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IRREVERSIBLE INVESTMENTS, UNCERTAINTY, AND THE RAMSEY POLICY

Gaute Torsvik
Trond E. Olsen

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Center for Economic Studies
University of Munich
Ludwigstr. 33
8000 Munich 22
Germany
Telephone: 089-2180-2747
Telefax: 089-397303

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Abstract

The main goal for the tax reforms that have been enacted in several countries in the past decade has been to reduce the diversity of tax rates on income from different capital types. Capital types differ among other things, with respect to their degree of irreversibility. In this paper we show that if there is uncertainty about the government's future revenue requirement it is in general not optimal to tax income from investment projects of different durability at the same tax rate.

The conventional wisdom concerning the optimal distribution of the tax burden over time is also challenged. It is not optimal to smooth the tax burden over time if taxes can be levied on income from capital types which leave investors wtih different degrees of flexibility. We find that even if the expected revenue requirement in the future is much higher than the current revenue requirement it is optimal for the government to run a deficit in the first period.

Gaute Torsvik
Trond E. Olsen
Norwegian Research Centre in Organization and Management
Rosenbergsgt. 39
N-5015 Bergen
Norway

1 Introduction

The main goal for the tax reforms that have been enacted in several countries in the past decade has been to reduce the diversity of tax rates on income from different capital types. The theoretical argument in favour of a "levelled-playing-field" is that it provides a neutral tax system which then assures efficient production. The desirability of efficiency on the production side of the economy was pointed out by Diamond and Mirrlees [1971]. They showed that in the absence of profits - i.e., under constant returns to scale - or if profits can be taxed away by the government, then production efficiency should be maintained.

It is well known that there are situations where it can be optimal to deviate from the "level-the-playing-field" principle of taxation . Auerbach [1979], Christiansen and Kvinge [1989], Feldstein [1990] and Richter [1988] have shown that if there are other distortions on the supply side in the economy, for instance if there is one untaxed asset or if there is an intertemporal distortion caused for example by a general income tax, then it is in general optimal to differentiate taxation of capital income. In second best situations like this the government should e.g. take differences in investment responses to changes in after tax return, into account when tax rates are chosen. Optimal differentiated tax rates will be contingent on differences in the slope of marginal productivity curves across capital types. Responsive investments should be taxed more leniently than less responsive investments (the inverse elasticity rule).

In this paper we present a *new argument in favour of heterogeneous taxation* of capital income. We argue that it can be optimal for the government to tax income from different capital types at different tax rates even in cases where all capital types have the same slope of the marginal productivity curve. What distinguishes capital types in our model is their degrees of irreversibility. That is, we consider a situation where individuals can put their resources into projects which give them different degrees of flexibility in the future. It is well known from the theory of real and financial "options"

that flexibility - the possibility to reallocate resources in the future - is valuable if there is uncertainty about the future returns of current choices. This means that the aggregate level of irreversible investments will be very sensitive to uncertainty about future returns and, as is pointed out by R. Pindyck, it therefore becomes;

> "..*important to understand how investment depend on factors that are at least partly under government control..*" (Pindyck [1991];1141).

In this paper we study the problem of optimal policy design in presence of uncertainty in a model where individuals make irreversible allocation decisions.

Based on the option argument it is easy to understand that if irreversible and reversible capital types are exposed to the same risk about future returns there will be a bias towards reversible projects and little investment in irreversible capital. If there is uncertainty about future government spending this is exactly the effect of taxing according to the "level-the-playing-field" principle. Investors then know that all capital types will be taxed at the same rate in the future, but they are uncertain about the *level* of these tax rates. Big spending requires large tax revenues and hence high tax rates on all types of capital, while the opposite is true if spending is low. Homogeneous taxation will, because of the option argument, discriminate against irreversible investments. Uncertainty about government spending seems therefore to provide an argument in favour of differentiated taxation. This conjecture is confirmed in our analysis. The government should differentiate taxation and the differentiation should be made on the basis of capital types' degrees of irreversibility.

How, then, should optimal taxes be differentiated? What is the optimal "slope" of the "playing field"? The argument in favour of different taxes on capital types with different degrees of irreversibility is based on the effect uncertainty about future taxes has on current investment. Uncertainty about future tax rates has no effect on current investments in reversible capital but it clearly has a negative incentive effect on current investments in irreversible capital. It seems therefore natural to conjecture that irreversible capital types should be made less exposed to tax uncertainty by letting more of the uncertainty about future taxes be allocated to reversible projects. In the light of this argument the tax policy we actually find to be optimal is surprising. For in section three we demonstrate that this conjecture is false; it is not optimal to levy more of the tax rate risk on income from reversible capital types. On the contrary, relative to a "level-the-playing-field" policy, the government should *increase* the differences in tax rates between high and low states of government spending on irreversible investment, and *decrease* the variability in taxes on reversible capital types. This policy in itself will, compared with a levelled playing field, reinforce the distortion of investments away from irreversible capital types. There is, however, a remedy, and that is to reduce *current* taxation on irreversible projects. In this way the government makes investment in such assets more attractive.

The way taxes are differentiated between capital types with different degrees of irreversibility has also important implications for *optimal debt policy*. The point usually made with respect to optimal intertemporal distribution of taxes, is that a government should *smooth* the tax burden over time if its revenue requirement is expected to change in the future (See for example the articles by Robert Barro [1979], Robert Lucas and Nancy Stockey [1983] and Lucas [1986]). The neoclassical model views debt accumulation as a way to spread over time the costs of distortionary taxation. For example, if there is a probability of a sharp increase in the the government's need of revenue in the future, the advice usually given by economists is that the government should run a surplus in periods before the increase is expected. Or put differently, in the words of Robert Lucas;

> *" One implication of the Ramsey principle is that tax rates ought to be smoothed over periods of differing government demands, relative to endowment. In practice this principle requires deficit spending during wars and depressions, balanced (in present value sense) by surplus in times of relative peace and prosperity. This is a common sense principle which has long been recognised"*

(Lucas [1986]; 130).

Our results on optimal debt policy contradicts this common sense principle of tax smoothing. In a two period model we find that it can be optimal for the government to

run a *deficit* in the first period even in cases where the expected revenue requirement in the second period is considerably higher than what it is in the first period.

Why do we get this result? Usually when the tax smoothing principle is discussed there is only one tax base (see Barro [1979] and Bohn [1990]). In our model the government collects its revenue from two tax bases, it is taxing income from both irreversible and reversible projects. Our result follows from the link between present and future capital supply that is induced by investment opportunities that are irreversible.

We think our normative results have theoretical interest as well as practical relevance. Time inconsistency is, however, a problem. The tax policy we find to be optimal from an ex ante perspective is not time consistent. Announced taxes distort investment in irreversible projects, but when the future arrives irreversible investment decisions are irrevocably made. At this stage increased taxation has only income effects. A government trying to minimize the excess burden of taxation will take advantage of the possibility to get distortion free tax-revenue by increasing taxation of irreversible projects.

