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DIRECT AND INDIRECT TAXATION WHEN HOUSEHOLDS DIFFER IN MARKET AND NON-MARKET ABILITIES

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Abstract

How are optimal taxation rules affected when households differ in their non-market, as well as in their market skills? Where income taxation is concerned, the policy prescription applying in the case where households are differentiated only by the wage rate (namely, that high-wage households should be taxed more than low-wage households, but that the marginal income tax rate on the former should be zero) may be reversed when there are also differences in household production skills. Such a reversal is not very likely, however, because there are efficiency gains in subsidizing households with a comparative advantage in non-market activities. Indeed, simulations with a wide range of parameter values show that redistribution is always in favour of low-wage households, and that the amount redistributed increases with the non-market skills of the latter. If households differ in both market and non-market skills, we also find that the introduction of indirect taxation alongside income taxation may be welfare-improving, because it helps relax the incentive constraint, even when the utility function is separable in labour. This contradicts the Atkinson-Stiglitz theorem. The shift from direct to indirect taxation/subsidization allows the optimal marginal rate of income tax on potential mimickers to be different from zero. This is confirmed by simulation experiments.

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1 Introduction

Imagine the following situation. There are two individuals. One, a tax expert, can earn a great deal of money selling professional advice, but is not much good at anything else. The other can conjure up a delicious meal out of practically nothing (“can do wonders with an egg”), but is not so good at making money. The government steps in with its usual armoury of taxes and subsidies. Whatever its objective in so doing (raising revenue, redistributing welfare or pursuing efficiency) the government will have to take into account the fact that the two individuals differ in their domestic, as well as market ability. How should it go about it?

Following Mirrlees (1971), optimal taxation models have tended to assume that individuals differ only for their wage rate (market ability). Other authors have considered additional sources of heterogeneity,¹ but the first to investigate the case where people differ in both market and domestic ability is Sandmo (1990).² He, however, makes special assumptions about the structure of the household decision model that somewhat cloud the issue, and restricts the scope of the analysis by limiting his attention to the determination of a linear income tax.³ In the present paper, we re-examine the issue using a more standard Becker-style model of household production, and extend the analysis in two directions.

The first is to allow for the possibility that the optimal income tax schedule may be non-linear, and that the government may not be able to observe the personal characteristics of individuals. Since Stiglitz (1982), we know how to deal with both these issues when individuals differ only for their

¹Atkinson-Stiglitz (1976) and Stiglitz (1982) discuss the case where tastes, as well as wage rates, differ. Cigno (1986) examines the case where households differ in their reproductive capacity, as well as in the husband’s and wife’s earning abilities. More recently, Mayshar-Yithaki (1996) deal with tax reforms when households differ in abilities and needs; Rochet (1991) and Cremer-Pestieau (1996) study social insurance and optimal taxation in a model with differences in wages and proneness to illness; Cremer *et al.* (1998) focus on indirect taxation when people differ in wages and endowments.

²Christiansen (1984) includes a model of optimal taxation with household production, but the focus of his paper is on a completely different question (namely, on which goods should indirect taxes be levied given that a general income tax is in place). Cigno (1986) has production of children, and allows for differences in the maximum number of children a couple can have, as well as in the wage rates of the parents, but the focus is on how the optimal choice of policy instruments varies as the government becomes more egalitarian. Jacobsen *et al.* (1998) study optimal taxation with household production, but only in a one-household framework.

³Indirect taxation is considered in the case where there is only one individual.

market ability. Here, we see what happens if individuals differ also for their non-market (household production) ability.

The second is to examine the possible advantages of introducing a commodity tax alongside income taxation. There is now a substantial literature on the direct-indirect tax mix, which develops arguments in defence of commodity taxation. Boadway *et al.* (1994) focus on the fact that the income tax is easier to evade, Cremer and Gahvari (1995) explore the role of wage uncertainty, Cremer *et al.* (1998) consider differences in endowments as well as wage rates. All of these are, in some measure, attempts at refuting the Atkinson-Stiglitz theorem that indirect taxes are redundant, if an income tax is in place, when the utility function is at least weakly separable in leisure (or, equivalently, labour) and other goods.⁴ By contrast, Edwards *et al.* (1994) and Nava *et al.* (1996) adopt a more conventional framework, but simply drop the separability assumption. We attempt to extend this body of knowledge by allowing for differences in both market and non-market skills.

2 Basic assumptions

A suitable framework for examining the optimal tax implications of differences in household production ability is that of Becker (1965), where the consumer derives utility from non-transferable goods, domestically produced by the same consumer using own time and commodities purchased from the market.

