CES Working Paper Series

HOTELLING CONFRONTS CAPM:
A TEST OF THE THEORY OF
EXHAUSTIBLE RESOURCES

Margaret E. Slade
Henry Thille

Working Paper No. 73

1994

*Center for Economic Studies
University of Munich
Schackstr. 4
80539 Munich
Germany
Telephone & Telefax:
++49-89-2180-3112*

We would like to thank Denise Young for letting us have access to her data. We are also grateful to the following people for thoughtful comments: Avner Bar-Ilan, Richard Blundell, Burton Hollifield, Robert Marks, Costas Meghir, Usha Mittoo, Stylianos Perrakis, Makoto Saito, Raman Uppal, Denise Young, and participants in seminars at the AERE meetings in Boston, the Canadian Natural-Resources Study Group in Ottawa, the University of British Columbia and University College London. Finally, we would like to thank CES at the University of Munich for support.

HOTELLING CONFRONTS CAPM:
A TEST OF THE THEORY OF
EXHAUSTIBLE RESOURCES

Abstract

A model of pricing of natural-resource commodities that integrates financial and product markets is derived and tested. In the model, price-taking firms extract from a deposit of an exhaustible resource; production is risky, and investors own shares in the mine. Investors can diversify their risk by holding a portfolio of other assets. The model thus unifies two strands of the economic and financial literature: one that builds on the Hotelling model of resource depletion and the other that extends the Capital-Asset-Pricing Model. Tests of the predictions make use of shadow prices that were estimated from a panel of fourteen small Canadian copper mines. The rate of growth of shadow prices is related to technological variables such as the rate of change of costs due to depletion and to financial variables such as the risk-free rate of return, the rate of return on the market portfolio, and the rate of growth of macroeconomic aggregates. Our tests fail to reject the theoretical restrictions that are implied by the Hotelling/CAPM model.

Key words: Exhaustible resources, Hotelling model of resource depletion, capital-asset pricing model (CAPM), unbalanced panel, generalized method of moments (GMM).

JEL Classification: C23, G12, L71, Q30

*Margaret E. Slade
Henry Thille
Department of Economics
The University of British Columbia
Vancouver, B. C.
Canada V6T 1Z1
(604) 822-4608
Fax: (604) 822-5915*

I: Introduction

In 1931, Hotelling published his classic article on the economics of exhaustible resources. Since that time, scores of authors have generalized and extended the basic model.¹ Moreover, there is really no competing theory that explains the supply and pricing of nonrenewable resources. By any standard, therefore, Hotelling's effort has been successful.

Unfortunately, when we turn to the empirical side, the picture is not as rosy.² Indeed, the majority of tests of the Hotelling model produce negative results. It is interesting, however, that one of the few models that is not rejected by the data originates in the finance literature. Miller and Upton's hypothesis (1985a) concerning the asset value of a mine receives some empirical support.

Our paper contains an empirical test that is based on the Capital-Asset Pricing Model or CAPM (Sharpe 1964 and Lintner 1965), which also originates in the finance literature. It builds on the theoretical work of Hartwick and Yeung (1988) and Gaudet and Khadr (1991) that extends the Hotelling model to include risky-resource production and the possibility of diversifying risk by holding a portfolio of other assets.

The organization of the paper is as follows. In the next section, we modify the model of Gaudet and Khadr (1991) to include nonconstant returns to scale and stock effects in the cost function. In other words, extraction cost is allowed to vary with the rate of extraction and as the reserve base is depleted. This involves changing only the resource-extraction sector of the economy. We do not discuss efficient production of the composite commodity or the consumer's portfolio allocation for two reasons. First, these sectors of the model would remain unchanged, and second, we derive our estimating equation as a no-arbitrage condition that holds in a somewhat broader context than a competitive equilibrium.

Section III describes previous empirical tests of the Hotelling model. We begin with papers that examine the time-series behavior of prices, p , and are thus not direct tests of the Hotelling model. We then move to models that assess the rate of growth of prices, $\dot{p}$, which are appropriate

¹ Extensions include extraction costs that vary with cumulative extraction (Levhari and Levitan 1977), uncertainty with respect to the size of the reserve base (Loury 1978) or the magnitude and timing of discoveries (Pindyck 1980), and strategic behavior in resource markets (Salant 1976), to name just a few.

² Empirical articles include Barnett and Morse (1963), Heal and Barrow (1980), Smith (1981), Slade (1982), Stollery (1983), Halvorsen and Smith (1984 and 1991), Farrow (1985), Miller and Upton (1985), and Young (1992).

when marginal-extraction cost is zero. Finally, we look at models that exploit duality theory to evaluate the rate of growth of shadow prices, $\dot{\lambda}/\lambda$. These models are appropriate when marginal cost depends on the rate of extraction and/or cumulative extraction.

Section IV describes our data and estimation technique. We make use of the cost function for copper mining and milling that was estimated by Young (1992).¹ The basic data consist of a panel of input and output prices and quantities for fourteen small Canadian copper mines. Our econometric model, a stochastic difference equation, relates the rate of growth of shadow prices to technological variables such as extraction cost and to financial variables such as the risk-free rate of return, the rate of return on the market portfolio, and rates of growth of macroeconomic aggregates. Our estimating equation, which nests both the Hotelling model and the CAPM, can be used to assess these theories. Indeed, each model implies particular restrictions on the parameters of this equation. In addition to the basic CAPM, we consider extensions that allow us to evaluate the arbitrage-pricing theory of Ross (1976), Merton's (1973) intertemporal CAPM, and Brennan's (1970) after-tax CAPM.

The fourteen copper mines were fairly short lived, mostly less than twenty years, and their years of operation did not coincide perfectly. We thus have an unbalanced panel. Other econometric complications arise due to the presence of measurement error in the shadow prices and depletion effects, unobservable expectation variables, and nonconstant variances of expected returns. We use a generalized method of moments (GMM) technique for dynamic-panel data to obtain efficient estimates in the presence of these complications.

Section V presents the empirical results and section VI summarizes and concludes. To anticipate, we cannot reject the joint Hotelling/CAPM model. Moreover, the financial and macroeconomic variables that distinguish our test from previous are significant in most versions of the model that we estimate. It therefore seems that the omission of risk and the possibility of risk diversification provides at least a partial explanation for the poor empirical performance of Hotelling's theory.

¹ These data were provided by Denise Young.

II: The Theoretical Model

IIa: The Hotelling Model

The simplest Hotelling model predicts that, in the absence of extraction cost, the price p of an exhaustible-resource commodity will rise at the rate of interest r , $\dot{p} = r$. When marginal-extraction cost is nonzero, it is the shadow price, $\lambda = p - C_q$, that rises at the rate of interest, $\dot{\lambda} = r$, where C is total-cost and a subscript denotes a partial derivative. Finally, when there are depletion effects so that extraction cost depends on the level of remaining reserves R , the shadow price rises at a rate that is less than the rate of interest. This lower rate of shadow-price appreciation, which is given by $\dot{\lambda} = r + C_R/\lambda$, reflects the user cost associated with deterioration of the quality of ore mined in the future.

These standard results from the theory of exhaustible resources apply to resource commodities sold in competitive markets under conditions of certainty. We maintain the competitive-market assumption here but relax the certainty or perfect-foresight assumption. When price or extraction cost is a random variable, the mine is a risky asset, and when there are other risky assets in the economy, mine owners can diversify their risk. We therefore have a problem in portfolio choice. Before developing this problem formally, however, we examine an asset-pricing model without extraction.

IIb: The Capital-Asset Pricing Model (CAPM)

In the simplest version of CAPM, risk-averse investors have homogeneous beliefs and two-fund separation holds in financial markets.¹ Under two-fund separation, portfolio decisions can be separated into the choice of an optimal combination of a (single) risky asset and the riskless asset (Tobin 1958). Moreover, a combination of two-fund separation and homogeneous beliefs implies that it is possible to price any risky asset using a convex combination of the risk-free rate of return r and the expected rate of return on the market portfolio r^m . In other words, all investors hold the same portfolio of risky assets, the market portfolio.

¹ Sufficient conditions for two-fund separation are that traders have preferences that are quadratic in wealth or that all risky-asset returns are normally distributed.

Formally, the basic CAPM relationship says that if r^j is the expected rate of return on risky asset j , then¹

$$r^j = r + \beta^j (r^m - r). \quad (1)$$

In other words, the rate of return on asset j consists of the risk-free rate r plus an asset-specific risk premium $\beta^j(r^m - r)$. Moreover, the 'beta' coefficient, β^j , that determines the risk premium is the ratio of the covariance of r^j and r^m to the variance of r^m , $\beta^j = \sigma_{jm}/\sigma_m^2$.

There are a number of restrictive assumptions that underlie the derivation of the CAPM. Perhaps the most important from our point of view is the static nature of the model, which must be combined with a dynamic resource-extraction problem. We also consider relaxing the assumptions that imply two-fund separation and introducing nonneutral taxes. These extensions are of interest because they have empirical implications.

Intertemporal CAPM (ICAPM)

A dynamic version of CAPM in which investors trade continuously was developed by Merton (1973). In Merton's model, asset prices are assumed to evolve according to the relationship

$$\frac{dp_t^j}{p_t^j} = r_t^j dt + \sigma_j dz_j, \quad j=1, \dots, N, \quad (2)$$

where r_t^j is the instantaneous (certain) rate of price appreciation for the j th asset and dz_j is the increment of a stochastic process z_j that obeys Brownian motion. In other words, z_j is a Wiener process.² Equation (2) therefore implies that current return is known exactly, but that uncertainty about future returns grows with the time horizon.³

¹ For a survey of CAPM, see Brennan (1987).

