

# *CES Working Paper Series*

ON TESTING THE SUSTAINABILITY  
OF GOVERNMENT DEFICITS IN  
A STOCHASTIC ENVIRONMENT

Henning Bohn

Working Paper No. 3

*Center for Economic Studies  
University of Munich  
Ludwigstr. 33  
8000 Munich 22  
Germany  
Telephone: 089-2180-2747  
Telefax: 089-397303*

## ON TESTING THE SUSTAINABILITY OF GOVERNMENT DEFICITS IN A STOCHASTIC ENVIRONMENT

### Abstract

In recent years, a number of empirical studies have examined the long-run sustainability of U.S. debt policy. Some studies conclude that U.S. fiscal policy has been sustainable, others disagree. This paper argues that the issue should be reexamined, because the traditional sustainability tests explicitly or implicitly assume that the rate of return on government debt is "on average" above the rate of economic growth, a condition that does not hold for historical U.S. data. The paper derives and implements a new test for sustainability that does not rely on a particular relation between interest rates and growth rates. I conclude that U.S. fiscal policy has historically satisfied a sufficient condition for sustainability.

*Henning Bohn*  
*Department of Finance*  
*The Wharton School*  
*University of Pennsylvania*  
*Philadelphia, PA 19104-6367*  
*USA*

## 1. Introduction

In recent years, a number of empirical studies have examined the question whether U.S. fiscal policy has historically been on a sustainable path, i.e., whether it is consistent with the government's intertemporal budget constraint. Some conclude that U.S. fiscal policy has been sustainable, others disagree. (See Hamilton and Flavin 1986, Hakkio and Rush 1986, Kremers 1989, Trehan and Walsh 1988 and 1991, and Wilcox 1989.)<sup>1</sup>

This paper reexamines the sustainability issue and implements a new test for the sustainability of U.S. fiscal policy. A reexamination is needed because the existing tests are based on theoretical models that explicitly or implicitly assume an interest rate on government bonds above the average growth rate of the economy. In fact, interest rates on U.S. government bonds have been below the average growth rate for long periods, including the periods that were studied in the literature. The sustainability condition tested in this paper does not require a specific inequality between average growth rates and interest rates.

The empirical analysis is based on the theoretical work of Bohn (1990), which derives constraints on sustainable policies for stochastic, dynamically efficient economies. A sufficient condition for the sustainability of government policy in such economies is that the level of the primary deficit responds positively to marginal changes in the debt-GNP ratio. The empirical analysis shows that such a positive response exists in U.S. fiscal policy, which suggests that U.S. government policy has historically been sustainable.<sup>2</sup> The empirical part of the paper also comments on inference problems created by high deficits during war periods and suggests that the resulting jumps in debt may have been a source of confusion in interpreting fiscal data.

The paper is organized as follows. Section 2 reviews the theoretical issues and derives the sufficient condition for sustainability. Section 3 examines the empirical evidence. Section 4 concludes.

## 2. Conditions for Sustainability

This section reviews the government's intertemporal budget constraint and the transversality condition and then derives a sufficient condition for sustainability.

### 2.1. What is the appropriate Sustainability Condition?

The first issue is the appropriate form of the intertemporal constraint on fiscal policy. Traditionally, empirical studies have simply asserted that the path of government debt has to satisfy a constraint of the form

$$\lim_{N \rightarrow \infty} \frac{1}{(1+r)^N} \cdot E_t[D_{t+N}] = 0 \quad (1)$$

where  $r$  is the safe interest rate,  $D_{t+N}$  government debt at the start of period  $t+N$ , and  $E_t$  the conditional expectation at time  $t$ . As empirical proxies for the safe interest rate, some studies use the average return on government debt (e.g., Hamilton and Flavin 1986), others use the  $N$ -period compound actual return on government debt (e.g., Wilcox 1989).<sup>3</sup>

However, such an approach is questionable for two reasons. From a theoretical perspective, the question whether or not a transversality constraint has to hold is a general equilibrium issue. In asserting a constraint without providing a general equilibrium setting, one has to rely implicitly on some other body of theory that may or may not be appropriate for the empirical analysis. From an empirical perspective, we know that real returns on U.S. government debt have been rather low, at least prior to 1980. The average real return on U.S. Treasury bills, for example, has been 0.23% for 1929-88 and 1.11% for 1954-1988 (Bohn 1990). Both values are well below the average U.S. growth rate.

The relation between growth rate and interest rate is important, because equation (1) will be violated whenever government debt grows at a rate above the safe interest rate. But in an economy in which the safe interest rate is below the growth rate of

aggregate income (GNP) on average, this constraint will be violated even if the debt-GNP ratio is constant or slightly falling. That is, sustainability tests based on equation (1) would find a policy with constant debt-GNP ratio non-sustainable. Such a conclusion is clearly unwarranted,<sup>4</sup> suggesting that constraint (1) is not a necessary condition for sustainability. Unfortunately, equation (1) is the condition tested in the literature (Hamilton and Flavin 1986, Hakkio and Rush 1986, Kremers 1989, Trehan and Walsh 1988 and 1991, Wilcox 1989, Corsetti and Roubini 1991).<sup>5</sup>

One might add that problems with equation (1) should not really come as a surprise, because the deterministic and certainty-equivalence models in which a constraint like (1) can be derived just do not allow steady states with dynamic efficiency combined with a real interest rate below the rate of economic growth. However, dynamic efficiency and a low real interest rate are both key characteristics of historical U.S. data.<sup>6</sup> Thus, models that would justify equation (1) are not an appropriate theoretical framework for an empirical analysis of U.S. fiscal policy.

To derive constraints on government policy that apply even if the safe interest rate is relatively low, Bohn (1990) has examined stochastic dynamically efficient economies populated by risk-averse individuals. In this more general analytical framework, the safe interest rate can be below the rate of economic growth even though the economy is dynamically efficient by construction.<sup>7</sup> The relevant constraint on the path of government debt turns out to be

$$\lim_{N \rightarrow \infty} E_t[u_{t,N} \cdot D_{t+N}] = 0, \quad (2)$$

where  $u_{t,N}$  is the marginal rate of substitution between periods  $t$  and  $t+N$ . The key difference between (1) and (2) is that, according to (2), the rate at which debt  $D_{t+N}$  is discounted depends on the probability distribution of debt across states of nature and its correlation with the marginal rate of substitution, but not on the interest rate on government debt.<sup>8</sup> For example, if government policy is such that the amount of

outstanding debt is asymptotically proportional to aggregate income, then the correct discount rate in the transversality condition is the rate at which claims on future income would be discounted. Since the discount rate of future income will be above the rate of income growth (as an implication of dynamic efficiency), a debt policy with stable debt-GNP ratio will be found sustainable in the sense of equation (2).<sup>9</sup>

Overall, the point is that equation (2) and not equation (1) is the sustainability condition that should be tested empirically.

