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A Comment to Sefton and Weale

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Abstract

Weitzman shows that capital gains should be excluded from NNP. This result is somewhat deceptive since, with his assumption of a constant interest rate, there are no aggregate capital gains, while capital gains in each open economy should be included fully. Here, the implications of Weitzman welfare foundation is explored in the case of non-constant interest rates, a case often encountered in resource models. It is established that the conventional measure of NNP must be adjusted for capital gains and interest rate effects. This result is of importance for the distribution of NNP between resource rich and resource poor countries.

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1. Introduction

How should the notion of Net National Product (NNP) be adjusted for the depletion of natural and environmental resources? Most, if not all, contributions that attempt to give a theoretical solution to this problem base their analysis on the welfare foundation of NNP due to Martin Weitzman. Weitzman (1976) analyzes the case with a constant interest rate and concludes that, for a closed economy, NNP should equal consumption plus the sum of net investments, while capital gains should not be included. However, it will be argued in the present paper that this exclusion of capital gains is somewhat deceptive since, with the assumption of a constant interest rate,

(i) there are no aggregate capital gains, while

(ii) capital gains in each open economy should be included fully.

The major challenge is therefore to extend Weitzman foundation to the case where interest rates are not constant; a case which is often encountered in resource models.

If the interest rates are not constant, there is a term structure of interest rates. It turns out that in order to generalize Weitzman's (1976) analysis, it is the infinitely long term interest rate that is significant. The present paper shows, under the assumption of constant returns to scale, that NNP satisfying the Weitzman foundation equals NNP as conventionally defined, plus capital gains, plus an adjustment for the growth rate of the infinitely long term interest rate. This result is of importance for the distribution of NNP between resource rich and resource poor countries. Through this result, this paper comments on and complements the interesting analysis by Sefton and Weale (1994) (subsequently referred to as SW).

The paper is organized as follows: Sections 2 (and 3) follows Weitzman (1976) by using a problem of welfare (and wealth) maximization as the point of departure when defining a concept of NNP that satisfies the Weitzman foundation. Section 4 uses this definition to derive an expression for this concept of NNP in a closed economy, and investigates two special cases in which the expression coincides with the conventional NNP. Sections 5 derives an expression for NNP for both open and closed economies under the assumption that the technology exhibits constant returns to scale in the capital goods. Section 6 illustrates the results of Section 5 by using a numerical example of a two country world, and discusses the empirical relevance of the results for the measurement of NNP in open economies. Section 7 relates the results to the analyses of SW as well as my own paper on capital gains and NNP in open economies (Asheim, 1994). Some of the formal analysis is contained in an appendix. Throughout it is assumed that there is no population growth.

Finally, note that, with non-constant interest rates, the Weitzman foundation yields an income definition that differs from that of Schanz-Haig-Simons. A comparative analysis of the two income definitions is, however, outside the scope of the present paper.

2. The Weitzman foundation in terms of utility

Consider an economy (which may be closed or open) maximizing at time t

$$\int_t^\infty \lambda(s)u(x(s))\,ds \tag{1}$$

over all feasible consumption paths. Here, $x(s)$ denotes consumption at time s, $u(\cdot)$ is a time-invariant, convex and differentiable utility function, and $\lambda(s)$ is the positive utility discount factor applicable at time s. Expression (1) divided through by $\lambda(t)$ measures the current *welfare* of the economy at time t. If $\lambda(s) = \lambda(0)e^{-\varrho s}$, there is a constant utility interest rate: $\varrho = -\dot{\lambda}(t)/\lambda(t) = \lambda(t)/\int_t^\infty \lambda(s)ds$ for all t. Otherwise, there is not a constant utility interest rate. This gives rise to a term structure of utility interest rates. The instantaneous (very short term) utility interest rate is $\varrho_i(t) := -\dot{\lambda}(t)/\lambda(t)$. The infinitely long term utility interest rate is $\varrho_\ell(t) := \lambda(t)/\int_t^\infty \lambda(s)ds$, provided that $\int_t^\infty \lambda(s)ds$ exists.[1] Note that, along a maximizing path, $1/\varrho_\ell(t)$ is the price at time t, in terms of utility, of a utility annuity from time t on. Note also that $\dot{\varrho}_\ell(t)/\varrho_\ell(t) = \varrho_\ell(t) - \varrho_i(t)$. Hence, $\varrho_\ell(t)$ is decreasing if and only if $\varrho_i(t) > \varrho_\ell(t)$.