² If z is a Wiener process, then for any partition of the time interval, $t_0, t_1, t_2, \dots$, the random variables $z(t_1) - z(t_0), z(t_2) - z(t_1), \dots$ are independently and normally distributed with mean zero and variances $t_1 - t_0, t_2 - t_1, \dots$, respectively.

³ Processes such as (2) are called Ito processes and are continuous but not differentiable. For an introduction to stochastic control and the Ito calculus, see Kamien and Schwartz (1981).

If investors maximize the expected utility of their lifetime consumption, and if the investment-opportunity set is constant over time, the ICAPM relationship is

$$r_t^i = r_t + \beta^i(r_t^m - r_t), \quad \beta^i = \sigma_{jm}/\sigma_m^2. \quad (3)$$

Equation (3) is the simplest version of the CAPM that is compatible with a dynamic model of resource extraction. Since it is virtually identical to the static CAPM, in what follows we refer to (3) as CAPM.

More generally, we expect current asset demands to be affected by the possibility of future changes in the investment-opportunity set. In particular, investors will demand assets as vehicles to hedge against unfavorable shifts in future investment possibilities. However, if $K-1$ state variables are sufficient to describe changes in the opportunity set, then there are K fundamental assets (portfolios or mutual funds) that span the space of asset returns. We assume that K is much less than the total number of assets N .

Unfortunately, ICAPM is silent about not only the identity of the fundamental assets or factors, but also about their number K . The theory, however, does suggest properties that the factors should possess. A hedging asset should be correlated with expected future changes in investment opportunities, which suggests choosing the rate of return on a long-term bond as a factor.

*Arbitrage Pricing Theory (APT)*¹

The arbitrage-pricing theory due to Ross (1976) was developed to deal with the restrictive assumptions that underlie two-fund separation (normality of returns or quadratic utility). Again we suppose that there are K fundamental assets (portfolios or mutual funds) that span the space of returns. If the unexpected part of each asset's return is a linear combination of the unexpected parts of the returns on the K fundamental assets, then the expected return of each asset is the same linear combination of the expected returns on the fundamental assets. CAPM is therefore a special case of APT with $K = 1$. APT, however, is more general in that it is an arbitrage relationship that holds outside of equilibrium.

ICAPM and APT can be integrated. For example, Chamberlain (1988) develops an intertemporal asset-pricing model that shows that, at

¹ For a survey of APT, see Huberman (1987).

each time t , asset prices obey the APT formula. Moreover, this formula is identical to the CAPM when $K = 1$ or to the ICAPM when $K > 1$. In our empirical work, we therefore make no distinction between the two multifactor models.

Just as with ICAPM, APT does not identify the factors or their number. Nevertheless, it suggests properties that they should possess. In particular, the time-series movements of the factors should explain a substantial fraction of the time series movements of the returns on the traded assets. In practice, rates of change of financial and macroeconomic aggregates are often used as factors (see, for example, Chen, Roll, and Ross 1986).

After-Tax CAPM

If firms pay dividends, the rate of return on the market portfolio is

$$r^m = \frac{dp^m + \text{div}}{p^m}, \quad (4)$$

where p^m is the price of one share in the market portfolio, and div is the dividend that is received by the owner of that share.

A complication arises, however, when dividends are taxed more heavily than capital gains, as was true in the U.S. and Canada.¹ This complication led to the development of an after-tax CAPM (Brennan 1970) and an empirical literature investigating the existence of tax effects on stock prices.² When unfavorable tax treatment causes investors to demand higher pre-tax returns on dividend-paying stocks, capital gains and dividend yields should be separated. Moreover, each should be multiplied by $(1 - T_i^d)$ where T_i^d is the marginal rate of taxation on asset class i .

In our empirical tests, we extend the Hotelling/CAPM model (HCAPM) that is derived below to include hedging, APT, and tax factors. The empirical debates that surround CAPM, however, are not our primary focus; instead we wish to see how alternative specifications of the financial model affect HCAPM.

¹ A brief discussion of Canadian taxation of capital gains, dividends, and resource rents can be found in appendix C.

² For example, Blume (1980) tests tax effects using U.S. data, whereas Morgan (1980) uses Canadian data. Both find evidence of dividend aversion.

IIC: The Hotelling/Capital-Asset-Pricing Model (HCAPM)

Modeling Uncertainty

When the assumptions that underlie CAPM hold, we can use a modified version of this relationship to determine the equilibrium rate of resource-shadow-price appreciation. We derive the expected rate of return to owning a mine when production is risky and owners and investors are risk averse. In other words, we obtain the appropriate risk-adjusted-discount rate for cash flows from mining. We analyze the optimization problem faced by the manager of a mine from which ore is extracted and sold in a competitive market at a parametric price p .

There are many ways to introduce uncertainty into natural-resource markets, some of which are mentioned in the first footnote. In this paper, we focus on continuous technology and demand shocks. In our model, mining cost depends on the rate of extraction, q , the level of remaining reserves, R , and a random productivity shock, Θ ,

$$TC = C(q, R, \Theta). \quad (5)$$

Furthermore, the rate of technical progress consists of a deterministic portion plus stochastic increments. In other words, Θ is an Ito process defined by

$$\frac{d\Theta}{\Theta} = \mu_{\Theta} dt + \sigma_{\Theta} dz_{\Theta}, \quad (6)$$

where μ_{Θ} is the instantaneous rate of technical change.

With uncertain extraction, price p and the consumer's marginal utility of income M are also random variables. Following Gaudet and Khadr (1991), we begin with an exogenous specification of the evolution of p and M . Gaudet and Khadr go on to show that, when the technology shock Θ is a geometric Brownian motion (GBM) as in (6), in a competitive equilibrium, both p and M evolve as GBMs. We do not, however, reproduce their results here.

Moreover, although our arbitrage condition (equation 11) is derived in a partial-equilibrium context, it can also be obtained in a general-

equilibrium framework.¹ However, as this is a standard problem in finance, it is omitted here. Readers who are interested in the details are referred to Gaudet and Khadr (1991), who derive the envelope condition in a natural-resource context. The principal difference between our problem and theirs arises from allowing nonconstant returns to scale and including R in the cost function. The latter modification leads to a different risk-free rate of return ($r + C_R/\lambda$ versus r). An advantage to our approach is that our equilibrium condition holds in a broader set of circumstances. It is, however, consistent with a competitive equilibrium

We assume that price follows an Ito process of the form

$$\frac{dp}{p} = \mu_{pt}dt + \sigma_p dz_p. \quad (7)$$

In (7), μ_{pt} , the instantaneous rate of price appreciation, varies over time as *industry* reserves change. Industry reserves, however, are exogenous to the firm. The price-taking firm knows the current rate of price appreciation exactly, but uncertainty about future returns grows with the time horizon.

Pindyck (1981) was perhaps the first economist to assume that resource prices follow a GBM. The assumption has a number of advantages. For example, when there is no uncertainty ($\sigma_p = 0$), prices evolve according to the standard Hotelling rule. Moreover, when the resource is not exhaustible ($\mu_{pt} = 0$), prices evolve according to the rule for commodities that are traded in efficient markets. For example, it is often assumed that percent changes in the prices of agricultural commodities are white noise. Finally, (7) allows for unforeseen events such as entry and exit or, as Pindyck (1981) points out, even irrational behavior on the part of a resource cartel.²

Finally, consumers receive utility from consuming a composite good. We let M be the marginal utility of income of (identical) risk-averse agents; M transforms dollars into utils from consuming the composite

¹ Specifically, one can introduce the consumer's optimal-portfolio choice and derive the 'envelope' condition that relates the share of her portfolio that is held in the form of the risky asset to its excess return over the safe asset and the variance of its return. Furthermore, it is possible to obtain a consumption beta that is related to her rates of relative-risk aversion and intertemporal substitution.

² Clearly, a resource cartel violates a competitive-market assumption. In that sense, our partial-equilibrium framework is more general. We only require that the firms of interest are price takers and need not specify how the expected-price trajectory is determined.

commodity. For example, it could be inversely proportional to the dollar price of the consumption good. In other words, when the price of consumption is high due to scarcity of the composite commodity, dollars have more value. Since we can think of M as an exchange rate -- the price at which dollars from extraction can be transformed into utils -- it seems reasonable to assume that M also follows a GBM,¹

$$\frac{dM}{M} = \mu_M dt + \sigma_M dz_M. \quad (8)$$

Just as with the resource price process, we take the evolution of M to be exogenous. However, we know from Gaudet and Khadr that our assumption is consistent with a competitive equilibrium.

We allow the three diffusion processes, (5), (7), and (8) to be correlated.

Efficient Resource Extraction

The manager chooses an extraction path $\{q_t\}$, $0 < t < \infty$, to maximize the expected-present value of the utility of profits. This stochastic-control problem has a value function V that is given by

$$\begin{aligned} V(R, p, \Theta, M) &= \max_{\{q_\tau\}} E_t \left\{ \int_t^\infty e^{-r(\tau-t)} M_\tau [p_\tau q_\tau - C(q_\tau, R_\tau, \Theta_\tau)] d\tau \right\}, \\ &=: \max_{\{q_\tau\}} E_t \left\{ \int_t^\infty e^{-r(\tau-t)} M_\tau \pi_\tau d\tau \right\}, \end{aligned} \quad (9)$$

subject to

$$\dot{R}_\tau = -q_\tau, \quad R_\tau \geq 0, \quad q_\tau \geq 0, \quad (10)$$

(5), (7), and (8). Instantaneous profit π is assumed to be concave in q .

The first-order conditions for this problem can be manipulated to yield

¹ If v is the dollar price of the composite good and M is proportional to $1/v$, then if v evolves according to a GBM, which is a common assumption for prices, $1/v$ also follows a GBM.

