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# TAX POLICY IN A MODEL OF LEISURE, SAVINGS, AND ASSET BEHAVIOUR

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# TAX POLICY IN A MODEL OF LEISURE, SAVINGS, AND ASSET BEHAVIOUR

**Abstract**

Existing qualitative analysis of tax policy in an intertemporal setting has generally ignored at least one of three primary decisions facing households, namely, the choice of leisure, savings and the asset-mix. The present paper posits a simple two-period life-cycle model where all these decisions are endogenous. For simplicity, however, we consider the case where agents possess a fixed skill level, and thus face a constant wage rate (i.e., there is no learning or human capital appreciation). Uncertainty is introduced via the rate of return in the capital market. In this framework, we contrast the allocative effects of alternative broad based tax policies such as the income tax, the consumption tax and their equivalents. We consider both where lump-sum transfers are made available to the household such that either expected utility or the budget balance remains unchanged, and also where individual do bear the revenue loss as well as the pure distortionary consequences of increased taxation.

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## I. Introduction

Choice of leisure, savings and the asset mix are examples of primary decisions (along with say, education and bequests) that each household faces. Traditionally, these have been treated separately in analytical work on taxation (e. g., the text by Atkinson-Stiglitz, 1980). Everyone recognizes the highly interdependent nature of these decisions, yet, to our knowledge, no existing analysis explores the effect of taxation in a framework where these decisions are made jointly. Evidently, this is due to the risk of complicating the model such that it yields little insight. This paper presents a simple two-period life-cycle model where the supply of effort, savings and the choice between safe and risky assets are all endogenous. All savings are for future consumption and hence there are no bequests. For simplicity, we allow work-leisure choice in period one only implying a constant skill level (i. e., no learning). The resulting analysis is tractable, and sheds new light on many issues.

In analyzing the allocative effects of taxation it is important to decide how the revenue collected is spent. Given the ultimate interest in comparing alternative policies, we start off by considering the Hicks compensated tax perturbation whereby lump-sum (non-random) transfers are made to the household such that expected utility remains unchanged. Thus the loss of utility for all taxes compared is set at zero.

The above procedure may be contrasted to a *balanced budget* analysis where, for each tax, the actual revenue collected is returned in a lump-sum (stochastic, where appropriate) manner to the taxpayer.[1] However, under conditions of *certainty*, it can be easily seen that, for small tax changes, this mechanism involves no deadweight loss and the resulting analysis is strictly equivalent to the Hicks compensated process noted above. Given *uncertainty*, there is an important difference. Risk-sharing by the state gets eliminated, as already noted by Gordon (1985), even when, in principle, we have a full loss-offset tax policy. Thus there would indeed be some loss of utility even for small changes in the tax rate. It turns out that the analytics of a balanced budget analysis are easily recoverable from the Hicks compensated exercise; hence we are able to report these additional results

[1] For example, Kotlikoff and Summers (1979) and Gordon (1985) follow this approach.



rather conveniently. Evidently, the utility compensated analysis retains, where relevant, the risk-sharing element which we believe to be an important aspect of the tax code in most countries.[2]

Finally, we move on to discuss the *specific* (or, uncompensated) effects of taxation, where the implicit assumption is that the tax revenue is spent on pure public goods that introduce no new distortion. Most existing analysis of taxation is of this latter variety which would make it somewhat easier for us to compare the results with the literature.[3]

We consider four broad based taxes (namely, wage, consumption, wealth and income) and arrive at several new results of significance. Foremost is the effect of taxation on risk-taking. The standard literature based on simple portfolio choice models (e. g., Atkinson-Stiglitz (1980), Stiglitz (1969) gives the impression that risk-sharing by the state (via loss offset policies) is responsible for the well known result that risk-taking may increase due to increased taxation of (capital) income. With endogenous savings, we need to distinguish between the total demand for the risky asset (i. e. *total* risk-taking) *and* the proportion of savings held in the risky form. Should the latter, call it portfolio riskiness (or, *proportional* risk-taking), be taken to measure risk-taking, we find that it typically rises due to increased taxation. Significantly, however, this need not have anything to do with risk-sharing by the state. Increased wage taxation, for example, leads to a portfolio at least as risky as before. Since wages are non-random, there is a total absence of risk-sharing on the part of the government; the result obtains as taxation lowers the risky asset demand relatively less than savings. This result holds for all preferences described by non-decreasing relative risk-aversion (NDRRA) and for all three types of analysis (namely, utility compensated, balanced-budget and specific).

Total risk-taking, the other measure, is seen to be typically discouraged

[2] It may be argued that the government does not ultimately bear any risk nor should it be willing to do so. In practice, presumably it shifts the risk of random capital income revenue on to other agents by adjusting its other disbursements (or, indeed by borrowing). Whatever the mechanism, the fact remains that an individual facing a capital risk is able, given loss offset tax policies, to transfer part of the risk on to other agents via the state.

[3] Income effects, indeed, cannot be ignored in the discussion of tax reform. A given dollar of revenue would have different income effects depending on whether it is raised as an income tax or as a consumption tax.

by taxation (at the uncompensated, and, *a fortiori*, at the balanced-budget levels). The uncompensated result is contrary to the standard result in the literature (e. g., Atkinson-Stiglitz, 1980, p. 107). Focusing on the balanced-budget analysis, our finding is also in contrast to Gordon (1985) who reported that taxation of income arising out of the corporate sector (again, in a model without leisure choice) had no effect on total risk-taking.

The significance of the inter-linkage of markets becomes immediate once we analyze wage taxation. Increased taxation renders both current consumption and savings (including its risky and safe components) relatively expensive vis-a-vis leisure. Thus a utility compensated tax increase not only allows more leisure, we also witness a lower level of current consumption, savings and risky asset demand. Since all remaining taxes also involve effective taxation of earnings, these distortions are retained throughout which alter the subsequent analysis in a fundamental way. For example, we find that current consumption is discouraged by a Hicks compensated increase in the consumption tax. Absent leisure behaviour, the latter analysis is known to leave the net consumption path unaffected.

Insofar as the specific effects are concerned, the results obtained here for the consumption and income taxes differ significantly from the literature. For constant relative risk-aversion (CRRA) preferences, the income tax has been known to discourage both savings and total risk-taking while the consumption tax leaves them unaffected (Ahsan; 1989, 1990). Given endogenous leisure, we now find that the consumption tax itself encourages both savings and the risky asset demand for plausible values of the CRRA parameter while the income tax discourages these incentives for low to moderate values of the same statistic. Thus, at least in the latter scenario, the preference for consumption over income taxation remains intact if savings incentive is taken as a criterion.[4]

The paper is organized as follows. The underlying choice model is discussed in section II where we present some results on risk-aversion and wealth elasticities relevant to the expanded framework adopted here. Utility compensated and balanced-budget analysis of taxation are performed in section III, while the following section reviews the specific effects. Some

[4] We should note that endogenous labour supply in the context of a model of human capital accumulation may give rise to a different result (as already alluded to, among other, by Hamilton).



concluding remarks are offered in section V. The relatively more technical aspects of the derivation are grouped together in an appendix at the end of the paper.

## II. The Model

In this section we provide a brief description of the choice model in the absence of taxation and discuss some comparative static results relating wealth effects and risk-aversion. As explained in the introduction, individuals choose labour supply ($L$) in period 1 along with the saving ($S$) as well as the asset composition ($a$, $m$) of their portfolio. Second period consumption equals investment proceeds (principal and returns). The budget equations are therefore

(2.1) $$C_1 = \{Y - w(L_o - L) - (a + m)\}$$

(2.2) $$C_2 = [(1 + r)\{Y - w(L_o - L) - C_1\} + az]$$

where $(wL_o + B)$ is denoted by $Y$ ("full income") and $B$ may be taken to represent either a lump-sum component of work compensation or, more commonly, the endowment of non-human capital. $L_o$ is the human capital stock which is exogenous. The rates of return are $r$ on the safe asset, $x$ (with $z = (x - r)$ measuring the net rate) on risky asset and $w$, the wage rate. The random variable $x$ is assumed to possess a well defined distribution function with $x \in [-1, \infty)$.

Agents arrive at life-cycle work, saving and wealth allocation decisions by solving

(2.3) $$\max_{(L, C_1, a)} Eu(L, C_1, C_2) = f(L_o - L) + g(C_1) + Eh(C_2).$$

In a choice framework as general as this, additive separability is necessary for tractability.[5] Throughout the paper we shall assume individual preferences to be guided by decreasing absolute and non-decreasing relative risk-aversion (DARA and NDRRA, respectively) hypotheses. Solving the

[5] Even authors who set up the problem in the general form usually find it necessary to impose conditions of a similar (or often more restrictive) nature when it comes to evaluating comparative static results. See, for example, the paper by Kotlikoff and Summers.

maximization problem where $C_2$ is given by eq. (2.2) leads to the following first-order conditions (FOC's):

$$(2.4) \qquad (-)\, f' + w(1 + r)E(h') = 0$$

$$(2.5) \qquad g' - (1 + r)E(h') = 0$$

$$(2.6) \qquad E(zh') = 0$$

with respect to (wrt) labour supply, current consumption and demand for the risky asset, respectively. These conditions have a straightforward interpretation. The second-order conditions (SOC's) are discussed in the appendix (section A.1) where we find that given DARA and NDRRA preferences, concavity of $f$, $g$ and $h$ functions are sufficient for the former to be satisfied.

Under these preferences, it is well known that both the demand for the risky asset and consumption in each period are normal goods. Adding work-leisure choice, the same conditions guarantee that leisure too is normal (appendix, section A.1).

For the class of NDRRA preferences, it is also possible to obtain some additional restrictions on wealth effects. Even with CRRA, given that there are three distinct parameters (namely, the elasticity of marginal utilities), we do not necessarily assume them to be the same. Evidently, the sharpest results occur when indeed they do coincide. Denoting these elasticities $\theta^R$, $\theta^1$ and $\theta^2$, respectively, for leisure, current and future consumption, the appendix (section A.1) gives a brief outline of the following result.

**Theorem 2.1**

(a) *Given NDRRA preferences,*

$$\eta^R \geq \eta^c \geq \eta^a \geq \bar{\eta}$$

*(with the strict equality holding if CRRA obtains);*

(b) *For the specific case of CRRA,*

*1) should* $\theta^1 = \theta^2 = \theta$,

$$\eta^c,\ \bar{\eta} \gtrless 1 \text{ as } \theta \lessgtr \theta^R;$$

*2) further, if* $\theta = \theta^R$

*(i)* $\eta^c = \bar{\eta} = 1$; *and*

*(ii)* $\left\{(-)\left(\frac{\ell}{1 - \ell}\right)\eta^L\right\} = 1$

*where* $\eta$ *denotes the wealth elasticity (i. e., wrt Y), and the superscript identifies the appropriate choice variable. The elasticity* $\eta^c$, *for example, refers to current consumption and* $\bar{\eta}$ *denotes an arbitrary positive number.*

In the context of preferences described here, it is more reasonable to believe that $\theta^1 = \theta^2$, but they differ from $\theta^R$. The interesting aspect of the above result is that even when $\theta \neq \theta^R$, we still obtain nice bounds on the magnitude of wealth elasticities, and it can be viewed as a new result. Part (b)(2), on the other hand, is a generalization of similar results available in models of pure consumption allocation over time.

Naturally the above result is quite powerful and helps the discussion of the comparative static effects of taxation. Indeed in much of the literature on risk-aversion, the connection between hypotheses on the former and the implied restriction on preferences become evident from the analysis of wealth effects. It is of interest to note, by contrast, that choice under certainty generally lack a theory of wealth effects.

A final remark on the results relating wealth effects is in order. While the actual terms of the simple wealth effects vary from one tax regime to another, it may be easily checked that the elasticity results (namely, theorem 2.1) holds in its exact form for all tax regimes.

## III. Compensated Effects of Taxation

In this section we consider both the utility compensated as well as the balanced budget effects of taxation. Lump-sum compensation in the Hicks sense provide individuals with non-random transfers eliminating income effects, and thus allows us to focus on the substitution effects associated with price distortions created by the tax policy.

