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THE DESIRE FOR LAND: STRATEGIC LENDING WITH ADVERSE SELECTION

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## Abstract

This paper is concerned with two consequences of the attachment to land. First, it established precise conditions under which debt contracts secured by land are mutually preferred to trades of cash against (future) delivery of land. Second, by extending a standard model of adverse selection, it explains how different debt contracts can coexist. When an 'inside' and an 'outside' lender compete, the latter placing a lower value on the collateral, a separating equilibrium may exist in which the 'insider' offers a contract which is attractive only to 'risky' borrowers, whereas the 'outsider' offers a pooling contract. This holds even when borrowers are not rationed ex ante.

Keywords: Rural credit markets, adverse selection

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# 1. Introduction

This paper is concerned with several consequences of the peasant's legendary attachment to his land and, more generally, with the consequences of differences in the value of a particular asset to different borrowers and lenders. It will be argued that such differences underlie the observation that the market for land is often thin, and yet, rather paradoxically, that there is also extensive indebtedness, with loans secured by land. It will be argued, too, that differing valuations of an asset that is used as collateral can explain the phenomenon whereby different lenders can offer different credit contracts and yet stay in business, despite the fact that some of these contracts are evidently more attractive to borrowers than others, as is vividly demonstrated by the evidence for rural areas arrayed and analysed in Hoff et al. (1993). It will be established that, under adverse selection, it may be optimal for some types of lender to offer contracts that are attractive only to "risky" borrowers, which runs exactly counter to the standard argument [Stiglitz and Weiss (1981) and Clemenz (1986)].

The peasant will sell his land only in dire need. He will lease out his land if circumstances warrant, preferably to a relative, and even then on a short-term basis. If his need for cash is especially pressing, he will sometimes mortgage his land, often on the condition that he lease it back on a usufruct basis, and he will do everything he can to redeem the mortgage.. He is not, however, the only member of rural society who has a strong desire for land. The moneylender and trader, who are often one and the same, also value land, not only to improve their portfolios of assets, but also for the social prestige that its ownership confers. Their own shortcomings as cultivators, moreover, will be no bar to profitable ownership if there is an ample supply of capable tenants. The problem, for the moneylender and trader, is that the peasant is extremely reluctant to sell -- at a price they find acceptable. Hence, the market for land as an asset is usually very thin, even though that for tenancies is quite active [see, for example, Binswanger and Rosenzweig (1986)].

Yet land does change hands, often in times of distress, when crops have failed or commodity prices have collapsed. In such cases, much of the land in question had been pledged as collateral to secure a loan, or mortgaged outright. The lender therefore achieved indirectly through a loan what he was unable to do profitably through a straight purchase. There were several episodes in South Asia in the late nineteenth and early twentieth centuries in which many peasants lost their land to moneylenders in this way (Rothermund, 1993, pp. 46-48). Some of these episodes followed changes in the law that effectively transformed land into a commercial commodity. It might be argued, therefore, that such alienation on a large scale was simply the result of the peasants' failure to understand that, by pledging their land as collateral, they had put it at risk.

That is not, however, the whole story. In his classic study of the punjabi peasantry, Darling (1925) demonstrated that the peasants responded to good times by going into debt, using their land as security. When bad times followed, many of them lost their land. Darling took pains to understand what motivated peasants and lenders to enter into such contracts. For the peasant, a loan secured by land meant that he was still the owner, and would remain so if all turned out as he expected. As for the moneylender, he advanced credit not only for the interest it would yield, but also with an eye to acquiring the land itself. That is to say, he lent with a strategic aim in view. It can be argued, therefore, that a further consequence of this attachment to land is the extensive use of debt.

We begin, in section 2, by establishing precise conditions under which the parties will enter into debt contracts secured by land when the loan is used as working capital, in preference to either straight trades of cash against (future) delivery of land, or no contract at all. Differences in the parties' valuations of the land play a key role in these results. In section 3, the optimum contract of a monopolistic lender is characterised when information is symmetrically held by the two parties, a characterisation which also forms part of the groundwork for the model of adverse selection developed in section 4.

In section 4, we explore and explain the phemenon whereby different lenders offer contracts with different terms and yet stay in in business. In order to accomplish this, we extend the models of adverse selection developed by Stiglitz and Weiss (1981) and Clemenz (1986) to incorporate the considerations that arise in sections 2 and 3. These extensions take the form of a loan whose size can vary and of differing valuations of the collateral. When an 'inside' lender competes with an 'outsider', the latter placing a lower value on the collateral, we prove that there can exist a separating equilibrium in which the 'insider' offers a contract which is attractive only to borrowers whose probability of default is high, whereas the 'outsider' offers a pooling contract aimed at the "average" borrower. Such an equilibrium can exist, moreover,

even when there is no rationing of loans ex ante, in the sense that the two lenders have sufficient funds between them to ensure that all borrowers can obtain an acceptable contract. In such an equilibrium, the insider's choice of strategy is swayed by his (relatively) strong desire for land, and that strategy is runs wholly counter to the standard results in earlier models.

# 2. 'Strategic Lending' Versus Straight Exchange

There are two sorts of agents, borrowers and lenders, and two assets, land and money. Borrowers have initial endowments of land, but no money; lenders have initial endowments of money, but no land. If there were perfect spot markets for land in all periods, a lender could always buy land at the going rate for current use and, if he wished, sale at a later date. This possibility deprives the granting of a loan against the security of the borrower's land of any 'strategic' value -- though moneylending on such terms may still be a more attractive option than the alternative of earning a riskless return in a safe placement.

Suppose, therefore, that there are no spot markets for land, but that individuals can enter freely into bilateral trades of land for money. If the deal takes the following form of a futures contract, it will be equivalent to a secured loan on which the borrower defaults with certainty, but retains exclusive rights to the return. The lender gives the borrower a sum in one period, which the borrower invests to yield a risky return in the next. The borrower keeps the entire return, in exchange, the lender acquires the land which has been pledged as security. The question is whether there exists a sum such that both parties find the deal acceptable. If there does not, then no voluntary trade will occur.

Consider, next, the case of a standard debt contract in which the loan is secured by the borrower's land and there is some probability of default, which arises from the riskiness of the borrower's investment. This contract can be viewed as a convex combination of a bilateral trade of the kind just described above and a loan yielding the denominated rate of interest with certainty. Even if there exists no mutually acceptable bilateral trade, there remains the possibility that there exists a mutually acceptable standard debt contract. In that case, one can legitimately speak of the loan as having 'strategic' value to the lender; for there exists no direct way of acquiring land on acceptable terms. This interpretation of the debt contract is also

consistent with accounts in the institutionalist and anthropological literature, which emphasize farmers' reluctance to sell outright, but also tell of their grudging willingness to offer land as collateral in order to obtain loans. In the latter case, there is always the prospect of repaying the loan, and so of holding onto their land.

Having established that are there circumstances in which lending may be construed as having a strategic motive, we turn to a formal statement of the problem, which we have reduced to the bare essentials. The potential borrower is a farmer who owns a plot of land A. If he borrows the amount B, the farmer will produce an output whose cash value is x, where

(1) 
$$x = \begin{cases} q + f(B) & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi. \end{cases}$$

Thus, there are just two states of nature, and the probability of a successful outcome,  $\pi$ , is independent of the farmer's investment of working capital. It is assumed that f(B) is increasing, strictly concave and twice-differentiable in B, with f(0) = 0. This latter case may be interpreted as a traditional technique, in which output is producible without resort to external finance. Inputs must be committed before the state of nature is revealed.

