Can bilingualism be dynamically stable?

A simple model of language choice*

by

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0. Abstract

A model is developed in which parents choose the language or languages in which their children are brought up. Their choice of language community into which children are socialized depends both on the practical value of the language as a means of communication and on the emotional attachment of the parents to the language as a carrier of cultural identity. In the model, two languages are considered, and children can be brought up as monoglots or bilinguals, that is be socialized into both linguistic communities. The dynamic structure of the model is investigated and dynamically stable equilibria are characterized. It is shown that the behavior of bilingual parents is the crucial factor in determining the survival chances of bilingualism in society. *Journal of Economic Literature* Classification Numbers: C72, J10, J15, Z10.

1. Introduction

In many corners of the world we observe the coexistence of two or more language groups in the same location. If language only served as a means of communication, this would be very inefficient. Efficiency (in the long run, at least) would require that only one language be used on one location. The situation can, in a certain sense, be even more inefficient if, like in Wales for instance, a part of the population is bilingual (in Welsh and English) and the other monoglot (in English). In such a case, it is individually irrational to bring up bilinguals if language only serves as a means of communication.

Of course, an important aspect of language, maybe the most important one, is that it is one of the main carriers of cultural identity. For members of a certain language group the possibility to preserve their culture through the transmission of the language to the next generation can be of highest importance. The focus of this essay is on the determination of conditions under which such an emotional attachment to a language can guarantee its survival over several generations.

There are some studies of the problem of survival of minority languages related to the practical advantage of their use. Grin (1992) models in some detail factors determining the actual use of a minority language by bilinguals with an emotional attachment to it and derives thresholds for this usage to increase over time. The number of speakers able to use the minority language is, however, largely exogenous to the model.

Some authors discuss the learning of languages for interethnic communication. Also here the focus is on the practical advantage of learning a language. Pool (1991) analyzes a model with two language groups and three languages, the two ethnic ones and one interethnic (Esperanto), where languages, other than the native one, are learned for the purpose of communication between members of the two language groups. He demonstrates that the equilibrium level of speakers of Esperanto can be anywhere between zero and one under different assumptions on the language aptitudes, costs and benefits of the individuals. Selten and Pool (1991, 1995) and Güth *et al*. (1997) also address the same problem, analyzing under what conditions different equilibria can exist. In those three contributions the dynamic aspect is totally absent. Church and King (1993) discuss the same efficiency issue in a model with only two languages and come to the conclusion that the majority language should be promoted.

A general model of the evolution of conventions can be found in Goyal and Janssen (1997). Their model permits the interpretation that the society consists of two groups of monoglots and one group of bilinguals Here the size of the payoffs by the adoption of one convention *(i. e*. language) or the other or both determine the possible equilibria. Also here the practical advantage of adopting a certain convention is the driving force of the model.

In our model, we want to focus both on the practical advantage of choosing a certain language and on the emotional attachment to the language as the carrier of cultural identity captured in the desire of parents that their culture be carried on by their children. For this purpose we postulate a trade-off between the efficiency aspect of language as means of communication, on the one hand, and its importance as a carrier of culture, on the other hand. This trade-off is the basis for the parents when they decide in which language to bring up their children. We allow for three groups, monoglots in one of two languages and bilinguals. The communication costs are positive between two monoglots in different languages and zero between a monoglot and a bilingual person or between two bilinguals or between two monoglots in the same tongue. With monoglot we understand a native speaker of only one language. Such a person might be able to communicate very well in the other language, but not with the ease or comfort of a native speaker. That is, the communication costs are not only the lack of comprehension linguistically, but also of psychological nature. A bilingual person is, hence, someone, who is fully socialized into both languages groups.

