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THE VALUE OF GENETIC INFORMATION IN THE LIFE INSURANCE MARKET

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Abstract

This paper analyzes the effects of additional information in a life insurance market under adverse selection. It is shown that individuals have an incentive to acquire information about their risk type if their informational status cannot be observed by insurers. In aggregate, the existence of a testing opportunity has an effect on the equilibrium premium. We describe the conditions under which, from an ex ante standpoint, all individuals gain, all lose or in which some gain and some lose from the existence of the test.

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1 Introduction

The purpose of this paper is to determine whether information such as that obtained from genetic screening has positive or negative value in a life insurance market which displays adverse selection. In many countries existing or proposed legislation prohibits the use of genetic tests for ratemaking purposes and so assessing the impact of such information in the context of a model of adverse selection in which information about risk type is both private and increasing is an important exercise. Also, other types of health tests generate what is effectively private information for insureds.

The question of the private and social value of additional information in insurance markets has been analyzed by Doherty and Thistle (1996) in the context of the usual insurance model of adverse selection. As we do, they analyze the case where insurers cannot observe whether consumers have obtained a test. They show that, in this context, acquiring information usually has a positive private value since by taking the test the market possibilities (especially the price of insurance) do not change for the individual but better information allows consumers to make a more informed choice; that is, to adjust the amount of insurance they buy to what is optimal for their risk type. However, the social value of the testing opportunity is negative. If there were no asymmetric information before the test, all individuals would insure for a medium premium. If insurers could observe test results then, depending on the outcome of the test, some (those with good news) could buy insurance cheaper and for those with bad news, the premium would increase. By the martingale property of conditional expectation, the *expected* premium would be the same as the medium premium before, but since the premium is a random variable and individuals are risk averse, all are worse off from an ex ante perspective. Asymmetric information after the test aggravates this problem, since low risks (those with good news) cannot buy full insurance for the low premium but must signal their type by purchasing only partial coverage.

Our model departs from Doherty and Thistle's in that we analyze the same problem in a life insurance model. The fundamental difference between life insurance and other insurance policies is, from an institutional point of view, that individuals can buy life insurance from as many companies as they want and therefore price–quantity contracts are not a feasible means against adverse selection; insurance companies can only quote a uniform price for all life insurance contracts.[1] A second important difference between life insurance and other insurance is that there is no *natural* choice for the size of loss.

[1] Insurance companies also don't generally share information about the amount of insurance purchased by their customers. For a discussion about this possibility, see Hellwig (1988).

In most models of insurance there is a fixed insurable loss l and this loss is independent of its probability of occurring. Thus, a risk averse individual when faced with an actuarially fair premium will purchase full coverage insurance regardless of her probability of loss. Increasing symmetric information about risk type will therefore not have any potential for increased consumption efficiency in terms of the amount of insurance desired. In the context of life insurance, however, this is not the case as a change in the probability of death can, and as we show generally does, lead to changes in the amount of insurance demanded even when these probabilities are symmetrically observed. Thus, information of the type we study here has an added possible dimension for providing positive social value by allowing for better informed consumption choices.

Not only is life insurance (as well as long term disability insurance, for which the same is true) different from other types of insurance, our model is also very relevant for applications as the influence of genetic tests on life insurance markets will undoubtedly grow in the coming years. Even if the target date of 2005 for completion of the *Human Genome Project*, which is a massive international effort to map and sequence the entire human genome, is not met, recent success in the discovery of disease genes and development of associated genetic screening tests suggests that this technology will become much more frequently used and at much lower cost. The fact that many countries ban the use of information from genetic screening tests for ratemaking purposes[2] makes the assumption of private information an appropriate one for this application and the fact that only a small proportion of known genetic diseases are amenable to effective medical treatments makes the use of a pure adverse selection model applicable.[3]

Consistent with Doherty and Thistle, we find that the private value of becoming informed is positive in our model. More surprisingly, the social (ex ante) value of information may be either negative (as in the normal insurance model) or positive and we describe the scenarios and construct examples for both possible sets of cases. The intuition why additional information might lead to a Pareto improvement is as follows. Suppose there are three groups, high risks, low risks and initially uninformed individuals who may be high or low risks. Suppose that in the reference situation only high risks buy life

[2]The insurance industries in France, the Netherlands and the U.K. have established a moratorium on using results from genetic screening tests for establishing insurance prices. Several other countries, including Austria, Belgium and Norway, either have regulations on the use of genetic information in place or are in the process of developing such policies. For a discussion of legal restrictions on the use of genetic information, see Lemmens and Hahamin (1996).

[3]For elaboration on these points see Holtzman, et al. (1997) and Rowen, et al. (1997).

insurance (this is possible since the premium might be unacceptably high for low risks and uninformed individuals); hence the premium will be based on the high risks' probability of death. Now suppose a test for uninformed consumers becomes available. If consumers were not to adapt their insurance purchases to their new information, their ex ante expected utility would be unchanged. However, although uninformed consumers who test negative (i.e. learn that they are low risks) will still not buy life insurance, those testing positive (the high risks) will. Therefore, the ex ante expected utility for uninformed consumers is increased by the new testing opportunity.

Besides there being a possible positive value of insurance for those who take the test, there is also the possibility of positive price spillover effects for those who don't take the test. To see how this can occur, suppose there is a range of risk types so that an uninformed individual who takes a test may be determined to be one of a number of higher or lower risk types. Suppose such a person initially does not buy insurance and upon testing discovers she is of a high risk type but that she carries a risk level less that the average of those who initially purchase insurance. If she now purchases insurance the result will be a lower equilibrium price of insurance. Negative spillover effects from increased information, which are generally stressed for standard insurance models such as that of Doherty and Thistle (1996), are also possible in our model. Such a situation arises in our model when the initially uninformed individual discovers herself to be a risk type higher than the average risk of those originally purchasing insurance.

The model for the life insurance market we use is similar to that developed in Villeneuve (1996) to explain the effect of adverse selection on the markets for life insurance and annuities. Our model is developed in such a way as to stress the life insurance purchasing decision of individuals for the purpose of replacing lost income due to *premature* death.

We proceed as follows. First, we present a simple model of life insurance demand and some results concerning the comparative statics of this model. We then show that if insurers cannot observe whether an individual took a test, then individuals have an incentive to do so (i.e., acquire costless information) in this framework. Finally, we analyze for different test scenarios the ex ante welfare implications of the testing opportunity, taking account of the fact that the equilibrium premium for life insurance will change if individuals can gather information about their risk type before buying life insurance. Depending on the scenario, it is possible i) that all individuals in this market lose (in comparison to the situation where the test is not available), ii) that some gain and some lose or iii) that all individuals gain.

