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The Eurozone as an Inflation Target Zone

Abstract

In the revised monetary policy strategy of the European Central Bank (ECB), "price stability is best maintained by aiming for two per cent inflation over the medium term", with "symmetric commitment" to this target. "Symmetry means that the Governing Council considers negative and positive deviations from this target as equally undesirable". In this article, we therefore analyse this policy strategy through a model of inflation target zone, with a central value and symmetric upper and lower bounds on inflation, within which the central bank may decide not to intervene, provided inflation is expected to fluctuate around the central value. We show that the policy benefits guaranteed by a target zone can be dissipated if market agents are uncertain about its width.

JEL-Codes: E310, E420.

Keywords: European Central Bank, monetary policy strategy, inflation target zones.

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1. Introduction

In July 2021 the European Central Bank (ECB) announced the long-awaited revision of policy strategy. The relevant documents (ECB 2021a, 2021b) cover a number of issues, but the kernel of the revised strategy, namely the operational (re)definition of "price stability", can be summarised in two points (ECB 2021a):

- "the Harmonised Index of Consumer Prices (HICP) remains the appropriate price measure for assessing the achievement of the price stability objective" (p. 1), with the intention to recalibrate the index with the inclusion of the costs of the owner-occupied housing, and to downgrade the weight of the most volatile components such as energy prices;
- "price stability is best maintained by aiming for two per cent inflation over the medium term" (p. 2), with "symmetric commitment" to this target. "Symmetry means that the Governing Council considers negative and positive deviations from this target as equally undesirable" (p. 2).

The second point introduces the most apparent modification with respect to the previous definition of price stability as a year increase of the HICP "below but close to 2%" dating back to 2003. In order to provide an appropriate background, Table 1 summarises essential statistical evidence of the past record of the Eurozone monthly observations of the year rates of change of the HICP (see also Figure 1 for the frequency distribution).

Table 1. Eurozone monthly observations of the year rates of change of the HICP, 1999:1-2022:5

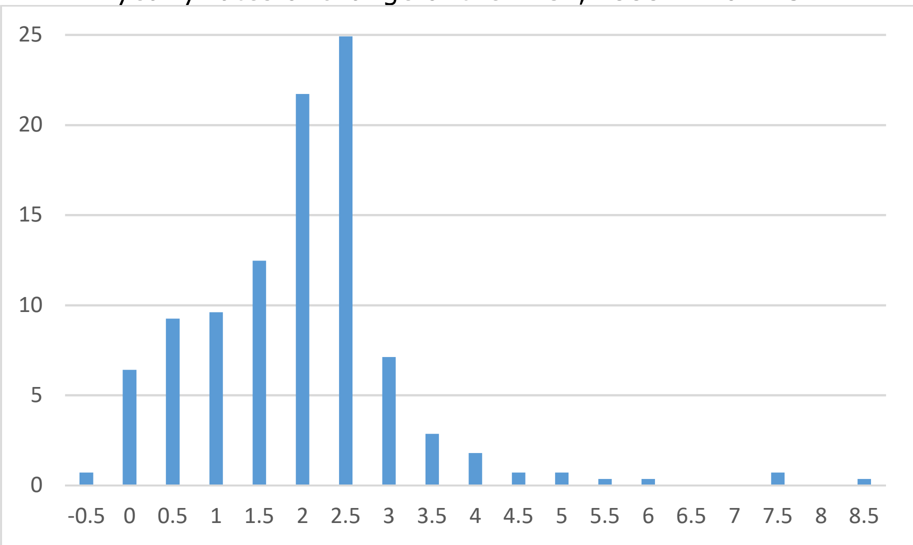
Mean	1.71%
Min., Max	-0.6%, 8.1%
Variance	1.44
Obs. < 2%	60.1%
Obs. <1%, 3%>	75.8%
Obs. <0.5%, 3.5%>	78.6%

Source: Elaborations on ECB, Statistic Warehouse, HICP series

At first glance, an average year inflation of 1.71% seems consistent with the definition of "below but close to 2%". Nonetheless, 60.1% of observations below 2% provide clear evidence of a downward bias (see also Figure 1) corresponding, as is well known, to the twelve years (2009-21) between the Great recession and the COVID-19 pandemic, when inflation remained *systematically* below 2%.

Several commentators have welcomed the new definition, with a clear-cut target value of 2%, as an improvement in view of a more consistent and transparent application of inflation targeting as a general framework for monetary policy conduct and communication (e.g. Wyplosz, 2021; Demertzis, 2021; Darvas and Martins, 2021, Blot et al., 2021). There remain, however, non-trivial margins of ambiguity, which are being brought to the fore by the rapid inflationary evolution of the post-pandemic scenario.

Figure 1. Percent frequency distribution of the Eurozone's monthly observations of the yearly rates of change of the HICP, 1999:1-2022:5



Source: Elaborations on ECB, Statistic Warehouse, HICP series

On the one hand, the new definition of price stability "is now symmetric and it allows for temporary overshooting as needed" (Wyplosz, 2021, p. 6). This interpretation rests on the two statements reported above: that upward and downward deviations are equally undesirable, *and* that the 2% target is to be achieved over the medium term, implying that deviations will not be corrected immediately. The ECB's non-intervention mode on the policy rate (unlike, for instance, the US Federal Reserve and the Bank of England) *vis-à-vis* inflation pointing above the 2% target since the second semester of 2021 may provide a ready-made example.[1]

On the other hand, the new strategy "remains vague regarding the margin of tolerance and the time allowed for overshoot" (Wyplosz, 2021, p. 6). Indeed, the ECB has not announced the width of the tolerance bands (though past experience may suggest a range like 1%-3% or 0.5%-3.5%, see Table 1). As a matter of fact, the Governing Council on July 21, 2022, announced a first increase in the policy rate of 50 basis point, possibly followed by further 25-50 basis points later, as "the new staff projections foresee annual inflation at 6.8% in 2022, before it is projected to decline to 3.5% in 2023 and 2.1% in 2024 – higher than in the March projections" (ECB, 2022b).

[1] This ECB's stance has been motivated, consistently with the first point of the new strategy mentioned above, by an assessment of the acceleration of inflation almost entirely due to the "non core" components of the HICP, and of the transitory nature of their sharp increase (ECB, 2022a).

According to Demertzis (2021), "the most important feature of a tolerance band [around the 2% target] is that it provides a very clear framework for evaluating central bank performance" (p. 4), while it dispenses with identifying the time horizon of deviations explicitly. "For as long as inflation is within the tolerance band, then it is also at 2% on average" (p. 3). This statement, however, is not warranted, since inflation might well remain within a band centred on 2% without averaging to 2% to the extent that it remains above or below most of the time (see Figure 1). As we shall explain later, a specific mechanism of reversion to the mean (possibly not reliant on benevolent features of shocks) is also necessary.

It is widely understood that central banks are reluctant to tying their own hands to a sharp numerical definition of tolerance bands of inflation. The ECB itself, at least since the outbreak of the pandemic, has increasingly made appeal to "flexibility" as a key element in its policymaking process justified by the surge of "radical" uncertainty affecting model-based projections and by the use of a variety of information inputs. This view revives the long-standing rules *vs.* discretion debate–as to whether or not in-flexibility is the right attitude in the face of uncertainty (Lohman, 1992).

