household sector, the more productive the woman is in the market sector relative to the household. A tax increase causes a larger shift from more productive to less productive sectors the greater the disparity in productivities. To see this, recall the expression for female labour supply in (19) and

differentiate it with respect to the marginal tax rate¹² to obtain

$$\frac{\partial l_f^h}{\partial \beta_f} = - \frac{1}{k_h} \frac{\partial y_f^h}{\partial p_f^h} \frac{w_f}{k_h} \quad (23)$$

This derivative is clearly larger the smaller is k_h , for two reasons. First, for a given demand derivative $\partial y_f^h / \partial p_f^h$, the lower the productivity the greater is the induced change in the domestic time allocation and so the larger the change in market labour supply. Secondly, the higher the ratio of productivities in the two sectors, w_f / k_h , the greater the effect of the tax change on the price of the domestic good and so the larger the change in market labour supply.

We now apply these results to an analysis of the welfare effects of tax reforms.

3 Tax Reform with Individual Taxation

We characterise any tax reform as a *revenue neutral* pair of infinitesimal changes in the marginal tax rates. We hold the lump sum α constant throughout. The government budget constraint is

$$\sum_{h=1,2} \sum_{i=f,m} n_h (1 - \beta_i) w_i l_i^h - n\alpha - R = 0 \quad (24)$$

where $R \geq 0$ is the revenue requirement, n_h is the number of households of type h and n is the total number of households. Revenue neutrality requires

$$\sum_{h=1,2} \sum_{i=f,m} n_h \left[\sum_{j=f,m} (1 - \beta_j) w_j \frac{\partial l_j^h}{\partial \beta_i} - w_i l_i^h \right] d\beta_i = 0 \quad (25)$$

implying

$$d\beta_f = \mu d\beta_m \quad (26)$$

with

$$\mu \equiv - \frac{\sum_h n_h [\sum_{j=f,m} (1 - \beta_j) w_j \frac{\partial l_j^h}{\partial \beta_m} - w_m l_m^h]}{\sum_h n_h [\sum_{j=f,m} (1 - \beta_j) w_j \frac{\partial l_j^h}{\partial \beta_f} - w_f l_f^h]} \quad (27)$$

¹²For simplicity we ignore the possible effect on f 's income share, which is unlikely to have much effect on the argument.

It seems reasonable to assume that the incremental tax revenues resulting from the changes in labour supplies, the first terms in the numerator and denominator, do not exceed the respective tax bases, the second terms in the numerator and denominator, and so we will in everything that follows consider only the case where $\mu < 0$. A revenue neutral tax reform that reduces one tax rate must increase the other.

Consider now the changes in individual utilities induced by a tax reform. To derive these we take the total differentials of the indirect utilities, using (16) - (18), and recalling that $p_f^h = \beta_f w_f / k_h$, and $p_m = \beta_m w_m$:

$$dv_f^h = \lambda_f^h [-(y_{ff}^h - y_f^h) \frac{w_f}{k_h} d\beta_f - y_{fm}^h w_m d\beta_m + w_f l_f^h d\beta_f + d\alpha_f] \quad (28)$$

$$dv_m^h = \lambda_m^h [-y_{mf}^h \frac{w_f}{k_h} d\beta_f - (y_{mm}^h - y_m^h) w_m d\beta_m + w_m l_m^h d\beta_m + d\alpha_m] \quad (29)$$

We assume a revenue neutral tax reform for which $d\beta_f < 0 < d\beta_m$, so that female marginal tax rates are raised and male marginal tax rates lowered. For example, this would correspond to a move toward a flatter rate structure in a progressive anonymous individual tax system where women have lower wage incomes. Alternatively it would correspond to a move toward joint taxation in a selective individual tax system in which women have lower marginal rates. Suppose for the moment that there are no changes in intra-household transfers, so that $d\alpha_i = 0$. Then we have immediately:

Result 1: Intra-household terms of trade effects. The first two terms in the brackets in (28) are negative, those in (29) are positive. The price of the household good women sell (men buy) has fallen, that of the good women buy (men sell) has risen, and so all women are worse off, all men are better off, other things being equal. These effects will be greater:

- the larger is μ in absolute value, for a given $d\beta_m$;
- the larger is the volume of trade within the household;
- the lower is household productivity k_h , since this determines the size of the fall in price of f 's domestic good. This implies that women's utility falls more in the low productivity households, other things being equal. Since these are also the households with lower utilities, this could be thought of as a regressive effect of the tax reform.