## 2.2. A Strategy for Empirical Testing

At first sight, one might think that tests of condition (2) might proceed analogously to tests of condition (1), using estimates of the marginal rate of substitution instead of the safe interest rate. Unfortunately, there is a more fundamental inference problem that suggest a different empirical strategy.<sup>10</sup>

The main problem is that if real interest rates are as low as they have been in the U.S., the debt-GNP ratio will have a downward drift in expectation even if the government is simply rolling-over all debt with interest. A policy of always rolling over debt with interest clearly violates equation (2), which one can see most easily by examining the intertemporal budget constraint associated with equation (2), which is

$$D_t = \sum_{n \geq 0} E_t[u_{t,n} \cdot (T_{t+n} - G_{t+n})] \quad (3)$$

This constraint shows that initial debt must be balanced by later primary surpluses in at least some periods and some states of nature. The problem with the rolling-over strategy is that there are some sample paths along which economic growth is (unexpectedly) below the interest rate. However, these sample paths may have very low probability. Unless one of these sample paths has actually occurred, the government will be able to sustain a rolling-over strategy (or something similar) for ever. In the data, one will often

observe a downward drift in the debt-GNP series. But that does not provide information on whether the government pursues a sustainable policy.

To clarify this point with a simple example, suppose government policy is to let debt grow at the rate of interest unless and until the debt-GNP hits an upper bound, at which point taxes will be increased. Because of the upper bound on the debt-GNP ratio, the policy is sustainable in the sense of (2). Assuming the interest rate is below the average growth rate, debt will grow more slowly than aggregate income in expectation. A tax increase will occur only very rarely. Unless one of the rare states of nature with a tax increase occurs along the sample path that is realized, an outside observer cannot distinguish this policy from the non-sustainable policy of always rolling over debt.<sup>11</sup>

More generally, the point is that the single sample path of the economy that we observe does not always provide enough information about the distribution of fiscal variables across different possible sample paths to assess the sustainability of government policy.<sup>12</sup> Since average returns on U.S. government debt have historically been below the U.S. growth rate, the possibility of such inference problems cannot be dismissed easily. To draw empirical inferences about the sustainability of government policy, one has to hope that actual government policy belongs to a class of policies for which inferences can be made. Inferences about sustainability will have to be conditional on the assumption that actual government policy falls into the relevant class of policies.

The empirical analysis in this paper will derive such conditional results. The empirical strategy is to exploit the fact that policy parameters can be estimated from a finite sample, if the policy variables are linear functions of a small number of state variables and if there is sufficient sample variation in these state variables. A single sample path will then provide enough information to assess sustainability. The analysis will therefore focus on a class of linear policy rules. The next section will derive sufficient conditions for the sustainability of such policies. If an estimated policy satisfies these conditions, one may conclude that government policy is sustainable.<sup>13</sup> If not, one

might have to examine whether there is a less stringent sufficient condition that the policy satisfies, or consider the possibility that government policy may be non-linear.

### 2.3. Sufficient Conditions for Sustainability

The set of sufficient conditions for sustainability stated below is motivated by the fact that the government cannot run permanent primary budget deficits (see (3)). If the government runs primary deficits and if debt grows relative to GNP, the government must eventually respond by reducing the deficit. If this reaction happens only at high levels of debt, we may not see it along the observed sample path. That is the inference problem. But if taxes and/or spending respond linearly to higher initial debt, one should see a positive reaction of the primary budget surplus to changes in the debt-GNP ratio at all levels of the debt-GNP ratio. One should be able to estimate it. The proposition below will show that such a positive reaction is sufficient for sustainability, provided the other determinants of the ratio of primary surpluses to GNP are bounded.

To state the result formally, some notation is useful. Denote aggregate income (GNP) by  $Y_t$ , the "tax rate" by  $\tau_t = T_t/Y_t$ , the ratio of government spending to income by  $g_t = G_t/Y_t$ , the debt-income ratio (at the beginning of a period) by  $d_t = D_t/Y_t$ , and the primary surplus relative to income by  $s_t = \tau_t - g_t$ . Let  $R_t$  be the return on government debt and  $y_t$  be the growth rate of income. Then the usual budget equation in levels

$$D_{t+1} = (1+R_{t+1}) \cdot [G_t - T_t + D_t] \quad (4)$$

implies a budget equation in ratio form

$$d_{t+1} = x_{t+1} \cdot [g_t - \tau_t + d_t] = x_{t+1} \cdot [d_t - s_t] \quad (5)$$

where  $x_{t+1} = (1+R_{t+1})/(1+y_{t+1})$ . Note that the return on debt has to satisfy the Euler condition  $E_t[u_{t+1} \cdot (1+R_t)] = 1$ . With this notation, the sufficient condition is as follows. The proof is in the appendix.



PROPOSITION 1:

If the stream of aggregate income  $Y_t$  has a finite present value and if the primary surplus can be written as

$$s_t = \mu_t + \rho \cdot d_t \quad (6)$$

where  $\mu_t$  is a bounded stochastic process and  $\rho > 0$ , then government policy satisfies constraints (2) and (3), i.e., policy is sustainable in this sense. ♦

The key element in this statement is the requirement that the government responds to increased initial debt by increasing the primary surplus. The assumption of a finite present value of income seems reasonable, because otherwise the economy would have infinite wealth. The boundedness assumption could be replaced by a stationarity assumption combined with a restriction on the correlation between  $\mu_t$  and the income process (see the appendix). But since the boundedness assumption does not appear to be very stringent—the bound may be as wide as one wishes—more complicated conditions would only be distracting. Thus, Proposition 1 is far from being necessary for sustainability, but it should suffice for empirical analysis. Note that in contrast to the conditions derived in the literature, this condition does not require a positive rate of return on government debt or a positive safe interest rate. Also, it does not require government bond returns or safe interest rates above the rate of economic growth.

The objective of not imposing restrictions on interest rates also motivates why the proposition has been written in terms of the primary surplus as function of debt rather than as condition on the path of debt (cf. Kremers 1989). If the primary deficit satisfies (6), the debt-income ratio will have the law of motion

$$d_{t+1} = x_{t+1} \cdot (1-\rho) \cdot d_t + x_{t+1} \cdot \mu_t \quad (7)$$

In models where the interest rate exceeds the growth rate on average in the sense that  $E[x_{t+1}] = x > 1$ , it may be instructive to examine the path of debt and its unit root properties, because one might be able to infer that  $\rho > 0$ , if one finds a coefficient on lagged debt (i.e.,

an estimate of  $x(1-\rho)$  less or equal to one. However, such an argument cannot easily be made here, because  $x$  may well be less than one. The debt-GNP ratio will then have a downward drift even if the government is rolling over debt with interest (recall Section 2.2). Since real interest rates on U.S. government bonds have historically been below the growth rate (at least up to 1980), the case  $x < 1$  may in fact be the empirically most relevant case.<sup>14</sup> Then a precise estimate of how far the regression coefficient is below one and a precise estimate of the value of  $x$  would be required to infer a positive value of  $\rho$  from (7). The empirical analysis will therefore focus on equation (6).<sup>15</sup>

If the process  $\mu_t$  is stationary, the policy rule (6) links the time series properties of the primary surplus and of debt. If debt has a unit root, the primary surplus should also have one, and  $(s_t, d_t)$  should be co-integrated with vector  $(1, -\rho)$ . If debt is stationary, the primary surplus should be stationary, too. Since the literature on co-integration tests has emphasized such links (e.g., Trehan and Walsh 1988, 1991), unit root issues will be reviewed in the empirical section.<sup>16</sup>

Finally, a note on potential additional constraints on the debt-income ratio may be appropriate. As in deterministic models (McCallum 1984), the transversality condition (2) does not require a bounded or even stationary debt-income ratio. However, there may be economies in which the government faces an upper bound on the debt-income ratio or on tax revenues (Blanchard 1984, Kremers 1989, Bohn 1991b). If there are such additional restrictions on government policy, constraint (2) will be necessary but not sufficient for the feasibility of government policy. Fortunately, in the case of  $x < 1$ , a policy with  $\rho > 0$  should have a debt-income ratio that is usually declining rather than growing (see (7)), which suggests that constraints on the debt-income ratio may well be satisfied. In any case, the empirical analysis will focus on testing the necessary condition (2). The question whether U.S. government policy satisfies more stringent additional conditions would be beyond the scope of this paper and is left for future research.<sup>17</sup>

### 3. Empirical Evidence

This section will examine two types of empirical evidence. First, a structural model of government behavior based on Barro's tax-smoothing approach will be used to estimate a process for the primary surplus. Second, I will examine the time series properties of the fiscal data.

