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CAREERS IN ONGOING HIERARCHIES

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Abstract

The probability that an unskilled worker can be succesfully trained to be a manager depends on the effort of the firm. With positive hiring costs, a firm prefers to train its own managers. However the optimal size of the firm for productive efficiency may conflict with efficient managerial husbandry. How a firm copes with the above constraint generates stochastic layoffs, lateral mobility, promotions, diverse earnings profiles, fast track jobs and up or out rules.

Keywords: Hierarchies, on-the-job training, careers, up or out rules, fast track jobs.

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CAREERS IN ONGOING HIERARCHIES

1. Introduction

The standard model of human capital accumulation and earnings growth assumes that intertemporal labour market behaviour is mostly driven by supply considerations.[1] Although successful, the model is silent about several intermittent career events such as layoffs, promotions, lateral mobility and mandatory retirement. In this paper, we explain those events by integrating standard concerns about supply with the structure of demand in labour markets.[2]

Within a competitive, symmetric information and complete contracting environment, we focus on the need of ongoing hierarchies to produce new managers to replace retiring managers. Often, it is efficient for a firm to train unskilled workers on the job to become managers. Then the size of the firm required by productive efficiency may conflict with the optimal number of unskilled workers that can simultaneously be trained to become managers for the future. How a firm copes with this tradeoff generates personnel decisions that were previously inexplicable within a competitive, symmetric information and complete contracting framework. Our model generates heterogenous career profiles for identically endowed workers. These include stochastic promotions, dismissals, lateral mobility within the firm, upward mobility across firms and variations in earnings across identically skilled workers.

Our model also explains two labour market phenomena, fast track jobs and

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[1] E.g. Becker (1976), Ben Porath (1967), and Mincer (1974).

[2] Other models do not deviate from the supply side emphasis of earlier work but add asymmetric information and incomplete contracting to labour markets (e.g. Akerlof and Yellen (1986), Hall and Lazear (1984), Kahn and Huberman (1988), Lazear (1979), Weiss and Wang (1990)).

up or out rules. Many hierarchies have fast track jobs. These are entry level positions in which an occupant remains for a short time. Then he is either promoted to more senior positions ahead of the cohort that entered the firm with him. Or he is moved to another lateral position in the firm and loses attention from the management of the firm. Other hierarchies practice up or out rules. In this case, all junior workers are either promoted or dismissed from the firm after some finite period.[3]

We explain these practices as follows. In a hierarchy, some junior positions have two functions. First, an occupant in one of these positions is a direct factor in current production. Second, the occupant is also trained and or screened, or engaged in some other ways on the job to become a productive senior worker. When these junior slots are scarce, the firm may dismiss a junior worker from his position, even if he is capable, in order to give his slot to another candidate who is more promising for the senior rank. If there are other unskilled slots that are not used for developing managers, an unsuccessful trainee can move to such a position and not have to leave the firm. With lateral mobility, the dual role junior slots that we have just described resemble fast track slots. In some hierarchies, when a trainee fails, lateral mobility is not optimal. Then the up or out rule applies and unsuccessful trainees leave the firm. So whether the fast track or up or out holds depends on the nature of managerial and output production of the firm.

In this paper, we describe an environment where the above explanation

[3] These rules are puzzling because with heterogenous workers, some workers are best suited for these jobs and should be able to stay in them indefinitely.

for the two types of institutions is operative.[4] In the model, workers live for two periods. A firm can choose to train some of its unskilled workers on the job to become managers.[5] Firms that hire managers from the outside have to pay hiring costs.[6] So if a young trainee is successful, he is promoted to management ahead of the unskilled workers who entered with him and the firm saves on hiring cost. If he is not successful, he has to leave the training slot for an unskilled slot in the firm or elsewhere. No firm would be willing to train him at the prevailing wage of a young trainee because he has to retire after the second period and a firm cannot reduce its future hiring cost. Whether he can remain in the firm depends on the availability of non-training unskilled slots. This availability depends on the difference between the optimal number of total unskilled workers employed in production and the optimal number of trainees cum unskilled workers. When the difference is positive, the hierarchy has fast track jobs. When the difference is zero, the hierarchy practices up or out. Depending on the circumstances, a firm and or the industry may move from one regime to the other.

In the basic model, we assume that firms make all wage offers. An

[4] Rosen (1990), in discussing the careers of executives, wrote: "No doubt differences in style and technology make it (up *or out rules*) a poor approximation for many firms, yet essential elements remain common to all of them. Promotions focus an executive's career attention on those discrete points when a 'window of opportunity' opens for possible advancement. Typically these windows fit some predetermined rough outline of the firm's organization structure and there is competition for them from contenders, both within and without the firm." We show when up or out rules apply, when the 'window of opportunity' opens, provide an endogenous organizational structure of the firm, and characterize the competition for these positions.

[5] We build on on-the-job training models (e.g. Becker, Ben Porath and Mincer for non-stochastic models; Cooter and Restrepo (1979), Morrison and Schmittlein (1981), Sicherman and Galor (1990) for stochastic models).

[6] The importance of hiring cost, a large part of which is investment in specific capitial, is documented in Oi (1962) and Rosen (1968).

extension of the basic model comes from redistributing the bargaining power between the firm and its successful trainees. Due to the positive hiring cost, a successful trainee has some bargaining power versus the firm. Wages of internally promoted managers are on average higher than those hired from the outside. However, the distribution of bargaining power between the firm and its successful trainees has no implication for quantities or the value of the firm. So we have an insider-outsider model of the labour market which, unlike Lindbeck and Snower (1988), has no implication for market efficiency. This paper counsels caution in using evidence of wage differentials, unexplained by ability differences, to imply labour market inefficiency.

Many of the issues discussed here have been previously explored in the literature. Studies of careers in hierarchies where slots are scarce include Doeringer and Piore (1971), Rosen (1986), Rosenbaum (1984), Stern (1987), White (1970) and Tuck (1954). Economists have also studied career dynamics in hierarchical, incomplete contracting and asymmetric information environments (e.g. Carmichael (1983), Costa (1988), Cremer (1986), Ferrall (1989), Huberman and Kahn (1988), MacLeod and Malcomson (1988), Malcomson (1984), Rosen (1990), Waldman (1990)). Their emphasis on asymmetric information and incomplete contracting is very different from ours. The literature on cohort effects are related to our work in their concern for the interactions between old skilled workers cum teachers and new unskilled workers (e.g. Stapleton and Young (1988) and Welch (1979)). Finally, we consider related issues in O'Flaherty and Siow (1989, 1990), Demougin and Siow (1991).

Section 2 of the paper sets up the model. The fast track and up or out are discussed in Sections 3 and 4 respectively. Market equilibrium is considered in Section 5. Section 6 has extensions and a conclusion.

2. The Model

Consider a stationary competitive industry that sells a homogenous good. Output is supplied by a continuum of firms who all have access to identical technology. We normalize the measure of firms to 1. Let aggregate output in the industry in period t be Q_t. The price per unit in period t is P_t where P_t is determined by the inverse demand schedule, $Q^D(P_t,d_t) = Q_t$, $Q^D_P < 0$. d_t is a non-stochastic demand shifter for the industry. For now, we assume that the price of output is sufficiently high such that no firm will choose to leave the industry and we also restrict entry into the industry.

A firm produces output with a fixed quantity of skilled workers (hereafter managers), which we normalize to 1, and unskilled workers.[5] Workers in this economy are active in the labour market for only two periods. Thus a firm has to continuously replenish its work force.

The alternative wage of an unskilled worker in the economy in period t is A_t. Let a representative firm at date t hire one manager and n_t unskilled workers. m_t of those unskilled workers receive training where $m_t \le n_t$. Unskilled workers that do not receive training will never become skilled; if they are hired by a firm in period t, they will be paid the alternative wage A_t. If a trainee is successful, he will become a manager in the next period (when he is old). His managerial talent is equally productive in any company in the industry - however, we will assume that when a manager switches firms within the industry, the hiring firm must pay a hiring cost. In period t, the wage of a manager in the industry is $A_t + S_t$. S_t is interpreted as the skill premium which is to be determined by the model. If a trainee fails, he

[5] While the one manager assumption is analytically convenient, many firms are two level hierarchies with one manager (who may also be the owner).

remains an unskilled worker in his second period, t+1. If he stays in the industry, he will receive A_{t+1} or, alternatively, he can leave the industry for A_{t+1} elsewhere. Workers have no moving cost when they leave a firm or their industry.

