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HUMAN CAPITAL INVESTMENT, GOVERNMENT AND ENDOGENOUS GROWTH

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Abstract

The paper studies the role of government policy in an optimizing model of endogenous growth with human as well as physical capital accumulation. The government incurs expenditure on education and training, levies taxes on income from capital and labor, and charges positive or negative "tuition fees" for education services. On the assumption that the "education" sector is characterized by a Leontief technology, the optimal capital income tax rate is found to equal the fraction of GDP absorbed by government spending on education services. The optimal tuition fee and the optimal labor income tax rate are shown to be either positive or negative, depending among other things on the strength of the positive externalities from education. However, when these externalities are negligible and tuition fees are tied to the general wage level, we find that a comprehensive income tax and full-cost tuition fees will enable the government to implement a first-best optimal growth path. It is also shown that the government can achieve a first-best allocation without making any use of lump sum taxes and subsidies, if charges for education are related to the wage rate of each individual, and there are no external effects from education and training.

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1. Introduction.

In the theory of economic development it has long been recognized that investment in human capital through education and training tends to promote economic growth. There is considerable empirical evidence to support this view. For instance, according to the most recent World Development Report there has been a systematic tendency for developing countries with higher average levels of schooling to grow faster than LDCs with lower levels of shooling (World Bank, 1990, pp. 79-81).

For many years human capital theorists have elaborated on the ideas that human as well as physical capital can be accumulated, and that human capital investment is likely to respond to economic incentives[1]. Yet these ideas have only recently been incorporated into macroeconomic models of economic growth. However, at least since the influential contribution by Lucas (1988), human capital investment is now seen as one of the important engines of growth in the burgeoning literature on "endogenous" growth[2]. As readers familiar with this literature will know, the characteristic of these new growth theories is that they seek to explain the economy's steady state growth rate, in contrast to traditional neoclassical growth theory where the long term "natural" growth rate is exogenous.

The present paper can be seen as an elaboration of the analysis of Lucas (op.cit.). Our main extension of Lucas' model is to introduce a government sector which levies taxes

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[1]See, for instance, the 1976 special issue of the Journal of Political Economy (volume 84, Number 4, Part 2) which contains a number of contributions to the theory of human capital.

[2]A useful survey of this literature is provided by Sala-i-Martin (1990). Some of the most recent contributions to the theory of endogenous growth can be found in the special issues of the Journal of Political Economy (vol. 98, no. 5, part 2, October 1990), The Quarterly Journal of Economics (vol. CVI, Issue 2, May 1991), and the European Economic Review (vol. 35, no. 2/3, April 1991).

on income from capital and labor and offers subsidies to or charges "tuition fees" for education. The purpose of this extension is to investigate how government policy will affect the economy's long term growth rate through its impact on the incentives for human capital accumulation, and to identify policy packages which could ensure that the economy will follow an optimal growth path[3].

In Lucas' model the only resource input into the process of human capital accumulation is the time spent by households on education and training. In our modified model, it is assumed that the production of labor skills also requires a complementary resource input which might be thought of as school buildings, research laboratories, teachers' services, etc. The existence of such complementary inputs turns out to have important implications for the design of optimal incentives for human capital investment.

In the next section, we develop our formal framework of analysis. Following this, we solve our model of endogenous growth for the economy's steady state growth rate and analyze how this growth rate will vary in response to changes in various parameters, including government policy instruments. We then characterize the economy's first-best optimal growth path and investigate whether and how the government might be able to design its policies so as to ensure coincidence between the actual and the optimal growth path. Finally, the concluding section provides a summary of our main results and suggests some directions for future research.

2. A model of human capital investment and endogenous growth.

Following Lucas (1988, section 4), we focus on a closed economy producing a single good which can be used for consumption as well as physical capital investment. The business sector consists of a large number of identical competitive firms, and the household sector is comprised of a large number of identical households optimizing the time path of consumption over an infinite horizon. The subsections below provide a formal description of our model economy. Unless otherwise indicated, variables which are not explicitly dated are implicitly understood to refer to the current time period t.

[3]Lucas (1990) studies optimal tax policy in a model with human capital accumulation, but his formal analysis focuses only on steady states and does not allow for subsidies to or charges for education. Nerlove et alia (1990) investigate the effects of income taxes on the desired stocks of human and physical capital, but they do not account for secular growth and do not characterize the optimal government policy.