2. The model

The three language groups in society are denoted by $L \in \{A, B, C\}$: the speakers of language A, of language B, and those who are bilingual, denoted by C. There further exists a large number, $N(t)$, of agents at time t. The number of agents belonging to language group L is given by $P_L(t)$. Suppressing the dependence on time, we have

$$N = \sum_L P_L \ . \tag{2.1}$$

We normalize the number of agents to one through division by N and defining

$$p_L := \frac{P_L}{N} . \tag{2.2}$$

Ignoring the fact that there are two distinct sexes, there are six possible combinations of parents, family types: $F \in \{AA, AB, AC, BB, BC, CC\}$. It is assumed that mating is random, giving rise to the following frequency distribution of the various family types, $\mu(F)$:

F	$\mu(F)$
AA	p_A^2
AB	$2p_A p_B$
AC	$2p_A p_C$
BB	p_B^2
BC	$2p_B p_C$
CC	p_C^2

. (2.3)

The expected number of children resulting from a certain family type is given by $2\beta(F)$, and the distribution of the children on the language characteristics by the functions $\alpha_L(F;\, p_A,\, p_B)$, such that:

$$\sum_L \alpha_L(F; p_A, p_B) = 1. \tag{2.4}$$

These functions are the crucial ingredients of our model. The dependence on the variables p_L captures the practical advantage of belonging to a certain language group, since they measure the frequency with which an individual encounters another individual in group A, B and C, respectively. The basic hypothesis is that there are positive communication costs if an A and a B individual encounter one another, but no communication costs if two A, two B, two C, an A and a C, or a B and a C meet. The function α_A is, hence, assumed to be non-decreasing in p_A and non-increasing in p_B. Similarly, α_B is non-increasing in p_A and non-decreasing in p_B, and α_C is non-decreasing in both variables. The dependency on the variable F captures the cultural transfer through the family. It is hypothesized that the emotional attachment in the family to a certain language, and, hence, the frequency of its transmission to the children, is determined by its strength among the parents. This leads to a monotonicity assumption; see section 3, below. A further analysis of these functions based on utility maximization, is given in the appendix.

We can now specify the number of children in the various language groups:

$$\Pi_L(t) = 2\sum_F \alpha_L(F, p_A, p_B)\beta(F)\mu(F), \tag{2.5}$$

as well as the distribution of the children over the language groups:

$$\pi_L(t) := \frac{\Pi_L}{\sum_L \Pi_L} = \frac{\sum_F \alpha_L(F, p_A, p_B)\beta(F)\mu(F)}{\sum_F \beta(F)\mu(F)}. \tag{2.6}$$

With a suitable choice of the time unit, we can write the dynamics of the system as

$$\dot{p}_L(t) = \pi_L(t) - p_L(t), \tag{2.7}$$

for all L, and a stationary distribution obtains if

$$p_L(t) = \pi_L(t). \tag{2.8}$$

3. Assumptions

The first part of the following assumption is purely technical and simplifies the presentation of the analysis considerably. The second part, which also was discussed above, can be derived as a result of utility maximization, see the appendix.

Assumption 3.1: The $\alpha_L(F; p_A, p_B)$ are continuous and differentiable in p_A, and p_B. Furthermore, for the partial derivatives with respect to p_A and p_B we have $\alpha_{AA} \geq 0$, $\alpha_{AB} \leq 0$, $\alpha_{BA} \leq 0$, $\alpha_{BB} \geq 0$, $\alpha_{CA} \geq 0$, and $\alpha_{CB} \geq 0$.

The following substantial assumptions both strengthen and facilitate the analysis.

Assumption 3.2: $\alpha_A(AA; p_A, 0) = \alpha_B(BB; 0, p_B) = 1$ for all p_A and $p_B \in [0, 1]$.