In light of these considerations, we propose that the ECB's new policy framework may conveniently be analysed and assessed as a *symmetric inflation target zone*. In a target zone (TZ), the target variable, which is typically subject to random shocks, is kept within a band determined by a "floor" and a "ceiling" with a central value. In a symmetric TZ the floor and the ceiling are equally distant from the central value.

TZs are generally associated with exchange-rate systems, where they may also be set officially, as was the case with the European Monetary System. Less common is the association of TZs to monetary policy, though several central banks do make reference to range of values of inflation, rather than point values, in their policy communications (Castelnuovo et al., 2003; Chung et al., 2020). In Section 2 we begin with an overview of the taxonomy of "inflation zone targeting" proposed by Chung et al.

Across different uses and meanings, typical of inflation TZs is that the policymaker is *committed to intervening* to keep the target variable within the band when it is expected to breach either the floor or the ceiling, but he/she *may decide not to intervene* as long as the variable is expected to fluctuate within the band. The rationale of this policy framework is to anchor expectations to the central value of the target variable, while granting the policymaker some "slackness" in the face of shocks *on the premise that systematic intervention is costly*.

As recalled above, one of the fields where the TZ modelling technique has extensively been developed is exchange-rate theory. Key references for our purposes are Krugman (1991), Krugman and Rotemberg (1990, 1992), Bertola and Caballero (1992). As far as we know, this technique has not been applied to inflation targeting.[2] As will be seen, novel insights can be gained beside the available discussions of pros and cons of the inflation TZs.

In the model we present in Section 3, the target variable is the inflation rate, determined by a New-Keynesian Phillips Curve expressed in continuous time, which consists of the output gap, a forward-looking expectation component, and an exogenous component (e.g. energy costs) hit by random shocks following a Brownian motion. The output gap is determined by a standard New-Keynesian IS function, that is to say a negative relationship with the gap between the nominal policy rate controlled by the central bank net of expected inflation, and the equilibrium "natural" real interest rate.

As is typical in TZ models, the interplay between "the fundamentals" and the expectation component of the target variable is key to the functioning of the TZ. To see this, we first derive the rule that governs the policy rate in the case of the central bank's commitment to keeping inflation on target all the time . We show that systematic inflation targeting would imply a (moving) target value of the policy rate, determined by inflation shocks as well as the *expected rate of inflation acceleration*, or in other words, by the extent to which inflation expectations are de-anchored from the target.

This result sets the stage for the key policy issue underlying the adoption of the inflation TZ. To the extent that systematic inflation targeting succeeds in anchoring inflation expectations, the dynamics of the policy rate reflects the intrinsic volatility of prices, which may be transmitted to other policy-relevant variables (the most commonly considered being output). Allowing for a tolerance band may limit the transmission of the volatility of prices to the policy rate and the rest of the economy. Yet the question is whether this arrangement does deliver smoother paths for the policy rate, and does keep inflation dynamics centred on the target. A key implication of our model is that the answer depends on whether or not market agents are certain about the boundaries of the TZ.

After proving in Section 4 that boundaries known with certainty do deliver the desired results (the so-called "honeymoon effect" in the TZ literature), in Section 5 we introduce uncertainty regarding the boundaries of the TZ, which is the current situation in the Eurozone. We then model a scenario in which agents identify an upper bound of inflation that is with some probability above or below the true one held by the central bank. If the probability attached to the former case is sufficiently high, and inflation expectations de-anchor, the inflation process can take a divergent path that actually pushes inflation to breach the true upper bound in a self-fulfilling manner (known as "divorce effect" in the literature). The central bank should consequently intervene for *smaller* shocks, and with *higher* policy rate.

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[2] Della Posta (2018, 2019) and Della Posta and Tamborini (2022) have applied TZ models to the case of speculative attacks on public debt in the Eurozone.

In Section 6 we summarise and conclude with some policy implications.

2. Taxonomy of inflation target zones

While no central bank has an official inflation TZ, reference to ranges of values around a point inflation rate is quite common practice, also known as "thick inflation targeting" (Castelnuovo et al., 2003; Chung et al., 2020). This practice may have different motivations, for which Chung et al. (2020) provide the following useful taxonomy: (i) *uncertainty ranges*, "that acknowledge uncertainty about inflation outcomes", (ii) *operational ranges*, "that define the scope for intentional deviations of inflation from its target"; (iii) *indifference ranges*, "over which monetary policy will no react to inflation deviations" (p. 1).

These three motivations share the common notion that inflation is a volatile phenomenon which can hardly be pinpointed at its target value all the time, a *caveat* that central banks also wish to communicate to the public. Implicitly, the idea is that no matter how great the benefits may be in keeping inflation at bay, there are also costs to be borne (ranging from frequent or volatile use of the appropriate instrument(s) to side effects on particular sectors of the economy as a whole).

The fact that large and frequent changes in the policy rate may be a disturbance to central banks is witnessed by the literature on the practice of "interest-rate smoothing" (e.g. Sack and Wieland, 2000; Lei and Tseng, 2019). The main negative by-product of the policy-rate volatility induced by point inflation targeting arises in the context of "flexible" inflation targeting, i.e. when the central bank attaches some value to output stability in addition to price stability. A further source of concern relates to financial stability. After the earlier consensus that price stability was a necessary and sufficient condition for financial stability collapsed with the global financial crisis, central bankers' conventional wisdom seems now turned upside down. The ECB pedagogy about its various asset purchases programmes hinges on financial stability as a precondition for price stability (Lane, 2020; Schnabel, 2021). By the same token, however, inflation-targeting activism may run counter financial stability, triggering "financial fragmentation" as it is now dubbed in the ECB vocabulary (Wyplosz, 2021; Schnabel, 2021).

Orphanides and Wieland (2000) provide a theoretical foundation for the (revealed) central banks' preference "to emphasize containing inflation within a target range rather than aiming for a point target" (p. 1352), which they name "zone-quadratic preferences".[3] The result is that "it may be sensible for the policymaker to ignore small deviations of

[3] In essence, the authors reformulate the standard quadratic loss function with arguments the deviations of inflation and output from the respective targets by introducing a "slack" factor in the inflation deviations (equation 15).

inflation from its target rather than incur the higher-order costs required to bring inflation back to its target" (p. 1363).

Considering Chung et al. (2020) comparative analysis of central banks' practices, one may conclude that uncertainty ranges are the most common practice, operational ranges the least common, with indifference ranges somewhere in between (with a few conceptual as well as practical overlaps with uncertainty ranges).

In the Orphanides-and-Wieland setup the inflation TZ is a no-intervention area, i.e an instance of the Chung et al. indifference ranges. The latter authors examine the pros and cons of indifference ranges by means of a simulation calibrated on the US data. The model we propose in this paper is in the same line.

The main advantage of our modelling strategy is that it allows for greater analytical tractability, clearer identification and discussion of policy implications, and comparison with the above-mentioned works. There are, of course, also some drawbacks of which we are aware. One is that the well documented evidence of central banks' preference also for smooth adjustments may be ill represented by a system where interventions are activated in one shot just at the margins of the TZ.[4] Another is that a central bank (the ECB in particular) may be unwilling to advertise a no-intervention area conveying to the public the idea of being passive *vis-à-vis* the mandate of price stability (yet, as recalled above, this happened at the early stage of the post-pandemic inflation runup). Finally, the technology of our model does not allow to deal with the quite common motivation for inertia due to the temporary nature of the shocks. However, this can be seen as a strength as we show that the TZ can be beneficial in controlling inflation even when shocks are not temporary.