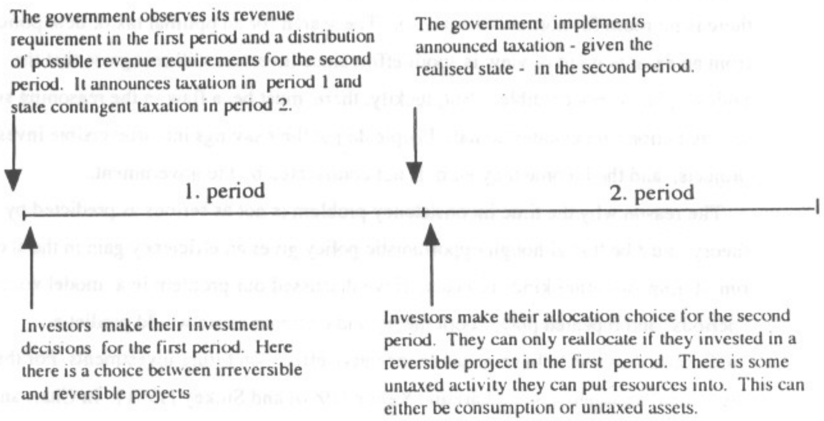
If this is how capital taxation and investments are determined in the real world, then there is no room for normative analyses. The search for an optimal tax or debt policy from an ex-ante point of view is spoilt effort since investors will recognise that the optimal plan is not credible. But, luckily, there must be a flaw in the reasoning since its predictions are counterfactual: People do put their savings into irreversible investment projects, and the income they earn is not confiscated by the government.

The reason why the time inconsistency problem is not as serious as predicted by theory must be that although opportunistic policy gives an efficiency gain in the short run, it imposes other kinds of costs. If we discussed our problem in a model with many "periods" and repeated play, both theory and common sense would predict a confiscatory policy to have negative incentive effects on future investments. For the theoretical arguments, see Chari and Kehoe [1990] and Stokey [1991]. In Chari and Kehoe a two period model is repeated over and over. Their stage game is a simpler version of our's, since they have only one tax base. We could also embody our two period model into a more dynamic game. In this way the Ramsey policy could be made credible. The commitment assumption we make in this paper can therefore be interpreted as an assumption to the effect that deviations from an announced policy generate long run costs that are large enough to make the announced policy credible.

2 The setting

The model includes two periods. Investors can choose to invest in assets with different durability in the beginning of the first period. We consider a situation where there is uncertainty about the future tax burden, modelled here as there being uncertainty about the tax burden in the second period. This uncertainty could be induced by uncertainty about the government's expenditures in the future. We study a second best situation where investors can allocate resources into some untaxed "activity" between the first and the second period. The "escape asset" can either be interpreted as consumption or some untaxed investment possibility, e.g. investments abroad (the asset need not be untaxed, we only require that it is taxed at a fixed rate whatever state is realised in the second period.)

The basic structure of the situation we study is given below.



A model

In the beginning of period i there are fixed endowments e_i that can be allocated to three different projects a, b and c. The economy produces an aggregate output Y using

these three different types of capital. Let k^a, k^b and k^c be the amounts of capital invested in each project. Capital is transformed into output according to the following simple aggregate production function,

$$Y_i = f(k_i^a) + f(k_i^b) + k_i^c r \qquad i = 1,2. \tag{1}$$

We assume that income from capital type, c , cannot be taxed. If we let $\phi_i^j \in (0, 1)$ be the tax rate levied on income from project $j = a,b$ in period $i = 1,2$ we can write the government's budget constraint as,

$$g_i = \phi_i^a f'(k_i^a)k_i^a + \phi_i^b f'(k_i^b)k_i^b. \tag{2}$$

The problem we study is quite general, but for expositional reasons we have chosen this simple form of the production function and the government's budget constraint. In order to have a well behaved optimization problem, we assume that the elasticity of the marginal product of capital, $\varepsilon(k) = -(f''/f')k$, is non-decreasing; $\varepsilon'(k) \geq 0$, and satisfies $0 < \varepsilon(k) \leq 1$.

Our results would not change if there was decreasing returns on the untaxed "activity" but it simplifies a lot to assume constant returns since all (uninteresting) income effects of taxation are eliminated by this assumption. One interpretation of the model is that k^c is the amount of capital allocated to the world capital market where the return on capital is given by r, and this capital income cannot be taxed by the government.

None of our results are dependent on the assumption that both taxable projects have an output function of the same functional form. We could have used a more general functional form where different capital types had different slopes of their marginal productivity curves, ($f^{a''}(k^a) \neq f^{b''}(k^b)$, for $k^a = k^b$), or where the cross derivatives of the marginal productivities of capital types were either positive or negative (not zero as in our additive production function (1)). With a more general technology we would have to consider arguments in favour of differentiated taxation based on differences in ex ante supply (or cross) elasticities of different capital types. These arguments are, however, well understood and need not be analysed here, see for example Feldstein [1990]. The point is that if the output function of projects a and b had the same functional form it would be optimal to level-the-playing-field if the capital types were of the same durability (i.e., if investment in a and b imposes the same degree of irreversibility). So if we find that it is optimal to differentiate taxation of capital income in our model this must be based on differences in projects' degree of irreversibility.

In a model with decreasing returns to scale there are profits. We have assumed that profits are left untaxed. We make this assumption in order not to get too many tax instruments to handle. Alternatively we could let the government tax away all pure rents. This would change the government's budget constraints but not our main conclusions. Another possibility would be to include labour as a production factor and to let projects a and b have constant returns to scale with respect to labour and capital. What is profits in our specification, $f(k) - f'(k)k$, would then be the wages received by the workers. In this case we could model labour income as just another reversible tax base (that is not done here).

We consider a situation where the government's future revenue requirements are uncertain, that is, g_2 is a random variable which can take values both higher and lower than the revenue requirement in the first period. For simplicity we assume a two point distribution where the revenue requirement takes the value $\overline{g}$ with probability p and $\underline{g}$ with probability $(1 - p)$, where $\overline{g} > g_1 > \underline{g}$.

The level of government consumption is not a choice variable in our model. In principle we could let government spending be determined in the model. Uncertainty about future government consumption could e.g. correspond to a situation with uncertainty about future governments' preferences; "will a left wing government with preferences for high government consumption be elected in the future"? Modelled like this we would have to tackle the problem of strategic choice of economic policy, where the government making policy today knows that it might be replaced by another government in the future and thus has incentives to strategically manipulate state variables - such as debt - in order to influence future policy choices. (See e.g. Persson

and Svensson [1989] and Alesina and Tabellini [1990]). Such strategic aspects of economic policy would make our discussion less clear-cut. For that reason we consider a case where government consumption is exogeneously given.

The way we model uncertainty about the future level of public spending corresponds for example to a situation where there is a probability of a "war" at some date in the future, (Lucas and Stokey [1983]). It could also correspond to a case where government *consumption* is constant over time but consumption can be financed by taxation of capital and labour income (not modelled here) *and* by lump sum revenues of uncertain magnitude. The tax burden levied on capital income will then be contingent on the realisation of these lump sum revenues. This is a good description of the situation in countries like Norway and Britain where the uncertain lump sum element of revenue is income from oil reserves in the North - Sea.

The capitalists' allocation problem

Capitalists are risk neutral and allocate the fixed capital stock in order to maximize the net return of capital. In order to capture irreversibility in a stark form we assume that it is impossible to reduce the level of capital invested in project a, hence $k_1^a \leq k_2^a$. Each capitalist is assumed to be small and she takes the marginal productivity of capital in project a and b as given when she makes an allocation choice. Their objective is to choose period one investments, $k_1^j, j = a,b,c,$ and state contingent period two investment policies $k_2^a(k_1^a, \phi_2^a, \phi_2^b)$ so as to maximize;

$$\begin{aligned} v = {} & f'(k_1^a)(1 - \phi_1^a)k_1^a + f'(k_1^b)(1 - \phi_1^b)k_1^b + rk_1^c \\ & + \delta E[f'(k_2^a)(1 - \phi_2^a)k_2^a + f'(k_2^b)(1 - \phi_2^b)k_2^b + rk_2^c], \end{aligned}$$

where E denotes expectation with respect to the probability distribution for the second period tax rates. Total resource constraints are given by $e_i = k_i^a + k_i^b + k_i^c$, $e_2 = e_1(1 + r)$, and the irreversibility constraint is given by $k_1^a \leq k_2^a$. δ is the capitalists' and the government's discount factor which is equal to $[1/(1 + r)]$.