Let $\{x^*(s)\}_{s=t}^\infty$ be a consumption path maximizing (1) over all feasible consumption paths. In particular, $\int_t^\infty \lambda(s)u(x^*(s))ds$ exists. Let $v(t)$ denote the utility-NNP at time t. Weitzman (1976, p.160) writes: "Net national product is what might be called the *stationary equivalent* of future consumption, and this is its primary welfare interpretation." Hence, for $v(t)$ to satisfy the Weitzman foundation of NNP, $v(t)$ has to be the level of utility, that if sustained indefinitely, would yield the same welfare as the welfare maximizing path. Hence, $\int_t^\infty \lambda(s)v(t)\,ds = \int_t^\infty \lambda(s)u(x^*(s))ds$, or,

$$v(t) = \int_t^\infty \lambda(s)u(x^*(s))ds/\int_t^\infty \lambda(s)ds = \varrho_\ell(t)\cdot\int_t^\infty \frac{\lambda(s)}{\lambda(t)}u(x^*(s))ds. \tag{2}$$

Hence, utility-NNP is the infinitely long term utility interest rate times current

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[1] The integral exists provided that the instantaneous utility interest rate is positive and does not fall "too fast".

welfare. As noted by Weitzman (1976, p.159), a constant utility flow equal to $v(t)$ is, in general, not actually attainable; rather, it is potentially attainable if the actual utility path $\{u(x^*(s))\}_{s=t}^{\infty}$ could be changed to a constant utility path without changing the supporting prices. In particular, if $\{x^*(s)\}_{s=t}^{\infty}$ is the unique path maximizing (1) and $\{x^*(s)\}_{s=t}^{\infty}$ does not yield constant consumption, then a constant utility flow equal to $v(t)$ is not attainable. Hence, the Weitzman foundation gives an upper bound for what is usually called Hicksian income (Hicks, 1946, Ch.14).

It turns out that $\dot{v}(t) = (\lambda(t)/\int_t^{\infty}\lambda(s)\,ds)\cdot(v(t) - u(x^*(t)))$. Hence, $v(t)$ is a solution to the differential equation

$$\dot{v}(t) = \varrho_{\ell}(t)\cdot(v(t) - u(x^*(t))). \tag{3}$$

Writing (3) as $v(t) - u(x^*(t)) = \dot{v}(t)\cdot(1/\varrho_{\ell}(t)$ yields the following interpretation: The difference between the stock of utility annuities at time t and the actual utility level at time t equals the rate at which utility annuities can be accumulated times the price of such annuities.

In contrast, SW determines utility-NNP (or 'welfare income') as the solution to the differential equation $\dot{V}(t) = \varrho_i(t)\cdot(V(t) - u(x^*(t)))$, which yields $V(t) = \int_t^{\infty}(-\dot{\lambda}(s))u(x^*(s))ds/\lambda(t)ds = \int_t^{\infty}\varrho_i(s)\frac{\lambda(s)}{\lambda(t)}u(x^*(s))ds$. If there is one constant utility interest rate, then $\varrho = \varrho_i(s) = \varrho_{\ell}(s)$ for all s; hence, $v(t) = V(t)$. Also if $\{x^*(s)\}_{t=s}^{\infty}$ is a constant consumption path, so that $u(x^*(s)) = u^*$ for all s, then $v(t) = V(t) = u^*$ since $\varrho_{\ell}(t)\cdot\int_t^{\infty}\frac{\lambda(s)}{\lambda(t)}ds = 1$ and $\int_t^{\infty}\varrho_i(s)\frac{\lambda(s)}{\lambda(t)}ds = 1$. However, in general, if there is not a constant utility interest rate or if utility is not constant, then $V(t)$ cannot be based on the Weitzman foundation.

3. The Weitzman foundation in terms of consumption

Under regularity conditions (see the Appendix) there exists a path of present value prices $\{p(s)\}_{s=t}^{\infty}$ such that maximization of (1) implies the maximization of

$$\int_t^\infty p(s)x(s)\,ds \tag{4}$$

over all feasible consumption paths. Expression (4) divided through by $p(t)$ measures the current *wealth* of the economy at time t. It must hold for all s that $x^*(s)$ maximizes $\lambda(s)u(x(s)) - p(s)x(s)$; hence, if $x^*(s)$ is interior, then $\lambda(s)u'(x^*(s)) = p(s)$. In analogy with $\lambda(s)$, $p(s)$ determines a term structure of consumption interest rates. The instantaneous (very short term) consumption interest rate is $r_i(t) := -\dot{p}(t)/p(t)$. The infinitely long term consumption interest rate is $r_\ell(t) := p(t)/\int_t^\infty p(s)ds$. In analogy, $1/r_\ell(t)$ is the price at time t, in terms of consumption, of a consumption annuity from time t on. Note also that $\dot{r}_\ell(t)/r_\ell(t) = r_\ell(t) - r_i(t)$. Hence, $r_\ell(t)$ is decreasing if and only if $r_i(t) > r_\ell(t)$.