2 The Model

Each individual faces the possibility of death, with probability denoted p. If no insurance is purchased then if this person dies (the death state) the surviving family members earn income from assets in amount K while if the person lives (the life state) income is made up of earned income Y as well as income from assets K. The individual may purchase insurance in amount L at price λ and so transform income in the state in which he lives to $K + Y - \lambda L$ and income in the state in which he dies to $K + (1 - \lambda)L$. Letting $u(.)$ represent utility in the life state and $v(.)$ utility for surviving members in the death state, then expected utility as a function of insurance purchases is[4]

$$EU(L) = (1 - p)u(K + Y - \lambda L) + pv(K + (1 - \lambda)L) \tag{1}$$

Only nonnegative amounts of insurance can be chosen, $L \geq 0$. Note that the life state and death state could also represent situations in which the individual is sufficiently healthy to earn income or not, respectively. Thus, the model can be used to understand disability insurance as well.

Individuals have different probabilities of death and we assume that these are not observed by insurers. As in Villeneuve (1996), we assume that the insurers also do not observe total insurance purchases and so quantity rationing as a self-selection device is not relevant; a realistic assumption for life insurance markets. Thus, insurers sell whatever quantity is demanded at a single price. The industry is assumed to be perfectly competitive and the only costs are from paying out claims. Thus, insurance is offered at the pooled actuarially fair rate. As in Villeneuve (1996), it will be shown here that the market will partition into individuals who do not participate and individuals who do participate. If more than one risk type participates in the market it follows that at any given price, demand for insurance is higher the greater is an individual's probability of death and so the market price is always greater than the actuarially fair rate for the participant with the lowest probability of death. It is also possible that only the highest risk type participates in the market.

All of our results can be derived in a simple framework with four possible types indicated by their loss probabilities $0 < p_L < p_M < p_H < p_{HH} < 1$, and often we only need three types. One can think of the L-types as the low risk class, the M-types as the medium risk class, the H-types as the high risk class

[4]One motivation for this form could be that the household decides on the level of insurance to purchase for either of one or two income earners and so $v(.)$ is utility of the surviving household members.

and the HH-types as the very high risk class. Note that individuals who are initially "uninformed" and for whom a test is available must belong to one of the middle risk types (either p_M or p_H) before the test; since every nontrivial test will convey positive or negative information, they cannot belong to one of the most extreme types before the test.

An individual with probability of death p and facing price of insurance λ solves the following problem:

$$\max_L EU(L) \tag{2}$$

subject to $L \geq 0$, with first-order-condition

$$-\left\{(1-p)\lambda u'(K+Y-\lambda L) - p(1-\lambda)v'(K+(1-\lambda)L)\right\} \leq 0 \tag{3}$$

$$L \geq 0, L\left\{.\right\} = 0$$

and second-order-condition

$$(1-p)\lambda^2 u''(.) + p(1-\lambda)^2 v''(.) < 0 \tag{4}$$

which is always satisfied. Let $L(p,\lambda)$ denote the optimal insurance demand as a function of p and λ; sometimes we also use L_t^* or L_t to denote the optimal insurance demand of type t.

Assumption 1 *We assume that at zero insurance purchases the marginal utility of income is higher in the death state than in the life state (i.e., $u'(K+Y) < v'(K)$).*

Applying Assumption 1 to the first order condition (3) we can conclude that an individual will purchase insurance if the price of insurance is less or equal to the actuarially fair price. We state this as

Lemma 1 *If $\lambda \leq p_t$ then an individual of risk type t will participate in the market (i.e., $L(p_t,\lambda) > 0$).*

One implication of Lemma 1 is that the highest risk class always purchases a positive amount of insurance.

Lemma 2 below indicates the condition under which an individual of risk type p_t will not participate in the market. This result also follows directly from the individual's first-order condition.

Lemma 2 *An individual of risk type t will not participate in the market if and only if*

$$\frac{p_t}{1-p_t} \leq \frac{\lambda}{1-\lambda}\frac{u'(y+K)}{v'(K)}$$

As a direct result of Lemma 2 we can see that there will be a critical value of p, call it p_c, such that the market will segment into those individuals who do purchase insurance ($p_t > p_c$) and those who do not ($p_t \leq p_c$).[5] Note that the set of individuals who purchase insurance is never empty but the set of individuals who do not purchase insurance may be empty and so all individuals participating in the market is a possibility. We present this as Lemma 3.

Lemma 3 *There exists a value p_c such that $L(p, \lambda) = 0$ if $p \leq p_c$ and $L(p, \lambda) > 0$ if $p > p_c$.*

Lemma 4 below indicates how demand for insurance varies for individuals as the risk of death, p, varies, given a fixed price of insurance λ. The higher is the risk of death for an individual, the higher is his demand for life insurance:

Lemma 4 *For market participants (i.e., those for whom $p > p_c$),*

$$\frac{\partial L(p,\lambda)}{\partial p} = -\frac{\lambda u' + (1-\lambda)v'}{(1-p)\lambda^2 u'' + p(1-\lambda)^2 v''} > 0. \tag{5}$$

This follows from applying the implicit function theorem to the first order condition (3).

Now that we have explored the individual's behavior for exogenous p and λ, we show that individuals have an incentive to become informed by a costless test if this does not change their market opportunities; this will be the case if insurers cannot use test results for pricing either for legal reasons, as described in the introduction, or simply because they cannot observe that an individual took a test.

For concreteness, we analyze a test for H-types which renders them HH-types with probability ρ and M-types with probability $1-\rho$, but it should become clear that the result that there is a positive value of information for the individual holds more generally. Consistency (i.e., the martingale property of conditional expectation) requires

$$p_H = (1-\rho)p_M + \rho p_{HH}. \tag{6}$$

[5] A similar result can be found in Villeneuve (1996).

In the absence of the test the H-types obtain expected utility $EU_H^* = EU_H(L_H^*)$ where

$$EU_H^* = (1-p_H)u(Y+K-\lambda L_H^*) + p_H v(K+(1-\lambda)L_H^*). \tag{7}$$

Substituting for p_H from equation (6) above we can write EU_H^* as

$$EU_H^* = (1-\rho)EU_M(L_H^*) + \rho EU_{HH}(L_H^*) \tag{8}$$

If an H-type takes the test and discover he is type M, then he will choose a level of insurance more appropriate to type M (denoted L_M^*), while if he discovers he is type HH, he will choose a level of insurance more appropriate for that type (L_{HH}^*). Thus, since $EU_M(L_M^*) \geq EU_M(L_H^*)$ and $EU_{HH}(L_{HH}^*) > EU_{HH}(L_H^*)$ by the definition of an optimal insurance demand,[6] expected utility of taking the test, conditional on the probabilities held ex ante to taking the test, is

$$EU_H^{**} = (1-\rho)EU_M(L_M^*) + \rho EU_{HH}(L_{HH}^*) > EU_H^* \tag{9}$$

We have thus established our first major result which is parallel to a result of Doherty and Thistle (1996) in their model.