While declaredly using a household production framework, Sandmo (1990) makes two special assumptions that effectively change the nature of the model. The first is that the individual derives utility from three goods, of which one (“leisure”) is made up of time only, another can be bought ready-made from the market, and only the third is actually produced with own time and a market input. By itself, that is a pretty innocuous assumption: it merely takes to extremes the idea that a product may be intrinsically more time-intensive than another. It becomes less innocuous, however, when taken together with the second assumption, that the good bought from the market is a perfect substitute for the one produced at home. Since the individual is indifferent between working for himself and working for a wage (he only distinguishes between leisure, which gives him direct utility, and other uses of his time, which do not), the model could just as well be interpreted as saying that the agent sells the domestically produced good to the

⁴*cf.* Atkinson and Stiglitz (1976), and Stiglitz (1982).

market, and then uses his total income (from the sale of this product, and from the sale of his labour) to buy factors of production and final consumption goods. That makes it a conventional income-leisure model, with an occupational choice problem (employment at a given wage rate *versus* self-employment) grafted on. While interesting in its own right, this does not seem the most appropriate vehicle for investigating the effects of differences in non-market abilities.

Like Sandmo, we allow for one of the goods to be bought ready-made from the market, but do not take this to be a perfect substitute for the one produced at home. Furthermore, we stay closer to the Becker spirit by not including leisure in the list of utility-yielding goods (but nothing of great substance changes, if we allow for the time employed in household production to enter the utility function, thereby implying that the agent is not indifferent to how he uses his time). The preferences of each individual are described by a utility function $u(x, y)$, where x is the good bought from the market, and y the one produced at home. The two are substitutable for each other, but at a diminishing marginal rate. The domestic production function for the second good is $y = y(h, z; b)$, where h is the amount of time dedicated to domestic production, z an input bought from the market, and b a productivity parameter. The budget constraint is $x + Pz = B$, where P is an after-tax price and B is after-tax income. The latter is given by $B = Y - T$, where $Y = wl$ is pre-tax income, $l = 1 - h$ the labour supply, w the wage rate, and T an income tax (possibly a function of Y). For simplicity, we assume that each household consists of one individual,⁵ so that we can interchangeably talk of one or the other. We further assume that there are only two household types ($i = 1, 2$), differentiated by domestic and market abilities (b^i and w^i), and that there is the same number of households for each type.

In section 3, we assume that the government armoury holds only a (non-linear) income tax. In section 4, we see what happens if the government is empowered to use also a commodity tax. Pre-tax prices are assumed to be invariant with direct and indirect taxes, and normalized to unity. For simplicity and comparability with the work of others (including Sandmo, 1990), we take it that the government objective is to maximize the sum of the utilities of the two types of individuals. We assume that the government

⁵ Assuming otherwise would raise problems of intra-household resource allocation that go beyond the scope of the present exercise. Optimal income taxation of couples is studied by Apps-Rees (1997) and Schroyen (1997).

knows individual preferences (the same for everybody), and the distribution of personal characteristics (equal numbers of type-1 and type-2 households), but does not know who is who. More specifically, we assume that the government observes the income (Y), but not the personal characteristics (b, w), or the allocation of time between domestic and market activities (h, l) of any given person. Therefore, the government does not know, and has no way of inferring from observed behaviour, whether a person is type-1 or type-2. This assumption, widely used in conventional optimal taxation models where individuals differ only in their wage rate, seems all the more appropriate in a model where individuals differ also in their (more difficult to ascertain) domestic ability.

In conventional optimal-taxation models, the non-observability of the household type raises the following problem. Since, normally, *laissez-faire* utility increases with the wage rate, the government may wish to redistribute from high- w to low- w types. Not knowing which is which, however, all it can do is tax higher incomes more heavily than lower incomes (*progressive taxation*). But, that may create an incentive for high- w types to masquerade as low- w by reducing their labour supply to such an extent, that their income is the same as that of a person with a low wage rate. To guard against that, the government will then choose the tax schedule subject to the self-selection constraint, that a high- w individual must not be able to increase his post-tax utility by choosing to earn as little as a low- w type.⁶

In our model, *laissez-faire* utility is an increasing function of both b and w . This introduces two additional complications. One is that, while behaviour and thus income depend on comparative advantage, utility increases with absolute advantage. In certain circumstances, income and utility could thus move in opposite directions. If it wants to redistribute in favour of less advantaged households, the government might then find itself taxing lower incomes more heavily than higher incomes (*regressive taxation*)! The other complication is that the government faces not one, but two self-selection constraints (a type-1 household should have no interest in being taken for type-2, and a type-2 should have no interest in being taken for type-1), and

⁶The possibility that a self-selection constraint may operate in the opposite direction is ruled out by imposing the “single-crossing” condition (Stiglitz, 1982) that, in the (B, Y) plane, the indifference curves of the high-wage person are always flatter than those of the low-wage person. That is, for any given increase in pre-tax income, it takes a larger amount of additional disposable income to keep a low-wage person on the same indifference curve, than a high-wage person.

that there is no way of telling *a priori* which one is going to be binding.⁷

3 Income taxation

Suppose that the government has the powers to tax incomes, but not commodities. The problem of choosing an income tax schedule can be equivalently stated as that of determining two pairs of pre-tax and post-tax incomes (Y^i, B^i) . For any given tax schedule, the decision problem of household type i can be solved in two steps. The first is to choose z^i to maximize

$$u(B^i - Pz^i, y(z^i; 1 - (Y^i/w^i), b^i)) \quad (1)$$

taking pre-tax and post-tax incomes as given. This gives us the demand for the market input, $z(Y^i, B^i, P; w^i, b^i)$, and the maximized value of utility, $v(Y^i, B^i, P; b^i, w^i)$, as functions of (Y^i, B^i) . The second step is to select the (Y^i, B^i) pair (*i.e.*, since $Y^i = w^i l^i$, the labour supply) that yields the higher v .