3. The model

We define the inflation rate in continuous time $\pi(t)$ as the rate of change of the (log of) the relevant price index $p(t)$, say the consumer price index (CPI), in the (infinitesimal) time unit t, i.e.

(1) $\quad \pi(t) \equiv dp(t)/dt$

The CPI motion process is represented by a standard Phillips Curve (PC) consisting of an expectation component, denoted by the generic superscript (^{e}), a first "fundamental" variable $y(t)$ (e.g. the log-rate of deviation current output from potential output per unit of time), and a second exogenous "fundamental" $z(t)$ (e.g. the log-rate of deviation of costs

[4] Some earlier models of exchange-rate TZs also treated the case of inframarginal interventions (e.g. Bertola and Caballero, 1992). We leave this extension for future work.

and other relevant variables for price formation from their normal level). The CPI motion process is therefore[5]:

(2) $dp(t) = \beta dp^e(t) + \kappa y(t)dt + z(t)dt$ $\qquad 0 < \beta < 1$

The fundamental variable z shifts stochastically according to the following driftless geometric Brownian motion process:[6]

(3) $z(t) = \sigma W(t)$

where σ is the instantaneous standard deviation of the Brownian motion W, which is so characterized:

(4) $dW(t) = \chi\sqrt{dt}$,

with χ being a random variable independently, identically and normally distributed, with 0 mean and variance equal to 1.

For notation simplicity we shall drop the time index *t*. Equation (2) can then be reformulated in terms of rates of change per unit of time by dividing by *dt*, obtaining the following motion law of inflation:

(5) $\pi = \beta\pi^e + \kappa y + z$

If the central bank has an inflation target, this should be consistent with the steady-state of the inflation motion law. If the target is positive, say $\pi^* > 0$, for $\pi = \pi^e = \pi^*$, since $E(z) = 0$, it follows that $\pi^* = \kappa(1-\beta)^{-1} y^*$. That is to say, the central bank also has an (implicit) target on a nonzero output gap equal to:

(6) $y^* = \pi^*(1-\beta)/\kappa$.[7]

Still following standard macro-models, the central bank can control y by means of the nominal interest rate *vis-à-vis* expected inflation, according to a so-called IS function where the rate of deviation of output from the target is negatively correlated with deviations of the real value of the policy rate from the value r^* which keeps the output gap in line with the target, and which for simplicity we assume to be constant:

(7) $y = y^* - \alpha(i - \pi^e - r^*)$

Therefore, substituting (6) and (7) into (5), we obtain that inflation is driven by

(8) $\pi = (1-\beta)\pi^* + (\beta + \alpha\kappa)\pi^e - \alpha\kappa(i - r^*) + z$ $\qquad (\beta + \alpha\kappa) < 1$

Let us now express expected inflation in the "accelerationist" form, measuring the extent to which the rate of change in the CPI in an instant of time is expected to accelerate, decelerate, or remain equal with respect to the current one:

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5 Examples are Shone (2002), Chiarella et al. (2009, ch. 1),

6 Some TZ models consider instead a Brownian motion process with drift (e.g. Krugman and Rotemberg, 1992; Bertola and Caballero, 1992). In this context, the drift would not add further insights, and we can therefore avoid its use here.

7 Note that y^* is larger, the larger the degree of price stickiness (small κ). As explained by Woodford (2003, ch. 2), the nonzero inflation target generally adopted by central banks is the mirror image of the positive output gap that, from the social welfare point of view, is necessary to achieve in order to compensate for the distortions created by the "market frictions" embedded in the parameter κ.

(9) $\dot{\pi}^e \equiv \pi^e - \pi = \frac{dp^e}{dt} - \frac{dp}{dt}$

Using (9) to express π^e, equation (8) can be rewritten as follows:

(10) $\pi = [(1-\beta)\pi^* - \alpha\kappa(i - r^*) + (\beta + \alpha\kappa)\dot{\pi}^e + z](1 - \beta - \alpha\kappa)^{-1}$

We can further redefine the variables in order to obtain a simpler specification for equation (10):

(11) $\pi = \pi^* - \gamma\hat{\imath} + \delta\dot{\pi}^e + \xi$,

with:

$\hat{\imath} = i - i^*$

$i^* = r^* + \pi^*$

$\gamma = \alpha\kappa(1 - \beta - \alpha\kappa)^{-1}$

$\delta \equiv (\beta + \alpha\kappa)(1 - \beta - \alpha\kappa)^{-1}$

$\xi = z(1 - \beta - \alpha\kappa)^{-1}$

The commitment to keeping inflation on target systematically means that at any point in time the inflation process (11) should satisfy $\pi = \pi^*$. As a consequence, the policy rate should match the time-varying target value:

(12) $\tilde{\imath} = i^* + \frac{\delta}{\gamma}\dot{\pi}^e + \frac{1}{\gamma}\xi$

The target policy rate the central bank should aim at is centred on $i^* = r^* + \pi^*$, also known as non-accelerating-inflation rate of interest (NAIRI).[8] In fact, this is the rate that prevails in quiet states ($\xi = 0$, $\dot{\pi}^e = 0$). V*is-à-vis* inflation shocks, the target interest rate should be increased or reduced to offset the shock as well as the expected rate of inflation acceleration (deceleration). The more $\dot{\pi}^e > 0$ makes π deviate from π^*, the higher $\tilde{\imath}$ should be. The reactivity of the policy rate is governed by the three parameters α, κ, β. Note that the reactivity should be stronger the smaller are the policy-rate impact on output α (a "flat" IS), and the output impact on inflation κ (a "flat" PC).

4. Introducing the inflation target zone

Against the previous background, one possible rationale for a TZ for inflation is that the central bank enjoys *some* margin of "slackness" in steering the policy rate *vis-à-vis* deviations of inflation from its target. As said in Section 2, such a rationale implies that complying systematically with the target policy rate (12) is costly to the central bank.

As a paradigmatic example, the output-gap equation (7) suggests that, to the extent that the central bank is fully compliant with its target policy rate (12), output, too, is subject to inflation volatility. In fact, substitution of (12) into (7) yields:

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[8] It also appears as the "intercept" in the standard formulation of the Taylor Rule.

(13) $\quad \tilde{y} \ = y^* - \alpha(\frac{\delta}{\gamma} - 1)\dot{\pi}^e \ - \frac{\alpha}{\gamma}\xi$

which shows that keeping inflation systematically aligned with the target comes at the cost of creating output gaps in inverse proportion to inflation shocks and the acceleration (deceleration) rate of inflation expectations.

Consideration of the welfare loss due to output volatility leads to the class of reaction functions with optimal balancing between inflation and output stabilisation also known as "flexible" inflation targeting. As shown by Orphanides and Wieland (2000), however, this conventional treatment is not sufficient to obtain an inflation TZ, at least in the sense of a no-intervention area. Simply, for any observed inflation shock at any point in time the central bank would move the policy rate less than it would do in the pure inflation targeting equation (12).