If $p_B = 0$, there are only individuals of the A and C types in the society. That is, there is no practical advantage in bringing up a B type child, since an A or C type can fully communicate with everyone in this society, indeed, if $p_A > 0$, there would be a practical disadvantage in bring up a B type, since it would have communication difficulties with the A types. By the same token there is no practical advantage in bringing up a C type child as compared to an A type, and *vice versa*. Hence, only an emotional attachment to language B can motivate educating a child in it, *i. e.*, socializing the child into group B or C. The assumption states that such an emotional attachment is not to be found in a family where both parents are monoglot A speakers. The corresponding argument, of course, applies if $p_A = 0$. In short, if the parents are monoglot in the same language, there is no emotional attachment in the family to the other language, and only practical advantages could motivate its learning. This assumption is, strictly speaking, not necessary. However, if it is not made, monolingual communities would not necessarily exist, and we would make it very easy for ourselves answering the question of the title. The interesting question is whether bilingualism can be stable in spite of assumption 3.2.

Assumption 3.3: $\alpha_A(BB; p_A, p_B) = \alpha_B(AA; p_A, p_B) = \alpha_A(BC; p_A, p_B) = \alpha_B(AC; p_A, p_B) = 0$ for all p_A and $p_B \in [0, 1]$.

It is not unreasonable to assume that no monoglot A-speakers emerge from families, where both parents are monoglot B-speakers or where one parent is a monoglot B-speaker and the other one is bilingual and *vice versa* for monoglot B-speaking children. Again, the assumption makes results showing the stability of bilingualism stronger.

Assumption 3.4: $\alpha_A(AC; p_A, p_B) \geq \alpha_A(AB; p_A, p_B)$ and $\alpha_B(BC; p_A, p_B) \geq \alpha_B(AB; p_A, p_B)$ for all p_A and $p_B \in [0, 1]$.

That is, the frequency of monoglot speakers of any one language among children is an non-decreasing function of the strength of that language in their family type, a fairly innocent assumption.

These last three assumptions can, for the sake of clarity, be collected into one monotonicity assumption relating the frequency of monoglot children in a certain language to the strength of that idiom among the parents in the family type of the child:

Assumption 3.5: $1 = \alpha_A(AA;\, p_A,\, 0) \geq \alpha_A(AA;\, p_A,\, p_B) \geq \alpha_A(AC;\, p_A,\, p_B) \geq \alpha_A(AB;\, p_A,\, p_B) \geq \alpha_A(BC;\, p_A,\, p_B) = \alpha_A(BB;\, p_A,\, p_B) = 0$ and $1 = \alpha_A(AA;\, p_A,\, 0) \geq \alpha_A(AA;\, p_A,\, p_B) \geq \alpha_A(AC;\, p_A,\, p_B) \geq \alpha_A(CC;\, p_A,\, p_B) \geq \alpha_A(BC;\, p_A,\, p_B) = \alpha_A(BB;\, p_A,\, p_B) = 0$ for all p_A and $p_B \in [0, 1]$ and *mutatis mutandis* for α_B.

4. Monolingual equilibria

In section 2, the dynamics of the system was characterized. Because of the normalization in (2.2), we can eliminate the variable p_C, setting it equal to $1 - p_A - p_B$, and reduce the relevant variable space to a two-simplex. We thus have

$$\dot{p}_L = f_L(p_A, p_B), \text{ for } L \in \{A, B\}, \tag{4.1}$$

where f_L is found from (2.6) and (2.7) after the appropriate substitution for p_C in $\mu(F)$:

$$f_L(p_A, p_B) = \frac{\sum_F \alpha_L(F, p_A, p_B)\beta(F)\mu(F)}{\sum_F \beta(F)\mu(F)} - p_L\,. \tag{4.2}$$