A. The wage tax. If a proportional tax on wage at the rate ($t_w$) were to apply also to the lump-sum element, $B$, say measuring the value of fringe benefits, we obtain

$$(3.1) \qquad C_1 = [(1 - t_w)(Y - w(L_o - L)) - (a + m)].$$

Consequently,

$$(3.2) \qquad C_2 = [(1 + r)\ \{(\ 1 - t_w)[Y - w(L_o - L)] - C_1\} + az]$$

The new FOC's are:

$$(3.3) \qquad (-)f' + w(1 - t_w)(1 + r)E(h') = 0;$$

$$(3.4) \qquad g' - (1 + r)E(h') = 0;$$

(3.5) $E(zh') = 0;$

where evidently the $(g')$ and $(h')$ function's are defined over $C_1$ and $C_2$ as given by (3.1) and (3.2). Totally differentiating this system, the coefficient of the changes in the exogenous variables (namely, $t_w$ and $Y$) when transferred on the rhs (with dependent variables arranged on the lhs) appear as follows. From each of (3.3), (3.4) and (3.5), respectively, we obtain

(3.6) $d_1 = \left\{[(f')(1 - t_w)^{-1}] + (w(1 - t_w)(1 + r)^2)[Y - w(L_e - L)]E(h'')\right\}dt_w$

$- [(w)(1 - t_w)^2(1 + r)^2E(h'')]dY;$

(3.7) $d_2 = (-)\left\{(1 + r)^2[Y - w(L_e - L)]E(h'')\right\}dt_w + [(1 - t_w)(1 + r)^2E(h'')]dY;$

(3.8) $d_3 = \left\{(1 + r)[Y - w(L_e - L)]E(zh'')\right\}dt_w - [(1 - t_w)(1 + r)E(zh'')]dY$

From (3.2) and the FOC's it follows that for expected utility to remain constant, the lump-sum transfer required by the wage tax is given by

(3.9) $dY = \{(1 - t_w)^{-1}[Y - w(L_e - L)]dt_w\}$

On substitution of this condition in eqs. (3.6) - (3.8), we obtain the new rhs vector relevant to the analysis of utility compensated effects of wage taxation:

(3.10) $[\{(f')(1 - t_w)^{-1}\} \quad 0 \quad 0\,]'$

These derivatives appear as follows:[6]

(3.11) $\left.\frac{\partial L}{\partial t_w}\right|_{Eu} = \left\{\frac{1}{D(w)}\right\}\left[\left\{(f')(1 - t_w)^{-1}\right\}\left\{g'' E((z)^2h'') + (1 + r)^2(K)\right\}\right] < 0;$

(3.12) $\left.\frac{\partial C_1}{\partial t_w}\right|_{Eu} = \left\{\frac{1}{D(w)}\right\}\left[(w(1 + r)^2)(f')(K)\right] < 0;$

(3.13) $\left.\frac{\partial a}{\partial t_w}\right|_{Eu} = (-)\left\{\frac{1}{D(w)}\right\}\left[(w(1 + r))(f')(g'')E(zh'')\right] < 0;$

where $D(w) < 0$ is the determinantal value of the appropriate Hessian matrix and $(K) > 0$ (as explained in the appendix, section A.1).

[6] A brief outline of all important derivations is given in the appendix.



From (3.1), we also recognize that:

(3.14) $S = (a + m) = [(1 - t_w)(Y - w(L_e - L)) - C_1]$

In view of (3.9)

(3.5) $\left.\frac{\partial S}{\partial t_w}\right|_{Eu} = \left\{w(1 - t_w)\left.\frac{\partial L}{\partial t_w}\right|_{Eu} - \left.\frac{\partial C_1}{\partial t_w}\right|_{Eu}\right\} \gtrless 0,$

which is generally ambiguous. However, substituting (3.11) and (3.12), the above becomes determinate in sign:

(3.16) $\left.\frac{\partial S}{\partial t_w}\right|_{Eu} = \left\{\frac{1}{D(w)}\right\}\left[(w)(f')(g'')E((z)^2h'')\right] < 0$

Clearly the wage tax makes leisure more affordable and hence the labour supply declines (eq. 3.11) which is well known. In the present context, we also observe that along with labour supply, current consumption, saving and even risk-taking are all discouraged by taxation. An intuitive explanation would proceed as follows. Examining the budget equations (3.1) and (3.2), we note that increased leisure affects both current and future consumption negatively. Now, (3.2) may also be restated as

(3.17) $C_2 = [(1 + r)S + az],$

and thus with $C_2$ both its components, risk-taking and aggregate savings, decline. It is interesting to note that savings undergo two opposing forces; increased leisure tends to depress savings directly while reduced current consumption pulls it the other way (see eq. (3.15)). Evidently, the former dominates [compare (3.11) and (3.12)].

Denoting the share of risky assets in the portfolio by $\lambda(= [a/S])$, it would appear that $\partial\lambda/\partial t_w|Eu$ would be ambiguous since both risk-taking and savings decline due to taxation. However, note that

(3.18) $\left.\frac{\partial \lambda}{\partial t_w}\right|_{Eu} = \left(\frac{1}{S}\right)^2\left[S\left.\frac{\partial a}{\partial t_w}\right|_{Eu} - a\left.\frac{\partial S}{\partial t_w}\right|_{Eu}\right],$

and given (3.13) and (3.16)

$$\left.\frac{\partial \lambda}{\partial t_w}\right|_{Eu} = (-)\left\{\frac{(wf')(g'')}{(S)^2D(w)}\right\}\left[(1 + r)SE(zh'') + aE((z)^2h'')\right].$$

Further, in view of $C_2$ [given by (3.17)],

$$\text{(3.19)} \qquad \left.\frac{\partial \lambda}{\partial t_w}\right|_{Eu} = (-) \left\{\frac{(wf')(g'')}{(S)^2 D(w)}\right\} E\{zC_2h''\} \geq 0$$

In the appendix it was seen that $E\{zC_2h''\} \leq 0$ for NDRRA, and hence *proportional* risk-taking cannot decrease due to wage taxation in the present framework.[7] Thus even though the volume of the risky asset and savings both decline, the riskiness does not go down. In effect this is due to the fact that under NDRRA, the relative decline in savings is more severe compared to the demand for the risky asset.

The wage tax therefore, under expected utility compensation, leads to an increased value of utility of leisure, $f$, which, to be fully offset, requires both $g$ and the $E(h)$ functions to decline. To recapitulate the discussion so far, we may state the following:

**Proposition 3.1** *(a) An expected utility compensated increase in the wage tax leads to decreased levels of labour supply, current and future consumption. Since the latter is effected via savings and risk-taking, they both decline as well.*

*(b) The optimal $(C_1/C_2)$ path is also distorted except when $\eta_c^1 = \eta_c^2$* (e. g. CRRA).

*(c) The above tax increase also leads to a portfolio at least as risky as before.*

We should also note that if the tax merely applied to wages (namely, $wL$), the budget equations would modify slightly, but the comparative static results highlighted here appear exactly as above.

**B. The Consumption Tax.** With a proportional tax on consumption in each period (at rate $t_c$), we obtain

$$\text{(3.20)} \qquad C_1 = (1 + t_c)^{-1} \left[Y - w(L_o - L) - (a + m)\right],$$

and

$$\text{(3.21)} \qquad C_2 = \{(1 + t_c)^{-1} \left[(1 + r)\{Y - w(L_o - L)\} + az\right] - (1 + r)C_1\}$$

Combining these two constraints,

$$\text{(3.22)} \qquad \{C_1 + C_2(1 + r)^{-1}\} = (1 + t_c)^{-1} \left[\{Y - w(L_o - L)\} + az\,(1 + r)^{-1}\right];$$

we observe that the tax effectively falls on labour income (inclusive of $B$) as

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[7] Elsewhere this measure was introduced and called *proportional* risk-taking (Ahsan; 1989, 1990).



well as the present value of the net return from risk-taking ("capital gain/loss"). In terms of the effects of taxation, we would thus expect the results of the wage tax to reappear, along with the risk-sharing aspects of the taxation of net return to risk-taking. The latter is known to induce individuals to hold a riskier portfolio, a result originally due to Domar and Musgrave (1944). Since the intertemporal terms of trade remains unaffected, we do not expect any additional distortion in the consumption allocation decision.

Given (3.21), the FOC's are modified only slightly:

$$\text{(3.23)} \qquad -(f') + w(1 + t_c)^{-1}(1 + r)E(h') = 0;$$

$$\text{(3.24)} \qquad g' - (1 + r)E(h') = 0;$$

$$\text{(3.25)} \qquad (1 + t_c)^{-1} E(zh') = 0.$$

When we differentiate the above system, the relevant rhs vector comparable to (3.6) - (3.8), now appears as follows. From each of (3.23) - (3.25), respectively, we have

$$\text{(3.26)} \quad d_1 = \Big[\Big\{(1 + t_o)^{-1}(f') + \Big\{(w)(1 + t_c)^{-3}(1 + r)^2\Big\} \Big[Y - w(L_o - L)\Big]E(h'')$$

$$+ \{(a)(w)(1 + t_c)^{-3}(1 + r)\}E\{zh''\}\Big]dt_c$$

$$(-)\Big[\Big\{(w)(1 + t_c)^{-2}(1 + r^2)\Big\}E(h'')\Big]dY;$$

$$\text{(3.27)} \quad d_2 = (-) \Big[\Big\{(1 + t_c)^{-2}(1 + r)^2\Big\} \Big[Y - w(L_o - L)\Big]E(h'')$$

$$+ \Big\{a(1 + t_c)^{-2}(1 + r)\Big\}E\{zh''\}\Big]dt_c + \Big[\Big\{1 + t_c)^{-1}(1 + r)^2\Big\}E(h'')\Big]dY;$$

$$\text{(3.28)} \quad d_3 = \Big[\Big\{(1 + t_c)^{-3}(1 + r)\Big\}\Big[Y - w(L_o - L)\Big]E(zh'')$$

$$+ \Big\{a(1 + t_c)^{-3}\Big\}E((z)^2h'')\Big]dt_c(-)\Big[\Big\{(1 + t_c)^{-2}(1 + r)\Big\}E(zh'')\Big]dY.$$

From the budget equations and the FOC's, it can be verified that for expected utility to remain unchanged in the face of rising taxes, the required lump-sum compensation is

(3.29) $$dY = \left\{(1 + t_c)^{-1}\right\}\left[Y - w(L_e - L)\right]dt_c$$

Given the modified vector of $d$'s (in light of 3.29), compensated effects themselves are:

(3.30) $$\left.\frac{\partial L}{\partial t_c}\right|_{Eu} = \left\{\frac{1}{D(c)}\right\}\left[\left\{(1 + t_c)^{-3}(f')\right\}\{g''E((z)^2h''\} + (1 + r)^2(K)\right\}\Big] < 0;$$

(3.31) $$\left.\frac{\partial C_1}{\partial t_c}\right|_{Eu} = \left\{\frac{1}{D(c)}\right\}\left[(1 + t_c)^{-4}(w)(1 + r)^2(f')(K)\right] < 0;$$

(3.32) $$\left.\frac{\partial a}{\partial t_c}\right|_{Eu} = \left\{(a)(1 + t_c)^{-1}\right\} - \left\{\frac{1}{D(c)}\right\}\left[\left\{(1 + t_c)^{-3}(w)(1 + r)\right\}\left\{(f'(g'')E(zh'')\right\}\right] \gtrless 0.$$

Savings are defined by (3.20):

(3.33) $$S = [Y - w(L_e - L) - (1 + t_c)C_1],$$

thus (using (3.29) and the above derivation)

(3.34) $$\left.\frac{\partial S}{\partial t_c}\right|_{Eu} = \{S(1 + t_c)^{-1}\} + \left\{\frac{1}{D(c)}\right\}[(w)(1 + t_c)^{-3}(f')(g'')E((z)^2h'')] \gtrless 0$$

where $D(c)$ denotes the determinantal value of the appropriate Hessian (see the expression A3.1 in the appendix).

Interpreting the above results, we note that the impact of the consumption tax on labour supply and current consumption is essentially identical to that of wage taxation. The fact that an expected utility compensated increase in the consumption tax would decrease current consumption is somewhat counter-intuitive. But evidently, such intuition is derived from our familiarity with models of fixed labour supply. Indeed, analyzing the consumption tax in a consumption-saving framework with fixed labour supply, Ahsan (1989) found the compensated effect to be zero for current consumption.

Turning to the effect on risk-taking we find that a positive term, namely the Domar-Musgrave phenomenon, adds ambiguity to the negative effect operating through increased leisure. Note that the risk-sharing via taxation, by itself, yields the effect such that the net of tax amount of the risky asset demand (i.e., $[a/(1 + t_c)]$) remains unchanged due to increased taxation.



On surface, savings appear to be affected by increased consumption taxation much like the risk-taking equation. On a closer examination it appears that risk-sharing is not the explanation. The negative effect evidently is due to the effective taxation of earnings under the consumption tax. The positive effect arises due to the fact that net consumption, absent work-leisure choice, does not change in response to the tax. From (3.33), it follows that

(3.35) $$C_1 = [(Y - w(L_o - L)) - S](1 + t_c)^{-1}.$$

In other words, if $C_1$ were to remain constant, net earnings also remain constant due to expected utility compensation (see 3.29) and thus savings must adjust in the manner of eq. (3.34) (the first term) so that net savings remain unaltered.

From (3.21), it is also seen that

$$\left.\frac{\partial E(C_2)}{\partial t_c}\right|_{Eu} = (-)(1 + t_c)^{-1}E(C_2) + (1 + t_c)^{-1}\left[E(z)\left.\frac{\partial a}{\partial t_C}\right|_{Eu} + (1 + r)\left.\frac{\partial S}{\partial t_c}\right|_{Eu}\right].$$

Substituting in (3.32) and (3.34), the above expression simplifies to

(3.36) $$\left.\frac{\partial E(C_2)}{\partial t_c}\right|_{Eu} = \left\{\frac{(1 + t_c)^{-4}}{(f'')D(c)}\right\}\left[(1 + r)(f'')(g'')\{E[(z)^2(h'')] - E(z)E[zh'']\}\right]$$

which is always negative. It is thus of interest to note that even though we are, in general, unable to determine the sign of the risk-taking and savings functions, future consumption is expected to decline.