A firm must choose a training intensity for its m_t trainees[6]. A training intensity generates for each trainee a probability of success, which is independent across trainees. Let γ_t denote this probability. A firm will be able to attract trainees as long as they get the same expected income from undergoing training as otherwise. Let β denote the common discount factor of workers and firms. In the first period of their working life, all workers are unskilled. At date t, there is an elastic supply of trainees at the wage $W(\gamma_t, S_{t+1})$ which satisfies the arbitrage condition:

$$A_t + \beta A_{t+1} = W(\gamma_t, S_{t+1}) + \beta[\gamma_t(A_{t+1}+S_{t+1}) + (1-\gamma_t)A_{t+1}] \qquad (2.1)$$

The left hand side (LHS) of (2.1) is the present value of the income stream of a young worker who does not receive any training. The right hand side (RHS) is the expected discounted present value of income from being in the training program. The equation can be rewritten to yield the training wage offered by a firm as a function of the alternative wage, the skill premium and the likelihood of a success generated by its training program:

$$W(\gamma_t, S_{t+1}) = A_t - \beta\gamma_t S_{t+1} \qquad (2.2)$$

The term $\beta\gamma_t S_{t+1}$ denotes the wage reduction the firm can obtain by hiring a trainee instead of a non-trainee. The firm obtains this reduction because the expected future earning of a trainee is higher than that of a non-trainee. This also implies that a second generation unskilled worker will never accept a training position.

If a firm is unsuccessful in training a manager from within for period t, it can hire a manager trained by other firms by paying a hiring cost of $P_t H$. We assume hiring cost is proportional to the price of output because it consists of direct hiring costs as well as the output that is lost when the newly hired manager is learning how his new firm operates. Since a firm may successfully train more than one manager, the excess managers leave to work in firms that did not manage to promote from within. Because of the hiring cost, $P_t H$, firms prefer to promote from within. We initially assume that firms make all wage offers.[7]

We now discuss the cost of training. Let the cost of the training program per trainee be $P_t C(\gamma_t)$, where $C(.)$ satisfies:

$$C(0) = 0 \ , \ C'(0) = 0 \ , C' \geq 0 \ , C'' > \beta H \qquad (2.3)$$

Thus, training cost starts at zero and is convex in γ_t. The restriction on C" will discussed later. We make the opportunity cost of training depends on output price, P_t, because we interpret this cost as the foregone output of the firm when a trainee takes time off from production to learn and managers take time off from production to teach.

We are now ready to discuss the optimization problem of a representative firm. Let the gross production function of the firm be $F(n_t)$. We assume that

[6] We assume that all the trainees are treated equally and receive the same quality and quantity of training.

[7] Section 6 shows that the particular bargaining assumption between the firm and its managers is irrelevant to the firm's quantity choices.

F satisfies the following restrictions:

$$F' \geq 0 \ , \ F'' < 0 \ , \ F(\infty) < \mu < \infty \tag{2.4}$$

Given the path of prices, $\{P_j, S_j\}_{j=t}^{\infty}$, the objective of the firm is to choose current and future quantities of employment, and the size and intensity of its training program in order to maximize the expected present value of discounted profits. In this environment, the representative firm faces a stochastic dynamic programming problem. Let V_t denotes the maximum expected present value of the firm at time t after the hiring decision of a manager has been taken. It is formally defined as:

$$V_t = \max_{n_t, m_t, \gamma_t} P_t[F(n_t) - m_t C(\gamma_t)] - (A_t + S_t) - (n_t - m_t)A_t - m_t W(\gamma_t, S_{t+1}) - \beta P_{t+1} H(1-\gamma_t)^{m_t} + \beta V_{t+1} \tag{2.5}$$

$$0 \leq \gamma_t \leq 1 \tag{2.5.1}$$

$$0 \leq m_t \leq n_t \tag{2.5.2}$$

$F(n_t) - m_t C(\gamma_t)$ is the output of the firm after subtracting training cost. $A_t + S_t$ is the wage paid to the current manager. $(n_t - m_t)A_t$ is the wage bill for unskilled workers who do not receive training. $m_t W(\gamma_t, S_{t+1})$ is the wage bill for trainees. The term $\beta P_{t+1} H(1-\gamma_t)^m$ is the discounted expected hiring cost to be paid in t+1 for a manager. $H(1-\gamma_t)^m$ is the expected loss of output in t+1 due to having possibly to hire from the outside in t+1. V_{t+1} is the maximum expected present value of profits for the firm at t+1 after any hiring cost for t+1 has been paid. Note that V_{t+1} as we have

defined it, is independent of actions taken at time t.[8] The constraints are consistency requirements. (2.5.1) guarantees that the probability of success is in the unit interval. (2.5.2) insures that the number of trainees is less than the total number of unskilled workers hired by the firm. We assume that the price of output is sufficiently high such that every firm will want to produce positive output in each period.

When demand is stationary, we will show later that there is an equilibrium where the prices, P_t and S_t, are constant for all t. We will discuss this equilibrium here. In this case, from (2.5), the firm will choose the same values for n_t, m_t, and γ_t for all t and we can dispense with the time subscript. We also substitute the wage function for trainees, equation (2.2), to obtain the following definition:

$$\pi(n,m,\gamma) = P[F(n) - mC(\gamma)] - (A+S) - nA + \beta mS\gamma - \beta PH(1-\gamma)^m \tag{2.6}$$

In any period, π denotes those parts of the profit stream which are influenced by current decisions. It includes the profit of that particular period and the implication for hiring in the following period. We note that a firm will always hire at least one trainee, indeed:

$$\forall\, n \geq 0 \quad \pi(n,0,0) = \pi(n,1,0)$$
$$\text{and} \quad \frac{\partial \pi}{\partial \gamma}(n,1,0) = \beta S + \beta PH > 0$$

Thus, as long as $S > 0$, it is better for the firm to have at least one trainee receive a little bit of training than no trainee. This implies that the lower constraints on γ and m will never be binding. Since V_{t+1} is

[8] Although correct, our definition of V_t is not standard dynamic programming notation because we include a t+1 payoff, $\beta P_{t+1} H(1-\gamma_t)^{m_t}$, in date t's return.

independent of time t decisions, the problem of the firm at time t can be reduced to a one period Lagrange maximization problem:

$$\max_{n,m,\gamma} \pi(n,m,\gamma) + \lambda\gamma + \nu(1-\gamma) + \mu m + \eta(n-m) \tag{2.7}$$

For a choice variable x, let x* denote the optimal value of x. m* and $\gamma^* > 0$ guarantees that the multipliers λ and μ are zero. Thus, the first-order condition of the firm's problem simplify to:

$$\left.\begin{array}{ll} (1) & PF'(n*) - A + \eta = 0 \\ (2) & -PC(\gamma*) + \beta S\gamma* - \beta PH(1-\gamma*)^{m*}\ln(1-\gamma*) - \eta = 0 \\ (3) & -Pm*C'(\gamma*) + \beta Sm* + m*\beta PH(1-\gamma*)^{m*-1} - \nu = 0 \end{array}\right\} \tag{2.8}$$

We can still distinguish different cases depending on whether either of the two other constraints are binding or not. We first consider the situation where the optimal number of trainees is strictly less than the total number of unskilled workers demanded by the firm, i.e. the case where $\eta = 0$.

3. The Fast Track: $m^* < n^*$

Under the fast track, the firm's unskilled workforce exceeds the number of trainees. So some if not all unsuccessful trainees may move laterally in the firm. Whether they move laterally or not, unsuccessful trainees have to leave the training slots. That trainees have to be young and unsuccessful trainees have to leave the training slot correspond to our interpretation of Rosen's comments on the "window of opportunity" in footnote 4.

Under the fast track, the firm's gross output decision becomes separable from its training resolution. We first show that $\gamma^* < 1$. Suppose to the

contrary. In this case, (2.8.2) and (2.8.3) further simplify. Eliminating the term βS, we can solve for ν:

$$\nu = P[\frac{C(1)}{1} - C'(1)] < 0 \tag{3.1}$$

Since C is convex, (3.1) is a contradiction because the lagrange multiplier must be non-negative. Thus, the convexity of the cost function in γ guarantees that if the firm does not train all its unskilled worker, it will never choose the highest possible training intensity.

Under the fast track, $\eta = 0$ and unskilled employment is determined only by the real wage, $a = A/P$, as can be seen in (2.8.1). Moreover, dividing (2.8.2) and (2.8.3) through by P, we see that training policies, γ^* and m^*, only depend on the real skill premium, $s = S/P$.

In the appendix, we prove the following result.