#### 3.1. Estimating the Determinants of the Primary Surplus

The basic framework for the structural analysis is Barro's (1979, 1986a, 1986b) tax-smoothing model.<sup>18</sup> The model considers an optimizing government that minimizes the the cost of tax collection by smoothing marginal tax rates over time. Key features of the optimal policy are that tax rates should only depend on permanent government spending and on initial debt, i.e., not vary over the business cycle or with temporary fluctuations in spending. If one subtracts current non-interest spending from taxes to obtain the primary surplus (all relative to GNP), the model implies that the level of temporary government spending GVAR and a business cycle variable YVAR are the determinants of the non-debt components of primary surplus,  $\mu_t$ . When an approximation error  $\varepsilon_t$  is included, one has

$$\mu_t = \alpha_0 + \alpha_G \cdot \text{GVAR}_t + \alpha_Y \cdot \text{YVAR}_t + \varepsilon_t$$

and

$$s_t = \alpha_0 + \alpha_G \cdot \text{GVAR}_t + \alpha_Y \cdot \text{YVAR}_t + \alpha_d \cdot d_t + \varepsilon_t \quad (8)$$

where the  $\alpha$ 's are coefficients and  $\alpha_d$  is the estimator for  $\rho$ . The variables GVAR and YVAR are taken from Barro (1986a). The empirical analysis will focus on the question whether the coefficient  $\alpha_d$ , which estimates  $\rho$ , is positive.<sup>19</sup>

Estimates of equation (8) are in Table 1.<sup>20</sup> Column 1 shows the results for the sample period 1916-1983, which is the full sample for which Barro's regressors are available. Column 2 has the sample period 1920-82 excluding 1941-47, which is the period Barro (1986a) used. Column 3 has the postwar sample 1954-83, and column 4 uses 1960-83,

which is close to the sample period used in Hamilton and Flavin (1986) and Wilcox (1989). As one can see,  $\alpha_d$  is significantly positive in all regressions. The variables GVAR and YVAR enter negatively, as the Barro model predicts.<sup>21</sup>

Table 2 displays sample averages of some of the variables. For all four sample periods, the return on government bonds  $r = E[R_t]$  is below the average growth rate  $y = E[y_t]$ .<sup>22</sup> The average primary surplus is negative overall and only slightly positive in some subperiods.<sup>23</sup> The fact that  $r < y$  holds in the data is noteworthy for two reasons. First, it underscores the doubts about the applicability of models in which dynamic efficiency is linked to the inequality  $r > y$ . Tests based on such models would find U.S. government policy non-sustainable, even if it had a stable debt-GNP ratio.<sup>24</sup> Second, the fact that  $r - y < 0$  and also  $\alpha_d > 0$  suggests that the debt-income ratio will have mean reversion (see (7)). This is confirmed in Table 3. Table 3 reports regressions similar to those in table 1, but with  $s_t$  replaced by  $\Delta d_{t+1}$ . The coefficient on  $d_t$  is negative in all cases.<sup>25</sup>

The mean reversion result for debt is interesting in light of Barro's (1979) prediction that the debt-income ratio should be non-stationary, i.e. that  $\rho \approx r - y$ .<sup>26</sup> Here, one would have to be seriously concerned about sustainability if the equality  $\rho \approx r - y$  were indeed true, because that would imply  $\rho < 0$ . Still, one may ask whether  $\rho \approx r - y$  implies a rejection of the tax-smoothing model. Fortunately (for the model), the prediction  $\rho \approx r - y$  does not extend to a stochastic setting with risk aversion. Though a general analysis of optimal taxation in stochastic models is clearly beyond the scope of this paper, the appendix shows for a special case that the optimal policy will satisfy  $\rho > 0$  irrespective of the sign of  $r - y$ .

Overall, the estimates suggest that the sufficient condition for sustainability is satisfied. The primary deficit responds positively to increased debt. Moreover, there is evidence for mean reversion in the debt-income ratio.

### 3.2. *Time Series Properties of Fiscal Data*

This section will discuss the time series properties of the debt and deficit series. It will also examine how the above results relate to the literature that uses unit root and cointegration results to draw inferences about sustainability. The main purpose of the section is to defend the above conclusions against potential objections based on unit root tests.

The non-structural analysis of fiscal variables has been deferred so far, because the raw data are difficult to interpret without some reasonable null hypotheses derived from a structural model. A theoretical framework is particularly helpful here, because the statistical tests cannot easily distinguish a process with unit root from a stationary series with high autocorrelation. Unfortunately, these are the relevant alternatives for the series of debt-income ratios. An additional problem for data analysis is created by the war periods, which introduce significant heteroskedasticity into the government spending, deficit, and debt series. In effect, all the big movements in these series occurs over a few years that are special in many ways.

In the literature, inferences about unit roots are usually based on augmented Dickey-Fuller (ADF) tests that test the null hypothesis that the series are integrated of order one (Dickey and Fuller 1981, Fuller 1976). Such test statistics are displayed in Table 4. Panel A has the tests on  $d_t$  and  $s_t$  for the sample period 1916-1983, for the longer sample period 1800-1988, and for subperiods that exclude the major wars. The 1800-1988 sample is provided because the unit roots issue concerns the long-run properties of the data, about which a long sample should be most informative.<sup>27</sup> Using the full 1916-83 and full 1800-1988 samples, one cannot reject the null hypotheses that government debt is  $I(1)$  and that the deficit is  $I(0)$ , where  $I(\cdot)$  denotes the order of integration. Similar results have been obtained in the literature (e.g., Trehan and Walsh 1988, 1991).<sup>28</sup>

The result that debt and the primary deficit have different orders of integration is somewhat disturbing, because it is difficult to reconcile with the sustainability condition

(6), unless one assumes that the  $\mu_t$  process is non-stationary but still bounded. However, I will argue that the unit root result for  $d_t$  should not be accepted at face value. The unit root result for  $d_t$  is rather odd in light of equation (5) and in light of the previous finding that the interest rate on government debt is below the growth rate, i.e., that  $E[x_{t+1}] = x < 1$ . If  $s_t$  is stationary and  $x < 1$ , then equation (5) implies that  $d_t$  should not have a unit root. However, since  $x$  is close to 1, the debt-income ratio should be highly autocorrelated. Thus, a highly autocorrelated stationary process is the appropriate null hypothesis for  $d_t$ . But this is not the hypothesis tested in the Dickey-Fuller tests, which have in fact notoriously low power against highly autocorrelated alternatives. Given the overwhelming evidence against a unit root in  $s_t$ , the failure to reject the null hypothesis of the ADF test should therefore not be seen as evidence that  $d_t$  is  $I(1)$  but rather as evidence that the ADF test has insufficient power against the relevant highly autocorrelated alternative.

