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Abstract

This paper examines both the desirability and feasibility of technology transfers in a setting where institutions governing the security of output or income are imperfect. Based on a guns-versus-butter model involving two countries (a technology leader and a technology laggard), our analysis characterizes how global efficiency and the countries' preferences over transfers depend on the nature of technology, as well as on the initial technological distance between them and the degree of output security. In the case of a general-purpose technology the leader might refuse a transfer, whereas in the case of a sector-specific technology the laggard might have such an incentive. Notably, for both types of technology, our analysis reveals the possible emergence of a "low-technology trap," wherein a technology transfer to the laggard is more likely to be blocked precisely when the laggard's initial technology is sufficiently inferior to its rival. We explore how the degree of output security and the laggard's capacity to absorb state-of-the-art technology affect the range of technological distances that generate such traps for each type of technology.

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1 Introduction

In the midst of ongoing geopolitical tensions between the U.S. and China lie concerns about China’s actual or potential use of American technology to further its own military objectives. While U.S. trade and investment relations with China have always been formulated with national security issues in mind, the expansion of technologies having both military and civilian applications has introduced new complexities (Olson, 2019). Indeed, heightened by China’s alleged practice of “forced technology transfers” and by its recent policy shift to foster dual-use infrastructure and resource sharing between the military and civil government, research institutes, and companies, these concerns have prompted the U.S. to impose sanctions on China.¹ Likewise, even before Russia’s invasion of Ukraine, the U.S. imposed a series of sanctions intended to block access by Russia’s defense sector to western technologies in the aerospace, marine, and electronic sectors, that allegedly facilitate dual-use technology.² The most direct consequence of such sanctions would seem clear enough—that is, to limit the transfer of technology. But, although the benefits of technology transfers on productivity have been widely studied, less is known about their potential drawbacks based on security considerations. Could the presence of conflict between countries make technology transfers costly to render them undesirable and thus enhance the appeal of sanctions? Shedding light on this issue has important policy implications insofar as the diffusion of technology is a significant factor that explains variation in income levels across countries (e.g., Parente and Prescott, 1994; Caselli and Wilson, 2004).

This paper examines both the desirability and feasibility of technology transfers in a setting where institutions governing the security of output or income are imperfect.³ More precisely, building on a single-period, guns-versus-butter model involving two countries (a technology leader and a technology laggard), our analysis identifies the conditions under

¹These sanctions were imposed under the Trump administration in November 2020 with [Executive Order \(E.O.\) 13959](#), and were subsequently broadened by the Biden Administration in June 2021 with [E.O. 14032](#). According to Sykes (2021), although the notion of “forced technology transfers” encompasses involuntary transfers through the actual theft of intellectual property (e.g., corporate espionage), it also includes more consensual sorts of transfers through the application of “corporate structure requirements” (CSRs), which require foreign investors to form a joint venture with Chinese firms or give them a controlling equity stake; CSRs effectively allow Chinese firms to demand a technology transfer as a condition for establishing a partnership. See the U.S. Department of State’s interpretation of China’s recent policy, once called the “Chinese Civic-Military Fusion policy”: <https://www.state.gov/wp-content/uploads/2020/05/What-is-MCF-One-Pager.pdf>. In October 2022, the Biden administration imposed new and more comprehensive restrictions on the sale of U.S. semiconductor technology (having military applications) to China (see <https://www.nytimes.com/2022/10/07/business/economy/biden-chip-technology.html>).

²These sanctions were authorized in April 2021 with [E.O. 14024](#) and extended in March 2022 as described in the U.S. Treasury’s press release: <https://home.treasury.gov/news/press-releases/jy0692>.

³Multiple studies have found reduced-form evidence that the quality of institutions is positively related to per-capita income (e.g., Acemoglu et al., 2001) and that differences in institutions and government policies explain variation in productivity levels across countries (e.g., Hall and Jones, 1999). Also, see Rodrik et al. (2002), who find that institutions contribute more to per-capita income than trade and geography.

which a technology transfer enhances global efficiency (or the sum of their payoffs) and the conditions under which either a technology leader or a technology laggard would choose to block a transfer. Both sets of conditions depend on the type of technology considered, the extent to which countries' technologies differ initially and the degree of output insecurity.

The model is structured as a two-stage, complete information game. In stage 1, the technology leader declares whether it will make its technology available to the laggard (or not) at some exogenously determined cost. At the same time, the laggard states whether it will accept this technology. In stage 2, given those choices, countries choose simultaneously how to allocate their respective resources to the production of butter (or consumables) and to the production of guns (or arms). As in the standard guns-versus-butter framework with decisions made by each country to maximize its own payoff, the allocation to guns is motivated by the insecurity of output or butter. In our simple setting, if butter output were perfectly secure, no resources would be allocated to guns and neither country would object to technology transfers. But, provided that at least some fraction of output is subject to expropriation by one's rival, countries generally do arm, and each country's arming decision depends not only on the degree of output security, but also on the levels of technology that both countries possess and the nature of the technology itself. We study two distinct possibilities: (i) a general-purpose or dual-use technology that affects the countries' ability to transform their respective resource endowments into butter for consumption and into guns for contesting output; and (ii) a sector-specific or civilian-use technology that affects only their ability to produce consumables. For greater clarity, the analysis considers the implications where countries differ at most in one of these two types of technology.

In accordance with the subgame-perfect, Nash equilibrium concept, we first characterize the outcome of the second stage in terms of arming choices and payoffs as they depend on the technological distance between the two countries and the degree of output security, as well as the nature of the technology considered. Not surprisingly, regardless of whether the laggard experiences an exogenous improvement in the general-purpose technology or in the sector-specific technology, both countries tend to enjoy a positive direct payoff effect, given their arming choices, in terms of increased output and thus a larger prize in the contest between them.