According to these authors, for the central bank to set a no-intervention area it should be the case that the cost associated to the inflation gaps ($\pi - \pi^*$) exceeds the cost associated to the creation of the policy-rate gaps ($\tilde{\imath} - i^*$) only beyond a certain (symmetric) threshold of shocks $|\xi| \geq \varepsilon$. The symmetric TZ is then identified by three parameters: its lower bound $\underline{\pi}$, its upper bound $\bar{\pi}$, and its central value π^*:

$$\bar{\pi} = \pi^* + \varepsilon, \underline{\pi} = \pi^* - \varepsilon$$

where ε is the symmetric size of the TZ above and below the central value. An example of determination of the boundaries of the TZ along these lines is provided in the Appendix A.1, with a numerical exemplification in Appendix A.4.[9]

As a point of reference, let us first examine the case in which the central bank ignores output gaps as in the policy-rate equation (12), in the no-intervention area sticks to the NAIRI *i**, and inflation expectations remain static, $\dot{\pi}^e_t = 0$. According to equation (11), inflation will only deviate from target randomly with the stochastic motion of z:

(14) $\quad \pi \ = \pi^* \ + \xi$

Given that $E\xi \ = 0$, it follows that $E\pi = \pi^*$, which is indeed consistent with the hypothesis of anchored (rational) expectations.

This relationship is represented by the straight line L in Figure 2, where $\bar{\xi}$ is the largest shock that can be accommodated in the no-intervention area within the TZ. Clearly $\bar{\xi} = \varepsilon$, ($\underline{\xi} = -\varepsilon$). [10]

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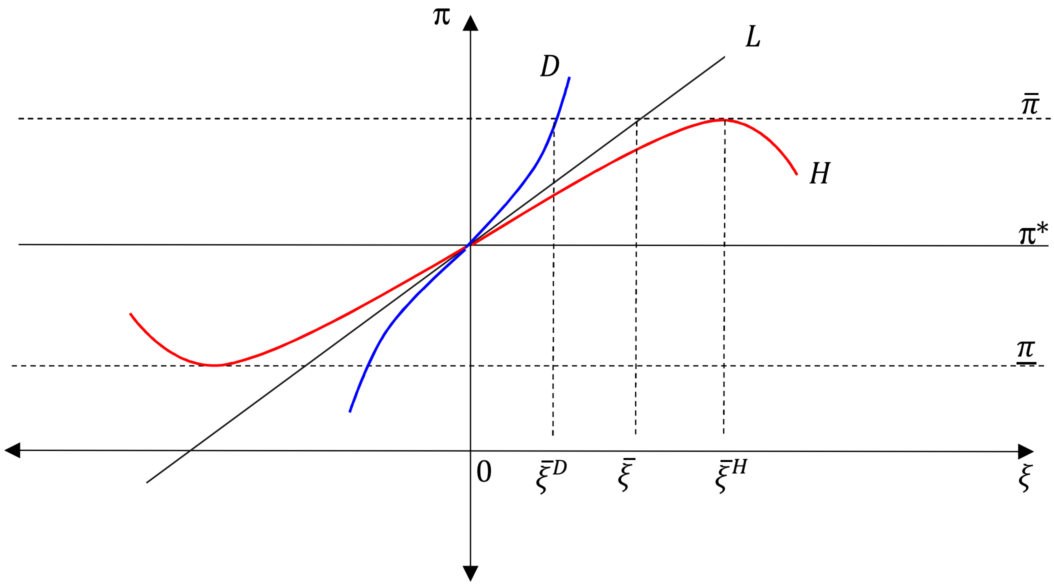
[9] It is also shown that nonzero weight assigned to output-gap loss values is neither necessary nor sufficient in order to obtain an inflation TZ. On the other hand, if the central banks does assign a weight to output-gap loss values, the result is a higher absolute value |ε| and a lower policy rate in the intervention area as in standard flexible inflation targeting.

[10] To the extent that the inflation process has to be controlled by the policy rate, and the TZ has to be symmetric, the choice of the width of the bands is constrained by the zero lower bound of the policy rate. According to (12), the lower bound of the TZ should satisfy $\tilde{\imath} \geq 0$. This condition clearly depends on the NAIRI *i**, i.e. the sum of the equilibrium real interest rate *r** and of the inflation target itself. As is well known,

Figure 2. The linear (L), honeymoon (H) and divorce (D) inflation patterns in the TZ



If we now go back to the more general case in which inflation expectations are not necessarily anchored, what is the effect of the existence of the TZ?

The inflation process (11) in the no-intervention area ($\hat{\imath} = 0$) is a first-order differential equation, which has the following general solution (see Appendix A2):

(15) $\pi = \pi^* + \xi + A_1 e^{\lambda_1 \xi} + A_2 e^{\lambda_2 \xi}$

with $\lambda_{1,2} = \pm\sqrt{2/\delta\hat{\sigma}^2}$, $\hat{\sigma}^2 = \sigma^2(1-\beta-\alpha\kappa)^{-1}$

The parameters $A_{1,2}$ are indeterminate, and in order to determine them and close the model, it is necessary to analyse the behaviour of equation (15) at two determined points.

The first is given by the value of the function at $\xi = 0$, which should match the central value of the TZ, i.e. $\pi = \pi^*$, and therefore

(16) $A_1 + A_2 = 0$

Setting $A_1 = A, A_2 = -A$, and $\lambda_1 = \lambda, \lambda_2 = -\lambda$, it is possible to reformulate (15) as follows:

(17) $\pi = \pi^* + \xi + A(e^{\lambda\xi} - e^{-\lambda\xi})$

The second point is given by the value of the function as inflation approaches the upper bound of the TZ. The key contribution of the TZ literature in this respect is that the result depends on the central bank's commitment to "defending" the TZ (Krugman, 1991; Bertola and Caballero, 1992). In case of perfect commitment of intervention at the upper bound, it has been shown that the expectation component of the target variable is curbed so that, at the intervention point, the inflation process is "smoothly pasted" with the upper bound and is "reflected" within the TZ – determining the so-called "honeymoon effect".

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the growing concern among central banks with the zero lower bound is traced back to the evidence of r^* falling in negative territory (Lane, 2020). We shall not deal with this problem explicitly.

Let $\bar{\xi}^H > 0$ be the value of ξ at the instant inflation hits the upper bound of the TZ. "Smooth pasting" obtains with $\pi(\bar{\xi}^H)$, as given by (17), being tangent to the upper bound of the TZ, i.e.

$$\frac{d\pi}{d\xi}|_{\bar{\xi}^H} = 1 + \lambda A\left(e^{\lambda\bar{\xi}^H} + e^{-\lambda\bar{\xi}^H}\right) = 0$$

which yields

(18) $$A^* = -[\lambda\left(e^{\lambda\bar{\xi}^H} + e^{-\lambda\bar{\xi}^H}\right)]^{-1} < 0$$

Then $A^* < 0$, (or $A_1 < 0$, $A_2 > 0$) is a sufficient condition to obtain the typical within-the-band S shaped function of the target variable found in the TZ literature, labelled H in Figure 2. See also Appendix A.4 and Figure 4 for a numerical simulation.