<u>Proposition 1</u>: If there is a triplet (n^*, m^*, γ^*) which satisfies the first-order conditions of the maximization problem (2.7) and the constraints, with $\eta = 0$, it is the unique solution to the firm's problem. Comparative statics with respect to nominal prices are:

$$\left.\begin{array}{llll} (1) & \frac{\partial n^*}{\partial P}(S,P) \geq 0 & (2) & \frac{\partial n^*}{\partial S}(S,P) = 0 \\ (3) & \frac{\partial \gamma^*}{\partial P}(S,P) \geq 0 & (4) & \frac{\partial \gamma^*}{\partial S}(S,P) \leq 0 \\ (5) & \frac{\partial m^*}{\partial P}(S,P) \leq 0 & (6) & \frac{\partial m^*}{\partial S}(S,P) \geq 0 \\ (7) & \frac{\partial(\gamma^* m^*)}{\partial P}(S,P) \leq 0 & (8) & \frac{\partial(\gamma^* m^*)}{\partial S}(S,P) \geq 0 \end{array}\right\} \tag{3.2}$$

(3.2.8) says that the expected number of successful trainees, which is also the supply of managers, is increasing in the skill premium, S. (3.2.4) and (3.2.6) together say that when S increases, it is cheaper at the margin to increase the expected number of managers by decreasing the probability of success and increasing the number of trainees. This adjustment is determined by two factors. First, the marginal cost of increasing the number of trainees is constant whereas that of increasing intensity is increasing.[9] Second, a marginal increase in the number of trainees lowers the expected hiring cost more than a marginal increase in training intensity.

Since n^* does not depend on S, S can adjust solely to clear the managerial market. For the managerial market to clear, on average each firm must reproduce its own manager, i.e. $\gamma^*(S,P)m^*(S,P) = 1$. For a given P, we denote S(P) the market clearing skill premium. S(P) is implicitly defined by the managerial market clearing equation:

$$\gamma^*(S(P),P)m^*(S(P),P) = 1 \qquad (3.3)$$

Denote $\Gamma(P) = \gamma^*(S(P),P)$ and $M(P) = m^*(S(P),P)$. These functions represent, for a given output price, optimal training policies when the skill premium has adjusted to clear the managerial market. Although in principle, both functions depend on P, they are invariant in the output price because S adjusts to any change in P to maintain the equilibrium real skill premium that clears the managerial market.[10] Denote P^+ the output price such that $n^*(S,P^+) = M(P^+)$. Since n^* is non-decreasing in the output price, the

[9] This result is of course due to our specific assumption about training costs.

[10] Training policies depend only on the real skill premium, $s = S/P$. Denote with s^* the real skill premium which clears the managerial market. S(P) will change to keep s^* constant.

proposition also implies that for all $P \geq P^+$ the fast track rule applies. Otherwise, up or out rules.

4. Up or Out: $m^* = n^*$

When the number of trainees is equal the number of unskilled labour working within, firms practice the up or out rule. All trainees that are unsuccessful must leave their firm and the industry. Even if a trainee was willing to stay on his position at the same wage as before, the firm would not allow him to do so.[11] The reason is simply that the firm cannot promote him after the second period because he will retire from the market. Thus, the firm could not save on hiring cost by promoting a manager from within. The firm will keep an unsuccessful trainee only if he takes a pay cut. If we just consider his contemporaneous productivity in the firm, his dismissal will be a puzzle because he is no less productive than in the foregoing period or any other unskilled worker.[12]

Because of the market clearing condition for skilled workers, which in the present case requires $\gamma n = 1$, we assume throughout the remaining of this section that $\gamma^* < 1$. This implies that the multiplier $\nu = 0$ in (2.8). In

[11] No unsuccessful trainee will want to stay because there is no moving cost. If a worker incurs a moving cost of μ when he leaves the firm, there may be involuntary layoffs. A firm will pay one incumbent manager only $A+S-\mu$ to stay since all other managers have to leave and they get A+S elsewhere and have to incur moving cost. To get trainees, a firm will have to pay $W'(\gamma,S) = A - \beta(\gamma S - \mu)$. If $A - \mu < W'(\gamma,S)$, there will be involuntary layoffs of unsuccessful trainees. Positive moving cost will make the firm choose a larger γ^* and smaller m^*.

[12] This layoff is related to the issue of age discrimination and mandatory retirement. A firm may choose to layoff an older worker or promote a younger worker, not because the older worker is currently less productive than his younger colleague, but because the younger worker has more promise for the future. See also Stern.

this case, there remain only two unknowns, n and γ. Eliminating η from (2.8), we can simplify the first-order condition of the firms's problem[13]

$$\left.\begin{array}{ll}(1) & P[F'(n*) - C(\gamma*)] - A + \beta S\gamma* - \beta PH(1-\gamma*)^{n*}\log(1-\gamma*) = 0 \\ (2) & -PnC'(\gamma*) + \beta Sn* + n*\beta PH(1-\gamma*)^{n*-1} = 0\end{array}\right\} \quad (4.1)$$

Unlike the fast track case, we no longer have separability between the decisions on unskilled employment and training. Since m* = n*, we know:

$$\frac{\partial\pi}{\partial n}(n^*,m^*,\gamma^*) = P[F'(n^*) - A] = -\eta < 0 \quad (4.2)$$

The concavity of the production function implies that the firm is hiring more unskilled workers than can be justified from just maximizing current profits. Thus, training considerations push the optimal span, which is the ratio of junior workers to senior workers, of the hierarchy to be larger than that which would be predicted by production considerations alone.

This result is counter to the standard beliefs about career mobility in hierarchies. Standard models of hierarchies derive the span of the hierarchy from production considerations alone (e.g. Benopoulos (1990), Calvo and Wellisz (1979), Rosen (1982), Waldman (1984)). Given this exogenous span, career mobility in hierarchies have a zero sum flavour since the firm has less and less slots as one moves up the hierarchy (e.g. Doeringer and Piore, Rosen (1986, 1990), Rosenbaum (1984, 1989)). Our view is quite different. Firms

[13] (4.1.1) and (4.1.2) are just the first-order conditions of the simplified version of the firm's problem:

$$\max_{\gamma,n} P[F(n) - nC(\gamma)] - (A+S) - nA + n\beta S\gamma - \beta PH(1-\gamma)^n$$

may hire more unskilled workers cum trainees than can be justified by production considerations. Firms are also better off if more trainees become successful. Our view of the pyramidal structure of the hierarchy is like that of the schooling system. In schools, not all students move up to the next grade. Success is due to effort, selection and luck. Schools and the society are better off if more students, holding effort constant, are able to move up in grade. Likewise, our firms may distort the production decision in order to get more successful promotions.

In the appendix, we prove the following result:

Proposition 2: In the case where a firm practices the up or out rule ($n^* = m^*$, $\eta > 0$), the comparative static with respect to the nominal prices are:

$$\left.\begin{array}{llll}(1) & \frac{\partial\gamma^*}{\partial P}(S,P) \gtreqless 0 & (2) & \frac{\partial\gamma^*}{\partial S}(S,P) \gtreqless 0 \\ (3) & \frac{\partial n^*}{\partial P}(S,P) \gtreqless 0 & (4) & \frac{\partial n^*}{\partial S}(S,P) \geq 0 \\ (5) & \frac{\partial(\gamma^* n^*)}{\partial P}(S,P) \gtreqless 0 & (6) & \frac{\partial(\gamma^* n^*)}{\partial S}(S,P) \geq 0\end{array}\right\} \quad (4.3)$$

The reason that n* and γ* react in an indeterminate way to a change in the output price is because changes in P affect the real wage, the real skill premium, real hiring costs and the cost of training in different ways. Consider an increase in P. It reduces the real wage and real training costs which affect employment and training intensity positively. It also reduces the real skill premium and expected hiring cost, which lead to lower employment and training intensity. n* is increasing in S because the firm earns more from each trainee when S rises and also it allows the firm to optimally hire more trainees which it wants to do. γ* does not necessarily rise when S rises because there are two opposing effects. Training intensity is high relative to the case where the optimal number of trainees is not constrained by the number of unskilled workers (See the paragraph below). Thus an increase in n^* allows the firm to lower its optimal intensity. This effect opposes the increase in γ^* which is due to the increased return to γ^* from a rise in S. As in the fast track case, (4.3.6) says that the expected number of successful trainees is increasing in the skill premium.

