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THE ITALIAN CONTINUOUS TIME MODEL: RESULTS OF THE NONLINEAR ESTIMATION

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Abstract

Continuous-time econometric models (i.e., models specified as systems of stochastic differential equations) are by now well established both in theory and in practice. Several macroeconometric continuous-time models exist for various countries. Among these the Italian continuous-time model developed by Gandolfo and Padoan over the years is in its fifth version. It is a medium-term disequilibrium model of real and financial accumulation of an open economy capable of generating both long-run growth and cycles. It has been successfully used for various purposes, amongst which the determination of the exchange rate (where it has been capable of outperforming the random walk in out-of-sample forecasting experiments).
Up to now, the estimates of the parameters have been obtained using a linearized version of the model. Although for simulation purposes the original nonlinear version can be (and has actually been) used, it would be highly desirable to obtain the parameter estimates through a nonlinear estimation method. Such a method has been made available recently by Clifford R. Wymer (1993a,b) through his ESCONA program. The purpose of this paper is to apply such a method to our Italian model, and to compare the results with those of the linear approximation.

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1. Introduction

The advantages of continuous time econometric modelling have been treated in depth elsewhere (see, for example, Gandolfo, 1993a), hence we will not deal with them again. We would only like to point out two of them. As we know, we can use econometric methods and models for two main purposes, partly overlapping. One is to test a theory empirically and to discriminate between competing theories. The other is for policy purposes.

As regards the first point, we know that much theoretical debate is based on different a priori assumptions on the speeds of adjustment of markets and on the *relative* speeds of adjustment of different variables (e.g., quantities and prices). A neo Keynesian, for example, will assume that quantities adjust much more rapidly than prices, so that the latter can be taken as constant in short–run dynamics (the fix–price approach). On the contrary, a new classical macroeconomist will make the opposite assumption that prices adjust much more rapidly than quantities, so that markets clear continuously. These antagonistic assumptions are usually imposed *a priori*. But when there is a debate between competing theories, we do think that the respective fundamental antagonistic assumptions should be confronted with the data. However, should one wish to do so by using standard discrete methods, one would not be able to discriminate between speeds of adjustment such that the adjustment occurs in a smaller amount of time than the unit period inherent in the data. The issue is so important that the availability of rigorous estimates of the adjustment speeds independently of the observation interval should be welcome. And, in fact, with continuous time econometrics it is always possible to obtain asymptotically unbiased estimates of the adjustment speeds.

As regards the second point, suppose that we have a macroeconometric model and the sample set consists of quarterly observations. In discrete time, after estimating the parameters of the model we can use it to perform policy simulations and forecasts just for three–month intervals. But with the continuous time approach, once obtained the estimates of the parameters of the differential equation system, we can solve it and obtain the *continuous* paths of the endogenous variables. This might be of paramount importance for the policy authorities. Suppose, for example, that –given a discrete time model– a simulation or optimization tells the monetary authorities that they should increase the money supply from 100 today to 102 next period (say, the quarter). But how is M to be brought from 100 to 102 (sudden jump, gradually, etc.) over the quarter the model does not say. With the continuous time methodology we get the continuous path from 100 to 102.

The Italian continuous time model, whose construction was initiated in 1979 (the first version was published in 1980; the current version is the fifth, published in 1990) is nonlinear. The method previously available required a linearization for estimation purposes. Although the estimated values of the parameters could be used in the original nonlinear version for simulation and similar exercises, it would be desirable to have nonlinear estimates. This is now possible thanks to Wymer's new ESCONA program. The purpose of this paper is to present these nonlinear estimates and compare the results with the previous linear estimates.

In sect. 2 we give a succinct overview of the model, which has been presented at length elsewhere (Gandolfo and Padoan, 1990). Section 3 summarizes the empirical results of the linearized model. The nonlinear features of the model are discussed more in depth in sect. 4. The results of the nonlinear estimation and their comparison with the linear estimates are examined in section 5. Section 6 concludes the paper.

2. An overview of the model

Our model (for a detailed treatment see Gandolfo and Padoan, 1990; the equations of the model are given in the Appendix to the present paper for reference purposes) is a medium term disequilibrium model specified and estimated in continuous time as a set of stochastic differential equations which stresses real and financial accumulation in an open and highly integrated economy. The Mark V version includes a detailed specification of the financial sector as well as the endogenous determination of the exchange rate.

It considers stock–flow behaviour in an open economy in which both price and quantity adjustments take place. Stocks are introduced with reference to the real sector (where adjustments of fixed capital and inventories to their respective desired levels are present) and to the financial sector which includes the stock of money, the stock of commercial credit, the stock of net foreign assets and the stock of international reserves. Real and financial feedbacks are, therefore, taken into account in the model. Government expenditure and revenues (taxation) are also present so that the effects of endogenous public deficits are included.

Quantity behaviour equations are considered for the traditional macroeconomic variables in real terms (private consumption, net fixed investment, imports and exports of goods and services, inventory changes, net domestic product). Expectations operate through various mechanisms concerning expected real output, the effects of monetary disequilibria on the expected domestic price level, and exchange rate expectations..

A price block is included, which determines the domestic price level, the nominal wage rate and the export price level. Endogenous determination of the latter was considered crucial for for an export led economy such as Italy's, while wage–price spiral effects are explicitly taken into account. The specification of the financial sector was completed by the inclusion of an interest–rate determination equation. The exchange rate is endogenously determined, on which more below (sect. 4).

Although the model is a closely interlocked system of simultaneous differential equations, the following causal links may be singled out. Their description also allows a better understanding of the view of the economy which underlies the model itself. Let us start with the real side.

The growth process is both export–led and expectations–led. Given foreign demand and prices, real exports grow according to domestic competitiveness and to supply constraints. Export growth enhances output growth which in turn modifies real expectations and, consequently, real capital formation. Output growth also influences real imports, aggregate public consumption, direct taxes and the level of private consumption (through the determination of disposable income) which feed back on output. Changes in inventories, whose desired level is linked to expected output, act as a buffer in output determination.

The performance of real aggregates is also deeply influenced by money and by price behaviour. The latter depends on cost–push (including exchange rate effects) and monetary mechanisms as well as on expectations. Prices also enter into the determination of financial variables whose behaviour is closely connected with that of real variables.

A central place in the model is occupied by money and credit. As regards the former, monetary disequilibria influence consumption demand, the interest rate, and price expectations. As regards credit, its expansion, as determined by the behaviour of banks, influences real capital accumulation as well as exports of goods and services.

An important role is also played by the rate of interest, for it influences the demand for money (and hence real consumption), credit expansion, and the accumulation of net foreign assets. The rate of interest is determined by both market forces and policy intervention.

Policy actions are represented by policy reaction functions for the money supply, the interest rate (in part), and international reserves.

In sum, our model stresses real and financial accumulation in an advanced open economy in which aggregate demand and supply on the one hand and liquidity (i.e. money and credit availability) on the other play crucial roles, together with expectations.