Differences in the payoff effects across the two types of technologies derive from differences in their influence on arming choices that determine the sign of the indirect, strategic payoff effects. In the case of an exogenous improvement in the laggard's general-purpose technology, both countries arm by more. While the direct positive effect noted earlier always dominates the negative strategic effect for the laggard, the opposite can hold true for the leader. In particular, when their technological distance is large initially and the degree of output security is sufficiently low, the laggard employs such improvements intensively in

the production of guns such that the resulting strategic effect dominates, thereby reducing the leader's payoff. By contrast, an exogenous improvement in the laggard's sector-specific technology raises that country's opportunity cost of arming to induce it to arm by less and produce more butter, implying that the leader's payoff always rises. However, such an improvement increases the leader's arming. Although the resulting adverse strategic payoff effect can be swamped by the positive direct effect so that the laggard's payoff increases like the leader's payoff, this need not be the case. In particular, when the initial difference in their sector-specific-technology is sufficiently pronounced and the degree of output security is relatively low but not zero, the adverse strategic effect dominates.

Whether countries differ only in terms of the general-purpose technology or only in terms of the sector-specific technology, exogenous improvements in the degree of output security never decrease the leader's payoff. The laggard's payoffs however, could fall with such improvements. A necessary condition for this outcome is that the laggard's technological distance from the leader is sufficiently large. While this negative relationship holds for all initial security levels in the case of the general-purpose technology, it holds for the case sector-specific technology only when output is sufficiently secure to start.

Interestingly, the results described above hint at the possibility of a "low-technology trap," wherein a country that starts out at a low level of technology remains in such a state.⁴ In our model, such a trap could materialize because either the leader or the laggard blocks the technology transfer. To explore this possibility further, we study the equilibrium of the extended game of technology transfers, drawing from our analysis of how exogenous improvements in technology and security affect arming and, consequently, payoffs. Along the lines of the technology transfer literature surveyed by Keller (2004), we admit the possibility that the laggard's capacity to implement the superior technology held by the leader might be limited. For both types of technology, we find that a low-technology trap is more likely to be observed when the technological distance between the leader and the laggard is sufficiently large to start and when the limits on the laggard's ability to absorb state-of-the-art technology are greater. In the case of the general-purpose technology, the reason for this is that the leader can enjoy a higher payoff by refusing the transfer (or imposing a sanction) and thus preserving its own advantage in predation. But, the greater is the degree of output security, the more likely it is for the leader to consent to the technology transfer. In the case of the sector-specific technology transfers, it is the technology laggard who might choose to reject the transfer because the rejection induces the leader to arm less

⁴This self-reinforcing outcome, which is related to but distinct from a poverty trap, can be viewed in our analysis as the result of the failure of institutions to support perfect output security. See the survey on poverty traps by Azariadis and Stachursky (2005). Also, see Gonzalez's (2012) insightful discussion of why imperfect property rights are problematic for economic development and why they tend to persist over time in less developed countries.

and produce more butter. We find further that the trap is more likely to be observed when the degree of output security is moderate.

Admittedly, the process by which technology is diffused is much more complicated than the way we have captured it in this paper, likely depending on trade and especially foreign direct investment (e.g., Keller, 2004; Anderson et al., 2019; Perla et al., 2021). We chose to abstract from such interactions in our analysis to isolate the effects of imperfect output security on the desirability and feasibility of technology transfers. Nonetheless, it is worth pointing out how some of our findings mirror previous results in the literature on the welfare effects of productivity improvements. In particular, Samuelson (2004) demonstrates that technological progress in a less developed nation can reduce the technology leader’s welfare. By contrast, Jones and Ruffin (2008) find, in a different setting, that the leader might gain from transferring the technology from its exporting sector to another country. Taken together, these results are close to our finding that transfers of the general-purpose technology can potentially increase or decrease the leader’s payoffs. However, in those papers, the welfare effects of productivity improvements operate through a terms-of-trade channel, whereas the effects we identify here are due to insecurity and operate through arming incentives. Along similar lines, Bhagwati’s (1958) finding of “immiserizing” growth, where the direct positive effect of income growth is dominated by an adverse terms-of-trade effect, and Johnson’s (1967) finding that economic growth could immiserize a country in the presence of distortions (such as tariffs) resemble our finding that sector-specific technology improvements can adversely affect the laggard’s payoff by increasing the size of the contested prize and thus inducing the leader to produce more arms.

It is also worthwhile to point out an important distinction between the transfers of technology we consider and resource transfers intended to support peace. For example, in Beviá and Corchón (2010) and Garfinkel and Syropoulos (2021) who study peace as the preservation of the status quo, it is the less affluent country that tends to have an incentive to arm and declare war (modeled as a “winner-take-all contest”) against its richer rival, since it perceives a larger potential gain from doing so. Naturally, a transfer of resources (a rival and excludable good) made by the richer country has the effect of evening out the distribution of endowments and, therefore, reduces the potential relative gain from war for the less affluent country.⁵ By contrast, a transfer of technology (a non-rival but excludable good) does not imply a direct loss of technological know-how for the donor. Nonetheless, it does tend to equalize the two countries’ capabilities. What’s more, when the transfer involves the sector-specific technology, it has a similar effect of easing the severity of conflict as reflected in lower aggregate arming. While a transfer of the general-purpose technology

⁵Garfinkel and Syropoulos (2020) show a more affluent country could “burn” its own resources to similarly support peace, but this alternative is Pareto dominated by offering a resource transfer.

also tends to even out the countries' capabilities, it tends to induce both countries to arm by more, thereby intensifying the conflict between them.

Our analysis relates to a number of earlier papers that highlight the effects of rent-seeking activities to hinder the development and adoption of new technologies and products. For example, Parente and Prescott (1999) show how monopoly rights held by labor raise the costs and thus reduce the incentive of firms to adopt a superior technology.⁶ Along similar lines, Desmet and Parente (2014) argue that craft guilds in Europe blocked technological progress and economic growth before the 18th century; but eventually, as markets expanded, firms' profits became sufficiently large to overcome the guilds' resistance to new technologies. With a different focus in the context of an R&D-based growth model, Dinopoulos and Syropoulos (2007) explore how incumbent firms can safeguard their monopoly rents due to their past innovations through costly activities that slow down the rate by which other firms successfully innovate to become the new incumbent. Also see Dinopoulos and Syropoulos (1998), who show how competition between duopolistic firms for property rights over technology can hinder the diffusion or sharing of that technology through licensing agreements. We contribute to this literature by suggesting a complementary barrier to technological progress—namely, costly international interactions to establish property rights over insecure output.

