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## REGULATION OF ENVIRONMENTAL QUALITY UNDER ASYMMETRIC INFORMATION

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## REGULATION OF ENVIRONMENTAL QUALITY UNDER ASYMMETRIC INFORMATION

### Abstract

We analyze the impact of a standard about environmental quality on the efficiency of local contracts. The standard is set nationwide by a federal government but the quality can be renegotiated on a local level. We consider the case that at the local level, one party has private information. We show that, for that case, the allocation will be distorted away from the efficient solution in the direction of the threat point. The threat point is defined by the standard (the definition of property rights). From a normative perspective, we prove that the federal government can do better than allocate the property right to either of the players. It is shown that efficiency can be increased by setting the standard between either extremes. For a specific example, conditions for the welfare-maximizing standard are discussed.

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## 1. Introduction

Preferences for environmental quality vary across different regions. Similarly, regulations about the quality of environment will affect profits of local firms across regions in different ways. Therefore, it would not be efficient to impose a nationwide standard for environmental quality by a central agency since in each region the quality level should allow for specific regional characteristics. Under complete information, leaving the definition of regional quality to a decentralised bargaining process should result, according to Coase Theorem (Coase, 1960), in an efficient outcome: Rational agents with complete information about all feasible efficiency gains should be expected to agree upon an efficient contract in a bargaining game.

In general, however, the parties involved in the bargaining game (the polluters and the pollutee) have private information about their own characteristics. In that case, the outcome of the bargaining process will generally be distorted relative to a Pareto-optimal solution (Samuelson (1985), Farrell (1987)). In particular, the bargaining solution will depend on which outcome will be realized in the absence of an agreement: the threat point of the bargaining game has an impact on the solution. A similar observation has been made by Illing (1991).

In this paper, we argue that for this reason, it is not optimal to leave the decision about environmental quality entirely to the bargaining process at the local level. Rather, the federal government should define a nationwide standard, but also allow for renegotiation between the parties affected at the local level, thus making it possible to take into account regional differences. We analyze the effect of a nationwide standard on the efficiency of local contracts. For simplicity, we only consider the case where the local government has private information about her disutility of pollution.

It is shown that, in this case, the definition of property rights has a predictable impact on the outcome. There are essentially three cases. First, the case where the firm "owns" the right to pollute. In this case, clean environment becomes a good and the firm will be willing to sell a higher quality of the environment against cash. However, the asymmetric information structure yields standard inefficiency results; for almost all type of local governments the polluter underproduces quality of the environment except for the local government which values quality of the environment the most, the inefficiency is decreasing in type and the local government extracts a positive rent. This is the standard case studied in the literature on asymmetric information, see for example Baron and Myerson (1982) and Maskin and Riley (1984).

Second, the case where the local government owns the right to a clean environment. In this case, it is pollution which becomes a good and the firm will offer to buy the right to pollute. Again, the private information structure results in inefficiencies. We will prove, however, that now the polluter overproduces quality of the environment, efficiency results for the local government which values quality the least and inefficiency is increasing in type. Thus, switching the threat point also switches the direction and the slope of the inefficiency.

The third case is when the federal government sets an intermediate standard for the quality of the environment. We show that, depending on the type of the local government, quality of the environment will either be over or underproduced compared to the first-best solution. The slope of the inefficiency also switches. Initially, inefficiency is first increasing, then decreasing and then again increasing in type. There are three types of local governments which yield a pareto efficient outcome; the local government which values quality of

environment the most, the one which values it the least and an intermediate case. From a theoretical perspective, we believe these are novel results.

According to the Coase Theorem, the distribution of property rights has no effect on the efficiency of the outcome. Numerous authors have already pointed out that this result does not hold in the presence of asymmetric information (see Samuelson (1985), Farrell (1987), Schweizer (1988)). We show that, in the presence of one-sided asymmetric information, the allocation will be distorted in the direction of the threat point. Behind this result is the following intuition: because property rights define the threat point of the game, and the threat point determines the individual rationality constraints, the solution is sensitive to a change in the distribution of property rights. They determine the flow of transfers.

From a normative perspective, we prove that the federal government can do better than allocate the property right to either of the player. We suggest that the federal government sets a nationwide standard and simultaneously allows the parties to bargain away from that standard. The standard defines the threat points of either parties. We show that efficiency can be increased by setting the standard between either extremes. For a specific example, the optimality condition for the welfare-maximizing standard is given.

The paper is closely related to Illing (1991) which discusses the case where types can only take two values. In this case, the results are not completely satisfactory because first-best efficiency can be obtained in either state. The present model generalizes Illing (1991) to the case of a continuous distribution and points out that his result is an artifact of the two-point distribution.

Our paper is also related to Sinn and Schmolzi (1981) who analyze the Coase Theorem

in the context of market power. They derive a similar result that efficiency increases when property rights are allocated closer to the efficient outcome. Our paper obviously differs in that we study an asymmetric information problem. It also suggests a justification for market power as assumed in Sinn and Schmolzi (1981).

The remaining of the paper is organized as follows. In the next section, we present the model. In the following section we discuss the problem of the polluter as the principal. In section 4, we solve for the two extreme cases; the firm has the right to pollute and the local government owns the right to a clean environment. Section 5 derives the main result. In section 6, we discuss the choice of an optimal standard by the federal government. The last section offers some concluding remarks.