In addition, the failure to reject a unit root for debt seems to be driven largely by the war periods. Table 4 shows that a unit root in debt can be rejected strongly, when the two world wars are excluded from the 1916-83 sample and when the three major wars (Civil War, World War I and World War II) are excluded from the 1800-1988 sample. The unit root rejections for  $s_t$  become even stronger.

The impression that the debt-GNP ratio is indeed stationary and that the war periods deserve special scrutiny is visually reinforced by the graph of debt shown in Figure 1. The series has a visible downward drift during peacetime periods interrupted by upward jumps during the wars. The wartime jumps in the debt-GNP ratio are in line with the tax-smoothing model, which predicts that temporary military spending should be largely debt-financed. But these movements create problems in the unit root tests, because high deficits often occur in the later years of a war, i.e., right after debt has already been driven up by the deficits in the early years of the war. This produces a positive link between the change in debt and the level of debt during war years, which obscures the

longer run mean reversion—or at least raises the standard errors so much that a unit root cannot be rejected. In contrast, a nice feature of the structural model of Section 3.1 is that the GVAR variable absorbs most of the short-term disturbances created by wars.

An alternative method to remove some war-related movements from the debt series—instead of simply excluding wars—is to split off a component of debt that is related to military spending. Define

$$\gamma_t = \sum_{i \geq 0} [(\prod_{j=1}^i x_{t-j}) \cdot (1-\rho)^i \cdot m_{t-i}]$$

and  $b_t = d_t - \gamma_t$ . The variable  $\gamma_t$  can be interpreted as the cumulative effect of past military spending on current debt in the sense that  $b_t$  is the amount of debt that the country would have if military spending had been zero for ever and if the primary surplus had been higher by that amount. The stationarity of  $b_t$  can be tested without computing  $\gamma_t$ , if one notes that a regression of  $\Delta b_{t+1}$  on  $b_t$  is algebraically equivalent to a regression of  $\Delta d_{t+1}$  on  $d_t$  and  $(x_{t+1} \cdot m_t)$ . Since military spending  $m_t$  is clearly stationary (with and without war periods, see Table 4), the null hypothesis of a unit root in  $b_t$  can be tested by examining the coefficient of  $d_t$  in the latter regression. As Table 4 shows, a unit root in  $b_t$  is strongly rejected.<sup>29</sup> Thus, the stationarity question about  $d_t$  reduces to the question whether  $\gamma_t$  is stationary, i.e., whether the government pays off past war-spending fast enough that it does not accumulate from one war to the next.

This final question is somewhat difficult to answer, because we have only three observations on major wars, which does not allow strong inferences about the asymptotic movements of  $\gamma_t$ . One should therefore acknowledge that, depending on ones prior beliefs about the frequency of future wars, one may arrive at different judgements. For example, those who doubt the stationarity of the debt-GNP series may argue that the peak debt-GNP ratios increased from Civil War to WWI to WWII. On the other hand, the fact that the process for military spending  $m_t$  is stationary combined with  $E[x_{t+1}] < 1$  and  $(1-\rho) < 1$  suggests that  $\gamma_t$  is stationary.

Overall, there is no convincing evidence for a unit root in the debt-GNP ratio  $d_t$ . Failures to reject a unit root for periods that include wars are likely due to the inability to distinguish between high autocorrelation and a unit root. The only qualification of this result is that one should hesitate to make definitive statements about a variable that is strongly affected by war-related spending, given that one has only three observations on major wars.

This conclusion about the debt-GNP ratio is significant for several reasons. First, the finding that both  $s_t$  and  $d_t$  are stationary assures that the results of Section 3.1 do not suffer from a spurious regression problem. It also implies that those regressions cannot be interpreted as co-integrating. The difficulty of drawing inferences in the context of war-related problems explains why a structural model was used for estimating the effect of  $d_t$  on  $s_t$ .

Second, the unit root results provide a connection to Kremer's (1989) study of the sustainability of government debt.<sup>30</sup> He finds a unit root in the logarithm of the debt-income ratio. The contrast suggests that the choice of levels versus logs may matter for small sample inferences. Also, note that if one took logs in equation (7) and if  $\mu_t$  is relatively small, one would likely find a unit root in  $\log(d_{t+1})$ , because the multiplicative term involving  $d_t$  would (approximately) be transformed into a sum with a unit coefficient on  $\log(d_t)$ . This will be true even if  $\rho$  is far above zero.

Third, the result that there is neither a unit root in the primary deficit nor in government debt, provided the variables are defined as GNP shares, raises some questions about the interpretation of sustainability tests based on co-integration. Most of these tests—e.g., Trehan and Walsh (1988,1991), Hakkio and Rush (1986)—use real levels of fiscal variables and find unit roots in government spending, debt, and taxes. From the results here, it appears that the unit root in real debt is either not really there or due to a unit root in GNP. The unit roots in spending and taxes seem to exist in both definitions (see Bohn 1991a).<sup>31</sup>



For long sample periods (say, 1800-1988), an analysis in GNP-shares is clearly preferable to an analysis in levels, because there would be extreme heteroskedasticity if levels of fiscal variables were used. For shorter sample periods, this heteroskedasticity problem may not be as obvious, but it remains troubling. If the analysis is done terms of GNP-shares, the stationarity of  $s_t$  and  $d_t$  implies that regressions involving debt and deficits cannot be interpreted as co-integrating regressions. The non-stationarity of  $\tau_t$  and  $g_t$  together with the stationarity of  $s_t$  implies co-integration between  $\tau_t$  and  $g_t$ . This can be exploited to obtain insights about the behavior of governments in response to high deficits (Bohn 1991a), but it does not provide information about sustainability.<sup>32</sup> Overall, theorems about co-integration do not appear to be particularly helpful for assessing the sustainability of government policy.<sup>33</sup>

#### 4. Conclusions

The paper has derived a sufficient condition for the sustainability of government policy that is applicable even if the interest rate on government bonds is below the average rate of economic growth. Based on this condition, the paper concludes that U.S. fiscal policy has historically been sustainable. The paper also suggests that some previous tests of the sustainability of government debt may have to be interpreted cautiously, because they rely on models that implicitly assume a safe interest rate above the average rate of economic growth, an inequality which is not satisfied in the data.

### Footnotes

\* The paper was written while I was visiting the Center for Economic Studies at the University of Munich. Financial support from the Geewax-Terker Program in Financial Instruments is gratefully acknowledged.

<sup>1</sup> Corsetti and Roubini (1991) examine the same question for a cross-section of countries.

<sup>2</sup> It should be emphasized, however, that the paper studies a long-run sample of fiscal policy variables that ends in 1983. It does not attempt to assess whether U.S. fiscal policy of the 1980s was different. This may be an interesting topic for future research.

<sup>3</sup> The exact definition does not matter much for the argument below, as long as the discounting is done at a rate somewhere near the average rate of return on government bonds. I will use the label "safe interest rate" as shorthand for the precise definitions.

