CENTER FOR ECONOMIC STUDIES

SUSTAINABILITY AND CONSTANT CONSUMPTION PATHS IN OPEN ECONOMIES WITH EXHAUSTIBLE RESOURCES

John M. Hartwick

Working Paper No. 61

UNIVERSITY OF MUNICH

CES

Working Paper Series

CES Working Paper Series

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Center for Economic Studies
University of Munich
Schackstr. 4
80539 Munich
Germany
Telephone & Telefax:
++49-89-2180-3112

This paper was written while I was a visitor at the Center for Economic Studies, University of Munich. Thanks to them for hospitality and support and to Ray Rees for helpful discussions. The SSHRCC, Government of Canada, also provided support. Geir Asheim and Cees Withagen provided invaluable comments on an earlier draft. An earlier version was presented in seminars at the Universities of Munich and Tilburg.

CES Working Paper No. 61
June 1994

SUSTAINABILITY AND CONSTANT CONSUMPTION PATHS IN OPEN ECONOMIES WITH EXHAUSTIBLE RESOURCES

Abstract

We review some of the historical background to the capital theory approach to sustainability. We then turn to sustainability in a group of countries trading flows from an exhaustible resource. We derive an adjusted invest-resource-rents rule which leaves each country, in a group of trading countries, on a constant consumption path. Oil importers invest a fraction (greater than unity) of the rents ascribable to the current use of their own oil stocks and oil exporters invest a fraction (less than unity) of the rents ascribable to their current use of their own oil stocks. Each country's value of imports equals its value of exports. In a partial equilibrium model of a small open oil exporting country, we observe that the exact invest-resource-rents does leave the country's consumption constant over time.

John M. Hartwick
Queen's University
Dept. of Economics
Kingston, ON K7L 3N6
Canada

Sustainability and Constant Consumption Paths in Open Economies with Exhaustible Resources

Introduction

Since there are at least three good surveys of theoretical aspects of sustainability (namely, Solow [1991], Hammond [1994], Pezzey and Toman [1994]) available, I will not attempt a cannibalized fourth. Instead I will make some brief general remarks about the background of theory of sustainability and then turn to an area of current research, namely open economy aspects of sustainability. With this approach I can still present the references to the literature which I know about, an invaluable part of a good survey, and also introduce a reader to the core of the theory because I need this material as the stalk to graft on my open economy analysis.

There are at least three distinct ideas tied up in the economic theory of sustainability which I am dealing with today. There is first the idea that if exhaustible resource stocks are depleted today in the course of producing final goods, one need not immediately contemplate a permanent shrinkage in future production possibilities because producible or machine capital can be "over-accumulated" in order to "compensate" for the current reduction in the stock of natural capital. This idea is mentioned in Pigou [1935] and in Hayek [1941; p.88]. An important variant of this idea is of "over-accumulating" knowledge capital in order to "balance-off" the current diminution of the stock of natural capital (Robson [1980]). More generally, technical progress may allow smaller and smaller flows from exhaustible resources to maintain say a non-shrinking set of production possibilities (as in for example Stiglitz [1974]). The second idea that comes to mind is that sustainability suggests non-shrinking production possibilities as time passes. A simple indicator of non-shrinking production possibilities is of course the observed aggregate consumption level not declining over time.[1] For the case of multiple consumption goods one turns to THE utility of the current consumption vector not declining over time. Rawls maximin criterion, a moral injunction, is a polar case in this line of thought and in part inspired the classic Solow [1974] paper on sustainability. The injunction is of course: do for others who will occupy period t+1 what we would have preferred back in t-1, what others who occupied period t-1 would do for us, the occupants of period t. The third idea involves linking the first two ideas together. The simplest variant is of course: consume at a level which results in no shrinking of one's "capital". For an individual, this is not too difficult to contemplate since everything can be measured in dollars but at the level of the nation satisfactory measures of what "capital" is being maintained intact are generally elusive. Hicks [1942] and Pigou [1941] debated aspects of the meaning of "maintaining capital intact". This was a final exchange in the long-running debate on the links between capital and national accounting, a debate in which Pigou and Hayek sparred and Hicks assisted in clarifying matters. A primary legacy was Hicks' [1939; Chapt. 13] notion that INCOME be defined as POTENTIAL CONSUMPTION which if "withdrawn" from current production leaves capital intact.[2] In Solow [1974], the problem of measuring "capital intact" was reduced to, given oil stocks being run down in accord with the Hotelling efficiency condition, and given the level of consumption unchanging, how much $\dot{K}$ is currently needed to "support" this program, at least for another period. This is in fact one kind of investment balancing off

[1] Asheim [1988] [1991] has axiomitized the concept of non-declining U(C) in an economy with exhaustible resources. There is no simple way to rank two distinct efficient candidate paths. See also Pezzey [1993].