Both agents are risk neutral. The lender has a hoard of cash,  $y_0$ , and access to a safe placement yielding the rate i. The farmer and lender put cash (numéraire) values on the plot at V and v, respectively. It is highly plausible that V>v. First, the farmer will have a psychological and social attachment to the land, both as the current owner and as a member of his community. Let the value of this attachment be denoted by  $\alpha$ . Second, having farmed the land in the past, he will have some private knowledge of how to get the best out of it. The value of the land to the farmer is the sum of  $\alpha$  and the expected value of the output he can produce from it:

$$(2) V = \alpha + \pi q.$$

In what follows, it will be assumed that farmers differ only in  $\pi$  and q.

Since the plot is indivisible, all of it is pledged as collateral to secure a loan, or all of it is exchanged in a straight bilateral trade. One justification for this simplication is that plots of land are legally registered and subdivision is costly. Subdivision beyond a certain point may be impossible also on technical grounds.

In the case of a standard debt contract (B, r, A), in which the amount B is lent at the rate of interest r and secured by A, the farmer's and lender's respective endowment vectors upon the realization of output, when the loan matures, are

$$Z = \begin{cases} \left[ A, q + f(B) - (1+r)B \right] & \text{with probability } \pi \\ \left[ 0, 0 \right] & \text{with probability } 1 - \pi, \end{cases}$$

and

$$z = \begin{cases} \left[ 0, (1+r)B + (1+i)(y_0 - B) \right] & \text{with probability } \pi \\ \left[ A, (1+i)(y_0 - B) \right] & \text{with probability } 1 - \pi. \end{cases}$$

The levels of final wealth of the borrower and lender are, respectively,

(3a) 
$$\Omega = \begin{cases} [q + f(B) - (1 + r)B + V] & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi, \end{cases}$$

and

(3b) 
$$\omega = \begin{cases} [(1+r)B + (1+i)(y_0 - B)] & \text{with probability } \pi \\ [\nu + (1+i)(y_0 - B)] & \text{with probability } 1 - \pi. \end{cases}$$

Let E denote the expectations operator, and let I:=(1+i) and R:=(1+r). The contract

(B, R; A) therefore yields the following levels of expected utility:

(4a) 
$$E\Omega = \pi [q + f(B) - RB] + \pi V$$

and

(4b) 
$$E\omega = \pi RB + (1 - \pi)\nu + I(y_0 - B)$$
.

We turn now to a bilateral trade (A,S), under which the farmer receives S and the land changes hands when production is complete. Their endowment vectors are, respectively,

$$Z' = \begin{cases} [0, q + f(S)] & \text{with probability } \pi \\ [0, 0] & \text{with probability } 1 - \pi, \end{cases}$$

<sup>&</sup>lt;sup>1</sup>A further possibility is that a market of sorts for land exists, but that it is thin; so that a buyer's chances of encountering a seller depend on the state of the world. A loan then possesses a strategic advantage insofar as it increases the probability that a lender will be able to acquire land at a particular point in time. For if the loan is secured by land, the lender is at the head of the queue of potential buyers should the borrower be forced into a 'distress' sale.

and

$$z' = [A, (1+i)(y_0 - S)]$$
 with certainty.

The first step is to establish conditions under which one form of the above two contracts will always Pareto-dominate the other. The following proposition establishes the central role played by the parties' valuations of the land.

<u>Proposition 1</u>. A standard debt contract is Pareto-superior to a bilateral exchange if V > v. The converse holds if v < V.

<u>Proof.</u> Consider any (A,S). In a standard debt contract, set B=S and RB=V. Then the farmer will have indentical pay-offs (measured in terms of the numéraire) in each state of nature under both forms of contract. The lender's final endowment vector will be as follows: If the project fails, he will obtain  $[A,I(y_0-B)]$  under both contracts. If it is successful, he will obtain  $[0,RB+I(y_0-B)]$  under a standard debt contract. If V>v, he prefers the latter to  $[A,I(y_0-B)]$ , which he would obtain in a bilateral trade; for RB=V. If v>V, an analogous argument establishes that a bilateral exchange will dominate a standard debt contract. Q.E.D.

As argued above, V > v is strongly plausible, and it yields an explanation for the prevalence of indebtedness over bilateral trading in land. The intuition for this result is that when the borrower values the land more highly than the lender, a standard debt contract has the advantage that the former will retain possession of the land some of the time. Conversely, when the lender's valuation is the higher of the two, a bilateral (forward) trade will give him possession always. It should be noted that a bilateral trade can mimic any standard debt contract if a state-contingent payment is made upon realisation of the output. A standard debt contract still has the advantage, however, that the lender does not need to observe the state of nature if RB is less than V; for in this case, the borrower will have an incentive to repay the loan when the project succeeds, and he will be unable to repay when it does not. If RB exceeds V, the lender's position will be the same under both arrangements [see also Gale and Hellwig (1985) on this point].

The next step is to establish conditions under which a loan contract secured by land will be Pareto-superior to the agents' reservation alternatives of not contracting at all.

<u>Proposition 2</u>. Let  $B^* = \arg\max_{B} \left\{ \pi \left[ f(B) - IB \right] \right\}$ . Then there exists a standard debt contract which is Pareto-superior to not contracting at all if and only if

(5) 
$$\left\{\pi \left[f(B^*)\right] - IB^*\right\} - (1-\pi)(V-\nu) > 0.$$

Proof: The agents' reservation levels of expected utility are, respectively,

(6a) 
$$\Omega = \pi q + V$$

and

(6b) 
$$\underline{\omega} = Iy_0$$

We therefore ask whether there exists a contract (B,R;A) such that  $E\Omega \ge \underline{\Omega}$  and  $E\omega \ge \underline{\omega}$ , with at least one holding as a strict inequality<sup>2</sup>. From (4a), (4b), (6a), (6b), the required conditions may be written as:

(7a) 
$$\pi [f(B)-RB]-(1-\pi)V \ge 0$$

and

(7b) 
$$(\pi R - I)B + (1 - \pi)v \ge 0$$
.

Let (7a) hold with equality. Substituting for RB in (7b) yields the condition

(8) 
$$\pi [f(B)-IB]-(1-\pi)(V-\nu)>0.$$

Then (7a) and (7b) will hold if and only if (5) holds. Q.E.D.

Remark 1: Condition (5) states simply that the maximal expected profits from applying finance to production on the farm must exceed the expected loss that arises in connection with the property changing hands in favour of the lender, whose valuation of it is lower than the

<sup>&</sup>lt;sup>2</sup>It is assumed that the lender's initial endowment of cash,  $y_0$ , is so large that the choice of (B,R;A) is not otherwise constrained.

borrower's. Given the assumptions on f(B), a necessary, but not sufficient, condition for (5) to hold is  $f'(0) > I/\pi$ . We assume henceforth that this condition is fulfilled. Since  $f(\cdot)$  is strictly concave and smooth,  $B^*$  is unique and satisfies  $f'(B^*) = I/\pi$ .

To sum up, Propositions 1 and 2 state necessary and sufficient conditions for trading in land to be absent, and yet for landowners to use their land as collateral to secure loans, so that land will indeed change hands when they are unable to repay.