2.1. *Production and factor pricing*

The output Y_i of the i'th firm is given by the Cobb-Douglas production function

$$Y_i = K_i^{\alpha} N_i^{1-\alpha} H^{\epsilon}, \quad 0 < \alpha < 1, \quad \epsilon \geq 0 \tag{2.1}$$

where K_i is the physical capital stock invested in the firm, N_i is the effective input of labor into the firm's production process, and H is an index of the average skill level of workers in the economy, i.e. the stock of "human capital" possessed by the representative worker. When the parameter ϵ is positive, the output of the individual firm is seen from (2.1) to vary positively with the average skill level in the economy, reflecting an assumption of positive externalities stemming from education and training. On the other hand, when ϵ is zero, no such externalities are assumed to exist.

The effective labor input in firm "i" depends on the particular skill level H_i of workers employed in that firm. Thus,

$$N_i = n_i H_i \tag{2.2}$$

where n_i is the number of hours worked in the i'th firm.

Because all workers are identical, they all work the same hours and all have the same skill level. Using this fact along with the assumption that all firms are identical, we may drop the subscripts referring to a particular firm and write output in the representative firm as

$$Y = K^{\alpha}(nH)^{1-\alpha} H^{\epsilon} \tag{2.3}$$

Given perfect competition, factor rewards are determined by marginal productivities. From (2.3) the real rate of interest (r) is therefore given by

$$r = \alpha K^{\alpha-1}(nH)^{1-\alpha} H^{\epsilon} \tag{2.4}$$

and the wage rate per unit of effective labor input (w) is equal to

$$w = (1-\alpha) K^{\alpha}(nH)^{-\alpha} H^{\epsilon} \tag{2.5}$$

By definition, the hourly wage rate equals the total wage bill wnH divided by the number of hours worked n and is therefore equal to wH.

2.2. *Human capital accumulation*

The total time endowment of households is normalized at unity. Households can allocate their time between work in the labor market and time spent on education and training. For the representative worker, the time spent on the acquisition of skills - during which no wages are earned - is thus equal to $1 - n$.

In an interesting empirical study, Rosen (1976) found that the pattern of lifetime earnings of U.S. male high school and college graduates was reasonably consistent with the predictions of a model of optimum human capital investment in which individuals accumulate human capital subject to the constraint

$$\dot{H}/H = g(1-n), \quad g' > 0 \tag{2.6}$$

where the dot indicates the derivative with respect to time. Equation (2.6) has the interesting implication that a constant (relative) rate of growth of the skill level can be maintained as long as the individual is willing to spend a constant amount of his time $1 - n$ on the acquisition of new skills.

Lucas (1988, sec. 4) adopted the following simple linear version of (2.6)

$$\dot{H}/H = \delta(1-n), \quad \delta > 0 \tag{2.7}$$

where δ is a constant parameter. While analytically convenient, the specification (2.7) ignores the fact that, in addition to the input of "student" time, the production of human capital typically also requires some complementary infrastructure such as school buildings, laboratory equipment, and teaching services. In the present one-good model, these complementary inputs will be represented by an amount of output G devoted to "education", and the growth rate of human capital will be assumed to be determined by a Leontief-technology of the form

$$\dot{H}/H = min[\delta(1-n), \eta(G/Y)], \tag{2.8}$$

where δ and η are (positive) fixed input coefficients. Implicit in (2.8) is the idea that, for a given number of "student" hours $1-n$, a certain number of teaching hours, classrooms, and laboratory facilities etc. will have to be provided to make the students' time productive. The postulate in (2.8) that there are no substitution possibilities at all between the two inputs in the production of human capital should be seen only as a rough approximation intended to capture the strong input complementarities in the educational sector. For instance, a university student is not likely to benefit much from a course if she does not acquire the relevant textbooks. As another primitive example, if two professors were to show up in the same lecture room to give the same lecture, the resulting "educational output" will hardly be greater than if only one professor had appeared.

Note from (2.8) that the complementary input G is measured relative to total output Y. Thus, a constant growth rate of skills can be maintained only if G increases pari passu with Y. Again, this seems a reasonable first approximation, since teaching services will be a major component of G, and since the real wages of teachers are likely to grow at the same rate as total real income Y in the long run.

Note also that, because constant input levels $1-n$ and G/Y are sufficient to maintain a constant relative growth rate of labor skills, the possibility of self-sustained endogenous growth is more or less assumed from the outset rather than being proved[4]. This does not mean that there is no upper bound on the growth rate, however. Since the amount of time spent on education and training cannot exceed unity, the specification (2.8) implies an upper limit δ to the growth rate of human capital.