Further note that should $\theta^R = \theta > 1$, both risk-taking and savings would be encouraged by the tax increase. In such an event,

(3.37) $$\left.\frac{\partial a}{\partial t_c}\right|_{Eu}^{\theta^R = \theta} = \left\{\frac{a}{1 + t_c}\right\}\left[1 - \left\{\frac{w(L_e - L)}{Y}\right\}\left\{\frac{1}{\theta^R}\right\}\right]$$

(3.38) $$\left.\frac{\partial S}{\partial t_c}\right|_{Eu}^{\theta^R = \theta} = \left\{\frac{S}{1 + t_c}\right\}\left[1 - \left\{\frac{w(L_e - L)}{Y}\right\}\left\{\frac{1}{\theta^R}\right\}\right].$$

Evidently for $\theta > 1$, the standard impact of consumption taxation dominates the component arising out of adjustment of work-leisure choice. (Note that the coefficient of $(1/\theta^R)$ is always strictly less than unity).

Turning to the riskiness of the portfolio, following (3.18) we evaluate

$$\frac{\partial\lambda}{\partial t_c}\bigg|_{Eu} = (-)\left\{\frac{(1 + t_c)^{-3}(wf')(g'')}{(S)^2\{D(c)\}}\right\}[S(1 + r)E(zh'' + aE(z^2h'')]$$

Again, from (3.20) and (3.21), we note that

$$C_2 = (1 + t_c)^{-1}[(1 + r)S + az],$$

or,

$$\frac{\partial\lambda}{\partial t_c}\bigg|_{Eu} = (-)\left\{\frac{(1 + t_c)^{-2}(wf')(g'')}{(S)^2\{D(c)\}}\right\} E(zC_2h'') \geq 0. \tag{3.39}$$

Thus even if we were unable to determine the direction of change in savings and in total risk-taking, portfolio riskiness is not discouraged by an expected utility compensated increase in the consumption tax. Focusing on the CRRA case, in light of (3.37) and (3.38), we observe that for any value of $\theta^R$, portfolio riskiness remains unchanged. Interestingly enough, even when both savings and risk-taking rise (e. g. when $\theta^R \geq 1$) they do so in equal proportions and thus riskiness cannot change.

To summarize the discussion of the consumption tax, we may state the following.

**Proposition 3.2.** a) *An expected utility compensated increase in the consumption tax leads to a decline in labour supply, current and expected future consumption. The $(C_1/E(C_2))$ path is actually distorted, except in the case of CRRA.*

*b) The effect on risk-taking and savings are ambiguous in general; for the case of CRRA with $\theta^R = \theta > 1$, however, they are both encouraged.*

*c) The asset-mix, however, remains at least as risky as before.*

**C. The Wealth tax.** A proportional tax on labour income (inclusive of the lump-sum component, B) as well as the net return to risk-taking, it was noted in the introduction, may be described as a "wealth" tax. Such a tax is equivalent to a consumption tax in the sense that they lead to identical lifetime budget sets. In the former, savings are deductible, i. e., cash-flows are taxed, while in the wealth tax case, savings are made out of net-of-tax earnings but subsequent capital income (except capital gains/losses) are tax-exempt.[8]

Denoting the tax rate by $t_h$, the budget equations are given by

[8] Ahsan (1990), discussed these two taxes in the context of the debate on cash-flow vs. pre-payment versions of expenditure tax in the tax-reform literature.



$$C_1 = [(1 - t_h)[Y - w(L_e - L)] - (a + m)];$$

$$C_2 = [(1 - t_h)\{(1 + r)(Y - w[L_e - L]) + az\} - (1 + r)C_1]; \text{ or}$$

$$[C_1 + C_2(1 + r)^{-1}] = (1 - t_h)[\{Y - w(L_e - L)\} + az(1 + r)^{-1}]. \tag{3.40}$$

Comparing (3.40) with (3.22), the equivalence between the consumption and wealth taxes become apparent. The relative price distortions are identical to that of the consumption tax. The time-path of tax revenue is different however, which implies that even though the consumption profile remains intact between the two taxes, savings are generally higher in the consumption tax. To an extent, individuals save in order to pay future taxes. Indeed this is what we find happening once we examine the comparative statics.

Since the impact of the tax on the three primary variables are practically identical to those of the consumption tax, we merely note

$$\frac{\partial L}{\partial t_h}\bigg|_{Eu} < 0, \quad \frac{\partial C_1}{\partial t_h}\bigg|_{Eu} < 0; \quad \frac{\partial a}{\partial t_h}\bigg|_{Eu} \gtrless 0. \tag{3.41}$$

The explanation is also identical. For savings,

$$S = [(1 - t_h)(Y - w(L_e - L)) - C_1] \tag{3.42}$$

$$\frac{\partial S}{\partial t_h}\bigg|_{Eu} = \left[\{(1 - t_h)w\}\frac{\partial L}{\partial t_h}\bigg|_{Eu} - \frac{\partial C_1}{\partial t_h}\bigg|_{Eu}\right] \tag{3.43}$$

or,

$$\frac{\partial S}{\partial t_h}\bigg|_{Eu} = \left\{\frac{1}{D(h)}\right\}[w(1 - t_h)^2(f')(g'')E((z)^2h'')] < 0. \tag{3.44}$$

In obtaining (3.43) we make use of the fact that expected utility compensation requires

$$(1 - t_h)dY|_{Eu} = [Y - w(L_e - L)]dt_h. \tag{3.45}$$

Evidently, savings decline under wealth taxation. Comparing (3.44) and (3.34), we note that even though the consumption tax had an ambiguous impact on savings, there was a positive force (the first term in (3.34)) counteracting the negative influence arising out of increased leisure. The latter is common to both taxes. Similarly to the consumption tax, expected future consumption is also decreased by the wealth tax. This reinforces the earlier remark that the consumption-path is similar between the two taxes.

What about portfolio riskiness? From (3.40)

$$C_2 = [(1 + t_h)az + (1 + r)S],$$

and

$$\text{(3.46)} \qquad \frac{\partial\lambda}{\partial t_h}\bigg|_{Eu} = \left\{\frac{1}{S}\right\}^2\left[\left\{\frac{aS}{1 - t_h}\right\} - \left\{\frac{(1 - t_h)(wf')(g'')}{D(h)}\right\}E(zC_2h''\right] > 0.$$

This is rather intuitive; riskiness did not decline in the consumption tax case even when savings was higher. Now, under wealth taxation, savings are discouraged unambiguously while total risk-taking is affected identically. Portfolio riskiness therefore goes up. Note that this is true even if CRRA holds. In the latter event, just as in the consumption tax, the total demand for the risky asset might even be higher (for $\theta^R > 1$). For other values of RRA, or, indeed if increasing RRA holds, total risk-taking, if it declines, it does so less than proportionately vis-a-vis savings.

We therefore observe the following.

**Proposition 3.3** *a) Increased taxation of wealth that preserves expected utility at a given level induces increased leisure and decreased levels of current and (expected) future consumption, and savings.*
*b) Total demand for the risky asset may go either way. When it declines, it does so less than proportionately compared to savings, and thus portfolio riskiness strictly rises.*

**D. The Income Tax.** This is the final of the four broad-based taxes considered in this paper. Letting $t_n$ denote the rate of tax on both labour and capital income, and with full loss-offsetting, we have

$$\text{(3.47)} \qquad C_1 = \{(1 - t_n)[Y - w(L_o - L)] - (a + m)\},$$

and, therefore,

$$\text{(3.48)} \qquad C_2 = \{(1 + \tilde{r})[(1 - t_n)\{Y - w(L_o - L)\} - C_1] + a\tilde{z}\}$$

where we have denoted $\tilde{r}$ and $\tilde{z}$ the net of tax yields on investment (i. e., $\tilde{r} = \{(1 - t_n)r\}$ and $\tilde{z} = \{(1 - t_n)z\}$). Rewriting (3.48), the allocation of disposable labour income is seen to equal

$$\text{(3.49)} \quad (1 - t_n)[Y - w(L_o - L)] = \{C_1 + (1 + \tilde{r})^{-1}C_2 - (1 + \tilde{r})^{-1}(\tilde{z})a\}.$$

From this expression we evidently recognize the distortion of the intertemporal terms of trade (the price of future consumption being increased) brought on by taxation of capital income. Since net risky income is a means of transferring consumption to the future, it also undergoes a re-valuation; not only is the net return (gain/loss) reduced by the income tax, when



converted into consumption units, it is worth even less. In terms of economic effects, therefore, we have to contend with a host of complexities. Taxation of all income, regardless of source, has the direct effect of distorting the work-leisure choice and that of the asset-mix. The indirect effects work via the discount rate in altering the time path of consumption and discouraging risk-taking. Absent asset behaviour, this latter aspect of income (or interest income) taxation would be lost.

The relevant FOC's are:

$$\text{(3.50)} \qquad -(f') + (\tilde{w})(1 + \tilde{r})E(h') = 0;$$

$$\text{(3.51)} \qquad (g') - (1 + \tilde{r})E(h') = 0;$$

$$\text{(3.52)} \qquad E((\tilde{z})h') = 0;$$

where we also define $\tilde{w}$ as the net wage rate.

Total differentiation of these conditions, given (3.48), allows us to identify the coefficients of the exogenous variables as follows (transferred to the rhs):

$$\text{(3.53)} \qquad d_1 = \Big[(1 - t_n)^{-1}(f')\{(1 + 2\tilde{r})(1 + \tilde{r})^{-1}\}$$
$$+ \{\tilde{w}(1 + \tilde{r})\}E\{h''[(1 + 2\tilde{r})\{Y - w(L_o - L)\} - rC_1 + az]\}\Big]dt_n$$
$$- [(1 - t_n)(\tilde{w})(1 + \tilde{r})^2E(h'')]dY;$$

$$\text{(3.54)} \qquad d_2 = -\Big[\{(1 + \tilde{r})^{-1}(rg') + (1 + \tilde{r})E\{h''[(1 + 2\tilde{r})\{Y - w(L_o - L)\}$$
$$- rC_1 + az]\}\Big]dt_n + [(1 - t_n)(1 + \tilde{r})^2E(h'')]dY;$$

$$\text{(3.55)} \qquad d_3 = \Big[E\{(\tilde{z})h''[(1 + 2\tilde{r})\{Y - w(L_o - L)\} - rC_1 + az]\}\Big]dt_n$$
$$- (1 - t_n)(1 + \tilde{r})E((\tilde{z})h'')dY.$$

To see the expected utility compensated effects of taxation, we impose

$$\text{(3.56)} \qquad (1 - t_n)(1 + \tilde{r})dY\Big|_{Eu} = \{(1 + 2\tilde{r})[Y - w(L_o - L)] - rC_1\}dt_n$$

on these $d$-terms above (eq. (3.56) defining the appropriate lump-sum transfer), and obtain the following.

(3.57) $$\left.\frac{\partial L}{\partial t_a}\right|_{Eu} = \left\{\frac{(f')}{(1 - t_a)D(n)}\right\} [(1 + 2\tilde{r})(1 + \tilde{r})^{-1}(g'')E((\tilde{z})^2h'') + (1 + \tilde{r})^2 (\tilde{K})] < 0;$$

(3.58) $$\left.\frac{\partial C_1}{\partial t_a}\right|_{Eu} = \left\{\frac{1}{D(n)}\right\} [(1 + \tilde{r})^2(wf')(\tilde{K}) - (rg')(1 + \tilde{r})^{-1}(f'')E((\tilde{z})^2h'')] \gtrless 0;$$

(3.59) $$\left.\frac{\partial a}{\partial t_a}\right|_{Eu} = \left(\frac{a}{1 - t_a}\right) - \left\{\frac{1}{D(n)}\right\}[(1 + 2\tilde{r})(wf')(g'')E((\tilde{z})h'') + (rg')(f'')E((\tilde{z})h'')] \gtrless 0;$$

The expressions $(\tilde{K}) > 0$ and $D(n) < 0$ are defined by eqs. (A5.2) and (A5.1), respectively, of the appendix.

From (3.47) and (3.49), the effect on savings is seen to equal

$$\left.\frac{\partial S}{\partial t_a}\right|_{Eu} = \left(\frac{rS}{1 + \tilde{r}}\right) + \left\{\tilde{w}\left.\frac{\partial L}{\partial t_a}\right|_{Eu} - \left.\frac{\partial C_1}{\partial t_a}\right|_{Eu}\right\}$$

or (given the above),

(3.60) $$\left.\frac{\partial S}{\partial t_a}\right|_{Eu} = \left(\frac{rS}{1 + \tilde{r}}\right) + \left\{\frac{1}{D(n)}\right\} [(wf')(g'')E((\tilde{z})^2h'') + (rg')(1 + \tilde{r})^{-1}[(f'') + (\tilde{w})^2(g'')]E((\tilde{z})^2h'')] \gtrless 0$$

Comparing the effect of income tax on labour supply with that of a wage tax, we note the absence of any influence other than the one introduced by the reduced net wage. Thus, as is intuitive, labour supply declines.