## 3. The Monopoly Contract Under Symmetric Information

We begin by establishing the shapes of the agents' preference maps in the space of (B,R). For the borrower, total differentiation of (4a) yields

$$(9) \qquad \frac{dR}{dB}\Big|_{E\Omega} = \frac{f'-R}{B}.$$

A section of the locus of pairs of (B,R) satisfying f'(B) = R is simply the notional demand schedule for credit. As  $f(\cdot)$  is strictly concave, this schedule is downward-sloping. The (limit) point of this schedule where demand vanishes is the pair thereon that also satisfies (7a) with equality. That is, it is the intersection of the level surface  $E\Omega(B,R) = \Omega$  with the locus defined by f'(B) = R. In Figure 1, this limit point is denoted by  $(\tilde{B}, \tilde{R})$ . At the other extreme,  $\lim_{B \to \infty} R = 0$ . Each of the borrower's level surfaces in the space of (B,R) is upward-sloping up to the point at which it crosses this schedule and is downward-sloping thereafter. Differentiating (9) once more, we obtain

(10) 
$$\frac{d^2R}{dB}\bigg|_{E\Omega} = \frac{Bf''-2(f'-R)}{B^2}.$$

It is clear from (9) that these level surfaces are quasi-concave. It follows from (10) that they are strictly concave to the left of the locus defined by f'=R, and hence to the left of the notional demand schedule.

Turning to the lender, total differentiation of (4b) yields

(11) 
$$\frac{dR}{dB}|_{E\omega} = -\frac{\pi R - I}{\pi B}.$$

These level surfaces are upward- or downward-sloping according as  $I \gtrsim \pi R$ . The nature of the map can be characterised precisely as follows. By writing (4b) in the form

$$[\pi R - I]B = E\omega - [(1 - \pi)\nu + Iy_0],$$

it is seen that the map is a family of rectangular hyperbolae, with origin  $[0,(I/\pi)]$  in (B,R)-space. For any given  $(I,\nu,y_0,\pi)$ , the map is generated by varying  $E\omega$ . For the level surface corresponding to  $E\omega = [(1-\pi)\nu + Iy_0]$ , we must have  $R = I/\pi$   $\forall B$ . This horizontal line is a sort of 'watershed' in the map: for all  $E\omega > [(1-\pi)\nu + Iy_0]$ , the level surfaces slope downwards; in the converse case, they slope upwards. All are, of course, asymptotic to the said horizontal line.

The lender's problem is to

(12) maximize  $E\omega$  subject to  $E\Omega \ge \underline{\Omega}$ ,

where  $E\Omega$ ,  $E\omega$  and  $\Omega$  are given by (4a), (4b) and (6a), respectively. If  $V>\nu$  and condition (5) is satisfied, this problem will have a solution  $\left(B^0,R^0\right)$  such that  $B^0>0$  and  $E\omega^0>\underline{\omega}$ . Two possibilities are depicted in panels (a) and (b) of Figure 1. Since the allocation will be fully efficient under the circumstances assumed here, with R employed solely to redistribute utility between the two parties, it follows that  $E\Omega=\Omega$  at the lender's optimum. Hence, the contractual variables take the values  $B^0=B^*$  and  $R^o=R^*$ , where  $R^o$  satisfies

(13) 
$$\pi [f(B^*) - R^0 B^*] - (1 - \pi)V = 0$$

As noted above, the uniqueness of  $B^*$  is assured by the strict concavity of  $f(\cdot)$ , and that of  $R^0$  then follows from (13).

Remark 2: It follows from the definition of B\* that B\* is increasing in  $\pi$ .

In order to complete the groundwork for section 4, we shall need the effects of changes in  $\alpha$ , q and  $\pi$  upon the lender's expected wealth at his optimum. By writing out problem (12) in full, it is readily checked that the Envelope Theorem yields the following results:

- (14)  $\partial E\omega^{\circ}/\partial\alpha = -(1-\pi)<0$ ,
- (15)  $\partial E\omega^{\circ}/\partial q = -(1-\pi)\pi < 0$ ,

and

(16)  $\partial E\omega^{\circ} / \partial \pi = [f(B^{*}) + (V - v)] - (V - \alpha)[(1/\pi) - 1],$ 

which may take either sign, and is the more likely to be negative, the closer are the two valuations of the land. That  $E\omega^{\circ}$  should fall as  $\alpha$  and q increase is as expected.

#### 4. Adverse Selection

In this section, we provide an explanation for the coexistence of what we shall call 'inside' and 'outside' lenders, and for the possibility that, in equilibrium, 'insiders' offer contracts that yield lower expected payoffs to borrowers than those yielded by the contracts offered by 'outsiders'. To this end, we propose a model of adverse selection which is based on one developed previously to explain credit rationing (Clemenz 1986). Our analysis is not, however, confined to the case in which borrowers are rationed ex ante.

The main idea of the standard adverse selection model is as follows: Borrowers with a relatively high probability of success may have more profitable alternatives to taking a loan than borrowers with a small probability, in which case, the former will drop out from the market first as the loan rate rises. As a consequence, the average probability of default of the remaining loan applicants increases and the lender's expected return per loan is reduced [see Clemenz (1986)]. In the first subsection, we begin by extending this model, which was developed for a fixed loan size, by allowing B to vary. We then show that strategic lending may turn the above argument on its head, that is, lenders may set a high loan rate in order to get only the "risky" borrowers. In the second subsection, we extend the model further by introducing a second, 'outside' lender, who places a lower value on the land than the first. In this situation, an equilibrium with the properties described above may exist.

#### 4.1 One lender

Suppose the lender faces two types of farmers as potential borrowers, henceforth called types 1 and 2. All farmers own one plot, to which they have the same psychological and social attachment. Their private information is their probability of success and the associated output,  $q_i$ , i=1,2. The value that a farmer places on his plot depends on his type only to the extent that  $\pi$  and q vary:  $V_i$  is given by (2), with  $\pi$  and q appropriately subscripted. Let  $\pi_2 > \pi_I$  and  $q_2 > q_I$ , so that  $V_I > V_2$ . For simplicity, we assume that both types have access to the same technology, so that (1) becomes

(1') 
$$x_i = \begin{cases} q_i + f(B) & \text{with probability } \pi_i \\ 0 & \text{with probability } 1 - \pi_i. \end{cases}$$
  $i = 1, 2.$ 

First, we establish a condition such that the reservation expected-wealth contour of a type-1 borrower lies above that of a type-2 borrower in (B,R)-space. The expected wealth yielded by the contract (B,R) to a type-i farmer is

(17) 
$$E\Omega_i(B,R) = \pi_i[q_i + f(B) - RB + V_i], \qquad i = 1,2.$$

As  $\pi_1 < \pi_2$  and  $q_1 < q_2$ ,  $E\Omega_1(B,R) < E\Omega_2(B,R)^3$ . By rewriting (17) in the form

(18) 
$$f(B) - RB = [(E\Omega_i / \pi_i) - (V_i + q_i)],$$

it is seen that the maps are identical up to a renumbering, which is a consequence of the assumption that both types have access to the technology f(B) to augment what they can produce without finance. That the maps have this property precludes the use either of collateral or of the loan-size as a screening device, as proposed by Bester (1985) and Milde and Riley (1988). For the reservation contour  $E\Omega_i = \Omega_i$ , the r.h.s. of (18) is  $[(1/\pi_i) - 1]V_i$ , so that the reservation contour of a type-1 borrower lies above that of a type-2 if and only if

$$(1-\pi_2)V_2/\pi_2 > (1-\pi_1)V_1/\pi_1$$

or, from (2),

(19) 
$$[(1-\pi_2)q_2 - (1-\pi_1)q_1] > \alpha [(1/\pi_1) - (1/\pi_2)].$$

By choosing  $q_2$  sufficiently greater than  $q_1$  so as to make the l.h.s. positive, (19) can always be satisfied by making  $\alpha$  sufficiently small.