Finally, it must be pointed out that even if (2.8) gives a correct description of human capital accumulation for the individual agent, it would not describe the evolution of the average skill level in an economy with mortal households, unless accumulated skills and knowledge can to some extent be passed on from one generation to the next. With a constant demography and no knowledge transfers across generations, the average skill level would tend to stay constant, even if the skills of each individual were increasing with age. In this paper we assume that the growth of individual skill levels as well as additions to the average skill level are determined by a function of the form (2.8), implying that at least part of the skills and knowledge accumulated by a generation can be transferred to succeeding generations[5].

Assuming production efficiency in the "education" sector, it follows from the "production function" (2.8) that

$$\delta(1-n) = \eta(G/Y) \quad \Longleftrightarrow \quad G = \beta(1-n)Y, \quad \beta \equiv \delta/\eta \tag{2.9}$$

[4] This was correctly pointed out to me by Hans-Werner Sinn. However, it can be shown that if one allows for the positive interaction between human capital investment and technological innovations, it will be possible for the economy to generate sustained endogenous growth, even if additional time spent on education will only raise the absolute (but not necessarily the relative) rate of growth of human capital. See Sørensen (forthcoming).

[5] A similar assumption, which seems quite realistic, is made by Lucas (1988) and Stokey (1991).

It also follows from (2.8) that, as long as (2.9) is satisfied, the accumulation of human capital may indeed be described by equation (2.7).

2.3. *Government*

The government sector is assumed to supply the educational services G. The associated public expenditure is financed through a capital income tax (levied at a rate τ), a labor income tax (imposed at a rate θ), "tuition fees" F (positive or negative), and a lump sum tax T (positive or negative). The lump sum tax is assumed to be adjusted endogenously to ensure continuous budget balance. Thus the government budget constraint reads

$$G = \tau rK + \theta wnH + F + T \tag{2.10}$$

In a growing economy it is natural to assume that tuition fees and/or subsidies to educational activities are related to the level of earnings obtainable in the labor market. In the analysis below, we shall consider two alternative hypotheses regarding the determination of educational charges or subsidies. Under the first hypothesis, tuition fees or subsidies are related to the wage level of the individual worker/"student". More specifically, the government is assumed to charge a fee or to grant a subsidy amounting to some fraction c of the income which the individual might have earned in the labor market during the time $1-n$ spent on education or training. In other words,

$$F = cwH(1-n) \tag{2.11}$$

where it is recalled that wH is the real wage rate and where c may be positive or negative.

Under the second hypothesis, fees or subsidies are related to the average level of earnings in the economy $(wH)_a$ so that

$$F = c(wH)_a(1-n) \tag{2.12}$$

In the present model where all households are identical, we have $wH = (wH)_a$, but the two alternative hypotheses will nevertheless have different implications for economic behavior. When F is determined by (2.11), the individual worker will take account of the fact that, if he decides to increase his skill level through more education and training today, his future tuition fees or grants will increase. By contrast, when fees/subsidies are tied to the average level of earnings, as in (2.12), each atomistic agent will act on the assumption that his own training activities will have no impact on the fees/subsidies he will face in the future.

2.4. *The representative household*

Our representative infinitely-lived atomistic household should be thought of as representing a dynasty of overlapping families linked by altruistic bequests. At time zero, the household maximizes a utility function U of the form

$$U(0) = \int_0^\infty u(C(t))e^{-\rho t}dt, \quad \rho > 0 \tag{2.13}$$

where $u()$ is instantaneous utility, $C(t)$ is the level of consumption during time t, and ρ is the constant utility discount rate. Dropping the time variable for notational convenience, we specify instantaneous utility as

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad \sigma > 0 \tag{2.14}$$

where $-\sigma$ is the constant elasticity of marginal utility, and σ^{-1} is the constant intertemporal elasticity of substitution.

The maximization of discounted lifetime utility (2.13) with respect to consumption $C(t)$ and w.r.t. the amount of time spent in the labor market $n(t)$ takes place subject to two constraints. The first one is (2.7) which governs the accumulation of skills, given our assumption of production efficiency (2.9). The second one is the dynamic budget constraint

$$\dot{K} = r(1-\tau)K + wH(1-\theta)n - T - F - C \tag{2.15}$$

(2.15) is a simple savings definition stating that the accumulation of non-human wealth - which must take the form of physical capital accumulation $\dot{K}$ - equals the sum of after-tax income from capital and labor minus lump-sum taxes and tuition fees (positive or negative) and minus consumption.