Without the irreversibility constraint, $k_1^a \le k_2^a$ it would be trivial to find optimal taxes. If all projects are fully reversible there is no need to announce taxes for the second period. When capital can freely be re-allocated between periods the choices made today have no implications for what choices can be made tomorrow and the decisions made today will therefore not be contingent on the policy announced for the future. Given our specification of production functions it is, in the case with no irreversibility constraint, optimal to "level-the-playing-field" between capital types within both periods (and within both states in the second period). If $\bar{\phi}_2^a$ and $\bar{\phi}_2^b$ denote the tax-rates that are implemented in the high revenue requirement state, and $\underline{\phi}_2^a$, $\underline{\phi}_2^b$ are the tax-rates imposed if the revenue requirement is low in the second period, the optimal tax policy would imply:

$$\bar{\phi}_2^a = \bar{\phi}_2^b > \phi_1^a = \phi_1^b > \underline{\phi}_2^a = \underline{\phi}_2^b.$$

It is considerably more interesting to study the problem of finding optimal taxes if making an investment in one of the taxable capital types, here a, is an irreversible decision. That is what we will do in the next section.

First we consider a situation where there is some constitutional amendment requiring the public budget to be balanced in every period (and for every realised revenue-requirement in the second period) .

3 Balanced budgets and optimal tax policy

Given the structure of the revenue requirements where $\bar{g} > g_1 \ge \underline{g}$ it seems natural first to consider tax policies where $\bar{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$. In particular, "level-the-playing-field" is, as we have seen, a policy in this class. Note, however, that such policies expose investors to risks and therefore generate option values associated with holding reversible capital. If an irreversible investment decision is made, there is no possibility to re-allocate resources to consumption or untaxed assets (c) if the high tax

state ($\bar{\phi}_2^a$) is realised. Uncertainty about future taxes will therefore have a negative impact on *current* investments in irreversible capital.

The uncertainty about future tax rates is under the control of the government. The government can reduce, or even eliminate, the uncertainty about tax rates on irreversible capital income , e.g. by announcing a fixed - state-independent - tax rate on irreversible investments (a). In this case all unexpected revenue must be financed by taxes on income from project b. This would increase future uncertainty about tax rates for project b but since this is a fully reversible project, increased uncertainty about future returns has no effect on current investment in asset b. Given the asymmetric impact uncertainty about future tax rates has on current investments in a and b, it is not quite clear whether the optimal tax policy has a structure where $\bar{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$. We shall therefore first prove that such a structure is indeed optimal.

If the government announces a tax plan where $\bar{\phi}_2^a > \phi_1^a > \underline{\phi}_2^a$ the supply of capital to different projects is given implicitly by the first order conditions to the capitalists' maximization problem:

$$f`(k_1^a) = f`(\bar{k}_2^a) = \frac{r(1 + \delta p)}{(1 - \phi_1^a) + \delta p(1 - \bar{\phi}_2^a)} \quad (3)$$

$$f`(\underline{k}_2^a) = \left(\frac{r}{1 - \underline{\phi}_2^a}\right) \quad (4)$$

$$f`(k_i^b) = \left(\frac{r}{1 - \phi_i^b}\right) \text{ for } i = 1,2 \quad (5)$$

Relation (5) says that investments in each period are carried on to the point where marginal returns on taxed and untaxed capital are equalised. This is the standard condition for optimal investment if capital is fully reversible. From (3) we can see how the irreversibility of type a capital distorts the investment made in this project in the first period. For this tax structure, the investment decision is based on taxes in the first period and on the tax rate announced to be implemented in the bad state in the second period. In this case the investors do not take account of the tax rate announced for the good state. The reason is of course that by investing in c they have an option to increase investment in a *if* the good state with low taxes is realised in the second period. But, since a is an irreversible investment project, there is no offsetting possibility for a reduction in the capital supplied to project a if the bad state is realised. This asymmetry gives rise to a portfolio distortion and an efficiency loss in the first period.

In its design of the tax policy the government can eliminate the asymmetry created by the possibility to increase but not reduce investment in a. The government can do so by announcing taxes for the second period such that investors *never* find it optimal to increase investment in project a in the second period. Announcement of a fixed tax-rate in the second period - the same in every state - which is higher than the tax-rate in the first period would be an example of such a tax policy.

More generally, in order to make the first period investment decision bind in all states in the second period, the government must announce taxes that give,

$$k_1^a = \bar{k}_2^a = f'^{-1}\left(\frac{r(1+\delta p)}{(1-\phi_1^a)+\delta p(1-\bar{\phi}_2^a)}\right) \geq f'^{-1}\left(\frac{r}{1-\underline{\phi}_2^a}\right) = \underline{k}_2^a$$

In terms of tax rates this implies $\underline{\phi}_2^a \geq \phi^* = \dfrac{\phi_1^a + \delta p \bar{\phi}_2^a}{1+\delta p}$. Our first result, stated as Lemma 1, shows that such a tax structure will never be optimal if the revenue requirements satisfy $\bar{g} > g_1 \geq \underline{g}$.

Lemma 1

If $\bar{g} > g_1 \geq \underline{g}$ it is never optimal to announce taxes such that the first period investment is binding in the good state. That is, it is never optimal to announce

$$\underline{\phi}_2^a \geq \phi^* = \frac{\phi_1^a + \delta p \bar{\phi}_2^a}{1+\delta p}.$$

This lemma is proved formally in Appendix 1.

It follows from Lemma 1 that optimal taxes are set such that investors will increase investment in a if the good state is realised. It is relatively easy to understand why such a policy is optimal. For consider a situation where the opposite is true, that is, where $\underline{\phi}_2^a > \phi^*$, and the investment decision made in period 1 thus is strictly binding whatever state is realised in the future. In this case a reduction in $\underline{\phi}_2^a$ will have a positive incentive effect on investment in project a in the first period and hence in both states in the second period. The government can therefore reduce taxation of income from project b both in the first period and in the high revenue requirement state in the second period. On the other hand, taxation of capital income of type b must be increased if the good state is realised in the second period. But since this tax rate is low initially ($\underline{\phi}_2^a$ is high) the cost of such an increase, in terms of higher excess burden, is low. A reduction in $\underline{\phi}_2^a$ therefore improves efficiency, and this argument can be repeated for every $\underline{\phi}_2^a > \phi^*$.

Setting $\underline{\phi}_2^a = \phi^*$ cannot be optimal either. In this case a reduction in $\underline{\phi}_2^a$ has no effect on the first period investment in project a . And if the good state is realised in period two it is optimal to have the same amount of capital in projects a and b. It is then optimal to set $\underline{\phi}_2^a = \underline{\phi}_2^b$ which is lower than ϕ^*.

According to Lemma 1, the government should set taxes such that the supply of capital of type a will be based on considerations where future tax rates enter asymmetrically. Only the tax rate in the bad state in the second period is taken into account. One should expect this asymmetry to have implications for optimal taxes. In particular one should expect that it is not optimal to level the playing field, as we know would be optimal if both projects a and b were perfectly reversible.