The firm trains its workers more intensely under the up or out rule. The reson is because under up or out, the firm trains less workers than it would if it is not constrained by the size of the firm.[14] In other words, if production consideration does not constrain the number of trainees, the promotion rate will even be lower. So a firm practices up or out, not because unskilled workers are easy to motivate with the threat of dismissal, but because it is so difficult to train managers.[15]

5. Market Equilibrium

Since S is endogenous, we let $S^* = S(P)$ to clear the market for managers. So we only have to consider equilibrium in the market for output. We use Figure 1 to bring the results of the sections 3 and 4 together. In the case where $P < P^+$, the firm behaves according the up or out rule. In this situation, all the unskilled workers are being trained, $M(P) = N(P)$, and the firm employs more unskilled workers than warranted by production consideration alone.

$$n^{**}(P) < N(P) \quad \text{where} \quad F'(n^{**}) = \frac{A}{P} \tag{5.1}$$

As P increases, total employment and the number of trainees, rises. After P exceeds P+, the fast track applies and $N(P) = n^{**}(P) > M(P)$.[16] Employment is increasing in the output price, but training policies become invariant to P. The total employment, the number of trainees and the probability of success jointly determine the aggregate supply.

$$Q^s(P) = F(N(P)) - M(P)C(A(P)) - H(1-A(P))^{M(P)} \tag{5.2}$$

In the appendix, we prove the following result.

<u>Proposition 3</u>: If $A/((1-\beta)P) > H$, the aggregate supply function is increasing in the output price everywhere. For any non-stochastic demand shifter d, there exists a market clearing price P^*.

Empirically, the discounted alternative real wage over the infinite life of the firm, $A/((1-\beta)P)$, is much larger than the real hiring cost, H. So the restriction on H in proposition 3 is satisfied for empirically relevant parameter values.

To conclude this section, consider serially uncorrelated demand shifts

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[14] In order to obtain the result analytically, one can implicitly differentiate (4.1.2):

$$\frac{d\alpha^*}{dn^*} = \frac{d\alpha^*}{dm^*} = -\frac{n*\beta PH(1-\gamma*)^{n*-1}\log(1-\gamma*)}{-Pn*C''(\gamma*)-n*(n*-1)\beta PH(1-\gamma*)^{n*-2}} \leq 0$$

[15] Our explanation for up or out is completely different from that of Kahn and Huberman, and Waldman (1990), although as pointed out in O'Flaherty and Siow (1990), the two motivations can be complimentary in explaining the data.

[16] Many large law firms in North America in the last decade, after unprecedented growth in profits and size, have switched from pure up or out to the fast track. Smaller firms have not switched. See Galanter and Palay (1990), Gilson and Mnookin (1989).

in the output market. If the fast track is operative, shifts in transitory demand, d_t's, will only affect current output and the price of current output. S_{t+1} adjusts to keep $s^*_{t+1} = S_{t+1}/P_t$ constant to clear the managerial market in t+1. The firm will not change its training policies. When the up or out rule is binding, things are slightly more complicated. Consider an unaticipated demand shift out in period t. Firms will not react before period t. The price of output will rise in period t. According to proposition 3, firms will expand production in period t which means that more trainees are also hired. The real skill premium in the next period s_{t+1}^* must fall to maintain equilibrium in the managerial market in t+1.[17] Note that changes in S_{t+1} have no impact on decisions in t+1 since all firms will hire one manager in each period. Optimal training effort, γ_t^*, falls.

Finally, consider the case where the growth rate of demand is initially zero. New firms are allowed to enter the industry. We assume that entry cost is zero except for the hiring cost of a new manager. So the equilibrium price of output adjusts to the level where a potential entrant is indifferent between entering and staying out (or when an existing firm who has to hire a new manager from the outside is earning zero profits). Now consider a small permanent positive growth rate of demand. In order to maintain a stable price in the output market, new firms must enter the industry in each period. This entry means that each existing firm must produce an average of more than one manager per period.[18] Under the fast track, the real skill premium must rise

[17] The result can be obtained by substituting $\Gamma(P)$ into (A.24.1) in the appendix and differentiating the resulting expression with respect to $s = S/P$.

[18] This comparative static is relevant for restaurants. Existing restaurants do not increase output enough to satisfy increases in permanent demand. Instead new restaurants open.

in order to induce $\gamma^* m^*$ to rise above one. The number of trainees grows and training effort falls. With an increase in s^*, by the envelope theorem, profits per firm (at the equilibrium price with zero growth) will fall. So P^* has to rise to maintain the zero profit constraint for a potential entrant. With a rise in P^*, the firm will also hire more unskilled labor. So with an increase in the permanent growth rate of demand, the real skill premium rises, the price of output rises, firm size grows, output per firm is larger, the number of trainees grows and training effort per trainee falls. Under up or out, the results are less clear. As before, in order to maintain a stable equilibrium price of output, each existing firm has to produce an average of more than one manager per period. As such, real resources are used up which means that the output price has to rise to maintain the zero profit constraint for a potential entrant. The rise in the output price will induce more output and trainees for each existing firm. But the direction of change in the skill premium and training intensity are ambiguous.

6. Extensions

We first consider a change in the bargaining power between the firm and its workers. In the second extension, we show that on-the-job screening is a special case of the model that we have already studied.

In the basic model, the firm has all the bargaining power and make all wage offers. We now assume that workers make wage offers and firms either accept or reject the offers. Managers promoted from within have some bargaining power because a firm can save on hiring cost if it can hire a manager from within. If a firm is unable to promote any trainee from within, it will have to pay A+S+PH to hire a manager from the outside. The probability of this event occurring is $(1-\gamma)^m$.

Consider the case in which only one trainee out of m is successfully trained. Then he can demand a wage of A+S+PH because a firm would have to spend that much to replace him. The probability that he will be the only successful trainee is $\gamma(1-\gamma)^{m-1}$. With m trainees, the probability that a firm has exactly one successful trainee is $m\gamma(1-\gamma)^{m-1}$.

Now suppose the firm has more than one successful trainees. With successful trainees (managers) competing with each other to work for the firm the unique Nash Equilibrium wage offer by all these managers is A+S.[19] The probability of this case occurring is $1 - (1-\gamma)^m - m\gamma(1-\gamma)^{m-1}$.

From the point of view of a potential trainee, he is now indifferent between working in this firm or elsewhere if:

$$A + \beta A = W'(\gamma,S) + \beta\{ \gamma[A+S + (1-\gamma)^{m-1}PH] + (1 - \gamma)A \} \qquad (6.1)$$

Rearrange (6.1) to get:

$$W'(\gamma,S) = A - \beta\gamma(S + (1-\gamma)^{m-1}PH) \qquad (6.2)$$

The trainee cares about how many other trainees there are in the firm because his bargaining power depends on how many successful trainees there are. The wage he is willing to accept in (6.2) reflects that concern.[20]

Now consider the value of the firm who has an incumbent manager as a cost of A+S.

$$V' = \underset{(n,m,\gamma)}{\text{Max}}\ P(F(n) - mC(\gamma)) - (A + S) - (n - m)A \qquad (6.3)$$
$$- mW'(\gamma,S) - \beta PH((1 - \gamma)^m + m\gamma(1-\gamma)^{m-1}) + \beta V'$$

[19] The managers play a Bertrand game in wages.

[20] This concern is similar to the tournament literature where contestants worry about how many of them there are (e.g. Rosen (1990)).

one. Upon substituting (6.2) into (6.3) we see that the definition of V' is the same as that of V in (2.5). This equivalence shows that the value of the firm is exactly the same as when the firm makes all the wage offers. The firm also optimally chooses the same quantities as before. Thus changing the bargaining power between workers and firms does not change the value of the firm. However the earnings profiles of trainees are now different. Their starting wage $W'(\gamma,S)$ is lower than before. More importantly, their wages if they are successfully trained may differ across managers. The manager who is the only one to be successful in his firm will get a higher wage.

The above result can be extended to the case where a lone successful trainee gets ω share of the rents, PH. So we have a model that generates wage differentials across identical workers. In particular, equally productive managers that are promoted from within may get different wages. This result provides a cautionary note to concluding that wage differentials across identical workers implies inefficient labour markets.[21] Unlike Lindbeck and Snower (1988), we have an insider-outsider model of the labour market that does not imply any labour market inefficiency.