From the formal point of view, the model is specified in logarithmic form for the analytical reasons explained in Gandolfo (1981, sect. 2.2.3). Let us now come to the *qualitative properties* of the model. One of the features of the continuous time approach, as stressed in Gandolfo (1981), is that it is not only a method of estimation, but an integrated approach which also involves the study of the qualitative properties of the model according to the methods in use in mathematical economics. Thus it can be shown (Gandolfo and Padoan, 1990) that the model has an analytical steady state with standard (neoclassical) properties and with economically sensible comparative dynamics results. Naturally, since the steady state solution turns out to be stable, the economy will move towards the steady state itself. But, given the relatively long adjustment lags (relatively low adjustment speeds: see the estimates of the α parameters in Table 1 below) we never actually observe the steady state results. This is very important, because it shows that it is wrong to assume long–run equilibrium conditions to hold in the actual course of events when a model is used for descriptive purposes and estimation.

Finally, the study of local stability and sensitivity (by sensitivity analysis we mean the study of the partial derivatives of each eigenvalue with respect to each parameter) requires the application of the Liapunov–Perron–Poincaré theorem. According to this theorem the usual procedure of expanding in Taylor's series (and neglecting all higher–order terms) is valid if and only if the resulting linear model is a *uniformly good* approximation to the original nonlinear model. When the model is in log form, it turns out that the usual procedure does yield a uniformly good approximation. The results of our analysis are given in Gandolfo and Padoan (1990).

We would now like to stress that the present paper is *not* aimed at discussing the economics of model (this has been extensively done elsewhere, see Gandolfo and Padoan, 1990), but at presenting the results of the nonlinear vs linear estimation. Since our experience shows that readers (especially those who are unfamiliar with continuous time models) who simply look at the equations of the model may form an incorrect view of the underlying economics, we strongly urge any reader who is interested in the model as such to read our 1990 paper (Gandolfo and Padoan, 1990) before examining the remainder of the present paper. Here we shall, however, give a brief explanation of the equations where the nonlinearities of the model are present (Sect. 4) and where the main differences in estimation results are to be found (Sect. 5).

3. The linearized model

The (log)linear version of the model was obtained linearizing the nonlinear equations around the sample means. This version was estimated with a sample of quarterly data (running from 1960 to 1984 inclusive) using Wymer's program RESIMUL, which gives FIML estimates of the parameters. The estimated version was then subjected to extensive validation. In addition to the standard procedures (Carter–Nagar system test, standard errors of the parameters, in–sample and out–of–sample dynamic forecasts) we examined the stability properties of the model by computing the eigenvalues of the linear approximation about the steady state. It is interesting to note that the model has both real and complex roots, thus giving rise to a cyclical behaviour around the steady state, which is stable. Hence the model shows a cyclical growth behaviour, which is consistent with the stylized facts of the Italian economy in the period under consideration.

Since the eigenvalues come out of a matrix whose elements depend on the estimated parameters, it is also possible to compute the standard errors of the real and imaginary parts of the eigenvalues. Thus it is possible to check whether stable eigenvalues are indeed significantly negative. The computation of the partial derivatives of each eigenvalue with respect to each parameter (sensitivity analysis) is important for several reasons, amongst which the fact that it immediately shows the crucial parameters, i.e. those which have an important effect on stability (either stabilizing or destabilizing).

All this study of the model (reported in Gandolfo and Padoan, 1990) convinced us that it was a reasonably good model, and hence that we could use it for forecasting and policy simulations. It should be pointed out that, in general, once the estimates of the parameters are obtained through the linear approximation, these values can be substituted in the original nonlinear formulation, which can then be used for any kind of exercise. Any exercise thus amounts to solving a nonlinear differential system numerically, which we did using Wymer's program APREDIC.

The two studies which have shown the usefulness of the model and, in general, of the continuous time approach, are (i) out–of–sample forecasts of the lira/dollar exchange rate and (ii) the simulation of the consequences of the liberalization of capital movements in the Italian economy which was to take place in 1990.

As regards point (i), the poor out–of–sample predictive performance of all the standard structural models of exchange rate determination is well known after the studies of Meese and Rogoff (1983a,b), subsequently confirmed by several other studies. These models failed to outperform the random walk in out–of–sample forecasting experiments even when the actual values of the exogenous variables were used (ex post forecasts). In various papers (e.g. Gandolfo et al., 1990a,b; 1993) we repeated the Meese and Rogoff exercise as regards the lira/dollar exchange rate and found similar results: all of the standard structural models failed to outperform the random walk in out–of–sample ex post forecasts, even when we used error–correction specifications and time–varying–coefficients regressions. However, when we used our model to perform a similar exercise, we obtained forecasts vastly superior to those obtained through the random walk. This is important not only for our model but also for economic theory in general, for it shows that economic theory, after all, is useful.

As regards point (ii), it is a perfect example of the importance of sensitivity analysis. In our model, international capital flows are modelled according to the portfolio approach, whereby they are given by the adjustment of the actual to the partial equilibrium stock of net foreign assets. The adjustment speed reflects the presence of controls on these flows: high controls give rise to a low adjustment speed, while in the absence of controls this speed tends to infinity. The low value of the estimated adjustment speed was perfectly consistent with the presence of capital controls in the Italian economy in the sample period. Now, the partial derivative of one eigenvalue with respect to α_{24} (the parameter that denotes the adjustment speed of the actual to the desired flow of net foreign assets) turned out to be positive and particularly large in relative terms (on the notion of "sufficiently small" and "particularly large" in sensitivity analysis see Gandolfo, 1992). This pointed out to a possible destabilizing effect of capital liberalization in the Italian economy, *ceteris paribus*. In a series of papers (e.g. Gandolfo and Padoan, 1988; 1990a,b; 1991) written well before the phenomenon took place, we used simulation analysis to show that full capital liberalization would cause a major exchange–rate crisis after a period of 2–3 years of apparent stability, if no offsetting measures (which were indicated in our papers) were taken. No such measures (amongst which we had indicated, and simulated, the now *a' la mode* Tobin tax) were taken, and after a period of about two years and a half of apparent stability (which led the monetary authorities to believe that all was going well) we had the crisis of the lira in September 1992, precisely two years and three quarters after the liberalization (for a summing up see Gandolfo and Padoan, 1992).

These results confirm the validity of the model. Although we are envisaging a MARK VI version with many improvements, we feel that the nonlinear estimation of the current version might give useful insights in both the model's structure and the feasibility of the estimation program. Before turning to the estimation results, however, we would like to give a brief overview of the nonlinearities in the model, that warrant a nonlinear estimation.

4. The nonlinearities of the model

Since the model was built with a view to the then available linear estimation procedure, most of the equations are linear in the logarithms. There are, however, significant nonlinearities in eqs. (1), (16), (17), (20) and (24), that require a detailed explanation.

In eq. (1), real private consumption, C, adjusts to its desired level, $\hat{C}$, which is given by the average propensity to consume (which is variable according to several effects) applied to real disposable income, $(Y-T/P)$. The second term in the equation represents the effects of monetary disequilibrium on consumption. Given the logarithmic formulation of the equation, the term $\log(Y-T/P)$ is nonlinear in the logarithms.