Our analysis is closest to Gonzalez (2005), who similarly studies the role of appropriative conflict in understanding why superior technologies might not be adopted, and can be seen as complementing his analysis in a number of ways. Most importantly, we study not only a sector-specific technology as he does, but also a general-purpose technology that directly expands the abilities of the contenders to arm. Studying these two types of technologies within a unified framework allows for reasonable comparisons and reveals sharp differences as described above. In addition, Gonzalez (2005) studies the decision makers' simultaneous choices to adopt a superior technology when their initial sector-specific technologies are identical. His finding that, in equilibrium depending on parameter values, neither agent chooses the superior technology or maybe just one does, but never both, points to technological backwardness. By contrast, we consider a two-sided game of technology transfer in which initial differences in countries' technological know-how play a decisive role and both actors must consent to a transfer, finding that, depending on parameter values, differences

⁶See Atkin et al. (2017) who present experimental evidence suggesting that misaligned incentives between workers and firm owners prevented the adoption of a superior technology among soccer-ball producers in Pakistan. In a related analysis similarly emphasizing the winners and losers of innovation, Krusell and Ríos-Rull (1996) develop a dynamic, political economy model wherein those that benefit from the current technology might attempt to block the adoption of superior technologies, with the outcome depending on the distribution of skills among voters. More recently, Milner and Solstad (2021) study how government policies that influence the adoption of new technologies depend not only on domestic politics, but also international factors including global competition and relations with other countries.

in technology can persist. Finally, our simple parameterization of output insecurity allows us to study how the degree of imperfect property rights determines countries’ arming choices and payoffs, and ultimately the diffusion of technology.

In what follows, the next section describes the theoretical model of output disputes that we use to explore countries’ incentives to arm for any given type of technology difference between countries after establishing the existence of a unique equilibrium in arming in the second stage of the game. In Section 3, we study how exogenous improvements in each of the two kinds of technology and in the security of output matter for countries’ arming choices, their payoffs and global efficiency. In Section 4, we turn to the first stage of the game to consider the possibility of technology transfers, demonstrating the possible emergence of a low-technology trap. Section 5 considers possible extensions of the analysis to explore the robustness of the baseline model. Finally, Section 6 offers some concluding remarks, including other possible extensions of the analysis. Technical details appear in appendices.

2 A Model of Output Disputes

Consider an environment with two risk-neutral countries $i = 1, 2$. Each country i holds a secure endowment of R^i units of a resource (“labor”) that it can allocate to the production of arms (or “guns”) and a consumption good (“butter”). We assume that these countries have solved their collective action problems, so that each country’s decision maker acts in the interests of its respective country as a whole.⁷

A key feature of the model is that it admits possible differences across countries with respect to two distinct types of technologies in production. The first is a general-purpose or dual-use technology, reflected in the parameter α^i , that transforms country i ’s resource endowment R^i into its “effective endowment” or human capital, $H^i = \alpha^i R^i$, and is applicable to activities that are socially valuable (producing butter) as well as to activities that are redistributive (producing guns). This technology would be positively related to the country’s infrastructure, the quality of its educational system, institutions, and so on. The second sort of technology, captured by β^i , is specific to the production of butter (i.e., civilian uses) which we refer to as a “sector-specific” technology. To be more precise, let G^i and X^i denote country i ’s output of guns and butter, respectively, and suppose that the associated technologies exhibit constant returns to scale. Country i ’s resource constraint implies that, for any quantity of guns produced $G^i \in [0, H^i]$, the maximal quantity of butter it can produce is $X^i = \beta^i (H^i - G^i)$. To fix ideas, we assume henceforth that country 1 is more efficient or productive than country 2 in the following sense: $\alpha^1 \geq \alpha^2 (> 0)$ and $\beta^1 \geq$

⁷Alternatively, the model can be thought of capturing the interactions between individuals or groups of individuals (that again have solved collective action problems) within a single country.

$\beta^2 (> 0)$, with at least one inequality holding strictly.⁸ For obvious reasons, we will refer to country 1 as the technology leader and country 2 as the technology laggard.

Let us denote the aggregate production of butter across the two countries by $\bar{X} \equiv \sum_{i=1,2} X^i$. To study the importance of the degree of output insecurity, we suppose that a fraction $\sigma \in [0, 1)$ of each country i 's butter X^i is secure; the remainder, $(1 - \sigma) X^i$, goes into a common pool $(1 - \sigma) \bar{X}$ that is contested. The degree of output security would depend on a number of factors including formal and informal international institutions of governance that mediate and govern disputes.⁹ Country i 's share of this pool depends on both countries' arming according to the following conflict technology:

$$\phi^i \equiv \phi^i(G^i, G^j) = \begin{cases} G^i/\bar{G} & \text{if } \bar{G} > 0 \\ X^i/\bar{X} & \text{if } \bar{G} = 0 \end{cases}, \quad i, j \in \{1, 2\}, i \neq j, \quad (1)$$

where $\bar{G} \equiv \sum_{i=1,2} G^i$ denotes aggregate arming across the two countries. The function in (1) has a number of important properties.¹⁰ First, ϕ^i is a symmetric function in the sense that $\phi^i(G^i, G^j) = \phi^j(G^i, G^j)$ for $i, j \in \{1, 2\}$, $i \neq j$.¹¹ Second, ϕ^i is strictly increasing and concave in country i 's arming G^i (i.e., $\phi_{G^i}^i \equiv \partial\phi^i/\partial G^i > 0$ and $\phi_{G^i G^i}^i < 0$). By contrast, ϕ^i is strictly decreasing and convex in rival j 's arming (i.e., $\phi_{G^j}^i = \partial\phi^i/\partial G^j < 0$ and $\phi_{G^j G^j}^i > 0$). Finally, $\phi_{G^i G^j}^i = (\phi^i - \phi^j) \phi^i \phi^j / (G^i G^j) \geq 0$ as $G^i \geq G^j$. As will become evident, these properties ensure the existence of a unique subgame perfect equilibrium in arming.¹²

Under risk neutrality, each country i 's payoff is defined as

$$U^i(G^i, G^j) = \phi^i(1 - \sigma) \bar{X} + \sigma X^i, \quad i, j \in \{1, 2\}, i \neq j, \quad (2)$$

for all $G^i \in [0, H^i]$, where $\phi^i = \phi^i(G^i, G^j)$ is shown in (1) and as previously defined $X^i =$

⁸The restriction $\alpha^2 \in (0, \alpha^1]$ requires, among other things, that both players be capable of producing something. We could admit the possibility that country 2 is unable to produce butter (in which case we would consider all $\beta^2 \in [0, \beta^1]$). But, for technical reasons, we focus on $\beta^2 \in (0, \beta^1]$.