Let $y(t)$ denote the consumption-NNP at time t. For $y(t)$ to satisfy the Weitzman foundation of NNP, $y(t)$ has to be the level of consumption, that if sustained indefinitely, would yield the same wealth as the wealth maximizing path. Hence, $\int_t^\infty p(s)y(t)ds = \int_t^\infty p(s)x^*(s)ds$, or

$$y(t) = \int_t^\infty p(s)x^*(s)ds/\int_t^\infty p(s)ds = r_\ell(t)\cdot\int_t^\infty \frac{p(s)}{p(t)}x^*(s)ds. \tag{5}$$

Hence, consumption-NNP is the infinitely long term consumption interest rate times current wealth. Since $\dot{y}(t) = (p(t)/\int_t^\infty p(s)ds)\cdot(y(t) - x^*(t))$, $y(t)$ is a solution to the differential equation

$$\dot{y}(t) = r_\ell(t)\cdot(y(t) - x^*(t)). \tag{6}$$

In contrast, SW determines consumption-NNP (or 'income') as the solution to the differential equation $\dot{Y}(t) = r_i(t)\cdot(Y(t) - x^*(t))$, which yields $Y(t) =$

$\int_t^\infty(-\dot{p}(s))x^*(s)\,ds/p(t) = \int_t^\infty r_i(s)\frac{p(s)}{p(t)}x^*(s)\,ds$. If there is one constant consumption interest rate, then $r = r_i(s) = r_\ell(s)$ for all s; hence, $y(t) = Y(t)$. Also if $\{x^*(s)\}_{t=s}^\infty$ is a constant consumption path, so that $x^*(s) = x^*$ for all s, then $y(t) = Y(t) = x^*$ since $r_\ell(t)\cdot\int_t^\infty\frac{p(s)}{p(t)}ds = 1$ and $\int_t^\infty r_i(s)\frac{p(s)}{p(t)}ds = 1$. However, in general, if there is not a constant consumption interest rate or if consumption is not constant, then $Y(t)$ cannot be based on the Weitzman foundation.

Ideally, one would want to have that $v(t) = u(y(t))$. Because then, consumption-NNP would be an exact welfare indicator. However, (2) and (5) and the property that $\lambda(s)u'(x^*(s)) = p(s)$ for all s imply that $\int_t^\infty\lambda(s)(v(t) - u(y(t)))\,ds$ $= \int_t^\infty\lambda(s)[u(x^*(s)) - u(y(t)) + u'(x^*(s))(y(t) - x^*(s))]ds$, where the term in the brackets is non-negative by the concavity of u. Therefore, since $\lambda(s) > 0$ for all s, $v(t) \geqq u(y(t))$, and $y(t)$ is a lower bound for an exact welfare indicator. On the other hand, repeating the argument of Section 2 implies that $y(t)$ is an upper bound for Hicksian income.

Through (5), consumption-NNP is expressed as the infinitely long term consumption interest rate times the discounted value of future consumption, the latter term having been interpreted as wealth. This is not the form that national accountants find useful, since wealth – especially as given by (5) – is not easily measured (see e.g. Usher, 1994, Section IV). Therefore, combine (5) and (6) to yield

$$\begin{aligned} y(t) &= x^*(t) + \dot{y}(t)/r_\ell(s) \\ &= x^*(t) + \frac{d}{dt}\Big(\int_t^\infty\frac{p(s)}{p(t)}x^*(s)\,ds\Big) + (\dot{r}_\ell(t)/r_\ell(t))\cdot\int_t^\infty\frac{p(s)}{p(t)}x^*(s)\,ds. \end{aligned} \tag{7}$$

Hence, consumption-NNP equals consumption plus the growth of current wealth plus the rate of change of the infinitely long term consumption interest rate times current wealth. The next two sections will consider cases where expression (7) can be evaluated using current prices and quantities only. Finally, note that (5) and $\dot{r}_\ell(t)/r_\ell(t) = r_\ell(t) - r_i(t)$ imply that the last term of (7) equals $(1 - r_i(t)/r_\ell(t))\cdot y(t)$. Hence, the interest adjustment can alternatively be expressed multiplicatively: $y(t) = (r_\ell(t)/r_i(t))\cdot[x^*(t) + \frac{d}{dt}(\int_t^\infty\frac{p(s)}{p(t)}x^*(s)\,ds)]$.

4. Expressions for consumption-NNP in a closed economy

The conventional measure of NNP is $x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$, where $\mathbf{k}^*(t)$ is the vector of capital stocks at time t, and $\mathbf{Q}(t)$ are the competitive prices of the capital stocks in terms of current consumption. In words, the conventional NNP measures consumption plus the sum of net investments. In the present analysis, it is assumed that all capital goods are contained in the vector $\mathbf{k}^*(t)$, including stocks of knowledge accumulated through learning and research activities as well as stocks of natural and environmental resources. Moreover, competitive prices for such stocks are assumed to be available. Hence, both *endogenous* technological progress as well as resource depletion are assumed to be taken into account in the conventional measure. Such an expansion of the conventional measure has been suggested by Weitzman (1976), Hartwick (1990), and others. I will hence refer to this expanded conventional measure as the Weitzman-Hartwick NNP. The question that is being posed is the following: Does the Weitzman-Hartwick NNP satisfy the Weitzman foundation under such idealized assumptions concerning the availability of prices?