Eq. (16) is the monetary authorities' reaction function on international reserves, and is quite complicated because of the change in regime (from fixed to floating

exchange rates) which occurred in the sample period. This equation has been specified to reflect the regime change, but also contains elements which are independent of the regime in force and which we may call permanent elements in the monetary authorities' behaviour. Let us begin with the permanent elements, which are the leaning–against–the–wind policy and the desired reserve ratio. The Italian monetary authorities have generally followed a policy of leaning against the wind (even during the Bretton Woods era), to smooth out the path of the exchange rate and/or to prevent excessive fluctuations. This policy implies that international reserves move in the opposite direction to the exchange rate, hence the term $-\delta_7 \mathrm{Dlog} E$. As regards the second permanent element, The Bank of Italy has always paid a great deal of attention to the months of financial covering of imports, that is how long the current flow of imports can be maintained if the existing stock of reserves are used up for this purpose: this implies the existence of a desired ratio (γ_{15}) of R to the value of imports. Thus the term $\delta_8 \log(\hat{R}/R)$ can be interpreted as a partial adjustment equation of R towards $\hat{R}$.
Let us now come to the elements relating to the exchange–rate regime. During the Bretton Woods era, there was an obligation to maintain the exchange rate within $\pm 1\%$ of the official parity E_c. Therefore when the actual exchange rate tended to exceed the parity, international reserves were used up (and vice versa);hence the term $b\delta_5 \log(E_c/E)$, where b is a variable which takes on the value 1 during the Bretton Woods era and zero subsequently. During the managed float regime, one constant concern of the monetary authorities has been the competitiveness of domestic goods in foreign markets, which is measured by the ratio $(PXGS/E\cdot PF_f)$. Therefore, assuming a competitiveness target γ_{14}, and given $PXGS$ and PF_f we can define a target exchange rate $\hat{E}$ as that exchange rate which fulfills the competitiveness target, hence $\hat{E} = PXGS/\gamma_{14}PF_f$ International reserves are used to guide the actual exchange rate towards its target value (we could also see this as a form of PPP rule), hence the term $(1-b)\delta_6 \log(\hat{E}/E)$. Other terms could have been included, but we preferred not to burden an already overcrowded equation with additional terms.
The presence of the multiplicative switching variable b is an important nonlinear feature of the model.

Equations (17), (20) and (24) are definitional equations, linear in the natural values but nonlinear in the logarithms.
Eq. (17) defines the change in inventories (DV) as a residual in the goods market. Eq. (20) defines the change in the public sector borrowing requirement (DH). Finally, eq. (24) is the balance–of–payments definition. This last equation plays a fundamental role in our model: that of determining the exchange rate.

In order to put this point into proper perspective, we introduce the distinction between models where there is a specific equation for the exchange rate and models where the exchange rate is implicitly determined by the balance–of–payments equation (thus the exchange rate is obtained by solving out this equation). From the mathematical point of view the two approaches are equivalent, as can be seen from the following considerations.

Let CA denote the current account, NFA the stock of net foreign assets of the private sector, R the stock of international reserves. Then the balance–of–payments equation simply states that

$$\mathrm{CA} + \Delta\mathrm{NFA} + \Delta\mathrm{R} = 0, \qquad \text{(a)}$$

Introduce now the following functional relations:

$$\mathrm{CA} = \mathrm{f}(\mathrm{E},...), \qquad \text{(b)}$$

$$\Delta\mathrm{NFA} = \mathrm{g}(\mathrm{E},...), \qquad \text{(c)}$$

$$\Delta\mathrm{R} = \mathrm{h}(\mathrm{E},...), \qquad \text{(d)}$$

$$\mathrm{E} = \varphi(...), \qquad \text{(e)}$$

where E is the exchange rate and the dots indicate all the other explanatory variables, that for the present purposes can be considered as exogenous. Given that system (a)–(e) contains five equations in four unknowns, we can either drop equation (e) and use equation (a) to determine the exchange rate, or keep eq. (e) and drop, say, eq. (c) or eq. (d); then eq. (a) can be used to determine the capital movements balance (ΔNFA) or the reserve change (ΔR) residually.

It should be stressed that if one uses the balance–of–payments definition to determine the exchange rate one is not necessarily adhering to the traditional or "flow" approach to the exchange rate, as was once incorrectly believed. A few words are in order to clarify this point. If one follows this approach one is simply using the fact that the exchange rate is determined in the foreign exchange market, which is reflected in the balance–of–payments equation, under the assumption that this market clears instantaneously (as it actually does, if we include the monetary authorities' demand or supply of foreign exchange as an item in this market; in our approach this item is given by eq. (d), which defines the monetary authorities' reaction function). In fact, no theory of exchange–rate determination can be deemed satisfactory if it does not explain how the variables that it considers crucial (whether they are the stocks of assets or the flows of goods or expectations or whatever) actually translate into supply and demand in the foreign exchange market which, together with supplies and demands coming from other sources, determine the exchange rate. When all these sources —including the monetary authorities through their reaction function on the foreign exchange market, eq. (d)— are present in the balance–of–payments equation, this equation is no longer an identity, but becomes a market–clearing condition. Thus it is perfectly legitimate (and consistent with any theory of exchange–rate determination) to use the balance–of–payments equation to calculate the exchange rate once one has specified behavioral equations for <u>all</u> the items included in the balance of payments.

A final observation: the balance–of–payments equation –eq. (a) above, which corresponds to eq. (24) in the model– is a *nonlinear* implicit function in the exchange rate, which then turns out to be a *nonlinear* function of all the other endogenous variables (implicit function theorem). Here we have a case in which there is a strong theoretical motive for a variable (the exchange rate) to be nonlinear.

5. Results of the nonlinear estimation

The model, after a few manipulations required by the estimation program (see Appendix 2), was estimated using Wymer's ESCONA program for nonlinear estimation (Wymer, 1993a). This requires initial parameter values, which have been set equal to the parameter estimates obtained in the linearized version of the model. The data set is of course the same as that used in the estimation of the linearized model (1960–I to 1984–IV) and fully described in Gandolfo and Padoan (1990, Appendix 2).

The initial estimation runs of the model have been performed on the mainframe (an IBM 3090) of the University of Rome "La Sapienza". Due to the enormous computation (CPU) time required, the estimation was later transferred to an IBM RISC/6000 Unix workstation. The increase in computation time was only about 20%, and thanks to the processor's internal precision, the estimates are much more precise. Every run usually takes several days of uninterrupted and fully dedicated (i.e., the machine is not being used for anything else) computation before obtaining the first stable parameter estimates. For instance, the last run that gave rise to the estimates reported in this paper took about 160 hours after the initialization to produce the Hessian matrix (which took about 50 hours).

The estimation results are presented in Tables 1 and 2. For ease of comparison the estimates obtained through the linearized version (Gandolfo and Padoan, 1990) are also given in these tables. All parameters are, of course, more or less different. If we adopt $\pm 2\sigma$ intervals around the linear estimates, about half (52%) of the parameters are different; this proportion however falls to 42% if we adopt a higher confidence interval ($\pm 3\sigma$). At any rate, given that these intervals are only asymptotically meaningful, we feel that it is more important, to extract some economic sense from the numbers, to concentrate on those equations where major differences exist. By major differences we mean those that entail the loss of significance of a previously significant parameter and vice versa, or a significant change in a parameter's sign, and hence a qualitatively different economic explanation. We shall also examine minor differences, i.e. where the parameter sign does not change, but the numerical difference is important from the economic point of view. It should be noted that, since most γ's are simply scale parameters, their numerical differences are generally to be considered as not relevant.