If a household wishes to misrepresent its type in the hope of gaining a more favourable tax treatment, it can do so by earning the same pre-tax income as the other household type. If that is the case, we say that the household is a “mimicker” (variables referring to mimickers are distinguished by an apex). If, for example, type-2 households wanted to masquerade as type-1 households, whose tax treatment is identified by the pair (Y^1, B^1) , mimickers would be choosing their z so as to maximize $\hat{u}(B^1 - P\hat{z}^2, y(\hat{z}^2; 1 - Y^1/w^2, b^2))$. The solution to this problem gives us the indirect utility function, $\hat{v}(Y^1, B^1, P; w^2, b^2)$, and the conditional demand function, $\hat{z}(Y^1, B^1, P; w^2, b^2)$. Similar expressions apply, with indices interchanged, if type-1 households are the mimickers.

The government chooses the income tax schedule (*i.e.*, a menu of (Y, B) pairs) to maximize the social welfare function,

$$\sum_{i=1}^2 v^i(Y^i, B^i, P; b^i, w^i) \quad (2)$$

⁷Strictly speaking, there are two self-selection constraints also in the case where households differ for their wage rate only. However, with only one differentiating characteristic and two household types, we can tell *a priori* which one is going to be stringent (normally the one which says that high- w individuals must not be able to gain from appearing low- w). With two differentiating characteristics, by contrast, that is not possible (low- w persons could have a b sufficiently high, to make them want to be taken for high- w). Put another way, imposing the single-crossing condition (see previous note) is reasonable when households are differentiated by only one characteristic, but not when they are differentiated by two.

subject to the government budget constraint,

$$\Sigma_{i=1}^2 (Y^i - B^i + (P - 1)z^i) = \bar{R} \quad (3)$$

where $\bar{R}$ is some pre-set revenue requirement (which may be zero), and to the self-selection constraints,

$$v^i(Y^i, B^i, P; w^i, b^i) \geq \hat{v}^j(Y^j, B^j, P; w^j, b^j), \quad i, j = 1, 2. \quad (4)$$

The first order conditions are

$$\begin{aligned} (1 + \sigma_1)v_B^1 - \sigma_2\hat{v}_B^2 + \mu \left(-1 + (P - 1)\frac{\partial z^1}{\partial B^1} \right) &= 0 \\ (1 + \sigma_1)v_Y^1 - \sigma_2\hat{v}_Y^2 + \mu \left(1 + (P - 1)\frac{\partial z^1}{\partial Y^1} \right) &= 0 \\ (1 + \sigma_2)v_B^2 - \sigma_1\hat{v}_B^1 + \mu \left(-1 + (P - 1)\frac{\partial z^2}{\partial B^2} \right) &= 0 \\ (1 + \sigma_2)v_Y^2 - \sigma_1\hat{v}_Y^1 + \mu \left(1 + (P - 1)\frac{\partial z^2}{\partial Y^2} \right) &= 0 \end{aligned} \quad (5)$$

where μ and σ_i are the Lagrange multipliers associated with, respectively, the government budget constraint and the self-selection constraint relative to the mimicker of type i .

Since the implicit marginal tax rate is given by $T'(Y) = 1 + v_Y/v_B$, simple manipulation of (5) yields⁸

$$\begin{aligned} T'(Y^1) &= \frac{\sigma_2}{(1 + \sigma_1)v_B^1}(\hat{v}_Y^2 + \hat{v}_B^2) - (P - 1)\frac{\partial \bar{z}^1}{\partial Y^1}, \\ T'(Y^2) &= \frac{\sigma_1}{(1 + \sigma_2)v_B^2}(\hat{v}_Y^1 + \hat{v}_B^1) - (P - 1)\frac{\partial \bar{z}^2}{\partial Y^2}, \end{aligned} \quad (6)$$

where the upper bar denotes a compensated demand. With $P = 1$ (recall that we have no indirect taxation yet) the last r.h.s. term vanishes, and the signs of the marginal tax rates thus depend on the mimicker's marginal rate of substitution of post-tax for pre-tax income, $MRS_{BY} = |v_Y/v_B|$. The marginal rate of income tax on type- i households will be positive if, at

⁸cf. Stiglitz (1982), Edwards *et al.* (1994), Nava *et al.* (1996). The derivation uses the fact that $\frac{\partial \bar{z}}{\partial Y} = \frac{\partial z}{\partial Y} - \frac{\partial z}{\partial B} \frac{v_Y}{v_B}$. This follows from the comparative statics of the household optimization problem.

the social optimum, the *MRS* of type- j households at the socially optimal (B^i, Y^i) is less than unity ($i, j = 1, 2$).