Current consumption becomes indeterminate as the increased price of future consumption now induces additional current consumption (the second term on the rhs of (3.58)). As usual, we also retain the direct effect of a reduced wage via increased leisure that discourages consumption.

In the risk-taking equation we observe three distinct forces. The stimulative effect (the first term in (3.59)) is due to the Domar-Musgrave phenomenon brought on the direct taxation of capital income. Wage taxation also has a negative effect on risk-taking as encountered earlier (the first term within square brackets). Finally a second element that also discourages



risk-taking is the indirect effect working through the increased price of future consumption.

Turning to savings behaviour, the first term encouraging savings is similar to the consumption tax case and arises as follows. Comparing the budget equations it is evident that the income tax is similar (indeed, equivalent) to a consumption or wealth tax *except* for the distortion in the intertemporal rate of time preference. Thus abstracting for a moment from this as well as the work-leisure adjustment, an expected utility compensated increase in proportional taxation of consumption in each period induces a savings behaviour necessary to retain the net-of-tax value of the consumption path unchanged. We may refer to this effect as the pure effect of effective taxation of consumption in each period. The combined influence through labour supply (i. e., direct effect of wages being taxed) and current consumption (both direct and indirect) is however negative. This is evident by examining terms within the square brackets in eq. (3.60), which is positive. The overall outcome becomes ambiguous.

In order to evaluate the effect on proportional risk-taking, we first note that the terms within square brackets in (3.60) may be rewritten as

(3.61) $$(1 + 2\tilde{r})(1 + \tilde{r})^{-1}(wf')(g'')E((\tilde{z})^2h'') + (rg')(1 + \tilde{r})^{-1}(f'')E((\tilde{z})^2h'')$$

Thus, given (3.54) and (3.60),

(3.62) $$\left.\frac{\partial \lambda}{\partial t_a}\right|_{Eu} = \left(\frac{1}{S}\right)^2 \Big[(aS)((1 - t_a)(1 + \tilde{r}))^{-1} - [(D(n))^{-1}(1 + \tilde{r})^{-1}]((rg')(f'') + (1 + 2\tilde{r})(wf')g'')E(\tilde{z}C_2h'')\Big] > 0$$

While there are more terms, this is comparable to the result of the wealth tax (see eq. (3.46)). Even though the general outcome on both total risk-taking and savings is ambiguous, portfolio riskiness is higher with increase income taxation. The intuition is as follows. Riskiness rises due to two distinct forces, the first being the dominance of the Domar-Musgrave effect over the pure effect of the "consumption tax element" of the income tax. This is evident from a comparison of the first term in each of (3.59), (3.60) and (3.62). The second effect arises such that the remaining distortions (via labour market and rate of time preference), though lowering both total risk-taking and saving, lowers risk-taking proportionately less. This is captured by the second term in (3.62) which is non-negative in the present model, the first being strictly positive. The second effect, also appeared in the case of consumption and wealth taxes. It is interesting to note that the additional distortion of time-preference does not upset the earlier finding.

From the budget equations we find

$$E(C_2) = [(1 + \tilde{r})S + E(\tilde{z})\ a]$$

or

$$\text{(3.63)} \qquad \left.\frac{\partial E(C_2)}{\partial t_n}\right|_{Eu} = \left\{- [(rS) + aE(z)] + (1 + \tilde{r})\left.\frac{\partial S}{\partial t_n}\right|_{Eu} + E(\tilde{z})\left.\frac{\partial a}{\partial t_n}\right|_{Eu}\right\}$$

On proper substitution and simplification,

$$\text{(3.64)} \qquad \left.\frac{\partial E(C_2)}{\partial t_n}\right|_{Eu} = \left[\left\{\frac{1}{D(n)}\right\} \{[(1 + 2\tilde{r})(wf')(g'') + (rg')(f'')]\} \right.$$

$$\left. [E((\tilde{z})^2 h'') - E(\tilde{z})E((\tilde{z})h'')]\right] < 0$$

Once again even though savings and risk-taking, taken separately, respond to income taxation in an ambiguous manner, expected future consumption, effectively a sum of the two, actually declines! From (3.63), (3.59) and (3.60) it is evident that the direct effect of a tax increase on future consumption, $\{(-)[rS + aE(z)]\}$, cancels out the positive elements in the risk-taking and saving functions (namely the Domar-Musgrave and the pure effect of consumption taxation, respectively). Thus the terms appearing in (3.64) mutually reinforce the negative elements of risk-taking and savings functions. The principal finding for the income tax analysis then is as follows.

**Proposition 3.4** *(a) An expected utility preserving increase in the rate of income tax leads to a decreased supply of labour and a reduced level of expected future consumption;*

*(b) It has an ambiguous effect on total risk-taking and savings, but portfolio riskiness actually rises.*

**E. Balanced Budget Effects of Taxation.** Should the government return the incremental revenue (due to the tax increase) to the taxpayer as a lump-sum rebate, the risk ultimately remains within the private sector. This is the



principal attraction of this procedure. The rebate is given in each period to match the revenue obtained in the same period. In such a context, it is standard to evaluate the derivatives at zero initial taxes (see e.g., Kotlikoff and Summers, 1979). Given the preceding analysis, it can be easily verified that under the new procedure the analysis of the wage tax is left unchanged as the revenue base in question is risk-free.

For the consumption tax, the tax perturbation removes the D-M effect from the risk-taking equation and also the positive element in the saving incentive. The latter phenomenon arises since in the new transfer mechanism, savings remain unchanged in order to maintain the fixed net consumption path. Consequently, both risk-taking and savings decline. Not surprisingly, the absence of risk-sharing also renders the behaviour of risk-taking in the wealth tax case fully analogous to the consumption tax. Indeed, given the identical outcome on labour supply from the earlier analysis, we observe that *under the balanced budget exercise the allocative effects of consumption and wealth taxes are identical.*

Much like the analysis of the consumption tax, increased taxation of income also leads to the elimination of the positive elements in both risk-taking and savings equations. These effects also imply obvious changes in the effect on portfolio riskiness. We summarize the main conclusion as follows.

**Proposition 3.5** *(a) A balanced budget increase in the tax rate discourages labour supply, total demand for the risky asset, savings and expected future consumption for all tax bases analyzed here;*

*(b) Portfolio riskiness is at least as high as before the tax increase, again, for all taxes;*

*(c) The allocative effects of the consumption and wealth taxes, unlike the Hicks compensated analysis, are identical.*

**F. A Sum-up.** Before concluding this section let us draw some inferences based on a comparison of the four taxes analyzed above and attempt to put them in the context of the existing literature.

**(i) On Tax Equivalence.** The presumed equivalence of consumption and wage taxation does not hold once asset choice is modeled. This points out the separate role of capital market risks as opposed to alternative sources of risk such as the wage rate (or indeed the return to human capital).[9]

The wealth tax, as pointed out by Ahsan (1989, 1990), is indeed equivalent to a consumption tax even in the expanded framework of this paper. His earlier finding that both these taxes lead to an identical consumption path but due to differing time-path of revenue obligations, savings are lower under wealth taxation, is again confirmed.

**(ii) Effect on Labour-Supply.** All broad-based taxes discussed here involve effective taxation of wage earnings, and hence the distortion is identical in nature. Our result that the compensated labour supply declines due to increased taxation is standard in models with a known wage rate. Although many authors do not carefully identify compensated and uncompensated effects, Kotlikoff-Summers (1979) effectively report a result similar to ours.

**(iii) Riskiness of the Portfolio.** Perhaps the most significant result of the preceding exercise is that increased taxation generally leads to a riskier asset-mix. The traditional literature on taxation and risk-taking with a fixed level of savings gives the impression that this was essentially due to the risk-sharing by the state through loss-offset tax policies (the Domar-Musgrave phenomenon). In an intertemporal world with endogenous savings, we find the earlier assertion to be false. Focusing on the wage tax, with no evident risk-sharing, riskiness is at least as high with increased taxation. The explanation, as we saw, that given NDRRA the labour market distortions produced a smaller relative decline in total demand for the risky asset compared to savings. Interestingly enough with a tax on interest income, the new distortion also created an identical pattern of relative decline in these functions. The income and wealth taxes, however, also introduce the Domar-Musgrave element and hence allow an unambiguous increase in portfolio riskiness even if CRRA holds.[10] We wish to emphasize that even with risk-sharing, especially for the class of increasing RRA preferences, riskiness would be higher in all cases (including the analysis of balanced budget perturbation).

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[9] Contrast the present formulation with Hamilton.

[10] In the case of the consumption tax, the Domar-Musgrave effect is exactly offset by a component of the savings incentive thus yielding the weaker version of the result.



**(iv) Total Risk-Taking.** Recall that consumption, wealth and income taxes all involve risk-sharing by the state. Should this be ignored (as, for example, would occur if we implemented lump-sum rebates that matched the revenue due to increased taxation), the total demand for the risky asset would decline in all cases. This is in contrast to models that ignore endogenous leisure where it is seen that only the income tax led to a decline, while consumption and wealth taxes left the risky asset demand unaffected under a similar tax perturbation.[11]

**(v) Consumption and Savings.** In some cases we were not able to determine the direction of the compensated effect without further restrictions. However, for the general case of NDRRA, it was of interest to find that expected future consumption declined in all cases. Recall that the latter is the sum of gross savings (principal plus interest) and net risky income. Generally speaking this result is due to the dominance of the negative influence of taxation in both savings and in the total demand for the risky asset over other terms (such as the Domar-Musgrave effect in risk-taking, and the effective taxation of consumption in each period in the savings function). In other words, even in cases we are unable to determine whether savings actually declined (e. g. the consumption and income taxes), we know that savings plus interest actually declined due to utility compensated tax increases. Likewise, the value of the capital gain (loss) declines due to the tax even when actual demand for the risky asset remains ambiguous.

## IV. Specific Effects of Taxation

We now review the *specific effects* of taxation such that no transfers are made back to the tax-payer. Implicitly, however, we assume that government spends the revenue in providing a pure public commodity which does not cause any further distortion in behaviour. Most treatment of the allocative effects of taxation, as it happens, are of this type. Hence we should be able to compare our results to the existing literature somewhat better than in the preceding section.

**A. The Wage Tax.** From section A.2 of the appendix we find that the specific effects are of the form:

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[11] See Ahsan (1989, 1990) for details.

(4.1) $$\frac{\partial X}{\partial t_w} = \left\{\frac{\partial X}{\partial t_w}\Big|_{Eu} - T(w)\frac{\partial X}{\partial Y}\right\}$$

where $X = L$, $C_1$, $a$, and $S$, and

$$T(w) = \left[(1 - t_w)^{-1}\{Y - w(L_e - L)\}\right] = \frac{dY}{dt_w}\Big|_{Eu}.$$

In other words, $T(w)$ is the rate at which lump-sum transfers would be required for expected utility compensation (see eq. (3.9) above). In particular we have

(4.2) $$\frac{\partial L}{\partial t_w} \gtrless 0,\ \frac{\partial C_1}{\partial t_w} < 0,\ \frac{\partial a}{\partial t_w} < 0 \text{ and } \frac{\partial S}{\partial t_w} < 0$$

(see eqs. (A2.6), (A2.9), (A2.10) and (A2.11) in the appendix). Although the effect on labour supply is ambiguous in general, for CRRA preferences (with $\theta^R = \theta$), eq. (A2.8) reveals that

(4.3) $$\frac{\partial L}{\partial t_w}\Bigg|^{\theta = \theta^R} \gtrless 0 \text{ as } \theta \gtrless 1$$

The above result is an extension of a similar analysis in the simple one-period work-leisure choice model presented in Atkinson-Stiglitz (1980, pp. 35-36). Evidently, for $\theta > 1$, the income effect dominates the substitution effect (leisure being a normal good in the present model). From (A.2) it is also evident that future consumption also declines unambiguously with increased taxation.

In order to see the effect of wage tax on portfolio riskiness, note that simplifications arise since the compensated effects (eqs. (3.13) and (3.16)) are actually proportional to simple wealth effects (eqs. (A2.3) and (A2.4)). We obtain

(4.4) $$\frac{\partial \lambda}{\partial t_w} = \left\{\frac{a}{SY}\right\}\left[\left\{\frac{wf'}{(1 - t_w)f''}\right\} - T(w)\right](\eta^a - \eta^S).$$

Thus for the general class of NDRRA, by theorem 2.1 (a),

(4.5) $$\frac{\partial \lambda}{\partial t_w} \geq 0$$

For CRRA, however, regardless of the values of $(\theta, \theta^R)$ there is no change. In eq. (4.4), the first term within square brackets arises out of the substitution effect. Thus, given the sign of $(\eta^a - \eta^S)$, both substitution and income effects reinforce each other in their influence on $\lambda$.