Our neutrality result stems from two sources. Firstly, we assume that workers and firms both know that they are playing a dynamic game. Workers become insiders and acquire bargaining power over time in the firm. They pay for that advantage in the competitive unskilled labour market. Secondly, we assume that the successful trainees know how many of them there are and that there is no collusion among them. If these assumptions fail, neutrality may

[21] There is a large literature debating whether unexplained wage differences in earnings regressions reflect unobserved ability differences or other factors (e.g. Dickens and Katz (1987), Kreuger and Summers (1988), Murphy and Topel (1987), Gibbons and Katz (1989)).

fail. We believe that these assumptions are reasonable. Forward looking workers and firms, as well as a competitive market for unskilled labour are uncontroversial. Successful trainees, who are in the same cohort at the firm, should recognize how many of them there are. Collusion seems difficult to sustain because it requires the manager that is retained to pay off the others. An important point is that the neutrality result is not due to our assumption of one manager per firm. Demougin and Siow (1991) shows that the neutrality result generalizes to the case of a standard concave production function in skilled and unskilled workers.

Now we show that on-the-job screening is a special case of our model. Let there be two types of workers in the economy, B and D. Both types of workers are equally productive in the unskilled job. Type D's are capable managers and type B's are not. The proportion of type D's in the population is γ. A firm wants to hire a type D to be a manager. However initially, the type of any worker is unknown to both the firm and the worker. The firm can spend some effort and elicit the type of an unskilled worker on the job without error.[24] So firms hire unskilled workers, screens some of them, and if it is able to find a type D, the firm will promote him in the next period to be a manager. If the firm is unable to find any type D's within the firm, it hires from the outside. Let the cost of screening an unskilled worker in period t be P_tC. The wage that an unskilled worker is willing to accept is $W(\gamma, S_{t+1})$. If the firm screens m_t unskilled workers, the only change for the problem of the firm is to replace $m_tC(\gamma)$ in (2.4) with m_tC. The conditions for equilibrium in the managerial and output market remain the same. The

[24] O'Flaherty and Siow (1989, 1990) studies the case of imperfect sequential screening.

Analysis of this new problem is simpler than before because γ is no longer a choice variable. No new issue arises.[25]

Finally, the prediction that unsuccessful trainees leave the training slot, under the fast track or up or out, is not specific to workers having two period lives. As long as workers are finite lived, firms have more to gain by training younger workers rather than older workers because of hiring cost. Young unskilled workers are also willing to accept a lower wage to receive training than older unsuccessful trainees. If unskilled workers improve in productivity as they gain market experience, the firm is faced with finding an optimal layoff date by trading off the gain in unskilled worker productivity and the opportunity cost of the training slot as an unsuccessful trainee ages. This extension is beyond the scope of this paper.

In conclusion, we presented a model of career profiles in hierarchies. In common with Demougin and Siow (1991) and O'Flaherty and Siow (1989,1990), this model adds demand constraints at the firm level on the supply of on-the-job training or screening. This addition generates career profiles and labour market institutions that were previously inexplicable within a competitive, symmetric information and complete contracting framework.

[25] With data on hirings, promotions and layoffs, an economist may be unable to tell the different between on-the-job screening and on-the-job training. If workers live longer than two periods, and the firm uses sequential screening or training strategies, then the two models can be distinguished (See O'Flaherty and Siow (1990)).

REFERENCES

Akerlof, George and Janet Yellen (1986), Efficiency Wage Models of the Labour Market, Cambridge University Press.

Becker, Gary (1975), Human Capital, 2nd. ed., NBER

Benopoulos, Ioanis (1990), "Incentive Limits to Scale: Optimal Span of Control and the Role of Monitoring", Columbia University manuscript.

Ben-Porath, Yoram (1967), "The Production of Human Capital and the Life Cycle of Earnings", Journal of Political Economy, 75, 352-365.

Calvo, Guillermo and Stanislaw Wellisz (1979), "Hierarchy, Ability and Income Distributions", Journal of Political Economy, 87, 991-1010.

Carmichael, H. Lorne (1983), "Firm Specific Human Capital and Promotion Ladders", Bell Journal of Economics, 14, 251-258.

Cooter, Robert and Rodrigo Restrepo (1979), "A Closed Model of Careers in a Simple Hierarchy", Bell Journal of Economics, 10, 526-548.

Costa, Joan E. Ricat I (1988), "Managerial Task Assignments and Promotions", Econometrica, 56, 449-466.

Cremer, Jacques (1986), "Cooperation in Ongoing Organizations", Quarterly Journal of Economics, 101, 33-50.

Dickens, William and Lawrence Katz (1987), "Inter-Industry Wage Differences and Industry Characteristics", in Lang, Kevin and Jonathan Leonard ed., Unemployment and the Structure of Labour Markets, Basil Blackwell.

Doeringer, Peter B. and Michael J. Piore (1971), Internal Labour Markets and Manpower Analysis, D.C. Heath and Co.

Demougin, Dominque and Aloysius Siow (1991), "Managerial Husbandry and the Dynamics of Ongoing Hierarchies", University of Toronto manuscript.

Ferrall, Christopher (1989), "Agents becoming Principals: Theory and

...imation of a Tournament Model of Promotions within Partnerships", ...versity of Minnesota manuscript.

...nter, Marc and Thomas M. Palay (1990), "Why the Big get Bigger: The ...otion-to-Partner Tournament and the Growth of Large Law Firms", Virginia ... Review, Vol. 76, No. 4, 747-811.

...bons, Robert and Lawrence Katz (1989), "Does Unmeasured Ability Explain ...er-Industry Wage Differences?", MIT manuscript.

...lson, Ronald J. and Robert H. Mnookin (1989), "Coming of Age in a Corporate ... Firm: The Economics of Associate Career Patterns", Stanford Law Review, ..., 567-595.

...ll, Robert and Edward Lazear (1984), "The Excess Sensitivity of Layoffs and ...ts to Demand", Journal of Labour Economics, 2, 233-258.

...n, Charles and Gur Huberman, "Two-sided Uncertainty and "Up or Out" Contracts", Journal of Labour Economics, 6, 423-444.

Krueger, Alan and Lawrence Summers (1988), "Efficiency Wages and the Inter-Industry Wage Structure", Econometrica, 56, 259-93.

Lazear, Edward (1979), "Why is there Mandatory Retirement?", Journal of Political Economy, 87, 1261-84.

Lindbeck, Assar and Dennis J. Snower (1988), The Insider-Outsider Theory of Employment and Unemployment, MIT Press.

MacLeod, W. Bentley and James M. Malcomson (1988), "Reputation and Hierarchy in Dynamic Models of Employment", Journal of Political Economy, 96, 832-854.

Malcomson, James M. (1984), "Work Incentives, Hierarchy, and Internal Labour Markets", Journal of Political Economy, 92, 486-507.

Mincer, Jacob (1974), Schooling, Experience and Earnings, NBER.

Morrison, Donald G. and David C. Schmittlein (1981), "A Model of Careers in a

Simple Hierarchy: Generalizing the Junior Professional's Decision Rule", Bell Journal of Economics, 12(1), 310-320.

Murphy, Kevin and Robert Topel (1987), "Unemployment, Risk and Earnings", in Lang, Kevin and Jonathan Leonard ed., Unemployment and the Structure of Labour Markets, Basil Blackwell.

O'Flaherty, Brendan and Aloysius Siow (1989), "On the Job Screening, Up or Out Rules and Firm Growth", University of Toronto manuscript.

O'Flaherty, Brendan and Aloysius Siow (1990), "Up or Out Rules in the Market for Lawyers", University of Toronto manuscript.

Oi, Walter (1962), "Labour as a Quasi-Fixed Factor", Journal of Political Economy, 70, 538-555.

Rosen, Sherwin (1968), "Short-run Employment Variation on Class I Railroads in the US, 1947-1963", Econometrica, 511-529.

Rosen, Sherwin (1982), "Authority, Control and the Distribution of Earnings", Bell Journal of Economics, 13, 311-323.

Rosen, Sherwin (1986), "Prizes and Incentives in Elimination Tournaments", American Economic Review, 76, 701- 715.

Rosen, Sherwin (1990), "Contracts and the Market for Executives", NBER working paper no. 3542.

Rosenbaum, James E. (1984), Career Mobility in a Corporate Hierarchy, Academic Press.

Rosenbaum, James E. (1989), "Organization Career Systems and Employee Misperceptions", in Arthur, Michael et. al., ed., Handbook of Career Theory, Cambridge University Press.

Sicherman, Nachum and Oded Galor (1990), "A Theory of Career Mobility", Journal of Political Economy, 98, 169-192.

...leton, David and Douglas Young (1988), "Educational Attainment and Cohort ...", Journal of Labour Economics, 6, 330-361.

...n, Steven (1987), "Promotion and Optimal Retirement", Journal of Labour ...omics, 5, Part 2, 107-123.

...k, R. H. (1954), An Essay on the Economic Theory of Rank, Basil Blackwell.