If the lender has limited funds, he must decide on which reservation contour he will offer a contract. This depends on the expected return, which is decreasing in q, but may be increasing or decreasing in  $\pi$  [recall (15) and (16)]. He will choose a contract on the reservation contour of a type-1 borrower if and only if

(20) 
$$(1-\pi_1)\nu + \pi_1 R_1^0 B_1^0 + I[y_0 - B_1^0] > (1-\overline{\pi})\nu + \overline{\pi} \overline{R}^0 \overline{B}^0 + I[y_0 - \overline{B}^0],$$

<sup>&</sup>lt;sup>3</sup>It follows that if, contrary to our assumption, production were impossible without a loan, then  $V_1 = V_2 = \alpha$ , and the reservation iso-expected utility contour of a type-2 would lie above that of a type-1 in this space.

<sup>4</sup>This property also holds under other assumptions about the technologies. One such example is that in which there is a discrete difference between the technology without finance and a common techology with finance.

where  $\overline{\pi}$  denotes the average probability of a success in the population,  $\left(B_1^0,R_1^0\right)$  is the lender's optimal contract if he faces a type-1 borrower with certainty, and  $(\overline{B}^0,\overline{R}^0)$  is his optimal contract if he aims at the "average" borrower, so that  $(\overline{B}^0,\overline{R}^0)$  must be just attractive to a type-2 borrower. We shall henceforth refer to these contracts as the separating and pooling contracts, respectively. As  $\pi_1 < \overline{\pi}$ , it follows from Remark 2 that  $B_1^0 < \overline{B}^0$ .

Remark 3: It is straightforward to establish that (19) and (20) can be made compatible. It is clearly possible to choose a positive set of parameters  $(\alpha, q_1, q_2, \pi_1, \pi_2)$  satisfying (19). Condition (5) can be satisfied by making f(B) sufficiently weakly concave, and it ensures the existence of a contract  $(B_1^0, R_1^0)$  such that the l.h.s. of (20) is positive. Suppose (20) is not satisfied for that set of parameters. Increasing  $q_2$  will preserve (19) and will leave the l.h.s. of (20) unchanged, while reducing the r.h.s. of (20). Let  $q_2$  be such that  $\overline{R}^0 = 0$ . As  $\pi_1 < \overline{\pi}$  and  $B_1^0 < \overline{B}^0$ , it follows at once that (20) will then hold.

## 4.2 Competition between an inside and an outside lender

We now introduce a second lender, who can be thought of, in some sense, as an 'outsider'. The insider may, for example, be a member of the local community, whereas the outsider is not. In that case, owning land therein may confer little or no social prestige on her, and if she does obtain land, managing its cultivation will be more costly for her than for the insider<sup>5</sup>. The distinction between the insider and outsider may also be institutional. For example, the outsider may be a bank, which has no interest in managing tenancies; whereas the insider may be a trader or an agent, who, though he does not live in the village itself, has better opportunities to make profitable use of land. Although the outsider knows that there is someone in the community who is willing and able to pay a price up to  $\nu$  for each plot, it may be impossible for her to get this amount in the event that one of her borrowers defaults. There are transactions costs involved, and if there is bilateral bargaining, she is unlikely to be in a

position to force the insider to his reservation level. For these reasons, the value that the outsider places on the land offered as collateral,  $v_o$ , will be lower than that of the insider.

We must now specify what the outside lender does with the land that comes into her possession when one of her borrowers defaults. One possibility is that she retains it. We shall deal with this case only in in passing. For it is much more plausible that she sells the plot to the insider. For simplicity, we assume that she does so at her reservation price  $v_o$ . The insider therefore has two ways of acquiring land: first, through default by one of his own borrowers; and second, indirectly, through default by one of the outsider's borrowers, in which case, he makes a profit of  $(v - v_o)$ . In this respect, the presence of the outsider is an advantage to the insider, inasmuch as he can "freeride" on her foreclosures. For simplicity, we assume that the insider places no value at all on additional plots beyond the first. Thus, he purchases a plot from the outsider, when she offers one, only if he has not acquired a plot through his own lending operations.

Finally, we assume that the inside lender has limited funds and is willing to grant at most one loan per period. This may be true for a trader-lender or a landlord who is lending out of own assets. An outsider may also have limited funds. If she is an intermediary, she may have to impose credit rationing because there is a "bank-optimal" loan rate of interest which does not yield a sufficiently high expected return for her to offer a deposit rate that will attract enough savings for her to satisfy the entire demand for loans [for details of such a model, see Clemenz (1986)]. As we shall see below, the assumption that both lenders have limited funds is crucial for the properties of the equilibrium.

We focus on situations in which the local lender would prefer the separating contract were he the only lender, whereas the outsider would not, that is, (20) is satisfied for the local lender, but not for the outsider. We begin by noting that the argument of Remark 3 cannot be used to establish the existence of such a case. The reason for this is that the outsider must make non-negative expected profits by offering the pooling contract, which implies that the r.h.s. of (20) must be strictly positive (by virtue of the fact that  $v > v_0$ ). This condition imposes an upper limit on  $q_2$ . The following proposition nevertheless holds, and it plays a key role in the proofs of the propositions that deal with competition between the two lenders.

<sup>&</sup>lt;sup>5</sup>Outsiders are also usually less well informed about the characteristics of loan applicants, and find it more difficult to monitor the actions of those to whom they do give loans. These particular forms of asymmetric information will not, however, be dealt with in this paper.

<u>Proposition 3</u>. There exists a non-empty set of positive parameters  $\{\alpha, q_1, q_2, \pi_1, \pi_2, \nu, \nu_0\}$  and an increasing, strictly concave and twice-differentiable function f(B) such that the following conditions are simultaneously satisfied:

- (i)  $V_2 > V_1 > v > v_0 \ge 0$ ;
- (ii)  $E\omega[B_1^0, R_1^0; \nu] \ge \underline{\omega};$
- (iii)  $E\omega[\overline{B}^0, \overline{R}^0; \nu_0] \ge \underline{\omega};$
- (iv)  $E\omega[B_1^0, R_1^0; v] > E\omega[\overline{B}^0, \overline{R}^0; v];$
- (v)  $E\omega[\overline{B}^0, \overline{R}^0; \nu_0] > E\omega[B_1^0, R_1^0; \nu_0]$ ; and
- (vi) condition (19).

Proof: see Appendix. Q.E.D.

The content of this proposition can be grasped by considering two islands which are identical in all respects, save that one is inhabited by an insider and the other by an outsider. Proposition 3 asserts that there exist islands such the separating contract is observed on the former and the pooling contract on the latter. In certain circumstances, however, the proposition tells us more than that. Suppose one of the lenders moves to the other island and that they then engage in competition. If the outsider retains those plots that fall into her hands as a result of default, and if her funds are so limited that the insider is still certain to obtain at least one application if he continues to offer the separating contract, then Proposition 3 establishes that there are circumstances in which there is an equilibrium in pure strategies in which the insider offers the separating contract and the outsider the pooling contract.

We now investigate whether this result also holds in the case where the outsider is willing to sell such plots to the insider at her reservation price. To show that it does, consider the following game: There are two type-1 and one type-2 borrowers, and one lender of each sort. Each lender can offer at most one loan. In stage one, both lenders choose whether or not to offer a contract. In stage two, all borrowers may apply for contracts, but it remains private information for which contracts, if any, each borrower has applied. In stage three, contracts are allocated among applicants as follows. If both lenders offer the same contract, all applicants are allocated at random between the two lenders. If the contracts differ, one application for the contract preferred by borrowers is accepted at random, and the remaining contract is allocated

at random among the other applications, if any. In stage four, "nature" determines the returns of all borrowers, which are assumed to be stochastically independent.