Table 1: Estimated Adjustment Parameters

Parameter	Entering equation number	Linear Estimates: Point estimate (a)	Linear Estimates: Asymptotic standard error (b)	Linear Estimates: (c) = (a) / (b)	Non-Linear Estimates: Point estimate (a)	Non-Linear Estimates: Asymptotic standard error (b)	Non-Linear Estimates: (c) = (a) / (b)
α1	(1)	1.147	0.214	5.37	2.181	0.227	9.60
α2	(1)	0.114	0.041	2.77	0.363	0.013	28.67
α3	(2)	1.003	0.178	5.64	0.109	0.023	4.74
α4	(2)	0.112	0.014	8.14	0.137	0.043	3.17
α5	(4)	0.500	0.157	5.90	1.035	0.059	17.43
α6	(4)	0.329	0.125	2.62	0.094	0.033	2.82
α7	(4)	0.021	0.010	2.05	0,000a		
α8	(5)	0.869	0.142	6.09	0.917	0.101	9.12
α9	(5)	0.618	0.219	2.81	-5.941	0.796	7.46
α10	(6)	1.879	0.273	6.88	3.367	0.020	172.04
α11	(6)	0.573	0.058	9.88	0.462	0.002	270.88
α12	(7)	0.493	0.111	4.44	0.105	0.005	22.92
α13	(7)	0.313	0.114	2.75	0.382	0.004	97.65
α14	(7)	0.122	0.058	2.07	0.017	0.003	5.66
α15	(8)	0.340	0.113	3.02	0.359	0.026	13.99
α16	(9)	0.383	0.103	3.71	0.644	0.180	4.29
α17	(10)	0.030	0.007	4.17	-9.73E-06	1.24E-04	0.08
α18	(10)	0.073	0.003	25.68	-0.004	0.004	1.20
α19	(10)	0.181	0.007	27.13	0.052	0.005	9.92
α20	(10)	0.049	0.005	13.78	-0.059	0.003	17.38
α21	(10)	0.085	0.004	21.73	-0.008	0.002	4.39
α22	(11)	0.140	0.026	5.35	0.036	0.005	7.37
α23	(11)	-2.904	0.516	5.63	2.425	0.398	6.09
α24	(12)	0.091	0.032	2.84	0.059	0.002	25.09
α25	(12)	-0.182	0.029	6.15	-0.124	0.010	12.07
α26	(13)	4.095	0.938	4.36	4.735	0.020	239.89
α27	(14)	0.155	0.055	2.81	0.076	0.059	1.29
α28	(15)	0.401	0.110	3.61	0.638	0.039	16.42
α29	(15)	-3.410	0.654	5.21	-4.753	0.216	22.00
α'	(2)	0.059	0.004	15.71	0.027	0.005	5.33
η	(3)	0.083	0.025	3.27	0.106	0.004	28.20

a = value imposed.

Table 2: Other Estimated Parameters

Parameter	Entering equation number	Linear Estimates: Point estimate (a)	Asymptotic standard error (b)	(c) = (a) / (b)	Non-Linear Estimates: Point estimate (a)	Asymptotic standard error (b)	(c) = (a) / (b)
β1	(1)	-1.296	0.299	4.33	2.205	0.009	232.29
β2	(1)	0,000a			0,000a		
β3	(1),(7),(10)	2.237	0.968	2.31	-0.330	0.641	0.52
β4	(1),(7),(10)	1.135	0.109	10.43	1.025	0.059	17.50
β5	(1),(7),(10)	1.480	0.164	9.01	1.245	0.016	77.51
β6	(4)	0.672	0.140	4.81	0.522	0.021	24.74
β7	(4)	0.547	0.101	5.41	0.413	0.021	20.13
β8	(4)	1.166	0.158	7.38	1.497	0.006	251.32
β9	(5)	0.378	0.082	4.58	0.985	0.163	6.03
β10	(5)	0.500	0.128	5.14	0.465	0.015	30.94
β11	(5)	0.945	0.112	8.49	1.125	0.032	35.55
β12	(7)	0.330	0.047	6.94	0.432	0.045	9.64
β13	(7)	0.501	0.046	10.87	0.510	0.019	26.82
β14	(7)	0,000a					
β15	(8)	0.480	0.081	5.92	0.133	0.052	2.58
β16	(8)	0.404	0.070	5.79	0.811	0.037	22.20
β17	(9)	0.629	0.041	15.45	0.816	0.031	25.98
β18	(11)	-2.340	0.294	7.96	-5.587	2.104	2.66
β19	(12)	9.026	3.281	2.75	6.171	0.063	93.66
β20	(12)	1.000a			1.000a		
β21	(12)	1.000a			1.000a		
β22	(14)	1.101	0.048	22.92	1.000a		
λ4	(9)	0.019	0.001	16.94	0.012	4.46E-04	27.97
δ1	(13)	0.103	0.049	2.08	0.110	0.001	96.64
δ2	(13)	-0.092	0.080	1.14	-0.081	7.35E-04	110.60
δ3	(13)	0.367	0.081	4.51	0.003	0.002	1.79
δ4	(13)	0.065	0.031	2.02	0.092	7.52E-04	122.10
δ5	(16)	-0.390	0.376	1.04	0,000a		
δ6	(16)	-0.126	0.200	0.63	0.333	0.042	8.02
δ7	(16)	0.927	0.188	4.92	1.861	0.028	67.44
δ8	(16)	0.282	0.038	7.39	0.043	0.004	12.07
γ1	(1)	0.764	0.005	152.72	0.939	0.439	2.14
γ2	(1),(7),(10)	0.0006	0.0007	0.83	0.033	0.002	16.79
γ3	(2)	6.011	0.288	20.84	2.622	0.080	32.77
γ4	(4)	0.089	0.107	0.82	0.005	5.50E-04	90.84
γ5	(4),(6)	1.009	0.061	16.60	1.581	0.050	31.66
γ6	(5)	0.199	0.197	1.01	9.894	0.240	41.28
γ7	(7)	1.264	0.226	5.58	1.023	0.485	2.11
γ8	(8)	1.061	0.022	48.88	1.084	0.113	9.58
γ9	(9)	0.742	0.017	42.51	0.727	0.026	27.63
γ10	(11)	0.0095	0.0004	20.67	266.400	11.809	22.56
γ11	(12)	0.00036	0.00007	4.95	omitted		
γ12	(14)	0.134	0.048	2.80	0.173	0.016	10.73
γ13	(15)	0.252	0.007	33.94	0.334	0.009	38.71
γ14	(16)	1,000a			0.832	0.153	5.44
γ15	(16)	1.147	0.213	5.37	1.695	0.249	6.81

a = value imposed.