In order to confirm this conjecture we must solve the government's maximization problem. The government's aim is to maximize expected aggregate output subject to the state contingent budget constraints, and subject to the constraints that capital will be allocated by investors in order to equalize expected net returns. The government's objective function is thus,

$$Y = f(k_1^a) + f(k_1^b) + (e - k_1^a - k_1^b)r + \delta\{p[f(\bar{k}_2^a) + f(\bar{k}_2^b) + (e_2 - \bar{k}_2^a - \bar{k}_2^b)r] + (1-p)[f(\underline{k}_2^a) + f(\underline{k}_2^b) + (e_2 - \underline{k}_2^a - \underline{k}_2^b)r]\}; \tag{6}$$

where e_2 is equal to $e(1 + r)$ since c is assumed to be the marginal project. The budget constraints are:

$$g_1 = \phi_1^a f'(k_1^a)k_1^a + \phi_1^b f'(k_1^b)k_1^b \tag{7}$$

$$\bar{g} = \bar{\phi}_2^a f'(\bar{k}_2^a)\bar{k}_2^a + \bar{\phi}_2^b f'(\bar{k}_2^b)\bar{k}_2^b \tag{8}$$

$$\underline{g} = \underline{\phi}_2^a f'(\underline{k}_2^a)\underline{k}_2^a + \underline{\phi}_2^b f'(\underline{k}_2^b)\underline{k}_2^b \tag{9}$$

The constraints that capital owners allocate their capital in order to maximize their after tax income are given by equations (3), (4) and (5).

Derivation of optimal tax rates

If the good state is realised in the second period we know, from Lemma 1, that the optimal tax structure implies that investors will increase their holdings of type a capital. In this state it is "as if" both projects are fully reversible. Thus, it is optimal to tax income from both projects at the same tax-rate; $\underline{\phi}_2^a = \underline{\phi}_2^b$.

The derivation of optimal taxes for the first period and for the bad state in the second period is more involved and is relegated to an appendix. Here we give an intuitive

argument. We define μ_1 to be the multiplier associated with the government's budget constraint in the first period. Let $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the high and the low revenue requirement constraints, respectively, in the second period. Define $\bar{\lambda}_2$ to be the current value of the multiplier associated with the high revenue requirement in the second period, i.e. the current shadow price of government revenue if this state is realised,

$$\bar{\lambda}_2 = \frac{\bar{\mu}_2}{\delta p}$$

Given the tax structure implied by Lemma 1 the stocks of irreversible capital (a) in the first period and in the high revenue requirement state in the second period are equal. The effect on investment in a of increased taxation in the bad state in the second period can therefore always be neutralised by an appropriate reduction in the tax rate in the first period. As shown below it follows from this that the shadow price of government revenue in the first period, μ_1, and the current value of the shadow price in the bad state in the second period, $\bar{\lambda}_2$, must be equal in optimum. It will become clear that the equality between these two shadow prices has some quite interesting implications.

To see that $\mu_1 = \bar{\lambda}_2$, suppose one is at an optimum and suppose the government conducts the following experiment: It increases the tax rate in the first period, ϕ_1^a, and decreases the tax rate in the bad state in the second period such that the first period supply of capital of type a is left unchanged. If there is a one unit increase in the tax rate in the first period, the offsetting reduction in the second period (bad state) tax rate must be equal to $-1/\delta p$ in order to leave the capital stock unchanged (see equation (3)). This experiment induces an increase in the government's revenue of one dollar in the first period and a loss in revenue equal to $-1/\delta p$ in the second period (in the bad state). There is no effect on the excess burden of taxation of the policy reform since the supply of capital is unchanged. The marginal value of increased revenue in the first period is equal to μ_1, while the value of a loss in revenue of $-1/\delta p$ dollar in the high revenue state in the second period is equal to $-1/\delta p\ \bar{\mu}_2$, which is precisely the shadow price of government revenue measured in current value terms, $\bar{\lambda}_2 = \bar{\mu}_2/\delta p$. The government is free to make an experiment where tax rates are changed in the opposite direction, that is, increase $\bar{\phi}_2^a$ and decrease ϕ_1^a such that the supply of capital is constant. It cannot be an optimum if the government can gain something by making an "experiment" in either of the two directions. Hence in optimum μ_1 must be equal to $\bar{\lambda}_2$ [1].

We can now see how optimal tax rates should be set. The tax rates for the good state are trivial since these are the same as for the fully flexible case. There are then four tax rates (ϕ_1^a, ϕ_1^b, $\bar{\phi}_2^a$, $\bar{\phi}_2^b$) and three capital stocks (k_1^a, k_1^b, $\bar{k}_2^b$) left to be determined. These capital stocks affect expected production via the following terms

$$(1 + \delta p)[f(k_1^a) - rk_1^a] + f(k_1^b) - rk_1^b + \delta p[f(\bar{k}_2^b) - r\bar{k}_2^b] \qquad (10)$$

The two budget constraints that are of interest is those corresponding to g_1 and $\bar{g}$. In optimum we know that the value of a dollar of tax income collected in the second period is δp times the value of a dollar collected in the first period (the shadow prices satisfy $\mu_1 = \bar{\mu}_2/\delta p$). The two budget constraints can therefore, in the optimization problem, be combined to a single constraint of the following form

$$g_1 + \delta p\,\bar{g} = \phi^* (1 + \delta p) f`(k_1^a)k_1^a + \phi_1^b f`(k_1^b)k_1^b + \bar{\phi}_2^b\, \delta p f`(\bar{k}_2^b)\bar{k}_2^b \qquad (11)$$

where $\phi^* = (\phi_1^a + \delta p\, \bar{\phi}_2^a)/(1 + \delta p)$

The capitalists' investment rule for the irreversible project depends on the tax rates ϕ_1^a and $\bar{\phi}_2^a$ only via their average ϕ^* (see equation (3)). But then we see that the optimal taxation problem reduces to a problem of finding the vector of tax rates (ϕ^*, ϕ_1^b, $\bar{\phi}_2^b$) that maximizes the production function (10) subject to fullfilling the budget constraint (11). Since $\alpha f(\cdot)$ has the same elasticity as $f(\cdot)$ for any positive constant α, it follows directly from the elasticity rule for taxation that it is optimal to set the tax rates ϕ^*, ϕ_1^b and $\bar{\phi}_2^b$ all equal; thus $\phi_1^b = \bar{\phi}_2^b = (\phi_1^a + \delta p\, \bar{\phi}_2^a) / (1 + \delta p)$.

[1] A formal proof is given in appendix 2.

This means that investments in *a* and *b* should be taxed at different rates both in the first period and in the bad state (high revenue requirement) state in the second period.