<u>Proposition 4.</u> Suppose conditions (i) - (vi) of Proposition 3 are satisfied. Then there exists a unique equilibrium in pure strategies. This equilibrium will be separating, with the insider offering the contract  $(B_1^0, R_1^0)$ , if

(21) 
$$\left[ (1-\pi_1)\nu + \pi_1 R_1^0 B_1^0 - I B_1^0 \right] > \left[ (1-\overline{\pi})\nu + \overline{\pi} \overline{R}^0 \overline{B}^0 - I \overline{B}^0 \right] + (\pi_2 - \pi_1)(1-\overline{\pi})(\nu - \nu_0) / 3;$$

otherwise, both lenders will offer  $(\overline{B}^0, \overline{R}^0)$ .

<u>Proof</u>: (a) We prove uniqueness by showing that each lender has one (weakly) dominant strategy.

As the insider grants at most one loan, the outsider is sure to get one application when she makes the offer  $(\overline{B}^0, \overline{R}^0)$ . It is readily checked that such an offer would go to the type-2 borrower with probability one-third, which implies that the expected probability of default is  $(1-\overline{\pi})$ . By conditions (iii) and (v) of Proposition 3, it then follows that  $(\overline{B}^0, \overline{R}^0)$  is a dominant strategy for the outside lender.

Given that the outsider will always offer  $(\overline{B}^0, \overline{R}^0)$ , it is clear that  $(B_1^0, R_1^0)$  dominates all contracts attracting only type-1 borrowers and that  $(\overline{B}^0, \overline{R}^0)$  dominates all other pooling contracts. We first show that, for the insider, the separating contract strictly dominates the option of offering no contract. This requires that

$$(22) Iy_0 + (1 - \overline{\pi})(\nu - \nu_0) < [(1 - \pi_1)\nu + \pi_1 R_1^0 B_1^0 - IB_1^0 + Iy_0] + \pi_1 (1 - \overline{\pi})(\nu - \nu_0)$$

The l.h.s. comprises the total return from the safe placement plus the expected gain from purchasing a plot from the outsider in the event that her borrower defaults. The expression in brackets on the r.h.s. is the total expected return from the separating contract. In addition, the insider has the chance of buying a plot from the outsider. Since he acquires at most one plot, this sort of purchase occurs with probability  $\pi_1(1-\overline{\pi})$ . Rearranging (22), we obtain

(23) 
$$0 < [(1-\pi_1)\nu + \pi_1 R_1^0 B_1^0 - I B_1^0] - (1-\pi_1)(1-\overline{\pi})(\nu - \nu_0).$$

We now show that condition (iii) of Proposition 3 implies that (23) always holds. Condition (iii) may be written as

$$(24) \quad (1-\overline{\pi})v_0 + \overline{\pi}\overline{R}^0\overline{B}^0 - I\overline{B}^0 \ge 0.$$

Suppose (23) is reversed. Then condition (iv) of Proposition 3 implies that

(25) 
$$\left[ (1 - \overline{\pi}) v + \overline{\pi} \overline{R}^{0} \overline{B}^{0} - I \overline{B}^{0} \right] - (1 - \pi_{1}) (1 - \overline{\pi}) (v - v_{0}) \leq 0.$$

Subtracting (24) from (25), we obtain

(26) 
$$(1-\overline{\pi})(\nu-\nu_0) \leq (1-\pi_1)(1-\overline{\pi})(\nu-\nu_0),$$

which contradicts  $0 < \pi_1 < 1$ .

To complete the proof of uniqueness, note that the insider's expected utility from offering the pooling contract is

$$\left[ (1 - \overline{\pi}) \nu + \overline{\pi} \overline{R}^{\,0} \overline{B}^{\,0} + I y_0 \right] + \overline{\pi} (1 - \overline{\pi}) (\nu - \nu_0).$$

It then follows that (21) holds if and only if the separating contract dominates the pooling contract.

(b) Existence. If (21) does not hold, conditions (i) and (iii) of Proposition 3 ensure that both lenders offer the pooling contract and that all borrowers apply for loans. If (21) does hold, it remains to be shown that it is still compatible with conditions (i) - (iii), (v) and (vi) of Proposition 3. It is proved in the Appendix that there is a set of positive parameters  $\{\alpha, q_1, q_2, \pi_1, \pi_2, \nu, \nu_0\}$  and an increasing, strictly concave and twice-differentiable function  $f(\cdot)$  satisfying this requirement. Q.E.D.

### Remark 4: As

$$(1-\overline{\pi})(\nu-\nu_0) > \overline{\pi}(1-\overline{\pi})(\nu-\nu_0) > \pi_1(1-\overline{\pi})(\nu-\nu_0)$$

it follows that the insider's expected profits from indirect acquisition of land are highest when he is himself inactive as a lender, as intuition would suggest, and lowest when he offers the separating contract. Given that the conditions of Proposition 4 are fulfilled, the differences in these "freeriding" effects are not, however, strong enough to overturn the dominance of the separating contract in the presence of the outsider,

As noted in the Introduction, it is not uncommon for inside lenders to offer less attractive contracts than outsiders and yet remain in business. Proposition 4 provides an explanation for this phenomenon by an appeal to adverse selection when insiders and outsiders place different valuations on the land used to secure their loans and rationing arises from limited loanable funds. It is clear that different valuations are essential to this explanation. What remains to be explored is the extent to which the assumptions that lead to rationing exante can be relaxed.

For a separating equilibrium to exist, it appears to be essential that the presence of the outsider does not diminish too strongly the probability that the insider will actually conclude a separating contract, relative to the case in which the insider is the sole lender. To see the importance of this point, consider the variation of the above game in which there is only one type-1 borrower, with a suitable adjustment to the allocative mechanism. In this case, the following proposition holds.

<u>Proposition 5</u>. In the modified game with one borrower of each type, there exists no separating equilibrium.

<u>Proof:</u> To establish this claim, note that the separating contract continues to dominate the pooling contract if and only if

$$\frac{1}{2} \Big[ (1 - \pi_1) \nu + \pi_1 R_1^0 B_1^0 - I B_1^0 + I y_0 \Big] + \frac{1}{2} \pi_1 (1 - \pi_2) (\nu - \nu_0) + \frac{1}{2} \Big[ I y_0 + (1 - \pi_1) (\nu - \nu_0) \Big] \\
> \Big[ (1 - \overline{\pi}) \nu + \overline{\pi} \overline{R}^0 \overline{B}^0 - I \overline{B}^0 + I y_0 \Big] + \frac{1}{2} \Big[ \pi_1 (1 - \pi_2) + \pi_2 (1 - \pi_1) \Big] (\nu - \nu_0)$$

Rearranging, we obtain the condition

$$(27) \quad \frac{1}{2} \Big[ (1 - \pi_1) \nu + \pi_1 R_1^0 B_1^0 - I B_1^0 \Big] + \frac{1}{2} (1 - \pi_1) (1 - \pi_2) (\nu - \nu_0) > \Big[ (1 - \overline{\pi}) \nu + \pi \overline{R}^0 \overline{B}^0 - I \overline{B}^0 \Big].$$

Recalling the definitions of  $E\omega[B_1^0, R_1^0; v]$  and  $E\omega[\overline{B}^0, \overline{R}^0; v]$ , (27) may be written as

(28) 
$$E\omega[B_1^0, R_1^0; \nu] + (1 - \pi_1)(1 - \pi_2)(\nu - \nu_0) > 2E\omega[\overline{B}^0, \overline{R}^0; \nu] - Iy_0.$$