We thus obtain

**Proposition 4.1** *The specific effects of increased taxation of wage income are*

*(a) a decrease in the levels of current and future consumption, in the demand for the risky asset and in savings;*

*(b) labour supply, given CRRA, rises or falls as the RRA parameter exceeds or falls below unity;*

*(c) the portfolio mix remains as least as risky.*

**B. The Consumption Tax.** The derivation of the Slutsky equations are outlined in the appendix (section A.3):

(4.6) $$\frac{\partial X}{\partial t_c} = \left[\frac{\partial X}{\partial t_c}\Big|_{Eu} - T(c)\frac{\partial X}{\partial Y}\right]$$

where $X$ again denotes the choice variables ($L$, $C$, $a$ and $S$), and

$$T(c) = \left\{(1 + t_c)^{-1}[Y - w(L_e - L)]\right\} = \frac{dY}{dt_c}\Big|_{Eu}$$

(see 3.29). Note that the wealth effects are given by (A3.3) - (A3.6). From the discussion of the compensated effects, we observe that current consumption declines unambiguously due to increased taxation (see 3.31 and A3.4):

(4.7) $$\frac{\partial C_1}{\partial t_c} < 0$$

Another straightforward result turns out to be the effect on portfolio riskiness. Combining (A3.5) - (A3.6) with (3.32), (3.33) and (3.34)

(4.8) $$\frac{\partial \lambda}{\partial t_c} = \left\{\frac{a}{SY}\right\}\left\{\frac{wf'}{(1 + t_c)f''} - T(c)\right\}(\eta^a - \eta^S) \geq 0$$

The result at the compensated level therefore holds here too. It is also evident that both the income and substitution effects reinforce each other. In other words, income effects that lead to a decline in both savings and risk-taking does so relatively more for savings. The latter phenomenon in turn is explained by at least as high a wealth elasticity for savings than for the risky asset demand.

The labour supply effect, in light of (A3.3), (3.30) and the simplification of the type outlined for the wage tax (appendix eqs. (A2.7) - (A2.8)), appears as follows:

(4.9) $$\frac{\partial L}{\partial t_c}\Bigg|^{\theta = \theta^R} = \left\{\frac{f'}{(1 + t_c)(f'')}\right\}\left\{(1 + w\frac{\partial L}{\partial Y})\ (1 - \theta^R)\right\}$$

Thus

$$\left.\frac{\partial L}{\partial t_c}\right|^{\theta = \theta^R} \gtrless 0 \text{ as } \theta \gtrless 1$$

For CRRA, therefore, we obtain the identical conclusion as that of the wage tax which need not be surprising, given the identical nature of the labour market distortion induced by these taxes.

The effect on risk-taking differs significantly due to the emergence of the Domar-Musgrave (D-M) element under consumption taxation. Combining (A3.13) with (3.32) and (A3.5),

(4.11) $$\frac{\partial a}{\partial t_c} = \left[a(1 + t_c)^{-1}\right]\left\{(1 + \eta^a) + \left(\frac{wf'}{Yf''}\right)(1 - \theta^R)\eta^a\right\}$$

A number of inferences follow from the above derivation. First, we have:

(4.12) $$\frac{\partial a}{\partial t_c} \gtrless 0 \text{ as } \theta^R \gtrless 1 \gtrless \eta^a$$

This result arises (say, $\theta^R > 1 > \eta^a$) as the parameter values ensure that (a) the positive stimulus of D-M effect outweighs part of the income effect (the negative component), and (b) the remaining element of the income effect (measured by the coefficient of the $\theta^R$ term in 4.11) that encourages risk-taking also offsets the substitution effect arising out of the labour market distortion. When risk-taking declines both arguments are reversed (i. e., if $\theta^R < 1 < \eta^a$).

Perhaps the sharpest case would be that of CRRA (with $\theta^R = \theta$) since $\eta^a$ now becomes unity;

(4.13) $$\left.\frac{\partial a}{\partial t_c}\right|^{\theta^R = \theta} \gtrless 0 \text{ as } \theta \gtrless 1$$

Comparing the above expression with the analytics of the wage tax, we observe that this is effectively one part of the effect of wage taxation on risk-taking. The missing part (the component of income effect discouraging risk-taking equaling $(-)\eta^a$) is eliminated by the D-M phenomenon given CRRA. The condition on the RRA parameter implies that if the remaining substitution effect dominates (i. e., $\theta < 1$), increased consumption tax induces additional enjoyment of leisure and this leads to a reduced demand for the risky asset. Evidently such an interlinkage of markets are typically assumed away in most existing analysis.

Investigating the effect on savings closely, by a similar process we



obtain

(4.14) $$\frac{\partial S}{\partial t} = \left[S(1 + t_c)^{-1}\right]\left\{(1 - \eta^s) + \left(\frac{wf'}{Yf''}\right)(1 - \theta^R)\eta^s\right\}$$

Naturally,

(4.15) $$\frac{\partial S}{\partial t_c} \gtrless 0 \text{ as } \theta^R \gtrless 1 \gtrless \eta^s;$$

(4.16) $$\left.\frac{\partial S}{\partial t_c}\right|^{\theta^R = \theta} \gtrless 0 \text{ as } \theta \gtrless 1$$

and an identical explanation would also apply to this case.

Since we have already seen expected future consumption to decline at the compensated level (3.36), with the addition of income effects, this is *a-fortiori* so. To sum up the analysis of the consumption tax, we state the following.

**Proposition 4.2** *(a) The uncompensated effects of increased taxation of consumption in each period include a reduced level of both current and future consumption while the effects on labour supply, risk-taking and savings are ambiguous in general;*

*(b) For the class of CRRA preferences (with $\theta^R = \theta$), labour supply, total savings and the total demand for the risky asset rises, remains unchanged, or falls as the RRA parameter exceeds, equals or lies below unity;*

*(c) Riskiness of the portfolio, however, remains at least as high in all cases.*

**C. The Wealth Tax.** It can be easily checked that the specific effects of a tax on wealth are rather similar to that of the consumption tax. Indeed, the analysis is identical insofar as labour supply, current and future consumption and risk-taking are concerned. One notable change occurs in the savings behaviour; it declines unambiguously. We had already seen (section III) that the compensated effect was to discourage savings. Presently the income effect merely reinforces the same phenomenon. The explanation for this result is also identical to the one appliable to the compensated analysis, namely, the differing time path of revenue obligations between the consumption and wealth taxes.

One consequence of this result on savings is that proportional risk-taking strictly rises due to increased taxation:

$$(4.17)\qquad \frac{\partial\lambda}{\partial t_h} = \left(\frac{aS}{1 - t_h}\right) + \left(\frac{a}{SY}\right)\left[\frac{wf'}{(1 - t_h)(f'')} - T(h)\right](\eta^a - \eta^s) > 0,$$

where $T(h) = (a - t_h)^{-1}\{Y - w(L_o - L)\} = \left.\frac{dY}{dt_h}\right|_{Eu}$. Compared to the compensated effect, we recognize that the income effects further strengthen the encouragement of proportional risk-taking under wealth taxation.

To summarize therefore, we state the following.

**Proposition 4.3** *While the wealth tax affects the time path of consumption, labour supply and risk-taking behaviour identically to the consumption tax, unlike the latter, it leads to an unambiguous decline in savings and an increase in relative risk-taking.*

**D. The Income Tax.** The Slutsky equations for the income tax case are given in the appendix [eq.s (A5.12), (A5.15), (A5.16) and (A5.18)]. Evidently, in their standard form, they reveal little in the way of how taxation affects various incentives. This is largely due to the ambiguity in the compensated effects analyzed earlier. Simplification is also hard given the complexity of the coefficient of wealth effects, namely, the amount of lump-sum transfer needed to maintain constant expected utility.

**Labour Supply.** First we rewrite the compensated effect (eq. 3.57) in light of (A5.4) and (A5.7):

$$(4.18)\qquad \left.\frac{\partial L}{\partial t_n}\right|_{Eu} = \left\{\frac{f'}{(1 - t_n)f''}\right\}\left[\left\{1 + w\frac{\partial L}{\partial Y}\right\} + \left(\frac{r}{1 + \tilde{r}}\right)\frac{\partial S}{\partial Y}\right] < 0.$$

Also recognize that the lump-sum transfer relevant for utility compensation (see eq. (3.56)) may also be restated:

$$(4.19)\qquad T(n) = \left.\frac{dY}{dt_n}\right|_{Eu} = \left(\frac{1}{1 - t_n}\right)\left\{[Y - w(L_o - L)] + \left(\frac{rS}{1 + \tilde{r}}\right)\right\}.$$

Substituting (4.18) - (4.19) in (A5.12), we obtain

$$(4.20)\qquad \frac{\partial L}{\partial t_n} = \left\{\frac{f'}{(1 - t_n)f''}\right\}\left\{\left[1 + \left(\frac{\ell}{1 - \ell}\right)(\eta^L)(\theta^R)\right] + (1 - \theta^R)(w)\frac{\partial L}{\partial Y}\right.$$

$$\left. + \left[\frac{rS}{(1 + \tilde{r})Y}\right]\left[\eta^s + \left(\frac{\ell}{1 - \ell}\right)\eta^L\theta^R\right]\right\} \gtrless 0$$

which is evidently ambiguous. However, if CRRA prevails (with $\theta = \theta^R$),



$$(4.20a)\qquad \left.\frac{\partial L}{\partial t_n}\right|^{CRRA} = \left\{\frac{f'}{(1 - t_n)f''}\right\}\left[\left[1 + w\frac{\partial L}{\partial Y}\right] + \frac{rS}{(1 + \tilde{r})Y}\right](1 - \theta^R).$$

$$(4.20b)\qquad \left.\frac{\partial L}{\partial t_n}\right|^{CRRA} \gtrless 0 \text{ as } \theta^R \gtrless 1.$$

Given the obvious similarity between (4.18) and (4.20a), we observe that for the CRRA case the total effect of the income tax on labour supply is proportional to the compensated effect. For high $\theta^R$ (i. e., in excess of unity), the income effect dominates, the factor of proportionality becomes negative, and labour supply increases.

**Consumption.** In light of (4.19), the effect of the income tax on current consumption may be written as (see A5.15)

$$(4.17)\qquad \frac{\partial C_1}{\partial t_n} = \left.\frac{\partial C_1}{\partial t_n}\right|_{Eu} - \left[\left[\frac{1}{1 - t_n}\right]\left\{[Y - w(L_o - L)] + \left[\frac{rS}{1 + \tilde{r}}\right]\right\}\right]\frac{\partial C_1}{\partial Y}.$$

Next we note that the compensated effect may also be recast using the expression for wealth effects (see eqs. (3.58), (A5.5) and (A5.7)):

$$(4.22)\qquad \left.\frac{\partial C_1}{\partial t_n}\right|_{Eu} = \left\{\frac{wf'}{(1 - t_n)f''}\right\}\frac{\partial C_1}{\partial Y} - \left\{\frac{rg'}{(1 + t_n)(1 + \tilde{r})(g'')}\right\}\frac{\partial S}{\partial Y}.$$

These two expressions may now be combined to obtain

$$(4.23)\qquad \frac{\partial C_1}{\partial t_n} = \left\{\frac{wf'}{(1 - t_n)f''}\right\}\left[1 + \theta^R\left[\frac{Y}{w(L_o - L)} - 1\right]\right]\frac{\partial C_1}{\partial Y}$$

$$- \left\{\frac{rg'S}{(1 - t_n)(1 + \tilde{r})(Y)(g'')}\right\}[\eta^s - \theta_1\eta^c].$$

It can be verified that the first term on the rhs (i. e., $\partial C_1/\partial Y$ and its coefficient) arises out of wage taxation implicit in the income tax, which we have already seen to be negative. Here both income and substitution effects reinforce each other. The second term obviously is due to the distorted rate of time-preference which is ambiguous in sign due to the evident conflict between the income and substitution terms. (Note that the term involving $\eta^s$ measures the relevant substitution effect.) A sufficient condition for the second expression to be also negative appears to be interpretable if CRRA with $\theta = \theta^R$ holds:

(4.24) $$\left.\frac{\partial C_1}{\partial t_n}\right|^{CRRA} < 0 \text{ if } \theta_1 \geq 1.$$

Since in this case $\eta^s = \eta^c = 1$, larger than unitary value of $\theta_1$ guarantees the dominance of the second income effect over the corresponding substitution term, and, overall, current consumption declines.

Future consumption also declines. We have already seen that the negative impact of an increased price of future consumption (due to the income tax) at the compensated level. The income effects only exacerbate this tendency.