A monolingual equilibrium will obtain if p_A or p_B is equal to one. In view of assumption 3.5, such an equilibrium exists:

$$f_A(1,0) = f_B(0,1) = 0\,. \tag{4.3}$$

We will here investigate its stability. Denoting derivatives with respect to p_A and p_B with subscripts A and B, respectively, we have:

$$\begin{aligned}
f_{AA}(1,0) &= 2\frac{\beta(AC)}{\beta(AA)}\big(1-\alpha_A(AC;1,0)\big) - 1 \\
f_{BB}(1,0) &= 2\frac{\beta(AB)}{\beta(AA)}\alpha_B(AB;1,0) - 1 \\
f_{AB}(1,0) &= 2\frac{\beta(AC)}{\beta(AA)}\big(1-\alpha_A(AC;1,0)\big) - 2\frac{\beta(AB)}{\beta(AA)}\big(1-\alpha_A(AB;1,0)\big) + \frac{\partial\alpha_A(AA;1,0)}{\partial p_B} \\
f_{BA}(1,0) &= 0
\end{aligned}\,. \tag{4.4}$$

The eigenvalues of the matrix f_{LL} are non-negative (and, hence, the monolingual A-equilibrium, denoted by (A), is stable) if, and only if, the elements on the diagonal are non-positive. We then have the following proposition:

Proposition 4.1: A stable monolingual equilibrium (A) exists if, and only if, both of the following conditions hold:

$$\begin{aligned}
\alpha_A(AC;1,0) &\geq 1 - \frac{1}{2}\frac{\beta(AA)}{\beta(AC)} \\
\alpha_B(AB;1,0) &\leq \frac{1}{2}\frac{\beta(AA)}{\beta(AB)}
\end{aligned}\,. \tag{4.5}$$

Proof: See above.

The first of the two conditions is the more interesting one. It says that a second language cannot establish itself in a monolingual society if a sufficient proportion of the children of a monoglot and a bilingual (mutant) parent become monoglot. In reverse, a second language can enter into the society and survive without having a critical mass at the start if in families with a (mutant) bilingual parent the proportion of bilingual children is sufficiently high, *i. e.*, greater than one half if $\beta(AA) = \beta(AC)$. Since the practical value of the second language, of course, is zero in a monolingual society, this condition can only be met, if the cultural and emotional attachment of the bilingual parent to the second language is strong enough, if its status is high enough, and/or if the birth rate in the *AC* family is high enough in comparison to the *AA* family.

The second condition of proposition 4.1 is less interesting and would be met with any reasonable assumptions on parental behavior.

5. Bilingual equilibria

In the previous section, we found the general sufficient and necessary condition for the existence of monolingual equilibria. In this section, we will seek conditions for the existence of equilibria of the types (*AB*), (*AC*), (*BC*), (*C*), and (*ABC*). In order to facilitate the analysis, the following assumption is made:

Assumption 5.1: $\beta(F) = 1$ for all $F \in \{AA, AB, AC, BB, BC, CC\}$.

The rate of change in the size of language group *L* is then

$$\dot{p}_L = \sum_F \alpha_L(F; p_A, p_B)\mu(F) - p_L \ . \tag{5.1}$$

We obtain for language group *A*:

$$\begin{aligned}\dot{p}_A = \frac{1}{\nu}\Big[& p_A^2\alpha_A(AA; p_A, p_B) + 2p_A p_B\alpha_A(AB; p_A, p_B) + \\ & 2p_A p_C\alpha_A(AC; p_A, p_B) + p_B^2\alpha_A(BB; p_A, p_B) + \\ & 2p_B p_C\alpha_A(BC; p_A, p_B) + p_C^2\alpha_A(CC; p_A, p_B)\Big] - p_A.\end{aligned} \tag{5.2}$$

Corresponding expressions obtain for language groups *B* and *C*.