⁹While one could extend the analysis to admit differences in output security across countries, our focus on differences in technology alone allows us to highlight their importance in determining the severity of conflict in terms of diverting resources away from the production of butter and the possibility for technology transfers to mitigate such conflict. Interpreting agents as groups of individuals within a country, the degree of security would depend on domestic institutions that consist of regulatory and enforcement agencies, the police and the court system.

¹⁰This specification differs slightly from that which has been axiomatized by Skaperdas (1996) in that here, when $G^1 = G^2 = 0$, the conflict preserves the status-quo distribution of output. This modification, however, is inconsequential for the equilibrium analysis.

¹¹Below in Section 5 we also consider the possibility of international differences in the military-use technology and study the appeal of transfers of this technology from the leader to the laggard.

¹²Owing to our assumption that countries are risk neutral, the game we study could be interpreted equivalently as a "winner-take-all" contest over the pool of insecure butter, $(1 - \sigma)\bar{X}$, with ϕ^i representing the probability that country i emerges as the victor. Our interpretation of ϕ^i as country i 's share of $(1 - \sigma)\bar{X}$ could be thought of as a reduced form of bargaining. Indeed, the key insights of our analysis would remain intact if the contested butter were divided on the basis of Nash bargaining or other bargaining protocols in the shadow of destructive war (e.g., Anbarci et al., 2002).

$\beta^i(H^i - G^i)$ and $\bar{X} = \sum_{i=1,2} X^i$.¹³ The first term in the right-hand side (RHS) of (2) represents the portion of the contested pool of butter $(1 - \sigma)\bar{X}$ that is appropriated by country i on the basis of its relative strength reflected in ϕ^i . The second term represents country i 's secure butter. Since an increase in country i 's guns G^i diverts human capital away from its butter production, X^i falls with increases in G^i causing its payoff U^i to fall. However, an increase in G^i also raises country i 's share ϕ^i of the contested butter, and U^i is increasing in G^i on this count. Clearly, for any given G^j ($j \neq i$), an optimally behaving country i chooses its guns to balance the marginal benefit of arming against the corresponding marginal cost.¹⁴ Observe that the payoff for each country i also depends on country j 's arming choice. In particular, for any given and feasible G^i , an increase in the rival country j 's guns G^j reduces U^i (i.e., $U_{G^j}^i < 0$ for $i, j \in \{1, 2\}, i \neq j$), because it reduces country j 's contribution (X^j) to global output of butter (and thus the contested pool $(1 - \sigma)\bar{X}$) and because it reduces country i 's share of the contested pool ϕ^i .

Our central objective is to characterize the subgame perfect equilibrium of the following two-stage game.

Stage 1: The technology leader declares whether it will make its technology available to the laggard at some exogenously determined cost that could be bilateral, while the laggard declares whether it would accept this technology.

Stage 2: Given the first-stage choices, the countries choose their guns simultaneously and noncooperatively to maximize their respective payoffs.

Naturally, the two countries factor in how their first-stage decisions regarding the transfer of knowledge affect the distribution of contested butter and their respective payoffs.

To help set the stage for this analysis, let us consider what would happen if property rights were complete ($\sigma = 1$), so that butter output is perfectly secure. One can easily verify that, in this hypothetical case that we refer to as “Nirvana,” neither country would have an incentive to arm. Accordingly, each country i would devote all of its effective endowment H^i to the production of butter, so that $U_n^i = \beta^i H^i = \beta^i \alpha^i R^i$ for $i = 1, 2$ (the subscript “ n ” stands for “Nirvana”). In the presence of perfect output security, then, the more efficient/productive country would be indifferent between (freely) sharing and not sharing its superior expertise. By contrast, the relatively inefficient/unproductive country

¹³Let us add here that the analysis can be extended to allow for risk aversion and/or the possibility that the dispute over insecure output arises probabilistically. Either way, as in our current setup, the key mechanism through which technology transfers matter is the resolution of actual or potential conflict that hinges on countries' relative arming decisions.

¹⁴Allowing for the possibility that conflict destroys some fraction of both secure and insecure output would not affect equilibrium gun choices, but of course would affect payoffs. Even if conflict were to partially destroy insecure output only, the results to follow would remain qualitatively unchanged.

would necessarily value having access to the other country's superior technologies. As we will see shortly, the presence of conflict that motivates the two countries to arm alters their payoffs and thus their preferences over technology transfers.

2.1 Arming

We start with the arming subgame in stage 2. Partial differentiation of country i 's payoff in (2) with respect to G^i gives

$$U_{G^i}^i = (1 - \sigma) \bar{X} \phi_{G^i}^i - \beta^i [\sigma + (1 - \sigma) \phi^i], \quad i = 1, 2. \quad (3)$$

The first term in the RHS of (3) shows country i 's marginal benefit of arming (MB_G^i) due to the effect of an increase in G^i (given the opponent's choice G^j) to increase the share of contested output country i obtains in the conflict. The second term shows the marginal cost (MC_G^i) of arming due to the diversion of its resources available for the production of butter. Below we analyze how these terms depend on technologies and the degree of output security. For now, observe that, for any given G^j , an increase in country i 's guns G^i reduces MB_G^i due to the concavity of ϕ^i in G^i and the fact that the aggregate quantity of butter \bar{X} falls due to the fall in X^i . Furthermore, MC_G^i is increasing in G^i because ϕ^i is increasing in G^i . As such, U^i is strictly concave in G^i (i.e., $U_{G^i G^i}^i < 0$), thereby establishing the existence of a Nash equilibrium in pure strategies in the arming subgame. To proceed, we derive the equilibrium of this subgame, which establishes uniqueness.