Result 2. Wage-income effects. The term $w_f l_f^h d\beta_f$ will be negative and $w_m l_m^h d\beta_m$ will be positive, so this will be a further source of increase in male and decrease in female utilities. That is, before intra-household transfers, female full income falls and male full income rises. These effects will be greater the larger is μ in absolute value, given $d\beta_m$.

These results assume no compensating intra-household transfers are made. Without a more specific model of the determination of the intra-household welfare distribution we cannot characterise these specifically. However we can explore the question of the conditions under which such transfers could or could not compensate women for their utility losses and still leave their partners with a net welfare gain.

We obtain a very intuitive result on the conditions under which both partners in a household can potentially be better off after the tax reform.

Result 3. The male partner in the household can fully compensate the female for her utility loss and still be better off if and only if

$$\sum_{i=f,m} w_i l_i^h d\beta_i > 0 \quad (30)$$

as a result of the tax reform.

Proof: A woman can be just compensated for her utility loss iff

$$d\alpha_f = (y_{ff}^h - y_f^h) \frac{w_f}{k_h} d\beta_f + y_{fm}^h w_m d\beta_m - w_f l_f^h d\beta_f \quad (31)$$

Since $\sum_{i=f,m} d\alpha_i = 0$, substituting for $d\alpha_m = -d\alpha_f$ in (29) gives

$$dv_m^h = \lambda_m^h [w_m l_m^h d\beta_m + w_f l_f^h d\beta_f] \quad (32)$$

which gives the result.

The intuition underlying this result is straightforward, especially if we recall the analogy between the household and a small economy trading with the rest of the world at fixed prices, here the net of tax wage rates. A tax reform changes the household/economy's terms of trade leading to changes in production, consumption and welfares within it. but it can only be better off in the aggregate if the change is equivalent to an increase in its net income "from abroad".

This result can be expressed in elasticity terms, also taking account of the revenue-neutrality requirement, as follows. Condition (30) holds *iff*

$$(w_m l_m^h + \mu w_f l_f^h) d\beta_m > 0 \quad (33)$$

implying

$$\frac{w_m l_m^h}{w_f l_f^h} > -\mu = \frac{\sum_h n_h w_m l_m^h [1 - \frac{\sum_{j=f,m} (1-\beta_j) w_j}{w_m l_m^h} \frac{\partial l_j^h}{\partial \beta_m}]}{\sum_h n_h w_f l_f^h [1 - \frac{\sum_{j=f,m} (1-\beta_j) w_j}{w_f l_f^h} \frac{\partial l_j^h}{\partial \beta_f}]} \quad (34)$$

We can write the terms in square brackets in this condition as

$$\delta_i^h = 1 - \frac{(1-\beta_j)}{\beta_i} \frac{w_j l_j^h}{w_i l_i^h} \sigma_{ij}^h - \frac{(1-\beta_i)}{\beta_i} \sigma_i^h \quad h = 1, 2, \quad i, j = f, m, \quad i \neq j \quad (35)$$

where σ_i^h is i 's (uncompensated) labour supply elasticity with respect to the net wage, and σ_{ij}^h is the (uncompensated) elasticity of j 's labour supply with respect to i 's net wage. Noting further that the wage rates do not depend on h , we can finally rewrite (34) as

$$\frac{l_m^h}{l_f^h} > \frac{\sum_h n_h l_m^h \delta_m^h}{\sum_h n_h l_f^h \delta_f^h} \quad (36)$$

If the δ_i^h are all equal, this condition says simply that in the household whose ratio of male to female labour supply is above the ratio of average male to average female labour supply - the *traditional household* - the female can at least be compensated. The intuitive reason is simply that the gain in after tax male wage income more than compensates for the loss in after-tax female income because the female partner is supplying relatively little time to the market. By the same token, in households with a ratio of male to female labour supply below the ratio of the averages - *non-traditional households* - the male cannot compensate the female without making himself worse off.

The empirical evidence on labour supply elasticities¹³ suggests however that $\delta_m^h > \delta_f^h$, implying that it is possible that in *no* household could the woman be compensated, if the ratio of male to female labour supplies in traditional households is sufficiently close to the ratio of the averages. This evidence suggests

¹³For a good summary of this see Heckman (1993).

- $\sigma_f^h > \sigma_m^h$ for each household type: Women tend to have higher labour supply elasticities than men, especially in traditional households.
- $\sigma_{fm}^h \approx 0$. Variation in the after-tax wage rate of women has a negligible effect on male labour supply.
- $\sigma_{mf}^h < 0$. Variation in the male after-tax wage has a significant negative effect on female labour supply.

More realistically perhaps, since in reality there is a continuum of household types and a subset of households have zero female labour supply, we should express the point as saying that the set of households in which women cannot be compensated widens with the difference between δ_m^h and δ_f^h .