Proposition 1: When the mine is managed optimally, the evolution of the shadow price, $\lambda = p - C_q$, is governed by

$$\frac{1}{\lambda} E_t \frac{d\lambda}{dt} = \frac{\dot{\lambda}}{\lambda} = r + \frac{C_R}{\lambda} + \beta(r^m - r). \quad (11)$$

Proof: See appendix A.

Discussion

Equation (11) is the HCAPM relationship. We can compare it to the case where there is no uncertainty in the extraction sector. When $\sigma_\lambda^2 = 0$, λ is not a random variable, $\sigma_{\lambda m} = 0$, and we have the standard Hotelling relationship, $\dot{\lambda}/\lambda = r + C_R/\lambda$. When production is risky, in contrast, $\dot{\lambda}/\lambda$ is the average expected rate of change of the shadow price, which is equal to the risk-free rate of return for natural-resource assets, $r + C_R/\lambda$, plus the risk premium, $\beta(r^m - r)$. Finally, when $C_R = 0$, the average expected rate of resource-price appreciation obeys the standard CAPM relationship.

If β were negative (i.e., if the expected rate of shadow-price appreciation were negatively correlated with the expected rate of return on the market portfolio), investors would be willing to own shares in mines that yield less than the risk-free rate of return for natural-resource assets. This would be true because holding an asset with a negative beta would reduce the risk associated with holding the market portfolio. Normally, however, β will be positive and investors will demand a rate of return that exceeds $r + C_R/\lambda$. This is their payment for bearing risk.

Our condition (11) is derived as a no-arbitrage condition; we do not solve for the competitive equilibrium. The assumption of nonconstant-marginal cost is therefore important. Indeed, the mine manager's only control variable is extraction. If variations in q did not affect marginal cost, the manager could not ensure that (11) held. In particular, when marginal cost is constant, price must change until, in equilibrium, (11) holds. With our model, in contrast, even when (7) does not result from a competitive-equilibrium, (11) is valid.

Equation (11) forms the basis of our empirical tests.

III: Previous Tests of the Hotelling Model.

IIIa: Reduced-Form Tests

The Behavior of Prices

When we consider a century of data, a striking feature is the decline in the relative prices of the majority of mineral commodities. This stylized fact contrasts with the prediction of the simplest Hotelling model of exponential increases in the relative prices of natural-resource commodities.

Perhaps the first to analyze mineral-commodity prices formally were Barnett and Morse (1963), who looked at relative-price trends in an attempt to uncover evidence of natural-resource scarcity. They concluded that, because real prices had fallen over time, scarcity was not a problem. Moreover, in an update, Barnett (1979) reached the same conclusion.

Other researchers who have examined price trends are not in complete agreement with Barnett and Morse. For example, Smith (1979) looked at the stability of the coefficients of estimated price-trend relationships and decided that the data are too volatile to support definitive conclusions. And Slade (1982) found that, after substantial initial declines, in the 1970's there was some evidence of an upturn in the real-price paths of many mineral commodities. Since that time, however, prices have been increasingly volatile with large run ups followed by equally large declines (Slade 1991), but there is little evidence of sustained trends.

Other reduced-form tests were performed by Heal and Barrow (1980) and Smith (1981), who related $\dot{p}$ to r and other variables and found little support for the Hotelling model.

Given these stylized facts, it is of interest to examine the simple Hotelling model to see if it can be modified to produce prices that fall over substantial periods of time. There are several modifications that can do the trick.

First, in the simple model, the initial stock of the resource is known with certainty as of time zero. In practice, however, extraction of known deposits proceeds simultaneously with exploration for previously unknown ore bodies. Indeed, large discoveries increase the size of the stock and can cause prices to fall. This issue is examined by Pindyck (1980).

Second, the simple Hotelling model does not allow for technical change. The idea that new methods of extraction can lower costs, however, can be developed formally. This issue is explored by Slade (1982), who shows that prices can fall when cost-lowering mining techniques are introduced.

Finally, substitute materials can be found that cause the demand for mineral commodities to shift inwards. Several authors (Heal 1976 and Hanson 1980) have assessed the behavior of resource prices and rental rates when a backstop technology is introduced and have shown that rents can fall in the presence of obsolescence.

Periods of falling prices are therefore consistent with variants of Hotelling's model. Nevertheless, in the long run prices should rise.

The Hotelling-Valuation Principle

A second reduced-form model was developed by Miller and Upton (1985a and b), who based their test on a less widely known implication of Hotelling's analysis of the optimal time pattern of exploitation of an exhaustible resource. Miller and Upton call their implication the Hotelling-Valuation Principle.

More precisely, they show that with the competitive-market model, the value of the reserves in any currently operating, optimally managed mineral deposit should depend solely on the current spot price net of marginal-extraction cost, regardless of when the reserves are extracted.¹ They refer to this proposition as the Hotelling Valuation Principle. They test their model using stock-market valuations of the oil and gas reserves of a sample of U.S. companies and find that the data are consistent with their Principle. Unfortunately, there have been few subsequent tests of the Hotelling Valuation Principle.

IIIb: Structural Tests

More formal tests of the Hotelling model rely on estimates of the technology of production of the extractive firm combined with Euler equations associated with dynamic-profit maximization. Examples include Stollery (1983), Farrow (1985), Halvorsen and Smith (1984 and 1991), and Young (1992).

¹ The Hotelling Valuation Principle does not hold, however, when there are stock effects in the cost function

These researchers have estimated dual restricted-cost or profit functions for individual mines or mining industries. They assume that the technology of mining involves extracting unprocessed ore, n , which is combined with other inputs to produce metal, q . In other words, both mining and refining are modeled. n , which is transferred inside a vertically integrated firm, is treated as a fixed factor in the production of q .¹

Once the firm's technology is known, shadow prices or rental rates, λ , can be approximated by one of two methods. They can be calculated as i) the difference between price and marginal cost, $p - C_q$ (Stollery 1983, Farrow 1985, and Young 1992) or as ii) the shadow price of the unpriced ore to the vertically integrated metal producer, $-C_n$ (Halvorsen and Smith 1984 and 1991). These two estimates of λ do not measure the same thing. The first is the shadow price of one unit of contained metal *in situ*, whereas the second is the shadow price of one unit of ore of the grade that is currently being extracted, also *in situ*.

Notice that the structural approach is able to deal with some of the shortcomings of the reduced-form models. In particular, technical change and demand shifts can be modeled. Moreover, when the data pertain to firms whose reserves are known at the beginning of the estimation period, there are no new discoveries. Nevertheless, although some studies support the Hotelling model, its overall performance has been poor. In particular, estimated shadow prices for some commodities decline at an even faster pace than product prices.

We thus see that, with a few exceptions, there is little empirical support for the predictions of the Hotelling model. This can mean either that firms do not maximize profits or that the model is too simple to explain observed behavior. As the second explanation seems more fruitful, it is explored in our empirical tests.

IV: Data and Empirical Specification

IVa: Data and Preliminary Data Analysis

The raw data, which were collected by Young (1992), are a panel of observations on fourteen small Canadian copper-mining firms. These

¹ In other words, the market price of n is not relevant. It is assumed, however, that n is optimally chosen.

data consist of prices and quantities of inputs, outputs, and ore grades for each year of operation of each mine. The firms were chosen to conform as closely as possible to the assumptions of the Hotelling model. For example, all firms were small in the domestic as well as the international market and can therefore be considered to be price takers. Moreover, each operated in a small geographic area where reserves were known fairly accurately at the time that the mine opened. Finally, mines were excluded if their lifetimes were too short to enable calculation of cumulative-production variables.¹

The fourteen mines jointly span the 1956-1982 period. No single mine, however, operated in every year. Observations for each firm end when the firm ceases production, is acquired by another firm, or becomes involved in other ventures. In the latter two instances, the copper-cost data can no longer be separated from other cost data. These data are described in greater detail by Young (1992).

The data were used to estimate translog-cost functions for mining and milling. In other words, Young estimated a function, $C(n, q, R, v)$, where n is extraction, q is the output of the milling process, R is remaining reserves, and v is a vector of input prices. This function is a second-order approximation to an arbitrary technology.² Estimation of the HCAPM relationship (equation 11) requires firm-level values of $C_q = \partial C / \partial q$ and $C_R = \partial C / \partial R$. Young provided us with these partial derivatives for each firm in each year.³

We reemphasize the importance of using marginal rather than average costs. With constant returns to scale, the shadow price λ is $p - c^i$, where c^i is a mine-specific constant. The mine manager therefore has no control variable that can ensure efficient extraction. Moreover, when all mines have the same risk characteristics, efficiency requires that all mines have the same average costs. The assumption of identical average costs, however, is clearly violated in practice. When marginal costs vary with the rate of extraction, in contrast, the optimizing behavior of mine

¹ When there are no discoveries, remaining reserves, are $R_t = R_0 - X_t$, where X_t is cumulative production.

² Young estimated four versions of her model that differ according to whether the capital stock is assumed to be quasi fixed or in long-run equilibrium and whether the dynamic restrictions associated with the Hotelling model are imposed.

³ We chose to use the data from the cost function in which capital is a variable factor, since the restricted cost function yielded positive estimates of C_R . Moreover, we chose the version that does not impose Hotelling's rule, since this rule will be tested.

managers leads to equality of the marginal utilities associated with each unit produced, regardless of how price is determined.

Estimation of (11) also requires data for the rate of return on a risk-free asset, r_f , and on the market portfolio, r^m . For the former we use the Canadian 90-day Treasury Bill rate, converted to an annual rate of return by averaging the monthly numbers.

We use the return on the Toronto Stock Exchange (TSE) 300 index as a measure of r^m . The TSE, which is the largest Canadian stock exchange and the seventh largest in the world, handles about 75 percent of Canadian trading. It might be preferable to use a wider measure of the market return. However, the TSE 300 is the broadest generally available index for Canadian stocks.¹ Moreover, since the TSE 300 is made up of large firms, the problem of thin trading is reduced. Dividends associated with holding the TSE 300 are separated from capital gains by subtracting the share-price index from the total-return index. This is done so that tax effects can be assessed.