The creation of the inflation TZ brings some valuable effects. The first is that, even in the presence of some de-anchoring of expectations after a shock, and the central bank not intervening but at the upper bound, the dynamic path of inflation is curbed with respect to the linear path L. In fact, each point along the function H indicates that for the corresponding shock of size ξ inflation increases less than along L. The reason lies in the expectation component embodied in the function H which, thanks to the anticipation of the central bank's marginal intervention, decelerates and "pulls inflation down". This effect is stronger, the closer inflation is to the upper bound (Krugman, 1991; Bertola and Caballero, 1992). The second beneficial effect is that, as a consequence, the central bank can also accommodate a larger shock before intervening. This "honeymoon effect" is measured by the difference $(\bar{\xi}^H - \bar{\xi})$.[11] Finally, the upper bound operates as a "reflecting barrier", meaning that the inflation process is reflected towards the interior of the band.

We can conclude that, *under these conditions*, the main shortcomings pointed out in current discussions of inflation TZs, such as larger, on average, deviations from target, de-anchoring and self-confirming fluctuations in expectations (e.g. Castelnuovo et al., 2003; Chung et al., 2020) do not materialise, whilst the opposite benefits are obtained. These benefits, however, depend on the honeymoon climate, fundamentally that agents know the boundaries of the TZ with certainty and believe that the central bank will intervene.

5. Uncertainty about the boundaries of the target zone

As shown above, key to the well functioning of TZs is agents' certainty about the boundaries and the central bank's willingness/ability to "defend" the TZ, i.e. to intervene to keep the target variable within the boundaries.

[11] Quantitatively, this effect is larger when the parameter λ is smaller (see Appendix A.4). A smaller value of λ obtains for a larger value of β, the impact of expectations on inflation. Indeed, if expectations work to "pull inflation down" it is beneficial to have a large β. This is just a different viewpoint on the role of anchored expectations, and the general belief that a larger value of β in estimations of the Phillips Curve is welcomed as a sign of stability (e.g. Blanchard et al., 2015).

Two main sources of uncertainty may be distinguished, at least in theory. One is that the upper and lower bounds are common knowledge, but nonzero probability may be assigned to the central bank's unwillingness/inability to intervene.[12] In the case of excess inflation, the necessary increase of the interest rate may be deemed too costly regardless the formal commitment of the central bank. Note that a high upper bound makes the intervention less likely but at the same time more costly, since the increase in the policy rate should be larger at the intervention point. Another source of uncertainty arises if the width of the TZ is not announced by the central bank. This is indeed the present situation with the ECB.

To study the behaviour of the system at the upper bound of the inflation process (17), we shall follow the solution method of TZ "realignments" presented by Bertola and Caballero (1992).[13] This is based on an arbitrage argument, which lends itself to manageable treatment of either case of uncertainty.

As a first step of analysis we consider the case that agents do not know the boundaries of the TZ with certainty, assuming that they hold the same probabilistic belief which includes the true upper bound $\bar{\pi}$. Given a stochastic shock ξ, with probability p agents believe that inflation will be allowed to run above $\bar{\pi}$ by say the amount Δ^u. This event, therefore, is virtually equivalent to moving up to the centre of a higher TZ $\in [\bar{\pi}, \bar{\pi} + 2\Delta^u]$. With complementary probability (1-p), inflation will not be allowed to increase, and the central bank's intervention is such that inflation will remain at $\bar{\pi}$ or move below by say the amount Δ^d to the centre of the band $\in [\bar{\pi} - 2\Delta^d, \bar{\pi}]$. A noteworthy case occurs for $\Delta^d = \varepsilon$, i.e. if agents believe that inflation will return to π^* at the centre of the TZ. The probability distribution (p; 1-p) can be interpreted as a measure of uncertainty about the exact value of $\bar{\pi}$.[14] It plays a crucial role in the dynamic evolution of the system by conditioning the sign of the parameter A in the inflation process (17).

Given the shock $\bar{\xi}$, whereby inflation would reach the true $\bar{\pi}$ along the linear function L in Figure 2, in order to exclude unbounded speculative profits (losses) on the goods market, the actual level of inflation has to be equal to the *expected one* resulting from the probabilities assigned to the two different events (Δ^u, Δ^d). As we show in Appendix A.2, the value of A consistent with the above no-arbitrage condition is:

$$(19) \qquad A = [p\Delta^u - (1-p)\Delta^d]\left(e^{\lambda\bar{\xi}} - e^{-\lambda\bar{\xi}}\right)^{-1}$$

[12] A reference model for this case in the exchange-rate TZs was proposed by Krugman and Rotemberg (1990) for central banks being short of reserves, i.e. being *unable* to intervene, when the exchange rate is under attack at the upper bound of the TZ.

[13] The behaviour of the system at the lower bound is analogous and symmetric.

[14] Alternatively, if $\bar{\pi}$ were common knowledge, p would measure the distrust in the central banks' commitment to unconditional inflation stabilisation at $\bar{\pi}$.

Since the denominator is always positive, the sign of A depends on the value of p in the numerator, namely:

(20) $\quad A \gtrless 0 \text{ iff } p \gtrless \frac{\Delta^d}{\Delta^u+\Delta^d} \equiv p*$

We denote by $p*$ the critical level of p such that A = 0, yielding the linear case of the function L in Figure 2. The thrust of the result is that the inflation process may still display a honeymoon effect (concavity) if $p < p*$ and $A < 0$. However, if $p > p*$, i. e. the market belief is tilted towards an upper bound higher than $\bar{\pi}$, then $A > 0$, and the ensuing function, labelled D in Figure 2, becomes convex, bending above and to the left of the linear L. The consequence is that for any stochastic shock ξ, the actual level of inflation on the D function is greater than that on the L function. This scenario has been dubbed "*divorce"* in the TZ literature.

The intuition is that as inflation increases, the belief of larger tolerance accelerates inflation expectations, which "push up" inflation in a self-fulfilling process. Consequently, if $\bar{\pi}$ is in fact the true upper bound, the central bank is forced to intervene for *smaller* shocks and with *higher* interest rate (see (12) with $\dot{\pi}^e > 0$) This "divorce effect" is measured by the difference $(\bar{\xi}^D - \bar{\xi})$.[15] A numerical simulation is provided in Appendix A.4 and Figure 6.

The critical $p*$ in turn depends on inflation behaviour expected at $\bar{\pi}$. If inflation is expected to move up or down by the same amount, then $p*$ = 1/2.[16] The more inflation is expected to move up, $\Delta^u > \Delta^d$, the more $p*$ is reduced, meaning that also the chances of intervention at $\bar{\pi}$ should be higher (p lower) in order to keep the system on the linear track.

Therefore, whereas the inflation TZ may add beneficial flexibility to inflation targeting, uncertainty about the boundaries of the TZ may turn out to be detrimental if the belief takes hold that the upper bound is higher than the one the central bank has in mind.