Noting, again, the fact that

$$\sum_L p_L = 1, \tag{5.3}$$

we can analyze the dynamics of the p_L's in a two-dimensional (p_A-p_B)-phase diagram. We first find the dynamics of *A*. Expression (5.2) can, in view of assumption 3.3 be rewritten as

$$\begin{aligned}\dot{p}_A = p_C^2\alpha_A(CC; p_A, p_B) + p_A\big[p_A\alpha_A(AA; p_A, p_B) + \\ 2p_B\alpha_A(AB; p_A, p_B) + 2p_C\alpha_A(AC; p_A, p_B) - 1\big],\end{aligned} \tag{5.4}$$

or

$$\begin{aligned}\dot{p}_A = p_C^2\alpha_A(CC; p_A, p_B) - p_A\big[p_A\big(1-\alpha_A(AA; p_A, p_B)\big) + \\ p_B\big(1-2\alpha_A(AB; p_A, p_B)\big) + p_C\big(1-2\alpha_A(AC; p_A, p_B)\big)\big],\end{aligned} \tag{5.5}$$

where $p_C = 1 - p_A - p_B$.

If $p_A = 0$, this reduces to

$$\dot{p}_A = p_C^2\alpha_A(CC; 0, p_B) \tag{5.6}$$

if $p_B = 0$, in view of assumption 3.2, to

$$\dot{p}_A = p_C\big[\alpha_A(CC; p_A, 0) - p_A\big(1+\alpha_A(CC; p_A, 0) - 2\alpha_A(AC; p_A, 0)\big)\big] \tag{5.7}$$

and if $p_C = 0$, to

$$\dot{p}_A = -p_A\big[p_A\big(1-\alpha_A(AA; p_A, p_B)\big) + p_B\big(1-2\alpha_A(AB; p_A, p_B)\big)\big] \tag{5.8}$$

We can characterize the regions in the phase diagram where p_A (respectively p_B) is increasing, constant, or decreasing. We state the following lemmata:

Lemma 5.1: Under assumptions 3.3 and 3.4, $\dot{p}_A$ is a decreasing function of p_B.

Proof: Differentiate 5.4.

A consequence of this is that the values of p_B for which p_A is stationary ($\dot{p}_A = 0$) can be seen as a single valued function of p_A. In the phase diagram we have a curve that „doesn't bend over backwards".

Lemma 5.2: If $p_C = 0$ and if $\alpha_A(AC; 1, 0) < ½$, under assumptions 3.2, 3.3, and 3.4, $\dot{p}_A = 0$ for $p_A = 0$ or 1 and $\dot{p}_A < 0$ for $0 < p_A < 1$.

Proof: If $p_A = 0$ or 1, the result follows directly from (5.8) and assumption 3.2. The second part also follows from (5.8), the fact that both p_A and p_B are positive, and assumption 3.4.

One implication of this lemma is that a dynamically stable regime (AB) is impossible if $\alpha_A(AC; 1, 0) < ½$.

Lemma 5.3: If $\alpha_A(AC; 1, 0) < ½$ and if $\alpha_A(CC, 0, 0) > 0$, under assumptions 3.2 and 3.3 the following implications obtain:

There exists a positive number $p_A{}^{\circ+} < 1$ and a positive number $p_A{}^{\circ-} < 1$, such that $p_A{}^{\circ+}$ is the biggest and $p_A{}^{\circ-}$ the smallest $p_A{}^{\circ}$ solving the equation

$$p_A{}^{\circ} = \frac{\alpha_A(CC, p_A{}^{\circ}, 0)}{1 + \alpha_A(CC, p_A{}^{\circ}, 0) - 2\alpha_A(AC, p_A{}^{\circ}, 0)}. \tag{5.9}$$

Furthermore, if $p_B = 0$, then $\dot{p}_A = 0$, if $p_A = p_A{}^{\circ+}$ or $p_A{}^{\circ-}$, $\dot{p}_A < 0$, if $p_A{}^{\circ+} < p_A < 1$, and $\dot{p}_A > 0$, if $0 < p_A < p_A{}^{\circ-}$.

Proof: The result follows from the condition that $\alpha_A(CC, 0, 0) > 0$, from (5.7) and continuity.