<sup>4</sup> Bohn (1990) shows that a bounded debt-income ratio is a sufficient condition for sustainability.

<sup>5</sup> Kremers and Corsetti-Roubini also examine time series of debt-income (GNP or GDP) ratios, supposedly to test a stronger constraint. If the safe interest rate is below the average growth rate, a stationary debt-income ratio is in fact a much weaker constraint than (1) (to be discussed below).

<sup>6</sup> See Abel et al. (1989) on dynamic efficiency.

<sup>7</sup> That is, the relevant risky rate of return is above the growth rate on average; see Abel et al. (1989), Zilcha (1991).

<sup>8</sup> Equation (2) reduces to (1) if individuals are risk neutral. The expectation in (2) can also be interpreted as integral of debt times state-contingent claims prices over all states of nature.

<sup>9</sup> A proof of constraint (2) and more details on the model are provided in Bohn (1990). Note that the discount rate in (2) does not depend on whether the government issues securities that are safe or "risky" on a period-by-period basis.

<sup>10</sup> In the literature, three types of sustainability tests are common. A first set of papers estimates empirical proxies for the path of discounted government debt (e.g., Hamilton-Flavin 1986, Wilcox 1989). A second set of papers exploits co-integration properties of fiscal data that are implied by the intertemporal budget constraint (e.g., Hakkio and Rush 1986, Trehan and Walsh 1988, 1991). And a third set of papers examines the path of debt-GNP ratios (e.g., Kremers 1989). All three approaches test equation (1). The comments of this section apply directly to the first and third group of tests. Co-integration tests are discussed in Section 3.2.

11 The general inference problem as well as this example are discussed in more detail in Bohn (1990).

12 Similar inference problems have been noted in the context of the "Peso problem" in the foreign exchange literature and in the context of the question of "excess volatility" in stock prices.

13 That is, the policy is sustainable if it is indeed a member of the class of policies for which the condition applies. This qualification has to be added because even if the policy appears linear in the sample, one cannot exclude the possibility that it has a non-linearity in some "rare" states of nature that remain unobserved and that may make the policy non-sustainable.

14 However, given the shift to higher real interest rates in the 1980s, the process  $x_{t+1}$  may not even have an unconditional expectation and it may have a conditional expectation that sometimes switches signs. If that is the case, inferences from (7) would be even more difficult.

15 On the other hand, an estimate of  $x \cdot (1-\rho)$  at or above one—e.g., a unit root in the debt income ratio—might under some conditions be interpreted as evidence against  $\rho > 0$  if  $x < 1$ . Because of this potential evidence against  $\rho > 0$ , the properties of the debt-income ratio will also be examined.

16 A difference to this literature is that the variables are income-shares, not levels (cf. Hamilton-Flavin 1986, Trehan and Walsh 1988, 1991, Wilcox 1989). This is done partly for econometric reasons—to eliminate growth trends and potential heteroskedasticity—and partly for plausibility. If fiscal variables were defined as stationary stochastic processes in levels while GNP has a unit root component, the ratio might become implausibly small or large. (See below for more details.)

17 The main problem is that if one takes the notion of an upper bound on tax revenues serious, it implies an upper bound on the debt-income ratio for all states of nature, not just in expectation (Bohn 1991b; cf. Corsetti and Roubini 1991). Boundedness is a much more stringent condition than mere stationarity. Stationarity seems satisfied (see below), but as noted above, it does not assure sustainability. Therefore, is it not clear how to interpret time-series analyses of debt-income ratios (Kremers 1989, Corsetti-Roubini 1991).

18 This model was also used in Kremer's (1989) sustainability paper. It was originally derived as a partial equilibrium model (Barro 1979), but it can easily be embedded in a general equilibrium setting (see the appendix).

<sup>19</sup> I will not examine the boundedness of  $\mu_t$  in any formal way. The variable GVAR, the ratio of temporary spending to GNP, is by definition bounded in the interval  $[-1,1]$ . Boundedness of YVAR is plausible, because it is just the ratio of actual to potential GNP. The error term  $\epsilon_t$  is small in the data (see the footnote below).

<sup>20</sup> All regressions use Ordinary Least Squares (OLS) estimation. White's (1980) robust standard errors are provided to address potential heteroskedasticity problems.

<sup>21</sup> For completeness, residuals from equation (8) were subjected to unit root tests to verify the stationarity of  $\mu_t$ . A unit root was rejected strongly in all cases. AR(1) regressions on the residuals did not yield any significant autocorrelations. Concerning boundedness, the values of  $\epsilon_t$  always remain in relatively narrow intervals: -6.1% to +3.6% in the regression of Column 1, -2.8% to +3.3% for Column 2, and -1.2% to +1.4% for Column 3 and 4. For comparison,  $s_t$  takes values in  $[-24.8\%, +7.4\%]$ .

<sup>22</sup> The return estimates in the table are annual net federal interest payments divided by debt at the start of the fiscal year, minus inflation measured by the GNP-deflator. I also computed a series for  $x_{t+1}$  by dividing  $d_{t+1}$  by  $(d_t - s_t)$  and obtained a mean of 0.982, which implies  $r - y = -1.8\%$ . Further evidence that the safe interest rate has been below the average U.S. growth rate for long periods is collected in Bohn (1990).

<sup>23</sup> As explained in Bohn (1990), an average deficit is consistent with sustainability and it should not even be surprising when  $r < y$ .

<sup>24</sup> For the Barro data set used here, the debt-GNP ratio in 1983 is above the ratio in 1916, but below the ratios of 1920, 1954, and 1960. (The series is tabulated in Barro 1986a.) The frequently used Hamilton-Flavin debt series also implies a declining debt-income ratio for 1960-84 combined with an average return on government debt below the growth rate. (See Hamilton-Flavin 1986 and Wilcox 1989 for the debt and return data, respectively.)

<sup>25</sup> The YVAR variable enters positively in all regressions, as expected, but the temporary spending variable GVAR is significant only in the regression that includes the two world wars.

<sup>26</sup> In contrast to this paper, Barro finds no evidence for mean reversion in his studies, which use (scaled) changes in nominal government debt as left hand side variable and add a proxy for expected inflation on the right. One may wonder whether his inability to find mean reversion is due to problems associated with estimating inflation. (See also Kremers 1989.)

<sup>27</sup> The long run data are from Bohn (1991a). The sample period for the structural analysis was constrained by the availability of Barro's regressors.

<sup>28</sup> There is some concern that the results from ADF tests may be sensitive to the choice of lag length and subject to heteroskedasticity problems. But since the test results here merely confirm those in the literature, a more detailed analysis was not undertaken.

<sup>29</sup> Because of the presence of  $m_t$ , it is not clear to what extent Fuller's (1976) significance levels apply to this test. But it is instructive that the t-value on  $d_t$  is affected so drastically by adding  $m_t$ .

<sup>30</sup> In the structural analysis, Kremers also uses Barro's model as theoretical framework, and—like Barro—focuses on the growth rate of nominal debt, not on the primary surplus or the debt-income ratio as this paper. The above comments on Barro's approach apply analogously.

<sup>31</sup> Further evidence for a unit root in government spending is provided by Ahmed and Yoo (1989).