...man, Michael (1984), "Worker Allocation, Hierarchies and the Wage ...ribution", Review of Economic Studies, 51, 95-109.

...man, Michael (1990), "Up or Our Contracts: A Signalling Perspective", ...rnal of Labour Economics, 8, 230- 250.

...es, Andrew and Ruqu Wang (1990), "A Sorting Model of Labour Contracts: ...lications for Layoffs and Wage-Tenure Profiles", NBER working paper no. ...8.

...ch, Finis (1979), "Effects of Cohort Size on Earnings: The Baby Boom Babies Financial Bust", Journal of Political Economy, 87, S65-S97.

...ite, Harrison (1970), Chains of Opportunity, Harvard University Press.

APPENDIX

of Proposition 1: The problem is complicated because $\pi(n,m,\gamma)$ is not ...ave everywhere. We denote (n^*,m^*,γ^*) a solution of (2.7). Implicitly, we ... $n^{**}(S,P)$ as

$$PF'(n^{**}) - A = 0 \tag{A.1}$$

Consider the first-order condition (2.8). From (2.8.1), since $\eta \geq 0$, ...oncavity of F guarantees that $n^* \geq n^{**}$. In particular, if $n^* > n^{**}$ ... $\eta > 0$. If $\eta = 0$, then $n^{**} = n^*$. Consider figure 1. In order to keep ...figure simple, we suppress γ. Suppose there is a triplet $(n^{o*},m^{o*},\gamma^{o*})$ ... satisfies the system of first order conditions (2.8).

...1: $(n^{o*},m^{o*},\gamma^{o*})$ is the unique solution of (2.7)

First we show that any point which satisfies (2.8) is a strict local ...imum. Then we will show that this maximum is unique.

The Hessian matrix corresponding to (2.8) is:

$$H = \begin{pmatrix} PF'' & 0 & 0 \\ 0 & -\beta PH(1-\gamma)^m[\log(1-\gamma)]^2 & m\beta PH(1-\gamma)^{m-1}\log(1-\gamma) \\ 0 & m\beta PH(1-\gamma)^{m-1}\log(1-\gamma) & -PmC''-m(m-1)\beta PH(1-\gamma)^{m-2} \end{pmatrix} \tag{A.2}$$

This matrix is negative definite. To see this, note that the elements along the diagonal are all negative (the last term in the third element along the diagonal is negative because $m > 1$). The second subdeterminant is non-negative. The only difficulty is to show that $\det(H) < 0$.

$$\begin{aligned} \det(H) &= (\beta PH(1-\gamma)^m[\log(1-\gamma)]^2(PmC''+Pm(m-1)\beta H(1-\gamma)^{m-2} \\ &\quad -m^2\beta^2P^2H^2(1-\gamma)^{2m-2}[\log(1-\gamma)]^2)PF'' \\ &= m\beta P^2H(1-\gamma)^m[\log(1-\gamma)]^2(C''-\beta H(1-\gamma)^{m-2})PF'' \end{aligned} \tag{A.3}$$

Assumption (2.3), $C'' > \beta H$, guarantees that det(H) is negative. So we have shown that $(n^{o*},m^{o*},\gamma^{o*})$ is a local maximum.

To show uniqueness, suppose there is a point $(n^{o*},m^{1*},\gamma^{1*})$, corresponding to B in Figure 1, which yields a higher profit to the firm than $(n^{o*},m^{o*},\gamma^{o*})$ where $m^{o*} \neq m^{1*} \leq n^{**}$. Since $(n^{o*},m^{o*},\gamma^{o*})$ is a local maximum the firm must go through a saddle point in (m,γ) between the two points. But this ...impossible because any point which satisfies the first order conditions (2.8.2) and (2.8.3) also satisfies strict concavity.

Suppose there exist another local maximum, $(n^{2*},m^{2*},\gamma^{2*})$ which correspond to D in Figure 1 and where $m^{2*} > n^{**}$. In the proof of proposition 2, we also show that for any such point which satisfies the appropriate first-order conditions, the objective function is strictly locally concave. Thus if the firm's objective function takes a local maximum at $(n^{2*},m^{2*},\gamma^{2*})$ the objective function must also take a saddle point either in (m,γ) between A and C or in

$(m=n,\gamma)$ between C and D. Either case lead to a contradiction.

<u>Step 2:</u> Comparative static

In the case of an interior optimum, employment is determined from (2.8.1) by the real wage. Thus

$$\frac{\partial n^*}{\partial P}(S,P) = -\frac{F'(n^*)}{PF''(n^*)} \geq 0 \quad \text{and} \quad \frac{\partial n^*}{\partial S}(S,P) = 0 \tag{A.4}$$

Using the implicit function proposition on (2.8.2) and (2.8.3) yields:

$$\begin{aligned}\frac{\partial m^*}{\partial S}(S,P) &= -\frac{1}{\det(H)}\{(-PmC''-m(m-1)P\beta H(1-\gamma)^{m-2})\beta\gamma \\ &\qquad - (m\beta PH(1-\gamma)^{m-1}\log(1-\gamma))\beta m\} \\ &= \frac{1}{\det(H)}\{m\beta P\gamma(C''-\beta H(1-\gamma)^{m-2}) \\ &\qquad + m^2\beta^2 PH(1-\gamma)^{m-2}[\gamma+(1-\gamma)\log(1-\gamma)]\} \geq 0\end{aligned} \tag{A.5}$$

$$\begin{aligned}\frac{\partial \alpha^*}{\partial S}(S,P) &= -\frac{1}{\det(H)}\{(-m\beta PH(1-\gamma)^{m-1}\log(1-\gamma))(\beta\gamma) \\ &\qquad - (\beta PH(1-\gamma)^m[\log(1-\gamma)]^2)(\beta m)\} \\ &= \frac{m\beta^2 PH(1-\gamma)^{m-1}\log(1-\gamma)[\gamma+(1-\gamma)\log(1-\gamma)]}{\det(H)} \leq 0\end{aligned} \tag{A.6}$$

The sign follows because the square bracket in the last expression is non negative. In order to see this, define $g(\gamma) = [\gamma+(1-\gamma)\log(1-\gamma)]$, then, $g(0) = 0$ and for all γ in the unit interval $g'(\gamma) \geq 0$.

The derivatives with respect to P follow from (A.5) and (A.6) because differentiating the first-order conditions (2.8.2) and (2.8.3) with respect to S yields $\beta\gamma$ and βm respectively, whereas differentiating the same equations with respect to P yields $\beta\gamma(-S/P)$ and $\beta m(-S/P)$. Thus

$$\frac{\partial m^*}{\partial P}(S,P) = \frac{\partial m^*}{\partial S}(S,P)\left(-\frac{P}{S}\right) \leq 0 \tag{A.7}$$

$$\frac{\partial \gamma^*}{\partial P}(S,P) = \frac{\partial \gamma^*}{\partial S}(S,P)\left(-\frac{P}{S}\right) \geq 0 \tag{A.8}$$

...ining derivatives are important because γm is the average ...illed workers produced by firms, and since on average firms must ...ir own managers $\gamma m = 1$ is the equilibrium condition for the ...market. Therefore, the sign of the derivative of $(\gamma^* m^*)$ with the skill premium determines whether or not the market for skilled ...stable. Using (A.5) and (A.6) yields:

$$\begin{aligned}\frac{\partial(\gamma^* m^*)}{\partial S}(S,P) &= \frac{\partial \gamma^*}{\partial S}(S,P)m^* + \gamma^*\frac{\partial m^*}{\partial S}(S,P) \\ &= \frac{1}{\det(H)}(Pm\beta\gamma^2(C''-\beta H(1-\gamma)^{m-2} \\ &\qquad + m^2 P\beta^2 H(1-\gamma)^{m-2}[\gamma+(1-\gamma)\log(1-\gamma)]^2 \geq 0\end{aligned} \tag{A.9}$$

$$\begin{aligned}\frac{\partial(\gamma^* m^*)}{\partial P}(S,P) &= \frac{\partial \gamma^*}{\partial P}(S,P)m^* + \gamma^*\frac{\partial m^*}{\partial P}(S,P) \\ &= \left[\frac{\partial \gamma^*}{\partial S}(S,P)m^* + \gamma^*\frac{\partial m^*}{\partial S}(S,P)\right]\left(-\frac{P}{S}\right) \leq 0\end{aligned} \tag{A.10}$$