Consider a constant population economy with no *exogenous* technological progress as described by Dixit, Hammond, and Hoel (1980) (and reproduced in the Appendix). Then, as shown by SW (see also Asheim, 1994, proof of Proposition 1), the consumption-NNP suggested by SW coincides with the Weitzman-Hartwick NNP:

$$Y(t) = \int_t^\infty r_i(s)\frac{p(s)}{p(t)}x^*(s)\,ds = x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t). \qquad (8)$$

In the previous section it was established that the consumption-NNP of SW satisfies the Weitzman foundation in the special cases of a constant consumption interest rate and of constant consumption. It therefore follows from (8) that the Weitzman-Hartwick NNP satisfies the Weitzman foundation in these two special cases. Below an alternative demonstration of this latter result is provided by showing that in each of these two special cases, the second and third terms of the r.h.s. of (7) sum to $\mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$.

Special case 1: A constant consumption interest rate $(p(s) = p(0)e^{-rs}$; *i.e.,* $r = r_i(s) = r_\ell(s)$ *for all s).* This is the case considered by Weitzman (1976). Then, $\dot{r}_\ell(t) = 0$ implying that the third term of the r.h.s. of (7) is equal to zero. Furthermore, $p(s+\Delta)/p(t+\Delta) = e^{-r(s-t)}$ is a constant as a function of Δ; therefore, the second term of the r.h.s. of (7) equals $\int_t^\infty \frac{p(s)}{p(t)}\dot{x}^*(s)ds$. By Lemma 1 of the Appendix, this latter term equals $\mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$; hence, by (7), $y(t) = x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$. This amount to an alternative demonstration of Weitzman's (1976) original result: With no exogenous technological progress and a constant consumption interest rate, the Weitzman-Hartwick NNP satisfies the Weitzman foundation.

Special case 2: Constant consumption $(x^*(s) = x^*$ *for all s).* By (5) it follows that $y(t) = x^*$, while Hartwick's rule (Hartwick, 1977, and Dixit, Hammond, and Hoel, 1980; reproduced as Lemma 2 of the Appendix) implies that $\mathbf{Q}(t)\dot{\mathbf{k}}^*(t) = 0$. Hence, $y(t) = x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$. Note that, even though the second and third terms of (7) in this case cancel out, there are important resource models in which each of the terms differs from zero. A justly well known model of this kind is the one analyzed by Solow (1974), where his analysis shows that the consumption interest rates are decreasing along the constant consumption path.

The above analysis of a constant population economy with no exogenous technological progress is more appropriate for a closed economy than for an open economy. The 'technology' of an open economy trading in a competitive world economy must include its trade opportunities. Therefore, the assumption of no exogenous technological progress will be violated if its terms-of-trade are changing. In resource models, Hotelling's rule implies that a resource exporting economy will enjoy improving terms-of-trade, which within the context above correspond to exogenous technological progress. If such an economy follows a constant consumption path, its consumption level will not equal the Weitzman-Hartwick NNP (see Asheim, 1986, 1994, and Sefton and Weale, 1994, as well as Section 6 of the present paper).

5. Expressions for consumption-NNP with constant returns to scale

When the special cases of the previous section are not appropriate (as for open economies in resource models), can the consumption-NNP satisfying the Weitzman foundation be expressed in terms of current prices and quantities, and hence, be an operational measure of NNP? It can under the additional assumption that the technology exhibits constant returns to scale (CRS) in the capital goods. This entails that all factors of production, also labor ('human capital'), must be treated as capital goods, the market prices of which correspond to the present value of future earnings.

As shown in the Appendix, if the assumption of CRS is imposed for a closed economy with no exogenous technological progress, then for all s, $x^*(s) + \mathbf{Q}(s)\dot{\mathbf{k}}^*(s) + \dot{\mathbf{Q}}(s)\mathbf{k}^*(s) = r_i(s)\mathbf{Q}(s)\mathbf{k}^*(s)$ and $p(s)x^*(s) + \frac{d}{dt}[p(s)\mathbf{Q}(s)\mathbf{k}^*(s)] = 0$ so that $p(t)\mathbf{Q}(t)\mathbf{k}^*(t) = \int_t^\infty p(s)x(s)\,ds$ and $\mathbf{Q}(t)\mathbf{k}^*(t) = \int_t^\infty \frac{p(s)}{p(t)}x^*(s)\,ds$. Hence, the current value of the capital stocks equals current wealth. It now follows from (7) that

$$\begin{aligned} y(t) &= x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t) \\ &+ \dot{\mathbf{Q}}(t)\mathbf{k}^*(t) + (\dot{r}_\ell(t)/r_\ell(t))\cdot\mathbf{Q}(t)\mathbf{k}^*(t). \end{aligned} \tag{9}$$