[2] This leads to the idea that Net National Product be defined as some sort of "interest" on "national wealth" (Samuelson [1961], Weitzman [1976], Kemp and Long [1982], Lozada [1992], Asheim [1994] and Hartwick [1994]).

disinvestment in another stock. Dixit, Hammond, Hoel [1980] have labelled such paths as those with "zero net investment". Such paths are not in general those in which aggregate capital value (national wealth) are remaining constant because zero net investment is essentially changes in quantities of stocks at prevailing prices and changes in national wealth comprise both quantity changes and price changes - a chain rule calculation. The constant consumption model of Solow [1974] is of course a zero net investment model but it is not a constant wealth model. It is an increasing national wealth model. More on this below.[3]

When the stock of natural capital is re-generating itself as with say fish stocks, forest stocks, and environmental capital stocks, the notion of preserving capital intact is straight-forward in the steady state. In fact, the term sustainable yield has been around in the economics of the fishery and forestry much longer than it has been in the discussion of how any economy is performing (as in, for example, the Brundtland Report). There remains however the question of what course of action to take along the approach to the steady state (the transient trajectory) with renewable resources in the economy. If one is wedded to a constant consumption path over all time, then the investing of resource rents is the appropriate strategy off the steady state trajectory (Hartwick [1978], Becker [1982] Hamilton [1994]). This result contains the not-new suggestion that the exhaustible resource use problem is a special case of the renewable resource use problem in the sense that in the former, the economy has only a transient path to occupy. We now turn to some detailed analysis on constant consumption paths in open economies.

[3] I am indebted to Geir Asheim for clarifying this in conversation.

Open Economy Considerations

Consider splitting a closed economy with exhaustible resources, enjoying constant consumption over time, into two countries, one importing some oil (the exhaustible resource) from the other. We observe below that if each country saves exactly the resource rents ascribable to local resource stock flows, the importer's consumption level will be declining and the exporter's will be increasing (Asheim [1986]). We can describe this as the importer under-saving and the exporter over-saving relative to levels for constant consumption paths. Below, we characterize adjustment weights on each country's own resource rents which "neutralizes" the importer's under-saving and the exporter's over-saving. With "corrected" local savings levels, each country "ends up" on a constant consumption path, an intergenerational equity path (Solow [1974]).

The under and over saving takes the form of price changes on oil trade flows - opposite in sign but equal for each country. The adjustment weights on local own resource rents appear in offsets to these "capital gains" terms and one characterization is as an r percent rule on certain oil flows, not values. r is the rate of return, equal to the marginal product of capital in our model. We will work with an almost symmetric split of the one world into two countries. This makes the exposition straightforward and allows us to detour around special cases with corner solutions. The reader can easily develop the analysis for not nearly symmetric splits of the one world and for more than two countries. We comment on this in detail.

Under exact investment of resource rents, each country's change in consumption turns out to equal the exhaustible resource flow traded multiplied by its current price change. Thus the consumption shifts in each country can be interpreted as an adverse terms of trade shift for

oil importers and a favorable terms of trade shift for oil exporters. This becomes clear when we set out a model of an oil exporting nation facing constant world prices and interest rates, at the end. No terms of trade effects or consumption "wedges" are observed under the exact invest resource rents strategy. Thus "over-saving" and "under-saving" under exact savings of own oil use rents in the two country model are a consequence of endogenous terms of trade shifts, induced, of course, via oil price changes. The oil price changes are a consequence of asset equilibrium in the market for oil stocks (Hotelling's Rule). Our partial equilibrium model at the end has constant world oil prices; the r percent changes in resource rents operate via endogenous extraction cost shifts.