5.1 *Major differences*

5.1.1 *Interest rate*

The most important differences pertain to equation (10), the interest rate determination equation. For the reader's convenience we transcribe the equation here:

$$\mathrm{Di}_{TIT} = \alpha_{17}\log(\mathrm{M_d/M}) + \alpha_{18}[\mathrm{i_f}+\log(\mathrm{FR/E}) - \mathrm{i}_{TIT}] + \alpha_{19}\mathrm{DlogE} + \alpha_{20}\mathrm{Dr} + \alpha_{21}\mathrm{Dh}.$$

The interest rate equation is one of the most crowded equations in the model, reflecting the hybrid nature of this variable in the Italian economy in the sample period. The first effect is domestic and is represented by the excess demand for money, that can be seen either in in the traditional textbook context or in the context of the buffer stock notion discussed below (Sect. 5.1.4). The second term is related to the well–known interest–rate–parity conditions (see, for example, Gandolfo, 1995, Sect. 10.7 and Gandolfo and Padoan, 1990, pp. 100–101). Since in the Italian economy in the sample period the mobility of capital was far from perfect, a discrepancy between the two sides of the IRPC gives rise to a limited amount of capital flow –see Eq. (12)– and hence to a tendency for the domestic interest rate to move to close this discrepancy. The speed of adjustment is measured by a_{18} and depends, *inter alia*, on the authorities' control on capital movements. The term under consideration therefore also reflects policy considerations.
The third term, a_{19}DlogE, is the reflection on the interest rate of the monetary authorities' intervention to smooth the behaviour of the exchange rate and has to be seen in conjunction with the analogous term in Eq. (16). When the authorities follow a policy of leaning against the wind, they use both direct intervention in the foreign exchange market and interest–rate changes to pursue their goal. If, for example, they wish to contrast a devaluation in the exchange rate (DlogE $>$ 0), they will both consume international reserves [hence the term $-\delta_5$DlogE in Eq. (16)] and tend to increase the interest rate to favour capital inflows (or check capital outflows).
Similar considerations underlie the fourth term, α_{20}Dr: if, in the case under consideration, reserves are being used up at an increasing rate (Dr $>$ 0), the increase in the interest rate will tend to be greater, *ceteris paribus*. If, on the contrary, reserves accumulate at an increasing rate (Dr $>$ 0 as well, but DlogR being positive rather than negative), the interest rate can be decreased. Thus α_{20} can be either positive or negative on *a priori* grounds.

The fifth (and last) term in Eq. (10) represents the influence of the rate of change of the public deficit, a proxy for the acceleration of the stock of public debt. In the sample period what seems to have really mattered in Italy for institutional reasons is not the ratio of public debt to GDP or the rate of change of public debt, but the *acceleration* of public debt, whose effect on the rate of interest can be twofold. On the one hand, the higher this acceleration, the higher the interest rate that the authorities have to offer –*ceteris paribus*– to convince the public to buy an increasing amount of government bonds ($\alpha_{21} > 0$). On the other hand, when the stock of public debt is accelerating, the authorities try to decrease the interest rate to reduce the burden of interest payments, hence $\alpha_{21} < 0$.

Let us now come to the empirical results. As we can see from Table 1, α_{17}, the parameter associated with the excess demand for money, and α_{18}, the parameter associated with the interest rate differential are now not significantly different from zero. Both α_{20} and α_{21}, the parameters associated with the rate of change of international reserves and with the rate of change of the public sector's borrowing requirement, now show a significantly negative sign. The only parameter that has remained qualitatively unchanged is α_{19}, associated with the rate of change of the exchange rate. While the change in the sign of α_{20} and α_{21} is consistent with the economic interpretation of the equation (see above), the disappearance of any effect of the excess demand for money as well as of the interest rate differential is clearly due to data problems. In fact, the domestic interest rate that should be used to be consistent with the theory is the short–term interest rate (i_f is the short–term US interest rate). However, due to data limitations at the time of the linear estimates, the long–term Italian government bond yield was used as a proxy for the domestic interest rate. Hence, what the estimation results really show is that this proxy does function in a sufficiently small neighbourhood (the linear approximation) but does not when the exact (nonlinear) structure is used. In future versions of the model we shall certainly use more appropriate data.

5.1.2 *Exports*

Another equation in which one major difference appears is eq. (5), the export function:

$$\mathrm{DlogXGS} = \alpha_8 \log(\hat{\mathrm{XGS}}/\mathrm{XGS}) - \alpha_9 \mathrm{Da}, \qquad \text{where}$$
$$\hat{\mathrm{XGS}} = \gamma_4(\mathrm{PXGS}/\mathrm{PF_f}\cdot \mathrm{E})^{-\beta_2} \mathrm{YF}^{\beta_{10}} (\gamma_3 \mathrm{Y}/\mathrm{K})^{-\beta_{11}}.$$

In this equation, real exports of goods and services adjust to their partial equilibrium level determined by foreign demand for exports and by a supply constraint. Foreign demand, in turn, depends on relative prices and on on world income. Given the effect of foreign demand, the partial equilibrium level of exports also reflects the influence of domestic supply represented by the deviations of the output/capital ratio from its desired value ($1/\gamma_3$); these can be considered as a proxy for for the degree of capacity utilization. In other words, as the utilization of productive capacity increases, the negative impact of a supply constraint will be felt on exports.

The inclusion of the variation of the rate of change of bank advances is due to the observation that, in the sample period, when a credit squeeze occurred, Italian producers tried to increase the expansion of exports. The reason is that by so doing they could circumvent the credit squeeze which did not hold for export credits. Since $\mathrm{D}a = \mathrm{D}^2\log\mathrm{A}$ is the acceleration of bank advances, a positive value of $\mathrm{D}a$ is consistent with both a restrictive ($\mathrm{DlogA} < 0$) and an expansionary ($\mathrm{DlogA} > 0$) credit situation, hence α_9 can be either positive or negative on a priori grounds.

As we can see from Table 1, α_9 has changed sign, becoming significantly negative. This is consistent with the theory and confirms the existence of the effect that $\mathrm{D}a$ was meant to capture.

Let us also note another difference that, although minor according to our classification, pertains to the equation under consideration and hence is better examined here rather than in section 5.2. This pertains to the price–elasticity of exports, β_9, that has increased considerably. This shows that elasticity estimates by a linear approximation may be far from the true nonlinear estimates. Given the paramount importance of foreign trade elasticities (think, for example, of the Marshall–Lerner condition) and the fact that actual relative–price changes due to exchange–rate swings may be far from "sufficiently small", the possible unreliability of the usual (linear) estimates of these elasticities should be a matter of concern in all policy discussions of exchange–rate management and regimes.

5.1.3 *Monetary authorities' reaction function on international reserves*

A third equation where we find one major difference is equation (16). Since this equation has already been commented on in detail (see Sect. 4), we can examine the estimation results directly. From Table 2 we see that δ_6 (the parameter related to the management of the exchange rate under floating exchange rates), that was not significantly different from zero in the linear estimates, is now significantly positive, in conformity with our *a priori*. It has however not been possible to improve the situation as regards δ_5, which was, and remains, not significantly different from zero (in the nonlinear estimates it has actually been constrained to zero since in earlier stages of estimation it was never significantly different from zero, and the constraint improved the efficiency of the estimates).