When households differ in market ability only, maximizing the sum (or, more generally, a quasi-concave function) of household utilities⁹ implies re-distribution from high-wage to low-wage households (Stiglitz, 1982). Suppose, for example, that $w^2 > w^1$ and $b^1 = b^2$. As the self-selection constraint relative to type-1 households is then slack ($\sigma_1 = 0$), (6) simplifies (for $P = 1$) to

$$\begin{aligned} T'(Y^1) &= \frac{\sigma_2}{v_B^1}(\hat{v}_Y^2 + \hat{v}_B^2) > 0 \\ T'(Y^2) &= 0. \end{aligned} \tag{7}$$

We thus get the standard result that the marginal tax rate must be zero for high-wage individuals, because nobody is interested in mimicking them, but positive for low-wage persons to discourage mimicking.

In general, however, there is no way of telling in advance who may want to mimick whom. If market and domestic abilities are negatively correlated, it might in fact be socially desirable to tax low-wage individuals more than high-wage ones, and it might then be in the interest of the former to masquerade as the latter. The solution to the policy optimization problem determines simultaneously the implicit tax schedule, and which of the two self-selection constraints is binding. As it is rather difficult to show analytically how the optimal policy and the direction of re-distribution changes with the distribution of domestic and market skills, we proceed by numerical simulation.

3.1 A simulation model

For simulation purposes, we assume

$$u(x^i, y(z^i, h^i, b^i)) = \alpha \ln(x^i) + \beta(\gamma \ln(b^i + z^i) + (1 - \gamma) \ln(h^i)) \tag{8}$$

where α, β and γ are positive constants. The reason for using this functional form is that it facilitates computations, but also that it satisfies the separability property underlying the Atkinson-Stiglitz theorem (more about this in the next section). Notice that the domestic skill parameter, b , is a perfect substitute for the market input, z . This is a simple way of rendering the idea

⁹And, assuming single-crossing (see footnote 6).

that an increase in non-market skills constitutes a kind of factor-saving improvement in domestic technology. We also simplify the analysis by setting the revenue requirement equal to zero (purely redistributive taxation).

We have already recalled that, in the case where households only differ in their market abilities, the joint use of an additive utility function and an additive (utilitarian) social welfare function guarantees that redistribution will be from high- w to low- w individuals. We have also noted that, if there are differences in domestic as well as market ability, high-wage households are not necessarily better-off than low-wage households. Using (8), we found that, for any fixed triplet (w^1, w^2, b^2) , the difference between the *laissez-faire* utilities of type-1 and type-2 households increases monotonically, and passes from negative to positive, as b^1 increases. There is thus a critical value of b^1, b^0 , such that, for all $b^1 > b^0$, low-wage households have higher *laissez-faire* utility than high-wage households.

3.2 Simulation results

The exercise aims to answer the following question: how does the allocation of the tax burden between the two household types shift, as the distribution of b changes? To answer it, we computed the total tax paid by, and the marginal and average tax rates applicable to, the two household types in correspondence with different levels of b^1 . We did this for various values of the taste/home technology parameters (α, β, γ) , and of the skill parameters (w^1, w^2, b^2) being held constant. Contrary to what might be expected, we found that high-wage households are always taxed, and low-wage households subsidized. Indeed, we found that the tax on the former increases with the non-market skill of the latter.¹⁰ This contradicts a prediction of Sandmo (1990) that the opposite would be true.¹¹ Let us try to understand why.

Recall that the government is maximizing the unweighted sum of the utilities of the two household types. We have thus a tendency to equalize¹² the marginal utilities of post-tax incomes across household types. Since

¹⁰This is true even for values of b^1 ($b^1 > b^0$) such that low-wage households have higher *laissez-faire* utility than high-wage households.

¹¹What Sandmo actually says is that the marginal rate of income tax would be lower if w and b were negatively correlated, for in this case income inequalities overstate utility inequalities, than if they were positively correlated or uncorrelated. Since Sandmo uses a linear income tax, and the marginal rate of income tax is thus the same for everybody, this tantamounts to saying that, the higher the household production skill of low- w households, the lower the tax paid by high- w households.

¹²Strict equalization is prevented by the self-selection constraints.

the marginal utility of B , $[(\alpha + \beta)/(B + Pb)]$, is decreasing in B , the maximization of social welfare implies redistribution from the high-income type-2 household to the low-income type-1 household.¹³ Since the marginal utility of income is also decreasing in b , however, a rise in b^1 will reduce the marginal utility of B^1 , thus making redistribution in favour of type-1 households less desirable. That is, presumably, what lies behind Sandmo's result. But, there are other considerations.