<sup>32</sup> The analysis here sheds some new light on my results in Bohn (1991a). Since estimates of the co-integrating vector  $(1, -1, -r)$  of  $(\tau_t, g_t, d_t)$  were so imprecise that  $r=0$  could not be rejected, I estimated error-corrections models with  $r=0$ , i.e. without  $d_t$ , as well as with fixed positive values of  $r$ . Based on the traditional analysis of budget constraints—which I used in that paper—I was unable to justify a model with  $r=0$  except as a limiting case. The analysis here shows that error-corrections models that exclude  $d_t$  are legitimate. In any case, the results for  $r=0$  and  $r>0$  were qualitatively similar.

<sup>33</sup> Trehan and Walsh (1991) also raise doubts about the relevance of co-integration restrictions. Their doubts are based on the sensitivity of these restrictions to assumptions about interest rates. These doubts are certainly supported by this paper. However, it seems doubtful that the stationarity of primary or with-interest deficits provide convincing evidence on sustainability (cf. Trehan and Walsh 1991). For example, if the government sets  $s_t=0$  at all times—i.e., pursues a policy of rolling-over all debt with interest—and if the relation of  $r$  and  $y$  is such that  $x<1$ , one will likely find a stationary path of debt (see (5)) and therefore a stationary with-interest deficit. Both primary and with-interest deficit are stationary, but the policy is non-sustainable.

## References

- Abel, Andrew, Gregory Mankiw, Larry Summers, and Richard Zeckhauser, 1989, Assessing Dynamic Efficiency: Theory and Evidence, Review of Economic Studies 56, 1-20.
- Ahmed, S. and B.S. Yoo, 1989, Fiscal Trends and Real Business Cycles, Working Paper, Pennsylvania State University.
- Barro, Robert J., 1979, On the Determination of Public Debt, Journal of Political Economy 87, 940-971.
- Barro, Robert J., 1986a, U.S. Deficits since World War I, Scandinavian Journal of Economics 88, 195-222.
- Barro, Robert J., 1986b, The Behavior of United States Deficits, in R. Gordon, ed., *The American Business Cycle: Continuity and Change*, University of Chicago Press, Chicago, Ill.
- Blanchard, Olivier, 1984, Current and Anticipated Deficits, Interest Rates and Economic Activity, European Economic Review 25, 7-27.
- Bohn, Henning, 1990, The Sustainability of Budget Deficits in a Stochastic Economy, Rodney White Center for Financial Research Working Paper (6-90), University of Pennsylvania.
- Bohn, Henning, 1991a, Budget Balance through Revenue or Spending Adjustments? Some Historical Evidence for the United States, forthcoming Journal of Monetary Economics.
- Bohn, Henning, 1991b, The Sustainability of Budget Deficits with Lump-Sum and with Income-Based Taxation, forthcoming Journal of Money, Credit, and Banking.
- Dickey, D., and Wayne Fuller, 1981, Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root, Econometrica 49, 1057-1072.
- Fuller, Wayne, 1976, *Introduction to Statistical Time Series*, Wiley, New York.

- Hakkio, Craig, and Mark Rush, 1986, Co-Integration and the Government's Budget Deficit, Working Paper, Federal Reserve Bank of Kansas City.
- Hamilton, James, and Majorie Flavin, 1986, On the Limitations of Government Borrowing: A Framework for Empirical Testing, American Economic Review 76, 808-819.
- Kremers, Jeroen, 1989, U.S. Federal Indebtedness and the Conduct of Fiscal Policy, Journal of Monetary Economics 23, 219-238.
- Lucas, Robert, 1978, Asset Prices in an Exchange Economy, Econometrica 46, 1429-1445.
- McCallum, Bennett, 1984, Are Bond-financed Deficits Inflationary? A Ricardian Analysis, Journal of Political Economy 92, 125-135
- Trehan, Bharat, and Carl Walsh, 1988, Common Trends, The Government Budget Constraint, and Revenue Smoothing, Journal of Economic Dynamics and Control 12, 425-444.
- Trehan, Bharat, and Carl Walsh, 1991, Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Federal Budget and Current Account Deficits, Journal of Money, Credit and Banking 23, 210-223.
- White, H., 1980, A Heteroskedasticity-Consistent Covariance Estimator and Direct Test for Heteroskedasticity, Econometrica 48, 817-838.
- Wilcox, David, 1989, The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraint, Journal of Money, Credit and Banking 21, 291-306.
- Zilcha, Itzhak, 1991, Characterizing Efficiency in Stochastic Overlapping Generations Models, Working Paper, Tel-Aviv University.

## Appendix

### A1. Proof of Proposition 1

We have to show that  $z_n = E_t[u_{t,n} \cdot D_{t+n}]$  converges to zero as  $n \rightarrow \infty$ . That is, given any  $\epsilon > 0$ , there must be a value  $N^*$  such that  $|z_n| < \epsilon$  for all  $n \geq N^*$ . By iterating on (5) and (6), one obtains

$$d_{t+n} = (1-\rho)^n \left( \sum_{j=1}^n x_{t+j} \right) \cdot d_t + \sum_{i=1}^n (1-\rho)^{n-i} \cdot \left( \sum_{j=1}^i x_{t+j} \right) \cdot \mu_{t+i-1}$$

Using the relations  $z_n = Y_t \cdot E_t u_{t,n} \cdot \prod_{j=1}^n (1+y_{t+j}) \cdot d_{t+n}$  and  $E_t[u_{t+i,1} \cdot (1+R_{t+i+1})] = 1$ , this implies

$$z_n / Y_t = (1-\rho)^n \cdot d_t + \sum_{i=1}^n (1-\rho)^{n-i} \cdot E_t[a_{i-1}], \text{ where } a_k = u_{t,k} \cdot \prod_{j=1}^k (1+y_{t+j}) \cdot \mu_{t+k}.$$

By assumption, the present value of future income,  $V_t = Y_t \cdot \sum_{k \geq 1} E_t u_{t,k} \cdot \prod_{j=1}^k (1+y_{t+j})$  is finite. Finiteness of this sum implies that the elements in the sum must converge to zero, i.e.,  $E_t u_{t,k} \cdot \prod_{j=1}^k (1+y_{t+j}) \rightarrow 0$  as  $k \rightarrow \infty$ . Combined with a bound on  $\mu_t$ , this implies  $E_t a_k \rightarrow 0$  as  $k \rightarrow \infty$ . That is, for any  $\delta > 0$  there is an  $N$  such that  $|E_t a_k| \leq \delta$  for all  $k \geq N$ . Let  $\sum_{i=1}^N (1-\rho)^{n-i} \cdot E_t[a_{i-1}] = \Omega$ , then for  $n \geq N$  we have

$$|z_n / Y_t - [(1-\rho)^n \cdot d_t + (1-\rho)^{n-N} \Omega]| = \sum_{i=N+1}^n (1-\rho)^{n-i} \cdot |E_t[a_{i-1}]| \leq \delta / \rho$$

Since  $(1-\rho)^n \rightarrow 0$  as  $n \rightarrow \infty$ , the absolute value of  $z_n$  will be less than  $\epsilon$  for high enough  $n$ , provided one picks  $\delta < \epsilon \cdot \rho / Y_t$ . Q.E.D.