**Risk-Taking.** In order to simplify the expression for risk-taking, we first rewrite the compensated effect (eq. 3.59) in light of (A5.6) as:

(4.25) $$\left.\frac{\partial a}{\partial t_n}\right|_{Eu} = \left(\frac{a}{1-t_n}\right) + \left\{\frac{1}{(1-t_n)(1+\tilde{r})}\right\}$$
$$\left\{\left[\left(\frac{rg'}{g}\right) + (1+2\tilde{r})\left(\frac{wf'}{f}\right)\right]\frac{\partial a}{\partial Y}\right\}.$$

Combining the above with the income effects (see A5.16), and with some rearrangement we obtain:

(4.26) $$\frac{\partial a}{\partial t_n} = \left(\frac{a}{1-t_n}\right)\left[1 - \left\{1 - \left(\frac{w(L_o - L)}{Y}\right)\left(1 - \frac{1}{\theta^R}\right)\right\}\eta^a\right]$$
$$- \left[\left(\frac{a}{1-t_n}\right)\left(\frac{\tilde{r}}{1+\tilde{r}}\right)\left\{\frac{w(L_o - L)}{Y}\left(\frac{1}{\theta^R}\right) + \frac{C_1}{(1-t_n)Y}\left(\frac{1}{\theta}\right) + \frac{S}{(1-t_n)Y}\right\}\eta^a\right]$$

Note that the terms within the first set of curly brackets times the asset demand elasticity is positive for all values of $\theta^R$ and measures the effect of wage taxation implicit in the income tax (the substitution and income terms reinforcing each other). We had earlier seen that this element discourages risk-taking. The expression enclosed by the second set of square brackets is unambiguously positive, and it represents the combined forces of distorted discount rate and part of the effect of capital income taxation (again, both income and compensated aspects). This latter element also discourages risk-taking. The lone stimulus to increased demand for the risky asset comes from the other aspect of capital income taxation, namely the D-M phenomenon. A set of sufficient conditions of the latter to be upset by the disincentive due to wage taxation component of the income tax is given by



(4.26a) $$\frac{\partial a}{\partial t_n} < 0 \text{ if } \theta^R \leq 1 \text{ and } \eta^a \geq 1$$

Evidently, under these conditions the expression within the first set of square brackets becomes negative.

A slightly weaker set of conditions is obtained if we rearrange (4.26) a bit:

(4.27) $$\frac{\partial a}{\partial t_n} = \left(\frac{a}{1-t_n}\right)\left[(1-\eta^a) + \left(\frac{w(L_o - L)}{Y}\right)\left\{1 - \left(\frac{1+2\tilde{r}}{1+\tilde{r}}\right)\left(\frac{1}{\theta^R}\right)\right\}\eta^a\right.$$
$$\left. - \left(\frac{\tilde{r}}{1+\tilde{r}}\right)\left\{\left(\frac{1}{\theta}\right)\frac{C_1}{(1-t_n)Y} + \frac{S}{(1-t_n)Y}\right\}\eta^a\right].$$

Therefore

(4.27a) $$\frac{\partial a}{\partial t_n} < 0 \text{ if } \theta < \varphi,\ \varphi \in (1, 2) \text{ and } \eta^a \geq 1$$

where we have defined $\varphi = [(1+2\tilde{r})(1+\tilde{r})^{-1}]$ which lies in the open interval (1, 2). Elsewhere we have argued that such a range for the RRA parameter as moderate and rather reasonable.[12] It may be seen that imposition of CRRA yields little except that $\eta^a$ being unity drops out as a condition.

**Savings.** The derivation of the effect of taxation on savings is analogous to that of risk-taking. We obtain

(4.28) $$\frac{\partial S}{\partial t_n} = \left(\frac{S}{1-t_n}\right)\left[\left(\frac{\tilde{r}}{1+\tilde{r}}\right) - \left\{1 - \left(\frac{w(L_o - L)}{Y}\right)\left(1 - \frac{1}{\theta^R}\right)\right\}\eta^s\right.$$
$$\left. - \left(\frac{\tilde{r}}{1+\tilde{r}}\right)\left\{\left(\frac{w(L_o - L)}{Y}\right)\left(\frac{1}{\theta^R}\right) + \left(\frac{C_1}{(1-t_n)Y}\right)\left(\frac{1}{\theta}\right) + \frac{S}{(1-t_n)Y}\right\}\eta^s\right]$$

where the various components may again be interpreted as above. Naturally, by further rearrangement, we get a similar conclusion as for risk-taking:

(4.28a) $$\frac{\partial S}{\partial t_n} < 0 \text{ if } \theta < \varphi \text{ and } \eta^s \geq 1.$$

Although conditions (4.27a) and (4.28a) are identical, we note that the latter is slightly weaker since it is easier for $\eta^s$ to at least equal unity than for

12 Ahsan (1989) discusses various estimates found in the literature. Indications are that a value of $\theta$ between 1 and 2 is rather reasonable, while a value less than unity may be implausibly low.

$\eta^{s}$ (see theorem 2.1).

**Proportional Risk-Taking.** Given the result that portfolio riskiness rose under an expected utility compensated increase in the income tax, we would expect the same conclusion to hold given the addition of income effects. The latter being proportional to the relevant wealth elasticities are such that total risk-taking declines proportionately less than does savings (as $\eta^{s} \geq \eta^{a}$ under NDRRA). Combining the various elements, we obtain

$$(4.29) \qquad \frac{\partial \lambda}{\partial t_n} = \left\{\frac{a}{(1 - t_n)(1 + \tilde{r})S}\right\} \left[1 + \left(\frac{1}{Y}\right) (\eta^{a} - \eta^{s}) \left\{(1 + 2\tilde{r})\left(\frac{wf'}{f''}\right) + \left(\frac{rg'}{g''}\right) - (1 - t_n)(1 + \tilde{r})T(n)\right\}\right] > 0.$$

The principal findings for the income tax then are as follows:

**Proposition 4.4** *a) Increased rate of taxation of both labour and capital income leads to a decreased level of future consumption and an increased degree of riskiness of the portfolio;*

*b) Given CRRA, labour supply responds positively or negatively depending on whether θ exceeds or lies below unity;*

*c) Both the total demand for the risky asset and savings are also seen to decline if the constant RRA parameter lies in a moderate range.*

**E. Significance of Results.** The preceding analysis of specific incidence of taxation allows us to make a number of interesting observations.

The significant result is indeed the implication of the presence of endogenous labour supply on the remaining decisions, namely consumption-saving and the choice of the asset mix. In particular, the analysis of the consumption tax is now drastically altered. Absent leisure behaviour, it has been shown that the consumption tax leaves both savings and total demand for the risky asset unchanged given CRRA preferences (Ahsan; 1989, 1990). For plausible values of the RRA parameter (i. e., in excess of unity) we now find them both higher.

On the effect of income taxation, it has been earlier shown that both savings and total risk-taking declines unambiguously for all values of the CRRA parameter. Given endogenous labour supply, this is no longer so. For large values of θ, the outcome is ambiguous. These outcomes may have an important bearing on the choice of the tax base. Certainly for moderate values of the RRA parameter, preference for the consumption tax (over the



income tax) may indeed be based on stronger grounds than encountered in models that ignore leisure behaviour, especially if savings is included as a criterion.

## IV. Conclusion

We believe the addition of leisure behaviour in a model of household decisions to be an important one, and one that leads to nontrivial implications for the remaining choice variables. Evidently, an even richer description of this choice, possibly embedded in a theory of human capital accumulation, would be insightful. Given the relative tractability of the present exercise, this further extension should be worth pursuing.

A second concluding remark refers to the analysis of differential effects of taxation and the choice of tax base. At the utility compensated level, if we choose savings as a criterion, the consumption tax would be preferred (at least in the CRRA case). With the balanced budget tax perturbation, it follows that the wage tax is superior as it allows zero loss of welfare (for small taxes). Naturally, the question of the global desirability of a tax base can only be answered in a framework of differential analysis of tax alternatives such that discrete changes in either expected utility, expected revenue or some other benchmark remains constant. This latter exercise calls for explicit closed-form solutions of all decision variables and related maximum value functions (utilities, revenues, etc.). Such an exercise is feasible for the CRRA case (see Ahsan, 1989) and should be a useful complement to the qualitative analysis.

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# APPENDIX

### A.1. *Wealth Effects and Risk-Aversion.*

In this section of the appendix, we provide a brief outline of the proof of the results on wealth effects cited in the text, particularly, Theorem 2.1. We proceed by discussing the second-order conditions of the choice problem given by (2.3) and (3.3) of the text.

*SOC's.* Using $A_1$, $A_2$, and $A_3$ to denote the three *FOC*'s [equations (2.4)–(2.6)] and indexing the choice variables $L$, $C_1$, and $a$ by 1, 2, and 3, respectively, the elements of the corresponding Hessian matrix are given by the following.

$$A_{11} = (f'') + \{(w)^2(1+r)^2\}E\{h''\}$$
$$A_{12} = A_{21} = (-)\{w(1+r)^2\}E\{h''\}$$
$$A_{13} = A_{31} = \{w(1+r)\}E\{zh''\}$$
$$A_{22} = (g'') + (1+r)^2E\{h''\}$$
$$A_{23} = A_{32} = -(1+r)E\{zh''\}$$
$$A_{33} = E\{(z)^2h''\}.$$

It would also prove convenient to write out the principal minors of the Hessian for later reference. Given $A_{ij}$ as above, we denote the principal minor of the $ij$th element $D_{ij}$, and obtain the following

$$D_{11} = [A_{22}A_{33} - (A_{23})^2],$$
$$= (g'')E\{(z)^2h''\} + (1+r)^2\{K\};$$
$$D_{21} = D_{12} = [A_{12}A_{33} - A_{13}A_{23}],$$
$$= (-)\{w(1+r)^2\}\{K\};$$
$$D_{31} = D_{13} = [A_{12}A_{23} - A_{13}A_{22}],$$
$$= (-)\{w(1+r)\}(g'')E\{zh''\};$$
$$D_{22} = [A_{11}A_{33} - (A_{13})^2]$$
$$= (f'')E\{(z)^2h''\} + \{(w)^2(1+r)^2\}\{K\};$$
$$D_{32} = D_{23} = [A_{11}A_{23} - A_{21}A_{13}]$$
$$= (-)(1+r)(f'')E\{zh''\}$$
$$D_{33} = [A_{11}A_{22} - (A_{12})^2$$
$$= \{(f'')(g'')\} + (1+r)^2[(f'') + (w)^2(g'')]E\{h''\}$$

where the quantity $\{K\}$ used frequently above is given by

$$(A1.1) \qquad \{K\} \equiv (E\{h''\}E\{(z)^2h''\} - [E\{zh''\}]^2).$$

Given the above it is evident that the determinantal value of the Hessian may be obtained as

$$D = A_{11} \cdot D_{11} - A_{21} \cdot D_{21} + A_{31} \cdot D_{31}$$

$$(A1.2) \quad \text{or} \quad D = [(f'')(g'')E\{(z)^2h''\} + (1+r)^2[f'') + (w)^2(g'')]\{K\}]$$

The $SOC$'s are therefore:

$$(A1.3) \qquad A_{11} < 0, \quad D_{33} > 0, \quad \text{and} \quad D < 0.$$

Concavity of $\underline{f}$, $\underline{g}$, and $\underline{h}$ is sufficient for the first two of these conditions to be satisfied. The last can be guaranteed if we also assume that $\underline{K}$ be nonnegative. Elsewhere the author has shown that under NDRRA, $\underline{K}$ is indeed nonnegative (Ahsan, 1977).

*Simple Wealth Effects.* Before examining the wealth effects we also note another elementary result. Under DARA, the present model suggests that

$$(A1.4) \qquad E\{zh''\} > 0.$$

(Given that $\{(-)(h''/h')\}$ measures ARA in the present context, the above is easy to verify.)

Differentiating the $FOC$'s [equations (2.4)–(2.6)] wrt $Y$, we obtain

$$(A1.5) \qquad [A_{ij}] \begin{pmatrix} \partial L/\partial Y \\ \partial C_1/\partial Y \\ \partial a/\partial Y \end{pmatrix} = \begin{pmatrix} (-)\{(w)(1+r)^2\}E\{h''\} \\ (1+r)^2E\{h''\} \\ (-)(1+r)E\{zh''\} \end{pmatrix},$$

where we would often refer to the elements of the rhs vector by $d_j$ $(j = 1,2,3)$. Evaluating (A1.5), we immediately obtain the simple wealth effects:

$$(A1.6) \qquad \frac{\partial L}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[(w)(1+r)^2(g'')\{K\}] \qquad < 0;$$

$$(A1.7) \qquad \frac{\partial C_1}{\partial Y} = \left\{\frac{1}{D}\right\}[(1+r)^2(f'')\{K\}] \qquad > 0;$$

$$(A1.8) \qquad \frac{\partial a}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[(1+r)(f'')(g'')E\{zh''\}] > 0.$$



From equation (2.1), we obtain

$$\frac{\partial S}{\partial Y} = \left\{1 - \frac{\partial C_1}{\partial Y} + w\,\frac{\partial L}{\partial Y}\right\}.$$

Upon substitution of (A1.6) and (A1.7), and in light of (A1.2), the above simplifies

$$(A1.9) \qquad \frac{\partial S}{\partial Y} = \left\{\frac{1}{D}\right\}[(f'')(g'')E\{(z)^2h''\}] > 0.$$

It is thus evident that given DARA and NDRRA, all the wealth effects are signed so that leisure, consumption, the risky asset and savings are all normal goods.