When tuition fees are tied to the earnings of each individual, i.e. when F is given by (2.11), the first-order conditions for the solution of the consumer's problem turn out to be

$$C^{-\sigma} = \lambda_1 \tag{2.16}$$

$$\lambda_1 wH(1-\theta+c) = \lambda_2 \delta H \tag{2.17}$$

$$\dot{\lambda}_1 = [\rho - r(1-\tau)]\lambda_1 \tag{2.18}$$

$$\dot{\lambda}_2 = [\rho - \delta(1-n)]\lambda_2 - w[n(1-\theta) - c(1-n)]\lambda_1 \tag{2.19}$$

where λ_1 and λ_2 are the current-value co-state variables associated with K and H, respectively. Eliminating the co-state variables, it is possible after some manipulations to reduce (2.16) through (2.19) to the two conditions

$$\dot{C}/C = [r(1-\tau) - \rho]/\sigma \tag{2.20}$$

$$wH(1-\theta+c)r(1-\tau) = \dot{w}(1-\theta+c)H + w(1-\theta)(\frac{\dot{H}}{1-n}) \tag{2.21}$$

Equation (2.20) is the well-known Ramsey-condition for the optimal intertemporal allocation of consumption, stating that the consumer will plan to increase (reduce) his level of consumption over time when the net return to saving exceeds (falls short of) the rate of time preference.

The less familiar equation (2.21) is the condition for an optimal intertemporal allocation of time spent on education. To interpret this condition, suppose the consumer decides to engage in one extra hour of education today and to work one extra hour tomorrow instead. By doing so, he forgoes an amount of current labor income $wH(1-\theta)$, and his current tuition fees increase by cwH. If these two amounts had been invested in the capital market, they would have ensured the consumer an additional net capital income of $wH(1-\theta+c)r(1-\tau)$ one period later. Hence the left-hand side of (2.21) is the opportunity cost of switching an hour of work effort from the current to the next period, measured in terms of next period's income. The right- hand side measures the increase in next period's labor income (net of tuition fees) resulting from the intertemporal reallocation of work effort. First, if the real wage rate per unit of effective labor rises by $\dot{w}$ between today and tomorrow, this in itself will yield a gain of $\dot{w}H(1-\theta)$ in tomorrow's wage earnings when the consumer increases tomorrow's work effort at the expense of today's. Due to the rising real wage, the consumer will also save a future tuition fee of $\dot{w}Hc$ by taking an extra hour of education today rather than tomorrow. In addition, by increasing his level of skills by an amount $\dot{H}/(1-n)$ per hour of training today, the individual will be able to increase his hourly after-tax labor income tomorrow by the amount $w(1-\theta)\frac{\dot{H}}{1-n}$. In summary, condition (2.21) thus states that the total marginal benefit from education and training (the right-hand side) must equal its marginal opportunity cost (the left-hand side) at each point in time.

When tuition fees are instead tied to the average wage level in the economy, as in (2.12), the consumer's first-order conditions (2.16) through (2.18) remain unaffected, but (2.19) modifies to

$$\dot{\lambda}_2 = [\rho - \delta(1-n)]\lambda_2 - wn(1-\theta)\lambda_1 \tag{2.19.a}$$

In later sections we shall explore how the optimal size of taxes and tuition fees varies, according as consumer behavior is described by (2.19) or (2.19.a).

3. Endogenous steady state growth.

The dynamics of the model of endogenous growth described in section 2 are highly complex and will not be analyzed here. However, it is a fairly simple matter to derive the endogenous steady state growth rate implied by the model and to study how this long term growth rate responds to changes in various parameters, including those determined by government policy.

Along a steady state growth path, the real rate of interest must be constant. Taking logs of (2.4), differentiating with respect to time, and imposing the condition $\dot{r} = 0$, we obtain the steady-state relationship

$$g_h = (\frac{1-\alpha}{1-\alpha+\epsilon})g_k, \quad g_h \equiv \dot{H}/H, \quad g_k \equiv \dot{K}/K \tag{3.1}$$

where g_h and g_k are the steady-state growth rates of human and physical capital, respectively.