The two first terms are the Weitzman-Hartwick NNP. This measure must be adjusted for the capital gains $\dot{\mathbf{Q}}(t)\mathbf{k}^*(t)$ and the rate of change of the infinitely long term consumption interest rate. Since $y(t) = x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t)$ when there is a constant consumption interest rate (see Special case 1 of the previous section), (9) gives alternative demonstration of the result that I show in Asheim (1994, Proposition 1), namely that in a closed economy with CRS and no exogenous technological progress, a constant consumption interest rate implies that there are no capital gains.

Let the closed economy of the previous paragraph be split into open economies with constant subpopulations, each having access to the same CRS technology. Allow for free trade and sufficient factor mobility to ensure overall productive efficiency. Let superscript j correspond to the open economy j. Then $x^j(s) + \mathbf{Q}(s)\dot{\mathbf{k}}^j(s) + \dot{\mathbf{Q}}(s)\mathbf{k}^j(s) =$

$r_i(s)\mathbf{Q}(s)\mathbf{k}^j(s)$ is country j's budget constraint so that $\mathbf{Q}(t)\mathbf{k}^j(t) = \int_t^\infty \frac{p(s)}{p(t)} x^j(s)\,ds$. Hence, also in the case of an open economy it follows from (7) that

$$\begin{aligned} y^j(t) &= x^j(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^j(t) \\ &+ \dot{\mathbf{Q}}(t)\mathbf{k}^j(t) + (\dot{r}_\ell(t)/r_\ell(t))\cdot\mathbf{Q}(t)\mathbf{k}^j(t); \end{aligned} \tag{10}$$

i.e., an adjustment for capital gains and the rate of change in the long term consumption interest rate is needed.

It is now apparent why Weitzman's original result is somewhat deceptive. In the case of a constant consumption interest rate, Weitzman shows that in order to satisfy the Weitzman foundation, NNP for a closed economy should exclude capital gains. However, as argued above, in this special case, there are no aggregate capital gains. On the other hand, (10) implies that capital gains at a disaggregate level (e.g. in each open economy) should be included fully. Therefore, in the case Weitzman considers, a concept of NNP that includes capital gains is as correct at the aggregate level, and it has the added advantage of being generalizable to a disaggregate level.

With constant consumption (Special case 2 of Section 4), the Weitzman-Hartwick NNP correctly ignores non-zero aggregate capital gains, since the two last terms of (9) cancel out. This does not, however, imply that the corresponding terms of (10) – for each open economy – cancel out. In the context of resource models, an open economy with relatively large stocks of *in situ* resources will have large capital gains in relation to its wealth, while an open economy with relatively small such stocks will in comparison have small capital gains in relation to its wealth. The other adjustment term, however, *is* proportional to wealth. Hence, even when the Weitzman-Hartwick NNP need no adjustment at the aggregate level in order to satisfy the Weitzman foundation, the necessary adjustments at the disaggregate level will influence the distribution of NNP between different countries. In the next section, a numerical example is provided and the empirical relevance of this observation is discussed.

6. A numerical example and empirical evidence

To illustrate expression (10) of Section 5, a numerical example is presented based on Solow's (1974) model of manmade capital accumulation and resource depletion, in which a flow of a non-renewable resource, $-\dot{k}_R$, is combined with constant human capital, k_H, and manmade capital, k_C, in order to produce a consumption good, x. With its technology be described by $x + \dot{k}_C \leqq (k_H)^{1-a-b}(k_C)^a(-\dot{k}_R)^b$, $b < a < a + b < 1$, this model fits into the framework of Section 5. Choose $a = 0.20$ and $b = 0.16$. Solow (1974) shows that positive and constant consumption can be sustained indefinitely by letting accumulated manmade capital substitute for a diminishing resource extraction. Let $\mathbf{k}^*(t) = (k_H^*, k_C^*(t), k_R^*(t))$ denote the capital vector along such an efficient maximin path, with x^* being the corresponding consumption level, where variables without time dependence are constant. The investment in manmade capital is constant along this path, implying that total output is constant, with a fraction $1 - b$ going to consumption and a fraction b going to investment in manmade capital. By normalizing total output to 1, $x^* = 1 - b = 0.84$ and $\dot{k}_C^* = b = 0.16$. If the constant consumption path is implemented as a competitive equilibrium, interest rates are decreasing $(r_i(t) = a/k_C^*(t) = 0.20/k_C^*(t)$, $r_\ell(t) = (a - b)/k_C^*(t) = 0.04/k_C^*(t)$, and $\dot{r}_\ell(t)/r_\ell(t) = -b/k_C^*(t) = -0.16/k_C^*(t))$, competitive capital prices and price changes are given by: $Q_H(t) = \frac{1 - a - b}{a - b}\cdot(k_C^*(t)/k_H^*) = 16\cdot(k_C^*(t)/k_H^*)$, $\dot{Q}_H(t) = \frac{1 - a - b}{a - b}\cdot(b/k_H^*) = 2.56/k_H^*$, $Q_C = 1$, $\dot{Q}_C = 0$, $Q_R(t) = \frac{b}{a - b}\cdot(k_C^*(t)/k_R^*(t)) = 4\cdot(k_C^*(t)/k_R^*(t))$, $\dot{Q}_R(t) = \frac{b}{a - b}\cdot(a/k_R^*(t)) = 0.80/k_R^*(t)$, while resource extraction $-\dot{k}_R^*(t)$ equals $b/Q_R(t) = 0.16/Q_R(t)$.