We use a long-term bond rate for the hedging asset required by the intertemporal CAPM. This was chosen as the Canadian-Government 10-year bond rate and is denoted r^{lt} . The difference between r^{lt} and r_f is also a proxy for the term structure of interest rates.²

Finally, we must specify other "factors" for the Arbitrage Pricing Theory. As Huberman (1987) notes there are three methods of doing this. The first approach uses factor analysis to estimate the factor loadings. The second uses the estimated covariance of asset returns to choose factors in a somewhat more *ad hoc* fashion. Finally, the third approach, which is purely judgmental, uses a combination of economic theory and intuition to pick the factors. We follow Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) in choosing the third approach and make no clear-cut distinction between APT and ICAPM. In other words, we estimate a multi-factor model.

A priori, one expects general-economic variables to influence systematic or undiversifiable risk. Our additional factors are the growth rate of gross domestic product, the rate of inflation, and the Canadian/U.S.

¹ It is common to use a share or equally-weighted index of the stocks traded on an exchange as a proxy for r^m (see Eckbo, 1986, for example). Unfortunately, the data needed to construct such an index are not available as far back as 1956. We therefore follow Mehra and Prescott (1985) and use a less comprehensive index.

² Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) use this proxy.

\$ exchange rate.¹ The current rate of economic activity is often regarded as an indicator of the health of the economy. The variable that we use in the regression analysis is the deviation of GDP/GDP from trend (GDFT). Changes in the rate of inflation influence nominal cash flows as well as the nominal rate of interest. For the regression analysis we use deviations from the trend in the rate of inflation of the CPI (CDFT). Finally, the exchange rate influences foreign investors' willingness to hold Canadian stocks. We again construct deviation-from-trend growth in the Canadian/U.S. \$ exchange rate (EDFT).

Capital gains and dividend data were found in *TSE 300 Indices* (1991), the consumer and producer-price indices were taken from *CANSIM*,² and all other data came from the *Canadian Economic Observer - Historical Supplement* (Statistics Canada #11-210).

Trading takes place continuously; our data, however, are annual. We therefore replace the stochastic differential equation (11) with a stochastic difference equation in which rates of growth are calculated as

$$\left(\frac{\dot{x}}{x}\right)_t = \frac{x_t - x_{t-1}}{x_{t-1}} \quad (12)$$

All rates of growth are calculated from yearly-average data. In particular, the capital-gains portion of r^m is calculated as the rate of growth of the average share-price index. We do this to be consistent with the construction of λ and C_R . In addition, we construct the risk-free rate of return and the long-term bond rate for year t by taking averages over years t and $t-1$ (see Washburn and Binkley 1990 for a discussion of this issue). Finally, all estimated trends are linear.

Table I reports summary statistics for each variable. Variable mnemonics used in the tables are as follows:

LDOT	the growth rate of the shadow price, $\dot{\lambda}/\lambda$,
CRL	$C_R t/\lambda_{t-1}$,
RF	the risk free rate of return, r_f ,

¹ Chan, Chen, and Hsieh (1985) find that industrial production, time-varying risk premia, the term structure, and to a lesser extent the rate of inflation influence expected stock returns. We use GDP instead of industrial production because no consistent measure of industrial production exists for Canada over the period that we are studying. An exchange-rate variable is not often included in U.S. studies, but it seems more important for Canada.

² CANSIM is a computerized data base that is maintained by Statistics Canada.

RMSPI	the rate of return on the share-price index, $\dot{p}^m/p^m$,
DIV	dividend returns, div_t/p_{t-1}^m ,
RLT	the long-term bond rate, r^{lt} ,
GDOT	the growth rate of nominal GDP,
GDFT	deviation from trend GDOT,
CPIDOT	the rate of inflation,
CDFT	deviation from trend CPIDOT,
EDOT	the rate of change of the exchange rate, and
EDFT	deviation from trend EDOT.

Table I shows means, standard errors, minima, and maxima for each variable. The first two variables are averaged over the 144 observations in the panel, whereas all other variables are averaged over the 27 years that the panel spans. We see that, as expected, shadow prices are on average positive whereas depletion effects are negative. Moreover, shadow prices are extremely volatile, a fact that is not surprising given the volatility of copper prices. Finally, the rate of return on the market portfolio (the sum of capital gains RMSPI and dividends DIV) is higher and more variable than RF.

As a prelude to more detailed analysis, the final column in table I lists correlation coefficients between each explanatory variable and LDOT. We see that LDOT is positively correlated with RMSPI but negatively correlated with DIV. In other words, the covariances between the rate of growth of the shadow price and the two sources of returns on the market portfolio are similar in magnitude but opposite in sign. LDOT is also positively correlated with GDOT; all other correlation coefficients are statistically insignificant. Nevertheless, the correlation between LDOT and RF is marginally significant (at the 10% level) and negative. A negative relationship is opposite to that predicted by the Hotelling model but consistent with previous empirical findings. Finally, there is little trend in real shadow prices.

IVb: Empirical Specification

The most general model that we estimate can be written as

$$\begin{aligned} LDOT_{it} = & \alpha_0 + \alpha_1 CRL_{it} + \alpha_2 RF_{it} + \alpha_3 RMSPI_{it} + \alpha_4 DIV_{it} \\ & + \alpha_5 RLT_{it} + \alpha_6 GDFT_{it} + \alpha_7 CDFT_{it} + \alpha_8 EDFT_{it} + \varepsilon_{it}, \\ & i=1 \dots 14, t=t_{0i} \dots T_i. \end{aligned} \quad (13)$$

All other models can be obtained from (13) by setting some of the coefficients equal to zero.

β does not appear directly in the estimating equation; the method that we use to obtain β from (13) is described in appendix B. Equation (13) implies that β is constant over time and across assets. We test for and accept homogeneity across firms. Unfortunately, our time series are too short to allow modeling of time-varying betas.¹

One can estimate equation (13) by ordinary-least squares (OLS). There are a number of reasons, however, why OLS yields inefficient parameter estimates. These include the panel nature of the data, measurement error, unobservability, and nonconstancy of the variance of LDOT.

To illustrate, t_{0i} and T_i in (13) are the beginning and ending periods for firm i ; the unbalanced nature of the panel is reflected in the fact that the sample for firm i is indexed by i . Second, both C_q and C_R were generated from an earlier regression, and the variables LDOT and CRL are nonlinear functions of these generated variables. Nonlinear measurement error is therefore introduced.² Third, equation (13) contains realized values of unobservable expectational variables (i.e., rates of return on portfolios and rates of growth of macro aggregates). Finally, variances of expected returns are unlikely to be constant over time.³

To circumvent these problems, we use a generalized method of moments (GMM) estimator for unbalanced panels that is described in

¹ Bollerslev et. al. (1992) survey Arch models of time-varying betas. Campbell (1987) and Harvey (1989) use another approach in which conditional expectations are linear functions of instrumental variables.

² For a discussion of this problem, see Pagan (1984).

³ For example, financial markets experience periods of both relative calm and extreme volatility.

appendix D.¹ In addition to lagged values of variables in equation (13), we use the year, the rate of change of the producer-price index, and the rate of growth of aggregate consumption as instruments.

With respect to the unobservables, we assume that investors have rational expectations and use all information efficiently in forming forecasts. GMM minimizes the correlation between any information known at time t and the period- t innovations. Since our variables are rates of growth, variables dated $t - 2$ or earlier can act as valid instruments. As a check on the model, we present tests of instrument validity to ensure that our instrument set is uncorrelated with the residuals from our estimated models. In addition, each explanatory variable is regressed on the set of instruments to assess their explanatory power.

The use of averaged data induces an MA(1) structure in the error term (Working 1960). Moreover, heteroskedasticity can be introduced by measurement error and by nonconstant variances of asset returns. We correct for heteroskedasticity and serial correlation of an unknown form using Newey and West's (1987) procedure.

If we rewrite equation (13) as $y = X\alpha + u$, and let Z be a matrix of instruments, the GMM objective function is

$$\min_{\alpha} u'Z(M)^{-1}Z'u. \quad (14)$$

where

$$M = \sum_{i=1}^{14} \sum_{j=0}^2 \sum_{t=t_0+i+j}^{T_i} w_j \hat{u}_t \hat{u}_{t-j} (Z_t' Z_{t-j} + Z_{t-j}' Z_t), \quad (15)$$

$$w_j = 1 - j/3.$$

In (14), $Z'u$ is the vector of moment restrictions, and M is the Newey/West (1987) weighting matrix. We use an iterative procedure to minimize (14). In the first iteration, the weighting matrix, M , is replaced by an identity matrix, and in subsequent iterations, (15) is used to estimate M from the previous-iteration residuals, $\hat{u}$.² This process continues until M converges.

¹ GMM estimation was introduced by Hansen (1982). For general discussions, see Davidson and MacKinnon (1992).

² The standard practice is to iterate only twice. However, Kocherlakota (1990) suggests that the small-sample properties of two-iteration estimators are worse than those of multi-iteration estimators. Our iterative procedure is described in appendix D.

The various models and tests are as follows:

i) *Hotelling*

A priori coefficient restrictions: $\alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.

Test: $\alpha_0 = 0, \alpha_1 = 1, \alpha_2 = 1$.

ii) *CAPM*

A priori coefficient restrictions: $\alpha_1 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.

Test: $\alpha_0 = 0, \alpha_2 + \beta = 1$.

iii) *HCAPM*

A priori coefficient restrictions: $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$.

Tests:

Hotelling: $\alpha_0 = 0, \alpha_1 = 1$.

CAPM: $\alpha_0 = 0, \alpha_2 + \beta = 1$.