## 2. The Model

The federal government sets a nationwide standard  $y$  determining the minimum environmental quality  $x$  across all regions. It will be assumed that the quality level can be measured quantitatively. The level  $y$  will be enforced in all regions, unless polluter and the local council voluntarily agree to deviate from it. For simplicity, we assume that there is only one polluter at the local level. Given the standard  $y$ , the polluter and the council have the right to renegotiate. They can sign a contract specifying a different quality of the environment which takes into account the specific preferences for quality of the local population (represented by the preference of the local council) and the extent to which profits of the local producers are affected by regulations concerning the quality of the environment.

Ideally, the contract should be designed such that the allocation is pareto optimal given the specific preferences of the local council and the profit function of the firm. In general,

however, both the preferences and the profit function are private information of the respective participants. Thus, the contract has to take into account incentive compatibility constraints. In addition, as each party has to agree voluntarily, the payoff out of the contract cannot be lower than the payoff which the parties would get in case the standard  $y$  is imposed. In this way, the standard  $y$  determines the individual rationality constraints of both parties (the threat point of the game).

In the paper, we analyze how a change in the standard  $y$  affects the individual rationality constraints and thus has an impact on the outcome (the quality of environment which is obtained under the contract). For illustrative purpose, we consider the following game: the polluting firm offers a contract to the local council, and the council either accepts or disagrees. In the latter case, the standard  $y$  will be implemented.

### 2.1. The polluting firm

The polluting firm produces output  $q$ . In the absence of any restrictions concerning environmental quality, the firm would maximise revenue minus cost:  $\pi(q) = R(q) - C(q)$ . When a minimum environmental quality  $x$  has to be obeyed, the firm producing output  $q$  must incur additional costs  $\phi(x, q) > 0$  for all strictly positive pair  $(x, q)$  (the costs of keeping environmental quality at  $x$  when producing  $q$ ). A natural assumption is that  $\phi(x, q)$  is convex in  $q$  and  $x$  and that  $\pi$  is concave in  $q$ .  $\Pi(x)$  denotes the maximum profit of the firm when the quality level  $x$  has to be met, that is:

$$\Pi(x) = \max_q \pi(q) - \phi(q, x) \quad (1)$$

In appendix 1, we prove the following proposition:

*Proposition 1:*  $\Pi$  is decreasing concave in the quality level  $x$ .

## 2.2. The local council

The local council is concerned about environmental quality. Her preferences are  $u(x, a)$ ,  $a$  being a parameter which characterizes the extent to which good quality is appreciated by the local council. For simplicity, we represent the preference of the local council by a specific functional form  $u(x, a) = aU(x)$ , with  $U$  being increasing and concave in  $x$ . This restriction implies  $u_{xx} > 0$ : with stronger preferences for environmental quality the marginal utility increases for each quality level.<sup>1</sup>

The parameter  $a$  is private information of the local council. The firm has only information about the density of the characteristic  $a$ . Specifically, we assume that  $a \in [a_1, a_2]$  and  $a \sim f(a)$  with cumulative  $F(a)$ . For the purpose of this paper the information of the firm, thus,  $f$  and  $F$ , could be interpreted as subjective. Qualitatively, it would not affect any of the results. We impose the following restriction on the distribution of  $a$ :<sup>2</sup>

$$\forall \alpha \in [0, 1], \quad \frac{d}{da} \left[ \frac{F(a) - \alpha}{f(a)} \right] > 0 \quad (2)$$

## 2.3. The first-best solution

In order to examine the impact of private information and of the standard  $y$  on the

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<sup>1</sup> Throughout the paper, we use standard notations. A function with a subscript denotes a partial derivative with respect to that variable. Alternatively, when we write an optimal control problem, we use a  $\cdot^*$  to denote the first derivative of a function.

<sup>2</sup> This is a standard hazard rate assumption. Assumptions of this type are common in asymmetric information problems and are satisfied for the standard distributions.



outcome and, in particular, the efficiency of the contract, we first derive the optimal environmental quality under complete information. The first-best solution is defined for every state as:

$$x^*(a) = \underset{x}{\operatorname{Argmax}} \Pi(x) + aU(x) \quad (3)$$

The concavity of the functions  $\Pi$  and  $U$  guarantees that the first-best solution is increasing in the parameter  $a$ . As the appreciation of the local council for quality increases, the marginal utility curve is shifted upwards and thus optimal quality will increase.