We summarize our observations in Proposition 1 (the formal proof is given in Appendix 2),

Proposition 1

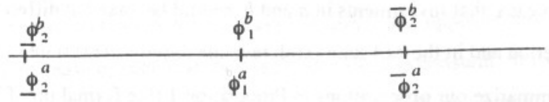
Suppose the revenue requirements satisfy $\bar{g} > g_1 \geq \underline{g}$. *Then optimal tax rates satisfy;* $\phi_1^b = \bar{\phi}_2^b = (\phi_1^a + \delta p\, \bar{\phi}_2^a) / (1 + \delta p)$ *and* $\bar{\phi}_2^a > \phi_1^a > \bar{\phi}_2^b = \phi_1^b > \underline{\phi}_2^a = \underline{\phi}_2^b$. *Given these tax rates equal amounts of capital are allocated to projects a and b in the first period as well as in in both states in the second period:*

$$k_1^a = k_1^b = \bar{k}_2^a = \bar{k}_2^b < \underline{k}_2^a = \underline{k}_2^b .$$

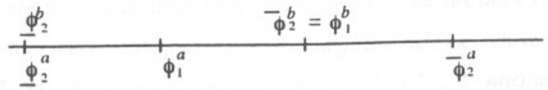
As we can see the tax system is *both state contingent and differentiated.* It is not optimal to level the playing field. In fact, relative to a tax policy where all capital income is taxed at the same tax rate (a "level-the-playing-field" tax policy), the optimal tax policy increases the difference between the tax rates in the low and in the high revenue state for the irreversible project *a* . It is thus optimal for the government to make investments in irreversible capital relatively more risky than investments in reversible capital. Ceteris paribus this makes the irreversible project even less attractive than under the levelled playing field regime. The remedy is, however, to attract investment in capital type *a* by a reduction in the tax rate for project *a* in the current period.

The picture of a "level-the-playing-field" tax structure and the optimal tax structure is as follows:

"level-the-playing-field"



Optimal tax structure.



To understand why taxes should be differentiated as indicated in Proposition 1 we start out with a tax policy that levels the playing field. That is, let income from both projects, a and b, be taxed at the same tax rate in both states in the second period. This yields a tax structure, where

$$\overline{\phi}_2^a = \overline{\phi}_2^b > \phi_1^a = \phi_1^b > \underline{\phi}_2^a = \underline{\phi}_2^b .$$

With homogeneous taxation of capital income, capital supply will be given by (see equations (3) - (5));

$$k_1^a < k_1^b,\ \overline{k}_2^a > \overline{k}_2^b \text{ and } \underline{k}_2^a = \underline{k}_2^b .$$

Assume now that the government undertakes a tax reform: The tax rate on project a in the bad state in the second period is increased while the first period tax rate on this project is reduced by such an amount that the first period investment in a is left unchanged. In the picture - with reference to a levelled playing field - this amounts to moving $\overline{\phi}_2^a$ to the right and ϕ_1^a to the left. The government now collects more revenue in the bad state in the second period and less revenue in the first period. To fulfil its revenue requirement it must reduce taxation of income from b in the second period (bad state) and increase taxation in the first period (moving $\overline{\phi}_2^b$ to the left and ϕ_1^b to the right). The tax reform's effect on welfare comes therefore solely from the impact it has on the supply of capital of type b. And as we can see, the change in tax rates will bring about a decrease in first period supply of b, and an increase in the supply in the bad state in the second period. If we take into consideration both the discounting and the concavity of

the production function we find that the increase in the second period capital of type b gives a welfare gain that dominates the first period loss. The concavity of the production function is important since, under a levelled playing field, the supply of capital b is higher in the first period than in the bad state in the second period.

This experiment indicates in what direction taxes should be differentiated. As we can see from Proposition 1, and from the picture of the optimal tax rates, $\overline{\phi}_2^a$ should be increased and ϕ_1^a reduced all the way until the first period and the second period - bad state - tax rate on b are equalised. Taxes are then chosen such that equal amounts of capital are allocated to projects a and b in the first period as well as in in both states in the second period.

The exact tax rates can now be found as follows. Let $k(\phi, r)$ denote the solution to $f'(k(\phi, r)) = r/(1 - \phi)$ and define $H(\phi, r) \equiv \phi f'(k(\phi, r))k(\phi, r)$. The common tax rate $\underline{\phi}$ for projects a and b in the low revenue state is then found by solving $\underline{g} = 2H(\underline{\phi}, r)$.

If ϕ^* denotes the common (effective) tax rate given in Proposition 1, then all capital stocks entering the budget constraints for the first period and for the high revenue state in the second state are equal, and given by $k(\phi^*, r)$. From these two budget constraints - (7) and (8) - we then obtain, $(g_1 + \delta p \overline{g})/(1 + \delta p) = 2H(\phi^*, r)$.

This equation determines ϕ^*, which now obviously exceeds $\underline{\phi}$. The tax rates for project a in the first period and in the high state in the second period can be found similarly by using the budget constraint and the relation $\phi^* = (\phi_1^a + \delta p \overline{\phi}_2^a)/(1 + \delta p)$.

Proposition 1 shows that optimal capital taxation changes dramatically when one of the capital types is irreversible. So does the allocation of capital and the volume of output. It therefore becomes interesting to compare the aggregated output level for the situation when all capital types are fully flexible to the situation where one of the capital types is irreversible. Proposition 2 shows that this kind of irreversibility leads to higher expected production and is therefore beneficial to the government .

Proposition 2

Given the government's current revenue requirement, and the distribution of its state contingent future requirements, then expected production will always be higher if one of the capital types is irreversible than if all capital types were perfectly flexible.

Proof:

From Proposition 1 we know that when one capital type (a) is irreversible then optimal tax rates and capital stocks satisfy $\phi_1^b = \bar{\phi}_2^b = (\phi_1^a + \delta p\, \bar{\phi}_2^a)/(1 + \delta p)$ and $k_1^a = k_1^b = \bar{k}_2^a = \bar{k}_2^b = k^*$. The revenue requirements in the first period and in the bad state in the second period then yield

$$\hat{g} = (g_1 + \delta p\, \bar{g})/(1 + \delta p) = 2\phi^* f`(k^*)k^* = 2[f`(k^*) - r]k^*. \qquad (12)$$

The last equality in (12) follows from the capitalists' allocation rule (3). Similarly the optimal tax rates (ϕ_i^f) and capital stocks (k_i^f) in the fully flexible case must satisfy $g_i = 2\phi_i^f f`(k_i^f)k_i^f = 2[f`(k_i^f) - r]k_i^f$ for $i = 1,2$. The last term is concave in k. This equation and (12) then imply that $k^* > \hat{k}$ where $\hat{k} = k_1^f/(1 + \delta p) + k_2^f \delta p/(1 + \delta p)$ The production function is concave in k, which implies that $f(\hat{k})(1 + \delta p) > f(k_1^f) + \delta p\, f(k_2^f)$. Since $k^* > \hat{k}$ and since the capital stocks in the good state are not affected by the irreversibility, it follows that expected production is higher when one capital type is irreversible.

Conclusions

In this section we have shown that if investment opportunities differ with respect to their degree of irreversibility, then it is not optimal to level the playing field in taxation of heterogeneous capital. It is not our intention to argue in favour of the existing hybrid structure of investment taxation . We think, however, it is important to be explicit about

intertemporal aspects when taxation of investments are discussed. What we have shown here is that if one such intertemporal aspect - the differences in irreversibility of capital types - is taken into consideration, then it is no longer true that harmonized taxation, i.e. equal tax rates in every period and in every state, is optimal.

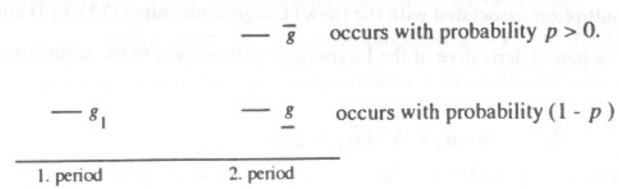
In the following section we discuss what implications capital types of different degrees of irreversibility have for the optimal distribution of the tax burden over time.

4 Optimal Debt policy

Assume now that the government is dispensed from the balanced budget requirement and that it can lend or borrow money in the first period. Obviously welfare is never lower in this case. It might be optimal to have a balanced budget in which case there is no change in welfare, but in general it will be optimal to run a deficit or a surplus, and then welfare will be higher. The issue we investigate here is *when* the government should borrow and *when* it should lend money.

