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RATIONAL BUBBLES DURING POLAND'S HYPERINFLATION: IMPLICATIONS AND EMPIRICAL EVIDENCE

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Abstract

It has been argued that the Polish hyperinflation at the end of the eighties was too excessive to be attributed to market fundamentals only. In this paper a range of indirect non-structural tests for rational, exploding deterministic and stochastic bubbles are carried out. We adopt a new strategy based on the work of Hall and Sola (1993a) which allows for the possibility of switching regimes in the time series properties of the data to allow an indirect test for the presence of stochastic bubbles.

Keywords: Rational Bubbles, Market Fundamentals, Non-stationarity, Integration, Cointegration

JEL Classification: C22, E31, E41

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1 INTRODUCTION

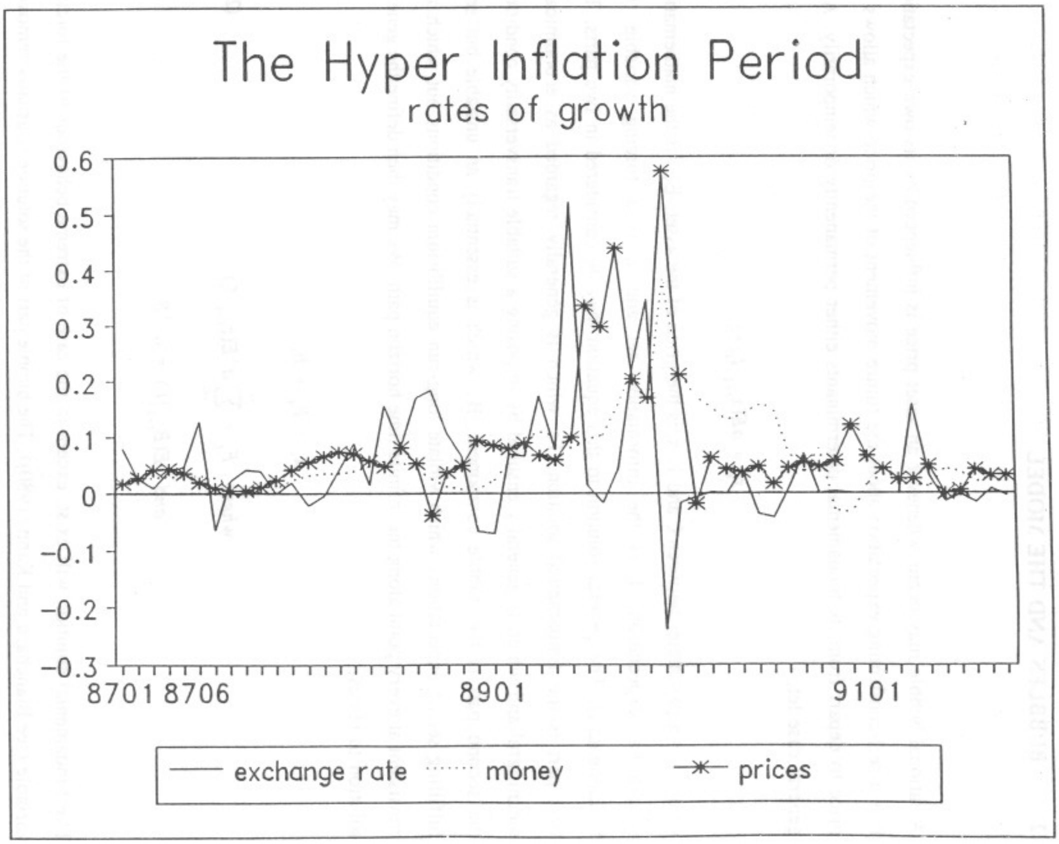
The possibility of the existence of rational bubbles has intrigued economic theorists for some time. Recently, considerable effort has been expended on an empirical search for evidence to determine the existence of this phenomenon (Shiller (1981), Diba and Grossman (1988a, 1988b, 1988c), Evans (1989) and Hamilton and Whitman (1985) for example). We would contend that the importance of this question to economies in transition may be considerable. Many of the Eastern European economies have been subject to periods of very high inflation during their transition from planned to market economies. The appropriate policy to deal with this inflation may be very different depending on the true nature of the underlying process generating the inflation. If rational bubbles are not present, then all that needs to be done is to take firm control of the market fundamentals. If, however, inflation is being driven by a bubble phenomenon, then positive action will be needed to shock expectations off the bubble path.

The case of Poland in the late 1980s is an interesting one. Late 1989 saw a burst of hyperinflation which could easily be interpreted either as the authorities loosing all control of the market fundamentals or as a classic bubble experience. Certainly, if we are ever to find sure evidence for the existence of such bubbles, it will be in a period of rapid upheaval such as this. Figure 1 details the development of the hyperinflation episode. It is worth noting that the black market exchange rate actually rose dramatically before either the price level or the money supply. We could then characterise the cause of the hyperinflation as being an autonomous rational bubble in the exchange rate which then lead to an increase in price inflation and finally a rise in the money supply. However if agents are rational, and were aware of the looseness of domestic monetary policy the episode might be seen as a rational response to the expected future rise in the money supply and prices. Thus, the exchange rate is seen as jumping to a true saddle path equilibrium in the face of future expected rises in the money supply. The appropriate policy response would differ depending on which of the two hypotheses is true. If it is a rational bubble, then we need simply work on the expectations mechanism to break the bubble. If, however, the cause is the setting of the fundamentals in the form of monetary policy, then this policy must be corrected.