Think of each household type as of a country producing two final goods, x and y . Think of the time endowment of each household type as of the country's labour force. Good x requires only labour, but good y requires also an imported input, z . If both final goods were tradeable, each country would allocate its labour force according to comparative advantage. In particular, Country 1 would produce only y , and import x and z . By distorting marginal incentives, income taxation would then cause an efficiency loss. And, the higher b^1 , the greater the distortion. In our model, however, y is not tradeable. The country with a comparative advantage in the production of this good will still tend to specialize in it, but not completely, because it must produce enough x to meet its own demand for this good, and to pay for the imports of z . Redistribution in favour of the country (household type) with the higher b may thus generate efficiency gains, which are to be set against the efficiency losses associated with the distortion of marginal incentives. This makes it possible that, as in our simulations, it is optimal to redistribute more and more income from type-2 to type-1 households as the latter become more and more skilful in domestic activities (and, thus, less and less disadvantaged relative to the former). Sandmo's contrary result reflects the fact, pointed out in our Section 2, that in his (1990) model all goods are effectively tradeable and, therefore, that there is no possible efficiency gain from redistributing in favour of households with greater non-market skills to be set against the efficiency loss from distorting marginal incentives.¹⁴

Table 1 reports summary simulation results for a particular set of parameter values. Type-2 (high-wage) households are always taxed. Although the average tax rate is obviously positive, the marginal tax rate is zero, because the binding constraint on the government choice of tax schedule is that households of this type should be indifferent between mimicking and not

¹³Recall that the income tax schedule is constructed subject to self-selection constraints, so that the rich have no interest in appearing poor, and the poor in appearing rich.

¹⁴"Taxes on income or consumption do lead to production inefficiencies and a less efficient division of labour in the economy" (Sandmo, 1990, p.90).

b^1	B^1	Y^1	B^2	Y^2	z^1	z^2	$T'(Y^1)$	$T'(Y^2)$	v^1	v^{2*}
0.5	3.73	3.46	4.79	5.06	0.91	0.93	.082	.000	.311	.517
1	3.66	3.35	4.77	5.08	0.55	0.92	.065	.000	.397	.515
1.5	3.58	3.24	4.75	5.09	0.19	0.92	.038	.000	.480	.509

Table 1:

Tax liabilities, incomes and time allocations for various values of $b^1(\alpha = \beta = \gamma = 0.5, P = 1, w^1 = 5, w^2 = 7, b^2 = 1; b^0 = 2.6)$ $v^2 = \hat{v}^2$

mimicking. Their marginal incentive to supply labour is thus undistorted by public intervention. By contrast, type-1 (low-wage) households are always subsidized, and their incentive to supply labour is reduced by a positive marginal rate of tax (forgone subsidy). As a result, $h^1 (= 1 - (Y^1/w^1))$ increases quite sharply as b^1 rises. At the values of $b^1 (< b^0)$ shown, type-1 households have lower *laissez-faire* utility than type-2 (and continue to have lower utility, $v^1 < v^2$, after government intervention). But, the pattern does not change when low-wage households become so skilful in non-market activities, as to be actually better-off than high-wage households (a fuller range of simulation results, for this and other parameter configurations, is shown in the Appendix).

4 Introducing commodity taxation

We now extend the model to allow for indirect taxation. We look, in particular, for conditions under which it is desirable to supplement the income tax with an excise tax on the market input, z (the tax on the other traded commodity, x , is normalized to zero). Having assumed that producer's prices are independent of taxes, choosing the tax on z is equivalent to choosing P .

The first-order conditions for the maximization of the social welfare function (2), subject to the government budget constraint (3) and self-selection constraints (4), are (5) and

$$v_P^1 + v_P^2 + \sigma^2(v_P^2 - \hat{v}_P^2) + \sigma^1(v_P^1 - \hat{v}_P^1) + \mu \Sigma_i \left((P - 1) \frac{\partial z^i}{\partial P} + z^i \right) = 0 \quad (9)$$

The marginal income tax rates are still given by (6). However, with $P \neq 1$, it is no longer true that the (potential) mimicker's marginal tax rate has to be zero. Using (5) and (9), we get

$$(P - 1)(\Sigma_{i=1}^2 \frac{\partial \bar{z}^i}{\partial P}) = \frac{\sigma^2 \hat{v}_B^2}{\mu}(z^1 - \hat{z}^2) + \frac{\sigma^1 \hat{v}_B^1}{\mu}(z^2 - \hat{z}^1). \quad (10)$$

This last equation gives us information on the sign of the indirect tax rate. Since the own-price Slutsky term $(\partial \bar{z}^i / \partial P)$ is negative, the tax rate on z , $P - 1$, will have opposite sign to the r.h.s. of (10). Let us see what that implies.

If $\sigma_1 = 0$ (i.e., the binding self-selection constraint relates to high-wage households), z should be taxed or subsidized according to whether a low-wage household buys less or more of it than the high-wage mimicker. The rationale is that, if the mimicker buys more of this commodity than the low-wage household, a tax hurts the former more than the latter, and thus helps to relax the binding self-selection constraint. A Pareto-improving change in the income tax schedule can then be operated. By contrast, in the case where $\sigma_2 = 0$, the optimal commodity tax is positive if a (low-wage) mimicker buys more of z than a type-2 person. Indirect taxation thus serves the purpose of alleviating the incentive problem associated with a non-linear income tax schedule. To do its job, the commodity tax must help discourage either type of mimicking.