Let this competitive world economy be split into two countries, between which human capital is distributed evenly $(k_H^1 = k_H^2 = \frac{1}{2}k_H^*)$, while only country 1 is endowed with the resource $(k_R^1(t) = k_R^*(t)$, $k_R^2 = 0)$, and only country 2 owns manmade capital $(k_C^1 = 0$, $k_C^2(t) = k_C^*(t))$. Productive efficiency is ensured by using half of the capital stock and resource flow in each country. As I show in Asheim (1994; Table 1, Case 3), each country keeps consumption constant if $\dot{k}_R^1(t) = \dot{k}_R^*(t)$, $\dot{k}_R^2 = 0$, $\dot{k}_C^1 = 0$, and

$\dot{k}_C^2 = \dot{k}_C^*$, with $x^1 = 0.48$ and $x^2 = 0.36$. The Weitzman-Hartwick NNP then becomes

$$x^1 + \mathbf{Q}(t)\dot{\mathbf{k}}^1(t) = x^1 + Q_R(t)\dot{k}_R^*(t) = 0.48 - 0.16 = 0.32$$
$$x^2 + \mathbf{Q}(t)\dot{\mathbf{k}}^2(t) = x^2 + \dot{k}_C^*(t) = 0.36 + 0.16 = 0.52.$$

Then turn to the adjustments for capital gains and the rate of change of the infinitely long term consumption interest rate, as specified by (10). No adjustment is needed for human capital since, for each $j = 1, 2$, $\dot{Q}_H(t)k_H^j(t) + (\dot{r}_\ell(t)/r_\ell(t)) \cdot Q_H(t)k_H^j(t) = 1.28 - (0.16/k_C^*(t)) \cdot 8k_C^*(t) = 0$. Since country 1 has no stock of manmade capital,

$$\dot{\mathbf{Q}}(t)\mathbf{k}^1(t) + (\dot{r}_\ell(t)/r_\ell(t)) \cdot \mathbf{Q}(t)\mathbf{k}^1(t) = \dot{Q}_R(t)k_R^1(t) + (\dot{r}_\ell(t)/r_\ell(t)) \cdot Q_R(t)k_R^1(t)$$
$$= 0.80 - (0.16/k_C^*(t)) \cdot 4k_C^*(t) = 0.16,$$

while, since country 2 has no resource stock,

$$\dot{\mathbf{Q}}(t)\mathbf{k}^2(t) + (\dot{r}_\ell(t)/r_\ell(t)) \cdot \mathbf{Q}(t)\mathbf{k}^2(t) = \dot{Q}_C(t)k_C^2(t) + (\dot{r}_\ell(t)/r_\ell(t)) \cdot Q_C(t)k_C^2(t)$$
$$= 0 - (0.16/k_C^*(t)) \cdot k_C^*(t) = -0.16.$$

Therefore, it follows from (10) that $y^1 = 0.32 + 0.16 = 0.48$ and $y^2 = 0.52 - 0.16 = 0.36$. For each country does the NNP based on the Weitzman foundation equal the constant consumption level.

This example indicates that the Weitzman-Hartwick NNP underestimate the NNP for resource rich countries (like country 1 in the example) and overestimates the NNP for resource poor countries (like country 2 in the example). Hartwick (1994) notes this problem and points to a study by Pearce and Atkinson (1993), who have calculated the Weitzman-Hartwick NNP for a number of countries. Japan turns out to consume much less that its Weitzman-Hartwick NNP and is ranked as the most sustainable of the 21 countries presented. Indonesia consumes more than its Weitzman-Hartwick NNP and is presented as an unsustainable economy. Given that Indonesia is the more resource rich country of the two, these conclusions may not survive adjustments for capital gains and interest rate effects.