Remark: $p_A{}^{\circ+} = p_A{}^{\circ-} = p_A{}^{\circ}$ if $\alpha_A(AC; p_A, 0)$ is a concave function in p_A and if

$$\frac{\alpha_A(CC, p_A, 0)}{p_A} > 2\frac{\partial \alpha_A(AC, p_A, 0)}{\partial p_A}. \tag{5.10}$$

Proof: Evaluating the derivative with respect to p_A of the expression in the square brackets in (5.7) where this expression equals zero, we readily find that this derivative is negative under the conditions of the remark. Continuity then implies that $p_A{}^{\circ}$ is unique.

Defining p_B* as min $\{p_B \mid \alpha_A(CC, 0, p_B) = 0\}$ or, if this set is empty, as 1, we note that if $\alpha_A(CC, 0, 0) > 0$, then $p_B* > 0$. Lemmata 5.1 to 5.3 allow us to characterize the dynamics of p_A in the phase diagram in the case that $\alpha_A(AC; 1, 0) < ½$ and $\alpha_A(CC, 0, 0) > 0$. In the diagram we have assumed that $p_A{}^{\circ}$ is unique. A similar diagram obtains for p_B. We can now prove the following propositions characterizing situations where different dynamically stable equilibria can exist. We only show the existence of various equilibrium constellations under the most interesting sufficient conditions and do not attempt to analyze all possible cases.

Proposition 5.1: Under assumptions 3.2, 3.3 and 3.4 if $\alpha_A(AC; 1, 0) < ½$, if $\alpha_A(CC, 0, 0) > 0$, if $\alpha_B(BC; 0, 1) < ½$, and if $\alpha_B(CC, 0, 0) > 0$, the following implications obtain:

1. If $p_B{}^{\circ-} > p_B*$, there exists a dynamically stable equilibrium constellation (BC) with $p_B = p_B{}^{\circ-} > 0$ and $p_C = 1 - p_B{}^{\circ-} > 0$.
2. *Mutatis mutandis* there exists a dynamically stable equilibrium constellation (AC).
3. If $p_A{}^{\circ-} < p_A*$ and if $p_B{}^{\circ-} < p_B*$, there exists a dynamically stable equilibrium constellation (ABC) with p_A , p_B and p_C >0.

Proof: Follows directly from the phase diagram.

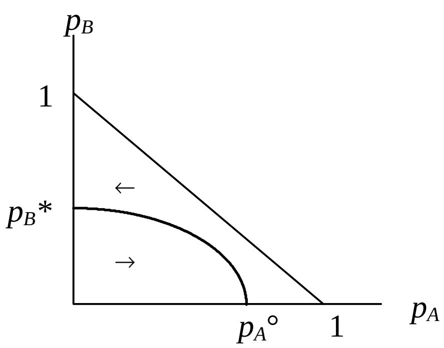

Figure 1: The dynamics of p_A.

Proposition 5.2: If $\alpha_A(AC; p_A, 0) > ½$ for $1 \geq p_A \geq 1 - \varepsilon$, and $p_A^* \leq 1 - \varepsilon$, where ε is some arbitrary positive constant, there exists a dynamically stable monolingual equilibrium constellation (*A*) and *mutatis mutandis* (*B*).

Proof: From equation 5.7 it is seen that, in this case, if $p_B = 0$, $\dot{p}_A > 0$ for all $p_A < 1$ and that $\dot{p}_A = 0$ for $p_A = 1$. Hence, $p_A^{\circ-}$ equals 1, and implication 1 of Proposition 5.1 applies.

Proposition 5.2, of course, is the same as proposition 4.1. Only here it is shown in terms of the dynamics of the phase diagram.

Proposition 5.3: If $\alpha_A(CC, p_A, p_B) = \alpha_B(CC, p_A, p_B) = 0$, for $p_A, p_B \leq \varepsilon$, if $\alpha_A(AC; 1, 0) < ½$, and if $\alpha_B(BC; 0, 1) < ½$, where ε is some arbitrary positive constant, there exists a dynamically stable equilibrium constellation (*C*).