The intuition for this result is also straightforward: the more elastic is female labour supply relative to that of males, the greater the net loss of tax revenue induced by the changes in labour supplies, and so the greater must be the increase in the female tax rate relative to the decrease in the male tax rate. This makes it harder for the male within a household to compensate the female.

The fact that women in some households *could* be compensated after the tax reform does not of course imply that they *will be*. Their net wage has fallen while that of men has risen, and this could imply a worsening of their relative position in the household¹⁴. Moreover it will now be shown that under plausible conditions each household's full income will fall as a result of the tax reform. This implies that under these conditions *the economy as a whole* cannot be better off after the tax reform.

Result 4: If:

- $l_f^h \leq l_m^h$, i.e. female labour supplies are no greater than male labour supplies;
- male and female labour supply elasticities are such that $\delta_m^h > \delta_f^h (> 0)$, then a revenue neutral tax reform with $d\beta_f < 0 < d\beta_m$ reduces the full income of every household.

Proof: Given the definition of full income we have

$$dY = T(w_f d\beta_f + w_m d\beta_m) = T(\mu w_f + w_m) d\beta_m$$

¹⁴For example in a Nash bargaining model of the household the threat points might well vary positively with the after-tax wage rates.

Thus for $d\beta_m > 0$ we will have $dY < 0$ iff

$$-\mu > \frac{w_m}{w_f} \quad (37)$$

that is if

$$-\mu = \frac{\sum_h n_h w_m l_m^h \delta_m^h}{\sum_h n_h w_f l_f^h \delta_f^h} > \frac{w_m}{w_f} \quad (38)$$

which is equivalent to

$$\sum_h n_h (l_m^h \delta_m^h - l_f^h \delta_f^h) > 0 \quad (39)$$

and this is ensured by (i) and (ii).

The intuition underlying the result is again straightforward: Given the relative sizes of the male and female labour supplies and the incremental tax revenues generated by the changes in marginal tax rates, an increase in the male after-tax wage requires such a decrease in the female after-tax wage to ensure revenue-neutrality that household full income falls as a result. The condition that women supply less market labour than men is of course quite plausible. The second assumption also fits the stylised facts about male and female labour markets, as detailed in the discussion of Result 3.

From the empirical standpoint the most relevant case to take seems to be that in which traditional households are better off and non-traditional households worse off as a result of the tax reform. The question then becomes that of evaluating this from the point of view of social welfare. The model of this paper suggests that the key factor in this evaluation is the nature of the relationship between the productivity parameter k_h and female labour supply, as summarised in the simple relationship (21). If female labour supply falls with increasing productivity, corresponding to the case of high elasticity of demand for f 's domestic good, then traditional households will be those with the higher utilities of household members. It follows that the type of tax reform just considered is unambiguously regressive in its impact, since it is increasing the utilities of those who are already better off. On the other hand, if female labour supply increases with domestic productivity, because the demand effect is weaker than the "time-releasing" effect of greater efficiency, then non-traditional households are those with higher utilities, and the tax reform considered improves the welfare of the less well off individuals.

subject always to the proviso that the women in the traditional households are actually compensated for the fall in their full incomes by transfers from their partners.

This can be put more generally and precisely by introducing the social welfare function

$$W = W(v_f^1, \dots, v_m^1, \dots, v_f^2, \dots, v_m^2, \dots) \quad (40)$$

and taking its total differential

$$dW = \sum_h n_h \sum_i W_i^h dv_i^h \quad (41)$$

with

$$dv_f^h = \lambda_f^h [-(y_{ff}^h - y_{fm}^h) \frac{w_f}{k_h} d\beta_f - y_{fm}^h w_m d\beta_m + w_f l_f^h d\beta_f + \sum_{i=f,m} \frac{\partial \alpha_f^h}{\partial \beta_i} d\beta_i] \quad (42)$$

$$dv_m^h = \lambda_m^h [-y_{mf}^h \frac{w_f}{k_h} d\beta_f - (y_{mm}^h - y_{fm}^h) w_m d\beta_m + w_m l_m^h d\beta_m + \sum_{i=f,m} \frac{\partial \alpha_m^h}{\partial \beta_i} d\beta_i] \quad (43)$$

Note that the social welfare function is defined, as it should be, on the utilities of all the *individuals* in the economy, and we assume the individual welfare weights $W_i^h > 0$. The expression for the change in social welfare involves both *within*-household and *across*-household distributional effects. There is no reason in general why the implicit welfare weights applied within a household would correspond to those that would be desired by a "social planner" - in an earlier paper¹⁵ we have called this the issue of *dissonance*. However here we focus on the across-household distributional effects and so we assume that the distributional preferences of the household are precisely those the planner would wish. It can then be shown¹⁶ that the marginal social utilities of income $W_i^h \lambda_i^h$ become the same for individuals in a given household and vary only across household types, and we now denote these by η_h . Then we have that the condition for social welfare to increase as a result of the tax reform becomes simply

$$dW = \sum_h n_h \eta_h (w_m l_m^h + \mu w_f l_f^h) d\beta_m > 0 \quad (44)$$

¹⁵ Apps and Rees (1988).