□

...<u>of Proposition 2:</u> We first prove that the second-order conditions of ...imization problem are satisfied. In principal, we should use a border ...n matrix. This approach is quite cumbersome though, because the border ...an is a 4X4 matrix. Instead, we examine the second-order condition of ...following optimization problem (see footnote 10):

$$\max_{\gamma,n} \; P[F(n) - nC(\gamma)] - (A+S) - nA + n\beta S\gamma - \beta PH(1-\gamma)^n \tag{A.11}$$

The first-order conditions of (A.11) are (4.1.1) and (4.1.2). For ... point which satisfy the first-order conditions, the Hessian matrix is:

$$H = \begin{pmatrix} PF''-\beta PH(1-\gamma)^n[\log(1-\gamma)]^2 & n\beta PH(1-\gamma)^{n-1}\log(1-\gamma) \\ n\beta PH(1-\gamma)^{n-1}\log(1-\gamma) & -PnC''-n(n-1)\beta PH(1-\gamma)^{n-2} \end{pmatrix} \tag{A.12}$$

The element along the diagonal are negative. We now show that the determinant of the Hessian matrix is indeed non-negative. This also concludes the argument in step 1 of proposition 1, because this shows that for any point which satisfies the first-order conditions of the firm's optimization problem, when n and m are restricted to be equal, the objective function of the firm is locally concave and thus, the point cannot be a local minimum.

$$\begin{aligned} \det(H) &= -(PF''-\beta PH(1-\gamma)^{n}[\log(1-\gamma)]^{2})(PnC''+n(n-1)\beta PH(1-\gamma)^{n-2}) \\ &\quad -(n\beta PH(1-\gamma)^{n-1}\log(1-\gamma))^{2} \\ &\geq C''n\beta P^{2}H(1-\gamma)^{n}[\log(1-\gamma)]^{2} \\ &\quad + n(n-1)\beta^{2}P^{2}H^{2}(1-\gamma)^{2n-2}[\log(1-\gamma)]^{2} \\ &\quad - n^{2}\beta^{2}P^{2}H^{2}(1-\gamma)^{2n-2}[\log(1-\gamma)]^{2} \\ &= n\beta P^{2}H(1-\gamma)^{n}[\log(1-\gamma)]^{2}(C''-\beta H(1-\gamma)^{n-2}) \geq 0 \end{aligned} \tag{A.13}$$

Using the implicit function theorem on (4.1.1) and (4.1.2) yields:

$$\begin{aligned} \frac{\partial n^{*}}{\partial S}(S,P) &= -\frac{1}{\det(H)}((-PnC''-n(n-1)P\beta H(1-\gamma)^{n-2})\beta\gamma \\ &\qquad -(n\beta PH(1-\gamma)^{n-1}\log(1-\gamma))\beta n) \\ &= \frac{1}{\det(H)}(n\beta P\gamma(C''-\beta H(1-\gamma)^{n-2}) \\ &\qquad + n^{2}\beta^{2}PH(1-\gamma)^{n-2}[\gamma+(1-\gamma)\log(1-\gamma)]) \geq 0 \end{aligned} \tag{A.14}$$

$$\begin{aligned} \frac{\partial \gamma^{*}}{\partial S}(S,P) &= -\frac{1}{\det(H)}((-n\beta PH(1-\gamma)^{n-1}\log(1-\gamma))(\beta\gamma) \\ &\quad + (PF''-\beta PH(1-\gamma)^{n}[\log(1-\gamma)]^{2})(\beta n)) \\ &= -\frac{1}{\det(H)}(PF''\beta n - \\ &\quad n\beta^{2}PH(1-\gamma)^{n-1}\log(1-\gamma)[\gamma+(1-\gamma)\log(1-\gamma)]) \gtreqless 0 \end{aligned} \tag{A.15}$$

$$\begin{aligned} \frac{\partial n^{*}}{\partial P}(S,P) &= -\frac{1}{\det(H)}((-PnC''-n(n-1)\beta PH(1-\gamma)^{n-2})(\frac{A}{P}-\frac{\beta S\gamma}{P}) + \\ &\qquad (n\beta PH(1-\gamma)^{n-1}\log(1-\gamma))(\beta Sn)) \\ &= \frac{1}{\det(H)}((nC''+n(n-1)\beta H(1-\gamma)^{n-2})A - \\ &\quad n\beta S(\gamma[C''-\beta H(1-\gamma)^{n-2}]+n\beta H(1-\gamma)^{n-2}[\gamma+(1-\gamma)\log(1-\gamma)])) \gtreqless 0 \end{aligned} \tag{A.16}$$

$$\begin{aligned} \frac{\partial \gamma^{*}}{\partial P}(S,P) &= -\frac{1}{\det(H)}((n\beta PH(1-\gamma)^{n-1}\log(1-\gamma))(\frac{A}{P}-\frac{\beta S\gamma}{P}) + \\ &\qquad \frac{\beta Sn}{P}(PF''-\beta PH(1-\gamma)^{n}[\log(1-\gamma)]^{2}) \\ &= \frac{1}{\det(H)}((n\beta H(1-\gamma)^{n-1}\log(1-\gamma)A+\beta SnF'') - \\ &\quad \beta Sn(\beta H(1-\gamma)^{n-1}\log(1-\gamma)[\gamma+(1-\gamma)\log(1-\gamma)])) \gtreqless 0 \end{aligned} \tag{A.17}$$

derivative guarantees that the market for skilled labour is stable.
(A.16) and (A.17) yield:

$$\begin{aligned} \frac{\partial(\gamma^{*}n^{*})}{\partial S}(S,P) &= \frac{\partial \alpha^{*}}{\partial S}(S,P)n^{*} + \gamma^{*}\frac{\partial n^{*}}{\partial S}(S,P) \\ &= \frac{1}{\det(H)}(Pn\beta\gamma^{2}(C''-\beta H(1-\gamma)^{n-2} \\ &\quad + n^{2}P\beta^{2}H(1-\gamma)^{n-2}[\gamma+(1-\gamma)\log(1-\gamma)]^{2} \geq 0 \end{aligned} \tag{A.18}$$

$$\frac{\partial(\gamma^{*}n^{*})}{\partial P}(S,P) = \frac{\partial \gamma^{*}}{\partial P}(S,P)n^{*} + \gamma^{*}\frac{\partial n^{*}}{\partial P}(S,P) \gtreqless 0 \tag{A.19}$$

□

Proposition 3: We first show that the aggregate supply function is
sing in the output price P. We distinguish between the fast track case,
and the up or out case, $P < P^{+}$.

Track: In this case $\Gamma(P)$ and $M(P)$ are constant. The argument follows
the market clearing requirement for the managerial market:

$$\gamma^{*}(S(P),P)m^{*}(S(P),P) = 1 \tag{A.20}$$

, using (A.7) and (A.8):

$$\frac{dS}{dP}(P) = -\frac{\frac{\partial\gamma^{*}}{\partial P}(S,P)m^{*}(S,P) + \frac{\partial m^{*}}{\partial P}(S,P)\gamma^{*}(S,P)}{\frac{\partial\gamma^{*}}{\partial S}(S,P)m^{*}(S,P) + \frac{\partial m^{*}}{\partial S}(S,P)\gamma^{*}(S,P)} = \frac{P}{S} \tag{A.21}$$

$$\frac{d\Gamma}{dP}(P) = \frac{\partial\gamma^*}{\partial S}(S(P),P)\frac{dS}{dP}(P) + \frac{\partial\gamma^*}{\partial P}(S(P),P)$$

$$= \left(-\frac{S}{P}\frac{\partial\gamma^*}{\partial P}(S,P)\right)\left(\frac{P}{S}\right) + \frac{\partial\gamma^*}{\partial P}(S,P) = 0 \qquad (A.22)$$

Since $\Gamma(P)$ is constant, (A.20) implies that $M(P)$ is also constant. Differentiating the supply function with respect to P yields:

$$\frac{dQ^s}{dP}(P) = F'(N(P))\frac{\partial n^*}{\partial P}(S(P),P) \geq 0 \qquad (A.23)$$

<u>Up or Out:</u> This case applies when $P < P^+$. It is more complicated because $\Gamma(P)$ and $M(P) = N(P)$ are not constant. Aggregate supply, employment and training are jointly determined by the following set of equations:

$$\left.\begin{array}{ll} (1) & P[F'(n) - C(\gamma)] - A + \beta S\gamma - \beta PH(1-\gamma)^n\log(1-\gamma) = 0 \\ (2) & -PnC'(\gamma) + \beta Sn + n\beta PH(1-\gamma)^{n-1} = 0 \\ (3) & \gamma n = 1 \end{array}\right\} \qquad (A.24)$$