The detailed economic events of the late 1980s are given in Wienicki (1990, 1991), which discusses the set of measures the government undertook to stabilise the inflation rate. Interestingly this involved not just a strong attempt to recapture control of the fundamentals but also a direct intervention in the expectations mechanism in the form of a highly punitive tax on future wage rises. We could interpret this tax as a direct attempt to burst a rational bubble and, certainly, making the currency convertible should have removed the bubble. It could, therefore, be argued that the authorities acted in a way which allowed for both possibilities and hence the subsequent control of inflation does little to help us understand the nature of the hyperinflation which took place. The aim of this paper is to see if more formal tests can be bought to bear to resolve the issue so that with a better understanding of the cause of these hyperinflation periods a more closely targeted set of polices may be designed.

The plan of the paper is as follows: Section 2 outlines the model and defines the type of bubbles we are dealing with; Section 3 then carries out a range of tests for deterministic bubbles; Section 4 broadens the testing procedure by investigating the possibility of stochastic bubbles using a switching regime model; Section 5 draws some general conclusions.

Figure 1



The Hyper Inflation Period
rates of growth
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
8701
8706
8901
9101
exchange rate
money
prices

2 BUBBLES AND THE MODEL

A rational bubble may occur whenever an asset price is influenced by its own expectation, it is a self-confirming expectation about the future movement of the price which allows the price to depart from its fundamental determinants either permanently or temporarily. As a general case let,

$$x_t = aE(x_{t+1}|I_t)+z_t \tag{1}$$

where a is a parameter between 0 and 1, x_t is the price of the asset, E(.) is the mathematical conditional expectation, I_t is the information set and z_t is a forcing variable (the fundamentals). The general solution to this equation may be considered in two parts. The first part is the fundamental solution, F_t, which is generally regarded as economically meaningful and which is generally ensured by imposing a suitable transversality condition; the second part is the bubble component, B_t, which is essentially an unstable but self-fulfilling set of expectations which violate long-run equilibrium conditions but which are consistent at every point along the infinite time horizon path. We may then define the general solution to (1) as,

$$x_t = F_t + B_t$$

$$\text{where } F_t = \sum_{\tau=0}^{\infty} a^{\tau} E(z_{t+\tau}|I_t) \tag{2}$$

$$\text{and } E(B_{t+1}|I_t) = a^{-1}B_t$$

The fundamental solution will exist, except in the case of extreme behaviour of the forcing variable (see Blanchard and Kahn (1980)). The bubble part of the solution is usually removed by the imposition of the transversality condition which prevents explosive departures from the fundamental path. If this transversality condition is not imposed, however, the bubble term may give rise to explosive solutions. In particular when (2) is deterministic, it implies that as time goes to infinity B will also approach infinity.

$$B_{t+1} = a^{-(t+1)}B_0 \tag{3}$$

So if B_0 is non-zero it will give rise to an ever decreasing, or increasing divergence between x and its fundamental solution.

The type of bubble described above is a deterministic bubble; it will go on forever and it has important implications for the integratedness of x and the cointegration of x and its fundamental determinants. If a deterministic bubble is present, it puts an explosive root into the time series behaviour of x, so x can not, in principal, be reduced to stationarity by simple differencing, hence x is not an integrated variable. x and its fundamentals will diverge over time by a non-stationary factor (B_t) and hence x and its fundamentals will not be cointegrated. For this reason tests of integration and cointegration have been proposed as a means of rejecting the existence of a bubble.

Such deterministic bubbles are, however, implausible. If a bubble goes on forever, it should be so evident in the data that a formal test is hardly needed and, typically, when we discuss the possibility of bubbles we think of them as temporary phenomena which eventually burst. A much more interesting class of bubble is that of stochastic or periodically collapsing and regenerating bubbles. If the difference equation for B_t is a stochastic equation, then at each point in time there is a probability that the bubble will burst. Evans (1991) has stressed that in the presence of such behaviour x will appear to be integrated and cointegrated with its fundamentals so that the cointegration tests are not able to detect such behaviour.

Recently, Hall and Sola (1993a) have proposed generalising the integration approach for testing for bubbles by introducing the possibility of different regimes within the sample period. This then allows for the possibility of stochastic bubbles which repeatedly collapse within the estimation period. The estimation strategy proposed relies on an application of the Hamilton (1990) filter. A remaining point of confusion, however, is that it may still be hard to distinguish between a bubble in the asset price itself and bubble-like behaviour in the determination of the fundamental series. One way of resolving this is to examine a range of series which have the same fundamentals. For example, using the illustration below, if the exchange rate and the price level are both fundamentally determined by the money supply and the money supply is expected in the future to grow rapidly, then the exchange rate and prices would both appear to have an explosive root develop at the same time. If we examine either series independently, we would be unable to distinguish between a rational bubble in that series developing and the possibility of a fundamental explanation. But when we observe the simultaneous change in behaviour of both series, it points to the presence of the fundamental cause of the problem. If we find that the change in behaviour is not synchronised across the two series, it sugests that it is not a rational response to the expected change in the fundamental and that the true explanation may be a pure rational bubble.