Proposition 1 *If information is private, an individual will always choose to have a costless test.*

The result follows from the fact that with no price effects (i.e., if there is no premium risk) knowing one's risk type allows one to make a more informed choice about the optimal level of insurance coverage. Each individual can consider the price of insurance as fixed with respect to her own decision whether to become informed or not. However, if all individuals of type H take the test then there will be market price effects as changes in demand by different risk types will lead to changes in the average cost of providing insurance. We address this issue next.

Up to now, we treated λ as exogenous. In the equilibrium, λ will be determined by the following equation (10) which indicates that the insurance industry must make zero profits:

$$\Pi(\lambda) = \sum_{t \in T}(\lambda - p_t)q_t L(p_t, \lambda) = 0. \tag{10}$$

Here, q_t is the proportion of individuals who belong to type t and $T = \{L, M, H, HH\}$ is the set of all types. If the premium λ satisfies (10), insurers

[6] Since HH types necessarily buy insurance, we have from Lemma 4 that $L_{HH}^* > L_H^*$ and consequently $EU_{HH}(L_{HH}^*) > EU_{HH}(L_H^*)$.

break even on average. Existence of a solution which lies between p_L and p_{HH} is guaranteed since $\Pi(\lambda)$ is continuous in λ and $\Pi(p_L) < 0$ and $\Pi(p_{HH}) \geq 0$. Note that it is not guaranteed that there is a unique value of λ which satisfies (10). If there exist multiple solutions of (10), we assume that the equilibrium premium is given by the smallest solution, $\underline{\lambda}$. The reason for this is as follows. Suppose we were in an equilibrium with a higher value of λ; then an insurer could deviate and charge a premium of $\underline{\lambda} + \epsilon$ (for ϵ small enough but positive) and make a profit;[7] hence there cannot be an equilibrium corresponding to a higher value of λ than $\underline{\lambda}$.

Sometimes it is useful to write (10) in the following form:

$$\lambda = \frac{\sum_{t \in T} p_t q_t L(p_t, \lambda)}{\sum_{t \in T} q_t L(p_t, \lambda)}. \tag{11}$$

λ must be equal to the weighted average risk of insurance buyers, where the weights are the respective insurance demands. The right hand side of (11) is also referred to as "average clientele risk". Note that by Lemma 4, higher risks buy more insurance. On the right hand side of (11), higher risks have a higher weight in the calculation of the average life insurance buyer's risk than what corresponds to their proportion in the population, and low risks have a lower weight than their proportion in the population. Hence, life insurance buyers' average risk (the average taken with respect to the weights given by demands) is greater than the average risk of the population (here the average taken with respect to the proportions of types in the population), even when all risk types participate in the market.

We now turn to our first result on the ex ante welfare effects of the testing possibility when we take account of the fact that the possibility to test changes the equilibrium premium. This result pertains to a scenario in which there is a test for a group which ex ante would not buy insurance. Clearly, this group must be (at least weakly) better off by the testing opportunity because it is still feasible for them not to buy insurance after any test result; however, in some cases it might be optimal to buy insurance after the test, and then the group for which the test is available is better off from an ex ante point of view. The effect for previous insurance buyers may be positive or negative: If those individuals who test positive and buy life insurance ex post have a higher probability of death than the average ex ante insurance buyer, the premium goes up and ex ante insurance buyers are worse off. However, it is also possible that those individuals who test positive are better risk types than average ex ante insurance buyers; then, the premium goes down and ex ante insurance buyers are better off. Let us state this formally.

[7] At $\underline{\lambda}$, $\Pi(\lambda)$ must cross the abscissa from below, otherwise $\underline{\lambda}$ could not be the lowest zero, taking account of the fact that $\Pi(p_L) < 0$. Hence $\Pi(\underline{\lambda} + \epsilon) > 0$.

Proposition 2 *Suppose without the test we have $L_{HH} > L_H > 0 = L_M$: type H and type HH individuals buy positive life insurance, but that type M individuals do not buy insurance without the test. Consider the following tests for M individuals:*

1. *Test 1 which renders individuals HH-types with probability ρ_1 and L-types with probability $(1 - \rho_1)$.*

2. *Test 2 which renders individuals H-types with probability ρ_2 and L-types with probability $(1 - \rho_2)$.*

M-types are better off with the test (for both tests) than without the test. If the test is of type Test 1, former H and HH-types are worse off (compared to the situation without the testing opportunity for M types). If the test is of type Test 2, former H and HH-types are better off.

Proof: That M-types are better off with the test is obvious, as argued above. We prove that under a test of type Test 2, λ decreases.

Denote the proportions of types before the test as q_t^0 and after the test as q_t^1. Given the testing scenario 2, we have $q_L^1 = q_L^0 + (1 - \rho_2)q_M^0$, $q_M^1 = 0$, $q_H^1 = q_H^0 + \rho_2 q_M^0$ and $q_{HH}^1 = q_{HH}^0$. Before testing, the profit function is

$$\Pi^0(\lambda) = (\lambda - p_H)q_H^0 L(p_H, \lambda) + (\lambda - p_{HH})q_{HH}^0 L(p_{HH}, \lambda) \tag{12}$$

and after testing, the profit function is

$$\Pi^1(\lambda) = \Pi^0(\lambda) + (\lambda - p_H)\rho_2 q_M^0 L(p_H, \lambda) \tag{13}$$

Evaluating $\Pi^1(\lambda)$ at λ^0, the lowest zero of $\Pi^0(\lambda)$, gives

$$\Pi^1(\lambda^0) = (\lambda^0 - p_H)\rho_2 q_M^0 L(p_H, \lambda^0) \tag{14}$$

which is positive since $p_H < \lambda^0 < p_{HH}$. Hence the lowest zero of Π^1, λ^1, must be to the left of λ^0; i.e., $\lambda^1 < \lambda^0$. The proof for case 1 is analogous and omitted. •

Examples 1 (see below) and A1 (see Appendix 1) illustrate the effects of both types of tests. In all of the examples we adopt the utility functions $u(x) = x^{1/2}$ and $v(x) = (ax)^{1/2}$, where a is a parameter with $0 < a < 1$. Using this pair of utility functions ensures that individuals faced with an actuarially fair price of insurance will purchase some positive amount of insurance and this amount will be such that income in the death state for surviving members of the family will be less than that in the life state. Such a formulation is consistent with maintaining equivalent income for a smaller sized family. This is not, however, required for our results. For more details on the demand properties of this formulation see Appendix 1.