By condition (iii) of Proposition 3, replacing  $Iy_0$  with  $E\omega[\overline{B}^0, \overline{R}^0; v_0]$  on the r.h.s. of (28) preserves the inequality:

$$E\omega\left[B_1^0,R_1^0;\nu\right]+\left(1-\pi_1\right)\left(1-\pi_2\right)\left(\nu-\nu_0\right)>2E\omega\left[\overline{B}^0,\overline{R}^0;\nu\right]-E\omega\left[\overline{B}^0,\overline{R}^0;\nu_0\right].$$

Subtracting inequality (v) of Proposition 3 then yields

$$E\omega\left[B_{1}^{0},R_{1}^{0};\nu\right]-E\left[B_{1}^{0},R_{1}^{0};\nu_{0}\right]+\left(1-\pi_{1}\right)\left(1-\pi_{2}\right)\left(\nu-\nu_{0}\right)>2\left\{E\omega\left[\overline{B}^{0},\overline{R}^{0};\nu\right]-E\omega\left[\overline{B}^{0},\overline{R}^{0};\nu_{0}\right]\right\}$$

$$(1-\pi_1)(\nu-\nu_0)+(1-\pi_1)(1-\pi_2)(\nu-\nu_0)>2(1-\overline{\pi})(\nu-\nu_0)$$

As  $v > v_0$ , this contradicts  $0 < \pi_1 < \pi_2 < 1$ . Q.E.D.

Starting with a situation in which there is only the inside lender, the entry of an outsider has two effects on the insider's expected profits. First, as noted earlier, the insider will have additional chances of acquiring land on favourable terms. Second, his pool of final applicants will be reduced, although this need not reduce his chances of concluding the contract he desires. In the original game, he can conclude the contract of his choice with certainty. In the modified game, however, he can do so only with probability one-half if he offers the separating contract.

This modified game with two borrowers motivates a third variant, in which there are (n-1) type-1 borrowers and one type-2, and the outsider has finance for exactly (n-1) pooling contracts. In this case, the probability that the insider can conclude the separating contract of his choice approaches one as n becomes large, and so re-establishes the possibility that a separating equilibrium will exist. Indeed, a stronger result holds.

Proposition 6. Suppose conditions (i) - (vi) of Proposition 3 are satisfied. Then in the third variant of the game, there will be a unique, separating equilibrium if n is sufficiently large.

Proof: (a) Uniqueness. The argument is similar to that of Proposition 4. It is clear that the

outsider will offer (n-1) pooling contracts  $(\overline{B}^0, \overline{R}^0)$ . The insider's expected utility from the separating contract is then

$$(1-1/n)[(1-\pi_1)\nu+\pi_1R_1^0B_1^0-IB_1^0]+Iy_0+[(1-1/n)\pi_1+1/n](1-\overline{\pi}^{n-1})(\nu-\nu_0),$$

since he concludes such a contract with probability (1-1/n) and concludes no contract with probability 1/n. His expected utility from the pooling contract is

$$\left[\left(1-\overline{\pi}\right)\nu+\overline{\pi}\overline{R}^{0}\overline{B}^{0}-I\overline{B}^{0}\right]+Iy_{0}+\overline{\pi}\left(1-\overline{\pi}^{n-1}\right)\left(\nu-\nu_{0}\right).$$

Hence, the former dominates the latter if and only if

$$(1-1/n) \left[ (1-\pi_1)\nu + \pi_1 R_1^0 B_1^0 - I B_1^0 \right] > \left[ (1-\overline{\pi})\nu + \overline{n} \overline{R}^0 \overline{B}^0 - I \overline{B}^0 \right] - (1-\overline{\pi}^{n-1}) (1+1/n - \pi_1 - \pi_2/n) (\nu - \nu_0)$$

Adding  $Iy_0$  to both sides and making n large then yields, in the limit,

$$E\omega[B_1^0, R_1^0; \nu] > E\omega[\overline{B}^0, \overline{R}^0; \nu] - (1 - \pi_1)(\nu - \nu_0),$$

which holds if condition (iv) of Proposition 3 holds.

The insider's expected utility from offering no contract is  $Iy_0 + (1 - \overline{\pi}^{n-1})(\nu - \nu_0)$ . He (weakly) prefers the pooling contract to this alternative if and only if

$$\left[\left(1-\overline{\pi}\right)\nu+\overline{\pi}\overline{R}^{0}\overline{B}^{0}-I\overline{B}^{0}\right]\geq\left(1-\overline{\pi}^{n-1}\right)\left(1-\overline{\pi}^{n-1}\right)\left(\nu-\nu_{0}\right)$$

0

$$\left[ \left( 1 - \overline{\pi} \right) v_0 + \overline{n} \overline{R}^0 \overline{B}^0 - I \overline{B}^0 \right] \ge - \overline{\pi}^{n-1} \left( 1 - \overline{\pi} \right) \left( v - v_0 \right).$$

Adding Iy<sub>0</sub> to both sides of this condition yields

$$E\omega\left[\overline{B}^{\,0},\overline{R}^{\,0};\nu_{0}\right]\geq Iy_{0}-\overline{\pi}^{\,n-1}(1-\overline{\pi})(\nu-\nu_{0}),$$

which holds for all n if condition (iii) of Proposition 3 holds. Hence, the separating contract is a dominant strategy for the insider if conditions (iii) and (iv) of Proposition 3 hold and n is sufficiently large.

(b) Existence follows at once from Proposition 3. Q.E.D.

Propositions 4, 5 and 6 establish results at the extremes where n is small and large, and (n-1) borrowers are of type 1. To complete the analysis, we consider the case where the outsider can offer n pooling contracts, so that all borrowers can obtain a loan of this sort.

Proposition 7. Suppose conditions (i) - (vi) of Proposition 3 are satisfied. If there are (n-m) and m borrowers of types 1 and 2, respectively, if n is large and m is sufficiently small, and if

the outsider can offer at least n pooling contracts, then there will be a pooling equilibrium in which the insider offers a contract that all borrowers find slightly more attractive than  $(\overline{B}^0, \overline{R}^0)$ , which is the contract offered by the outsider.

<u>Proof</u>: It is clear that the insider will get no applicants if he offers the separating contract. As  $v > v_0$ , he should offer a pooling contract which is slightly more attractive to borrowers than  $(\overline{B}^0, \overline{R}^0)$ , in order to be sure of concluding a contract. If n is large, the outsider will not find it worth her while to sweeten her offer of  $(\overline{B}^0, \overline{R}^0)$ ; for the ensuing expected loss accumulated over (n-1) contracts would then more than outweigh the expected profit from the additional contract — even on the implausible assumption that there would be no further response from the insider.

There remains, however, the question of whether the insider would prefer to become inactive. If he does so, his expected utility will be

$$Iy_0 + (1 - \pi_1^{n-m} \pi_2^{n-m}) (v - v_0)$$
.

If he offers a slightly more attractive contract than  $(\overline{B}^0, \overline{R}^0)$ , his expected utility will be arbitrarily close to

$$E\omega[\overline{B}^{0},\overline{R}^{0};\nu]+\overline{\pi}(1-\overline{\pi}^{n-1})(\nu-\nu_{0}).$$

The latter strategy weakly dominates the former if and only if

$$E\omega\left[\overline{B}^{\,0}\,,\overline{R}^{\,0}\,;\nu\right]\geq Iy_0+\left[\left(1-\overline{\pi}\right)+\left(\overline{\pi}^{\,n}-\pi_1^{\,n-m}\pi_2^{\,m}\right)\right]\left(\nu-\nu_0\right),$$

or

$$E\omega[\overline{B}^0,\overline{R}^0;\nu] \geq Iy_0 + (\overline{\pi}^n - \pi_1^{n-m}\pi_2^m)(\nu - \nu_0).$$

If m is sufficiently small, then  $(\bar{\pi}^n - \pi_1^{n-m} \pi_2^m) < 0$ ; so that the required condition will be fulfilled by virtue of condition (iii) of Proposition 3. Q.E.D.