Following recent studies on hyperinflation, a more concrete example of a bubble may be generated through the basic theoretical framework for money demand derived by Cagan (1956),

$$(m_t - p_t) = d_t - a\,(p^e_{t+1} - p_t) \qquad a > 0 \tag{4}$$

where m_t is the log of money at time t, p_t is the log of the general price level and time t, $\Delta p^e_t = (p^e_{t+1} - p_t)$ is the expected inflation rate between t and t+1 conditional on information known at time t, d_t is a set of extra forcing variables and a is the semielasticity of real money demand with respect to expected inflation[1]. The higher is expected inflation the lower will be the demand for real money balances[2]. Given the dynamics of money growth, equation (4) determines the dynamics of inflation. Rearranging gives,

$$p_t = \frac{1}{1+a}\,(m_t - d_t) + \left(\frac{a}{1+a}\right)\left(\frac{1}{a}\right)\sum_{i=1}^{\infty}\left(\frac{a}{1+a}\right)^i (m^e_{t+i} - d^e_{t+i}) \tag{5}$$

[1] Δ is the first-difference operator. An exhaustive discussion of rational bubbles is given in Blanchard and Fischer (1989). The rational expectations solution to the Cagan model was first obtained by Sargent and Wallance (1973). For expositional purposes, we have ignored a constant term of the money demand equation; a constant was, however, included in the econometric work.

[2] An important implicit assumption in this formulation is that other variables affecting money demand are constant and are thus included in the constant term a. A generalization of this hypotheses is given in section 4 below.

which has saddle point properties[3]. Equation (5), however, is not the only solution for the price level p_t. In the case that the transversality condition fails, there is a general rational expectations solution

$$p_t = \frac{1}{1+a}(m_t - d_t) + \frac{a}{1+a}\left[\frac{1}{a}\sum_{i=1}^{\infty}\left(\frac{a}{1+a}\right)^i (m^e_{t+i} - d^e_{t+i}) + \left(\frac{1+a}{a}\right)^t A_0 + \sum_{i=1}^{t}\left(\frac{1+a}{a}\right)^{t-i} u_i\right] \tag{6}$$

where A_0 is an arbitrary constant and u_i is a stochastic disturbance term with $E[u_{t+1} | I_t] = 0$. The last two terms in equation (6) capture deterministic and stochastic price-level bubbles.

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[3] The transversality condition is given by

$$\lim_{i \to \infty} E\{[\{a/(1+a)\}^i \mid I_t] p_{t+i}\} = 0.$$

3 TESTING FOR DETERMINISTIC BUBBLES

3.1 Integration tests

In the following empirical section of the paper we raise the question whether Cagan's (1956) model for money demand under conditions of hyperinflation is consistent with the Polish experience at the end of the 1980s. In other words, could market fundamentals be viewed as the only determinants of inflation, or were deterministic rational bubbles present as well? Several tests for deterministic rational bubbles now exist in the literature[4]. The majority of these tests are conducted in a rational expectations framework. The underlying rational expectations hypothesis, however, places strong restrictions both on the relation between inflation and market fundamentals and on the form that rational bubbles can take. Therefore, the correct specification of the tests is often doubtful[5]. As a consequence we first use a simple non-structural integration test. According to this test, rational bubbles à la Cagan strictly imply that the series is not integrated of any order. In a finite sample this is usually taken to mean that the price exhibits a higher order of non-stationarity [I(i)] than any of the underlying fundamentals $[I(j)_k]$ and $i > j$ for all k's. And so, the bubble hypothesis can be rejected if all series exhibit the same order of integrability[6]. This property can be thought of as a necessary condition for the absence of rational bubbles.

A number of unit root tests in the spirit of the classical Dickey-Fuller analysis now exist in the literature. Table I presents the results from the adjusted Dickey-Fuller (ADF), Phillips (1988) and Phillips and Ouliaris (1990) tests[7].

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[4] Flood and Garber (1980) were the first to carry out a direct test for inflationary bubbles in the context of Cagan's (1956) model of German hyperinflation in the interwar period.

[5] Hamilton and Whiteman (1985) and Diba and Grossman (1988) argue that all the empirical evidence in favour of rational bubbles may be interpreted as evidence of omitted fundamental variables in the relevant regressions, i.e. what appears to be a rational bubble could instead have arisen from agents responding to fundamentals not taken into account by the econometricians.

[6] This necessary condition for the absence of bubbles has first been proposed in Hamilton and Whiteman (1985).

[7] Details of the definitions and sources of variables are given in the Appendix.