In Example 1 we illustrate the effect of a Test 1 type in Proposition 2.[8] Individuals of risk type M are determined to be either type L or type HH, depending on whether the test results are negative or positive for the genetic disease. Since M-types don't purchase insurance ex ante to the test, and so a fortiori neither do L types, it follows that the impact of the test is to increase the proportion of individuals of type HH (as opposed to type H) who do purchase insurance. Thus, testing in this case leads to a higher price of insurance, from $\lambda = 0.06465$ to $\lambda = 0.06618$. From this perspective one might think that the adverse selection problem in this market is worsened. The amount of insurance purchased by each of types H and HH is indeed less after the implications of the increase of private information have worked their way through the market price. On the other hand, however, note that more individuals now participate in the market (i.e., those M-types who previously did not purchase insurance but having tested positive as HH-types now do purchase insurance). The result is that, for the overall population, per capita insurance demand has indeed increased. Thus, the market for life insurance may not shrink as a result of a *worsening* of adverse selection. However, those who purchased insurance previous to testing are made worse off as a result of testing while the M-types ex ante utility from testing is greater than if they do not test, and so there is a Pareto ambiguous result from testing.

Example 1 (Proposition 2, Test 1)
Basic Parameters:
$K = 100$, $Y = 100$, $a = 0.9$
Probabilities of death by type:
$p_L = 0.01$, $p_M = 0.02$, $p_H = 0.06$, $p_{HH} = 0.071$
Test probabilities of being assigned to HH/L type, respectively:
$\rho = 0.164$, $1 - \rho = 0.836$
If No Testing
Proportion of each risk type:
$q_L^0 = 0.2$, $q_M^0 = 0.4$, $q_H^0 = 0.3$, $q_{HH}^0 = 0.01$,
Insurance purchases: HH/H types purchase insurance at price $\lambda = 0.06465$
$L_{HH} = 119.3 \Rightarrow c_1 = 192.3$, $c_2 = 211.6$, $EU_{HH} = 8.620$
$L_H = 54.33 \Rightarrow c_1 = 196.5$, $c_2 = 150.8$, $EU_H = 8.754$
$L_M = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_M = 10.490$
$L_L = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_L = 10.956$

[8] In each example c_1 refers to consumption in the life state and c_2 refers to consumption in the death state.

With Testing

Proportion of each risk type:

$q_L^1 = 0.534$, $q_M^1 = 0$, $q_H^1 = 0.3$, $q_{HH}^1 = 0.166$

Insurance purchases: HH/H types purchase insurance at price $\lambda = 0.06618$

$L_{HH} = 109.0 \Rightarrow c_1 = 192.8$, $c_2 = 201.8$, $EU_{HH} = 8.557$

$L_H = 46.83 \Rightarrow c_1 = 196.9$, $c_2 = 142.6$, $EU_H = 8.726$

$L_M = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_M = 10.490$

$L_L = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_L = 10.956$

Ex ante expected utility for M types conditional on testing:

$EU_{M(test)} = (1-\rho)EU_{L(test)} + \rho EU_{HH(test)} = 14.056 > EU_{m(notest)} = 14.049$

Per capita demand for insurance:

28.23 with no testing, 32.14 with testing.

Using the same pre-test parameters as in Example 1 we can generate an example illustrating the implications of a Test 2 type in Proposition 2. This is done as Example A1 in Appendix 1.

Two remarks are in order concerning Proposition 2. First, we assumed in the proof of Proposition 2 that all M-types obtain the test (as this leads to an increase in their expected utility in our model), but even if only some M-types test, the result is unchanged. Second, the fact that there exist some types who do not participate in the insurance market is a feature which distinguishes our model of the life insurance market from the standard model of insurance. Hence, Proposition 2 cannot be replicated in the standard model of adverse selection in an insurance market (fixed loss, different probabilities of loss), since all types in these models buy at least some insurance. However, if we change the standard model a little in the direction of realism and assume that associated with all insurance policies is a fixed administrative cost (i.e., fixed with respect to the degree of coverage chosen), then it is easy to see that it is possible that low risk types do not find it worthwhile to incur the fixed costs and so choose not to insure. In this variant of the standard model, similar results as in Proposition 2 can be derived, so these results are not a peculiarity of our life insurance model.

Next we turn to the case in which life insurance buyers are a homogeneous group before they take the test. In this case we can show that the premium increase introduced by the test due to ex post adverse selection makes everybody worse off from an ex ante point of view. Before we can proceed, we need the following lemma:

Lemma 5 *Define $l(p) = L(p, \lambda = p)$. For those individuals who face a risk type specific actuarially fair price, a change in the probability of death has an* *ambiguous effect on the level of demand. That is,*

$$\frac{dl}{dp} = -\frac{(u'' - v'')l}{pu'' + (1-p)v''}$$

which can be greater, lower or equal to 0.

Proposition 3 *Assume that without a test all life insurance buyers are H-types ($L_H > 0$, $L_L = L_M = 0$ and $q^0_{HH} = 0$). Consider a test which renders H-types HH-types with probability ρ_3 and L-types (or M-types) with probability $(1 - \rho_3)$. All H-risks will take the test and they will be worse off from an ex ante perspective.*

Proof: Proposition 1 indicates that each H-risk will take the test. Consequently, λ will increase to p_{HH} after the test. We have to show that

$$\begin{aligned} EU(p_H, \lambda = p_H, l(p_H)) &> \rho EU(p_{HH}, \lambda = p_{HH}, l(p_{HH})) \\ &+ (1-\rho)EU(p_M, \lambda = p_{HH}, 0) \end{aligned} \tag{15}$$

Define

$$H(p) = (1-p)u(K + Y - pl(p)) + pv(K + (1-p)l(p)). \tag{16}$$

$H(p)$ is the value of expected utility which could be achieved if insurance were priced at risk-type specific fair prices.

Clearly

$$EU(p, \lambda, L(p, \lambda)) \leq H(p) \text{ for } \lambda \geq p \tag{17}$$

and given the setting of the proposition

$$EU(p_H, \lambda = p_H, l(p_H)) = H(p_H) \tag{18}$$

and

$$EU(p_{HH}, \lambda = p_{HH}, l(p_{HH})) = H(p_{HH}) \tag{19}$$

and

$$EU(p_M, \lambda = p_{HH}, 0) < H(p_M) \tag{20}$$

Since $p_H = \rho p_{HH} + (1-\rho)p_M$, it follows that for inequality (15) to hold it is sufficient to show that $H(p)$ is strictly concave.

$$\begin{aligned} H'(p) = -u(K + Y - pl(p)) &+ v(K + (1-p)l(p)) \\ &-(1-p)l(p)u'(K + Y - pl(p)) \\ &\quad -pl(p)v'(K + (1-p)l(p)) \end{aligned} \tag{21}$$

where the envelope theorem allows us to ignore $\frac{\partial H}{\partial l} \cdot \frac{\partial l}{\partial p}$. Differentiating equation (21) a second time yields

$$H''(p) = u'[l(p) + pl'(p)] + v'[-l(p) + (1-p)l'(p)] - [-l(p) + (1-p)l'(p)]u'$$
$$-v'[l(p) + pl'(p)] + (1-p)l(p)[l(p) + pl'(p)]u''$$
$$-pl(p)[-l + (1-p)l'(p)]v'' \tag{22}$$