Minor differences in this equation concern δ_7 and δ_8, whose numerical values suggest a greater importance of the leaning–against–the–wind policy and a smaller importance of the desired level of international reserves.

5.1.4 *Consumption function*

One major difference is also present in equation (1), the consumption function:

$$\mathrm{DlogC} = \alpha_1 \log(\hat{C}/C) + \alpha_2 \log(M/M_d), \qquad \text{where}$$

$$\hat{C} = \gamma_1 e^{\beta_1 \mathrm{DlogY}} (P/PMGS_f \cdot E)^{\beta_2} (Y - T/P), \qquad \beta_1 \gtrless 0,\ \beta_2 \gtrless 0.$$

In this equation, real private consumption adjusts to its desired level $\hat{C}$ which is given by the average propensity to consume applied to real disposable income. The propensity to consume, in turn, is a function of other variables. First, it may vary over the trade cycle either procyclically ($\beta_1 > 0$) or anticyclically $\beta_1 < 0$); this second possibility arises if a ratchet effect is operative. Second, in an open economy it may vary with the terms of trade (the Laursen and Metzler effect, that has an uncertain sign when both domestic and import prices vary at the same time: see Gandolfo and Padoan, 1990).

The second term in the equation represents the effect of monetary disequilibrium on consumption. This is an important issue and is related to the role of money as a buffer stock. As shown in Gandolfo and Padoan (1990, p. 96), this role implies that monetary disequilibrium (a discrepancy between actual and desired cash balances) will induce the agent to move towards his long–run target demand for money by changing both his consumption demand (hence the term under consideration) *and* the demand for bonds (hence the monetary disequilibrium term in the interest–rate equation, see above). To complete the summary of the role of money in the model, the importance of monetary disequilibrium in the inflationary process should be noticed. Suppose that we have a price equation of the type

$$\mathrm{DlogP} = f(\ldots) + c\pi$$

where the specification of the arguments of f needs not concern us here, and π represents inflationary expectations. If we hypothesize that inflationary expectations depend on the excess supply of money, $\pi = e\log(M/M_d)$, we get

$$\mathrm{DlogP} = f(\ldots) + \alpha\log(M/M_d), \qquad \alpha = ce$$

which is the formulation used in Eq. (7), that will be discussed below (Sect. 5.2).

Let us now come to the estimates. The difference consists in the fact that, while the linear estimate showed a significant anti–cyclical behaviour of the propensity to consume ($\beta_1 < 0$), the nonlinear estimate points to the opposite result of a significant pro–cyclical behaviour ($\beta_1 > 0$). Both results are, of course, consistent with the theory of the consumption function, as shown by the fact that in writing the theoretical model we allowed for $\beta_1 \gtrless 0$.

5.1.5 *Money demand*

Finally, one major difference is also to be found in the ubiquitous demand–for–money function (see above, on the role of monetary disequilibrium in the model):

$$M_d = \gamma_2 e^{-\beta_3 i_{TIT}} P^{\beta_4} Y^{\beta_5}, \quad \beta_3 \gtrless 0.$$

Traditional demand–for–money theory requires the sign of the partial derivative of money demand with respect to the interest rate to be negative. Hence in our specification the interest–rate semielasticity (β_3) should be negative. In our case, however, since the variable to be employed to represent the money stock (M_2) includes bank deposits whose demand is positively related to the rate of interest on these deposits (a rate that in Italy was closely related to i_{TIT}), the sign of β_3 could be either positive or negative on a priori grounds. The difference between the linear and nonlinear estimates shows in fact a significant change in the sign of β_3, that from positive becomes negative.

Let us now turn to the minor differences, excluding of course those that have already been examined in conjunction with the major ones.

5.2 *Minor differences*

These mainly concern the price–wage sector [Eqs. (7), (8), (9)]. In Eq. (7), i.e.

$$D\log P = \alpha_{12}\log(\hat{P}/P) + \alpha_{13}Dm + \alpha_{14}\log(M/M_d),$$

$$\hat{P} = \gamma_7 (PMGS_f E)^{\beta_{12}} W^{\beta_{13}} PROD^{\beta_{14}},$$

the domestic price level P adjusts towards a partial equilibrium level $\hat{P}$ which is basically determined by cost–push factors; but the easiness of monetary conditions and expectations also play a crucial role. The determinants of $\hat{P}$ are both domestic and foreign. Import prices capture both the effect of an increase in the cost of imported factors of production and a possible foreign competitiveness effect, which induces domestic producers to take into account the prices of of competing imported goods when they determine the prices of their products. The push from domestic costs is represented both by the level of the nominal wage rate W and, with an inverse relationship, by the level of productivity *PROD*.

The second and third term in Eq. (7) are both related to monetary factors, but have a completely different meaning and should be kept distinct. Although the basic determinants of $\hat{P}$ are cost push factors, the *speed* at which P adjusts to $\hat{P}$ is an increasing function of the easiness of monetary conditions (as represented by Dm), namely $\alpha_{12} = \varphi(Dm)$, $\varphi' > 0$. A suitable approximation then gives the second term under consideration. Finally, the third term represents the effect of inflationary expectations, as already clarified in Sect. 5.1.4.

Equation (7) has to be seen in conjunction with Eq. (8), the wage equation, for together they represent the wage–price spiral that was operative in the Italian economy in the sample period:

$$\mathrm{DlogW} = \alpha_{16}\log(\hat{W}/W), \qquad \hat{W} = \gamma_9 P^{\beta_{17}} e^{\lambda_4 t}.$$

In this equation, the nominal wage rate adjusts to its partial equilibrium level, which is a direct function of the domestic price level. Institutional factors (such as, for example, a dominant 'social pressure' in Hicks' sense) suggest that target nominal wages exceed the level that would be determined by by domestic price behaviour alone. These factors have been taken into account introducing a trend among the determinants of $\hat{W}$.

The introduction of a separate equation for export prices, i. e.

$$\mathrm{DlogPXGS} = \alpha_{15}\log(\hat{PXGS}/PXGS), \qquad \hat{PXGS} = \gamma_8 P^{\beta_{15}}(PF_f \cdot E)^{\beta_{16}}$$

is justified by the consideration that exporting firms adjust export prices taking account not only of the same elements that enter into the determination and adjustment of domestic prices (though with a different weight, hence the parameter β_{15}) but also of the foreign competition barrier. This is represented by the level of foreign competitors' export prices as defined in the export equation, which has an obvious positive effect on the level of $\hat{PXGS}$.

The estimation results concerning Eqs. (7) and (8) are best examined together. On the one hand, wages show a higher adjustment speed (α_{16} is significantly greater) to the desired level, which in turn is more elastic to domestic prices (β_{17} is significantly greater) and less dependent on exogenous factors (λ_4 is significantly lower). This points to a more dangerous functioning of the price–wage spiral in the sample period as regards the price–to–wage part. On the other hand, prices show a much lower adjustment speed (α_{12} is significantly much smaller) to the cost–push term, one of whose components is the wage rate. This points to a less dangerous functioning of the price–wage spiral. All in all, the price–wage–spiral subsystem remains stable, since the crucial stability condition (see Gandolfo and Padoan, 1990, p.112) $\beta_{13}\beta_{17} < 0$ is still fulfilled.