Deriving $\dot{w}/w$ from (2.5), inserting the resulting expression along with (2.7) into (2.21), and imposing the steady- state condition $\dot{n} = 0$, one also finds that

$$r(1-\tau) = \alpha(g_k - g_h) + \epsilon g_h + \delta(\frac{1-\theta}{1-\theta+c}) \tag{3.2}$$

Along a balanced growth path, consumption and physical investment must grow at the same rate, i.e. $\dot{C}/C = \dot{K}/K = g_k$. It then follows from (2.20) that

$$r(1-\tau) = \rho + \sigma g_k \tag{3.3}$$

We may now solve (3.1) through (3.3) for the steady-state growth rate of human capital:

$$g_h = [\frac{1-\alpha}{\sigma(1-\alpha)+\epsilon(\sigma-1)}][\delta(\frac{1-\theta}{1-\theta+c}) - \rho] \tag{3.4}$$

The first question one might ask is whether the long term growth rate determined by (3.4) is likely to be positive? Empirical estimates almost unanimously find an intertemporal elasticity of substitution $1/\sigma$ well below unity, implying $\sigma > 1$. The first square bracket in (3.4) will then be positive, and the growth rate will consequently be positive as long as ρ is not "too" high relative to δ. In other words, there will be secular growth provided the degree of consumer impatience (reflected in ρ) is not too high relative to the "productivity" of human capital investment (reflected in δ). The growth rate is seen from (3.4) to be higher, the lower the magnitude of σ, i.e. the higher the intertemporal elasticity of substitution (the lower the degree of risk aversion). These results all accord with intuition.

It is interesting to note that the capital income tax rate τ does not appear in (3.4). In contrast to certain other models of endogenous growth, e.g. those developed in Barro (1990), King and Rebelo (1990), and Rebelo (1991), the present model thus implies that capital income taxation will not hamper long run growth. However, in accordance with conventional neoclassical growth theory, our model does predict that a higher capital income tax rate will raise the real rate of interest and reduce the capital intensity of production in the long run[6]. This is easily seen: Since g_k is predetermined from (3.4) and (3.1), the right- hand side of (3.3) is independent of τ, so a rise in the capital income tax rate must induce an offsetting rise in the pre-tax rate of interest. This in turn requires a rise in the marginal productivity of capital which can be brought about only through a reduction in the capital-output ratio, given our assumption of a Cobb-Douglas technology.

Another striking implication of (3.4) is that the steady-state growth rate will also be independent of the labor income tax rate θ in the case where education is free, i.e. when $c = 0$. The reason is that in this case the private opportunity cost of education consists only of forgone after-tax wages. A change in the labor income tax rate will therefore cause an equiproportional change in the opportunity cost of education and in the higher future after-tax wages resulting from additional education. In other words, the relative rate of return to education will be left unaffected by the labor income tax when there are no educational fees or subsidies, and the desired level of human capital investment will therefore also be unaffected.

When c differs from zero, it follows from (3.4) that

$$\partial g_h / \partial \theta < 0 \quad for \quad c > 0 \tag{3.5}$$

and

$$\partial g_h / \partial \theta > 0 \quad for \quad c < 0 \tag{3.6}$$

[6] See Sinn (1987, chapters 8 to 10) for a lucid analysis of the effects of capital income taxation in a neoclassical intertemporal general equilibrium model of economic growth.

These results are not difficult to explain: When the tuition fee is positive, the opportunity cost of education will exceed the after-tax wage rate. A higher labor income tax rate will then reduce the opportunity cost of education relatively less than it reduces the after-tax wage gain resulting from a higher level of education. Because of this fall in the net rate of return to education, human capital investment will be discouraged. On the other hand, when education is subsidized ($c < 0$), the opportunity cost of education will be less than the after-tax wage rate, and a higher labor income tax rate will then increase the relative return to education, thereby stimulating the accumulation of skills.

Finally, it is seen from (3.4) that

$$\partial g_h / \partial c < 0 \tag{3.7}$$

Thus, higher "tuition fees" or lower rates of subsidies to education and training will discourage long term growth. This obvious result should need no further elaboration.

4. Optimal growth.

As demonstrated above, the government can affect the incentives for accumulation of human and physical capital through its choice of income tax rates and charges or subsidies for education and training. We now wish to investigate whether it is possible for the government to choose these policy instruments so as to ensure that the market economy will follow an optimal growth path. To answer this question, we must first characterize the optimal growth path.

4.1. *The social planning problem*

A first-best optimal growth path is a time path for consumption and work effort which maximizes the utility function of the representative household specified in (2.13) and (2.14) subject to the technological constraint governing human capital accumulation (2.8), and subject to the economy's resource constraint

$$\begin{aligned} \dot{K} &= Y - G - C \\ &= K^{\alpha}(nH)^{1-\alpha}H^{\epsilon}[1-\beta(1-n)] - C \end{aligned} \tag{4.1}$$

where we have used the production function (2.3) and the production efficiency condition (2.9) to arrive at the last equality in (4.1).