Using the first-order condition, equation (3), we have for $\lambda = p$ that $v' = u'$ and so the first four terms above cancel out; hence

$$H''(p) = (1-p)l^2u'' + pl^2v'' + l \cdot l' \cdot p(1-p)[u'' - v''] \tag{23}$$

Using the expression from Lemma 5 for l', we have

$$H''(p) = (1-p)l^2u'' + pl^2v'' - p(1-p)l^2\frac{[u''-v'']^2}{pu'' + (1-p)v''}$$

$$= \frac{l^2}{pu'' + (1-p)v''}\{(1-p)u''(pu'' + (1-p)v'') + pv''(pu'' + (1-p)v'')$$
$$-p(1-p)(u''^2 - 2u''v'' + v''^2)\}$$
$$= \frac{l^2}{pu'' + (1-p)v''}\{(1-p)^2u''v'' + p^2u''v'' + 2p(1-p)u''v''\}$$
$$= \frac{l^2u''v''}{pu'' + (1-p)v''} < 0 \tag{24}$$

Hence, $H(p)$ is concave and that establishes Proposition 3. •

This proposition illustrates the impact of one of the sources of possible negative effects of information about risk type. In this scenario a group of individuals initially do not have differential information about their specific risk types and so face an actuarially fair pooled price of insurance. The introduction of information about their risk types induces those who discover they are worse risks than previously anticipated to purchase more insurance and those who discover they are better risks to purchase less than they would at the original price. The result is an increase in the price. If lower risk types choose to exit the market, then the highest risk types face a price which reflects their higher probability of death. New lower risk types are at least as badly off as they would be if they received insurance at their actuarially fair rate. From an ex ante perspective the result of the information is worse than an actuarially fair lottery over prices. Thus, since the outcome is at least as bad as simply facing premium risk, there is an ex ante worsening of welfare as a result of the increase in information. Examples 2 and A2 (see Appendix 1) illustrate both types of outcomes for Proposition 3.

Example 2 (Proposition 3, Case with $L_L = 0$)

Basic Parameters:

$K = 100$, $Y = 100$, $a = 0.9$

Probabilities of death by type:

$p_L = 0.01$, $p_H = 0.1$, $p_{HH} = 0.5$

Test probabilities of being assigned to HH/L type, respectively:

$\rho = 0.184$, $1 - \rho = 0.816$

If No Testing

Proportion of each risk type:

$q_L^0 = 0$, $q_H^0 = 1$, $q_{HH}^0 = 0$

Insurance purchases: H types purchase insurance at price $\lambda = 0.1$

$L_H = 80.8 \Rightarrow c_1 = 191.9$, $c_2 = 172.7$, $EU_H = 27.150$

With Testing

Proportion of each risk type:

$q_L^1 = 0.816$, $q_H^1 = 0$, $q_{HH}^1 = 0.184$

Insurance purchases: HH types purchase insurance at price $\lambda = 0.5$

$L_{HH} = 84.2 \Rightarrow c_1 = 157.9$, $c_2 = 142.1$, $EU_{HH} = 9.373$

$L_L = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_L = 30.956$

Ex ante expected utility for H types conditional on testing:

$EU_{H(test)} = (1 - \rho)EU_{L(test)} + \rho EU_{HH(test)} = 26.992 < EU_{H(notest)} = 27.150$

Average demand for insurance:

84.2 with no testing, 15.5 with testing.

Finally, let us explore the case in which life insurance buyers are ex ante heterogeneous (belong to different risk types) and in which the test pertains to a group which is among the ex ante life insurance buyers. Intuitively, if the group for which the test is available is sufficiently small relative to the pool of individuals purchasing insurance, then adverse effects of testing on the equilibrium price will not outweigh the positive effects of a more informed choice for this group, so they will be better off. Of course, all other life insurance buyers just suffer from a price increase with no offsetting benefit, so they are worse off; no Pareto comparison can be made in this case. On the contrary, if the group for which the test is available is sufficiently large, we are almost in the situation of Proposition 3 and hence also the group for which the test is available will suffer more from increased adverse selection than it benefits from making a more informed choice. Also in this case, all other life insurance buyers suffer from a price increase, so they are worse off, too; hence in this case there is a Pareto worsening.

Proposition 4 *Suppose that initially H and M types (and possibly L-types) buy life insurance and nobody is identified as type HH; then suppose there is a test:*

1. *for H-types, rendering them HH-types with probability ρ or M types with probability $1-\rho$.*

2. *for H-types, rendering them HH-types with probability ρ or L types with probability $1-\rho$.*

3. *for M-types, rendering them HH-types with probability ρ or L types with probability $1-\rho$.*

   In all cases, λ increases and those who buy life insurance and cannot take the test are worse off. Those who take the test may be either worse off or better off. If the group which takes the test is sufficiently small, then this group is better off with the testing opportunity.

Proof: We formally prove the proposition for test 1. Although we assume L types do not participate in the market, either before or after the test, if they did the proof would be similar. The other cases follow analogously. First we prove that if testing occurs and the price λ does not change then the average clientele risk (the right hand side of (11)) increases. It then follows from Lemma 1 of Milgrom and Roberts (1994) that the new equilibrium premium must be higher. As a result of the increase in the price, those who don't test are made worse off. If the group that tests is small, then the advantage to these individuals of knowing more accurately their risk type and so making better informed insurance purchasing decisions exceeds the loss from the higher price.

Since only H and M-types are assumed initially to buy insurance we have that the average risk without testing is:

$$\lambda^0 = \frac{p_H L_H q_H^o + p_M L_M q_M^o}{L_H q_H^o + L_M q_M^o} \tag{25}$$

After the test (assuming all H-types receive the test), but without adjusting the premium, the average risk of life insurance buyers becomes:

$$\lambda^1 = \frac{p_{HH} L_{HH} \rho q_H^o + p_M L_M (q_M^o + (1-\rho) q_H^o)}{L_{HH} \rho q_H^o + L_M (q_M^o + (1-\rho) q_H^o)} \tag{26}$$

Note in equation (26) that the proportion of M types and HH-types is written in terms of pre-test proportions using the results $q_{HH}^1 = \rho q_H^o$ and $q_M^1 =$