Proof: Follows directly from assumption 3.4, equation 5.5, and the corresponding expression for $\dot{p}_B$.

5. Concluding remarks

Although it is difficult to isolate the factors determining the size of the p°'s, for some reasonable assumptions on the dependence of the functions $\alpha_A(AC; p_A, 0)$ and $\alpha_B(BC; 0, p_B)$ on p_A and p_B, respectively, one can say that, *ceteris paribus*, p_A° is monotonely increasing in the strength of monoglot *A*-speaking children in families of type *AC*, and correspondingly for p_B°. By the same argument, the p°'s are related in the same way to the strength of monoglots in families of type *CC* through the functions $\alpha_A(CC; p_A, 0)$ and $\alpha_B(CC; 0, p_B)$. This can be interpreted in the manner that p_A° increases with the social status of language *A* (α_A shifts upwards) and decreases with the strength of the emotional and cultural attachment of the bilinguals to language *B* (α_C, respectively α_B and α_C shift upwards). The size of the p^*'s is related to the strength of monoglots in families of type *CC*. Here, however, p_A^* is positively related to the strength of monoglot *B*-speaking children in families of the type *CC* through the function $\alpha_B(CC; p_A, 0)$. Also here the interpretation can be that p_A^* increases with the social status of language *B* (α_B shifts upwards), decreases with the social status of language *A* (α_A shifts upwards), decreases with the strength of the cultural identity for the *A* culture among the bilinguals (α_A and α_C shift upwards), and increases with the strength of the cultural identity for the *B* culture among the bilinguals (α_B shifts upwards).

We have seen that with the exception of the constellation (*AB*) all possible equilibria can exist and be dynamically stable. The crucial factors are the form of the functions $\alpha_A(AC; p_A, p_B)$, $\alpha_B(BC; p_A, p_B)$, $\alpha_A(CC; p_A, p_B)$, and $\alpha_B(CC; p_A, p_B)$. Based on the discussion above, we can draw some general conclusions. If one of the first two is too big, for instance if the language *A* or *B* has a high status in society in comparison to the other language, we get a monolingual equilibrium. If the second two are equal to zero for small values of the *p*'s, that is, if the emotional attachment to both idioms is strong enough, we get an equilibrium where everyone is bilingual (*cf*. the germanophone part of Switzerland). If the first one is relatively big, but not too big and the forth one sufficiently small, that is, language *A* has a relatively high status and language *B* a relatively low one but at the same time a high emotional value for those who speak it, an equilibrium with monoglots in language *A* and bilinguals can obtain (*cf*. the situation in Wales). Finally, if the first two are sufficiently small and the second two large enough, neither language has high status relative to the other, and an equilibrium with both, partially overlapping, coexisting language communities can exist (*cf*. the situation in Brussels). Of course, under certain parameter values two or more equilibria could be possible, *e. g.* both (A) and (BC). Which equilibrium prevails in these cases, depends on the initial values of the p's. A critical mass of A speakers would be necessary for (A) to be realized and a critical mass of B speakers or bilinguals would be required for the system to move to (BC).

Returning to the discussion in the first paragraph of this section, we find that an increase in the social status of language A, *ceteris paribus*, will increase $p_A°$ and p_B* and decrease p_A*. Similarly, an increase in the cultural identity of B speakers will decrease $p_A°$ and p_B* and increase p_A*. That is, the status of language A and the cultural identity of the B speakers tend to have contrary influences. If we stars out from a low value of the social status of language A and let the status increase, the equilibrium constellations could change from (BC) ($p_B° > p_B*$ and $p_A° < p_A*$) over (ABC) ($p_B° < p_B*$ and $p_A° < p_A*$) or {(BC), (AC)} ($p_B° > p_B*$ and $p_A° > p_A*$) to (AC) ($p_B° < p_B*$ and $p_A° > p_A*$) and (A) ($p_A° = 1$). An increase in the cultural identity of the B speakers will take us in the opposite direction.