Based on (3) and the resource constraint $G^i \in [0, H^i]$, we can write each country i 's (possibly constrained) best-response function, labeled $B^i(G^j)$, as:

$$B^i(G^j) = \min \left\{ \tilde{B}^i(G^j), H^i \right\}, \quad i, j \in \{1, 2\}, i \neq j, \quad (4a)$$

where $\tilde{B}^i(G^j)$ represents country i 's unconstrained best response that is implicitly defined by $U_{G^i}^i = 0$:

$$\tilde{B}^i(G^j) = -G^j + \sqrt{(1 - \sigma) G^j \left[H^i + G^j + \frac{\beta^j}{\beta^i} (H^j - G^j) \right]}, \quad i, j \in \{1, 2\}, i \neq j. \quad (4b)$$

One possibility is that neither country is constrained in its arming choice. In this case, the unconstrained best-response functions (4b) intersect at the following unique interior solution:

$$\tilde{G}^i = \frac{2(1 - \sigma) \beta^j (\beta^i H^i + \beta^j H^j)}{4\beta^i \beta^j + (\beta^i - \beta^j)^2 \sigma^2} \left[1 + \frac{\sigma \beta^i + (2 - \sigma) \beta^j}{\sqrt{4\beta^i \beta^j + (\beta^i - \beta^j)^2 \sigma^2}} \right]^{-1}, \quad (5)$$

for $i, j \in \{1, 2\}, i \neq j$.¹⁵ In the special case where $\beta^1 = \beta^2$, these solutions imply $\tilde{G}^i = \tilde{G}^j = \frac{1}{4}(1 - \sigma)(H^1 + H^2)$; otherwise, when $\beta^i \neq \beta^j$, we have $\tilde{G}^i \neq \tilde{G}^j$. But, regardless of whether or not the two countries possess identical sector-specific technologies, at least one country (say country i) could be resource constrained (i.e., $\tilde{B}^i(G^j) > H^i$), implying it is a pure predator that specializes in appropriation (i.e., $G^i = B^i(G^j) = H^i$).¹⁶

Letting a superscript “*” identify equilibrium values of variables, we can summarize the properties of the Nash equilibrium in arming as follows.

Proposition 1 (Arming.) *For any given degree of imperfect output security $\sigma \in [0, 1)$, effective resource endowments and technologies, a unique pure-strategy equilibrium of the arming subgame exists in which both countries produce positive quantities of guns. In this equilibrium, either (a) neither country is resource constrained, in which case $G^{i*} = \tilde{G}^i$ for $i = 1, 2$; or, (b) only one country (say country i) is resource constrained, in which case $G^{i*} = H^i$ and $G^{j*} = \tilde{B}^i(H^i)$, $i \neq j$.*

The reason both countries cannot be pure predators in the noncooperative equilibrium at the same time is simple and follows from (3). If both countries were to specialize completely in appropriation, no butter would be produced at all (i.e., $\bar{X} = 0$). But, then, the marginal benefit to arming would equal zero for both countries, implying that $U_{G^i}^i < 0$ for $i = 1, 2$, which is inconsistent with our supposition that both countries devote their entire effective endowments to arming. Nonetheless, one of the two countries could specialize in appropriation. The exact conditions under which this possibility arises depends on whether countries differ with respect to their general-purpose or their sector-specific technologies, as shown in the next section. At this point, it is worth mentioning that only the technology laggard ($i = 2$) might choose to specialize completely in appropriation.

In preparation for our forthcoming analysis of payoffs and their dependence on the countries’ available technologies and the degree of output insecurity $1 - \sigma$, let us temporarily suppose that the resource constraint in arming binds for neither country and focus on their best-response functions. Inspection of the first-order conditions (FOCs) $U_{G^i}^i = 0$ based on (3) for $i = 1, 2$ reveals that improvements in general-purpose technologies α^i or α^j , which expand respectively H^i or H^j , cause both countries’ marginal benefit of arming to rise and induce each country, given the rival’s arming, to produce more guns.¹⁷ While an improvement in country i ’s butter technology β^i raises its marginal cost and marginal benefit

¹⁵Although the unconstrained best-response functions also intersect at $(0, 0)$, this point does not represent a Nash equilibrium of the subgame in arming. In particular, if one country were to produce no guns, its rival would arm by an infinitesimal amount and thereby capture all of the contested output.

¹⁶The shape of the unconstrained best-response functions, as described in Appendix A assuming $\beta^j > 0$, implies that $\tilde{B}^i(G^j) > 0$ in the neighborhood of the Nash equilibrium where $G^j > 0$, whether or not country j is resource constrained. As such, it is not necessary to consider the possibility in (4a) that $\tilde{B}^i(G^j)$ can take on negative values.

¹⁷Of course, the equilibrium responses will, in general, differ because both countries’ arming also adjusts to

of arming, the expression in (4b) shows that the net effect on its own arming incentive, given G^j , is negative. Such improvements also increase country j 's marginal benefit, but without affecting its marginal cost; thus, an increase in β^i augments country j 's incentive to arm (given G^i). Finally, observe from (3) that, given G^j , improvements in security ($\sigma \uparrow$) reduce country i 's marginal benefit to arming and increase its marginal cost. As a consequence, this country's best response will fall.

2.2 Payoffs

Once again, keeping in mind that we will study the equilibrium responses to the changes in technology and output insecurity described above, in this section we delineate their channels of influence on payoffs. Let θ denote any one of the model's parameters (e.g., $\theta = \sigma$). Assume (arbitrarily for now) that country 1's arming is unconstrained by its effective endowment while rival 2's resource constraint on arming might or might not bind. Since $U_{G^1}^{1*} = 0$, we can invoke the envelope theorem for country 1 to obtain

$$\frac{dU^{1*}}{d\theta} = U_{\theta}^{1*} + U_{G^2}^{1*} \left(\frac{dG^{2*}}{d\theta} \right). \quad (6)$$

The first term in the RHS of (6) represents the direct effect of a change in θ on 1's payoff that can be found by partially differentiating U^1 in (2) and evaluating the resulting expression at the countries' optimizing guns choices G^{i*} ($i = 1, 2$). The second term in the RHS of (6) represents the indirect (or strategic) effect through the impact on rival 2's arming. Because a country's payoff generally depends negatively on its rival's guns (i.e., $U_{G^j}^i < 0$ for $i, j \in \{1, 2\}, i \neq j$) as argued earlier, the second term will be negative (positive) if $dG^{2*}/d\theta > 0$ ($dG^{2*}/d\theta < 0$) and will vanish if $dG^{2*}/d\theta = 0$.