$q_M^o + (1-\rho)q_H^o$. Thus, since all superscripts on the q_t parameters are 0 we drop these for convenience. The difference is:

$$\lambda^1 - \lambda^0 = (N_1 - N_0) \div Den \tag{27}$$

where,

$$N_1 = [L_H q_H + L_M q_M]\,[p_{HH} L_{HH} \rho q_H + p_M L_M (q_M + (1-\rho)q_H)]$$

$$N_0 = [L_{HH} \rho q_H + L_M (q_M + (1-\rho)q_H)]\,[p_H L_H q_H + p_M L_M q_M]$$

$$Den = [L_{HH} \rho q_H + L_M (q_M + (1-\rho)q_H)]\,[L_H q_H + L_M q_M]$$

Since the denominator (Den) is positive, the sign of $\lambda^1 - \lambda^0$ is the same as the sign of $N_1 - N_0$. Factoring out the term L_{HH} in $N_1 - N_0$ we see its coefficient is

$$p q_H \{p_{HH}[L_M q_M + L_H q_H] - p_M L_M q_M - p_H L_H q_H\} > 0$$

Hence replacing L_{HH} by L_H, noting that $L_{HH} > L_H > L_M$ as a consequence of the result stated in equation (12), using $p_H = \rho p_{HH} + (1-\rho)p_M$, and factoring gives

$$\lambda^1 - \lambda^0 > q_H^2 \rho (1-\rho) L_H (L_H - L_M)(p_{HH} - p_M) \div Den > 0$$

Thus, the average risk of life insurance buyers increases for a constant premium, and it follows from Lemma 1 of Milgrom and Roberts (1994) that the equilibrium value of λ rises. The remaining parts of the proof are straightforward. (See Appendix A2 for details of the algebraic steps.) •

In Proposition 4 individuals who test positive always are informed that they are the highest risk type possible. The result is an increase in the price of insurance, regardless of whether those who discover they are a lower risk end up purchasing insurance or not. The intuition for this outcome is that from Lemma 4 we know that demand for insurance is positively related to the perceived risk level, p. Consistency requires that the weighted average of the two groups' ex post perceived risk levels be the same as that of the tested individuals from the ex ante perspective. Thus, the effect of the changes in demand for insurance is an increase in the price. Thus, the average clientele risk rises as a result of testing even if those perceived to be better risks as a result of testing continue to purchase insurance. Such a case is illustrated by Example 3 where we see the impact of the information is a higher price ($\lambda = 0.05135$ compared to $\lambda = 0.05107$ in the no testing scenario). Notice that the price after testing is less than the actuarially fair price for either M-types or HH-types. Despite the increase in the price, the improved decisions on how much insurance to purchase dominates the impact of the increased price and so H-types are made better off by the testing opportunity in this example.

Example 3 (Proposition 4, Case with all types purchasing insurance before and after testing)

Basic Parameters:

$K = 100$, $Y = 100$, $a = 0.7$

Probabilities of death by type:

$p_L = 0.05$, $p_M = 0.052$, $p_H = 0.055$, $p_{HH} = 0.06$

Test probabilities of being assigned to HH/M type, respectively:

$\rho = 0.375$, $1 - \rho = 0.625$

If No Testing

Proportion of each risk type:

$q_L^0 = 0.8$, $q_M^0 = 0.1$, $q_H^0 = 0.1$, $q_{HH}^0 = 0$,

Insurance purchases: all types purchase insurance at price $\lambda = 0.05107$

$L_H = 64.3 \Rightarrow c_1 = 196.71$, $c_2 = 161.03$, $EU_H = 8.381$

$L_M = 46.1 \Rightarrow c_1 = 197.64$, $c_2 = 143.70$, $EU_M = 8.492$

$L_L = 34.5 \Rightarrow c_1 = 198.23$, $c_2 = 132.7$, $EU_L = 8.577$

With Testing

Proportion of each risk type:

$q_L^1 = 0.8$, $q_M^1 = 0.162$, $q_H^1 = 0$, $q_{HH}^1 = 0.038$,

Insurance purchases: all types purchase insurance at price $\lambda = 0.05135$

$L_{HH} = 94.83 \Rightarrow c_1 = 195.13$, $c_2 = 189.96$, $EU_{HH} = 8.227$

$L_M = 44.42 \Rightarrow c_1 = 197.72$, $c_2 = 142.14$, $EU_M = 8.488$

$L_L = 32.95 \Rightarrow c_1 = 198.31$, $c_2 = 131.26$, $EU_L = 8.574$

Ex ante expected utility for H types conditional on testing:

$EU_{H(test)} = (1-\rho)EU_{M(test)} + \rho EU_{HH(test)} = 8.390 > EU_{H(notest)} = 8.381$

Average demand for insurance:

38.61 with no testing, 37.14 with testing.

The case when not all types purchase insurance after testing is illustrated by Example A3 (see Appendix 1) and in this case the H-types are not made better off by the testing opportunity once the price effects are taken into account.

Proposition 4 has particular relevance to the current situation regarding knowledge and practice in genetic testing. Many genetic tests are just now being developed and so the *highest* risk types are not yet identified in the population. When a test, such as the genetic tests for the breast cancer genes BRCA1 and BRCA2 becomes available, it introduces new information to those who opt for the test, including the possibility of a much higher perceived risk of breast cancer.[9] In terms of the relevant risk perception parameters held before and after the test and depending on an individual's circumstances, this type of information may be consistent with any one of the three scenarios of Proposition 4. Moreover, this type of situation is also consistent with the presumption that only a small group may become tested. For example, it is plausible that in the ex ante situation only a subset of individuals may have a health episode which is symptomatic of a particular genetic disease and so induces these individuals to take a genetic test with some testing positive for the disease and others testing negative.[10] Alternatively, under private and differentiated health insurance schemes, some individuals may have access to costless testing while others do not.[11]

3 Conclusion

Additional information about life expectancy, especially through genetic tests, will become more and more important as the state of science progresses. We showed in this paper that individuals have an economic incentive to acquire such information if we assume either that insurers cannot observe whether an individual was tested or that legislation prevents insurers from using such information, both realistic possibilities. In a market where ex ante information is symmetrically distributed, the availability of the test decreases ex ante welfare; of course, in a more general model, this welfare loss would have to be weighed against possible gains due to the test since more information makes better medical treatment available. In a market with initial informational asymmetry, the welfare effects of a new test could go either way; we constructed examples for a Pareto improvement, a Pareto worsening and for a situation in which those who are tested gain and those who are not lose. At the moment, we believe that the last case is the most realistic one for genetic testing: Those who test positive for a certain gene causing a fatal illness receive very bad information, and are then probably worse risks than the average life insurance buyer, so the equilibrium premium will go up. Since only few people are tested currently, however, the price effect will be small. Hence those who are tested gain since they have the possibility to adjust their life insurance demand to their real risk type for an only moderately

[9] For a detailed discussion of relevant probabilities associated with BRCA1 and BRCA2 carrier status, see Easton, et al. (1995, 1997).