The Model

We look first at the structure of a closed one world economy. It has $S(t)$ tons of say oil left at date t. $-\dot{S}(t) = R(t)$ will be used in production of $Q(t)$ equal to $F(K(t), R(t))$ at date t. $K(t)$ is non-depreciating machine capital. $F(\cdot)$ is homogeneous of degree 1 in inputs and $K(t)$ and $R(t)$ are smoothly substitutable. $F(\cdot)$ is concave in its arguments. (Existence of constant consumption paths over infinite time requires $F(\cdot)$ to be Cobb-Douglas (see Solow [1974], Dasgupta and Mitra [1983] and Hamilton [1993]).) Population N, constant, only consumes.[4] We postulate the savings-investment rule (invest resource rents):

$$\dot{K}(t) = \lambda(t)R(t)F_R(t) \tag{1}$$

where $\lambda(t)$ moves exogenously through time, say near unity. $F_R(t)$ is the derivative $\partial F(\,)/\partial R$. We

[4] This is not an issue with a Cobb-Douglas production function but otherwise, putting N, a constant in the production function can introduce complicated scale effects as the economy's level of aggregate output, $Q(t)$, changes over time.

also take dynamic efficiency in exhaustible resource use as given, that is (Hotelling r% Rule):

$$\frac{\dot{F}_R(t)}{F_R(t)} = F_K(t). \tag{2}$$

Current consumption C(t) is given by $C(t) = F(\cdot) - \dot{K}$. If one differentiates this expression with respect to time, and does the same for (1), and one uses (1) and (2), one obtains (see the Appendix):

$$\dot{C}(t) = \frac{\bullet}{(1-\lambda(t))R(t)} F_R(t). \tag{3}$$

The central case of investing exactly exhaustible resource rents (namely $\lambda = 1$) yields $\dot{C} = 0$ (Hartwick [1977]. See also extensions[5] in Dixit, Hammond, Hoel [1980] and Cairns [1986].).

Consider the value of aggregate capital or national wealth W(t) in this economy at date t. We define $W(t) = K(t) + S(t)F_R(t)$. Observe that $\dot{W}(t) = \dot{K}(t) + S(t)\dot{F}_R(t) + F_R(t)\dot{S}(t)$ and $\dot{W}(t)$ is the change in wealth (aggregate capital value) in the economy at date t. The following result can be derived. If the economy is efficient, has net investment zero, and has constant returns to scale in F(K,R) then

$$C + \dot{W}(t) = W(t)F_K(t)$$

or $C + \dot{W}(t)$ is the interest flow from current wealth W(t). The demonstration requires simple substitution, i.e. $C = F(\cdot)-\dot{K}$, $F(\cdot) = KF_K + RF_R$, $\dot{K} = RF_R$, etc. This result is quite Hicksian since the income flow on the left is interest on capital on the right. The "logic" of Hicks' position suggests that the left hand side is net national product in this economy. Asheim [1994] seems to espouse this view. $\dot{W}(t)$ includes capital gains $S(t)\dot{F}_R(t)$ on oil stocks and these terms

[5]These include extending consumption C to a vector in U(C). Then U(C) remains constant and extending our two capital goods K and S to many capital goods. The $\dot{C}=0$ result was proved as an if and only if theorem. Our investing resource rents can be interpreted as aggregate or combined investment being zero. Another extension was to treat this combined investment as positive and constant.

have not been included in NNP in the modem stream of thought in national accounting, although some observers recommend land revaluations be placed in NNP (see Hartwick [1992] and references there). It turns out that these identical capital gains are in the WF_K term on the right. This suggests taht there is a more basic relation lying within ours above (it is $C - KF_K$) and that the claims for $C + \dot{W}(t)$, with its capital gains on current oil stocks, as the "formula" for NNP suspect. We end this discussion with the observation that $\dot{W}(t)$ above is not constant for the Solow [1974] constant consumption, zero net investment model. Thus maintaining capital value constant (capital "intact"?) is a separate matter from maintaining consumption constant over time or maintaining aggregate investment zero over time.

We now split the one world economy ($\lambda = 1$) into two price-taking, trading countries. We set $K_1(t) = K_2(t)$, given $K_1(t) + K_2(t) = K(t)$ above. We set $N_1 = N_2$ with $N_1 + N_2 = N$, above. We make country 1 (C1) less endowed with oil stocks, that is $S_1(t) < S_2(t)$ with $S_1 + S_2 = S(t)$, above. We assume $S_1(t) \cong S_2(t)$ so that country 1 will import $\in(t)$ a small amount of R(t) at each date. Since $K_1 = K_2$, efficiency requires that $R_1(t) + \in(t) = R_2(t)$ where $R_i(t)$ is use of exhaustible resource from stock $S_i(t)$. World prices are given from the one large country scenario earlier.