7. The analysis of Sefton and Weale

SW (Sections 1 and 2) claim that their measure of NNP has the advantage that it can be calculated using observable market prices. In contrast, my previous papers (Asheim, 1986, 1994) have measured the maximum sustainable consumption, using the prices that exist along such an efficient constant consumption path. Above we have seen, though, that the measure suggested by SW does not in general satisfy the Weitzman foundation, expect in special cases, one of which is the constant consumption case. In contrast to what SW claims, it turn out that the measure that I provide in Asheim (1986) *does* satisfy the Weitzman foundation also when using observable market prices along paths where consumption is not constant. To see this, substitute $\mathbf{Q}(t)\mathbf{k}^j(t) = \int_t^\infty \frac{p(s)}{p(t)} x^j(s)\,ds$ into (5) to yield $y^j(t) = p(t)\mathbf{Q}(t)\mathbf{k}^j(t) / \int_t^\infty p(s)\,ds$, the r.h.s. of which is identical to the r.h.s. of (S) of Asheim (1986). Even though (S) of Asheim (1986) is intended to measure consumption along an efficient constant consumption path, the present analysis shows that this expression in fact measures NNP satisfying the Weitzman foundation even at prices supporting non-constant consumption paths.

SW do not claim that their consumption-NNP satisfied the Weitzman welfare foundation as interpreted in the present paper. Their analysis is therefore correct on its own terms. In fact, their consumption-NNP has the following attractive features: For the world economy, it equals the Weitzman-Hartwick NNP. Furthermore, the Weitzman-Hartwick NNP is being split into NNPs for the open economies is such a way that (i) the consumption-NNPs of the open economies add up the Weitzman-Hartwick NNP for the world economy and (ii) if consumption is kept constant in an open economy, then NNP for this open economy equals its constant consumption level. Unfortunately, this decomposition is not neutral for changes in consumption paths that do not change welfare.

To see this, consider the numerical example of the previous section, and assume that the utility function of each country is linear. Then the welfare of each country is represented by $\int_t^\infty p(s)x^j(s)ds/p(t)$, where $p(s)$ is the competitive consumption discount factor (i.e., $r_i(t) := -\dot{p}(t)/p(t)$ and $r_\ell(t) := p(t)/\int_t^\infty p(s)ds$). Then, any consumption paths $\{x^1(s)\}_{s=t}^\infty$ and $\{x^2(s)\}_{s=t}^\infty$ that for each s, satisfy $x^1(s) + x^2(s) = 0.84$, the countries' budget constraints, and aggregate feasibility, will lead to the same maximal welfare for each of the two countries. The SW consumption-NNP, $\int_t^\infty(-\dot{p}(s))x^j(s)ds/p(t)$, will, however, be affected for the precise reason that the instantaneous consumption interest rate, $r_i(s) := -\dot{p}(s)/p(s)$, is not constant, since this directly implies that consumption at different times is not weighted in proportion to the welfare weights.

The measure of the maximum sustainable consumption that I suggest in Asheim (1994, Proposition 3) can be written as follows: $[x^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t)]\cdot[\int_t^\infty p(s))x^j(s)ds]/[\int_t^\infty p(s))x^*(s)ds]$. When this expression is being used to measure NNP in open economies at the observable market prices that exist when these economies do not follow constant consumption paths, it shares some of the attractive features of the SW consumption NNP. In particular, by summing over all the open economies one obtains the Weitzman-Hartwick NNP for the world economy which in turn equals the worldwide consumption-NNP of SW. However, the division of this total into NNPs for the open economies is different. In the measure that I suggest in Asheim (1994, Proposition 3), the total is divided between the different countries in proportion to their wealth. This implies that my measure has the additional attractive feature of being insensitive to the kind of welfare neutral changes in consumption paths considered above. Still, as for the SW consumption-NNP, this measure does not generally satisfy the Weitzman foundation. It does satisfy the Weitzman foundation if there is a constant consumption interest rate or if an efficient maximin path for the aggregate economy is implemented by each open economy choosing a constant consumption path.

Appendix

Following Dixit et al. (1980), $(x, \mathbf{k}, \dot{\mathbf{k}})$ is feasible if and only if $(x, \mathbf{k}, \dot{\mathbf{k}}) \in F$, where F is a smooth, convex, and time-invariant set and where $\mathbf{k}$ is a non-negative vector of capital stocks. The set F is assumed to satisfy free disposal of investment flows; i.e., if $(x, \mathbf{k}, \dot{\mathbf{k}}) \in F$ and $\dot{\mathbf{k}}' \leqq \dot{\mathbf{k}}$, then $(x, \mathbf{k}, \dot{\mathbf{k}}') \in F$.[2] A feasible path $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=t}^{\infty}$ is called *competitive* at present value consumption and capital prices $(p(s), \mathbf{q}(s))_{s=t}^{\infty}$ and utility discount factors $\lambda(s) > 0$ if and only if

(i) for each s, $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))$ maximizes instantaneous profit $p(s)x + \mathbf{q}(s)\dot{\mathbf{k}} + \dot{\mathbf{q}}(s)\mathbf{k}$ subject to $(x, \mathbf{k}, \dot{\mathbf{k}}) \in F$,

(ii) for each s, $x^*(s)$ maximizes $\lambda(s)u(x) - p(s)x$ over all x.