[10] Those who experience the health episode will temporarily be of intermediate risk type but the presumption is that the testing occurs soon enough after the health episode that decisions about insurance purchases would not be taken in the interim.

[11] Of course social costs should include the cost of the test in this instance, but they are not private marginal costs from the insured's perspective.

higher price. However, as genetic testing becomes available more widely and for less serious illnesses or also for certain other tests, the other scenarios we have investigated, in which testing can lead to either Pareto improvements or worsenings, are relevant.

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References

Doherty, N. and P. Thistle (1996): "Adverse Selection with Endogenous Information in Insurance Markets", *Journal of Public Economics*, 63, pp. 83-102.

Easton, D.F., et al. (1995): "Breast and Ovarian Cancer Incidence in BRAC1-mutation Carriers", *American Journal of Human Genetics*, 56, pp. 265-71.

Easton, D.F. et al, "Cancer Risks in Two large Breast Cancer Families Linked to BRCA2 on Chromosome 13q12-13", *American Journal of Human Genetics*, 61, pp. 120-128.

Hellwig, M.F. (1988): "A Note on the Specification of Interfirm Communication in Insurance Markets with Adverse Selection", *Journal of Economic Theory*, 46, pp. 154-163.

Holtzman, N.A., P.D. Murphy, M.S. Watson and P.A. Barr (1997): "Predictive Genetic Testing: From Basic Research to Clinical Practice", *Science*, 24, pp.602-605.

Lemmens, T. and P. Hahamin (1996): "Genetics in Life, Disability and Additional Health Insurance in Canada: A Comparative Legal and Ethical Analysis", Report submitted to the MELSI Advisory Committee of the CGAT Program, September, 1996.

Milgrom, P. and J. Roberts (1994): "Comparing Equilibria", *American Economic Review*, 84, pp. 441-459.

Rowen, L., G. Mahairas and L. Hood (1997): "Sequencing the Human Genome", *Science*, 24, pp. 605-607.

Villeneuve, B. (1996): "Mandatory Insurance and Intensity of Adverse Selection", CREST/INSEE working paper, mimeo.

Appendix 1

For all of our numerical examples we use $u(x) = x^{1/2}$ and $v(x) = (ax)^{1/2}$ for our utility functions in the states of life and death, respectively. In the case that insurance is priced at the actuarially fair price, $\lambda = p$, we have from equation (10) the following demand for life insurance:

$$L(p,p) = \frac{a(Y+K) - K}{1 - p + pa}$$

which implies income in the states of life (c_1) and death (c_2) are

$$c_1 = Y + K - pL = \frac{[(1-p)Y + K]}{1 - p + pa}$$

$$c_2 = K + (1-p)L = \frac{a[(1-p)Y + K]}{1 - p + pa}$$

and so the ratio $\frac{c_2}{c_1} = a$. Thus, for $a < 1$ we have that the demand for life insurance is such that income in the death state is less than income in the life state. This is consistent with a family which chooses its life insurance purchases to maximize the expected value of per capita consumption across the two states. That is, with fewer family members one requires less income to maintain equal per capita consumption. The smaller is a, the less is the degree of concern with income in the death state.

In order to make comparisons easier to read, affine transformations are made of the utility functions before reporting the values. For Examples 1, A1, A2, 3, and A3 the transformation is $(EU - 13) \times 10$ while for Example 2 the transformation is $(EU - 11) \times 10$.

Example A1 (Proposition 2, Test 2)
Test probabilities of being assigned to H/L type, respectively:
$\rho = 0.2$, $1 - \rho = 0.8$
If No Testing
Results same as for Example 1 above.
With Testing
Proportion of each risk type:
$q_L^1 = 0.52$, $q_M^1 = 0$, $q_H^1 = 0.38$, $q_{HH}^1 = 0.1$,
Insurance purchases: H/HH types purchase insurance at price $\lambda = 0.06397$
$L_{HH} = 124.1 \Rightarrow c_1 = 192.06$, $c_2 = 216.2$, $EU_{HH} = 8.650$
$L_H = 57.8 \Rightarrow c_1 = 196.3$, $c_2 = 154.1$, $EU_H = 8.768$
$L_M = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_M = 10.490$

$L_L = 0 \Rightarrow c_1 = 200,\ c_2 = 100,\ EU_L = 10.956$

Ex ante expected utility for M types conditional on testing:

$EUM_{test} = (1-\rho)EU_{L(test)} + \rho EU_{HH(test)} = 14.052 > EU_{M(notest)} = 14.049$

Average demand for insurance:

28.23 with no testing, 34.37 with testing.

In this case the price spillover effects of testing are beneficial (price falls from $\lambda = 0.06465$ to $\lambda = 0.06397$) and so all individuals, including those H and HH-types, are better off in the *testing* scenario.

Example A2 (Proposition 3, Case with $L_L > 0$)

Basic Parameters:

$K = 100,\ Y = 100,\ a = 0.9$

Probabilities of death by type:

$p_L = 0.09,\ p_H = 0.1,\ p_{HH} = 0.11$

Test probabilities of being assigned to HH/L type, respectively:

$\rho = 0.5,\ 1-\rho = 0.5$

If No Testing

Proportion of each risk type:

$q^0_L = 0,\ q^0_H = 1,\ q^0_{HH} = 0$

Insurance purchases: H types purchase insurance at price $\lambda = 0.1$

$L_H = 80.8 \Rightarrow c_1 = 191.9,\ c_2 = 172.7,\ EU_H = 7.150$

With Testing

Proportion of each risk type:

$q^1_L = 0.5,\ q^1_H = 0,\ q^1_{HH} = 0.5$

Insurance purchases: HH/L types purchase insurance at price $\lambda = 0.10562$

$L_{HH} = 97.3 \Rightarrow c_1 = 189.7,\ c_2 = 187.0,\ EU_{HH} = 6.860$

$L_L = 27.3 \Rightarrow c_1 = 197.1,\ c_2 = 124.4,\ EU_L = 7.286$

Ex ante expected utility for H types conditional on testing:

$EU_{H(test)} = (1-\rho)EU_{L(test)} + \rho EU_{HH(test)} = 7.073 < EU_{H(notest)} = 7.150$

Average demand for insurance:

80.8 with no testing, 62.3 with testing.