Table 1: Integration tests

Test for I(0)

	ADF(4)	ADF(12)	Z_t	Z_{alpha}	Z_{that}	Z_{ahat}
p	-0.6	-0.8	-0.1	-0.05	-0.1	-0.08
m	-0.7	0.2	0.13	0.1	0.14	0.11
e	-1.2	-1.4	-1.1	-1.0	-1.1	-1.0

Test for I(1)

	ADF(4)	ADF(12)	Z_t	Z_{alpha}	Z_{that}	Z_{ahat}
p	-1.9	-1.4	-2.1	-8.1	-1.4	-1.91
m	-1.8	-1.6	-1.7	-5.9	-2.4	-11.1
e	-2.3	-1.3	-2.3	-11.3	-6.0	-52.9

Test for I(2)

	ADF(4)	ADF(12)	Z_t	Z_{alpha}	Z_{that}	Z_{ahat}
p	-3.1	-1.9	-2.2	-10.0	-11.0	-50.1
m	-3.7	-2.4	-2.1	-10.2	-10.7	-71.1
e	-4.2	-2.9	-2.5	-12.4	-17.2	-66.0

Note: Sample period: 1987:2-1991:11. ADF(k): Adjusted Dickey-Fuller test with k lags in the dependent variable. Approximate 5 percent Dickey-Fuller critical value (for a sample size of 50) is -3.60. Blangiewicz and Charemza (1990) have recently computed critical values for the ADF test in small samples. Their 5 percent fractile for T=40 and T=50 is -2.70. Z_{alpha}, Z_t, Z_{ahat} and Z_{that} are the residual based tests suggested by Phillips (1988) and Phillips and Ouliaris (1990). The 5 percent critical values for Z_{alpha} and Z_{ahat} are -20.49 and for Z_t and Z_{that} -3.37.

The various test statistics indicate that all three time series are almost certainly stationary after second diferencing, so they are probably all I(2) series. In terms of the above indirect test for deterministic speculative bubbles, this conclusion suggests that the bubble-interpretation of Poland's hyperinflation should be precluded. However, we would add the warning that these tests do not have very strong power to detect non-integrated series as this was not the purpose they were designed for and so this can not be regarded as conclusive.

3.2 Cointegration tests

The fundamental idea underlying the non-structural cointegration tests employed in this paper comes from exploiting the difference between (5) and (6). If the latter is the correct specification, neglecting the bubble terms will lead to the result that the asset price and its fundamental determinants are not cointegrated, i.e. they will drift infinitely far apart[8]. The simplicity of this hypothesis and the possibility to avoid the structural specification of the process followed by the fundamentals are the major strengths of this sufficient test. We will begin by focusing on the Cagan relationship and see if a bubble could exist purely within the domestic price system. We will then introduce the exchange rate and see how the results alter.

We adopt the well known statistical framework of Johansen (1988, 1990,1991) for testing and estimating cointegrating vectors, but we will not present an exposition of this framework here. We assume that a given r-dimensional multiple time series is known to be generated by a Gaussian (normally distributed) VAR(k) process. In deriving the consistency properties of Johansen's ML-estimator a number of distributional assumptions were made. In practice it will rarely be known with certainty whether these assumptions are fulfilled for a specific dataset. Additionally, the LR-tests for the presence of cointegration may be quite sensitive to the order of the lags k. If, as is normally the case, the lag structure is prespecified, the test results may simply be a result of the imposed lag specification rather than of the data showing cointegration. One way out of the problem is to let the data determine the lag structure rather than imposing some arbitrary lag structure on the model. We therefore use statistical tools in order to determine the lag structure and in order to check the appropriateness of the distributional assumptions made. First, the unknown order k of the

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8 Note that this cointegration test is nonspecific with respect to expectations.

underlying data generating process is accomplished via the VAR order selection criteria AIC (Akaike's information criterion), BIC (Schwarz' information criterion) and HQC (Hannan and Quinn's information criterion). We assume an upper bound of k=18 for the VAR order and therefore set aside the first 18 observations[9]. The data up to 1991:11 are used for estimation so that the sample size is T= 47. The optimal lag length will be the one that minimizes the several statistical criteria. The results for the alternative criteria are given in table II below[10].

[9] In carrying out the AIC, BIC and HQC tests, k=18 was set to be the maximum lag structure. The number eighteen, we felt, mitigated against the potential problems of insufficient degrees of freedom and yet allowed for a sufficient number of lags to undertake a meaningful analysis with monthly data.

[10] A rigorous analysis of VAR order selection criteria is given in Lütkepohl (1991).

Table II: Estimation of the order of the bivariate VAR system

VAR order k	AIC	BIC	HQC
1	-11.400	-11.141	-11.223
2	-12.110	-11.590	-11.753
3	-12.424	-11.644	-11.890
4	-12.484	-11.444	-11.771
5	-12.611	-11.311	-11.720
6	-12.674	-11.114	-11.605
7	-12.906	-11.086	-11.658
8	-11.969	-10.889	-11.543
9	-12.494	-11.155	-11.891
10	-13.015	-11.416	-12.234
11	-13.211	-11.352	-12.252
12	-13.658	-11.539*	-12.520
13	-13.874	-11.494	-12.558*
14	-13.911	-11.272	-12.417
15	-14.246*	-11.347	-12.524
16	-13.412	-11.254	-12.512
17	-13.544	-11.125	-12.515
18	-14.007	-11.329	-12.801