### 3. The problem of the firm

The importance of the standard  $y$  under private information is illustrated by analyzing the following game: the firm proposes a contract to the local council. Then, the council either accepts or rejects the proposal. Thus, the firm acts as principal with the local council as the agent. According to the revelation principle, designing an optimal contract is (without loss of generality) equivalent to designing an optimal truth-revealing mechanism, see e.g. Myerson (1979). That is, the firm offers an incentive compatible, individually rational contract which induces the local council to reveal her true characteristic  $a$ . After revealing  $a$ , the quality level  $x(a)$  will be realised and the firm makes transfer payments  $t(a)$  to the local council. The transfer payments may be negative ( $t(a) < 0$ ). In that case, the firm receives payments from the council. The problem of the firm can be written as a control problem of the form:

$$\text{Max}_{x,t} \int_{t_1}^{t_2} \{\Pi(x(a)) - t(a)\} f(a) da$$

$$(4) \quad a \in \underset{r}{\text{Argmax}} \quad aU(x(r))+t(r)$$

$$(5) \quad aU(x(a))+t(a) \geq aU(y)$$

Equation (4) is the incentive compatibility constraint. It restricts the mechanism in such a way that it be optimal for the council to reveal her type truthfully. The second constraint, equation (5), guarantees that the council is never worse off by accepting rather than rejecting the mechanism. To solve the firm's problem, it is convenient to reformulate it by eliminating the transfer function. For this purpose, we define by  $V(a)$  the payoff from the council's reporting problem, i.e. the maximum payoff for a council with the characteristic  $a$ . The incentive compatibility constraint implies the following equality:

$$t(a) = V(a) - aU(x(a)) \quad (6)$$

Because the optimal contract anticipates the maximizing behaviour on behalf of the council, the first-order condition  $aU_{x_1}(a) + t_1(a) = 0$  will hold. This implies that the council's maximum payoff varies with her type according to the following equation:

$$\dot{V}(a) = U(x(a)) \quad (7)$$

Using these equations, the problem of the firm can be rewritten in terms of  $x$  and  $V$ . We follow standard practice. We, initially, substitute the first-order condition from the incentive compatibility constraint. We solve the resulting simplified problem. We, then, check that

truthful reporting is, indeed, globally optimal. The simplified optimization problem is:

$$\text{Max}_{x, V} \int_a^b \{\Pi(x) + aU(x) - V(a)\} f(a) da$$

$$(8) \quad \dot{V}(a) = U(x(a))$$

$$(9) \quad V(a) \geq aU(y)$$

This is a control problem with a restriction on the state variable.  $x$  is the control and  $V$  the state variable. We apply a standard sufficiency result from Seierstad and Sydsæter (1977) which for simplicity is rewritten in appendix 2. The Lagrangian of the above problem is:

$$L(x, V, \lambda, \eta, a) = \{\Pi(x) + aU(x) - V(a)\} f(a) + \lambda U(x) + \eta[V(a) - aU(y)] \quad (10)$$

Applying the above mentioned theorem yields the following set of conditions:

$$\{\Pi_x + aU_x\} f(a) + \lambda U_x = 0 \quad (11)$$

$$-f(a) + \eta = -\dot{\lambda} \quad (12)$$

$$\dot{V}(a) = U(x(a)) \quad (13)$$

$$\eta \geq 0 \quad (14)$$

$$\eta[V - aU(y)] = 0 \quad (15)$$

(11) is a standard Euler equation. (12) is the co-state equation. (13) is the law of motion, i.e. constraint (8). Inequality (14) imposes a non-negativity restriction on the Lagrange multiplier. The justification is the same as in the Kuhn-Tucker theorem. Finally, (15) is the complementary

slackness condition. We have omitted the boundary conditions at this time, as they depend on the size of  $y$ .

Before analyzing the general solution, we first characterize two natural extreme cases. First, the case where the firm owns the right to pollute and, second, the alternative case where the local council has the right to a clean environment. They also provide a good intuition for the general case.

#### 4.1. The firm has the right to pollute

If the firm owns the right to pollute, the quality standard should be  $y=0$ . More generally, we also identify with this case any quality standard  $y$  such that "clean environment" is a good, i.e. such that the solution of the bargaining process yields for every type a quality of the environment larger than the standard  $y$ .  $M = [x(a), t(a)]$  denotes the optimal mechanism for this case. Thus, analytically, we consider in this section the case where  $x_i(a) > y$  for all  $a$ .

*Proposition 2:* Define  $x(a)$  as the implicit solution of the following equation:

$$\Pi_x + aU_x + \frac{F(a)-1}{f(a)} U_x = 0 \quad (16)$$

If  $y < x(a_i)$ , then the solution to the firms problem is given by (16) for the optimal environmental quality and by the following equations:

$$V(a) = \int_{a_1}^a U(x_*(\bar{a})) d\bar{a} + a_1 U(y) \quad (17)$$

$$t_*(a) = V(a) - aU(x_*(a)) \leq 0 \quad (18)$$

$x_*(a)$  is increasing in  $a$ ;  $x_*(a) < x^*(a)$  for all  $a < a_2$  and  $x_*(a_2) = x^*(a_2)$ .