(a) The oil importer (C1)

We have the output balance

$$C_1(t) = F(K_1(t), R_1(t) + \in(t)) - \dot{K}_1(t) - \in F_{R_1}(t) \tag{4}$$

where $\in F_R(t)$ is payment for oil imports, $\in(t)$, and $\dot{K}_1(t)$ is own investment in $K_1(t)$. In keeping with each country "covering off" the economic depreciation of its own oil stock $S_i(t)$, we have

$$\dot{K}_1(t) = \lambda_1(t)\, R_1(t)\, F_{R_1}(t) \qquad (5)$$

where $\lambda_1(t)$ is a fraction, endogenous and presumably near unity for $\in(t)$, small. Our task is to characterize $\lambda_1(t)$ since (5) represents the "adjusted" invest resource rents rule. We also have $\dot{F}_{R_1}/F_{R_1} = F_{K_1}$. These derivatives will be the same as those in (2). If one differentiates (4) and (5) with respect to time and combines them, and uses (5) and (2), one obtains (see the procedure in the Appendix):

$$\dot{C}_1(t) = \frac{\bullet}{(1-\lambda_1(t))R_1(t)} F_R(t) - \in(t)\dot{F}_R(t). \qquad (6)$$

It follows that $\dot{C}_1(t) = 0$ if

$$\frac{\dot{\Delta}_1(R_1)}{\in(t)} = F_K(t), \qquad (7)$$

where $\dot{\Delta}_1(R_1) \equiv \frac{\bullet}{(1-\lambda_1)R_1(t)}$. (Recall that $\dot{F}_R/F_R = F_K$.) This condition for $\dot{C}_1 = 0$ is an r percent rule in quantities, since $\dot{F}_K(t)$ is the "rate of interest" here and $\in(t)$ and $(1-\lambda_1(t))\, R_1(t)$ are quantities of oil. This r percent rule defines the time path of $\lambda_1(t)$ and when combined with (4) becomes the adjusted invest resource rents rule.[6] Observe that if $\lambda_1(t) = 1$, then we would have the unadjusted invest resource rents rule and (6) would become

$$\dot{C}_1 = -\in(t)\dot{F}_R(t).$$

This is a rendering of the result in Asheim [1986], namely, if country i invests its resource rents, its $C_i(t)$ will not be constant. In this case, importers C1's $C_1(t)$ is declining because it is "under-saving" in covering its own economic depreciation in its stock $S_1(t)$ and in paying for imports, $\in(t)$. Thus $\lambda_1(0)$ must be greater than 1 and decrease toward 1 as time passes.

[6] Asheim [1986] and Asheim [1994a] contain expressions for country i's savings to cause C_i to remain constant. Their appearance and derivation are quite different from our adjusted resource rents expressions yielding $\dot{C}_i = 0$.

Observe that $\in(t)\dot{F}_R(t)$ is a quantity traded $\in(t)$ multiplied by a price change $\dot{F}_R(t)$ and is thus a terms-of-trade effect. $\in(t)\dot{F}_R(t)$ equals $\in(t)F_R(t)F_K(t)$. Hence the current decline of $C_1(t)$ from $C_1(0)$, given $\lambda_1(t)$ set at 1 is $\int_0^t \in(t)F_R(s)F_K(s)ds$ where $C_1(0)$ is a constant of integration. Since $\in(t) = -\dot{S}_\in(t)$ where $\dot{S}_\in(t)$ is the decline in C2's stock resulting from exporting $\in(t)$, we have[7]

$$C_1(0) - C_1(t) = -\int_0^t \dot{S}_\in(s)F_R(s)F_K(s)ds.$$

Wealth in C1 at date t is $W_1(t) = K_1(t) + S_1(t)F_R(t)$ and $\dot{W}(t) = \dot{K}_1(t) + S_1(t)\dot{F}_R + \dot{S}_1(t)F_R$. Given $C_1 = F(K_1,R_1) - \dot{K}_1 - \in F_R$, $\dot{K}_1 = \lambda_1 R_1 F_R$, constant returns to scale in $F(\cdot)$, and efficiency, one gets $C_1 + \dot{W}_1(t) = W_1(t)F_K(t)$ or $C_1 + \dot{W}(t)$ is interest on own wealth. This balance relation simplifes to $C_1 = K_1F_K + (1-\lambda_1(t))R_1F_R$. This contrasts with the closed economy analogue in which C equalled KF_K alone. Thus $(\lambda_1(t) - 1)R_1F_R$ is income "withdrawn" from K_1F_K to pay for the oil imports in C_1. The constant C_1 is less than interest on local K. The capital gains on oil stocks $S_1(t)\dot{F}_R(t)$ in $\dot{W}_1$ again cancel with such gains in $W_1(t)F_K$ and this suggests that $C_1 + \dot{W}_1(t)$ is not a satisfactory "formula" for NNP in this economy. More on defining NNP below.