The estimates concerning the partial equilibrium level of export prices show that its elasticity with respect to the domestic price level (β_{15}) is lower, while the elasticity with respect to foreign competitors' prices (β_{16}) is higher.

A final result that is worth mentioning concerns the investment function:

$$\mathrm{Dk} = \alpha_3[\alpha'\log(\hat{\mathrm{K}}/\mathrm{K}) - \mathrm{k})] + \alpha_4\mathrm{Da}, \qquad \hat{\mathrm{K}} = \gamma_3\tilde{\mathrm{Y}}.$$

The investment function implicit in this equation is an evolution of the capital stock adjustment principle, which in turn is sufficiently general to accommodate any investment theory. The traditional version of this principle states that actual net fixed investment, $I \equiv \mathrm{D}K$, is a function of the difference between the desired capital stock $\hat{K}$ and the actual capital stock K. In our opinion, this difference does not give rise directly to net investment, but rather is the determinant of *desired* investment $\hat{I}$. We then assume that the relevant economic agents adjust the actual rate of growth of the capital stock to the desired rate $\hat{k} \equiv \hat{I}/K$ according to a partial adjustment equation $\mathrm{D}k = \alpha_3(\hat{k} - k)$. A second modification that we introduce is that $\hat{K}$ is related not to current output, but to *expected* output $\tilde{Y}$. This gives rise to the first term in the investment equation (which is very similar to the second–order accelerator developed independently by Hillinger, 1992, Ch. 8). The second term in the investment function is related to the idea that the speed at which $\hat{k}$ adjusts to k is a function of credit conditions (that in the Italian economy have a great influence also on fixed investment), namely $\alpha_3 = \psi(\mathrm{D}a)$. A suitable approximation then gives the term under consideration.

In the nonlinear estimates, both α_3 and α' are significantly lower, thus reinforcing the results obtained with the linearized version, namely that stocks play the role of "order" variables –in the sense of synergetics (see Gandolfo and Padoan, 1990, pp. 92 and 110)– in the model. The value of γ_3 (see above) has decreased from 6.0 to a more reasonable 2.6.

5.3 Forecasting performance

Let us now consider the in–sample predictive performance of the model. Since our purpose is to compare the goodness of fit of the model with the two sets of parameter estimates, we have computed the in–sample single–period forecasts (see Gandolfo and Padoan, 1984, pp. 10–11, for the continuous–time equivalent of the standard discrete–time single–period forecasts).

The results are given in Table 3. Note that H is not present for the reasons explained in Appendix 2. It should be remembered that the variables are expressed in logarithms or as percentages: consequently, the root mean square error gives the average error as a proportion of the actual level of the endogenous variable. In most cases the fit is fairly good, and in a few cases very good. The very large RMSE displayed by the stock of inventories (V) can be justified on the basis of data problems. Data for changes in inventories in Italy are defined as a residual after the main components of national accounts have been computed, given the overall balance constraint. The variable that displays the next largest error is the rate of change of international reserves (r). This is not surprising given that the relative equation has to take account of the shift in the exchange rate regime over the sample period.

Table 3: Root Mean Square Errors of Static Forecasts

Variable	Linear Estimates	Non-Linear Estimates
C	0.03002	0.01892
k	0.00160	0.00122
MGS	0.06692	0.06012
XGS	0.05222	0.05952
Y	0.01605	0.01658
P	0.02720	0.01427
$PXGS$	0.02223	0.02080
W	0.03280	0.02117
i_{TIT}	0.01666	0.00715
A	0.01302	0.01340
nfa	0.08442	0.03545
m	0.01342	0.00957
T	0.04597	0.04669
G	0.04840	0.04704
R	0.04949	0.02674
V	0.29774	0.12047
K	0.01033	0.01340
M	0.00130	0.00087
a	0.01302	0.01340
r	0.15822	0.05350
h	0.00976	0.00832
E	0.09036	0.03867

The RMSEs obtained using the nonlinear estimates are generally better (and in some cases remarkably better) than those obtained using the linear estimates; just in a few cases the results are (only slightly) worse. Hence the nonlinear estimation procedure does provide parameter estimates that yield a closer fit of the model to the data.

We conclude by observing that the model, given the nonlinear estimates, might then be used for out–of–sample forecasting and simulation experiments as those that were carried out using the linear estimates (see above, Sect. 3). This is not, however, the purpose of the present paper. Furthermore, account being taken of the very high computational cost and of the fact that we are envisaging a new version of the model to be estimated with more recent data, we do not feel that such a work would be justified.

6. Conclusion

The nonlinear estimation of our nonlinear model has generally confirmed the qualitative robustness of the model. It has also pointed out, however, some interesting differences, especially as regards the monetary variables. These differences suggest that a nonlinear estimation of a nonlinear model does provide additional value as it can more fully extract the information contained in the data, bringing theory and facts closer together. This is also shown in the improvement in the in–sample performance of the model. It is however a matter of cost–benefit analysis whether this additional value justifies the extraordinarily larger computational time required on the present generation of computers.

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APPENDIX I

TABLE A.1 (Equations of the model)

Private consumption

$$D\log C=\alpha_1\log(\hat{C}/C)+\alpha_2\log(M/M_d), \tag{1}$$

where

$$\hat{C}=\gamma_1 e^{\beta_1 D\log Y}(P/PMGS_f\cdot E)^{\beta_2}(Y-T/P),\ \beta_1\lessgtr 0,\ \beta_2\gtrless 0;\ M_d=\gamma_2 e^{-\beta_3 i}TIT_P^{\beta_4}Y^{\beta_5},\ \beta_3\lessgtr 0, \tag{1.1}$$

Rate of growth in fixed capital stock

$$Dk=\alpha_3[\alpha'\log(\hat{K}/K)-k]+\alpha_4 Da, \tag{2}$$

where

$$\hat{K}=\gamma_3\tilde{Y}, \qquad \gamma_3=\kappa/u, \tag{2.1}$$

Expected output

$$D\log\tilde{Y}=\eta\log(\ddot{Y}/\tilde{Y}), \tag{3}$$

Imports

$$DlogMGS=\alpha_5 log(\widehat{MGS}/MGS)+\alpha_6 log(\widehat{V}/V)+\alpha_7 PCC, \quad (4)$$

where

$$\widehat{MGS}=\gamma_4 P^{\beta_6}(PMGS_f\cdot E)^{-\beta_7}Y^{\beta_8}, \qquad \widehat{V}=\gamma_5\tilde{Y}, \quad (4.1)$$

Exports

$$DlogXGS=\alpha_8 log(\widehat{XGS}/XGS)-\alpha_9 Da, \quad (5)$$

where

$$\widehat{XGS}=\gamma_6(PXGS/PF_f\cdot E)^{-\beta_9}YF^{\beta_{10}}(\gamma_3\tilde{Y}/K)^{-\beta_{11}}, \quad (5.1)$$