*Proof:* If for all types  $x_*(a) > y$ , then according to equations (8) and (9) the slope of the profit function is steeper than the slope of the constraint:

$$\dot{V}(a) = U(x_*(a)) > U(y) \quad (19)$$

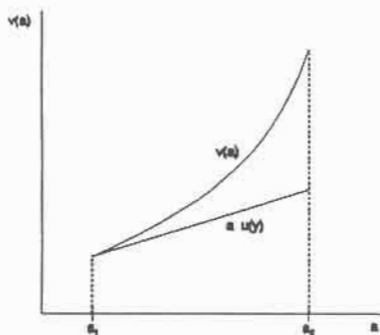


Figure 1

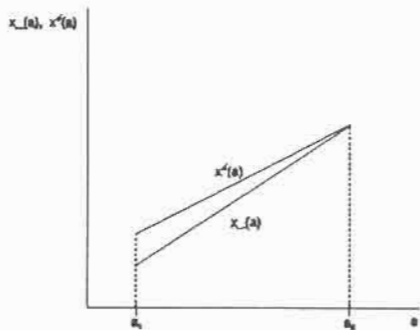


Figure 2

The firm tries to keep  $V(a)$  at the lowest feasible level. Since the function  $V(a)$  is always steeper than the function  $aU(y)$ , the individual rationality constraint  $V(a) \geq aU(y)$  will be binding at  $a_1$  (see figure 1), that is:

$$V(a_1) = a_1 U(y) \quad (20)$$

The complementary slackness, eq. (15) requires  $\eta = 0$  a.e. .The co-state equation, eq.(12), then implies  $\lambda = F(a) + \text{constant}$  a.e. . Since  $V(a_2)$  is free, the boundary condition implies  $\lambda(a_2) = 0$ ; therefore  $\lambda = F(a) - 1$ . Substituting this result into the law of motion yields the definition of  $x_*(a)$  given by equation (16). The slope of  $x_*(a)$  follows by implicit differentiation:

$$\frac{dx_*}{da} = - \frac{U_x + \frac{d}{da} \left[ \frac{F(a)-1}{f(a)} \right] U_x}{\Pi_{xx} + aU_{xx} + \left[ \frac{F(a)-1}{f(a)} \right] U_{xx}} > 0 \quad (21)$$

$a + [F(a)-1/f(a)] > 0$  follows from equation (16), because  $\Pi_x < 0$  and  $U_x > 0$ . Rewriting equation (16) proves by the concavity of  $\Pi + aU$  that  $x_*(a) \leq x^*(a)$  for all  $a$ .

$$\Pi_x(x) + aU_x(x) = \left[ \frac{1-F(a)}{f(a)} \right] U_x(x) \geq 0 \quad (22)$$

Figure 2 compares the outcome of the contract with the first-best solution. The figure also shows that the case discussed here is relevant if the standard is so low that  $x_*(a_1) \geq y$ . The definition of  $V(a)$ , eq. (17), follows from equation (13), and the observation that  $V(a_1) = a_1 U(y)$ . Equality (18) follows immediately from (6). We now prove that the transfers are always negative. From the incentive compatibility constraint we know that

$$t_1(a) = -a U_x x_*(a) < 0 \quad (23)$$

Since  $U$  and  $x$  are increasing,  $t$  is decreasing in  $a$ . Therefore it is sufficient to show that  $t(a_1) \leq 0$  to prove the claim. But  $t(a_1) = a_1 [U(y) - U(x_*(a_1))] \leq 0$ .

Finally, to conclude the proof we show that truthful reporting is, indeed, globally incentive compatible for the mechanism  $M$ . Define  $\bar{U}(a,r) = aU(x(r)) + t(r)$ , that is  $\bar{U}(a,r)$  denotes the utility of the local council when her type is  $a$  and she reports  $r$ . Thus the reporting problem of the local government is:

$$V(a) = \max_r \bar{U}(a,r)$$

By construction,  $\bar{U}_r(a,a) = 0 \forall a$ . Totally differentiating yields

$$\bar{U}_a(a,a) + \bar{U}_r(a,a) = 0 \quad (24)$$

Thus,  $\bar{U}_a = U_x x_a > 0$  for all pairs guarantees global incentive compatibility.  $\blacklozenge$

Behind the results lies a straightforward intuition: With a low enough standard, the council is willing to pay in order to increase the quality of the environment for all values of  $a$ . In that case, the firm is selling part of its right to pollute receiving, in return, compensation payments ( $t(a) \leq 0$ ). It behaves like a discriminating monopolist facing a council with unknown demand, see Maskin and Riley (1984).

The firm wants to minimize the informational rent extracted by the council. If the firm offered to sell to each type its efficient amount charging the council's reservation price, a council with high valuation would have an incentive to disguise as a low value type in order to catch a rent. Thus, to give an incentive for high value types to choose the contract designed for them, the firm has to bribe them by paying an information rent. This rent can be minimised by distorting the allocation (selling less than the efficient amount) for low types, thus making it less

attractive to pretend being a low-type council.

Only the council with highest valuation buys the efficient amount of the property right; she gets the highest information rent. In contrast, only the council with lowest valuation gets no rent at all. When the firm is the owner of the right to pollute, the council as a buyer has an incentive to understate her true valuation of the good. This tendency results in an inefficiently low quality of environment for almost all types.

#### 4.2 The local council has the right of a clean environment

The solution of this section will be denoted by by the mechanism  $M^* = [x^*(a), t^*(a)]$ . We identify with this case all such quality standard that pollution becomes a good - meaning that for all types of councils the transfers will all be positive and  $x^*(a) \leq y$ .