Allowing for the possibility that the resource constraint on country 2's arming binds, the effect of a change in θ on U^{2*} must be written as

$$\frac{dU^{2*}}{d\theta} = U_{\theta}^{2*} + U_{G^1}^{2*} \left(\frac{dG^{1*}}{d\theta} \right) + U_{G^2}^{2*} \left(\frac{dG^{2*}}{d\theta} \right). \quad (7)$$

The first two terms in the RHS of (7) represent the direct and indirect effects of a change in θ , along the lines discussed above in connection with country 1. If $dG^{2*}/d\theta = 0$, the third term will vanish regardless of whether country 2 specializes completely in appropriation (in which case $U_{G^2}^{2*} > 0$) or not (in which case $U_{G^2}^{2*} = 0$). Otherwise, if $dG^{2*}/d\theta \neq 0$, we must also take this effect into account.

each other's direct responses. Here we discuss only the direct effect on an individual country's best-response function.

3 How Technology Matters

To characterize the equilibrium of the arming subgame and deepen our understanding of how technology matters for payoffs, in this section we organize the presentation around two distinct cases: (i) one in which countries differ solely in their general-purpose technologies (so that $\alpha^1 > \alpha^2 > 0$ while $\beta^1 = \beta^2$); and (ii) another in which countries differ only in their productivity in butter (so that $\beta^1 > \beta^2 > 0$ while $\alpha^1 = \alpha^2$).¹⁸ With such a focus, we can perform several comparative statics exercises that throw valuable light on countries' incentives to participate, or not, in the types of technology transfers we consider later.

3.1 The Importance of General-Purpose Technologies

Under the assumption that $\beta^1 = \beta^2 = 1$, we explore in this section how differences in general-purpose technologies (α^i) across the two countries affect equilibrium arming and payoffs. To fix ideas, assume further that $R^i = \frac{1}{2}\bar{R} = 1$ ($i = 1, 2$), which implies $H^1 = \alpha^1$, $H^2 = \alpha^2$ and $\bar{H} \equiv \sum_{i=1,2} H^i = \alpha^1 + \alpha^2$. Thus, the impact of any change in α^2 given α^1 (which could arise due to technology transfers) operates through the implied changes in the effective resource endowments H^2 and \bar{H} .

3.1.1 Arming

Imposing these assumptions on (4a) and (5) and then rearranging the resulting expressions give respectively

$$B^i(G^j) = \min \left\{ -G^j + \sqrt{(1-\sigma)G^j(\alpha^1 + \alpha^2)}, \alpha^i \right\} \quad (8a)$$

$$\tilde{G}^i = \tilde{G} \equiv \frac{1}{4}(1-\sigma)(\alpha^1 + \alpha^2), \quad (8b)$$

for $i, j \in \{1, 2\}, i \neq j$. The expression in (8b) implies that there exists a unique threshold value of α^2 , conditioned on the leader's technology α^1 and the degree of output security σ , below which the laggard is resource constrained in its arming choice (i.e., $\tilde{G} > H^2 = \alpha^2$). This threshold, denoted by $\alpha_0(\sigma)$, is given by

$$\alpha_0(\sigma) \equiv \frac{1-\sigma}{3+\sigma}\alpha^1, \quad (9)$$

where $\alpha_0'(\sigma) < 0$, with $\alpha_0(0) = \alpha^1/3$ and $\alpha_0(1) = 0$. Thus, if $\alpha^2 \in (\alpha_0(\sigma), \alpha^1]$ which is more likely to hold as output security σ increases, then $G^{1*} = G^{2*} = \tilde{G} < \alpha^2$, implying that both countries produce butter as well as guns.¹⁹ Alternatively, if $\alpha^2 \in (0, \alpha_0(\sigma)]$, then

¹⁸We could allow for the possibility that countries differ along both of these technology dimensions. However, our focus on the two different cases outlined above enables us to isolate the distinct effects of each type of technology difference.

¹⁹Since $\alpha_0(0) = \alpha^1/3$ represents the maximum value that this threshold can take on for $\sigma \in [0, 1)$, any value of $\alpha^2 > \alpha_0(0)$ ensures that the equilibrium of the arming subgame is an interior solution.

the laggard becomes a pure predator, specializing in guns production, $G^{2*} = \alpha^2$, such that $G^{1*} = \tilde{B}^1(\alpha^2) = -\alpha^2 + \sqrt{(1-\sigma)\alpha^2(\alpha^1 + \alpha^2)}$; as one can easily verify, $G^{1*} < \alpha^1$ holds in this case, implying as argued earlier that both countries cannot be pure predators.²⁰

Based on the above, we can now establish the following:

Proposition 2 (General-purpose technology, arming and power.) *Given any degree of imperfect output security $\sigma \in [0, 1)$ and assuming the two countries have identical sector-specific technologies, the unique equilibrium of the arming subgame implies that the technology leader is at least as powerful as the laggard (i.e., $\alpha^2 < \alpha^1$ implies $\phi^{1*} \geq \phi^{2*}$ as a strict inequality when the laggard is resource constrained). Given $\alpha^1 (> \alpha^2)$:*