Our results are surprising as they are in sharp contrast to the tax smoothing principle advocated by Barro [1979] and several others. Consider for example the following situation. Let the expected revenue requirement in the second period be much higher than the revenue needed in the first period. Moreover let the revenue requirements satisfy $\bar{g} > g_1 = \underline{g}$ such that in no states in the second period are the required tax revenues lower than in the first period.

This case is illustrated by the figure below;



In this situation, should the government run a deficit or a surplus in the first period? One would expect an economist to answer " a surplus" . The excess burden of taxation is a convex function of the amount of taxes collected. Based on this it should be optimal to smooth the tax level over time. This would be the right conclusion if all tax bases where fully reversible. In our model where one capital type is irreversible " a surplus" is, however, the wrong answer. In this case it is optimal for the government to run a deficit.

The reason why the common principle of tax smoothing does not apply in our model is the irreversibility of project *a*, and the way optimal taxes are differentiated between this project and the reversible capital type (*b*). In the following we will show this formally. We also discuss what implications it has for optimal tax rates.

Let s be the public surplus accumulated in the first period. The government can lend and borrow freely at the world market interest rate r. The government's optimization problem is as before except for the budget constraints, which are now given by:

$$g_1 + s = \phi_1^a f'(k_1^a)k_1^a + \phi_1^b f'(k_1^b)k_1^b \tag{13}$$

$$\bar{g} - s\delta^{-1} = \bar{\phi}_2^a f'(\bar{k}_2^a)\bar{k}_2^a + \bar{\phi}_2^b f'(\bar{k}_2^b)\bar{k}_2^b \tag{14}$$

$$\underline{g} - s\delta^{-1} = \underline{\phi}_2^a f'(\underline{k}_2^a)\underline{k}_2^a + \underline{\phi}_2^b f'(\underline{k}_2^b)\underline{k}_2^b \tag{15}$$

Consider the Lagrangian for this problem. As before, let μ_1, $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the (new) budget constraints (13), (14) and (15) respectively. The partial derivative of the Lagrangian with respect to the surplus s is then given by:

$$L_s = -\mu_1 + \delta^{-1}(\bar{\mu}_2 + \underline{\mu}_2)$$

$$= -\mu_1 + p\,\bar{\lambda}_2 + (1-p)\,\underline{\lambda}_2 \quad \text{, where } \bar{\lambda}_2 \equiv \frac{\bar{\mu}_2}{\delta p} \text{ and } \underline{\lambda}_2 \equiv \frac{\underline{\mu}_2}{\delta(1-p)}$$

All other derivatives of the Lagrangian are the same as in the balanced budget case. From the previous section we therefore know that when $s = 0$ and taxes are set optimally, then the multipliers μ_1 and $\bar{\lambda}_2$ are equal (see the discussion preceding Proposition 1). This means that we can write the derivative of the Lagrangian with respect to s - evaluated at $s = 0$ - as,

$$L_s|_{s=0} = \mu_1[-1 + p] + (1-p)\underline{\lambda}_2 \tag{16}$$

The envelope theorem implies that equation (16) gives an expression for the welfare effects of a marginal change in s, given optimal choice of tax rates. Moreover, it is easy to see that for the tax structure given in Proposition 1, the state contingent current value shadow price, $\underline{\lambda}_2$, for the good state is lower than the shadow price for government revenue in period 1, μ_1[2]. The expression in (16) - which captures the marginal value of a surplus evaluated at $s = 0$ - is clearly strictly *negative* as long as $p < 1$. That is, it pays the government to run a (marginal) deficit in the first period.

The next two issues that are natural to discuss are: How large should the government deficit be and - second - how are optimal tax rates on income from capital *a* and *b* characterized when the government freely can distribute the tax burden over time?

The reasoning that lead to the conclusion that it is optimal to run a deficit if revenue requirements satisfy $\bar{g} > g_1 \geq \underline{g}$ suggests that it is optimal to run a deficit as long as the optimal tax structure is characterised by Proposition 1. That is, it suggests that the government should run a deficit as long as the effective tax revenues satisfy $\underline{g} - \delta^{-1}s \leq g_1 + s \leq \bar{g} - \delta^{-1}s$. Recall that Proposition 1 was based on Lemma 1. A closer inspection of the proof of this lemma shows that its conclusions are valid if the assumption $\bar{g} > g_1 \geq \underline{g}$ is replaced by the weaker assumptions $\bar{g} > g_1$ and $\underline{g} < (g_1 + \delta p\bar{g})/(1 + \delta p)$. (See the last line of the proof in Appendix 1.) Hence Proposition 1 is

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[2]The multipliers satisfy $(1 + \mu_1)\phi_1^b = \mu_1\varepsilon(k_1^b)$ and $(1 + \underline{\lambda}_2)\underline{\phi}_2^b = \underline{\lambda}_2\varepsilon(k_2^b)$. Given $\varepsilon'(k) \geq 0$ the inequality $\phi_1^b > \underline{\phi}_2^b$ then implies $\mu_1 > \underline{\lambda}_2$.

also valid under this weaker set of assumptions. In the present context this means that it is optimal to run a deficit as long as the effective tax revenues satisfy

$$\bar{g} - \delta^{-1}s > g_1 + s$$

$$\text{and} \tag{17}$$

$$\underline{g} - \delta^{-1}s < [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p)$$

For a sufficiently large deficit ($s^* < 0$) the inequality

$\underline{g} - \delta^{-1}s < [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p)$,

will be replaced by an equality.

One can show that further increases of the deficit will not increase welfare (see Appendix 3). Hence we have,

Proposition 3

• *If* $\bar{g} > g_1$, $\underline{g} < (g_1 + \delta p \bar{g})/(1 + \delta p)$ *the optimal tax and debt policy entails running a deficit of magnitude* (s^*), *where* $s^* < 0$ *is the solution to*

$\underline{g} - \delta^{-1}s = [g_1 + s + \delta p(\bar{g} - \delta^{-1}s)]/(1 + \delta p)$.

• *Optimal tax rates satisfy* $\phi_1^a < \bar{\phi}_2^b = \phi_1^b = \underline{\phi}_2^a = \underline{\phi}_2^b = (\phi_1^a + \delta p \bar{\phi}_2^a)/(1 + \delta p)$, *implying that capital stocks are equalised across all states and periods*:

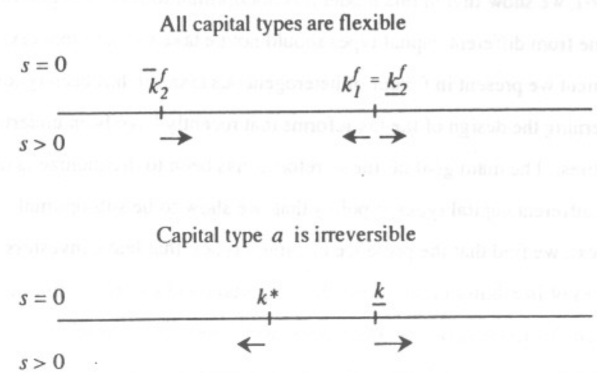
$k_1^a = k_1^b = \bar{k}_2^a = \bar{k}_2^b = \underline{k}_2^a = \underline{k}_2^b$.

A proof of Proposition 3 is given in Appendix 3.