Note: The two series are $(m-p)_t$ and Δp_t. * = Minimum

Based upon the results in table II the VAR order k=12 is chosen for the Johansen procedure[11]. To conserve space, only the LR-tests are reported. Given the rather small sample size we have also calculated the small-sample correction that has been proposed by Whittle (1953) and employed by Sims (1980). The correction reduces the likelihood ratio

[11] Contrary to the BIC and the HQC criterion the AIC criterion is not consistent i.e. it will asymptotically overestimate the true order of the VAR process with a non-zero probability. It is worth emphasizing that this result, however, does not necessarily mean that the AIC criterion is inferior to BIC and HQC in relative small samples. A comparison of the small sample properties of the three criteria is contained in Lütkepohl (1991), pp. 132-138 and pp. 382-384.

statistic by the factor [T-(p/q)]/T, where T is the number of observations, p is the number of parameters estimated and q is the number of equations in the system.

Table III: Test of number of distinct cointegrating vectors in the bivariate system $(m-p)_t$ and Δp_t

Number of vectors	Asymptotic LR-test	Small Sample LR-test	95% Critical Value
1	35.44	17.34	15.44
2	7.76	3.79	3.76

Note: There is an unrestricted constant in the VAR. The LR tests based on trace of the stochastic matrix test the hypothesis of reduced rank of r by setting up the hypothesis that a vector is not a cointegrating vector and then attempt to reject that hypothesis by exceeding the 95% quantiles of the limiting distribution. Estimation was carried out over the sample period 1988:1-1991:12 with k=12.

Table IV: Residual misspecification tests

	LB (8)	LB (16)	J-B-Normal
(m-p)	3.63	15.57	4.84
Δp	4.14	14.84	5.49

Note: The independence assumption is tested by the Ljung-Box Q-statistics for k=8 and k=16 [LB(8) and LB(16)]. The tests are asymptotically distributed $v^2(8)$ and $v^2(16)$. The Jarque-Bera normality test is distributed $v^2(2)$.

In table III, the likelihood ratio test statistics are calculated and compared to the 95% quantiles of the appropriate limiting distribution. The results for the small-sample LR-test indicate that the two variables are cointegrated[12]. This property can be thought of as a sufficient condition for the absence of self-fulfilling rational bubbles[13] in prices and money.

12 In most cases this ambiguity is due to the low power of the tests in cases when the cointegration relationship is close to the nonstationary boundary. This problem is often observed in empirical work when the speed of adjustment to the hypothetical steady-state is slow. See Johansen (1990) for details.

13 Since the satisfaction of the transversality condition requires the absence of bubbles the results can also be thought of as tests of the models' transversality condition.

We also calculated various diagnostic tests. The mispecification test statistics for autocorrelation and normality given in Table IV indicate that the VAR model provides a satisfactory approximation to the unknown DGP for the order of the distributed lag to be equal to twelve. Further evidence on the static long-run solution is available from Table V which gives details about how the variables get loaded into the dynamic system together with Wald tests that the loading weights are in fact zero. Table V demonstrates that under the assumption of one cointegrating vector(s) the cointegrating vector is linked with (m-p) and Δp. So in terms of causality this result strongly argues that there is feedback between the two variables under consideration. This statistical result reflects the macroeconomic situation in Poland at the end of the eighties. The year 1988 had ended with a wage explosion that carried over into 1989. The price liberalisation introduced in January 1990 was followed by a steep increase in prices. Budget expenditures also soared and the budget deficit exploded. The financing of this deficit by the monetary authorities gave rise to a surge in the supply of money and deep-seated inflationary expectations[14].

Table V: The loading weights for the estimated model

Loading weights under the assumption of one cointegrating vector:		
	(m-p)	Δp
α_{1i}	-0.26	0.29
W (1)	9.88	5.28

Note: The Wald tests [W(1)] for the α matrices are tests for the hypotheses that there are no cointegrating vectors in each equation. The W(1) statistics are asymptotically $\chi^2(1)$ variates.

And so we are unable to reject the standard Cagan model as an adequate description of the money demand process in Poland. The possibility remains, however, of a speculative bubble in the exchange market which may have influenced prices and caused the hyperinflation. One way to investigate this is to add the exchange rate to our set of variables and to look to see

[14] Note that the National Bank of Poland is not independent of the government. The new act approved in February 1992 changed this to some extent. Additionally, the effectiveness of monetary policy has been hampered by a rudimentary banking sector and a lack of instruments to control the supply of money.

if the number of cointegrating vectors increases. If purchasing power parity were maintained along with the Cagan equation we would expect to see two cointegrating vectors existing within the set of money prices and exchange rates. We therefore estimate a system for m, p and e and obtain the following results.

Table VI: Test of number of distinct cointegrating vectors in the trivariate system

Number of Vectors	Asymptotic LR-test	Small sample LR-test	95% Critical Value
1	51.3	33.09	29.68
2	16.21	10.70	15.41
3	1.37	0.90	3.76

Note: There is an unrestricted constant in the VAR. The LR test based on trace of the stochastic matrix tests the hypothesis of reduced rank of r by setting up the hypothesis that a vector is not a cointegrating vector and then attempt to reject that hypothesis by exceeding the 95% quantiles of the limiting distribution. Estimation was carried out over the sample period 1988:1-1991:11 with k=12.