- (a) *Improvements in the laggard’s general-purpose technology ($\alpha^2 \uparrow$) induce both countries to arm more heavily, with the following implications for the balance of power:*
 - (i) *When the laggard is resource constrained in its arming choice, it increases its guns by proportionately more, thus improving its power ($\phi^{2*} \uparrow$).*
 - (ii) *When the laggard is not resource constrained, the two countries make identical adjustments in arming choices and remain equally powerful (i.e., $\phi^{1*} = \phi^{2*}$).*
- (b) *Improvements in security ($\sigma \uparrow$) reduce the incentives for countries to arm given their rival’s choice, with the following implications for the balance of power:*
 - (i) *When the laggard is resourced constrained, the leader actually chooses fewer guns, implying an erosion of the leader’s power advantage ($\phi^{1*} \downarrow$).*
 - (ii) *When the laggard is not resource constrained, the two countries reduce their guns identically, thus leaving them equally powerful (i.e., $\phi^{1*} = \phi^{2*}$).*

The equilibrium in arming characterized in Proposition 2 is illustrated by point E in Fig. 1(a) where the countries’ best-response functions intersect each other, in the special case of totally insecure output (i.e., $\sigma = 0$) and technologically symmetric countries (i.e., $\alpha^1 = \alpha^2$, as well as $\beta^1 = \beta^2$). By the definition of equilibrium and due to complete symmetry in this benchmark case, point E lies on the 45° . The thick dotted green curve depicts the equilibrium pairs of guns that emerge for alternative values of α^2 (specifically, for technological regress associated with reductions in α^2 from α^1 towards 0). As shown in the figure, such regress that causes the value of the “prize” to fall induces each country to reduce its best response to any given arming choice by its rival. Initially, the shifts are symmetric so that the equilibrium remains symmetric and accordingly located on the 45° line, but closer to the origin. Hence, differences in general-purpose technologies need not imply

²⁰As shown in Appendix A, in the case where the two countries differ only in terms of their general-purpose technology, $\tilde{B}^1(G^2)$ reaches a maximum where the laggard is not resource constrained—i.e., at $G^{2*} = \tilde{G}$ that implies $G^{1*} = \tilde{B}^1(G^{2*}) = \tilde{G}$. As such, the technology leader chooses fewer guns and produces more butter when its rival produces no butter than when its rival produces some butter.

differences in arming and power across countries. However, when α^2 falls below the threshold $\alpha_0(\sigma)$ so that the laggard's resource constraint binds, the shifts become asymmetric. In particular, the laggard begins to specialize completely in appropriation; and, any additional reduction in α^2 brings about a one-for-one reduction in G^{2*} . The technology leader's best-response function shifts inward; and, since its own secure endowment has not changed, its production necessarily remains diversified. These adjustments, together with the strategic complementarity of G^1 for G^2 in the neighborhood of the constrained equilibrium, ensure that G^{1*} falls. However, as established in part (a.i) of the proposition and as indicated by the shape of the green locus, G^{1*} falls by less than G^{2*} ($= \alpha^2$) to induce a continuous rise in ϕ^{1*} above $\frac{1}{2}$.

The equilibrium adjustments in guns described above as α^2 varies are illustrated in Fig. 1(b) for two distinct values in the degree of security σ : $\sigma = 0$ and a value of $\sigma \in (0, 1)$. Proposition 2(b) shows that improvements in security ($\sigma \uparrow$) always induce the technology leader ($i = 1$) to produce fewer guns. The same is true for the laggard ($i = 2$) except, of course, when its effective endowment is exhausted by its production of guns. In the former case where neither country is resource constrained, such improvements have no effect on the balance of power; otherwise, they reduce the leader's power.

3.1.2 Payoffs

It should be clear from our analysis of the two countries' arming above that the leader always obtains a higher payoff than the laggard: $U^{1*} > U^{2*}$ as $\alpha^1 > \alpha^2$. Specifically, in the case where the laggard is resource constrained, the leader's arming is greater than its rival's arming, such that its share of the prize is larger (i.e., $\phi^{1*} > \phi^{2*}$); furthermore, since the leader (in contrast to the laggard) does not exhaust its effective endowment in the production of guns, it produces some secure butter for itself (i.e., $X^{1*} > X^{2*} = 0$). Even if the laggard is not resource constrained so that the two countries produce identical quantities of guns and thus enjoy equal shares of contested output (i.e., $\phi^{1*} = \phi^{2*}$), the leader's effective endowment is larger (i.e., $X^{1*} > X^{2*} > 0$), implying that the utility it derives solely from its secure output of butter is necessarily larger. Nonetheless, in either case, Hirshleifer's (1991) paradox of power holds here. That is to say, the laggard devotes a disproportionately larger share of its effective endowment to arming than its rival such that $1 \geq G^{2*}/G^{1*} > \alpha^2/\alpha^1$.

To examine the payoff effects of changes in α^2 , we start with the technology leader. Building on the analysis related to (6) for $\alpha^2 \in (0, \alpha^1]$ with (2), we obtain

$$U_{\alpha^2}^{1*} = (1 - \sigma) \phi^{1*} > 0, \tag{10a}$$

$$U_{G^2}^{1*} = (1 - \sigma) (\phi_{G^2}^{1*} \bar{X}^* - \phi^{1*}) < 0. \tag{10b}$$

Clearly, as can be seen from (10a), the direct effect of an improvement in the laggard's general-purpose technology α^2 on the leader's payoff U^{1*} is positive for all $\alpha^2 \in (0, \alpha^1]$. The reason is that an increase in α^2 for given guns expands the laggard's effective resource endowment, thereby inducing it to contribute more butter to the common pool $(1 - \sigma)\bar{X}$. By contrast, as indicated by (10b) and the result in Proposition 2 that $dG^{2*}/d\alpha^2 > 0$, the indirect effect is negative. In particular, an increase in α^2 induces the laggard to increase its arming that, given G^1 , reduces the leader's share of the contested pool while also reducing the size of that pool. Clearly, the direct and indirect effects push U^{1*} in opposite directions. Although it is unclear at this level of generality which effect dominates, we can resolve this issue as discussed shortly.