Is there more one can say about the sign of this tax? Consider, first, the case without differences in domestic productivity ($b^1 = b^2$), and recall that the demand for z is $z(B, Y, P; w, b)$. A high- w mimicker has then the same pre-tax and post-tax incomes as, but higher wage rate and lower labour supply than, a low- w household. Since the only difference between the mimicker and the mimicked is that the former dedicates more time to non-market activities, the first term on the r.h.s. of (10) is negative if z is a *complement* for h (in the precise sense that, all else being equal, the demand for the market input increases with the time allocated to non-market activities rises, $\partial z / \partial h > 0$). This reproduces the result of the standard model without domestic production (with time in domestic activities replacing leisure). By contrast, a low- w mimicker has the same pre-tax and post-tax incomes as, but lower wage rate and higher labour supply than, a high- w household. Hence, the second term on the r.h.s. of (10) is negative if z is a *substitute* for h . These results are summarized in Table 2.

In general, b^1 may differ from b^2 , as well as w^1 from w^2 . There are thus two reasons why the use of z may differ across household types. Let us focus again on the first term on the r.h.s. of (10). Suppose, for instance, that $\partial z / \partial b < 0$ and $\partial z / \partial h > 0$. If $b^1 > b^2$, a low-wage person spends less time in domestic activities, and is more skilful in the use of the market input,

	$\partial z/\partial h < 0$	$\partial z/\partial h > 0$
$\sigma_2 > 0, \sigma_1 = 0$	$P > 1$	$P < 1$
$\sigma_2 = 0, \sigma_1 > 0$	$P < 1$	$P > 1$

Table 2: Indirect tax rates with $b^1 = b^2$

	$b^1 < b^2$	$b^1 > b^2$
$\sigma_2 > 0, \sigma_1 = 0$	$P < 1$	$P > 1$
$\sigma_2 = 0, \sigma_1 > 0$	$P > 1$	$P < 1$

Table 3: Indirect tax rates with $\partial z/\partial h = 0, \partial z/\partial b < 0$

than a high-wage mimicker (pretending to be low-wage). The low-wager's demand for z is thus unambiguously lower than that of the mimicker. That is not necessarily true if $b^1 < b^2$, in which case it may happen that $z^1 > \hat{z}^2$.¹⁵ Similar reasoning helps us sign the second term on the r.h.s. of (10).

In order to better understand the role of the domestic productivity parameter, and to permit comparisons with the well-known results of Atkinson and Stiglitz (1976) and Stiglitz (1982), we now focus on the special case where the utility function and the home-production function take a weakly-separable form, such that

$$u = u(u_1(x), u_2(y_1(z; b), y_2(h))). \quad (11)$$

Separability ensures that the demands for z and x are independent of Y ($\partial z/\partial h = 0$). In the simpler case with $b^1 = b^2$, we then find that $z^1 = \hat{z}^2$ and $\hat{z}^1 = z^2$: indirect taxation is redundant. This verifies the Atkinson-Stiglitz theorem, according to which it is useless to tax commodities, when incomes are optimally taxed, if labour supply is separable from commodity demand. Suppose, however, that non-market skills differ across types, and that $\partial z/\partial b < 0$. If $b^1 < b^2$, a high-wage mimicker will then buy less z than a low-wage household, while a low-wage mimicker will buy more z than a high-wage household. In general, therefore, commodity taxation (positive or negative in accordance with (10)) may be desirable. Table 3 summarizes the results.

We have thus found that the Atkinson-Stiglitz theorem does not hold in household production models if we allow for differences in domestic skills:

¹⁵We have therefore a case in which the complementarity between z and home-time does not ensure that it is taxed even if $\sigma_1 = 0$. Jacobsen et al. (1998) obtain a similar result in a model with optimal linear income and commodity taxation, and one consumer.

b^1	z^1	z^2	$P - 1$	$T'(Y^1)$	$T'(Y^2)$	$T(Y^2) + (P - 1)z^2$	v^1	v^2
0.5	.91	.93	-.04	.087	.007	.28	.313	.517
1	.55	.92	.00	.065	.000	.31	.397	.515
1.5	.18	.90	.01	.035	-.003	.34	.483	.509

Table 4:

Indirect tax rates, effective tax liabilities and uses of z for various values of b^1 ($\alpha = \beta = \gamma = 0.5, w^1 = 5, w^2 = 7, b^2 = 1, b^0 = 2.6$)

it may be optimal to tax or subsidize commodities even when the utility function is separable in goods and non-labour time. That is because, so long as there are differences in the home-productivity parameters, the demand of the mimickers will differ from that of the mimicked, and commodity taxes thus help relax the incentive constraints on income taxation. This can be seen as an argument in favour of indirect taxation along the lines of the literature mentioned in the Introduction.

As was the case with the model of Section 3, the pattern of redistribution cannot be established analytically. To gain some insight into how this is affected by the introduction of indirect taxation, we turn again to our simulation model.