*Proposition 3:* Define  $x^*(a)$  as the implicit solution of the following equation:

$$\Pi_x + aU_x + \frac{F(a)}{f(a)} U_x = 0 \quad (25)$$

If  $y > x^*(a_2)$ , then the solution to the firms problem is given by (25) for the optimal environmental quality and by the following equations:

$$V(a) = a_2 U(y) - \int_{x^*(a)}^{y} U(x^*(\bar{a})) d\bar{a} \quad (26)$$

$$t^*(a) = V(a) - aU(x^*(a)) \geq 0 \quad (27)$$

$x^*(a)$  is increasing in  $a$ ;  $x^*(a) > x^*(a)$  for all  $a > a_1$  and  $x^*(a_1) = x^*(a_1)$ .



*Proof:* The proof parallels the proof of the last proposition. If for all  $a$ ,  $x^+(a) \leq y$ , then from (8) and (9) follows:

$$\dot{V}(a) = U(x^+(a)) < U(y) \quad (28)$$

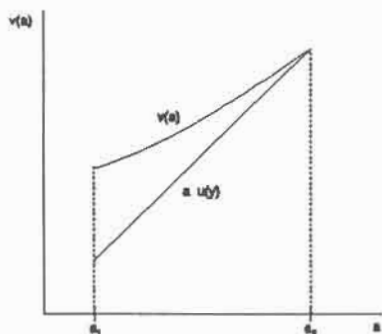


Figure 3

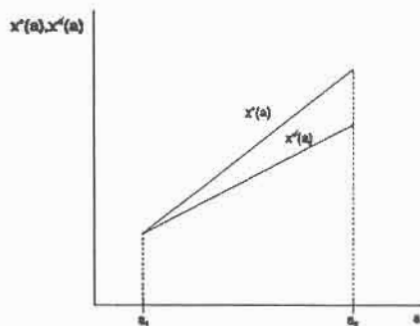


Figure 4

Since now the slope of  $V(a)$  is always less than the slope of the constraint  $aU(y)$ , the individual rationality constraint  $V(a) \geq aU(y)$  will be binding at the boundary point  $a_2$  (see figure 3). This implies that  $\eta = 0$  a.e. and thus,  $\lambda = F(a) + \text{constant}$ . Now, however,  $V(a_1)$  is free (see figure 1), implying  $\lambda(a_1) = 0$ . In conclusion  $\lambda = F(a)$ . This yields equation (25) as the definition of  $x^+(a)$ . Rewriting the Euler equation as in (29) shows that  $x^+(a) > x^*(a) \forall a \neq a_1$  and  $x^+(a_1) = x^*(a_1)$ .

$$\Pi_x(x) + aU_x(x) = - \left[ \frac{F(a)}{f(a)} \right] U_x(x) \leq 0 \quad (29)$$

Therefore, in the case discussed here environmental quality is always inefficiently high

except for the council with the lowest valuation (compare figure 4). From figure 4 it becomes also obvious that the case discussed holds only if the standard has been set so high that  $x^*(a_2) < y$ . The slope of  $x^*(a)$  follows from implicitly differentiating (25). Applying the same reasoning as in the foregoing proposition proves that  $x^*(a)$  is an increasing function of types. We omit the proof of the global incentive compatibility of the mechanism  $M^*$  as it is exactly the same as in proposition 2.

To conclude the proof, we show that transfers are always positive. As in the foregoing proof the fact that  $x^*(a)$  is increasing guarantees that  $t_1(a) < 0$ . Therefore, it is sufficient to show  $t(a_2) > 0$  in order to prove the claim. This follows immediately since  $y \geq x^*(a_2)$  and  $t(a_2) = a_2[U(y) - U(x(a_2))]$ .

♦

Behind the result is the following intuition: In the present case, the council owns the right to have a clean environment. Now, she is selling part of her rights to the firm. The firm pays transfers to the council. As a seller, she has an incentive to overstate the value of her good (the value of environmental quality). Therefore, the firm designs the contract in a way to make it unattractive to disguise as a high type council.<sup>3</sup> For that purpose, information rents are paid to low types and the allocation will be distorted in such a way that high types will sell less than the efficient amount (implying that quality will stay inefficiently high for almost all types). The contract sets an inefficiently high environmental quality except for the type  $a_1$  with the lowest valuation. Only the council with highest valuation, type  $a_2$  receives no rent.

One way to contrast proposition 2 and 3 is to think of either case as involving a different

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<sup>3</sup>The problem of the firm in the present case is formally equivalent to a regulator facing a monopolist with unknown cost (see Baron/Myerson 1982)

threat point. In section 4.1 the firm is in a strong position. Compared with the first-best solution quality of the environment is underproduced. In section 4.2 the local council is in a strong position. In this case, the quality of the environment is overproduced. Thus, switching the threat point also switches the direction of the inefficiency.

## 5. The general case

In the two foregoing sections, whether "clean environment" or pollution was a good and the implied direction of the inefficiency depended solely on the quality standard of the environment set by the federal government. In general, when the standard is in between either extreme, we will see that the specific type of the local council plays a determining factor. For some characteristics the quality of the environment will be too low and for others too high relative to the first-best solution.

There are five relevant cases to be analyzed, which can be distinguished by whether  $y$  belongs to a particular interval. The different cases follow from figure 5. The two extreme cases (I) and (V) have already been described in detail in the foregoing sections by the propositions 2 and 3.