(b) The oil exporter (C2)

C2's situation is the mirror image of that of the oil importer. Now C2's savings to replace her current oil use are $\lambda_2(t)R_2F_R(t)$, where $R_2(t)$ is current oil extracted in C2. $R_2(t)$ -

[7] The term $-\int_0^t \dot{S}(s)F_R(s)ds$ figured prominently in Hartwick [1994]. It was a key measure of wealth. The analogous expression for machine capital was also prominent. See also Solow [1986]. Here we are dealing with a gap between two flows, $C_1(0)$ and $C_1(t)$, not stocks. Hence the appearance of $F_K(s)$ under the integral.

$\in(t)$ is used in production in C2. Hence C2's replacement rule is

$$\dot{K}_2(t) = \lambda_2(t)R_2(t)F_R(t). \tag{8}$$

C2's value balance relation is

$$C_2(t) = F(K_2(t),R_2(t) - \in(t)) - \lambda_2(t)R_2(t)F_R(t) + \in(t)F_R(t). \tag{9}$$

We now differentiate (8) and (9) with respect to time, combine them, use (8) and (2) and obtain (see the procedure in the Appendix):

$$\dot{C}_2(t) = \frac{\bullet}{(1-\lambda_2(t))R_2(t)} F_R(t) + \in(t)\dot{F}_R(t). \tag{10}$$

This is the same as (6) with a sign change. (10) yields our principal savings rule result, now for C2, namely $\dot{C}_2(t) = 0$ if

$$\frac{\dot{\Delta}_2(R_2)}{\in(t)} = F_K(t) \tag{11}$$

where $\dot{\Delta}_2(R_2) \equiv -\frac{\bullet}{(1-\lambda_2(t))R_2(t)}$. (11) characterizes the time path of $\lambda_2(t)$ in the investment rule in (8). The rule is the same as that for C1 in (7) except in our case $\lambda_2(t)$ will be less than unity, and will increase toward unity as time passes. ($\lambda_1(t)$ was above unity and declined toward unity as time passed.)

For $\lambda_2(t)$ set equal to 1.0, $\dot{C}_2(t) > 0$ by current capital gains $\in(t)\dot{F}_R(t)$. C2 is in fact over-saving relative to a constant consumption scenario, and for this case

$$C_2(t) - C_2(0) = \int_0^t \in(s)\dot{F}_R(s)ds$$
$$= -\int_0^t \dot{S}_{\in}(s)F_R(s)F_K(s)ds.$$

Our crucial adjustment terms $\lambda_1(t)$ and $\lambda_2(t)$ are, in view of (7) and (11), not independent.

(7) and (11) imply

$$\dot{\Delta}_2(R_2) - \dot{\Delta}_1(R_1) = 0. \tag{12}$$

(12) indicates, roughly speaking, that for the case $\lambda_1 = \lambda_2 = 1$, C1's under-saving matches C2's over-saving. $\lambda_1(t)$ and $\lambda_2(t)$ ($\neq 1$) in (12) reflect this balancedness of the adjustments for over- and under-saving between our two countries. In fact $\lambda_1(t) - 1 = 1 - \lambda_2(t)$ because $\dot{K}_1(t) + \dot{K}_2(t) = \dot{K}(t)$ where $\dot{K}(t)$ is investment in the closed economy case and $R_1(t) + R_2(t) = R(t)$.

Again for C2's wealth defined in $W_2(t) = K_2(t) + S_2(t)F_R(t)$ we can obtain $C_2 + \dot{W}_2(t) = W_2(t)F_R(t)$, i.e. the left hand side is interest on local wealth. Again capital gains on oil stocks cancel on both sides to leave $C_2(t) = K_2(t)F_K(t) + (1-\lambda_2(t))R_2(t)F_R(t)$. The oil exporter enjoys a constant level of consumption above the income from interest on $K_2(t)$ because it receives extra income from exporting oil. (Note that $(1-\lambda_2(t))$ is positive.)