It is worth noting that it is the behavior of and cultural transmission through the bilingual parent(s) (C) that is crucial for the survival of bilingualism, be it in the form of a bilingual community coexisting with only one or with two monoglot groups.

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Appendix: Utility maximization

We assume that the behavior of each family can be described by the following utility function:

$$u(L; F; I; p_A, p_B) = w(L; F; i) + \lambda(L; p_A, p_B), \tag{A.1}$$

where

$$\lambda(L) = \begin{cases} g(1-p_B) & \text{for } L = A \\ g(1-p_A) & \text{for } L = B \\ g(1) & \text{for } L = C \end{cases} \tag{A.2}$$

and g is some monotonely increasing function. That is, λ is increasing in the fraction of the population one can interact with for a given choice of languages. This is the practical advantage of the language. The function $w(L; F; i)$ describes the subjective utility of the family of various language choices of the children. The latter also differs for different families through the parameter i. It is assumed that the characteristic i is randomly distributed among the families with a uniform distribution and that the distributions in different cohorts are stochastically independent. That is, the preference structure is stable and stationary over the various cohorts.

The family now chooses L to maximize u. The solution gives us the „demand“ functions

$$L = \hat{L}(F; p_A, p_B; i) \tag{A.3}$$

We are looking for the distribution of the language groups in the new generation, though. *I. e.*, we need to aggregate over i and F.

For the solution of this problem, a transition matrix from the various family types to the language choice of the children is formed. This matrix depends on the external opportunity costs (the practical disadvantages) of being monoglot in comparison to being bilingual and on the subjective utility of language choice, expressed in the function $w(L; F; i)$.

Let ξ be the external opportunity cost of being monoglot in A and χ that of being monoglot in B as compared to being bilingual, belonging to C. We can then find the distribution of the children on language choices:

$$\alpha_A(F;\chi,\xi) := \#\left\{ i \,\middle|\, \begin{matrix} [w(A;F;i) - w(C;F;i)] - \xi > 0 \;\wedge \\ [w(A;F;i) - w(C;F;i)] - \xi > [w(B;F;i) - w(C;F;i)] - \chi \end{matrix} \right\}, \tag{A.4}$$

$$\alpha_B(F;\chi,\xi) := \#\left\{ i \,\middle|\, \begin{matrix} [w(B;F;i) - w(C;F;i)] - \chi > 0 \;\wedge \\ [w(B;F;i) - w(C;F;i)] - \chi \geq [w(A;F;i) - w(C;F;i)] - \xi \end{matrix} \right\}, \tag{A.5}$$

and

$$\alpha_C(F;\chi,\xi) := \#\left\{ i \,\middle|\, \begin{matrix} [w(A;F;i) - w(C;F;i)] - \xi \leq 0 \;\wedge \\ [w(B;F;i) - w(C;F;i)] - \chi \leq 0 \end{matrix} \right\}, \tag{A.6}$$

where # denotes the relative measure.

That is, $\alpha_L(F; \chi, \xi)$ is the fraction of families of type F that would choose language group L for their children if the opportunity costs of choosing A in comparison to C are ξ and the opportunity costs of choosing B in comparison to C are χ. From the definition of the relative measure # it follows that

$$\sum_L \alpha_L(F;\chi,\xi) = 1\,. \tag{A.7}$$

It is also readily seen that α_A is non-increasing in ξ and non-decreasing in χ, α_B the opposite, and α_C non-decreasing in both ξ and χ.

From (A2) the opportunity costs are given by

$$\begin{aligned} \xi &= g(1) - g(1-p_B) \\ \chi &= g(1) - g(1-p_A) \end{aligned}\,. \tag{A.8}$$

It is easily seen that ξ and χ are monotonely increasing in p_B and p_A, respectively. Substitution into the α_L and a slight redefinition give us the functions of the main text and the properties of their dependency on the p_L's postulated there.