The difference between honeymoon and divorce can further be appreciated by means of Figure 7, which reproduces the first hundred draws from a Monte Carlo simulation of the inflation process in the honeymoon scenario of Figure 4 *vis-à-vis* the divorce scenario of Figure 6 (with probability of realignment of 60%), in response to the same set of shocks (see Appendix A.4 for details). The noteworthy results are two. First, the inflation process in the honeymoon scenario (solid line) always remain within the boundaries declared by the central bank (2% ± 1.5%), whereas in the divorce scenario (dotted line) we observe

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[15] The divorce effect is worse (it increases in absolute value) when the parameter λ is lower (see Appendix A.4). Hence, while a low λ enhances the honeymoon effect (see footnote 11), it worsens the divorce effect. As a matter of fact, if a low λ is due to a high β, a strong impact of expectations on inflation becomes harmful when they de-anchor and work to "push inflation up".

[16] Bertola and Caballero (1992) assume the probability of a symmetric upward or downward jump. See also Della Posta (2018)

recurrent violations of the boundaries after relatively larger shocks. Consequently, the observed mean, standard deviation and average square gap (from 2%) of the inflation process are lower in the honeymoon scenario than in the divorce scenario.

5. Concluding remarks

We examined the ECB's new definition of price stability in terms of an inflation TZ centred on the target value of 2% with allowance for symmetric tolerance bands above and below the target. This new policy framework implies that the systematic use of the policy rate to keep inflation on target *vis-à-vis* random shocks to prices is costly to the central bank, but it also entails the central bank's commitment to intervening on the policy rate (at least) at the margins of the TZ in order to keep inflation within the TZ and on target on average.

We showed that the most important policy implication is that the TZ may or may not deliver the desired results depending on the degree by which market agents are certain about the boundaries of the TZ. If this condition holds, the honeymoon effect takes place, the dynamic path of inflation after a shock is curbed so that the central bank can also accommodate larger shocks before intervening. The main shortcomings pointed out in current discussions of inflation TZs, such as larger, on average, deviations from target, de-anchoring and self-confirming fluctuations in expectations, only arise in the case of uncertainty about the true boundaries of the TZ.

Thus, while it may be tempting for the central bank to add further flexibility to the TZ by leaving its boundaries undetermined, the opposite divorce effect may occur if the belief takes hold that the upper bound is higher than the one the central bank has in mind. Inflation would accelerate towards the upper bound and the central bank would be forced to intervene for *smaller* shocks and with *higher* policy rate. In consideration of the past decade's persistence of low inflation, we wish to point out that these remarks equally apply for inflation shocks below target.

It may be argued that the central bank "communicates" the true boundaries of the TZ at the very moment when it starts intervening. This case may be matter of further investigation in a model where market agents hold heterogeneous beliefs and learn about the true boundaries over time. Yet, our intuition is that the central bank's formal communication remains important in a "forward guidance" spirit, so that agents are informed in advance about the macroeconomic scenario conditional on which the central bank is ready to intervene.

In this paper we concentrated on uncertainty about the true value of the boundaries of the TZ, while pointing out that uncertainty may also concern whether the announced boundaries are indeed "defendable" by the central bank. A few considerations on this case, too, are in order in light of our model of "realignments".

In the exchange-rate TZ literature, the typical event is when the central bank should defend the currency against a speculative attack at the upper bound of the TZ and finds itself short of reserves. In the case of excess inflation there seems to be no limits for the central bank to raise the policy rate as much as necessary once the margin of tolerance, up to which the cost of inflation does not exceed the cost of higher interest rates, has been exhausted (different is the situation of falling inflation owing to the zero lower bound of the policy rate). Nonetheless, unforeseen contingencies may upset the ex-ante comparative cost calculations on which the upper bound of inflation has been based. An example may be provided by the type of post-pandemic, war-time, *imported cost-push inflation* that is being experienced in Europe more than elsewhere (ECB, 2022c; Bonatti and Tamborini, 2022; Blanchard and Pisani-Ferry, 2022).[17] The cost of intervention may become substantially higher than in ordinary times. In cases like this, an upward realignment of the TZ may thus be warranted, perhaps temporarily, either by will of the central bank or under the market pressure of the divorce effect shown by our model. It is to be evaluated which alternative is better, and further study will be necessary, though our intuition is for the former, for the same motivation of transparency of the boundaries of the TZ put forward above.

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[17] This is a kind of s*upply-side shock* with a strong *real deterioration* of terms of trade, which may also be structural as far as energy sources and prices are concerned (Battistini et al., 2022). Central banking conventional wisdom maintains that monetary policy alone is ill suited to address this kind of shocks. First, because their typical consequence is a contraction of demand and economic activity (stagflation), independently of any monetary restriction. Second, because a monetary restriction would just inflict more demand cuts across the board of all sectors, whereas the correct reallocation response would require a shift of demand away from higher-price imported goods towards lower-price domestic goods. Likewise, if interest rates rise, they rise for all borrowers including those who should instead be induced to invest in the production of alternative energy technologies.

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Appendix

A.1. An example of determination of the boundaries of the TZ

As in Orphanides and Wieland (2000), assume that the central bank's loss function associated to inflation gaps $L(\pi, \pi^*, \theta)$ is parametrised on an indifference threshold θ. A simple specification of this function may be:

(A1) $L(\pi, \pi^*, \theta) \quad = (\pi - \pi^*)^2 - \theta$

Therefore, according to the inflation process (11), the loss associated to any deviation ε from target is given by

(A2) $L(\pi, \pi^*, \theta) \quad = \varepsilon^2 - \theta$

Let the loss associated to output gaps be given by the canonical quadratic function

(A3) $L(y, y^*) \quad = \omega(y - y^*)^2$

where ω is the relative weight assigned to output gaps. Given the output-gap equation in case of policy intervention (13), we can write:

(A4) $L(y, y^*) \quad = \omega(-\frac{\alpha}{\gamma}\varepsilon)^2$

In this policy setup, there may exist a symmetric range of deviations of inflation from target within which $L(\pi, \pi^*, \theta) < L(y, y^*)$, i.e. the inflation-gap loss value due to no-intervention is lower than the output-gap loss value due to intervention (Orphanides and Wieland, 2000, p.1364). The solution of the condition $L(\pi, \pi^*, \theta) = L(y, y^*)$ yields the boundaries of the TZ, the values $|\varepsilon|$ of the symmetric deviations of inflation from target within which the central bank opts for no-intervention. The solution is the following:

(A5) $\varepsilon = \pm\gamma\sqrt{\frac{\theta}{\gamma^2 - \omega\alpha^2}}$ $\qquad \omega < \gamma^2/\alpha^2$

Clearly, for $\theta = 0$ the TZ collapses to zero. By contrast, the central bank maintains a TZ even for zero weight assigned to output gaps ($\omega = 0$), i.e.

(A6) $\varepsilon = \pm\sqrt{\theta}$

Note, therefore, that nonzero weight assigned to the output-gap loss value is neither necessary nor sufficient in order to obtain a TZ. On the other hand, if the central bank does assign nonzero weight to the output-gap loss value, the result is a higher absolute value $|\varepsilon|$ and a lower policy rate in the intervention area as in standard flexible inflation targeting.[18]

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[18] The model can easily be extended to include an indifference threshold also for output-gap losses, e.g.