Example A3 (Proposition 4, Case with all types purchasing insurance before but not after testing)

Basic Parameters:

$K = 100$, $Y = 100$, $a = 0.9$

Probabilities of death by type:

$p_L = 0.045$, $p_M = 0.05$, $p_H = 0.066$, $p_{HH} = 0.071$

Test probabilities of being assigned to HH/M type, respectively:

$\rho = 0.7621$, $1 - \rho = 0.238$

If No Testing

Proportion of each risk type:

$q_L^0 = 0.8$, $q_M^0 = 0.1$, $q_H^0 = 0.1$, $q_{HH}^0 = 0$,

Insurance purchases: all types purchase insurance at price $\lambda = 0.05567$

$L_H = 156.089 \Rightarrow c_1 = 191.311$, $c_2 = 247.39$, $EU_H = 9.03$

$L_M = 44.17 \Rightarrow c_1 = 197.541$, $c_2 = 141.709$, $EU_M = 9.17$

$L_L = 15.37 \Rightarrow c_1 = 199.145$, $c_2 = 114.508$, $EU_L = 9.34$

With Testing

Proportion of each risk type:

$q_L^1 = 0.8$, $q_M^1 = 0.124$, $q_H^1 = 0$, $q_{HH}^1 = 0.076$,

Insurance purchases: only HH types purchase insurance at price $\lambda = 0.071$

$L_{HH} = 80.57 \Rightarrow c_1 = 194.279$, $c_2 = 174.85$, $EU_{HH} = 8.40$

$L_M = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_M = 9.09$

$L_L = 0 \Rightarrow c_1 = 200$, $c_2 = 100$, $EU_L = 9.33$

Ex ante expected utility for H types conditional on testing:

$EU_{H(test)} = (1 - \rho)EU_{M(test)} + \rho EU_{HH(test)} = 8.57 < EU_{H(notest)} = 9.03$

Average demand for insurance:

32.32 with no testing, 6.13 with testing.

Appendix 2

Here we present details on the algebraic steps used in the proof of Proposition 4. As in the main text, since we use the relationships $q^1_{HH} = \rho q^o_H$ and $q^1_M = q^o_M + (1-\rho)q^o_H$, all q_t values are in terms of the pre-test scenario (i.e., q^o_t's) and so for convenience we omit the superscripts. The demands for insurance, $L_H = L_H(\lambda, p_{HH})$, depend on the price, λ, and both λ^0 and λ^1 are evaluated at the same price. So, we have

$$\lambda^0 = \frac{p_H L_H q^o_H + p_M L_M q^o_M}{L_H q^o_H + L_M q^o_M}$$

and

$$\lambda^1 = \frac{p_{HH} L_{HH} \rho q^o_H + p_M L_M (q^o_M + (1-\rho) q^o_H)}{L_{HH} \rho q^o_H + L_M (q^o_M + (1-\rho) q^o_H)}$$

The difference is:

$$\lambda^1 - \lambda^0 = (N_1 - N_0) \div Den \tag{28}$$

where,

$$N_1 = [L_H q_H + L_M q_M]\,[p_{HH} L_{HH} \rho q_H + p_M L_M (q_M + (1-\rho) q_H)]$$

$$N_0 = [L_{HH} \rho q_H + L_M (q_M + (1-\rho) q_H)]\,[p_H L_H q_H + p_M L_M q_M]$$

$$Den = [L_{HH} \rho q_H + L_M (q_M + (1-\rho) q_H)]\,[L_H q_H + L_M q_M]$$

Arranging together the terms of $N_1 - N_0$ in which L_{HH} is a factor gives

$$L_{HH}\,\{p_{HH} \rho q_H L_H q_H + p_{HH} \rho q_H L_M q_M - p_H \rho q_H L_H q_H - p_M \rho q_H L_M q_M\}$$

where we can write the part inside the brackets $\{\}$ as

$$\{...\} = \{\rho q_H\,(p_{HH}[L_H q_H + L_M q_M] - p_H L_H q_H - p_M L_M q_M)\}$$

Since $\{...\} > 0$ and $L_{HH} > L_H$ we can replace L_{HH} by L_H in $N_1 - N_0$ and we will have that $N_1 - N_0 >$ the following expression

$$\begin{aligned}
&\rho q_H L_H\,(p_{HH}[L_H q_H + L_M q_M] - p_H L_H q_H - p_M L_M q_M) \\
&+ p_M L_M (q_M + (1-\rho) q_H)[L_H q_H + L_M q_M] \\
&- L_M (q_M + (1-\rho) q_H)[p_H L_H q_H + p_M L_M q_M]
\end{aligned}$$

Rewriting this expression gives:

$$\begin{aligned}
&\rho q_H L_H\,(p_{HH}[L_H q_H + L_M q_M] - p_H L_H q_H - p_M L_M q_M) \\
&+ L_H q_H [p_M L_M (q_M + (1-\rho) q_H) - p_H L_M (q_M + (1-\rho) q_H)] \\
&+ L_M q_M [p_M L_M (q_M + (1-\rho) q_H) - p_M L_M (q_M + (1-\rho) q_H)]
\end{aligned}$$

Noting that the third line of the above expression is zero, then upon substitution of $p_H = \rho p_{HH} + (1-\rho)p_M$ we get:

$$\begin{aligned}&\rho q_H L_H \left(p_{HH}[L_H q_H + L_M q_M] - p_M L_M q_M - (\rho p_{HH} + (1-\rho)p_M)L_H q_H\right)\\&+q_H L_H[p_M L_M(q_M + (1-\rho)q_H) - (\rho p_{HH} + (1-\rho)p_M)L_M(q_M + (1-\rho)q_H)]\end{aligned}$$

Upon expanding this expression and simplifying the result by factoring out q_H^2 and collecting terms with ρ and ρ^2 we get:

$$\begin{aligned}&q_H^2\{\rho[p_M L_H L_M + p_{HH} L_H^2 - p_M L_H^2 - p_{HH} L_H L_M]\\&-\rho^2[p_M L_H L_M + p_{HH} L_H^2 - p_M L_H^2 - p_{HH} L_H L_M]\}\end{aligned}$$

Upon expanding that part of the expression in Proposition 4 which is:

$$q_H^2 \rho(1-\rho)L_H(L_H - L_M)(p_{HH} - p_M)$$

we can see we get the same result.

Thus, treating the right hand side of the expression for λ^1 as a function of λ and letting $f(\lambda) = \lambda^1 - \lambda$, and also treating the right hand side of the expression for λ^0 as a function of λ and letting $g(\lambda) = \lambda^0 - \lambda$, we have that $f(\lambda) > g(\lambda)$. Since for $\lambda = 0$ we have $L_t > 0$ for all t, it follows that $f(0) > g(0) > 0$.

Letting λ_e^1 and λ_e^0, be the be the equilibrium prices in the testing and no testing scenarios, respectively, it follows from the definition of equilibrium price (see equation (10) and the discussion following) that:

$$\lambda_e^1 = \inf\{\lambda \mid f(\lambda) \leq 0\}$$

and

$$\lambda_e^0 = \inf\{\lambda \mid g(\lambda) \leq 0\}$$

¿From Milgrom and Roberts (1994) Lemma 1 we have $\lambda_e^1 > \lambda_e^0$. •