Table VII: Loading weights for the estimated trivariate model

Loading weights under the assumption of one cointegrating vector:			
	m	p	e
$ALPHA_{1i}$	-0.019	-0.005	-0.002
W (1)	39.05	0.50	0.03

Note: The Wald tests [W(1)] for the α matrices are tests for the hypotheses that there are no cointegrating vectors in each equation. The W(1) statistics are asymptotically $\chi^2(1)$ variates.

Applying the LR-test for cointegration due to Johansen (1988), the null non-cointegrating hypothesis is easily rejected at the 5% level. There is, however, still only one cointegrating vector, so the addition of the exchange rate has not added an extra vector to the system. Indeed the loading weights and the wald test also indicate that the one vector does not enter the exchange rate equation. So there is no cointegration between exchange rates and money and prices but only between money and prices in isolation. This, therefore, leaves open the possibility that there might be a bubble in the exchange rate which was the root cause of the inflationary process in Poland.

In the next section we pursue this further by applying the stochastic bubble tests mentioned earlier.

4 TESTING FOR STOCHASTIC BUBBLES

The problem with detecting stochastic bubbles, stressed by Evans (1991), is essentially that they will only exhibit characteristic bubble properties during the expansion phase. So if we test a complete sample we are unlikely to observe any systematic divergence between the asset price and its fundamentals. We must instead focus on the expansion phase only and look there for some systematic departures. This raises the problem of how we may detect the expansionary phase without prejudging the issue by choosing only those data points which fit out prior views as to what a bubble should look like. We do this by adopting a methodology proposed by Hall and Sola (1993a) which is based on the use of an extension of the standard ADF statistic to allow for the possibility of two regimes in the time series properties of the data using the Hamilton (1990) filter. This sets up the hypothesis that there may be two regimes governing the process generating the asset price and its fundamentals. So we might find that in one regime the data was non-stationary, perhaps even with an explosive root, and in the other regime it was stationary, collapsing back towards the fundamental solution. The option is always open to the filter to find that there is in fact only one significant regime. So the filter decides whether there is in fact more than one regime driving the data as a whole and it provides an endogenous split of the whole sample into the two regimes. We can then examine the two regimes to see if there is any detectable signs of bubble behaviour in each regime separately.

The basic Hamilton filter in its most general form is outlined in the appendix. Hall and Sola (1993a) applied this filter to the question of testing for stationarity in the presence of structural changes. In his applications, Hamilton (1988, 1989, 1990) implemented a very limited form of switching where only the constant and the variance of an equation was allowed to switch between regimes. Hall and Sola (1993b) discussed various generalizations of this which allowed the full parameter vector to switch and they also discussed the implications of various possible assumptions about the treatment of dynamic effects. Hall and Sola (1993a) proposed to use this methodology to specify a version of the standard ADF statistic which allows the DGP to switch between discrete regimes. We can think of this in the specific context of rational bubbles, where the motivation for the two regimes is quite obvious[15]. Or we may think of it in the more 'ad hoc' context of general structural breaks (as in Peron, 1989) where we could potentially increase the number of states to allow for any number of structural breaks. The switching parameters (S_t) of the ADF regression are discrete valued state variables. The state can take two possible values (0 or 1), in the example we will consider, although theoretically we can extend this to any number of states.

$$\Delta y_t = S_t(\alpha + \phi y_{t-1} + \sum_{i=1}^{n} \psi_i \Delta y_{t-i}) + (1-S_t)(\beta + \phi^d y_{t-1} + \sum_{i=1}^{n} \psi_i^d \Delta y_{t-i}) + (w_0 S_t + w_1(1-S_t))u_t \qquad (7)$$

So (7) will allow us to ask the question, are both regimes stationary or non-stationary? It will essentially supply us with two ADF statistics, one for each regime.

George Evans (1991) rightly argued that rational bubbles in stock prices are not reliably detectable through the use of standard tests to determine whether stock prices are 'more explosive' than dividends or whether stock prices and dividends are cointegrated. He demonstrated that periodically collapsing bubbles appear to be stationary on the basis of these tests, even though they have explosive conditional means, through a range of monte carlo experiments. Hall and Sola (1993a) demonstrate the effectiveness of the switching ADF test given above by taking the Evans data generation process and showing that it was in fact able to detect the two regimes and that it was able to reveal one of them to have an explosive root while the other was a stable process.