*Implications of CRRA: Theorem 2.1.*

Proof. Part (a) of the theorem follows immediately from the well-known lemma that of NDRRA

$$(A1.10) \qquad E\{(z)C_2h''\} \leq 0.$$

Using the expression for $C_2$ (equation 2.2) and dividing through by $D$ $(< 0)$, we obtain

$$(A1.11) \qquad \{(a)(f'')^{-1}(g'')^{-1}\}\,\frac{\partial S}{\partial Y} \geq \{S(f'')^{-1}(g'')^{-1}\}\frac{\partial a}{\partial Y} \quad \text{or} \quad \eta^S \geq \eta^a.$$

By definition, savings comprise the two assets, i.e.,

$$(A1.12) \qquad (\alpha+\beta)\eta^S = \alpha\eta^a + \beta\eta^m$$

where $\underline{\alpha}$ and $\underline{\beta}$ are the shares of $\underline{a}$ and $\underline{m}$ in $\underline{Y}$. Combining (A1.11) and (A1.12),

$$(A1.13) \qquad \eta^m \geq \eta^S \geq \eta^a \geq \overline{\eta} \quad \text{(say.)}$$

which is part (a) of the result. Q.E.D.

Proof of part (b) is much more laborious and requires restating the model where all choice variables are expressed as shares in a suitable manner.

The budget equations (2.1) and (2.2) are now written as:

$$(A1.14) \qquad b = \{1 - \omega(1-\ell) - (\alpha+\beta)\},$$

$$(A1.15) \qquad C_2 = (Y)[(1+r)\{1 - b - \omega(1-\ell)\} + \alpha z]$$

where $b = (C_1/Y)$, $\ell = (L/L_0)$ and $\omega = (wL_0/Y)$. Agents now choose $\{\ell, b$, and $\alpha\}$. Using the definitions

$$(A1.16) \qquad \begin{cases} \theta^R = [(-)L_0(1-\ell)(f'')/(f')], \\ \theta^1 = [(-)C_1(g'')/(g')], \\ \theta^2 = [(-)C_2(h'')/(h')] \end{cases}$$

and letting $\theta^1 = \theta^2 = \theta$, the rhs vector to a system of equations similar to (A1.5) becomes:

$$(A1.17) \qquad \begin{cases} d_1 = ([\{\theta L_0 f'\}/Y] - \{(\omega)^2(1+r)^2(1-\ell)\}(Y)E\{h''\}); \\ d_2 = \{(\omega)(1+r)^2(1-\ell)\}(Y)E\{h''\}); \\ d_3 = (-)\{(\omega)(1+r)(1-\ell)\}(Y)E\{zh''\}). \end{cases}$$

In this reformulation, the relevant Hessian matrix is also modified in an appropriate way. Evaluating the new system we obtain

$$(A1.18) \quad \eta^L\Big|^{\text{CRRA}} = \left\{\frac{\theta(1-\ell)}{(\theta^R)\ell}\right\}\left\{\left[\frac{(\omega^2)(1+r)^2)(Y)^6(g'')\{K\}}{D}\right]\left(\frac{\theta-\theta^R}{\theta}\right) - 1\right\};$$

$$(A1.19) \quad \frac{\partial b}{\partial Y}\Big|^{\text{CRRA}} = \left\{\frac{1}{D}\right\}[(\theta-\theta^R)(\omega)(1+r)^2(L_0 f')(Y)^3\{K\}];$$

and

$$(A1.20) \quad \frac{\partial \alpha}{\partial Y}\Big|^{\text{CRRA}} = (-)\left\{\frac{1}{D}\right\}[(\theta-\theta^R)(\omega)(1+r)(L_0 f')(Y)^3(g'')E\{zh''\}];$$

where we still use $D$ in a generic sense to denote the determinantal value of the new Hessian.

From (A1.19) and (A1.20), it follows immediately

$$(A1.21) \qquad \left\{\frac{\partial b}{\partial Y}, \frac{\partial \alpha}{\partial Y}\right\} \gtreqless 0 \quad \text{as} \quad \theta \gtreqless \theta^R.$$

Or, equivalently,

$$(A1.22) \qquad \{\eta^c, \overline{\eta}\} \gtreqless 1 \quad \text{as} \quad \theta \gtreqless \theta^R,$$

which establishes the first half of part (b) of Theorem 2.1. Finally, given $\theta^R = \theta$, from (A1.18) and (A1.22), the rest of the result also follows immediately. Q.E.D.



Having encountered the complexity of the model where the choice variables shares, we return to the earlier formulation from here on. The rhs expressions (A1.18)–(A1.20), as appearing above actually looked much more cumbersome in the absence of the CRRA specification.

*A2. The Wage Tax.*

In order to economize on notation, in the appendix we shall continue to denote the FOC's by $A_i$ ($i = 1, 2, 3$), and hence the elements of the appropriate Hessian by $A_{ij}$ and its determinantal value by $\underline{D}$ for all tax regimes. Since we discuss each tax separately in one section, this need not be confusing. In the text, however, since we often compare different taxes in a given section, we need to identify them.

*Wealth Effects.* From Equations (3.6)–(3.8) we can obtain the simple wealth effects by setting $dt = 0$ similar to the exercise in the preceding section. These are:

$$(A2.1) \qquad \frac{\partial L}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[\{(w)(1-t)^2(1+r)^2\}(g'')\{K\}] < 0;$$

$$(A2.2) \qquad \frac{\partial C_1}{\partial Y} = \left\{\frac{1}{D}\right\}[(1-t)(1+r)^2(f'')\{K\}] > 0;$$

$$(A2.3) \qquad \frac{\partial a}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[(1-t)(1+r)(f'')(g'')E\{zh''\}] > 0.$$

Again, from Equations (3.14), (A2.1), (A2.2), and (A2.5),

$$(A2.4) \qquad \frac{\partial S}{\partial Y} = \left\{\frac{1}{D}\right\}[(1-t)(f'')(g'')E\{(z)^2h''\}] > 0.$$

Note that here the relevant $\underline{D}$ is given by:

$$(A2.5) \quad D = [(f'')(g'')E\{(z)^2h''\} + (1+r)^2[(f'') + \{w(1-t)\}^2(g'')]\{K\}] < 0.$$

The utility compensated effects reported in the text are obtained by treating (3.10) as the rhs vector in a system of equations obtained by differentiating the FOC's. The second-order terms $A_{ij}$ are modified from those presented in the no-tax world in a straightforward manner. Hence the details are being omitted.

*Specific Effects.* Next, we discuss the uncompensated (or, specific) effects of taxation where no transfers are made back to the individual. These are obtained by setting $dY = 0$ in expressions (3.6)–(3.8). Given the structure of the new $[A_{ij}]$-matrix, we find

$$\frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\}[d_1 D_{11} - d_2 D_{21} + d_3 D_{31}],$$

where $D_{ij}$ are also derived in the manner postulated in the preceding section. Thus

$$\frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\}[\{(f')(1-t)^{-1}\}[(g'')E\{(z)^2h''\} + (1+r)^2\{K\}] + \{w(1-t)(1+r)^2\}[Y - w(L_0 - L)](g'')\{K\}],$$

or, using (3.11) and (A2.1)

(A2.6) $$\frac{\partial L}{\partial t} = \left.\frac{\partial L}{\partial t}\right|_{Eu} - (1-t)^{-1}[Y - w(L_0 - L)]\,\frac{\partial L}{\partial Y} \gtrless 0.$$

Equation (3.11) may also be restated in view of (A2.1):

(A2.7) $$\left.\frac{\partial L}{\partial t}\right|_{Eu} = \left\{\frac{f'}{(1-t)f''}\right\}\left[1 + w\,\frac{\partial L}{\partial Y}\right] < 0,$$

where it may be verified that $[1+(w)\partial L/\partial Y] > 0$ from the budget equations given our assumptions on risk-aversion. It turns out that the income term

$$[Y - w(L_0 - L)]\frac{\partial L}{\partial Y} = (-)(L_0 - L)\left[w\frac{\partial L}{\partial Y} - \left(\frac{\ell}{1-\ell}\right)\eta^L\right]$$

and should $\theta^R = \theta$, by theorem 2.1(b),

$$= (-)(L_0 - L)\left[1 + w\frac{\partial L}{\partial Y}\right].$$

Therefore,

(A2.8) $$\left.\frac{\partial L}{\partial t}\right|^{\theta^R=\theta} = \left\{\frac{f'}{(1-t)f''}\right\}\left[\left\{1 + w\frac{\partial L}{\partial Y}\right\}(1-\theta^R)\right] \gtrless 0.$$

Turning to current consumption, we obtain

$$\frac{\partial C_1}{\partial t} = \left\{\frac{1}{D}\right\}[-d_1 D_{12} + d_2 D_{22} - d_3 D_{32}]$$

or, simplifying,

$$\frac{\partial C_1}{\partial t} = \left\{\frac{1}{D}\right\}[(1+r)^2\{(wf') - [Y - w(L_0 - L)](f'')\}\{K\}] < 0.$$



(3.12) and (A2.2),

.9) $$\frac{\partial C_1}{\partial t} = \left[\left.\frac{\partial C_1}{\partial t}\right|_{Eu} - \left\{\frac{Y - w(L_0 - L)}{1-t}\right\}\frac{\partial C_1}{\partial Y}\right] < 0.$$

Similarly, for the risky asset demand,

$$\frac{\partial a}{\partial t} = \left\{\frac{1}{D}\right\}[d_1 D_{13} - d_2 D_{23} + d_3 D_{33}]$$

$$\frac{\partial a}{\partial t} = (-)\left\{\frac{(1+r)}{D}\right\}[\{(wf') - [Y - w(L_0 - L)](f'')\}(g'')E\{zh''\}] < 0.$$

Again using (3.13) and (A2.3),

2.10) $$\frac{\partial a}{\partial t} = \left[\left.\frac{\partial a}{\partial t}\right|_{Eu} - \left\{\frac{Y - w(L_0 - L)}{(1-t)}\right\}\frac{\partial a}{\partial Y}\right] < 0.$$

To determine the effect on savings, we return to equation (3.14), and given above derivation, obtain

$$\frac{\partial S}{\partial t} = \left\{\frac{1}{D}\right\}[\{(wf') - [Y - w(L_0 - L)](f'')\}(g'')E\{(z)^2h''\}]$$

given (3.11) and (A2.4),

A2.11) $$\frac{\partial S}{\partial t} = \left[\left.\frac{\partial S}{\partial t}\right|_{Eu} - \left\{\frac{Y - w(L_0 - L)}{(1-t)}\right\}\frac{\partial S}{\partial Y}\right] < 0.$$

### A3. *The Consumption Tax.*

Given (3.21), total differentiation of the FOC's [equations (3.23)–(3.25)] yield elements of the new Hessian, the $A_{ij}$-terms, as coefficients of the choice variables $\{dL, dC_1$ and $da\}$. The 'constant' terms $dt$ and $dY$ with their coefficients appear on the rhs. The expressions (3.26)–(3.28) are the rhs so obtained for each of the three FOC's, respectively. From the new $A_{ij}$-terms, we can compute the minors $D_{ij}$ and eventually the determinantal value of the Hessian. This can be seen to be as follows:

(A3.1) $$D = [\{(1+t_c)^{-2}(f'')(g'')\}E\{(z)^2h''\} + \{(1+t_c)^{-2}(1+r)^2\}[(f'') + (w)^2(1+t_c)^{-2}(g'')]\{K\}].$$

The compensated effects reported in the text [Equations (3.30)–(3.32)] follow immediately given (3.29).

*Wealth Effects.* In order to evaluate the wealth effects, we return to (3.26)–(3.28), set $dt = 0$, and obtain

$$(A3.2) \qquad [A_{ij}] \begin{pmatrix} \partial L/\partial Y \\ \partial C_1/\partial Y \\ \partial a/\partial Y \end{pmatrix} = \begin{pmatrix} -(w)(1+t_c)^{-2}(1+r)^2 E\{h''\} \\ (1+t_c)^{-1}(1+r)^2 E\{h''\} \\ (-)(1+t_c)^{-2}(1+r)E\{zh''\} \end{pmatrix}.$$

Consequently,

$$(A3.3) \qquad \frac{\partial L}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[\{(w)(1+t)^{-4}(1+r)^2\}(g'')\{K\}] \quad < 0;$$

$$(A3.4) \qquad \frac{\partial C_1}{\partial Y} = \left\{\frac{1}{D}\right\}[(1+t)^{-3}(1+r)^2(f'')\{K\}] \quad > 0;$$

$$(A3.5) \qquad \frac{\partial a}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[\{(1+t)^{-2}(1+r)(f'')(g'')E\{zh''\}] > 0.$$

Also from (3.33) and the above expressions, we obtain the effect on saving

$$(A3.6) \qquad \frac{\partial S}{\partial Y} = \left\{\frac{1}{D}\right\}[(1+t)^{-2}(f'')(g'')E\{(z)^2h''\}] > 0.$$

*Compensated Effects under CRRA.* The simplification of the risk-taking and saving equations as reported by (3.37) and (3.38) is obtained as follows. First we recognize that part of the utility compensated effects (those working via adjustments in labour supply) are indeed proportional to wealth effects. Thus from (3.32) and (A3.5)

$$(A3.7) \qquad \left.\frac{\partial a}{\partial t}\right|_{Eu} = \{a(1+t)^{-1}\}\left[1 + \left(\frac{wf'}{Yf''}\right)\eta^a\right].$$

Now we can restate the terms within [·] as

$$\left[1 - \left\{\frac{w(L_0 - L)}{Y}\right\}\left\{\frac{\eta^a}{\theta^R}\right\}\right].$$

Should $\theta^R = \theta$, $\eta^a = 1$ $(= \eta^S)$ and we obtain Equation (3.37). Savings equation is obtained similarly.