4.1 Simulation results

We start by noting that the utility function in (8) satisfies the restriction imposed by (11). Thus, commodity taxes can be helpful only if households differ in their non-market skills. Recall that our specification also satisfies $\partial z / \partial b < 0$. We then repeat the exercise carried out for the case of income taxation only, namely check whether the effective tax liability (now including indirect taxation) for household 2 increases or decreases as b^1 increases, for any given triplet (w^1, w^2, b^2) . Full results are reported in the Appendix.

As an example, we show in Table 4 the indirect taxation implications of the case presented in Table 1. $P - 1$ is the tax on z . The seventh column shows the total tax liability of high-wage households (equal to the total subsidy entitlement of low-wage households).

From Table 4, we see that:

- The marginal income tax rate on the household who would have had an interest in mimicking is different from zero when the indirect tax rate is different from zero. That is what (6) led us to expect.
- As b^1 increases, type-1 households buy less and less of z , not only because

their disposable income falls, but also because they become more skilful in the use of this input. Purchases of z by type-2 households also fall, because their disposable income falls, but less markedly, because their domestic skills do not improve (b^2 is fixed).

- Taxation of z becomes heavier (passing from negative to positive) as b^1 increases. The key to this result is that, while the demand for z by type-1 households falls sharply as b^1 grows larger, a mimicker's demand for it falls only a little (because its disposable income, equal to that of low-wage households, B^1 , falls, but the value of its b , b^1 , remains the the same). Consequently we observe a sharp change of tax regime as type-1 households become more skilful in non-market activities.

The combined effect of these facts is that, despite the shift from direct to indirect taxation, the overall tax liability of type-2 households (overall subsidy entitlement of type-1 households) is roughly the same as without the commodity tax (this can be seen by subtracting B^2 from Y^2 in Table 1). Thus, the tax bill of high-wage households still rises with the non-market skills of low-wage households. What changes, and here we see the benefit of introducing a second policy instrument, is the marginal rate of income tax on potential mimickers (type-2 households), which can be different from zero when the tax on z is different from zero.

5 Conclusion

We set out to discover how standard optimal taxation results change when households differ in their non-market, as well as in their market skills. Where income taxation is concerned, the policy prescription applicable to the case where households are differentiated by their wage rates only, namely that high-wage households should be taxed more than low-wage households (but the labour decisions of the former should not be distorted by a positive marginal rate of income tax), *may* be reversed when there are differences in domestic skills. May, not will, because there are efficiency gains from subsidizing households with a comparative advantage in non-market activities (to set against the efficiency losses associated with a positive rate of income tax on low-wage households) that make redistribution from high to low wage households desirable on efficiency grounds when market ability is negatively correlated with non-market skills. Indeed, simulations with a wide range of parameter values show that redistribution is always in favour of low-wage households, and that the amount redistributed increases with the

non-market skills of the latter. This contradicts a result of Sandmo (1990), possibly arising from the fact that, in Sandmo's model, all utility-yielding goods are effectively traded commodities, so that households specialize according to comparative advantage without any need for redistribution.

If households differ in both market and non-market skills, we also find that the introduction of a commodity tax alongside income taxation may be welfare-improving, because it helps relax the incentive constraint, even when the utility function is separable in goods and non-labour time. This contradicts the Atkinson-Stiglitz theorem. The shift towards indirect taxation allows the optimal marginal rate of income tax on potential mimickers to be different from zero. Simulation results indicate that the commodity tax rate increases (and changes sign, from negative to positive) with the non-market skills of low-wage households (holding those of high-wage households constant). The direction of redistribution is unaffected by the introduction of this new tax.

6 References

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7 Appendix

We report here the results of a range of simulation exercises. Those reported in tables 5 to 8 assume the same taste/technology parameters as tables 1 and 4 of the main text, but consider additional values of b^1 , and provide more detail. Tables 9 to 12 use different parameters, but yield the same results (except for the values of utility and welfare). It is difficult to find different price/parameter configurations that yield meaningful solutions.

b_1	$T'(Y^1)$	$T(Y^1)/Y^1$	$T'(Y^2)$	$T(Y^2)/Y^2$	v^1	v^2	$v^1 + v^2$
.5	.082	-.08	.000	.05	.311	.517	.83
1	.065	-.09	.000	.06	.397	.515	.91
1.5	.038	-.11	.000	.06	.480	.509	.99
2	.009	-.14	.000	.09	.56	.49	1.06
2.5	.009	-.14	.000	.09	.56	.49	1.11
3	.009	-.14	.000	.09	.67	.49	1.16

Table 5:

Direct taxation only: maginal and average income tax rates, utilities and social welfare for various values of b^1 ($\alpha = \beta = \gamma = 0.5, P = 1, w^1 = 5, w^2 = 7, b^2 = 1; b^0 = 2.6$) $v^2 = \hat{v}^2$

b_1	B_1	Y_1	B_2	Y_2	z_1	z_2	z_3	l^1	l^2
.5	3.73	3.46	4.79	5.06	0.91	0.93	0.58	.69	.72
1	3.66	3.35	4.77	5.08	0.55	0.92	0.55	.67	.73
1.5	3.58	3.24	4.75	5.09	0.19	0.92	0.52	.65	.73
2	3.62	3.16	4.65	5.11	0	0.88	0	.64	.74
2.5	3.62	3.16	4.65	5.11	0	.88	0	.64	.74
3	3.63	3.17	4.65	5.12	0	.88	0	.63	.74