Whether the government should run a deficit or a surplus in the first period does not depend on the expected magnitude of the future revenue requirement relative to the current revenue requirement. *It can be optimal for the government to run a deficit even if the expected revenue requirement in period two is ten times the revenue needed in the first period.* We have shown that this statement is true in cases where there is a positive

probability that the revenue requirement in the future will be higher than in the present period, and the government can collect taxes from both reversible and irreversible tax bases. This result is in sharp contrast to the common sense principle of tax smoothing, which says that since the excess burden of taxation is a convex function of the tax rate, the government should run a surplus in the first period if the expected revenue requirement in the future is higher than it is at present. It is, however, important to recognize that it is not the smoothing of tax rates in itself which gives higher welfare. In the situation we consider, all income effects of taxation are eliminated and the important thing to smooth between periods and states are tax bases - which are capital stocks - and not tax rates. In the traditional analysis there is only one fully reversible tax base. A higher tax rate levied on this tax base means less of it, and the conclusion of tax rate smoothing then follows. But it is really the smoothing of the tax base that drives the result.

We think the following argument explains why we do not get the standard tax smoothing result. Considered a case where $\bar{g} > g_1 = \underline{g}$. Let us first assume that the government's budget must balance, $s = 0$. If taxes are set optimally, which means a levelled playing field if all capital types are fully flexible, and taxes as indicated in Proposition 1 if a is an irreversible capital type, the allocation of capital will be as shown in the figure below:



Expected government outlay is assumed to be higher in the future than in the current period. The tax smoothing principle therefore prescribes a surplus in the first period. And from the figure we can see that in the flexible case, and given the concavity of the production function, the capital stocks move in the right directions when $s > 0$. The opposite is true in if capital type a is irreversible. If the government runs a surplus in this case it drives the capital stocks in the wrong directions. The optimal policy is therefore to run a deficit ($s < 0$). As we can see from Proposition 3, the optimal tax and debt policy generates perfect tax base smoothing; that is, in a second best optimum the capital stocks are equal across all periods and states. The tax rates are, however, far from smoothed between periods.

5 Concluding comments.

In this paper we have studied the problem of characterizing the optimal capital income tax when capital types differ with respect to their degrees of irreversibility, and there is uncertainty about the government's future revenue requirement. The distinction between irreversible and reversible capital types is important because it changes many of the conventional results about optimal taxation.

First, we show that in this model it is not optimal to level-the-playing-field. That is, income from different capital types should not be taxed at the same tax rate. The argument we present in favour of heterogeneous taxation has been ignored in the debate concerning the design of the tax reforms that recently have been undertaken in so many countries. The main goal of these reforms has been to harmonize taxation of income from different capital types, a policy that we show to be sub-optimal.

Next, we find that the presence of capital types that leave investors with different degrees of flexibility in the future, has important and surprising implications for the optimal distribution of the tax burden over time. The well known tax smoothing argument, which says that one should run a surplus (deficit) in the first period if the

required revenue in the future is much higher (lower) than in the current period, does not generalize to a model with tax bases of different durability. It is optimal to run a deficit in the present period even if the expected revenue future requirement is much higher - say ten times - than the current revenue requirement. This surprising result is driven by the way tax rates are optimally differentiated between irreversible and reversible capital types.

Appendix 1

Proof of Lemma 1.

Note first that $\underline{\phi}_2^a \geq \phi^*$ is equivalent to $\underline{\phi}_2^a \geq (\phi_1^a + \delta E\phi_2^a)/(1+\delta)$. Since the irreversibility constraint is binding in both states for $\underline{\phi}_2^a \geq \phi^*$, the capital allocated to project *a* will be given by

$$k_1^a = \bar{k}_2^a = f'^{-1}\left(\frac{r(1+\delta)}{(1-\phi_1^a) + \delta(1 - E\phi_2^a)}\right) \qquad (3')$$

Suppose now that is optimal to let $\phi_2^a > \phi^*$. Consider the maximization of expected production (6) subject to the revenue requirements (7) - (9) and the incentive constraint (3′), (4) and (5). Let μ_1, $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the revenue constraints (7), (8) and (9). To simplify the notation we use the following definitions in this appendix; $\bar{p} = \Pr[g_2 = \bar{g}]$ and $\underline{p} = (1 - \bar{p})$. Defining,

$$\bar{\lambda}_2 \equiv \frac{\bar{\mu}_2}{\delta\bar{p}} \text{ and } \underline{\lambda}_2 \equiv \frac{\underline{\mu}_2}{\delta\underline{p}},$$

we obtain the following first order conditions:

$$(1+\mu_1)\phi_1^b = \mu_1\varepsilon(k_1^b) \qquad (a,1)$$

$$(1+\bar{\lambda}_2)\bar{\phi}_2^b = \bar{\lambda}_2\varepsilon(k_2^b) \qquad (a,2)$$

$$(1+\underline{\lambda}_2)\underline{\phi}_2^b = \underline{\lambda}_2\varepsilon(k_2^b) \qquad (a,3)$$

$$(1+\mu_1)\phi_1^a + \delta E\phi_2^a + \bar{\lambda}_2\delta\bar{p}\,\bar{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\,\underline{\phi}_2^a = \qquad (a,4)$$
$$\{\mu_1[1 + \delta(1 - E\phi_2^a)] + \bar{\lambda}_2\delta\bar{p}\,\bar{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\,\underline{\phi}_2^a\}\varepsilon(k_1^a)$$

$$(1+\mu_1)\phi_1^a + \delta E\phi_2^a + \bar{\lambda}_2\delta\bar{p}\,\bar{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\,\underline{\phi}_2^a = \qquad (a,5)$$
$$\{\mu_1\phi_1^a + \bar{\lambda}_2[1+\delta - \phi_1^a - \delta\underline{p}\,\underline{\phi}_2^a] + \underline{\lambda}_2\delta\underline{p}\,\underline{\phi}_2^a\}\varepsilon(k_1^a)$$

$$(1+\mu_1)\phi_1^a + \delta E\phi_2^a + \bar{\lambda}_2\delta\bar{p}\,\bar{\phi}_2^a + \underline{\lambda}_2\delta\underline{p}\,\underline{\phi}_2^a = \qquad (a,6)$$
$$\{\mu_1\phi_1^a + \underline{\lambda}_2[1+\delta - \phi_1^a - \delta\bar{p}\,\bar{\phi}_2^a] + \bar{\lambda}_2\delta\bar{p}\,\bar{\phi}_2^a\}\varepsilon(k_1^a)$$

From the last two equations it follows that the current value of the multipliers for the two states in period 2 must be equal, $\bar{\lambda}_2 = \underline{\lambda}_2$. (Equating the expressions inside the curly brackets and rearranging yields $(\bar{\lambda}_2 - \underline{\lambda}_2)(1 - \phi_1^a + \delta(1 - E\phi_2^a)) = 0$, hence the claim follows.) From (a.4) and (a.5) it then follows that this common value for the period-2 multipliers must be equal to the multiplier for the period 1 revenue requirement:

$$\mu_1 = \bar{\lambda}_2 = \underline{\lambda}_2$$

Equations (a,1) - (a,4) then yield,

$$\phi_1^b = \bar{\phi}_2^b = \underline{\phi}_2^b = (\phi_1^a + \delta E\phi_2^a)/(1+\delta), \qquad (a,7)$$

and equal capital stocks across all states and periods ($k_1^a = \bar{k}_2^b = \underline{k}_2^b = k_1^b$).