The state variables in this dynamic optimization problem are $K(t)$ and $H(t)$, and the controls are $C(t)$ and $n(t)$. Applying the Maximum Principle, and letting λ_1 and λ_2 denote the co-state variables associated with K and H, respectively, one finds the following first-order conditions for an optimal growth path:

$$C^{-\sigma} = \lambda_1 \tag{4.2}$$

$$\lambda_1 K^{\alpha}(nH)^{-\alpha}H^{\epsilon}\{(1-\alpha)[1-\beta(1-n)]+\beta n\} = \lambda_2\delta \tag{4.3}$$

$$\dot{\lambda}_1 = \{\rho - \alpha K^{\alpha-1}(nH)^{1-\alpha}H^{\epsilon}[1-\beta(1-n)]\}\lambda_1 \tag{4.4}$$

$$\dot{\lambda}_2 = [\rho - \delta(1-n)]\lambda_2 - K^{\alpha}(nH)^{1-\alpha}H^{\epsilon-1}(1-\alpha+\epsilon)[1-\beta(1-n)]\lambda_1 \tag{4.5}$$

Equations (4.2) through (4.5), together with appropriate transversality conditions, implicitly describe the socially optimal evolution of $K(t)$ and $H(t)$ from any given initial values of these two stocks of capital. Let us now compare these conditions for optimal growth to the equilibrium conditions for the market economy with government involvement in education.

4.2. *Equilibrium growth and optimal policy rules*

We focus first on the case where charges or subsidies for education are tied to the wage rate of each individual, i.e. where (2.11) applies. Using the factor pricing equations (2.4) and (2.5), we may rewrite the first-order conditions for household utility maximization (2.16) through (2.19) as

$$C^{-\sigma} = \lambda_1 \tag{4.6}$$

$$\lambda_1 K^{\alpha}(nH)^{-\alpha}H^{\epsilon}(1-\alpha)(1-\theta+c) = \lambda_2\delta \tag{4.7}$$

$$\dot{\lambda}_1 = \{\rho - \alpha K^{\alpha-1}(nH)^{1-\alpha}H^{\epsilon}(1-\tau)\}\lambda_1 \tag{4.8}$$

$$\dot{\lambda}_2 = [\rho - \delta(1-n)]\lambda_2 - K^{\alpha}(nH)^{1-\alpha}H^{\epsilon-1}(1-\alpha)[1-\theta+c-(c/n)]\lambda_1 \tag{4.9}$$

The equilibrium conditions (4.6) through (4.9) are clearly of the same general form as the conditions for optimum growth (4.2) through (4.5). Indeed, the coefficients of λ_2 are identical in the two sets of equations, so if the government chooses its three policy instruments τ, θ, and c so as to equate the three coefficients of λ_1 in (4.7) through (4.9) with the corresponding coefficients in (4.3) through (4.5), it will be possible to ensure that the market economy will follow a first-best optimal growth path. Equating coefficients this way and rearranging terms, we arrive at the following optimal policy rules:

$$\tau = \beta(1-n) = \frac{G}{Y} \tag{4.10}$$

$$c = (\frac{n}{1-\alpha})[\beta n - \epsilon(1-\tau)] \tag{4.11}$$

$$\theta = c - (\frac{\beta}{1-\alpha})[2n - 1 + \alpha(1-n)] \tag{4.12}$$

In the conventional neoclassical growth model with infinitely- lived consumers, the optimal capital income tax rate can be shown to converge on zero as time goes to infinity (see Chamley, 1986). By contrast, in the present model we see from (4.10) that, for $\beta > 0$, the optimal capital income tax rate will always be positive and equal to the fraction of output absorbed by government spending on "educational infrastructure", G/Y.

The reason for this strong result is that there is a negative externality associated with physical capital accumulation in our model. When a private investor decides to accumulate an additional unit of physical capital, total output will increase by $\partial Y/\partial K = r$. However, given the Leontief technology reflected in (2.9), a fraction $\beta(1-n)$ of this additional output will have to be set aside for educational spending if society wishes to maintain the same quality of education, i.e. the same increase in skills per hour spent on education. In popular terms, additions to the physical capital stock will drive up the level of real wages, including the wages of teachers, and education therefore becomes more expensive for society. Hence the net social gain from an extra unit of physical capital investment will only be $r[1-\beta(1-n)]$. By setting the capital income tax rate in accordance with (4.10), the government can therefore ensure that the private return to investment $r(1-\tau)$ will equal the social return.

Of course, in practice the size of the negative externality from private capital formation described here is likely to be rather small, so our unusual rationale for a (modest) capital income tax is mainly of theoretical interest.