To generalize the proposition, suppose  $\mu_t$  follows the stationary stochastic process  $\mu_t = B^* + B(L)e_t$ , where  $e_t$  is white noise. No boundedness assumption is imposed. To show that  $E_t a_k \rightarrow 0$  still holds, one needs that the sum

$$E_t \sum_{n=0}^{k-1} u_{t,k} \cdot \prod_{j=1}^k (1+y_{t+j}) \cdot e_{t+k-n} \cdot B_n$$

converges to zero. (The  $B^*$ -part is unproblematic by the same argument used for  $\mu^*$  above.) Since  $B_n$  and  $E_t u_{t,k} \cdot \prod_{j=1}^k (1+y_{t+j})$  by themselves converge to zero, sufficiently low



covariances between the e-innovations and process of income growth times marginal rate of substitution will guarantee sustainability.

## A2. Tax Smoothing under Uncertainty

An example should be sufficient to demonstrate that tax-smoothing in a stochastic model does not necessarily imply a non-stationary path for the debt-income ratio. Consider the following simple Lucas (1978) exchange economy with government and with cost of tax collection (as in Barro (1979)). Consumers maximize  $\sum_{t=0} \beta^t \cdot u(c_t)$  subject to  $A_{t+1} = (1+R_{t+1}) \cdot [A_t + Y_t \cdot (1-\tau_t) \cdot h(\tau_t) - c_t]$ , where  $c$ =consumption,  $A$ =assets = claims on the government,  $R$  = return on government debt,  $Y$  = exogenous income,  $\tau$  = tax rate, and  $h(\tau) = h/2 \cdot \tau^2$  = cost of tax collection. Government debt satisfies  $D_{t+1} = (1+R_{t+1}) \cdot [D_t + G_t - Y_t \cdot \tau_t]$ . To simplify, assume that utility is CRRA with risk aversion  $\alpha$ , that  $G_t = g \cdot Y_t$  for all  $t$  and that income growth  $y_t$  is lognormal i.i.d. Then the individual first order condition for  $R_t$  is

$$E_t[(1+R_t) \cdot \beta \cdot (1+y_{t+1})^{-\alpha} \cdot ((1-g-h(\tau_{t+1}))/ (1-g-h(\tau_t)))^{-\alpha}] = 1. \quad (A1)$$

If the government were able to borrow on complete markets, it would be straightforward to show that the welfare maximizing policy for “small” values of  $h$  would be to issue income-indexed debt and to stabilize the tax rate at a fixed value at all times and for all states of nature. Therefore, to explain movements in tax rates, one has to impose restrictions on debt management. Specifically, I will impose the same assumption that Barro (1979) apparently imposes implicitly, namely that the government has to use safe debt with return  $R_{t+1} = r_t$ . Then the government’s first order condition is

$$E_t[(1+r_t + [D_t + G_t - Y_t \cdot \tau_t] \cdot \frac{dr_t}{dd_t}) \cdot \beta \cdot (1+y_{t+1})^{-\alpha} \cdot ((1-g-h(\tau_{t+1}))/ (1-g-h(\tau_t)))^{-\alpha} \cdot \tau_{t+1}] = \tau_t$$

using the the linearity of  $h(\cdot)$ . It is straightforward to show by differentiating (A1) that  $dr_t/dd_t$  will converge to zero in the limit as  $h$  becomes small. In the limit, optimal tax policy is characterized by

$$E_t[(1+r) \cdot \beta \cdot (1+y_{t+1})^{-\alpha} \cdot \tau_{t+1}] = \tau_t \quad (A2)$$

where  $r_t=r$  is determined by  $E_t[(1+r)\cdot\beta\cdot(1+y_{t+1})^{-\alpha}] = 1$ . To examine what this condition implies for debt service, consider the class of linear policies  $\tau_t = g + \rho \cdot d_t$ . Since this class of policies implies  $d_{t+1} = (1+r)/(1+y_{t+1}) \cdot (1-\rho) \cdot d_t$ , substitution into (A2) yields

$$(1-\rho) \cdot E_t[(1+r)^2 \cdot \beta \cdot (1+y_{t+1})^{-\alpha-1}] = 1, \quad (\text{A3})$$

For comparison, note that the optimal policy with complete markets will also fall into this linear class (suggesting that the linearity restriction is not unreasonable) with a parameter  $\rho_v = v/(1+v) > 0$ , where  $v$  is defined by  $E_t[(1+v)\cdot\beta\cdot(1+y_{t+1})^{1-\alpha}] = 1$ . The value  $v$  can be interpreted as ratio of income to the present value of income, which is positive because of dynamic efficiency. Using the lognormality assumption, one has

$$\log(1+r) = -\log(\beta) + \alpha x - \alpha^2/2 \cdot \sigma^2$$

$$\log(1+v) = -\log(\beta) + (\alpha+1)x - (\alpha+1)^2/2 \cdot \sigma^2$$

$$\log(1-\rho) + 2 \cdot \log(1+r) = -\log(\beta) + (\alpha-1)x - (\alpha-1)^2/2 \cdot \sigma^2$$

where  $(x, \sigma^2)$  are the mean and variance of  $\log(1+y_t)$ . After some algebra, this implies

$$\log(1-\rho) + \log(1+v) = -\sigma^2 > 0,$$

hence  $\rho > \rho_v > 0$ . That is, regardless of the interest rate  $r$ , the optimizing government will run a primary budget surplus that is at least as large as the surplus it would have run under the optimal policy with complete markets. In particular, the government will not try to exploit an interest rate on safe debt below the average growth rate to run primary deficits.

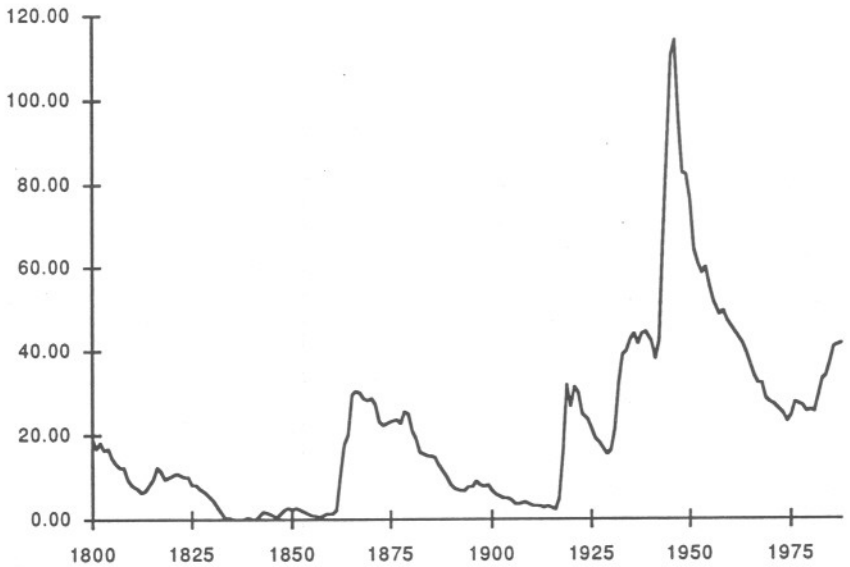
### A3. Description of the Data

Except for the budget surplus  $s_t$ , the data used for Tables 1 and 3 were taken from Barro (1986a). (An adjustment was made for YVAR in 1925 and 1930, however, where the values did not match those in Barro 1986b.) The primary budget surplus  $s_t$  for calendar years 1929-83 was constructed by dividing the difference of federal receipts and non-interest outlays<sup>1</sup> by nominal GNP, where all series were taken from the National Income

and Product Accounts (NIPA, taken from the WEFA database). The surplus for 1916-28 was obtained by interpolating the fiscal year surplus series that is described below.