Joint: $\alpha_0 = 0, \alpha_1 = 1, \alpha_2 + \beta = 1$.

iv) *K-Factor Model*

No a priori coefficient restrictions.

Tests:

Hotelling: $\alpha_0 = 0, \alpha_1 = 1$.

CAPM: $\alpha_0 = 0, \alpha_2 + \beta + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = 1$.¹

Joint: $\alpha_0 = 0, \alpha_1 = 1, \alpha_2 + \beta + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = 1$.

v) *After Tax CAPM*

No a priori coefficient restrictions.

RMSPI_t and DIV_t multiplied by one minus the relevant marginal rates of taxation for year t .²

Tests: Same as for K-Factor model.

For models iv) and v) we also test two-fund separation. The null hypothesis is $\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$, and rejection of the null is consistent with $K > 1$. In other words, it is evidence that additional factors contain information that is useful for pricing risky assets.³

¹ The appropriate test for the K-factor model is discussed by Huberman and Kandel (1987).

² See the appendix C for details.

³ We do not interpret rejection of the null as rejection of two-fund separation because our measure of r^m is somewhat limited. We only interpret rejection as consistent with $K > 1$.

V: Empirical Results

Table IIA contains OLS estimates of the equations that correspond to the different models. Standard errors are shown in parentheses under their respective coefficients, and standard errors corrected for heteroskedasticity of an unknown form (White 1980) are shown in square brackets. Finally, $\hat{\beta}$ is the estimate of beta (see appendix B).

In the CAPM and HCAPM equations, α_3 and α_4 , the coefficients of the components of the rate of return on the market portfolio, are significant but opposite in sign. In these equations, the corresponding betas are negative but small. With the K-factor models, in contrast, the coefficient of DIV becomes positive and insignificant, and the corresponding betas have reasonable values. For example, in the after-tax equation, beta is 0.43, which is comparable to the beta for gold mining reported in Guzman (1991). A low value for beta should not be surprising, since returns on real and financial assets are not highly correlated. Copper mines therefore command a lower rate of return on investment than the return on the market portfolio.

The additional factors in the K-factor models are significant and have reasonable signs. We see that shadow prices grow faster when GDP is growing, when the rate of inflation is falling, and when the Canadian dollar is appreciating. All of these results make intuitive sense. Furthermore, they indicate that financial and macro factors are important determinants of shadow prices.

Table IIB contains hypothesis tests for the various models. All tests are Wald χ^2 statistics, and P values for each test are shown in parentheses under the corresponding statistics. We see that, whereas the simplest Hotelling model is not rejected, this equation has no explanatory power (i.e., we cannot reject the hypothesis that all coefficients in the first equation jointly equal zero). In contrast, all other models have high overall explanatory power. With these equations, it makes sense to test the coefficient restrictions implied by the theory.

The χ^2 test statistics reveal that none of the theoretical restrictions implied by HCAPM (Hotelling, CAPM, and Joint in the table) is rejected. This regularity is very encouraging for the empirical relevance of the modified Hotelling model. We see that when financial variables that adjust for risk are added, the model receives considerable empirical support. Moreover, failure to reject the theoretical restrictions is not

vacuous, as with the simplest Hotelling model. Indeed, each of the last four equations contains coefficients that are individually significant as well as variables that jointly explain the data.

The restrictions corresponding to two-fund separation, in contrast, fare less well; they are rejected at any reasonable level of confidence. Rejection implies that investors use information contained in macro aggregates to price assets. This last finding is consistent with other tests of the APT (see Connor and Korajczyk 1992 for a survey).

A multi-factor model clearly outperforms all of the models with $K \leq 1$. For this reason, only the last equation in table IIA is estimated by GMM.¹ Table III contains the results from this estimation.

With the OLS estimates, correction for heteroskedasticity modifies the standard errors but leaves the estimated coefficients unchanged. With the GMM estimates, in contrast, the Newey/West correction changes both estimated coefficients and standard errors. This difference occurs because the second model is overidentified. For this reason, we show both first and final-stage estimates.

Comparing tables IIA and IIIA, we see that the coefficients of CRL and RMSPI have changed sign. Indeed, the depletion effect is positive in

IIIA, as predicted by theory, but the covariance between r^m and λ/λ is negative, contrary to expectation.² The GMM estimates therefore give more support to the Hotelling model but less to the CAPM and its extensions. With this exception, the tables are otherwise similar.

Turning to the GMM-hypothesis tests in table IIIB, we see that neither the Hotelling model, the CAPM, nor the HCAPM is rejected by the data. Moreover, the P values corresponding to the second two sets of coefficient restrictions are larger in table IIIB than in IIB. Finally, the evidence points to a model with K greater than one. Indeed, coefficient restrictions corresponding to the first three equations in table IIB are overwhelmingly rejected.³

¹ As with the OLS estimates, the GMM estimates of the before and after-tax K -factor models are very similar. For this reason, only one is shown.

² Whereas a negative beta is theoretically possible, it is very unusual.

³ Only the tests for two-fund separation are shown in the table. The other test statistics are also large.

Finally, our tests of instrument validity and explanatory power indicate that the instruments are uncorrelated with the errors and highly correlated with the explanatory variables.¹

VI: Summary and Conclusions

A model of pricing of a risky natural-resource commodity sold by risk-averse investors is derived and tested. This model, which unifies financial and product markets, differs from previous research in a similar vein (e.g., Hartwick and Yeung 1988 and Gaudet and Khadr 1991) in two respects. First, we allow for nonconstant returns and depletion effects in the cost function, and second, we derive our HCAPM as a no-arbitrage rather than a competitive-equilibrium condition.

The second modification has both strengths and weaknesses. The weaknesses are associated with the fact that we do not derive an equilibrium price process or endogenize the evolution of the marginal utility of income. The strength is that our equilibrium condition holds in a more general setting. For example, the behavior of price could result from the actions of producers who have market power. As long as the firms of interest are price takers, our condition is valid.

Our model receives substantial empirical support. Indeed, with the exception of the simplest Hotelling model, all versions that we estimate have high overall explanatory power. Moreover, the coefficients of the financial and macro factors tend to be individually significant and often have signs that are consistent with *a priori* expectations.

Our results can be contrasted with the overall poor performance of the Hotelling model in previous tests. On the one hand, we have reason for optimism; the Hotelling model is not rejected by the data. On the other hand, optimism must be tempered. Whereas financial and macro factors are significant determinants of returns to holding shares in a mine, in all but one version of the model estimated, traditional cost factors are not. Nevertheless, with the final version of the model, the "Hotelling" coefficients have signs that are consistent with *a priori* expectations, and depletion effects are marginally significant (at the ten-percent level).

¹ When the explanatory variables are regressed on the instruments, R^2 ranges from 0.48 for CRL to 0.99 for RLT.

It therefore seems that the addition of risk and the possibility of risk diversification improves the empirical performance of Hotelling's theory. This conclusion should not be very surprising. Uncertainty is pervasive in natural-resource markets, and resource-commodity prices are extremely volatile as a consequence. It therefore seems natural that medium-term risk considerations should be at least as important as long-term depletion factors when investors are making their portfolio and output decisions.

References Cited:

- Barnett, H.J. and Morse, C. (1964) Scarcity and Growth, Baltimore: Johns Hopkins.
- Barnett, H.J. (1979) "Scarcity and Growth Revisited," in Scarcity and Growth Reconsidered, V.K. Smith (ed.) Baltimore: Johns Hopkins, 163-217.
- Blume, M. (1980) "Stock Returns and Dividend Yields: Some More Evidence," Review of Economics and Statistics, 62: 567-577.
- Bollerslev, T., Chou, R.Y., and Kroner, K.F. (1992) "ARCH modeling in Finance: A Review of the Theory and Empirical Evidence," Journal of Econometrics, 52: 1-61.
- Brennan, M.J. (1970) "Taxes, Market Valuation, and Corporate Financial Policy," National Tax Journal, 23: 417-427.
- Brennan, M.J. (1987) "Capital-Asset Pricing Model," in The New Palgrave Dictionary of Economics, J. Eatwell, M. Milgate, and P. Newman (eds.), London: Macmillan Ltd.
- Campbell, J. (1987) "Stock Returns and the Term Structure," Journal of Financial Economics, 18: 373-399.
- Chamberlain, G. (1988) "Asset Pricing in Multiperiod-Security Markets," Econometrica, 51: 1283-1300.
- Chan, K.C., Chen, N., and Hsieh, D. (1985) "An Exploratory Investigation of the Size Effect," Journal of Financial Economics, 14: 451-471.
- Chen, N., Roll, R., and Ross, S.A. (1986) "Economic Forces of the Stock Market," Journal of Business, 59: 383-403.
- Connor, G. and Korajczyk, R.A. (1992) "The Arbitrage Pricing Theory and Multifactor Models of Asset Returns," Working Paper #139, Northwestern University.
- Davidson, R. and MacKinnon, J. (1992) Estimation and Inference in Econometrics, New York: Oxford University Press.
- Eckbo, B.E. (1986) "Mergers and the Market for Corporate Control: The Canadian Evidence," Canadian Journal of Economics, 19: 236-260.
- Farrow, S. (1985) "Testing the Efficiency of Extraction from a Stock Resource," Journal of Political Economy, 93: 452-487.
- Gaudet, G. and Khadr, A.M. (1991) "The Evolution of Natural-Resource Prices Under Stochastic Investment Opportunities: An Intertemporal Asset-Pricing Approach," International Economic Review, 32: 441-455.