Before presenting the main result of the paper, we require, in order to keep the notation compact, some further definitions:

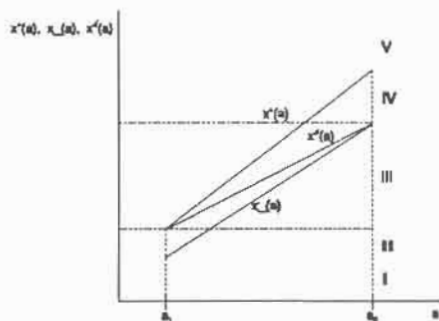


Figure 5

$$a_3(y) = \begin{cases} a_1 & y \leq x^*(a_1) \\ x^{-1}(y) & y \in [x^*(a_1), x^*(a_2)] \\ a_2 & y \geq x^*(a_2) \end{cases} \quad (30)$$

$$a_4(y) = \begin{cases} a_1 & y \leq x_-(a_1) \\ x^{-1}(y) & y \in [x_-(a_1), x_-(a_2)] \\ a_2 & y \geq x_-(a_2) \end{cases} \quad (31)$$

Geometrically,  $a_3$  is the closest point to the interval  $[x^*(a_1), x^*(a_2)]$ , whereas  $a_4$  is the closest point to the interval  $[x_-(a_1), x_-(a_2)]$ . In the appendix, we prove the following result:

*Proposition 4:* The optimal mechanism  $M = [x(a), t(a)]$  has the following form:

$$x(a) = \begin{cases} x^*(a) & y \geq x^*(a) \\ y & x^*(a) \geq y \geq x_-(a) \\ x_-(a) & y \leq x_-(a) \end{cases} \quad (32)$$

$$V(a) = \begin{cases} a_3(y)U(y) - \int_a^{a_3(y)} U(x(\bar{a}))d\bar{a} & y \geq x^*(a) \\ aU(y) & x^*(a) \geq y \geq x_-(a) \\ a_4(y)U(y) + \int_{a_4(y)}^a U(x(\bar{a}))d\bar{a} & y \leq x_-(a) \end{cases} \quad (33)$$

$$t(a) = V(a) - aU(x(a)) \quad (34)$$

For case (II), there is no trade in property rights for all types with a characteristic low enough,  $a \leq a_4$ ; for the remaining types  $a > a_4$ , the council pays in order to reach a higher quality of the environment. As in case I, quality remains inefficiently low except for type  $a_2$  with the strongest preferences for quality. The reverse holds for case (IV). In this instance, there is no trade in property rights for large types  $a \geq a_3$ ; for all other types, the firm gives compensation payments in order to increase its right to pollute and, thereby, reduces the quality of the environment.  $x$  is efficient only for type  $a_1$ , with the lowest preference, otherwise quality is inefficiently high. Both results follow the standard line of argument discussed in the foregoing sections.

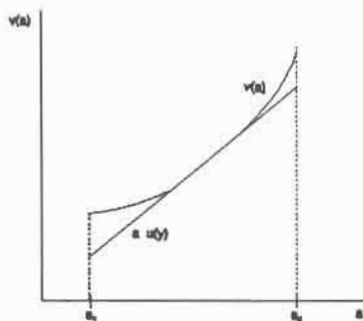


Figure 6

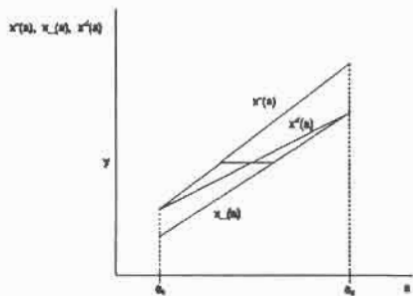


Figure 7

To our knowledge, case (III) is novel. Case (III) follows, when the federal standard is within the interval  $[x^*(a_1), x^*(a_2)]$ . In this situation, one can distinguish three subcases. For an interval with low preferences ( $a < a_3$ ), the firm pays the council in order to reduce the quality of the environment below the federal standard, whereas for a medium range of preferences there is no trade at all [ $t(a)=0$ ]. On the other hand, a council with strong enough preferences ( $a > a_4$ ) pays the firm to increase quality [ $t(a) > 0$ ]. The outcome is efficient for three different types<sup>4</sup>: Quality is efficient both for the lowest and the highest type. In addition, for each  $y$  in the interval  $[x^*(a_1), x^*(a_2)]$ , there exists a type  $a^*(y)$  such that  $x^*(a^*(y)) = y$ . Therefore, within the range of characteristics with no trade, there always exists some type  $a^*$  for which the efficient outcome happens to be the standard  $y$  set by the government. Of course, the efficient solution will be realized if, by chance, the council's characteristic turns out to be  $a^*$ .

Thus, with a federal standard set between the first-best extreme points inefficiency is, initially, increasing, then decreasing and finally again increasing in type. The slope of the inefficiency also switches. Depending on the type of the local council quality of the environment will either be under or overproduced compared to the first-best solution. In either case, the allocation is distorted in a predictable way: trade is inefficiently low relative to the first-best solution.