Table 6:

Direct taxation only: pre and post-tax incomes, input demands and labour supplies for various values of b^1 ($\alpha = \beta = \gamma = 0.5, P = 1, w^1 = 5, w^2 = 7, b^2 = 1; b^0 = 2.6$) $v^2 = \hat{v}^2$

b_1	$T'(Y^1)$	$T'(Y^2)$	$T(Y^2) + (P - 1)z^2$	v^1	v^2	$v^1 + v^2$
0	.12	.02	.27	.218	.518	.737
0.5	.087	.007	.28	.313	.517	.827
1	.065	-.01	.31	.317	.515	.907

Table 7:

Direct and indirect taxation: marginal income tax rates, effective tax liability of type-2 households, utilities and social welfare for various values of b^1 ($\alpha = \beta = \gamma = 0.5, w^1 = 5, w^2 = 7, b^2 = 1, b^0 = 2.6$)

b_1	B_1	Y_1	B_2	Y_2	$P - 1$	z_1	z_2	z_3	l^1	l^2
0	3.76	3.58	4.74	5.07	.063	1.34	1.02	.67	.72	.73
0.5	3.72	3.46	4.77	5.07	.025	.94	.97	.61	.70	.73
1	3.66	3.35	4.77	5.08	.000	.55	.92	.55	.73	.69

Table 8:

Direct and indirect taxation: pre and post tax incomes, indirect tax rate, input demands and labour supplies for various values of b^1 ($\alpha = \beta = \gamma = 0.5, w^1 = 5, w^2 = 7, b^2 = 1, b^0 = 2.6$)

b_1	$T'(Y^1)$	$T(Y^1)/Y^1$	$T'(Y^2)$	$T(Y^2)/Y^2$	v^1	v^2	$v^1 + v^2$
.5	.082	-.08	.000	.05	.62	1.04	1.66
1	.061	-.09	.000	.06	.40	.514	.91
1.5	.038	-.11	.000	.06	.96	1.02	1.98
2	.009	-.14	.000	.09	1.11	1.01	2.12
2.5	.009	-.14	.000	.09	1.25	1.00	2.25
3	.009	-.14	.000	.09	1.38	.99	2.37

Table 9:

Direct taxation only: maginal and average income tax rates, utilities and social welfare for various values of b^1 ($\alpha = \beta = 1; \gamma = 0.5, P = 1, w^1 = 5, w^2 = 7, b^2 = 1; b^0 = 2.6$) $v^2 = \hat{v}^2$

b_1	B_1	Y_1	B_2	Y_2	z_1	z_2	z_3	l^1	l^2
.5	3.73	3.46	4.79	5.07	0.91	0.93	0.58	.69	.72
1	3.66	3.35	4.77	5.08	0.55	0.92	0.55	.67	.73
1.5	3.58	3.24	4.75	5.09	0.19	0.92	0.52	.65	.73
2	3.62	3.16	4.65	5.11	0	0.88	0	.64	.74
2.5	3.62	3.16	4.65	5.11	0	.88	0	.64	.74
3	3.63	3.17	4.65	5.12	0	.88	0	.63	.74

Table 10:

Direct taxation only: pre and post-tax incomes, input demands and labour supplies for various values of b^1 ($\alpha = \beta = 1; \gamma = 0.5, P = 1, w^1 = 5, w^2 = 7, b^2 = 1; b^0 = 2.6$) $v^2 = \hat{v}^2$

b_1	$T'(Y^1)$	$T'(Y^2)$	$T(Y^2) + (P - 1)z^2$	v^1	v^2	$v^1 + v^2$
0	.12	.025	.27	.44	1.04	1.48
0.5	.087	.007	.28	.63	1.04	1.67
1	.065	-.01	.31	.80	1.04	1.84

Table 11:

Direct and indirect taxation: marginal income tax rates, effective tax liability of type-2 households, utilities and social welfare for various values of b^1 ($\alpha = \beta = 1; \gamma = 0.5, w^1 = 5, w^2 = 7, b^2 = 1, b^0 = 2.6$)

b_1	B_1	Y_1	B_2	Y_2	$P - 1$	z_1	z_2	z_3	l^1	l^2
0	3.76	3.58	4.74	5.07	.063	1.34	1.02	.67	.72	.73
0.5	3.72	3.46	4.77	5.07	.025	.94	.97	.61	.70	.73
1	3.66	3.35	4.77	5.08	.000	.55	.92	.55	.73	.69

Table 12:

Direct and indirect taxation: pre and post tax incomes, indirect tax rate, input demands and labour supplies for various values of b^1 ($\alpha = \beta = 1; \gamma = 0.5, w^1 = 5, w^2 = 7, b^2 = 1, b^0 = 2.6$)