Let k denote this common stock, let $h(k) \equiv f'(k)k$, and let $\hat{\phi}$ denote the common tax rate in (a.7). From the budget constraints we then have,

$$g_1 = (\phi_1^a + \hat{\phi})h(k)$$

$$\bar{g} = (\bar{\phi}_2^a + \hat{\phi})h(k)$$

$$\underline{g} = (\underline{\phi}_2^a + \hat{\phi})h(k)$$

By assumption $\underline{\phi}_2^a \geq \phi^* \equiv (\phi_1^a + \delta p\, \bar{\phi}_2^a)/(1 + \delta p)$. The budget constraints then imply $\underline{g} \geq (g_1 + \delta p\, \bar{g})/(1 + \delta p)$, which is a contradiction since $\underline{g} \leq g_1 < \bar{g}$.

This completes the proof.

Appendix 2
Proof of Proposition 1

The government maximizes expected production (6) subject to the constraints given by equations (3) - (9). Let μ_1 be the multiplier associated with the budget constraint in the first period. Let $\bar{\mu}_2$ and $\underline{\mu}_2$ be the multipliers associated with the high and low revenue requirement in the second period. Let p be the probability that $g_2 = \bar{g}$.

After some algebra the first order conditions for the tax rates for capital type b; ϕ_1^b, $\bar{\phi}_2^b$, $\underline{\phi}_2^b$ and a; ϕ_1^a, $\bar{\phi}_2^a$, $\underline{\phi}_2^a$ can be written in the following way:

$$(1 + \mu_1)\phi_1^b = \mu_1 \varepsilon(k_1^b) \qquad (a2,1)$$

$$(\delta p + \bar{\mu}_2)\bar{\phi}_2^b = \bar{\mu}_2 \varepsilon(k_2^b) \qquad (a2,2)$$

$$(\delta(1 - p) + \mu_1)\underline{\phi}_2^b = \mu_1 \varepsilon(\underline{k}_2^b) \qquad (a2,3)$$

$$(1 + \mu_1)\phi_1^a + (\delta p + \bar{\mu}_2)\bar{\phi}_2^a = \{\mu_1(1 + \delta p) + \bar{\phi}_2^a(\bar{\mu}_2 - \delta p \mu_1)\}\varepsilon(k_1^a) \qquad (a2,4)$$

$$(1 + \mu_1)\bar{\phi}_2^a + (\delta p + \bar{\mu}_2)\bar{\phi}_2^a = \{\bar{\mu}_2(1 + \frac{1}{\delta p}) + \phi_1^a(\mu_1 - \frac{\bar{\mu}_2}{\delta p})\}\varepsilon(k_1^a) \qquad (a2,5)$$

$$(\delta(1 - p) + \underline{\mu}_2)\underline{\phi}_2^a = \underline{\mu}_2 \varepsilon(\underline{k}_2^a) \qquad (a2,6)$$

By using the definition $\bar{\lambda}_2 \equiv \bar{\mu}_2/\delta p$ we can rewrite equations (a2,4) and (a2,5) to obtain,

$$(1 + \mu_1)\phi_1^a + \delta p(1 + \bar{\lambda}_2)\bar{\phi}_2^a = \{\bar{\lambda}_2(\delta p + 1) + \phi_1^a(\mu_1 - \bar{\lambda}_2)\}\varepsilon(k_a^1) \qquad (a2,7)$$

$$(1 + \mu_1)\phi_1^a + \delta p(1 + \bar{\lambda}_2)\bar{\phi}_2^a = \{\mu_1(1 + \delta p) + \bar{\phi}_a^2 \delta p(\bar{\lambda}_2 - \mu_1)\}\varepsilon(k_1^a) \qquad (a2,8)$$

The left side is the same in both equations. The expression in the parentheses on the right side of the equation must therefore also be equal. Using this information we get the following equation,

$$(\mu_1 - \bar{\lambda}_2)[1 - \phi_1^a + \delta p(1 - \bar{\phi}_2^a)] = 0.$$

Optimal tax rates are obviously lower than 100%. The first order conditions for optimal taxation therefore requires $\mu_1 = \bar{\lambda}_2$.

We can then write the first order conditions (a.2,1), (a.2,2) and (a.2,3) as follows,

$$(1 + \mu_1)\phi_1^b = \mu_1 \varepsilon(k_1^b) \qquad (a2,9)$$

$$(1 + \mu_1)\bar{\phi}_2^b = \mu_1 \varepsilon(k_2^b) \qquad (a2,10)$$

$$(1 + \mu_1)(\phi_1^a + \delta p\, \bar{\phi}_1^a) = \mu_1(1 + \delta p)\, \varepsilon(k_1^a) \qquad (a2,11)$$

Optimal tax rates in the first period and in the high revenue requirement state in the second period must therefore satisfy:

$$\frac{\phi_1^b}{\varepsilon(k_1^b)} = \frac{\bar{\phi}_2^b}{\varepsilon(\bar{k}_2^b)} = \frac{\phi_1^a + \delta p \bar{\phi}_2^a}{1 + \delta p} / \varepsilon(k_1^a) \qquad \text{(a2,12)}$$

For $\varepsilon'(k) \geq 0$ it is easy to see that, for any tax rates and capital stocks which satisfy (a2,12) and the relevant incentive constraints, the enumerators in (a2,12) must all be equal. Similarly, from (a.2,3) and (a.2,6) we obtain

$$\frac{\underline{\phi}_2^a}{\varepsilon(\underline{k}_2^a)} = \frac{\underline{\phi}_2^b}{\varepsilon(\underline{k}_2^b)} = \frac{\underline{\lambda}_2}{1 + \underline{\lambda}_2}$$

When $\varepsilon'(k) \geq 0$ this yields $\underline{\phi}_2^a = \underline{\phi}_2^b$. This completes the proof.

Appendix 3

Proof of Proposition 2.

The analysis in the text preceding Proposition 3 shows that welfare increases (strictly) with higher debt as long as $s > s^*$ (where $s^* < 0$) So, consider $s < s^*$. The effective revenue requirements $g_1(s) = g_1 - s$, $\underline{g}(s) = \underline{g} + \delta^{-1}s$ and $\bar{g}(s) = \bar{g} + \delta^{-1}s$ then satisfy

$$\underline{g}(s) > [g_1(s) + \delta \bar{g}(s)]/(1 + \delta p) \qquad (*)$$

From the proof of Proposition 1 in Appendix 2 it follows that in this case it is *not* optimal to implement a tax structure which induces the irreversibility constraint to bind only in the bad state. (If we solved the government's maximization problem under the revenue constraint (*) we would find that the first order conditions require $\underline{\phi}_2^a > \phi^*$, which implies that optimal second period tax rates must be binding in both states).

The proof of Lemma 1 in Appendix 1 shows that for the optimal such tax structure, where $\underline{\phi}_2^a > \phi^*$, the current value shadow prices of government revenue are all equal, μ_1

$= \bar{\lambda}_2 = \underline{\lambda}_2$. In this case the welfare effect of a marginal increase in the surplus s is given by

$$\begin{aligned} L_s &= -\mu_1 + \delta p\, \bar{\lambda}_2 \delta^{-1} + \delta(1-p)\, \underline{\lambda}_2 \delta^{-1} \\ &= 0 \end{aligned}$$

Hence $L_s < 0$ for all $s > s^*$ and $L_s = 0$ for all $s < s^*$ This means that setting $s = s^*$ maximizes welfare. The optimum is, however, not unique. Every $s < s^*$ gives the same welfare level. But for any such s the tax rates and the capital stocks will satisfy the conditions stated in Proposition 3.

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