$L(y, y^*) \quad = \omega((y - y^*)^2 - \phi)$

The solution of the boundaries of the TZ becomes:

$\varepsilon = \pm\gamma\sqrt{\frac{\theta - \phi\omega}{\gamma^2 - \omega\alpha^2}}$

Therefore, provided that ω satisfies the sign condition for $|\varepsilon|$ to be real-valued, in general the presence of ϕ may still yield $|\varepsilon| \neq 0$, except in the particular case of $\omega = \theta/\phi$

A.2. The general solution of the TZ model

In order to solve the inflation equation (11) in the no-intervention area ($\hat{\imath}_t = 0$), i.e.

(A7) $\pi \quad = \pi^* + \delta\dot{\pi}^e + \xi$,

let us assume a generic functional form for π:

(A8) $\pi \quad = f(\xi)$

We can now use this equation to calculate the expected variation of inflation. In order to do this, let us expand the equation in a Taylor-type series, by calculating Ito's differential:

(A9) $d\pi(\xi) \quad = f'(\xi)d\xi + \frac{1}{2}f''(\xi)(d\xi)^2$

From equations (1) to (4) in the text, it turns out that $E(d\xi) = 0$ and $E(d\xi)^2 = \hat{\sigma}^2 dt$. We obtain, then, Ito's Lemma:

(A10) $\frac{E(d\pi_t)}{dt} = \frac{1}{2}f''(\xi)\hat{\sigma}^2$

By replacing (A10) into (A7) we have:

(A11) $\pi \quad = f(\xi) = \pi^* + \xi + \delta[\frac{1}{2}f''(\xi)\hat{\sigma}^2]$

This is a differential equation of the second order whose generic solution is of the class (Bertola and Caballero, 1992, p.522):

(A12) $\pi \quad = \pi^* + \xi + A_1 e^{\lambda_1 \xi} + A_2 e^{\lambda_2 \xi}$.

where $\lambda_{1,2} = \pm\sqrt{2/\delta\hat{\sigma}^2}$ are the two roots of the characteristic equation.

Setting $A_1 = A, A_2 = -A$, and $\lambda_1 = \lambda, \lambda_2 = -\lambda$, we obtain

(A13) $\pi \quad = \pi^* + \xi + A(e^{\lambda\xi} - e^{-\lambda\xi})$

which is equation (17) in the text.

A.3. Uncertainty

To deal with uncertainty about the boundaries of the TZ we apply the Bertola and Caballero (1992) methodology of TZ "realignments".

Let us first consider that the inflation process can be reformulated in the form $\pi(f, \mathrm{c})$ where f is the value of the fundamental variable and c refers to the central value of the current TZ. Accordingly, (A13) can be rewritten as

(A14) $\pi(f, c) = f + A(e^{\lambda(f-c)} - e^{-\lambda(f-c)})$

Considering now the upper bound of the TZ, we have to examine what happens when the process ξ realises the highest level $\bar{\xi}$ that determines linearly, in the absence of any expectation effect, the largest acceptable inflation rate $\bar{\pi}$ by the central bank (see function L in Figure 2). In terms of (A14) we have:

(A15) $\pi(\bar{\xi}, \pi^*) = \bar{\pi} + A\left(e^{\lambda(\bar{\pi}-\pi*)} - e^{-\lambda(\bar{\pi}-\pi*)}\right) = \pi^* + \bar{\xi} + A\left(e^{\lambda\bar{\xi}} - e^{-\lambda\bar{\xi}}\right)$

Under uncertainty about the true upper bound of the central bank, a no-arbitrage argument provides the closing equation. Let agents believe that,

• with probability p, the central bank will let inflation grow further above $\bar{\pi}$ by say the amount Δ^u, at to the centre of a higher band $\in [\bar{\pi}, \bar{\pi} + 2\Delta^u]$

• with probability $1 - p$, inflation will remain at $\bar{\pi}$ or move below by say the amount Δ^d to the centre of the band $\in [\bar{\pi} - 2\Delta^d, \bar{\pi}]$.

Therefore, when the inflation process hits $\bar{\pi}$ the following no-arbitrage condition should hold:

(A16) $\pi(\bar{\xi}, \pi^*) = p\pi(\bar{\pi} + \Delta^u, \bar{\pi} + \Delta^u) + (1 - p)\pi(\bar{\pi} - \Delta^d, \bar{\pi} - \Delta^d)$,

That is to say:

(A17) $\pi^* + \bar{\xi} + A\left(e^{\lambda\bar{\xi}} - e^{-\lambda\bar{\xi}}\right) = p\left[\pi^* + \left(\bar{\xi} + \Delta^u\right)\right] + (1 - p)\left[\pi^* + \left(\bar{\xi} - \Delta^d\right)\right]$

from which it follows that:

(A18) $A = [p\Delta^u - (1 - p)\Delta^d]\left(e^{\lambda\bar{\xi}} - e^{-\lambda\bar{\xi}}\right)^{-1}$

which is equation (19) in the text.

A.4. Numerical simulation

The parameters that govern the inflation process, and the chosen empirical values referred to the Eurozone, are the following:

- β = coefficient of expected inflation in the Phillips Curve = 0.5: Amberger and Fendel (2017), Montoya and Doehring (2011)
- κ = coefficient of the output gap in the Phillips Curve = 0.3: Amberger and Fendel (2017), Riggi and Venditti (2014), Oinonen and Paloviita (2014)
- α = coefficient of the interest-rate gap in the IS function = 0.2: Smets and Wouters (2003), Laubach and Williams (2003), Garnier and Wilhelmsen (2005)
- $\hat{\sigma}^2$ = variance of the inflation process = 1.44: Table 1 in the text
- ε = width of TZ = ± 1.5: Table 1 in the text

With these data, and zero weight on output-gap losses, the indifference threshold towards inflation-gap losses introduced in A.1 above is θ = 2.25. Figure 3 displays the plot of the corresponding inflation-gap loss function (A2).

If output-gap losses, too, have a nonzero weight as in equation (A4), the width of the TZ, i.e. the value of $|\varepsilon|$ such that $L(\pi, \pi^*, \theta) < L(y, y^*)$ increases. With ω = 0.5, the boundaries are ε = ± 1.81, and with ω = 1 (i.e. the same as inflation-gap losses), the boundaries widen up to ε = ± 2.5.

Figure 3. The boundaries of the TZ

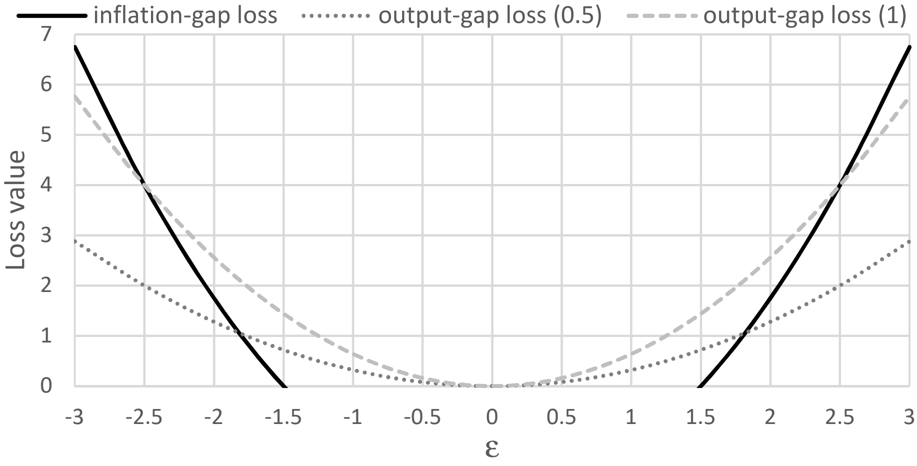


Figure 4 shows the inflation process in the honeymoon scenario, where A = -0.091, λ = 0.962, $\bar{\xi}^H$= 2.52, and $\bar{\xi}$ = 1.5. The honeymoon effect $(\bar{\xi}^H - \bar{\xi})$, the "extra shock" that the central bank can accommodate in the no-intervention area, is therefore 2.52 – 1.5 = 1.01. Figure 5 shows the inverse relationship between $\bar{\xi}^H$ and λ. A low λ (e.g. high impact of expectations on inflation β) enhances the honeymoon effect.