Corner Solutions and More than Two Countries

We have characterized the savings-investment rule which yields constant consumption paths for our two-country, trading world with an essential exhaustible resource. It is an adjusted invest-resource-rents rule. Our framework was two almost identical countries. This made trade flows small so that neither country was specialized and the two country assumption allowed us to sign the oil flows from exporter to importer. Clearly no part of our calculations depended on our assumption of $K_1 = K_2$ and $S_1 \equiv S_2$ with $S_1 < S_2$. Suppose, however, that C2 owned all the oil. In this case R_1F_R is zero and weigting this by λ_1 does not yield more saving. (An approach for this case is for C2 to have $\lambda_2(t) = 1$ and to transfer $\in(t)\dot{F}_R(t)$ to C1 in order to have $\dot{C}_1 = \dot{C}_2 = 0$. This was proposed by Asheim [1986].) However, as long as own oil use

$R_1(t)$ is infinitesimally positive, $\lambda_1 R_1 F_R$ ($= \dot{K}_1$) can be defined and our two-country results go through. (We require $C_1(t)$ and $\lambda_1(t)$ to remain positive.) Thus as long as each country holds some positive stock $S_i(t)$ at t, our adjusted saving-investment rule is relevant. (We require that each country owns sufficient capital K to have income to pay for imports of oil in order to rule out corner solutions.)

With say three countries, the pattern of oil flows in trade becomes more complicated. Suppose C1 is an oil importer and C2 and C3 are potential exporters, being equally "over" endowed with oil stocks. Suppose $K_1(0) = K_2(0) = K_3(0)$. In this case C1 should import equal amounts from both C2 and C3. It is not complicated to use our above reasoning to obtain appropriate $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ for this case. Our $\lambda(t)$ adjustment factors "work" for the many-country case. Note, also, that standard national accounting procedures "work" for each country in the trading system. In particular the value of exports equals the value of imports for each country. Also domestic NNP in each nation equals consumption $C_i(t)$ plus domestically financed investment. That is, $C_i(t) + \lambda_i(t)R_i(t)F_R(t) + X_i(t) - M_i(t)$ is $NNP_i(t)$ for country i, where $\lambda_i(t)R_i(t)F_R(t)$ is investment in i generated from current domestic production, $X_i(t)$ is current exports and $M_i(t)$ is current imports. All components are denominated in the numeraire commodity price, namely final goods output. $X_i(t) - M_i(t)$ equals zero in our framework. In the two country "example", $M_1(t)$ were oil imports and $F_1(\cdot) - C_1(t) - \lambda_1(t)R_1(t)F_R(t)$ were exports of the final good. This yields $NNP_1(t)$ in value-added in C1 as $F_1(\cdot) - M_1(t)$. Note that $F_1(K_1,R_1+\in)$ here is gross of oil import flow $\in$. Hence $F_1(\cdot) - \in F_R(t)$ is C1's valued-added derived from domestic factors of production. Hence $F_1(\cdot) - M_1(t)$ is domestic valued-added and equals C1's NNP(t).

In C2, $NNP_2(t) = C_2(t) + \lambda_2(t)R_2(t)F_R(t) + X_2(t) - M_2(t)$. Given $C_2(t)$ in (9), it follows that $NNP_2(t) = F_2(K_2,R_2-\in) + X_2-M_2$ is value-added and $X_2-M_2 = 0$. In each country, the value of exports equals the value of imports in "free trade". World NNP equals $NNP_1(t) + NNP_2(t)$ which in turn equals world value-added $F(K,R) \equiv F(K_1+K_2,\ R_1+R_2) = F_1(K_1,R_1+\in) + F_2(K_2,R_2 - \in)$.

An Oil Exporter Facing Constant Prices and Interest Rates

Our analysis above involved two country trade with endogenous prices, including the marginal product of capital, the interest rate. These prices were changing over time. Consider the case of a price-taking "oil republic" (OR) a country living off exports of oil. This is an autonomous problem. World oil prices will be constant at p per ton and the OR will have unchanging extraction costs. e(R) for R tons currently extracted from its stock, S(t). We assume $e(0) = 0$ and $e_R \equiv de/dR > 0$ and $e_{RR} \equiv d^2e/dR^2 > 0$. There is a constant population (say just consuming so that e(R) has no labor costs in it) and extraction is pursued to maximize discounted net profit. Hence

$$\frac{\dot{\overline{p-e_R(R)}}}{p-e_R(R)} = r \tag{13}$$

is satisfied (the Hotelling r% efficiency rule). r is the constant discount (interest) rate. We assume that the elders in this OR invest $R(t)\cdot[p-e_R(t)]$ abroad each period and live off current interest income rH(t) plus current producer surplus $L(t) = pR(t) - e(R(t)) - R(t)\cdot[p-e_R(t)]$. That is consumption