We proceed to apply the switching ADF test outlined in equation 7 to the three series money, prices and exchange rates for Poland. The results of this estimation exercise are shown in table IX, using the same notation as outlined in (7). The results for all three series are strikingly similar, but there is a marked difference between the two regimes. Most of the

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15 Evans bubbles start at period zero and burst time to time. When we say a bubble period we mean that this bubble (which by construction always exists) has a relevant size. This should be clear in section 3.

period is characterised by a unit root process with moderate drift and fairly simple dynamics. There is strong evidence, however, of a second regime in all three series which is characterised by an unstable root with much more complex dynamics. The last two lines of table VIII show the estimated parameter on the levels terms of the in the ADF formulation along with its associated 't' statistic which would be the normal formal ADF test. In the unit root regime we therefore find coefficients very close to unity with test statistics well below the conventional Dickey-Fuller significance levels but where the coefficients are on the stable side of the unit root. In the unstable regime we find positive coefficient estimates, which indicate an explosive root, and there 't' statistics are clearly well above the conventional Dickey-Fuller values. In the previous section we found that, treating the whole series as structurally stable suggested an I(2) characterization for all three series. Now allowing for the switch to an unstable regime produces, at most, an I(1) characterization of the data for most of the period but with the addition of the finding of an unstable sub-period. So we have strong evidence of a structural change in the data generation process over part of the period which certainly may suggest the presence of a rational bubble but it does not exclude the fundamental explanation.

Table VIII: The switching ADF test for the money, prices and exchange rates.

-	exchange rate	money	prices
α	0.18(2.3)	0.01(2.4)	0.014(0.8)
ψ_1	0.09(0.2)	0.42(4.3)	0.8(9.6)
ψ_2	-0.009(.1)	-0.23(3.9)	-0.05(0.7)
ψ_3	-0.06(.8)	0.17(3.1)	0.09(0.8)
ψ_4	-0.08(.91)	-0.09(2.0)	-0.02(0.18)
β	-1.08(2.3)	-6.1(12.3)	-3.03(73.4)
ψ_1^d	-4.27(17.9)	-2.2(15.4)	-1.49(141)
ψ_2^d	-1.2(9.3)	-0.89(10.1)	-0.33(32.5)
ψ_3^d	0.2(.4)	-1.37(9.2)	2.16(151)
ψ_4^d	-1.8(17.9)	-3.1(13.1)	-2.52(149)
σ	0.05(9.4)	0.02(9.3)	0.03(11.7)
σ^d	0.0006(3.7)	0.01(3.5)	0.00042(2.5)
p	0.88(5.5)	0.82(6.2)	0.88(11.6)
q	0.94(24.9)	0.98(43.9)	0.99(662)
ϕ	-0.02(1.9)	-0.00067(0.3)	-0.00058(0.29)
ϕ^d	0.27(4.2)	1.14(13.3)	0.37(78.3)

asymtotic 't' statistics in parenthesis

We turn now to Figure 2 to consider the timing of the regimes in the three series. This figure shows the probability of being in each of the two states at each point in time. When the probability is zero we are in the unit root regime and when it is unity we are in the unstable root regime. Unsurprisingly, the switch to the unstable regime occurs for all three series during the hyperinflation period. But the timing of the switch is interesting. The exchange rate has a short period of instability in late 1988 and then in 1989 it is again the first of the three to switch to the unstable regime. This is followed by prices and only later by the money stock. If prices and the exchange rate were both being driven by the money fundamentals, we would have expected to have seen a similar pattern of switching across the two series. In fact, the synchronisation of the two patterns is not very close and we would suggest that this points tentatively towards the conclusion that the main determination of the hyperinflation was a rational bubble in the exchange market. This conclusion must, however, remain tentative as it rests on the difference in the pattern of switching in the price level and the exchange rate and there is no formal way to gauge this difference and the data itself is subject to measurement problems.

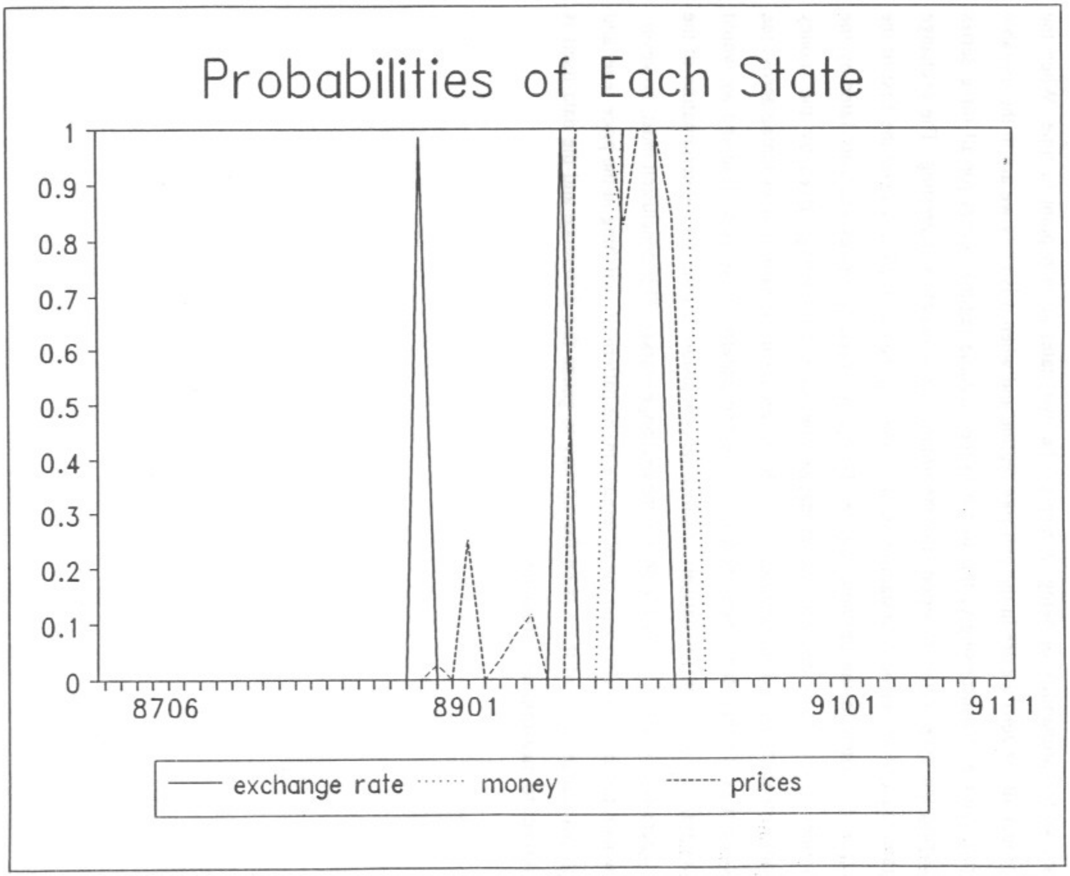
Probabilities of Each State
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
8706
8901
9101
9111
exchange rate
money
prices