The policy rule (4.11) is less simple than (4.10) but still has some intuitive appeal. Recall that $\beta \equiv \eta/\delta$. Thus, a high input coefficient η, reflecting a high requirement of teaching services etc. per hour of education, will make for a relatively high value of the optimal "tuition fee" c. On the other hand, a high degree of efficiency in the learning process (a high value of δ) and strong positive externalities from education and training (a high value of ϵ) will make for a low value of the optimal educational charge, according to (4.11). Indeed, since $\tau = G/Y < 1$, it will be optimal to grant a subsidy to education ($c < 0$), if the externalities are sufficiently strong.

The optimal labor income tax rate specified in (4.12) is sure to be lower than the optimal charge for education, as long as workers spend at least half of their (active) time in the labor market, i.e. provided $n \geq 0.5$. Thus the optimal tax on labor income may well be negative.

It should be stressed that the optimal taxes and charges identified here were derived on the assumption that the government has a lump-sum tax/subsidy instrument T at its disposal, as the government budget constraint (2.10) makes clear. In an important special case to be noted below, it will in fact be possible for government to implement a first-best optimal growth path even without relying on lump sum taxes or subsidies, but in the general case a first-best allocation cannot be achieved without use of the lump-sum instrument. However, the interesting theoretical point is that, in the present model with human capital accumulation, the government generally should not abstain from the use of taxes on income from capital and labor, even if it could in fact rely solely on lump sum taxes.

4.3. *Optimal policy in some benchmark cases*

To gain further understanding of the optimal policy rules, it is useful to consider the following special cases:

Benchmark 1: $\eta = \epsilon = 0$

Optimal policy rule:

$$\tau = c = \theta = 0 \tag{4.13}$$

Benchmark 2: $\eta = 0, \quad \epsilon > 0$

Optimal policy rules:

$$\tau = 0 \tag{4.14}$$

$$c = \theta = -(\frac{\epsilon n}{1-\alpha}) < 0 \tag{4.15}$$

Benchmark 3: $\epsilon = 0, \quad \eta > 0$

Optimal policy rules:

$$\tau = \frac{G}{Y} \tag{4.16}$$

$$F = cwH(1-n) = nG \tag{4.17}$$

$$\tau r K + \theta w n H + F = G \tag{4.18}$$

In the first benchmark case the input requirements in the education sector (apart from "student time") and the externalities from education are assumed to be negligible. In this rather unrealistic case a policy of complete laissez faire is seen from (4.13) to be optimal. The competitive market economy will automatically generate a first-best optimal growth path.

In the second benchmark case the complementary inputs required in the education sector are still negligible, but the externalities stemming from a higher level of education are positive. In this case, which corresponds to the one considered by Lucas (1988), we see from (4.14) and (4.15) that the government should levy no capital income tax, but that it should impose a lump sum tax in order to subsidize labor market activities and educational activities at identical rates. By following such a policy, the government will raise the private return to education and training and thereby stimulate human capital accumulation, compared to a situation of laissez faire.

The third benchmark case, in which the externalities stemming from a higher skill level are negligible, is perhaps the most interesting one[7]. First of all, the optimal capital income tax rate is now equal to G/Y, for the reasons explained in the previous section. Second, it is seen from (4.17) that tuition fees should be set so as to cover a fraction n of government expenditure on education services. This result, implying that the government should charge positive but less than "full cost" tuition fees, was derived from (4.11) by setting ϵ equal to zero and using (2.3), (2.5), and (2.9). Third, and perhaps most striking, one sees from (4.18) that the labor income tax rate should be chosen so as to balance the government budget, given the optimal values of τ and c determined from (4.16) and (4.17). In other words, there is no need to resort to lump sum taxes or lump sum subsidies to achieve the first-best allocation of resources!

The policy rule (4.18) can be derived from (4.12) by using (4.11) (with $\epsilon = 0$), (4.16), (4.17), (2.9) and (2.3) through (2.5). Utilizing (4.16), (4.17) and (2.3) through (2.5), one may also rewrite (4.18) as

$$\theta = \tau\left(\frac{1-\alpha-n}{1-\alpha}\right) \tag{4.19}$$

which shows that the optimal value of the labor income tax rate is lower than the capital income tax rate, and that it will even be negative if the proportion of time spent in the

[7] This is the case considered by King and Rebelo (1990) and Rebelo (1991), albeit within a somewhat different formal framework.

labor market (n) exceeds the wage share $(1-\alpha)$ of national income.