¹⁶ See the paper just cited.

Since type 1 households have higher productivity and so are better off we expect that $\eta_2 > \eta_1$. In the case assumed here, the term in brackets is positive for traditional and negative for non-traditional households, and so the condition is clearly more likely to be satisfied if the traditional households also have low productivity, that is, if domestic productivity and female labour supply are positively associated across households. In that case household wage income is positively associated with the utilities of household members. On the other hand, in the case where domestic productivity and female labour supply are inversely related, higher money income does not indicate higher utilities of household members.

Note finally that the strength of the relationship between domestic productivity and female labour supply is also important. Suppose for example that there is only a very small difference in the k_h values, but this difference induces a large increase in female labour supply. Then the welfare weights η_h will be very close in value since the utility difference between the household types is small, and the condition in (44) is unlikely to be satisfied, since the negative value for non-traditional households will outweigh the positive value for traditional households. In that case the reverse tax reform - widening the gap between marginal rates - would be welfare improving.

4 Tax Reform with Joint Taxation

We now extend the model of the previous section by allowing wage dispersion across households, and consider in this context issues raised by the reform of joint taxation. We assume there are two wage rates, w_1 and w_2 , with $w_1 > w_2$, and these again represent productivities in market production. The rest of the household model is as before. Both men and women can be of either market productivity type. There are then in principle eight possible household types. So much diversity is unnecessary for our purposes and so we shall make the assumption of *assortative mating*: a male and female in the same household have the same wage rate. There are then four possible household types each defined by a pair (w_i, k_j) , $i, j = 1, 2$. Clearly the household with (w_1, k_1) will have the highest utility possibilities and that with (w_2, k_2) the lowest. We assume that an increase in the market wage has at least as large an effect on household utility possibilities as an increase in domestic productivity and so the household with (w_1, k_2) is no worse off and may be better off than that with (w_2, k_1) .

There is clearly considerable scope for both vertical and horizontal inequity in a joint tax system based on household money income, in a way that depends on how labour supplies vary with the wage rates and domestic productivity. Thus suppose:

- male labour supplies are relatively invariant to changes in wage rates and domestic productivity;
- female labour supplies decrease sharply with an increasing male wage rate, to an extent that outweighs the positive effect of an increasing female wage rate, perhaps due to strong income effects on the demand for her domestic output;
- female labour supply decreases with increasing domestic productivity.

Then we have that:

- a household with (w_i, k_2) will have a higher money income and so pay more tax than the household with (w_i, k_1) , $i = 1, 2$, even though it has lower utility possibilities;
- the same is true for a household with (w_2, k_j) as compared to a household with (w_1, k_j) , $j = 1, 2$;
- the same is true for a household with (w_2, k_2) as compared to one with (w_1, k_1) .

Thus the ranking of households on the basis of tax burdens will be virtually the inverse of the ranking on the basis of utility possibilities. To these vertical inequities could be added the possible horizontal inequity:

- the households with (w_1, k_2) and (w_2, k_1) could have the same utility possibilities but different tax burdens, depending on the relative strengths of the effects of reducing wage rates and increasing domestic productivity on female labour supply.

If we on the other hand assume that a small increase in domestic productivity has a large effect in increasing female labour supply, two households with utility possibilities that are very close together could have significantly different tax burdens. These effects are all exacerbated in a joint tax system

with marginal rates that increase with household income. The basic point is that household money income could be a poor tax base in an economy consisting of two-person households with varying domestic as well as market productivities.

This is important for the evaluation of a tax reform in this economy that flattens the rate structure, reducing the higher and increasing the lower marginal rate. The *net* efficiency effects could well be quite minor, and so the reform would be evaluated in terms of its equity impact. Obviously the higher rate taxpayers will be better off and the lower rate taxpayers worse off, but it is then crucial to know whether the household's utility possibilities are positively or negatively associated with its market wage income. If this association is positive then the reform will tend to be regressive, since higher utility households obtain the tax reductions. If the association is negative then flattening the rate structure is desirable on distributional grounds.