Figure 4. The honeymoon effect

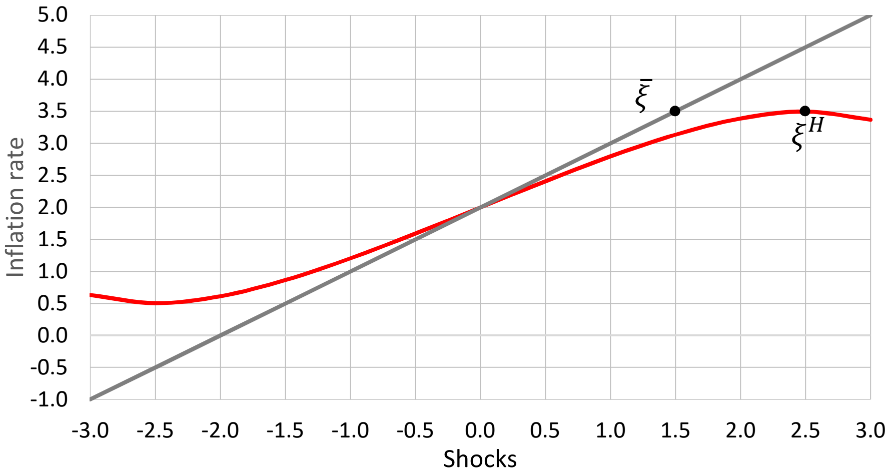


Figure 5. The relationship between $\bar{\xi}^H$ and λ

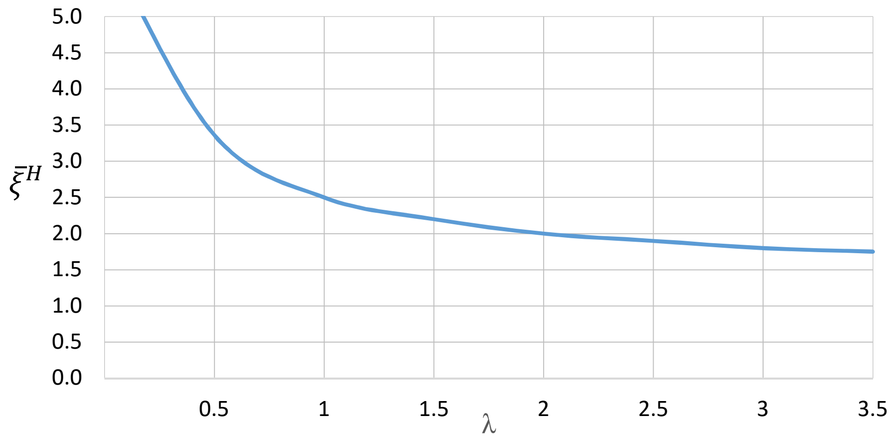
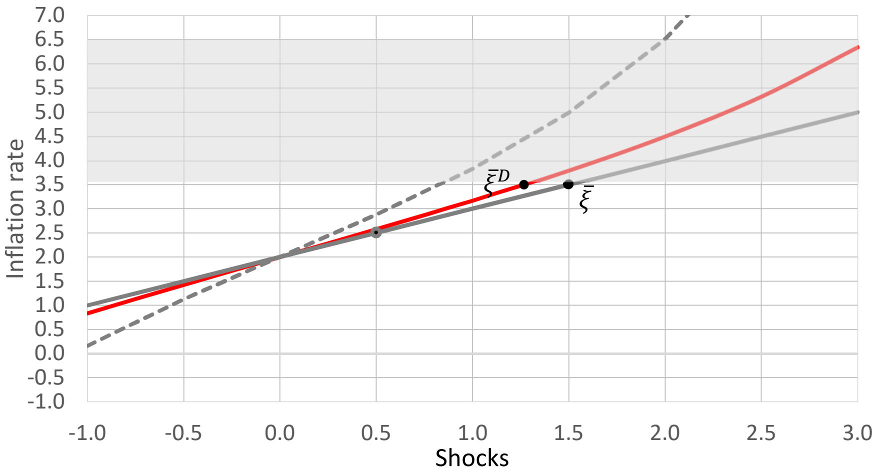


Figure 6 shows the divorce scenario due to uncertainty about the true value of the boundaries of the TZ. It represents the case where agents assign probability ($1 - p$ = 40%) to the true boundaries [0.5%, 3.5%] and probability p = 60% that they are [3.5%, 6.5%]. In terms of the general treatment in the text, as the inflation process hits the true upper bound, agents assign 60% probability to inflation being allowed to increase further by Δ^u = 1.5, and 40% probability that it will revert to the target of 2%, i.e. Δ^d = 1.5. Hence this is the case of symmetric "realignment" as in Bertola and Caballero (1992), so that the critical probability p^* which yields A = 0 (the linear process L) is 50% (see equation (20) in the text). Since $p > p^*$, A becomes positive, equal to 0.075, and the inflation process becomes convex bending to the left of the linear path L. The point of intervention of the central bank is when $\bar{\xi}^D$ =1.2.

Figure 6. The divorce effect with 60% probability of "realignment"



The shaded area represents the "realigned" TZ. The dotted function is given by 100% probability of "realignment"

The divorce effect is measured by $(\bar{\xi}^D - \bar{\xi}) = -0.3$, namely the anticipation of intervention due to uncertainty. It increases with the probability assigned to the "realignment" to the higher TZ, as is shown by the 100%-probability dotted line. For a given probability, $\bar{\xi}^D$ decreases with the parameter λ, i.e. the divorce effect *increases*. Therefore, while a low λ enhances the honeymoon effect, it worsens the divorce effect.

The difference between honeymoon and divorce can further be appreciated by means of Figure 7. It reproduces the first hundred draws from a Monte Carlo simulation of the inflation process in honeymoon scenario of Figure 4 *vis-à-vis* the divorce scenario of Figure 6 (with probability of realignment of 60%), in response to the same set of shocks extracted from a Normal distribution with zero mean and variance 1.44 (standard deviation 1.2). The sample mean is 0.26 with standard deviation 1.13. With honeymoon the inflation process displays a sample mean of 2.18 with standard deviation 0.81 (smaller than the standard deviation of shocks), and an average square gap (from 2%) of 0.69%.

With divorce, the sample mean rises to 2.32, the standard deviation to 1.4, and the average square gap to 2.06%.

Figure 7. Monte Carlo simulation of the inflation processes in Figures 5 and 6