4.4. *Optimal policy when tuition fees are tied to the average wage level*

The analysis above assumed that educational charges or subsidies were related to the wage rate of each individual worker. It is perhaps more realistic to assume that these charges or subsidies are tied to the average wage level prevailing in the economy and are thus exogenous to the individual agent. Consider therefore the case where F is given by (2.12) so that optimal consumer behavior is described by (2.19.a) rather than (2.19). Following a procedure quite similar to the one explained in section 4.2, it is then possible to derive the following optimal policy rules for the general case where β and ϵ are positive:

$$\tau = \beta(1-n) = \frac{G}{Y} \tag{4.20}$$

$$1-\theta = (\frac{1-\alpha+\epsilon}{1-\alpha})(1-\tau) \tag{4.21}$$

$$c = \frac{\beta n - \epsilon(1-\tau)}{1-\alpha} \tag{4.22}$$

It is seen that the optimal capital income tax rate is the same as before, whereas a comparison of (4.11) and (4.12) to (4.21) and (4.22) shows that the optimal tuition fee and the optimal labor income tax rate are now slightly different. Again, particular interest attaches to the benchmark case where the positive externalities from education are negligible ($\epsilon = 0$). In this case it can be shown that the policy rules (4.20) through (4.22) imply

$$\tau = \theta = \frac{G}{Y} \tag{4.23}$$

$$F = c(wH)_a(1-n) = G \tag{4.24}$$

$$\tau r K + \theta w n H + F = 2G \tag{4.25}$$

These results are quite remarkable. Equation (4.23) says that government expenditure (on education) should be financed by a comprehensive income tax, with no tax differentiation between income from capital and income from labor. Equation (4.24) implies that the government should charge full-cost tuition fees, and (4.25) finally says that the total amount of taxes and fees should equal twice the amount of government spending on education, enabling the government to return exactly half of its total revenue to the private sector in the form of lump sum subsidies.

5. Conclusions and caveats.

This paper analyzed a simple model of endogenous growth incorporating human as well as physical capital accumulation. Consumers could allocate their time between work in the labor market and time spent on education and training, and they were assumed to optimize the time path of consumption and work effort over an infinite horizon. The "education" sector of the model was characterized by a Leontief technology, while the competitive business sector was characterized by a Cobb-Douglas technology. The analysis deviated from previous work mainly by incorporating government spending on education and training, and by focusing on the effects of government policy on private incentives for the accumulation of human capital.

Not surprisingly, it was found that government subsidies to education and training will tend to increase the economy's long term growth rate. A tax on labor income was seen to have a positive impact on the steady state growth rate when education is subsidized, and a negative impact when the government charges positive "tuition fees". A capital income tax was shown to have no effect on the steady state growth rate, although it was found to reduce the capital intensity of production.

Even if the government had the possibility of relying solely on lump sum taxes, it was demonstrated that it should generally levy taxes on income from capital and labor and charge "tuition fees" to ensure that the economy would follow a first-best optimal growth path. The optimal capital income tax rate was shown to equal the fraction of GDP absorbed by government spending on education services. Crucial for this result was the assumption that a constant fraction of GDP will have to be devoted to education to maintain a constant growth rate of labor skills.

The optimal tuition fee as well as the optimal labor income tax rate were found to be either positive or negative, depending among other things on the strength of the positive external effects of education on the productivity of firms. When these externalities were negligible and tuition fees were tied to the general wage level, we obtained the strong results that a comprehensive income tax and full-cost tuition fees would be socially optimal. In the absence of positive externalities from education, we also found that the government could achieve a first-best allocation without making any use of lump sum taxes and subsidies, if charges for education were related to the wage rate of each individual worker/student.

In general, the analysis pointed to some hitherto unexplored interrelationships between education policy and optimal tax policy and suggested that there might be a positive efficiency role for a (modest) capital income tax. However, because of the many simplifying assumptions, the policy prescriptions derived from the model should obviously not be taken too literally.

There are several directions in which the present work could be extended. First, while our model focused on the effects of the tax-transfer system on private incentives for human capital investment, it may be that government has an even more important growth-promoting role to play in determing the productivity parameters δ and η in the education system through the setting of educational quality standards, by fostering competition among schools and universities, etc.

Second, because our model assumed all individuals to be identical, it abstracted entirely from equity considerations in the design of public policy, including education policy. In practice, concerns about equity often play a dominant role in determining tax policy and subsidies for education.

It would also be desirable to introduce second-best considerations by assuming that the government cannot make any use of lump sum taxes and subsidies. Further, since there is clearly an interaction between the accumulation of human capital and the invention of new products and production processes, it could be interesting to study the role of government policy in a richer model allowing for both of these sources of endogenous growth. An attempt at such an analysis is made in Sørensen (forthcoming).

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