More formally we now wish to analyse a move away from joint taxation, in which all households pay a single tax rate, by means of a revenue-neutral tax reform that reduces the marginal rate paid by secondary earners and increases that paid by primary earners. The analysis of the previous section is directly applicable, with just a change in notation. We number households so that:

$$\begin{aligned} h &= 1 \text{ for } (w_1, k_1) \\ h &= 2 \text{ for } (w_1, k_2) \\ h &= 3 \text{ for } (w_2, k_1) \\ h &= 4 \text{ for } (w_2, k_2) \end{aligned}$$

and take a revenue neutral tax reform for which $d\beta_m < 0 < d\beta_f$.

Proceeding exactly as before we obtain the condition that both members of a household can be better off if and only if

$$\frac{l_m^h}{l_f^h} < -\mu = \frac{\sum_h n_h w_h l_m^h \delta_m^h}{\sum_h n_h w_h l_f^h \delta_f^h} \quad h = 1, \dots, 4 \quad (45)$$

which will be satisfied for non-traditional but possibly not for traditional households. Again introducing a social welfare function defined on the utilities of the individuals in all four household types we obtain the condition for

a welfare improvement

$$dW = \sum_{h=1}^4 n_h \eta_h w_h (l_m^h + \mu l_f^h) d\beta_m > 0 \quad (46)$$

where $w_h = w_1$ for $h = 1, 2$ and $w_h = w_2$ for $h = 3, 4$. Again it is important to know which households - traditional or non-traditional - have the higher utility possibilities and therefore the lower welfare weights.

Here we wish to explore the following intuition. Suppose that at each wage rate there is only a small difference in domestic productivities, though this is enough to cause a significant increase in female labour supplies in the higher productivity households. It follows that non-traditional households are only slightly better off than traditional households at the same wage rate, but pay much more in tax. In that case the kind of tax reform considered here should be welfare increasing. A small increase in the tax rate on men will allow a large decrease in the tax rate on women, giving a net increase in overall labour supply. Traditional households are a little worse off, non-traditional households a lot better off, so overall there could be a welfare increase.

We show for a special case of this model, which can however easily be generalised, that this intuition is correct provided that the excess of female over male labour supply elasticities is sufficiently large relative to the differences in welfare weights between traditional and non-traditional households. Thus assume

- the traditional households, 2 and 4, have zero female labour supplies;
- the non-traditional households 1 and 3 have equal female labour supplies $l_f > 0$;
- all male labour supplies are equal, at $l_m > 0$;
- there is the same number of households of each type;
- the δ_m^h are all equal, as are the δ_f^h and their ratio is $\delta > 1$;
- the welfare weights satisfy:

$$\begin{aligned} \eta_2 &= \eta_1 + \epsilon_2 \\ \eta_4 &= \eta_3 + \epsilon_4 \end{aligned}$$

for $\epsilon_2, \epsilon_4 > 0$.

Then we have

Result 5: Given these assumptions, the tax reform considered here is welfare improving if and only if

$$\delta > 1 + \frac{\epsilon_2 w_1 + \epsilon_4 w_2}{2(\eta_1 w_1 + \eta_2 w_2)} \quad (47)$$

Proof: on the given assumptions we have

$$\mu = -2\delta \frac{l_m}{l_f} \quad (48)$$

Then inserting this in (46) and rearranging, recalling that $d\beta_m < 0$, gives the result.

This result says that if female labour supply elasticities are sufficiently greater than male, and the welfare weights on traditional and non-traditional households at a given wage rate are sufficiently close together, then reducing the tax rate on women and increasing that on men will be welfare improving. It makes traditional households a little worse off but non-traditional households so much better off that this offsets the somewhat higher welfare weights given to the former and is welfare-increasing overall.

5 Conclusions

This paper has used a simple and tractable model of the household to derive straightforward criteria for assessing the impact of a proposed tax reform on individual welfares, in tax systems corresponding to progressive individual taxation, selective individual taxation and joint taxation. The results show clearly that the effects of a tax reform involving the flattening of a progressive rate structure depend crucially on whether the pre-reform tax system is one of individual or joint taxation. We have stressed the importance of the patterns of specialisation and trade within a household, and of the way in which these vary across households, in determining the impact of a tax reform. Of central importance is the nature of the relationship between productivity in domestic production and female labour supply across households, in determining the progressivity or regressivity of the welfare effects induced by tax rate changes. The paper has sought to make a contribution not only in terms of substantive results, but also in suggesting a modelling approach which can extend the scope of the economic analysis of real-world issues in tax policy.

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