- Guzman, J. (1991) "Evaluating Cyclical Projects," Resources Policy, 17: 114-123.
- Halvorsen, R. and Smith, T.R. (1984) "On Measuring Natural-Resource Scarcity," Journal of Political Economy, 92: 954-964.
- Halvorsen, R. and Smith, T.R. (1991) "A Test of the Theory of Exhaustible Resources," Quarterly Journal of Economics, 106: 123-140.
- Hanson, D.A. (1980) "Increasing Extraction Costs and Resource Prices: Some Further Evidence," Bell Journal of Economics, 11: 335-342.
- Hansen, L.P. (1982) "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica, 50: 1029-1054.
- Hartwick, J. and Yeung, D. (1988) "Explaining Current Exhaustible Resource Prices with CAPM," Department of Economics D.P. No 726, Queen's University.
- Harvey, C.R. (1998) "Time-Varying Conditional Covariances in Tests of Asset-Pricing Models," Journal of Financial Economics, 24: 289-317.
- Heal, G.M. (1976) "The Relationship Between Price and Extraction Cost for Resource with a Backstop Technology," Bell Journal of Economics, 7: 317-378.
- Heal, G. and Barrow, M. (1980) "The Relationship Between Interest Rates and Metal-Price Movements," Review of Economic Studies, 47: 161-182.
- Hotelling, H. (1931) "The Economics of Exhaustible Resources," Journal of Political Economy, 30: 137-175.
- Huberman, G. (1987) "Arbitrage Pricing Theory," in The New Palgrave Dictionary of Economics, J. Eatwell, M. Milgate, and P. Newman (eds.), London: Macmillan Ltd.
- Huberman, G. and Kandel, S. (1988) "Mean-Variance Spanning," Journal of Finance, 42: 873-888.
- Kamien, M.I. and Schwartz, N.L. (1981) Dynamic Optimization, Amsterdam: North Holland.
- Kocherlakota, N.R. (1990) "On Tests of Representative Consumer Asset-Pricing Models," Journal of Monetary Economics, 26: 285-304.
- Levhari, D. and Levitan, N. (1977) "Notes on Hotelling's Economics of Exhaustible Resources," Canadian Journal of Economics, 10: 177-192.
- Lintner, J. (1965) "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics, 47: 13-37.

- Loury, G.C. (1978) "The Optimum Exploitation of an Unknown Reserve," Review of Economic Studies, 45: 621-636.
- Mehra, R. and Prescott, E. (1985) "Equity Premium: A Puzzle," Journal of Monetary Economics, 15: 145-161.
- Merton, R.C. (1973) "An Intertemporal Capital Asset Pricing Model," Econometrica, 41: 867-887.
- Miller, M.H. and Upton, C.W. (1985a) "A Test of the Hotelling Valuation Principle," Journal of Political Economy, 93: 1-25.
- Miller, M.H. and Upton, C.W. (1985b) "The Pricing of Oil and Gas: Some Further Results," Journal of Finance, 40: 1009-1020.
- Morgan, I. (1980) "Dividends and Stock-Price Behavior in Canada," Journal of Business Administration, 12: 91-106.
- Newey, W.K. and West, K.D. (1987) "A Simple, Positive-Semidefinite, Heteroskedasticity and Autocorrelation-Consistent Covariance Matrix," Econometrica, 55: 703-708.
- Pagan, A. (1984) "Econometric Issues in the Analysis of Regressions with Generated Regressors," International Economic Review, 25: 221-247.
- Pindyck, R.S. (1980) "Uncertainty and Exhaustible Resource Markets," Journal of Political Economy, 88: 1203-1225.
- Pindyck, R.S. (1981) "The Optimal Production of an Exhaustible Resource when Price is Exogenous and Stochastic," Scandinavian Journal of Economics, 277-288.
- Ross, S. (1976) "The Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory, 13: 341-360.
- Salant, S. (1976) "Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach," Journal of Political Economy 84: 1079-1093.
- Sharpe, W.F. (1964) "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, 19: 425-442.
- Slade, M.E. (1982) "Trends in Natural-Resource Commodity Prices: An Analysis of the Time Domain," Journal of Environmental Economics and Management, 9: 122-137.
- Slade, M.E. (1991) "Market Structure, Marketing Method, and Price Instability," Quarterly Journal of Economics, 106: 1309-1340.
- Smith V.K. (1979) "Natural-Resource Scarcity: A Statistical Analysis," Review of Economics and Statistics, 61: 423-427.
- Smith, V.K. (1981) "The Empirical Relevance of Hotelling's Model for Natural Resources," Resources and Energy, 3: 105-117.
- Solow, R.M. (1974) "The Economics of Resources or the Resources of Economics," American Economic Review, 64: 1-14.

- Stollery, K.R. (1983) "Mineral Depletion with Cost as the Extraction Limit: A Model Applied to the Behavior of Prices in the Nickel Industry," Journal of Environmental Economics and Management, 10: 151-165.
- Tobin, J. (1958) "Liquidity Preference as Behavior Towards Risk," Review of Economic Studies, 25: 65-86.
- Washburn, C.L. and Binkley, C.S. (1990) "On the Use of Period-Average Stumpage Prices to Estimate Forest Asset-Pricing Models," Land Economics, 66: 379-393.
- White, H. (1980) "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," Econometrica, 48: 817-838.
- Working, H. (1960) "A Note on the Correlation of First Differences of Averages in a Random Chain," Econometrica, 28: 916-918.
- Young, D. (1992) "Cost Specification and Firm Behavior in a Hotelling Model of Resource Extraction," Canadian Journal of Economics, 25: 41-59.

Appendix A:
Proof of Proposition 1

Proposition 1:

With the stochastic-control problem expressed by (9), if the mine is managed optimally, the evolution of the shadow price, $\lambda = p - C_q$, is governed by

$$\frac{1}{\lambda} \frac{dE_t \lambda}{dt} = \frac{\lambda}{\lambda} = r + \frac{C_R}{\lambda} + \beta(r^m - r). \quad (A1)$$

Proof:

The Bellman equation corresponding to (9) is¹

$$\begin{aligned} rV = \max_q [& M\pi - V_R q + V_p p \mu_p + V_\Theta \Theta \mu_\Theta + V_M M \mu_M \\ & + \frac{1}{2} V_{pp} p^2 \sigma_p^2 + \frac{1}{2} V_{\Theta\Theta} \Theta^2 \sigma_\Theta^2 + \frac{1}{2} V_{MM} M^2 \sigma_M^2 \\ & + V_{p\Theta} p \Theta \sigma_{p\Theta} + V_{pM} p M \sigma_{pM} + V_{\Theta M} \Theta M \sigma_{\Theta M}]. \end{aligned} \quad (A2)$$

We can partially differentiate (A2) with respect to R at the optimal q to obtain

$$\begin{aligned} rV_R = & -MC_R - V_{RR} q + V_{Rp} p \mu_p + V_{R\Theta} \Theta \mu_\Theta + V_{RM} M \mu_M \\ & + \frac{1}{2} V_{Rpp} p^2 \sigma_p^2 + \frac{1}{2} V_{R\Theta\Theta} \Theta^2 \sigma_\Theta^2 + \frac{1}{2} V_{RMM} M^2 \sigma_M^2 \\ & + V_{Rp\Theta} p \Theta \sigma_{p\Theta} + V_{RpM} p M \sigma_{pM} + V_{R\Theta M} \Theta M \sigma_{\Theta M}. \end{aligned} \quad (A3)$$

It is possible to simplify (A3) by applying Ito's lemma to the right-hand side, which yields

$$rV_R = -MC_R + \frac{1}{dt} E_t dV_R. \quad (A4)$$

¹ We are assuming incomplete exhaustion, i.e., $R_\infty > 0$. When cost rises as reserves are depleted, this seems natural.

Rearranging (A4), the stochastic Euler equation, we have

$$\frac{\frac{1}{dt} E_t dV_R}{V_R} = \frac{\dot{C}_R}{C_R} = r + \frac{MC_R}{V_R}. \quad (A5)$$

$V_R = \partial V / \partial R$ is the shadow price of R in utils. Let λ be the shadow price of R in dollars, $\lambda = \partial V / \partial R$. Since $\partial V / \partial R = (\partial V / \partial \$) \times (\partial \$ / \partial R) = M\lambda$, it follows that

$$\lambda = \frac{V_R}{M}. \quad (A6)$$

Substituting (A6) into (A5) we have

$$\frac{\frac{1}{dt} E_t d[M\lambda]}{M\lambda} = \frac{\dot{C}_R}{C_R} = r + \frac{C_R}{\lambda}. \quad (A7)$$

(A7) is Hotelling's rule in (risk-adjusted) utils. In other words, it states that the right-hand-side, the expected-average-rate of change of utility, should equal the risk-free rate of return for natural-resource assets. Not surprisingly, the rule is the same as the one obtained when there is no uncertainty. The expected-average-rate of change of the dollar-shadow price, $\dot{\lambda}/\lambda$, however, will not equal the risk-free-rate of return for natural-resource assets. Indeed, investors will be indifferent among assets only if their risk-adjusted rates of return are equal. The mine manager will therefore vary q until (A1) holds.¹ In other words, dollar returns must reflect undiversifiable risk.

¹ A no-arbitrage argument of this sort is used by Hotelling (1931) and Solow (1974).