#### 5. Concluding Remarks

We have shown, first, that differences in the value placed on land between lenders and borrowers can lead to mutually acceptable debt contracts secured by land even when straight

sales of land are not mutually acceptable and production is also possible without finance. Second, we have examined the case where valuations differ across both borrowers and lenders in the following ways: borrowers differ in their expected productivities, but not in their attachment to land; and lenders' valuations vary for both reasons. Characterising an 'inside' lender as one who places a relatively high value on the land offered as collateral, we have established the existence of equilibria in which the 'insider' offers a separating contract that will attract only "risky" borrowers, whereas the 'outsider' will offer a pooling contract. To accomplish the latter, we employed a model of adverse selection in which lenders can choose the size of the loan, but the lenders differ only in the value they place on the borrower's land.

Natural extensions of this framework involve different assumptions about the information available to lenders. First, the insider may be much better informed about the characteristics of applicants for loans. If he is able and willing to communicate this information to the outsider, the likelihood of a separating equilibrium may be greater if such communication leads to an increase in the expected number of applicants who approach him for separating contracts. Second, if there is moral hazard, the outsider will normally find it more difficult to monitor the actions of her borrowers, so that competition may lead to contracts that yield different expected utilities to borrowers, even when borrowers are identical. A third extension would involve relaxing the assumption that borrowers have identical attachments to land and that they produce identical additional outputs from the same amount of finance when successful. Such heterogeneity would bring about changes in the geometry of borrowers' preference maps, and so open the door to the use of self-selection mechanisms. These extensions await further work.

### Appendix

Proof of Proposition 3:

Step 1. We begin with conditions (iv) and (v). We have

(A1) 
$$E\omega[B_1^0, R_1^0; \nu] = (1 - \pi_1)\nu + \pi_1 R_1^0 B_1^0 + I(y_0 - B_1^0)$$

and

(A2) 
$$E\omega[\overline{B}^{0}, \overline{R}^{0}; v] = (1 - \overline{\pi})v + \overline{\pi}\overline{R}^{0}\overline{B}^{0} + I(y_{0} - \overline{B}^{0}).$$

The contract  $(B_1^0, R_1^0)$  is just a attractive to a type-1:  $E\Omega_1^0 = \underline{\Omega_1}$ . From (4a) and (6a), this condition may be written as

$$\pi_1 \Big[ f \Big( B_1^0 \Big) - R_1^0 B_1^0 \Big] - \Big( 1 - \pi_1 \Big) V_1 = 0.$$

Substituting for  $\pi_1 R_1^0 B_1^0$  in (A1), we obtain

(A3) 
$$E\omega[B_1^0, R_1^0; \nu] = [\pi_1 f(B_1^0) - IB_1^0] + Iy_0 + (1 - \pi_1)(\nu - V_1).$$

The contract  $(\overline{B}^0, \overline{R}^0)$  is just attractive to a type-2:  $E\overline{\Omega}^0 = \underline{\Omega}_2$ , which may be written as

$$\pi_2 \Big[ f \Big( \overline{B}^0 \Big) - \overline{R}^0 \overline{B}^0 \Big] - \Big( 1 - \pi_2 \Big) V_2 = 0.$$

Substituting for  $\overline{\pi}R^{0}\overline{B}^{0}$  in (A2), we have

(A4) 
$$E\overline{\omega} \left[ \overline{B}^{\,0}, \overline{R}^{\,0}; \nu \right] = \left[ \overline{\pi j} \left( \overline{B}^{\,0} \right) - I \overline{B}^{\,0} \right] + I y_0 + \left( 1 - \overline{\pi} \right) \nu - \overline{\pi} \left( \frac{1}{\pi_2} - 1 \right) V_2.$$

Condition (iv) may now be written as, after cancelling some terms,

$$\left[\pi_1 f\left(B_1^0\right) - I B_1^0\right] - \pi_1 \nu - \left(1 - \pi_1\right) V_1 > \left[\overline{\pi f}\left(\overline{B}^0\right) - I \overline{B}^0\right] - \overline{\pi} \nu - \overline{\pi} \left(\frac{1}{\pi_2} - 1\right) V_2.$$

or

(A5) 
$$(\overline{\pi} - \pi_1)v > \Delta + [(1 - \pi_1)V_1 - \overline{\pi}(1 - \pi_2)V_2 / \pi_2],$$

where

(A6) 
$$\Delta := \left[ \overline{\pi} f \left( \overline{B}^{0} \right) - I \overline{B}^{0} \right] - \left[ \pi_{1} f \left( B_{0}^{1} \right) - I B_{0}^{1} \right] > 0$$

by the strict concavity of  $f(\cdot)$  and the hypothesis that  $\pi_2 > \pi_1$ . It should be noted that the magnitude of  $\Delta$  can be varied by varying the concavity of  $f(\cdot)$ .

We now combine conditions (iv) and (v), using (A5), to yield the condition

(A7) 
$$(\overline{\pi} - \pi_1) v > \Delta + [(1 - \pi_1)V_1 - \overline{\pi}(1 - \pi_2)V_2 / \pi_2] > (\overline{\pi} - \pi_1)v_0$$

As  $v_0 \ge 0$ , we require that

(A8) 
$$Q := \Delta + \left[ (1 - \pi_1) V_1 - \overline{\pi} (1 - \pi_2) V_2 / \pi_2 \right] > 0.$$

By hypothesis,  $\pi_2 > \pi_1$  and  $q_2 > q_1$ . For any given positive vector  $\{\alpha, q_1, q_2, \pi_1, \pi_2\}$  satisfying condition (i), however, we may choose an  $f(\cdot)$  such that (A8) is satisfied.

Step 2. We now impose condition (vi), which holds if and only if

$$0 > -\overline{\pi} \left( \frac{(1 - \pi_2)V_2}{\pi_2} - \frac{(1 - \pi_1)V_1}{\pi_1} \right).$$

As  $\overline{\pi} > \pi_1$ , this implies that

$$[(1-\pi_1)V_1 - \overline{\pi}(1-\pi_2)V_2] < 0,$$

It follows that  $\Delta$  must be sufficiently large to preserve (A8), and hence (A7). This can be accomplished for any given positive  $\{\alpha, q_1, q_2, \pi_1, \pi_2\}$  by making  $f(\cdot)$  sufficiently weakly concave. We may, moreover, make Q sufficiently small for there to exist a v < V such that the l.h.s. of (A7) is satisfied. The r.h.s. of (A7) is clearly satisfied by choosing any  $\nu_0$  sufficiently close to zero.

Step 3. In Remark 1, we assumed that  $\pi f'(0) > I$ . This, together with  $\pi_1 < \pi_2$ , ensures that

$$\left[\overline{\pi}f\left(\overline{B}^{0}\right) - I\overline{B}^{0}\right] > \left[\pi_{1}f\left(B_{0}^{1}\right) - IB_{0}^{1}\right] > 0$$

independently of  $\{\alpha, q_1, q_2, \pi_1, \pi_2, \nu, \nu_0\}$ . This result is necessary, but not sufficient, for conditions (ii) and (iii) to hold.