Output

$$DlogY=\alpha_{10} log(\tilde{Y}/Y)+\alpha_{11} log(\widehat{V}/V), \quad (6)$$

Price of output

$$DlogP=\alpha_{12} log(\widehat{P}/P)+\alpha_{13} Dm+\alpha_{14} log(M/M_d), \quad (7)$$

where

$$\widehat{P}=\gamma_7(PMGS_f\cdot E)^{\beta_{12}}W^{\beta_{13}}PROD^{-\beta_{14}}, \quad (7.1)$$

Price of exports

$$DlogPXGS=\alpha_{15} log(\widehat{PXGS}/PXGS), \quad (8)$$

where

$$\widehat{PXGS}=\gamma_8 P^{\beta_{15}}(PF_f\cdot E)^{\beta_{16}}, \quad (8.1)$$

Money wage rate

$$DlogW=\alpha_{16} log(\widehat{W}/W), \quad (9)$$

where

$$\widehat{W}=\gamma_9 P^{\beta_{17}}e^{\lambda_4 t}, \quad (9.1)$$

Interest rate

$$Di_{TIT}=\alpha_{17}\log(M_d/M)+\alpha_{18}[i_f+\log(FR/E)-i_{TIT}]+\alpha_{19}D\log E+\alpha_{20}Dr+\alpha_{21}Dh, \qquad (10)$$

Bank advances

$$D\log A=\alpha_{22}\log(\hat{A}/A)+\alpha_{23}Dk, \qquad \alpha_{23}\gtrless 0, \qquad (11)$$

where

$$\hat{A}=\gamma_{10}e^{\beta_{18}i_{TIT}}M, \qquad \beta_{18}\gtrless 0, \qquad (11.1)$$

Net foreign assets

$$D\log NFA=\alpha_{24}\log(\widehat{NFA}/NFA)+\alpha_{25}\log(PMGS_f\cdot E\cdot MGS/PXGS\cdot XGS), \qquad \alpha_{25}<0, \qquad (12)$$

where

$$\widehat{NFA}=\gamma_{11}e^{\beta_{19}[i_f+\log(FR/E)-i_{TIT}]}(PY)^{\beta_{20}}(PF_f\cdot E\cdot YF)^{\beta_{21}}, \qquad (12.1)$$

Monetary authorities' reaction function on money supply

$$Dm=\alpha_{26}(\hat{m}-m)+\delta_3 Dh+\delta_4 Dr, \qquad (13)$$

where

$$\hat{m}=m^*+\{\delta_1[D\log(PY)-(\rho_P+\rho_Y)]+\delta_2 Di_{TIT}\}, \qquad \delta_1\gtrless 0, \quad \delta_2\gtrless 0, \qquad (13.1)$$

Taxes

$$D\log T=\alpha_{27}\log(\hat{T}/T), \qquad (14)$$

where

$$\hat{T}=\gamma_{14}(PY)^{\beta_{22}}, \qquad (14.1)$$

Public expenditure

$$D\log G=\alpha_{28}\log(\gamma_{13}Y/G)+\alpha_{29}D\log Y, \qquad \alpha_{29}\gtrless 0, \qquad (15)$$

Monetary authorities' reaction function on international reserves

$$DlogR=b\delta_5 log(E_c/E)+(1-b)\delta_6 log(\hat{E}/E)-\delta_7 DlogE+\delta_8 log(\hat{R}/R), \quad (16)$$

where

$$\hat{E}=PXGS/\gamma_{14}PF_f, \quad \hat{R}=\gamma_{15}PMGS_f\cdot E\cdot MGS, \quad b=\begin{cases}1 \text{ under fixed exchange rates,}\\ 0 \text{ under floating exchange rates,}\end{cases}$$

Inventories

$$DV=Y+MGS-C-DK-XGS-G, \quad (17)$$

Fixed capital stock

$$DlogK=k, \quad (18)$$

Rate of growth in money supply

$$m=DlogM, \quad (19)$$

Public sector's borrowing requirement

$$DH=PG-T, \quad (20)$$

Rate of growth in international reserves

$$r=DlogR, \quad (21)$$

Rate of growth in bank advances

$$a=DlogA, \quad (22)$$

Rate of growth in H

$$h=DlogH, \quad (23)$$

Balance of payments

$$PXGS\cdot XGS-PMGS_f\cdot E\cdot MGS+(UT_a-UT_p)-DNFA-DR=0. \quad (24)$$

TABLE A.2 (Variables of the Model)

Endogenous

A	= nominal stock of bank advances
a	= proportional rate of growth of A
C	= private consumption expenditure in real terms
E	= lira-dollar spot exchange rate
G	= public expenditure in real terms
H	= public sector borrowing requirement
h	= proportional rate of change of H
i_{TIT}	= domestic nominal interest rate
K	= stock of fixed capital in real terms
k	= proportional rate of change of K
M	= nominal stock of money (M2)
m	= proportional rate of change of M
MGS	= imports of goods and services in real terms
NFA	= nominal stock of net foreign assets
P	= domestic price level
PXGS	= export price level
R	= nominal stock of international reserves
r	= proportional rate of change of R
T	= nominal taxes
V	= stock of inventories in real terms
W	= money wage rate
XGS	= exports of goods and services in real terms
Y	= real net domestic product and income
$\bar{Y}$	= expected real net domestic product and income

Exogenous

E_c	= official lira-dollar parity under fixed exchange rates
FR	= forward exchange rate
i_f	= foreign nominal interest rate
PF_f	= foreign competitors' export price level (in foreign currency)
$PMGS_f$	= import price level (in foreign currency)
PROD	= labour productivity
t	= time
$(UT_a - UT_p)$	= net unilateral transfers, in nominal terms
YF	= real world income

APPENDIX 2

As stated in the text (Sect. 5) the model had to undergo a few manipulations before being fed to the computer for estimation. These are due to some limitations in the current version of the program, that cannot accept higher–order derivatives of the endogenous variables without specifying them as true endogenous, i.e., without specifying for them the relative behavioral equations or definitions.

In the model a number of second–order derivatives are present. In particular, Dk and Dm are explicit in Eqs. (2) and (12), whereas Da, Dr and Dh must be indirectly derived from the specification of the relative first–order derivatives (i.e., Eqs. (11), (16) and (20)). Whereas no problems are encountered for the explicit Dk and Dm, the following strategies have been adopted to circumvent the problem due to the presence of Da, Dr and Dh.

In estimation, Da has been substituted for a since the theory about the influence of credit on capital formation (see eq. 2) and on export dynamics (eq. 5) would allow this replacement (see Gandolfo and Padoan, 1990).

The solution for the case of the other two variables has been instead more complicated. First, (20) has been divided by H and then differentiated in order to obtain explicitly Dh (note that this causes H to disappear from the model as an endogenous variable). Second, DR has been isolated from (24). Next, both sides of the equation have been divided by R in order to obtain r (i.e., $DlogR$) on the left–hand side. Finally, the so–obtained equation has been differentiated to obtain Dr. By so doing, the value of the intercept term γ_{11} cannot be estimated since it disappears in the differentiation of eq. (12).

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