Appendix B: Calculation of Beta

Tests of CAPM and HCAPM involve β , the coefficient of $r^m = \frac{dp^m + div}{p^m}$, whereas the equation that is estimated involves β_1 and β_2 , the coefficients of dp^m/p^m and div/p^m , respectively. We use the following procedure to obtain an estimate of β . Equation (13) can be rewritten as

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \sum_{j=3}^k \beta_j x_j + \varepsilon, \quad (B1)$$

where $y = \text{LDOT}$, $x_1 = \text{RMSPI}$, $x_2 = \text{DIV}$, and $x_j, j=3, \dots, k$, are the other variables in (13). (B1) can be contrasted with

$$y = \tilde{\alpha} + \beta x_0 + \sum_{j=3}^k \tilde{\beta}_j x_j + \tilde{\varepsilon}, \quad (B2)$$

where $x_0 = x_1 + x_2$ is the rate of return on the market portfolio, r^m . We need to express β as a function of the coefficients in (B1). With this in mind, we suppose that $\tilde{\beta}_j = \beta_j, j=3, \dots, k$. This hypothesis was tested and not rejected.¹ Consider the OLS normal equations²

$$s_{yi} = \sum_{j=1}^k b_j s_{ji}, \quad (B4)$$

where

$$s_{yi} = \frac{\sum_m (y_m - \bar{y})(x_{im} - \bar{x}_i)}{n}, \quad (B5)$$

$$s_{ij} = \frac{\sum_m (x_{im} - \bar{x}_i)(x_{jm} - \bar{x}_j)}{n}, \quad (B6)$$

and b_j is the OLS estimate of β_j . (B4) can be rearranged to yield

¹ For example, with the K-factor model the χ^2 statistic is 0.66 with six degrees of freedom (a P value of 0.99).

² Equations B7-B9 hold in a broader context than OLS, but it is simpler to use the normal equations.

$$b_1 = \frac{s_{y1} - b_2 s_{12} - \sum_{j=3}^k b_j s_{j1}}{s_{11}}, \quad (B7)$$

$$b_2 = \frac{s_{y2} - b_1 s_{12} - \sum_{j=3}^k b_j s_{j2}}{s_{22}}, \quad (B8)$$

$$\hat{\beta} = b_0 = \frac{s_{y0} - \sum_{j=3}^k b_j s_{j0}}{s_{00}} \quad (B9)$$

$$= \frac{s_{y1} + s_{y2} - \sum_{j=3}^k b_j (s_{j1} + s_{j2})}{s_{00}} \quad (B10)$$

$$= \frac{b_1 s_{11} + b_2 s_{22} + (b_1 + b_2)s_{12}}{s_{00}}. \quad (B11)$$

In performing hypothesis tests involving β , we use the estimated coefficients b_1 and b_2 and the empirical covariances of the random variables.

Appendix C: Calculation of Tax Factors

In Canada prior to 1972, capital gains were tax exempt whereas the dividend-tax credit equaled only 20 % of dividend income. The tax reform of 1972, however, reduced the tax disadvantage of dividends by increasing their exemption and taxing capital gains at 50% of the regular income-tax rate. Nevertheless, capital gains still enjoy a more favorable treatment.

Tax factors $(1 - T_i)$ on dividends and capital gains are calculated using

$$T_i = (1 - \text{fraction exempt}) \times \text{marginal income-tax rate.} \quad (C1)$$

A marginal-income tax rate of 0.30 yields the following table.

<i>Tax Factors $(1 - T_i)$</i>		
<i>i</i>	$t \leq 1972$	$t > 1972$
p^m	1.0	0.85
d	0.76	0.78

Canadian taxation of mining profit is very complex. Nevertheless, as long as there are no major changes in tax schedules, taxation does not affect the calculation of λ . Indeed, the same factor appears in both numerator and denominator. In 1974, however, there were substantial increases in mining-profit taxes at both federal and provincial levels. The same year witnessed high profits due to record-high prices. These two factors approximately canceled each other, leaving the rate of after-tax shadow-price appreciation at zero. For this reason, LDOT is multiplied by zero in 1974.

Appendix D: Estimation and Tests

Estimation:

The model that we wish to estimate is

$$y = X\beta + u, \quad (D1)$$

and Z is a matrix of instruments. β is estimated as follows:

Step 1: Let the initial weighting matrix be

$$M^0 = I, \quad (D2)$$

where I is an N by N identity matrix.

Step 2: Let $\hat{\beta}^k$ be the solution to

$$\min_{\beta} (y - X\beta)'Z(M^k)^{-1}Z'(y - X\beta), \quad k = 0, 1, \dots \quad (D3)$$

and let the corresponding residuals be $\hat{u}^k$.

Step 3: Compute the new weighting matrix as

$$M^{k+1} = \sum_{i=1}^{14} \sum_{j=0}^2 \sum_{t=t_0i+j}^{T_i} w_j \hat{u}_t^k \hat{u}_{t-j}^k (Z_t'Z_{t-j} + Z_{t-j}'Z_t), \quad w_j = 1 - j/3, \quad (D4)$$

and compare it to M^k . If the difference is greater than some tolerance, repeat step 2 with k augmented. Otherwise, let $k^* = k$ and calculate the final estimate as.

$$\hat{\beta} = [X'Z(M^{k^*})^{-1}Z'X]^{-1}X'Z(M^{k^*})^{-1}Z'y. \quad (D5)$$

Finally, the variance of $\hat{\beta}$ is

$$\text{Var}(\hat{\beta}) = \frac{1}{N} [X'Z(M^{k^*})^{-1}Z'X]^{-1}. \quad (D6)$$

Tests of Linear Restrictions:

Let the null hypothesis be

$$R\beta - r = 0, \quad (D7)$$

where R is a Q by K matrix and r is a Q by 1 vector. The test statistic is

$$(R\hat{\beta} - r)'[R \text{Var}(\hat{\beta}) R']^{-1}(R\hat{\beta} - r), \quad (D8)$$

which is distributed χ^2 with Q degrees of freedom.

Tests of Overidentifying Restrictions:

The J statistic is calculated as

$$J = N \left[\frac{(y - X\hat{\beta})'}{N} Z(M^{k*})^{-1} Z' \frac{(y - X\hat{\beta})}{N} \right], \quad (D9)$$

which is distributed χ^2 with degrees of freedom equal to the number of instruments minus the number of parameters.

Table I:
Summary Statistics

<i>Variable</i>	<i>Mean %</i>	<i>Standard Error</i>	<i>Minimum</i>	<i>Maximum</i>	<i>Correlation with LDOT</i>
<i>LDOT</i>	5.6	48.3	-137.6	373.2	1.00
<i>CRL</i>	-1.9	2.5	-17.0	-0.1	-0.05
<i>RF</i>	6.1	2.7	3.0	15.8	-0.12
<i>RMSPI</i>	5.1	14.4	-24.0	36.1	0.25**
<i>DIV</i>	5.1	11.3	-14.2	38.2	-0.27**
<i>RLT</i>	7.3	2.1	3.9	14.7	-0.09
<i>GDOT</i>	9.9	4.2	3.5	19.4	0.34**
<i>CPIDOT</i>	5.4	3.7	0.8	12.4	0.02
<i>EDOT</i>	0.9	3.0	-3.3	7.8	0.03

** denotes significance at 1 percent.

Table IIA: OLS Estimates

Dependent Variable: $\hat{\lambda}/\lambda$

Model	Constant (α_0)	CRL (α_1)	RF (α_2)	RMSP1 (α_3)	DIV (α_4)	RLT (α_5)	GDFT (α_6)	CDFT (α_7)	EDFT (α_8)	$\hat{\beta}$	$\bar{R}^2$
			%	%	%	%	% change	% change	% change		
Hotelling	0.16 (0.10) [0.12]	-0.94 (1.64) [3.30]	-1.99 (1.47) [1.86]								0.00
CAPM	0.15 (0.10) [0.12]		-1.26 (1.41) [1.88]	0.82** (0.32) [0.29]	-1.00** (0.34) [0.37]					-0.01	0.10
HCAPM	0.12 (0.10) [0.11]	-1.20 (1.56) [3.18]	-1.21 (1.41) [1.90]	0.82** (0.32) [0.30]	-1.02** (0.34) [0.39]					-0.02	0.10
K-Factor	-0.01 (0.17) [0.20]	-0.51 (1.46) [2.71]	5.77 (4.74) [5.38]	0.57* (0.32) [0.27]	0.02 (0.41) [0.32]	-5.04 (5.67) [6.79]	10.19** (2.03) [1.77]	-9.47** (3.38) [2.28]	3.59** (1.52) [1.04]	0.32	0.23
After Tax	0.03 (0.17) [0.18]	-0.54 (1.46) [2.77]	8.01 (5.54) [5.98]	0.68* (0.36) [0.29]	0.03 (0.53) [0.41]	-7.39 (6.37) [6.86]	10.45** (2.09) [1.64]	-9.68** (3.27) [2.01]	3.57** (1.54) [1.21]	0.43	0.23

Standard errors in parentheses.

Standard errors corrected for heteroskedasticity of an unknown form in brackets.

* denotes significance at 5 percent.

** denotes significance at 1 percent.

Table II B:
OLS Hypothesis Tests

Test χ^2					
Model	Overall Explanatory Power	Hotelling	CAPM	Joint	2-Fund Separation
Hotelling	4.34 (0.23)	3.01 (0.39)			
CAPM	15.1** (0.00)		1.42 (0.49)		
HCAPM	17.0** (0.00)	1.80 (0.41)	1.31 (0.52)	1.85 (0.60)	
K-Factor	62.2** (0.00)	0.48 (0.79)	3.27 (0.20)	3.38 (0.34)	42.8** (0.00)
After Tax	66.4** (0.00)	0.95 (0.62)	4.95 (0.08)	5.68 (0.13)	53.0** (0.00)

P values in parentheses.

** denotes rejection at 1 percent.

Table III A:
GMM Estimates
Dependent Variable: $\hat{\lambda}/\lambda$

Model	Constant (α_0)	CRL (α_1)	RF (α_2)	RMSPI (α_3)	DIV (α_4)	RLT (α_5)	GDFT (α_6)	CDFT (α_7)	EDFT (α_8)	$\hat{\beta}$	J Statistic
			%	%	%	%	% change	% change	% change		
After Tax											
First-Step	0.27 (0.22)	5.43* (2.73)	31.82 (47.9)	-3.27* (1.58)	-2.67 (1.37)	-58.15 (74.7)	8.37 (8.27)	-44.67** (9.82)	6.94* (3.50)		
Newey	0.04	3.54	16.72	-0.55*	-1.13*	-15.47	9.30**	-22.20**	4.79**	-0.70	10.25
West	(0.08)	(2.45)	(5.06)	(0.26)	(0.47)	(29.02)	(3.11)	(3.61)	(1.68)		[0.74]

Standard errors in parentheses.