Figure 2.

5 CONCLUDING REMARKS

This paper has examined the Polish hyperinflation period of the late 1980s with a view to attempting to distinguish between the possibility of a fundamental explanation of inflation in terms of a loose money supply and an explanation based on a rational bubble developing in the prices of the system (either the exchange rate or domestic prices). We have tested the data for the exisence of deterministic bubbles and find that we can effectively rule out the possible existence of a deterministic bubble in the Polish case. We have then gone on to apply a new technique for examining the possible existence of a stochastic, or temporary, bubble. We find strong support for the notion that the basic regime of the price and money variables switched during the hyperinflation period. The timing of the switching in the different series leads us tentatively towards the conclusion that the main origins of the hyperinflation episode lay in the exchange market.

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APPENDIX A: Data Sources

All monthly data are in natural logarithms. The sample period is 1988:1 to 1991:11. The source of m and p is IMF International Financial Statistics, various issues and OECD database. The source of e is Koedijk and Kool (1992) and Pick's World Currency Yearbook, various issues. For the first year the published quarterly narrow money stock data have been transformed to monthly data using a time series process which is solved by the quadratic-linear form of the dynamic programming algorithm [for details, see, for example, Bertsekas (1976), 70-72].

m = Narrow money (billions of Zlotys, end of period, IFS line 34). Before 1989 the data incorporate the commercial banking operations of the NBP. Beginning in 1989, the data cover only the operations of the commercial banks, the number of which increased rapidly.

P = Index of retail prices, 1985=100 (Paasche index, IFS line 64).

e = Black market exchange rate of the Polish Zloty (number of currency units per U.S. dollar, end of period)

APPENDIX B: The General Hamilton Filter

The Hamilton (1990) Filter is a non-linear filter which allows the state equation to follow a particular non-linear restriction where the state variables follow a markov chain subject to a discreet adding-up restriction.

The general problem statement consists of the usual two parts, sets of measurement equations and state equations.

The general measurement equation may be written as

$$y_t = x_t^{\prime} B \xi_t + \omega_t$$

$$\textit{where } \omega \sim NIID(0, \xi_t \sigma^2),\ \xi_t \textit{ are the n state variables} \tag{8}$$

$$\textit{and } x_t^{\prime} B \textit{ are known}$$

the state equations take the usual form

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{9}$$

where F is an nxn matrix. Now if v_t were normally distributed this would be a standard state space model and the Kalman Filter would give optimal inference. The idea behind this model is however that each of the state variables represent the probability of being in a different state of the world (S_i, i=1...n). This implies that the state variables must sum to unity and, therefore, cannot be subject to the usual error process. The Hamilton Filter gives optimal inference and evaluates the likelihood function for this non-linear system. The motivation for this model is, of course, very strong as this system may be seen as a general regime switching framework. For example, if there are two state variables, this model encompasses the endogenous regime switching disequilibrium model.

The filter rests on the following statements, under the normality assumption made in the measurement equation,

$$f(y_t|S_t=i,X_t) = \frac{1}{(2\pi\sigma_i^2)^{1/2}} \exp\left(\frac{-(y_t-x_tB_i)^2}{2\sigma_i^2}\right) \tag{10}$$

and of course

$$f(y_t,S_t=i|x_t) = f(y_t|S_t=i,x_t).prob(S_t=i|x_t) \tag{11}$$

and

$$f(y_t|x_t) = \sum_{i=1}^{n} f(y_t,S_t=i|x_t) \tag{12}$$

The optimal inference about S_t may then be based on

$$\xi_{ti} = Prob(S_t=i|x_t,y_t) = \frac{f(y_t,S_t=i|x_t)}{f(y_t|x_t)} \tag{13}$$

The one-step-ahead prediction errors may then be derived by using the state equation to produce a forecast of the state variables in t+1 and the measurement equation may be evaluated in the usual way. The likelihood function may then be evaluated.