Note that (i) combined with the assumption of free disposal of investment flows implies that the vector $\mathbf{q}(s)$ is non-negative. Note that $\mathbf{Q}(s) := \mathbf{q}(s)/\mathrm{p}(s)$ are the current value capital prices used in Sections 4-7. A competitive path $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=t}^{\infty}$ is called *regular* at $(p(s), \mathbf{q}(s))_{s=t}^{\infty}$ and utility discount factors $\lambda(s) > 0$ if and only if

(a) $\int_t^{\infty} \lambda(s)u(x^*(s))dt$ exists

(b) $\mathbf{q}(s)\mathbf{k}^*(s) \to 0$ as $s \to \infty$.

A regular path $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=t}^{\infty}$ maximizes $\int_t^{\infty} \lambda(s)u(x(s))ds$ and $\int_t^{\infty} p(s)x(s)ds$ over all feasible paths $(x(s), \mathbf{k}(s), \dot{\mathbf{k}}(s))_{s=t}^{\infty}$ with initial stocks $\mathbf{k}(t) = \mathbf{k}^*(t)$.

Lemma 1. If $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=t}^{\infty}$ is regular at $(p(s), \mathbf{q}(s))_{s=t}^{\infty}$ (and $(\lambda(s))_{s=t}^{\infty}$), then $\int_t^{\infty} p(s)\dot{x}^*(s)ds = \mathbf{q}(t)\dot{\mathbf{k}}^*(t)$.

Proof. It follows from (i) that, for all s, $p(s) + \frac{d}{dt}(\mathbf{q}(s)\mathbf{k}(s)) \leqq p^*(s) + \frac{d}{dt}(\mathbf{q}(s)\mathbf{k}^*(s))$. Hence, by (b), $\int_t^{\infty} p(s)x(s)ds - \mathbf{q}(t)\mathbf{k}(t) \leqq \int_t^{\infty} p(s)x^*(s)ds - \mathbf{q}(t)\mathbf{k}^*(t)$. In particular, since F is time-invariant (i.e., no exogenous technological progress), $\int_t^{\infty} p(s)x^*(s+\Delta)ds$ -

[2] In the case of environmental capital, this assumption means that negatively valued waste products can freely be generated, not freely be disposed of.

$\int_t^\infty p(s)x^*(s)ds \leqq \mathbf{q}(t)\mathbf{k}^*(t+\Delta) - \mathbf{q}(t)\mathbf{k}^*(t)$, which implies that $\int_t^\infty p(s)[(x^*(s+\Delta) - x^*(s))/\Delta]ds \leqq \mathbf{q}(t)(\mathbf{k}^*(t+\Delta) - \mathbf{k}^*(t))/\Delta$ if $\Delta > 0$ and $\int_t^\infty p(s)[(x^*(s+\Delta) - x^*(s))/\Delta]ds \geqq \mathbf{q}(t)(\mathbf{k}^*(t+\Delta) - \mathbf{k}^*(t))/\Delta$ if $\Delta < 0$. By taking limits, this establishes the result, provided that $\dot{x}^*(s)$ exists for a.e. s and $\mathbf{q}(s)\dot{\mathbf{k}}^*(s) \to 0$ as $s \to \infty$. □

Hartwick's rule is a straightforward consequence of Lemma 1.

Lemma 2 (Hartwick, 1977; Dixit et al., 1980). If $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=t}^\infty$ *is regular at* $(p(s), \mathbf{q}(s))_{s=t}^\infty$ *(and* $(\lambda(s))_{s=t}^\infty$*), and* $\dot{x}^*(s) = 0$ *for all s, then* $\mathbf{q}(t)\dot{\mathbf{k}}^*(t) = 0$.

If the technology exhibits CRS, then F is a convex cone, implying that $(x^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))$ maximizes $p(s)x + \mathbf{q}(s)\dot{\mathbf{k}} + \dot{\mathbf{q}}(s)\mathbf{k}$ subject to $(x, \mathbf{k}, \dot{\mathbf{k}}) \in F$, only if $p(s)x^*(s) + \mathbf{q}(s)\dot{\mathbf{k}}^*(s) + \dot{\mathbf{q}}(s)\mathbf{k}^*(s) = 0$. Hence, $p(s)x^*(s) + \frac{d}{dt}(\mathbf{q}(s)\mathbf{k}^*(s)) = 0$ for all s, such that, by (b), $\mathbf{q}(t)\mathbf{k}^*(t) = \int_t^\infty p(s)x^*(s)ds$.

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