P values in brackets.

* denotes significance at 5 percent.

** denotes significance at 1 percent.

Table III B:
GMM Hypothesis Tests

<i>Test</i> χ^2					
<i>Model</i>	<i>Overall Explanatory Power</i>	<i>Hotelling</i>	<i>CAPM</i>	<i>Joint</i>	<i>2-Fund Separation</i>
<i>After Tax</i>	158.2** (0.00)	1.08 (0.58)	1.01 (0.60)	1.51 (0.68)	89.93** (0.00)

P values in parentheses.

** denotes rejection at 1 percent.

CES Working Paper Series

- 01 Richard A. Musgrave, Social Contract, Taxation and the Standing of Deadweight Loss, May 1991
- 02 David E. Wildasin, Income Redistribution and Migration, June 1991
- 03 Henning Bohn, On Testing the Sustainability of Government Deficits in a Stochastic Environment, June 1991
- 04 Mark Armstrong, Ray Rees and John Vickers, Optimal Regulatory Lag under Price Cap Regulation, June 1991
- 05 Dominique Demougin and Aloysius Siow, Careers in Ongoing Hierarchies, June 1991
- 06 Peter Birch Sørensen, Human Capital Investment, Government and Endogenous Growth, July 1991
- 07 Syed Ahsan, Tax Policy in a Model of Leisure, Savings, and Asset Behaviour, August 1991
- 08 Hans-Werner Sinn, Privatization in East Germany, August 1991
- 09 Dominique Demougin and Gerhard Illing, Regulation of Environmental Quality under Asymmetric Information, November 1991
- 10 Jürg Niehans, Relinking German Economics to the Main Stream: Heinrich von Stackelberg, December 1991
- 11 Charles H. Berry, David F. Bradford and James R. Hines, Jr., Arm's Length Pricing: Some Economic Perspectives, December 1991
- 12 Marc Nerlove, Assaf Razin, Efraim Sadka and Robert K. von Weizsäcker, Comprehensive Income Taxation, Investments in Human and Physical Capital, and Productivity, January 1992
- 13 Tapan Biswas, Efficiency and Consistency in Group Decisions, March 1992
- 14 Kai A. Konrad and Kjell Erik Lommerud, Relative Standing Comparisons, Risk Taking and Safety Regulations, June 1992
- 15 Michael Burda and Michael Funke, Trade Unions, Wages and Structural Adjustment in the New German States, June 1992
- 16 Dominique Demougin and Hans-Werner Sinn, Privatization, Risk-Taking and the Communist Firm, June 1992
- 17 John Piggott and John Whalley, Economic Impacts of Carbon Reduction Schemes: Some General Equilibrium Estimates from a Simple Global Model, June 1992
- 18 Yaffa Machnes and Adi Schnytzer, Why hasn't the Collective Farm Disappeared?, August 1992
- 19 Harris Schlesinger, Changes in Background Risk and Risk Taking Behavior, August 1992

- 20 Roger H. Gordon, Do Publicly Traded Corporations Act in the Public Interest?, August 1992
- 21 Roger H. Gordon, Privatization: Notes on the Macroeconomic Consequences, August 1992
- 22 Neil A. Doherty and Harris Schlesinger, Insurance Markets with Noisy Loss Distributions, August 1992
- 23 Roger H. Gordon, Fiscal Policy during the Transition in Eastern Europe, September 1992
- 24 Giancarlo Gandolfo and Pier Carlo Padoan, The Dynamics of Capital Liberalization: A Macroeconometric Analysis, September 1992
- 25 Roger H. Gordon and Joosung Jun, Taxes and the Form of Ownership of Foreign Corporate Equity, October 1992
- 26 Gaute Torsvik and Trond E. Olsen, Irreversible Investments, Uncertainty, and the Ramsey Policy, October 1992
- 27 Robert S. Chirinko, Business Fixed Investment Spending: A Critical Survey of Modeling Strategies, Empirical Results, and Policy Implications, November 1992
- 28 Kai A. Konrad and Kjell Erik Lommerud, Non-Cooperative Families, November 1992
- 29 Michael Funke and Dirk Willenbockel, Die Auswirkungen des "Standortsicherungsgesetzes" auf die Kapitalakkumulation - Wirtschaftstheoretische Anmerkungen zu einer wirtschaftspolitischen Diskussion, January 1993
- 30 Michelle White, Corporate Bankruptcy as a Filtering Device, February 1993
- 31 Thomas Mayer, In Defence of Serious Economics: A Review of Terence Hutchison; Changing Aims in Economics, April 1993
- 32 Thomas Mayer, How Much do Micro-Foundations Matter?, April 1993
- 33 Christian Thimann and Marcel Thum, Investing in the East: Waiting and Learning, April 1993
- 34 Jonas Agell and Kjell Erik Lommerud, Egalitarianism and Growth, April 1993
- 35 Peter Kuhn, The Economics of Relative Rewards: Pattern Bargaining, May 1993
- 36 Thomas Mayer, Indexed Bonds and Heterogeneous Agents, May 1993
- 37 Trond E. Olsen and Gaute Torsvik, Intertemporal Common Agency and Organizational Design: How much Decentralization?, May 1993
- 38 Henry Tulkens and Philippe vanden Eeckaut, Non-Parametric Efficiency, Progress and Regress Measures for Panel Data: Methodological Aspects, May 1993
- 39 Hans-Werner Sinn, How Much Europe? - Subsidiarity, Centralization and Fiscal Competition, July 1993
- 40 Harald Uhlig, Transition and Financial Collapse, July 1993
- 41 Jim Malley and Thomas Moutos, Unemployment and Consumption: The Case of Motor-Vehicles, July 1993

- 42 John McMillan, Autonomy and Incentives in Chinese State Enterprises, August 1993
- 43 Murray C. Kemp and Henry Y. Wan, Jr., Lumpsum Compensation in a Context of Incomplete Markets, August 1993
- 44 Robert A. Hart and Thomas Moutos, Quasi-Permanent Employment and the Comparative Theory of Coalitional and Neoclassical Firms, September 1993
- 45 Mark Gradstein and Moshe Justman, Education, Inequality, and Growth: A Public Choice Perspective, September 1993
- 46 John McMillan, Why Does Japan Resist Foreign Market-Opening Pressure?, September 1993
- 47 Peter J. Hammond, History as a Widespread Externality in Some Arrow-Debreu Market Games, October 1993
- 48 Michelle J. White, The Costs of Corporate Bankruptcy: A U.S.-European Comparison, October 1993
- 49 Gerlinde Sinn and Hans-Werner Sinn, Participation, Capitalization and Privatization, Report on Bolivia's Current Political Privatization Debate, October 1993
- 50 Peter J. Hammond, Financial Distortions to the Incentives of Managers, Owners and Workers, November 1993
- 51 Hans-Werner Sinn, Eine neue Tarifpolitik (A New Union Policy), November 1993
- 52 Michael Funke, Stephen Hall and Martin Sola, Rational Bubbles During Poland's Hyperinflation: Implications and Empirical Evidence, December 1993
- 53 Jürgen Eichberger and Ian R. Harper, The General Equilibrium Foundations of Modern Finance Theory: An Exposition, December 1993
- 54 Jürgen Eichberger, Bayesian Learning in Repeated Normal Form Games, December 1993
- 55 Robert S. Chirinko, Non-Convexities, Labor Hoarding, Technology Shocks, and Procyclical Productivity: A Structural Econometric Approach, January 1994
- 56 A. Lans Bovenberg and Frederick van der Ploeg, Consequences of Environmental Tax Reform for Involuntary Unemployment and Welfare, February 1994
- 57 Jeremy Edwards and Michael Keen, Tax Competition and Leviathan, March 1994
- 58 Clive Bell and Gerhard Clemenz, The Desire for Land: Strategic Lending with Adverse Selection, April 1994
- 59 Ronald W. Jones and Michihiro Ohyama, Technology Choice, Overtaking and Comparative Advantage, May 1994
- 60 Eric L. Jones, Culture and its Relationship to Economic Change, May 1994
- 61 John M. Hartwick, Sustainability and Constant Consumption Paths in Open Economies with Exhaustible Resources, June 1994
- 62 Jürg Niehans, Adam Smith and the Welfare Cost of Optimism, June 1994

- 63 Tõnu Puu, The Chaotic Monopolist, August 1994
- 64 Tõnu Puu, The Chaotic Duopolists, August 1994
- 65 Hans-Werner Sinn, A Theory of the Welfare State, August 1994
- 66 Martin Beckmann, Optimal Gambling Strategies, September 1994
- 67 Hans-Werner Sinn, Schlingerkurs - Lohnpolitik und Investitionsförderung in den neuen Bundesländern, September 1994
- 68 Karlhans Sauerheimer and Jerome L. Stein, The Real Exchange Rates of Germany, September 1994
- 69 Giancarlo Gandolfo, Pier Carlo Padoan, Giuseppe De Arcangelis and Clifford R. Wymer, The Italian Continuous Time Model: Results of the Nonlinear Estimation, October 1994
- 70 Tommy Staahl Gabrielsen and Lars Sjørgard, Vertical Restraints and Interbrand Competition, October 1994
- 71 Julia Darby and Jim Malley, Fiscal Policy and Consumption: New Evidence from the United States, October 1994
- 72 Maria E. Maher, Transaction Cost Economics and Contractual Relations, November 1994
- 73 Margaret E. Slade and Henry Thille, Hotelling Confronts CAPM: A Test of the Theory of Exhaustible Resources, November 1994