Let

$$S = \{\alpha, q_1, q_2, \pi_1, \pi_2 : \alpha > 0; 1 > \pi_2 > \pi_1 > 0; q_2 > q_1 > 0; and(19)\}.$$

It is clear that S is non-empty. Consider next the set  $\{\alpha, q_1, q_2, \pi_1, \pi_2\} \subset S$  and the set of all increasing, continuous, concave functions satisfying (A7) and condition (i). It was shown in Step 2 that that this set is non-empty. It should be noted that choosing f(.) in such a way as to make Q close to zero can be done independently of the condition on f'(0) in Remark 1.

Step 4. It remains to be shown that satisfying (A7) in this way is compatible with conditions (ii) and (iii). Conditions (ii) and (iii) may be written as

(A9) 
$$\left[ \pi_1 f(B_1^0) - I B_1^0 \right] - (1 - \pi_1) (V_1 - v) \ge 0$$

and

(A10) 
$$\left[ \overline{\pi} f \left( \overline{B}^{\,0} \right) - I \overline{B}^{\,0} \right] + \left( 1 - \overline{\pi} \right) \nu_0 - \overline{\pi} \left( \frac{1}{\pi_2} - 1 \right) V_2 \ge 0 .$$

We may choose  $v_0 = 0$ , the l.h.s. of (A7) being satisfied by a suitable choice of  $v < V_I$ . For any choice of f(.) satisfying (A7), therefore, Q > 0 implies

$$\left[\overline{\pi}f\left(\overline{B}^{0}\right)-I\overline{B}^{0}\right]-\overline{\pi}\left(\frac{1}{\pi_{2}}-1\right)V_{2}>\left[\pi_{1}f\left(B_{1}^{0}\right)-IB_{1}^{0}\right]-\left(1-\pi_{1}\right)V_{1}$$

Hence, (A10) will indeed be satisfied if

$$\left[\pi_{1}f(B_{1}^{0})-IB_{1}^{0}\right] \geq (1-\pi_{1})V_{1}.$$

Now observe that S contains vectors  $\{\alpha, q_1, q_2, \pi_1, \pi_2\}$  in which  $V_I$  is small. We may choose such a vector and a corresponding f(.) such that the above argument holds. By choosing f'(0) sufficiently large ( in the limit, by imposing the Inada condition), we also ensure that  $\left[\pi_1 f(B_1^0) - IB_1^0\right]$  is strictly bounded away from zero, again without prejudicing the choice of f(.) such that Q be sufficiently small and positive. Hence, there exists an  $\{\alpha, q_1, q_2, \pi_1, \pi_2\} \subset S$  and an f(.) such that conditions (i) and (iii)-(vi) are satisfied.

To complete the proof, recall from the proof of part (a) of Proposition 4 that (A10) implies (A9), that is, condition (iii) of Proposition 3 implies that condition (ii) holds. Q.E.D.

## Proof of part (b) of Proposition 4:

We begin by noting that (21) and (22) replace conditions (iv) and (ii), respectively, of Proposition 3.

Step 1. We show that (21) is compatible with conditions (i), (v) and (iv) of Proposition 3. Recalling (A1) and (A2), (21) may be written as

(A11) 
$$E\omega[B_1^0, R_1^0; \nu] > E\omega[\overline{B}^0, \overline{R}^0; \nu] + (\pi_2 - \pi_1)(1 - \pi_1)(\nu - \nu_0)/3.$$

Proceeding in exactly the same way as in the proof of Proposition 3, we obtain the counterpart of (A7):

 $(\overline{\pi} - \pi_1) > Q + (\pi_2 - \pi_1)(1 - \pi_1)(\nu - \nu_0)/3$ 

and

$$Q > (\overline{\pi} - \pi_1) \nu_0$$

Rearranging and simplifying, we obtain

(A12) 
$$(\overline{\pi} - \pi_1) \nu [1 - (1 - \pi_1)(1 - \nu_0 / \nu)] > Q > (\overline{\pi} - \pi_1) \nu_0.$$

Since  $0 < \pi_1 < 1$  and  $\nu > \nu_0 \ge 0$ , it is clear that condition (i) of Proposition 3 and (A12) can be satisfied by a suitable choice of  $f(\cdot)$ , given any positive vector  $\{\alpha, q_1, q_2, \pi_1, \pi_2\}$  that satisfies condition (vi).

Step 2. We now show that (22) and condition (iii) are compatible with choices that satisfy the foregoing requirements. Condition (22) may be written as

$$E\omega[B_1^0, R_1^0; \nu] \ge Iy_0 + (1 - \overline{\pi})(1 - \pi_1)(\nu - \nu_0),$$

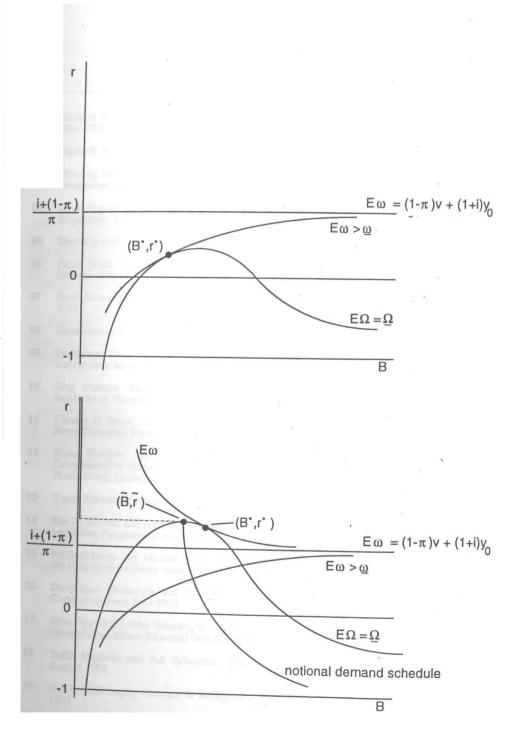
which is more stringent than condition (ii) of Proposition 3. Proceeding as in the final step of the proof of Proposition 3, we set  $v_0 = 0$  and use condition (iii) and Q > 0 to derive the implication that (22) will be satisfied if

$$\left[\pi_{1}f(B_{1}^{0})-IB_{1}^{0}\right] \geq \left(1-\pi_{1}\right)\left[V_{1}+(1-\overline{\pi})v\right],$$

since we have set  $v_0$  to zero. As  $v < V_1$ , the final argument of Step 4 of the proof of Proposition 3 then applies here. Q.E.D.

#### References

- Bester, H., (1985), "Screening vs. Rationing in Credit Markets with Imperfect Information", American Economic Review, 75: 850-55.
- Binswanger, H. P., and Rosenzweig, M. R., (1986), "Behavioural and Material Determinants of Production Relations in Agriculture", <u>Journal of Development Studies</u>, 22: 503-39.
- Clemenz, G., (1986), Credit Markets with Imperfect Information, Berlin: Springer-Verlag:
- Darling, M. L., (1925), <u>The Punjab Peasant in Prosperity and Debt</u>, London: Oxford University Press.
- Gale, D., and Hellwig, M., (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem", Review of Economic Studies, 52: 647-63.
- Hoff, K., Braverman, A., and Stiglitz, J. E., (1993), <u>The Economics of Rural Organisation</u>, Oxford: Oxford University Press.
- Milde, H. and Riley, J. G., (1988), "Signaling in Credit Markets", <u>Quarterly Journal of</u>
  Economics, 72: 101-29.
- Rothermund, D., (1993), An Economic History of India, London: Routledge.
- Stiglitz, J. E., and Weiss, A., (1981), "Credit Rationing with Imperfect Information, American Economic Review, 71: 393-410.



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