$$C(t) = rH(t) + L(t). \tag{14}$$

Since interest $rH(t)$ is being drawn off wealth abroad period by period, we have $H(t) = \int_0^t [p-e_R(s)]R(s)ds + H(0)$. Thus[8] $\dot{H}(t) = [p-e_R(t)]R(t)$. If one differentiates (14) with respect to time and uses (13) and $\dot{H} = [p-e_R]R$, one obtains $\dot{C}(t) = 0$. Hence investing oil rents abroad and living off the current interest on such, plus current producer surplus, yields a constant consumption path.[9] When $S(t)$ declines to zero at say T, there will be $H(T)$ dollars invested abroad and $C(T)$ will equal $rH(T)$ which will be the same value as was being enjoyed up to T. Clearly this policy of efficiently extracting oil and accumulating rent, net of interest, abroad is a savings-consumption strategy identical with selling off S_0 at market price $V(S_0) = \int_0^T [pR^*(t) - e(R^*(t))]e^{-rt}dt$ at $t=0$ and setting $C(t) = rV(S_0)$. (*'s indicate optimal values.) This is true because there are no market imperfections or uncertainties in our set-up, and the problem is autonomous.

Our autonomous, constant price and interest rate model for a single oil exporter differs from that for exporter C2 in our two country model in the sense that oil prices faced by C2 varied over time and generated terms of trade changes in $\in(t)\dot{F}_R(t)$. We had to "neutralize" these capital gains enjoyed by C2 with an adjusted invest resource rents savings rule. The constant oil price p eliminated capital gains in our autonomous model of the OR. In both models agents were acting with perfect foresight so that they could anticipate price and interest rate changes and optimize appropriately.

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[8] $H(t)$ is another instance of the index number mentioned in footnote 1. Clearly this index number is cumulative uncompounded or discounted rent. The lack of compounding occurs here because potential interest accumulation is "neutralized" by the period by period drawing off of current interest on the capital value.

[9] This argument was set out in detail in Hartwick and Hageman [1993] but no formal demonstration of $\dot{C}(t) = 0$ was given.

Concluding Remarks

There are indeed subtleties in moving from a unitized world system to a system of countries trading flows from their exhaustible resource stocks and each maintaining consumption constant over time. We derived the "wedges" that arise when our investment is financed in oil importing countries by own resource rents and derived adjustment weights for the own savings (resource rents). Oil importers should save more than resource rents ascribable to their own exhaustible resource flows and oil exporters should save less than resource rents ascribable to their own exhaustible resource case. Our subsequent model of a small open oil exporting nation, a PRICE-TAKER at a constant interest rate and commodity prices, revealed no "wedges" that were seen in the two country system with endogenous prices. Thus trade introduces subtleties to the derivation of constant consumption paths because prices are indeed moving over time and these price change effects show up as endogenous terms of trade effects. Relatively complicated savings-investment rules are needed in each country to neutralize these terms of trade effects on the simple invest-resource-rents rule, familiar for closed economies.

With our adjusted savings rule, we have been able to re-construct the closed economy set-up, given multiple countries in trade. This was our goal. We also noted that no new valuation issues were met and that traditional NNP measures "go through" in the open economy system. We were also able to relate constant consumption paths to interest-on-wealth expressions. These are compelling Hicksian notions of current national "income" being interest on national wealth. However constant consumption paths are not reflections of constant wealth paths. In no case was national wealth remaining constant over time.

Appendix: Derivation of Equation (3)

One differentiates $C(t) = F(K(t),R(t)) - \dot{K}(t)$ to obtain

$$\dot{C} = F_K\dot{K} + F_R\dot{R} - \ddot{K}(t) \qquad \text{(A1)}$$

One differentiates equation (1) to obtain

$$\ddot{K}(t) = \lambda(t)R(t)\dot{F}_R(t) + \lambda(t)F_R(t)\dot{R}(t) + R(t)F_R(t)\dot{\lambda}(t). \qquad \text{(A2)}$$

In A1, for F_K substitute $\dot{F}_R(t)/F_R$ from (2) and $\lambda(t)R(t)F_R(t)$ for $\dot{K}$. Also for $\ddot{K}(t)$ in A1 substitute the expression in A2. A1 reduces to $\dot{C} = \frac{\bullet}{(1-\lambda(t))R(t)} F_R(t)$, our expression in (3) in the text.

References

Asheim, G.B. [1986], "Hartwick's Rule in Open Economies", Canadian Journal of Economics, 19, pp. 395-402.

Asheim, G.B. [1988], "Rawlsian Intergenerational Justice as a Markov-Perfect Equilibrium in a Resource Technology" Review of Economic Studies, 55, pp. 469-484.

Asheim, G.B. [1991], "Unjust Intergenerational Allocations", Journal of Economic Theory, 54, pp. 350-371.

Asheim, G.B. [1994a], "The Concept of 'Net National Product' in an Open Economy", (mimeo).