The council's payoff is shown as a function of her characteristic in figure 7. All types with trade ( $t \neq 0$ ) receive an informational rent. The interval  $[a_3, a_4]$  is determined by the boundary conditions that quality has to be efficient at  $a_1$  and  $a_2$ . The proof of proposition 4 applies a theorem by Seierstad and Sydsæter (1977) which, for convenience, is reproduced in

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<sup>4</sup> In case (II) there is an extreme case, when  $y$  is just equal to  $x(a_1)$ , where quality is efficient for both the highest and the lowest type. The same result holds in case (IV) when  $y = x^*(a_2)$ .

appendix 1. The proof of the result follows in appendix 2.

The results of Illing (1991) are a special case of our model. When local council can be of only two types the solution described in proposition 4 yields always the first-best outcome. The result can be extended to the case with three types if the mechanism sets the quality standard equal to the first-best solution for the intermediary type. The results are obviously misleading and are an artifacts of the two and three point distribution.

## 6. Optimal Standard

From proposition 4 it follows immediately that the efficiency of the contract is strongly influenced by the standard set by the government. When the federal government has to decide which standard should be imposed on a national level, it, in general, has no specific information about the preferences of the local councils in different regions. It is natural to assume that the distribution of council's preferences across the regions is equal to the density function  $f(a)$ . In the absence of better information, the government's objective will be to maximize ex ante the expected welfare which is equivalent to minimizing expected inefficiencies.

Consider, for example, the case where types are uniformly distributed over the interval  $[a_1, a_2]$ . For every type the inefficiency can be characterized by the difference between the actual and the first-best level of quality of the environment. Thus, since each type has the same likelihood, the aggregate expected inefficiency can be represented in figure 7 as the sum of two areas: for all the characteristics  $a$  with  $x(a) < x^*(a)$ , the loss in efficiency due to quality being too low is the area below the function  $x(a)$  and above the function  $x^*(a)$ . On the other hand, for all characteristics with  $x^*(a) > x(a)$ , the sum of the losses can be represented by the area

above the function  $x^*(a)$  and below the function  $x^+(a)$ . With  $y$  increasing from  $x^*(a_1)$  to  $x^*(a_2)$ , expected efficiency first increases and then decreases. Thus, there is an interior optimum.

More generally, our model suggests some important policy conclusions: It cannot be optimal to set the quality standard for the environment in such a way that the property right is completely allocated to either the local council or to the firm. By giving the council the right to a clean environment, the quality of the environment agreed upon in the optimal contract between council and firm will almost always be inefficiently high, and the reverse is true when giving the firm the right to pollute. Thus, our model suggests that allocating the property right more equally, using an intermediate quality standard, is welfare enhancing. Independently of the distribution of types, the optimal quality standard must lie somewhere in the range between the efficient outcomes for the types with weakest and strongest preferences;  $y^* \in [x^*(a_1), x^*(a_2)]$ . A further specification of the optimal  $y$  depends on the precise distribution of types.

## 7. Concluding Comments

In the present paper, for simplicity, we have analyzed the case of one-sided private information. The results can be generalized to the case of two-sided asymmetric information when there is no correlation and one of the parties has complete bargaining power. Then this party acts as the principal, and as shown by Maskin and Tirole (1990), in the absence of income effects private information of the principal does not change the solution of the optimal contract. In the general case, if both parties have bargaining power, characterising the outcome of a bargaining game with two-sided information is fairly complex; in general there is no unique equilibrium. We, however, conjecture that also for general bargaining games, in sensible



(refined) equilibria the same results (the sensitivity of the bargaining outcome to the threat point) will be obtained for the following reason: The seller of a property right has always an incentive to overstate the value of his product whereas the buyer will understate her willingness to pay. Both effects work in the same direction: For both reasons, an inefficiently low amount of the property right will be traded. From that consideration, we conjecture that in general the allocation will be distorted in the direction of the threat point.

### Appendix

#### Appendix 1

*Proof of proposition 1:* Denote  $K(q, x) = \pi(q) - \phi(q, x)$ . Thus,  $K$  is concave in  $q, x$ . Applying the envelope theorem, we have:

$$\Pi_x(x) = -\phi_x(q^*(x), x) < 0 \quad (\text{A1})$$

where  $q^*$  denotes the profit maximizing production given the imposed quality level  $x$ . (2) proves the first claim that  $\Pi$  is decreasing. To conclude the proof, we define:

$$q_i = \underset{q}{\text{Argmax}} K(q, x_i)$$

The concavity of  $L$  and the definition of  $\Pi$  imply:

$$\begin{aligned} \Pi(\lambda x_1 + (1-\lambda)x_2) &\geq K(\lambda q_1 + (1-\lambda)q_2, \lambda x_1 + (1-\lambda)x_2) \\ &\geq \lambda K(q_1, x_1) + (1-\lambda)K(q_2, x_2) \\ &\geq \lambda \Pi(x_1) + (1-\lambda)\Pi(x_2) \end{aligned}$$