The first two equations are just the first-order conditions of the firm's optimization problem. The third equation is the market clearing requirement for skilled workers. Eliminating βS and substituting $n = 1/\gamma$ yields the implicit definition of $\Gamma(P)$:

$$F'\left(\frac{1}{\Gamma(P)}\right) - C(\Gamma(P)) + \Gamma(P)C'(\Gamma(P)) - \beta H(1-\Gamma(P))^{\frac{1}{\Gamma(P)}-1}[\Gamma(P) + (1-\Gamma(P))\log(1-\Gamma(P))] = \frac{A}{P} \qquad (A.25)$$

Applying the implicit function theorem yields:

$$d\frac{\Gamma}{dP}(P) = -\frac{A/P^2}{\Delta} \leq 0 \qquad (A.26)$$

where

$$\Delta = \frac{-F''}{\Gamma^2} + AC'' + \beta H\left(\frac{1}{\Gamma}-1\right)(1-\Gamma)^{\frac{1}{\Gamma}-2}[\Gamma + (1-\Gamma)\log(1-\Gamma)]$$

$$+\beta H(1-\Gamma)^{\frac{1}{\Gamma}-1}\frac{\log(1-\Gamma)}{\Gamma^2}[\Gamma + (1-\Gamma)\log(1-\Gamma)] + \beta H(1-\Gamma)^{\frac{1}{\Gamma}-1}\log(1-\Gamma) \qquad (A.27)$$

$$= \frac{-F''}{\Gamma^2} + \Gamma[C'' - \beta H(1-\Gamma)^{\frac{1}{\Gamma}-2}] + \frac{\beta H(1-\Gamma)^{\frac{1}{\Gamma}-2}}{\Gamma^2}[\Gamma + (1-\Gamma)\log(1-\Gamma)]^2 \geq 0$$

the up or out case $N(P) = M(P) = \Gamma(P)^{-1}$, the aggregate supply can be rewritten:

$$Q^s(P) = F(\Gamma(P)^{-1}) - \Gamma(P)^{-1}C(\Gamma(P)) - H(1-\Gamma(P))^{\Gamma(P)^{-1}} \qquad (A.28)$$

$$\frac{dQ^s}{dP}(P) = -\frac{1}{\Gamma^2}\frac{d\Gamma}{dP}(P)(F' + \Gamma C' - C - H(1-\Gamma)^{\frac{1}{\Gamma}-1}[\Gamma + (1-\Gamma)\log(1-\Gamma)])$$

$$= -\frac{1}{\Gamma^2}\frac{d\Gamma}{dP}(P)\left(\frac{A}{P} - (1-\beta)H(1-\Gamma)^{\frac{1}{\Gamma}-1}[\Gamma + (1-\Gamma)\log(1-\Gamma)]\right) \geq 0 \qquad (A.29)$$

cond equality follows by substitution for (A.25). Note that all the s in the last term in the curled bracket, except H, are less than 1. $A > PH$ guarantees that the slope of the supply function is positive.

We have seen that the supply function is increasing in all cases. In to conclude the proof, we just note that $Q^s(0)=0$ and as P goes to ty the supply grows without bound. Thus, there exists a price which the market.

□

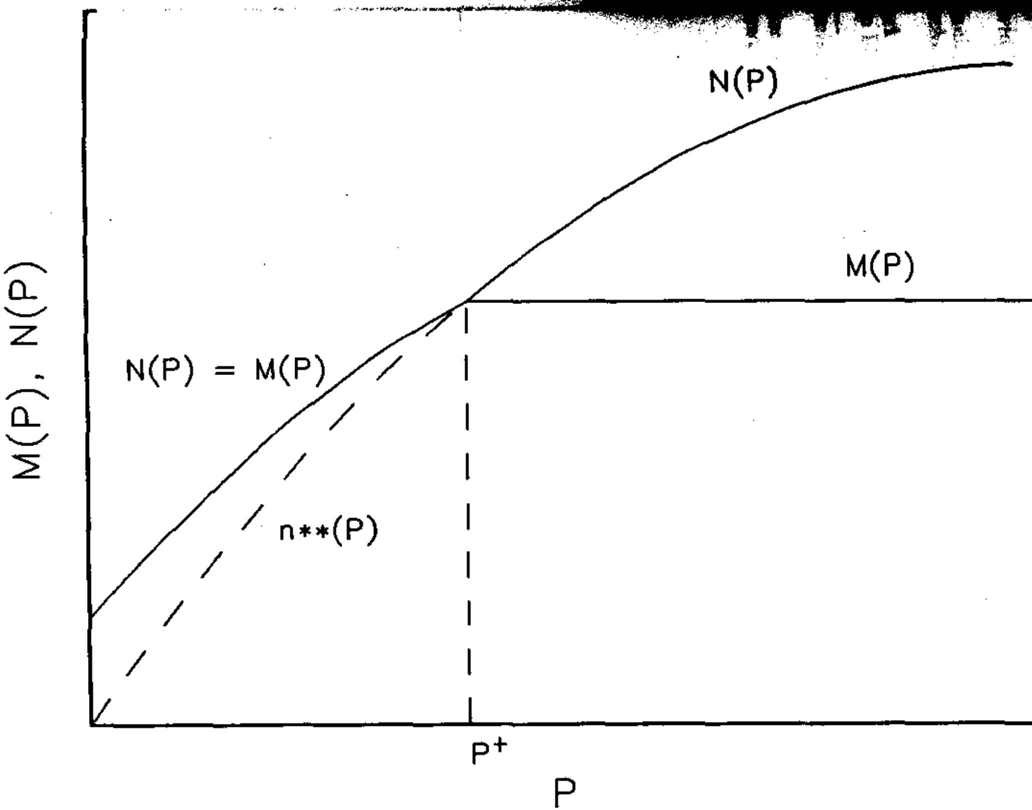
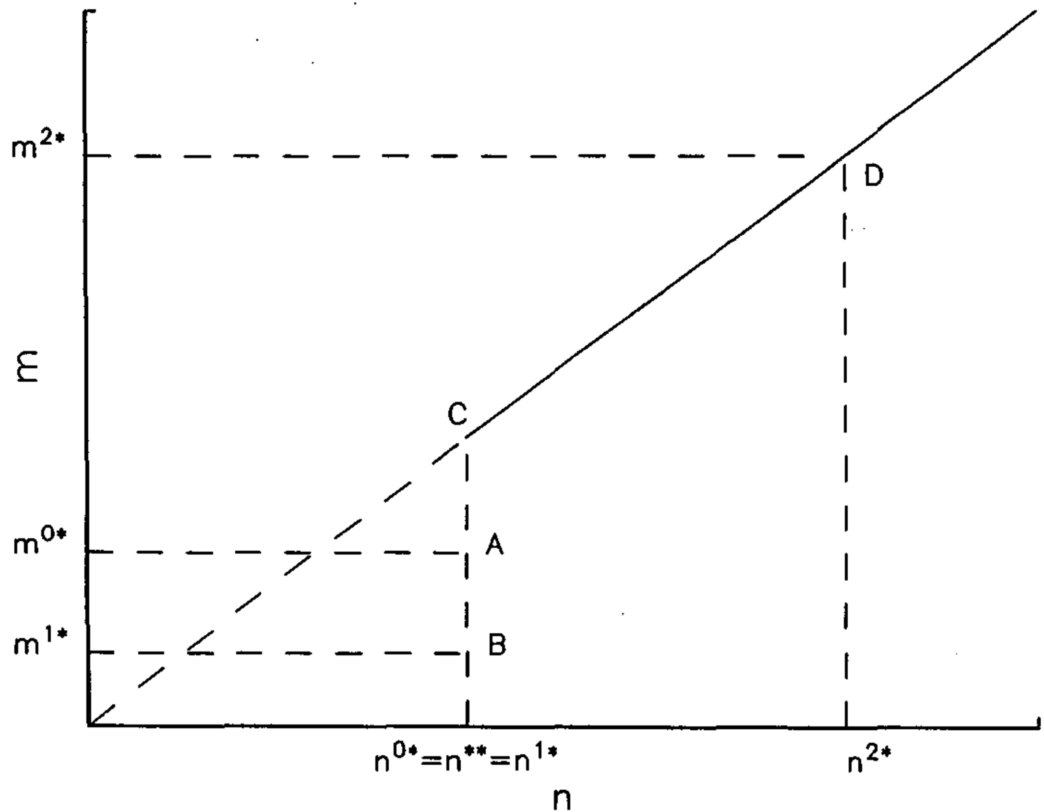

N(P)
M(P)
N(P) = M(P)
n**(P)
M(P), N(P)
P+
P

Figure A1



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