In Table 2,  $y$  is average real GNP growth from NIPA and  $r$  is the average ratio of federal interest outlays divided by initial debt on a fiscal year basis, minus the growth rate of the GNP deflator. (The GNP deflator was taken as inflation measure to maintain comparability to  $y$ .) All fiscal year series were taken from Bohn (1991a). They are largely based on the Historical Statistics of the United States and The Budget of the United States, various years. A description of this data set is available from the author.

**Figure 1: The Ratio of U.S. Government Debt to GNP**



**Legend:**

The figure shows the publicly held debt of the federal government divided by GNP at the start of each fiscal year (in percent). Data sources are described in the appendix.

**Table 1: Determinants of the Budget Surplus**Dependent Variable: Budget Surplus  $s_t$ 

Sample Periods:	1916-83	1920-40&48-82	1954-83	1960-83
Estimates:				
Constant	-0.019	-0.008	-0.010	-0.014
t-value	-5.018	-1.777	-2.096	-2.211
robust t	-4.687	-1.488	-1.933	-2.378
GVAR	-0.801	-0.533	-0.507	-0.386
t-value	-31.906	-1.777	-2.959	-1.988
robust t	-22.617	-1.488	-3.033	-2.038
YVAR	-1.430	-1.949	-2.189	-2.109
t-value	-3.344	-4.475	-4.578	-4.281
robust t	-4.124	-3.851	-3.829	-3.585
Debt $d_t$	0.056	0.031	0.027	0.048
t-value	6.090	2.881	2.148	2.251
robust t	4.735	2.231	2.283	3.096
$R^2$	0.942	0.635	0.635	0.731
Standard Error	0.015	0.011	0.011	0.007
DF on residuals	-6.457	-5.150	-5.150	-3.669

**Legend:**

The variable  $s_t$  is the primary U.S. budget surplus divided by GNP,  $d_t$  is the publically held debt divided by GNP at the start of the year, GVAR and YBAR are measures of temporary government spending and of cyclical variations in output, respectively, which are taken from Barro (1986a). All estimates are OLS with annual data. Details and data sources are described in the appendix.

**Table 2: Sample Averages**

Sample Periods:	1916-83	1920-40&48-82	1954-83	1960-83
Surplus $s_t$	-1.31%	0.40%	0.26%	0.08%
GNP-growth $y$	3.04%	2.68%	2.75%	2.91%
Real Returns $r$	0.32%	1.61%	0.27%	0.43%
Difference $r-y$	-2.72%	-1.07%	-2.48%	-2.49%
t-value on $r-y$	-2.44	-0.98	-4.52	-4.11

**Legend:**

Averages are taken over annual series. The variable  $s_t$  is the primary U.S. budget surplus divided by GNP, GNP-growth is the growth rate of real GNP. Real returns are payments on the government debt divided by initial debt, minus inflation. Data sources are described in the appendix.

**Table 3: Determinants of Changes in the Debt-Income Ratio**Dependent Variable: Change in the Debt-GNP Ratio  $\Delta d_{t+1}$ 

Sample Periods:	1916-82	1920-40&48-82	1954-82	1960-82
Estimates:				
Constant	0.037	0.017	0.011	0.019
t-value	4.766	1.757	1.235	2.628
robust t	3.278	1.838	1.269	3.172
GVAR	0.779	0.084	0.064	0.357
t-value	14.866	0.269	0.196	1.468
robust t	16.022	0.246	0.279	2.205
YVAR	1.237	2.357	3.187	3.010
t-value	1.360	2.523	3.437	4.861
robust t	1.755	2.602	6.470	8.056
Debt $d_t$	-0.133	-0.085	-0.069	-0.094
t-value	-6.964	-3.652	-2.932	-3.615
robust t	-3.621	-3.320	-2.445	-3.909
R <sup>2</sup>	0.794	0.371	0.585	0.773
Standard Error	0.031	0.024	0.013	0.0086
DF on residuals	-8.623	-7.373	-10.156	-7.528

**Legend:**

The variable  $d_t$  is the publically held debt divided by GNP at the start of the year, GVAR and YBAR are measures of temporary government spending and of cyclical variations in output, respectively, which are taken from Barro (1986a). All estimates are OLS with annual data. Details and data sources are described in the appendix.

**Table 4: Unit Root Tests**

**Panel A: Augmented Dickey-Fuller Tests on Government Debt and on the Budget Surplus**

Samples:	Calendar Years 1916-83		Fiscal Years 1800-1988	
	All Years	As in Barro <sup>†</sup>	All Years	Excl. Wars <sup>††</sup>
Debt $d_t$	1.790	3.149*	2.077	3.684**
Surplus $s_t$	4.147**	5.157**	6.211**	13.509**

**Panel B: Augmented Dickey-Fuller Tests on Other Variables**

Samples:	Fiscal Years 1800-1988	
	All Years	Excl. Wars <sup>††</sup>
Military spending $m_t$	4.839**	5.506**
Variable $b_t$	4.702**	

**Legend:** The test statistics are absolute t-values from augmented Dickey-Fuller regressions estimated with two lags. The critical values are 2.89 on the 5% level and 3.45 on the 1% level (based on 100 observations). The variable  $s_t$  is the primary U.S. budget surplus,  $d_t$  is the publically held debt at the start of the year, and  $m_t$  is U.S. military spending, all divided by GNP. The test statistic under  $b_t$  is refers to the t-value of a Dickey-Fuller type regression on  $\Delta d_t$  that includes  $m_t$  as regressor, as described in the text. The 1916-83 sample has calendar year variables (68 observations), the 1800-1988 sample has fiscal year values (189 observations). Details and data sources are described in the appendix.

<sup>†</sup> = sample period 1920-40 and 1948-82; 56 observations.

<sup>††</sup> = sample period 1800-1988 excluding 1861-65, 1917-19, and 1941-47; 174 observations.



and Product Accounts (NIPA, taken from the WEFA database). The surplus for 1916-28 was obtained by interpolating the fiscal year surplus series that is described below.

In Table 2,  $y$  is average real GNP growth from NIPA and  $r$  is the average ratio of federal interest outlays divided by initial debt on a fiscal year basis, minus the growth rate of the GNP deflator. (The GNP deflator was taken as inflation measure to maintain comparability to  $y$ .) All fiscal year series were taken from Bohn (1991a). They are largely based on the Historical Statistics of the United States and The Budget of the United States, various years. A description of this data set is available from the author.

## *CES Working Paper Series*

---

- 01 Richard A. Musgrave, Social Contract, Taxation and the Standing of Deadweight Loss, May 1991
- 02 David E. Wildasin, Income Redistribution and Migration, June 1991
- 03 Henning Bohn, On Testing the Sustainability of Government Deficits in a Stochastic Environment, June 1991
- 04 Mark Armstrong, Ray Rees and John Vickers, Optimal Regulatory Lag under Price Cap Regulation, June 1991
- 05 Dominique Demougin and Aloysius Siow, Careers in Ongoing Hierarchies, June 1991
- 06 Peter Birch Sørensen, Human Capital Investment, Government and Endogenous Growth, July 1991