♦

**Appendix 2:**

In this section, we prove proposition 4. For convenience, we restate theorem 8 of Seierstad and Sydsæter (1977) in terms of the present model. From section 3, we know that the principal's problem can be restated as an optimal control problem:

$$\text{Max}_{x, V} \int_{x_1}^{x_2} \{\Pi(x) + aU(x) - V(a)\} f(a) da$$

$$(A2) \quad \dot{V}(a) = U(x(a))$$

$$(A3) \quad V(a) \geq aU(y)$$

Following Seierstad and Sydsæter, we define the following functions:

$$H(x, V, \lambda, a) = \{\Pi(x) + aU(x) - V\} f(a) + \lambda U(x) \quad (A4)$$

$$H^*(V, \lambda, a) = \max_x H(x, V, \lambda, a) \quad (A5)$$

$$L(x, V, \lambda, a, \eta) = \{\Pi(x) + aU(x) - V(a)\} f(a) + \lambda U(x) + \eta [V(a) - aU(y)] \quad (A6)$$

Assume there exists a piecewise continuous function  $\lambda(a)$ , a function  $\eta(a) \geq 0$  and numbers  $\beta_i$ ,  $i=1, \dots, k$  such that for all  $a$  except for at a finite number of points the following conditions are satisfied:

$$H^*(V, \lambda, a) = \max_x H(x, V, \lambda, a) \quad (A7)$$

$$\dot{\lambda} = - \frac{\partial L}{\partial V} (x, V, \lambda, a, \eta) \quad (A8)$$

$$\begin{aligned} \lambda(a_i^-) - \lambda(a_i^+) &= \beta_i \\ \text{where } t_0 &< \tau_1 < \dots < \tau_k \leq t_1 \\ &\text{on the discontinuity points of } \lambda \end{aligned} \quad (A9)$$

$$\beta_i \geq 0 \quad ( = 0 \text{ if } V(a) - aU(y) > 0 ) \quad (A10)$$

$$\eta[V - aU(y)] = 0 \quad (\text{A11})$$

$$\eta \geq 0 \quad (\text{A12})$$

$$\lambda(a_1)[V(a_1) - a_1U(y)] = \lambda(a_2)[V(a_2) - a_2U(y)] = 0 \quad (\text{A13})$$

$$H^*(V, \lambda, a) \text{ is concave in } V \quad (\text{A14})$$

$$V - aU(y) \text{ is quasi-concave and differentiable in } V. \quad (\text{A15})$$

Then  $[V(a), x(a)]$  is an optimal pair.<sup>5</sup>

*Proof of Proposition 4:* (A15) is trivially satisfied. Further, applying the envelope theorem to (A5) immediately proves that the requirement (A14) is also satisfied. We have proven case (I) and (V) in the proposition 2 and 3. We first start by proving case (III). The relevant conditions are (A11), (A12), (A13) and:

$$\{\Pi_x + aU_x\}f(a) + \lambda U_x = 0 \quad (\text{A16})$$

$$-f(a) + \eta = -\dot{\lambda} \quad (\text{A17})$$

$$\dot{V}(a) = U(x(a)) \quad (\text{A18})$$

We define  $\alpha(a)$  such that  $\lambda(a) = F(a) - \alpha(a)$  and  $x(a)$  by equation (32) in the main text. This implies:

$$\lambda(a) = \begin{cases} F(a) & a_1 \leq a \leq a_3(y) \\ - \left\{ \frac{\Pi_x(y)}{U_x(y)} + a \right\} f(a) & a_3(y) < a \leq a_4(y) \\ F(a) - 1 & a_4(y) < a \leq a_2 \end{cases} \quad (\text{A19})$$

<sup>5</sup> We have slightly generalized the theorem in order to take into account that  $V$  is only constrained by an inequality condition at either boundary. In Seierstad and Sydsæter  $V$  is constrained at the lower bound by an equality constraint.

We note that  $\alpha'(a) = \eta(a)$ . Thus, for types outside of the interval  $[a_3(y), a_4(y)]$ , we have  $\eta(a) = 0$  and (A11) imposes no constraint on  $V$ . Further from (A19)  $\lambda(a_1) = \lambda(a_2) = 0$ , thus,  $V$  is neither constrained at the boundaries. Within the interval  $[a_3(y), a_4(y)]$ ,  $\alpha$  is implicitly defined by the following equality:

$$-\frac{\Pi_i(y)}{U_i(y)} = a + \frac{F(a) - \alpha}{f(a)} \quad (\text{A20})$$

We know  $\alpha(a_3) = 0$  and  $\alpha(a_4) = 1$ . Applying the implicit function theorem, we have:

$$\frac{d\alpha}{da}(a) = \frac{\frac{d}{da} \left[ \frac{F(a) - \alpha}{f(a)} \right] + 1}{f^{-1}(a)} > 0 \quad (\text{A21})$$

Thus, within the interval  $[a_3(y), a_4(y)]$ ,  $\eta > 0$  and (A11) implies that  $V$  is binding. All the necessary conditions are satisfied, therefore equation (32) gives the optimal quality. (33) follows from (A18) by integration and the appropriate boundary conditions at the points  $a_3(y)$  and  $a_4(y)$ . The cases (II) and (IV) follow from case (III). In case (II)  $a_3(y) = a_1$ , whereas in case (IV)  $a_4(y) = a_2$ .

♦

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