*Specific Effects.* Specific, or uncompensated effects of taxation are obtained by setting $dY = 0$ in (3.26)–(3.28). Still referring to the modified set of rhs expressions (A3.2), we obtain

$$\frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\}[d_1D_{11} - d_2D_{21} + d_3D_{31}],$$

$$(A3.8) \qquad \frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\}[\{(1+t)^{-3}(f')\}\{(g'')E\{(z)^2h''\} + (1+r)^2[K]\} + \{(w)(1+t)^{-5}(1+r)^2\}[Y - w(L_0 - L)](g'')\{K\}].$$

In view of (3.30) and (A3.3),

$$(A3.9) \qquad \frac{\partial L}{\partial t} = \left\{\left.\frac{\partial L}{\partial t}\right|_{Eu} - (1+t)^{-1}[Y - w(L_0 - L)]\frac{\partial L}{\partial Y}\right\} \gtrless 0.$$

Similarly, we obtain

$$(A3.10) \qquad \frac{\partial C_1}{\partial t} = \left\{\frac{1}{D}\right\}[\{(1+t)^{-4}(1+r)^2\}\{(wf') - [Y - w(L_0 - L)](f'')\}\{K\}] < 0.$$

$$(A3.11) \qquad \frac{\partial C_1}{\partial t} = \left\{\left.\frac{\partial C_1}{\partial t}\right|_{Eu} - (1+t)^{-1}[Y - w(L_0 - L)]\frac{\partial C_1}{\partial Y}\right\} < 0;$$

$$(A3.12) \qquad \frac{\partial a}{\partial t} = \left[\{a(1+t)^{-1}\} - \left\{\frac{1}{D}\right\}\{(1+t)^{-3}(1+r)[(wf') - (Y - w[L_0 - L])(f'')](g'')E\{zh''\}\}\right]$$

or,

$$(A3.13) \qquad \frac{\partial a}{\partial t} = \left\{\left.\frac{\partial a}{\partial t}\right|_{Eu} - (1+t)^{-1}[Y - w(L_0 - L)]\frac{\partial a}{\partial Y}\right\} \gtrless 0.$$

Turning to savings, from (3.33), (A3.8) and (A3.10),

$$(A3.14) \qquad \frac{\partial S}{\partial t} = \{S(1+t)^{-1}\} + \left\{\frac{1}{D}\right\}\{(1+t)^{-3}[(wf') - (Y - w[L_0 - L])(f'')](g'')E[(z)^2h'']\}$$

or,

$$(A3.15) \qquad \frac{\partial S}{\partial t} = \left\{\left.\frac{\partial S}{\partial t}\right|_{Eu} - (1+t)^{-1}[Y - w(L_0 - L)]\frac{\partial S}{\partial Y}\right\} \gtrless 0.$$

### *A.4. The Wealth Tax.*

The analytics of the wealth tax, being rather similar to that of the consumption tax, will be treated very briefly. First let us note the form of the determinantal value of the Hessian in this context.

$$D = [(1-t)^{-2}(f'')(g'')\}E\{(z)^2h''\} + \{(1-t)^2(1+r)^2\}[(f'')+(w)^2(1-t)^2(g'')]\{K\}] < 0. \tag{A4.1}$$

***Specific Effects.*** We now go on to state the specific effects of taxation. For the primary choice variables, we obtain:

$$\frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\} [\{(1-t)(f')\}[(g'')E\{(z)^2h''\} + (1+r)^2\{K\}] + \{(w)(1-t)^3(1+r)^2\}[Y - w(L_0 - L)](g'')\{K\}] \gtrless 0.$$

or

$$\frac{\partial L}{\partial t} = \left[\frac{\partial L}{\partial t}\bigg|_{Eu} - \{1-t)^{-1}[Y - w(L_0 - L)]\}\frac{\partial L}{\partial Y}\right] \gtrless 0; \tag{A4.2}$$

$$\frac{\partial C_1}{\partial t} = \left\{\frac{1}{D}\right\} [\{w(1-t)^2(1+r)^2\}(f')\{K\} - \{(1-t)^2(1+r)^2\}[Y - w(L_0 - L)](f'')\{K\}] < 0.$$

or

$$\frac{\partial C_1}{\partial t} = \left[\frac{\partial C_1}{\partial t}\bigg|_{Eu} - \{(1-t)^{-1}[Y - w(L_0 - L)]\}\frac{\partial C_1}{\partial Y}\right] < 0; \tag{A4.3}$$

$$\frac{\partial a}{\partial t} = \left\{\frac{1}{D}\right\}\left[\left\{\left(\frac{aD}{1-t}\right) - w(1-t)(1+r)(f')(g'')E\{zh''\}\right\} + (1-t)(1+r)[Y - w(L_0 - L)](f'')(g'')E\{zh''\}\right]$$

or,

$$\frac{\partial a}{\partial t} = \left[\frac{\partial a}{\partial t}\bigg|_{Eu} - \{(1-t)^{-1}[Y - w(L_0 - L)]\}\frac{\partial a}{\partial Y}\right] \gtrless 0. \tag{A4.4}$$

From equation (3.42) of the text, and (A4.2)–(A4.3) above, we find the effect on savings:

$$\frac{\partial S}{\partial t} = \left\{\frac{1}{D}\right\} [(1-t)^2(wf')(g'')E\{(z)^2h''\} - (1-t)^2[Y - w(L_0 - L)](f'')(g'')E\{(z)^2h''\}] < 0,$$



...iven (3.44)

$$\frac{\partial S}{\partial t} = \left[\frac{\partial S}{\partial t}\bigg|_{Eu} - \{(1-t)^{-1}[Y - w(L_0 - L)]\}\frac{\partial S}{\partial Y}\right] < 0.$$

### *The Income Tax.*

From the FOC's [equations (3.50)–(3.52)], the elements of the Hessian and ...rincipal minors can be obtained in the usual way. The value of the Hessian ...erminant is given by:

$$D = (f'')(g'')E\{(\bar{z})^2h''\} + (1+\bar{r})^2[(f'') + (\bar{w})^2(g'')]\{\bar{K}\} \tag{.1}$$

...ere

$$\{\bar{K}\} = [E\{h''\}E\{(\bar{z})^2h''\} - (E\{\bar{z}\}h''\}^2],$$

...{$\bar{r}$, $\bar{w}$, and $\bar{z}$} are evidently the net of tax values.

In obtaining expressions (3.53)–(3.55), it is necessary to refer to the differential ...ture consumption quite frequently:

$$dC_2 = [(1+\bar{r})\{(\bar{w})dL - dC_1\} + (\bar{z})da] - [(1+2\bar{r})[Y - w(L_0 - L)] - rC_1 + az]\,dt + (1-t)(1+\bar{r})dY. \tag{.3}$$

***...alth Effects.*** Setting $dt = 0$, given (3.53)–(3.55), and the [$D$]-marix, we obtain

$$\frac{\partial L}{\partial Y} = (-)\left\{\frac{1}{D}\right\}\left[\{(1-t)(\bar{w})(1+\bar{r})^2\}(g'')\{\bar{K}\}\right] < 0; \tag{.4}$$

$$\frac{\partial C_1}{\partial Y} = \left\{\frac{1}{D}\right\}\left[(1-t)(1+\bar{r})^2(f'')\{\bar{K}\}\right] > 0; \tag{A5.5}$$

$$\frac{\partial a}{\partial Y} = (-)\left\{\frac{1}{D}\right\}[(1-t)(1+\bar{r})(f'')(g'')E\{\bar{z}h''\}] > 0. \tag{.6}$$

Also, from (3.47),

$$\frac{\partial S}{\partial Y} = \left\{\frac{1}{D}\right\}[(1-t)(f'')(g'')E\{(\bar{z})^2h''\}] > 0. \tag{A5.7}$$

*Compensated Effects.* Imposing (3.56) on (3.53)–(3.55), the vector of $d$'s become

$$(A5.8)\quad \begin{cases} d_1|_{\mathrm{Eu}} = [\{(1+2\bar{r})(1+\bar{r})^{-1}\}(f')(1-t)^{-1}] \\ \qquad\qquad + (a)(1-t)^{-1}(\bar{w})(1+\bar{r})E\{\bar{z}h''\} \\ d_2|_{\mathrm{Eu}} = -[\{(1+\bar{r})^{-1}(rg')\} + \{a(1+\bar{r})(1-t)^{-1}E\{\bar{z}h''\}] \\ d_3|_{\mathrm{Eu}} = [(a)(1-t)^{-1}E\{(\bar{z})^2h''\}] \end{cases}$$

Given (A5.8), the compensated effects of taxation described by equations (3.57)–(3.59) can be obtained in the usual way. Some simplifications make use of the fact that

$$(A5.9)\qquad (f') = (\bar{w})(g'), \quad \text{[from (3.50)–(3.51)]},$$

and

$$(A5.10)\qquad [(1+2\bar{r})(1+\bar{r})^{-1}] = [1 + (\bar{r})(1+\bar{r})^{-1}].$$

*Specific Effects.* In order to obtain the specific effects of taxation, we return to (3.53)–(3.55) and set $dY = 0$, and work with the new vector of $d$'s so obtained. Given the $[D]$-matrix, we obtain

$$\begin{aligned} \frac{\partial L}{\partial t} = \left\{\frac{1}{D}\right\} \Bigg[ & \left(\frac{(f')}{1-t}\right) \{\{(1+2\bar{r})(1+\bar{r})^{-1}\} \\ & (g'')E\{(\bar{z})^2h''\} + (1+\bar{r})^2\{\bar{K}\}\} \\ (A5.11)\qquad & + \{(\bar{w})(1+\bar{r})[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1](g'')\{K\}\}\Bigg]. \end{aligned}$$

Given (3.57) and (A5.4), we may restate

$$\begin{aligned} (A5.12)\quad \frac{\partial L}{\partial t} = \Bigg\{ & \frac{\partial L}{\partial t}\bigg|_{\mathrm{Eu}} \\ & -\{(1-t)(1+\bar{r})\}^{-1}[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1]\frac{\partial L}{\partial Y}\Bigg\}. \end{aligned}$$

For consumption and risk-taking, we have, respectively,

$$\begin{aligned} (A5.13)\quad \frac{\partial C_1}{\partial t} = \left\{\frac{1}{D}\right\} \Big[ & (1+\bar{r})^2(wf')\{\bar{K}\} - (1+\bar{r})^{-1}(rg')(f'')E\{(\bar{z})^2h''\} \\ & -(1+\bar{r})[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1](f'')\{\bar{K}\}\Big]; \end{aligned}$$



$$\begin{aligned} \frac{\partial a}{\partial t} = \left\{\frac{1}{D}\right\} \Big[ & \{a(1-t)^{-1}\{D\} - (1+2\bar{r})(wf')(g'')E\{(\bar{z})h''\} \\ (A5.14)\qquad & - (rg')(f'')E\{\bar{z}h''\}\} \\ & + [(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1](f'')(g'')E\{(\bar{z})h''\}\Big]; \end{aligned}$$

In the derivation of wealth and compensated effects, we may also write

$$(A5.15)\quad \frac{\partial C_1}{\partial t} = \left\{ \frac{\partial C_1}{\partial t}\bigg|_{\mathrm{Eu}} - \{(1-t)(1+\bar{r})\}^{-1}[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1]\frac{\partial C_1}{\partial Y}\right\},$$

$$(A5.16)\quad \frac{\partial a}{\partial t} = \left\{ \frac{\partial a}{\partial t}\bigg|_{\mathrm{Eu}} - \{(1-t)(1+\bar{r})\}^{-1}[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1]\frac{\partial a}{\partial Y}\right\}.$$

For savings, given (3.47), (A5.11) and (A5.13), we obtain

$$\begin{aligned} (A5.17)\quad \frac{\partial S}{\partial t} = \{(1+\bar{r})^{-1}(rS)\} + \left\{\frac{1}{D}\right\} \Big[ & (1+\bar{r})^{-1}[(1+2\bar{r})(wf')(g'') \\ & + (rg')(f'')E\{(\bar{z})^2h''\}] \\ & - (1+\bar{r})^{-1}[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1](f'')(g'')E\{(\bar{z})^2h''\}\Big]. \end{aligned}$$

$$(A5.18)\quad \frac{\partial S}{\partial t} = \left\{ \frac{\partial S}{\partial t}\bigg|_{\mathrm{Eu}} - \{(1-t)(1+\bar{r})\}^{-1}[(1+2\bar{r})\{Y - w(L_0 - L)\} - rC_1]\frac{\partial S}{\partial Y}\right\}.$$

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