Asheim, G.B. [1994], "Net National Product as an Indicator of Sustainability", Scandinavian Journal of Economics, 96 (2).

Becker, Robert [1982], "Intergenerational Equity: The Capital Environment Trade-Off, Journal of Environmental Economics and Management, 9, pp. 165-185.

Cairns, Robert [1986], "Intergenerational Equity and Heterogeneous Resources", Scandinavian Journal of Economics, 88, pp. 401-416.

Dasgupta, S., and T. Mitra [1983], "Intergenerational Equity and Efficient Allocation of Exhaustible Resources", International Economic Review, 24, 1, pp. 133-153.

Dixit, A., P. Hammond, M. Hoel [1980], "On Hartwick's Rule for Regular Maximin Paths of Capital Accumulation and Resource Depletion", Review of Economic Studies, 47, pp. 551-556.

Hamilton, Kirk [1993], "Sustainability, the Hartwick Rule and Optimal Growth", (mimeo).

Hamilton, Kirk [1993], "The Greenhouse Effect and Constant Consumption Paths", (mimeo).

Hammond, Peter [1994], "Is There Anything New in the Concept of Sustainable Development?" in L. Camriglio, L. Pineschi, D. Siniscalo and T. Treves, eds., The Environment After Rio: International Law and Economics, London: Graham and Trotman.

Hartwick, J.M. [1977], "Intergenerational Equity and the Investing of Rents from Exhaustible Resources", American Economic Review, 66, pp. 972-974.

Hartwick, J.M. [1978], "Investing Returns from Depleting Renewable Resource Stocks and Intergeneration Equity", Economics Letters, 1, pp. 85-88.

Hartwick, J.M. [1992], "Deforestation and National Accounting", Environmental and Resource Economics, 2, pp. 513-521.

Hartwick, J.M. and A.P. Hageman [1993], "Economic Depreciation of Mineral Stocks and the Contribution of El Serafy" in Ernst Lutz, ed. Toward Improved Accounting for the Environment, Washington, D.C., World Bank.

Hartwick, J.M. [1994], "National Wealth and Net National Product", Scandinavian Journal of Economics, 96 (2), pp. 253-256.

Hayek, F.A. [1941], The Pure Theory of Capital, London: Routledge and Kegan Paul.

Hicks, John R. [1939], Value and Capital, Oxford: Oxford University Press.

Hicks, J.R. [1942], "Maintaining Capital Intact: A Further Suggestion", Economica, 9, pp. 174-179.

Kemp, M.C. and N.V. Long [1982], "On the Evaluation of Social Income in a Dynamic Economy" in G.R. Feiwel, ed., Samuelson and Neoclassical Economics, Boston: Kluwer-Nijhoff, pp. 185-189.

Lozada, Gabriel [1992], "Resource Depletion, National Income Accounting, and the Value of Optimal Dynamic Programs", Department of Economics, Texas A&M University, (mimeo).

Pezzey, John [1993], "The Optimal Sustainable Depletion of Non-renewable Resources I: Capital Resource Substitution", Department of Economics, University College, University of London, (mimeo).

Pezzey, John and M. Toman [1994], "Sustainability and Neoclassical Economic Theory", (forthcoming).

Pigou, A.C. [1935], "Net Income and Capital Depletion", Economic Journal, 45, pp. 235-241.

Pigou, A.C. [1941], "Maintaining Capital Intact", Economica, 8, pp. 271-275.

Robson, Arthur [1980], "Costly Innovation and Natural Resources", International Economic Review, 21, 1, February, pp. 17-30.

Samuelson, Paul A. [1961], "The Evaluation of 'Social Income'" Capital Formation and Wealth" in F.A. Lutz and D.C. Hague, eds. The Theory of Capital, New York: St. Martin's Press.

Solow, R.M. [1974], "Intergenerational Equity and Exhaustible Resources", Review of Economic Studies, Symposium, pp. 29-45.

Solow, R.M. [1986], "On the Intergenerational Allocation of Resources", Scandinavian Journal of Economics, 88, pp. 1441-1449.

Solow, R.M. [1991], "Sustainability: An Economist's Perspective", The Eighteenth J. Seward Johnson Lecture, Marine Policy Center, Woods Hole Oceanographic Institution.

Stiglitz, J.E. [1974], "Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths", Review of Economic Studies, (Symposium on the Economics of Exhaustible Resources), pp. 123-137.

Weitzman, Martin L. [1976], "On the Welfare Significance of National Product in a Dynamic Economy", Quarterly Journal of Economics, 90, pp. 156-162.

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