Turning to the technology laggard's payoff U^{2*} , we can study its dependence on α^2 by following the procedure outlined earlier for the leader, but this time using (7). Once again, the direct and indirect effects of changes in α^2 , shown as the first two terms on the RHS of (7), work in opposite directions:

$$U_{\alpha^2}^{2*} = (1 - \sigma)\phi^{2*} + \sigma > 0, \quad (11a)$$

$$U_{G^1}^{2*} = (1 - \sigma)(\phi_{G^1}^{2*}\bar{X}^* - \phi^{2*}) < 0. \quad (11b)$$

Specifically, equation (11a) shows that an increase in α^2 has a positive direct effect on the laggard's own payoff, while equation (11b) shows that the positive effect of an increase α^2 on country 1's arming (established in Proposition 2) has a negative impact. Additionally, from the last term in (7), when the laggard specializes completely in appropriation—namely, when $\alpha^2 \in (0, \alpha_0(\sigma))$ —we must also consider the indirect effect on U^{2*} through the laggard's own arming G^{2*} . This effect is positive and hence reinforces the direct effect, because in this case $U_{G^2}^{2*} > 0$ and, from Proposition 2, $dG^{2*}/d\alpha^2 > 0$. Of course, this effect vanishes for $\alpha^2 \in [\alpha_0^2(\sigma), \alpha_1]$, because $U_{G^2}^{2*} = 0$. Nonetheless, in general, due to the presence of a negative indirect effect whether the laggard specializes in appropriation or not, the sign of the net effect of an increase in α^2 on the laggard's payoff remains unclear at this point. Once again, as we will see shortly, it is possible to sign the net effect.

How does the degree of output security σ matter in this context? By partially differentiating the payoff functions (2) with respect to σ and evaluating the resulting expressions at the equilibrium of the arming subgame, one can obtain the direct effects for each country:

$$U_{\sigma}^{i*} = -\phi^{i*}\bar{X}^* + X^{i*} = (1 - \phi^{i*})X^{i*} - \phi^{i*}X^{j*}, \quad i, j \in \{1, 2\}, i \neq j. \quad (12)$$

Then, by summing the expressions above across countries $i = 1, 2$, one can confirm that the aggregate direct payoff effect $U_{\sigma}^{1*} + U_{\sigma}^{2*}$ is zero. Thus, when one country benefits from an increase in security, the other loses. Let us focus on country 1. When both countries are

unconstrained, then the two countries arm identically, $G^{i*} = \tilde{G}$ as shown in Proposition 2, such that $\phi^{1*} = \phi^{2*} = \frac{1}{2}$. Thus, we have $U_\sigma^{1*} = \frac{1}{2}(\alpha^1 - \tilde{G}) - \frac{1}{2}(\alpha^2 - \tilde{G})$, which is strictly positive whenever $\alpha^1 > \alpha^2$. When country 2 is resource constrained so that $G^{2*} = \alpha^2$, we have $U_\sigma^{1*} = (1 - \phi^{1*})(\alpha^1 - G^{1*})$, which is also positive since country 1 always diversifies its production. Accordingly, the direct effect of security improvements ($\sigma \uparrow$) is positive for the technologically advanced country and negative for the laggard.²¹ In addition, from Proposition 2, we know that $dG^{i*}/d\sigma \leq 0$ for $i = 1, 2$, implying that the indirect effect of an increase in σ on both countries' payoffs is non-negative. Clearly, then, the technology leader will always find security improvements appealing. However, due to the negative direct effect, it is unclear how the laggard would view such improvements.

We now argue that, despite the offsetting effects on equilibrium payoffs at work here, it is possible to identify more precisely their dependence on the initial values of the laggard's technology α^2 and of the security in property rights σ . The next proposition explains.

Proposition 3 (General-purpose technology and payoffs.) *Given any degree of imperfect output security $\sigma \in [0, 1)$ and assuming the two countries have identical sector-specific technologies, equilibrium payoffs of the arming subgame can be characterized as follows:*

- (a) *Improvements in the laggard's general-purpose technology ($\alpha^2 \uparrow$) always increase its own payoff U^{2*} , but their effect on the leader's payoff U^{1*} depends on the initial value of α^2 relative to α^1 and on σ that jointly determine whether or not the laggard is resource constrained:*
 - (i) *When the laggard is resource constrained, U^{1*} falls.*
 - (ii) *When the laggard is not resource constrained, U^{1*} rises.*
- (b) *Improvements in output security ($\sigma \uparrow$) always enhance the leader's payoff U^{1*} . Their payoff effect for the laggard depends on α^2 , given α^1 , and the initial value of σ that jointly determine whether or not the laggard is resource constrained:*
 - (i) *When the laggard is resource constrained, U^{2*} falls.*
 - (ii) *When the laggard is not resource constrained, U^{2*} rises.*

Taking into account both the direct and indirect payoff effects that possibly move in opposite directions as described earlier, Proposition 3 clarifies the remaining ambiguities. First, in contrast to the technology laggard that always benefits from an improvement in its own general-purpose technology ($\alpha^2 \uparrow$), the leader benefits only if, given the degree of output security, the initial value of α^2 is sufficiently large to induce the laggard to produce both guns and butter (i.e., when $\alpha^2 \in [\alpha_0(\sigma), \alpha^1)$). By contrast, if the laggard specializes completely in predation (i.e., when $\alpha^2 \in (0, \alpha_0(\sigma))$), the leader is made worse off due

²¹Of course, when $\alpha^1 = \alpha^2$, the direct payoff effect is 0 for both countries.

to the dominance of the negative strategic effect. Specifically, the laggard applies the entire increase in its effective endowment (due to $\alpha^2 \uparrow$) to arming, implying its butter production remains unchanged at zero.²² Second, while the leader always benefits from an improvement in output security ($\sigma \uparrow$), the laggard benefits only when it diversifies its production; otherwise, the direct negative effect of an increase in σ dominates to make the laggard worse off.

Fig. 1(c) depicts the dependence of both countries' payoffs on α^2 for $\sigma = 0$ and some $\sigma > 0$ as characterized in the proposition. This figure also shows the countries' payoffs under Nirvana U_n^i where $\sigma = 1$. As mentioned earlier, the leader's payoff in this special case U_n^1 is invariant to changes in α^2 , whereas the laggard's Nirvana payoff U_n^2 is increasing in α^2 . By contrast, in the presence of insecurity, the leader's payoff falls with improvements in the laggard's technology when the distance between their general-purpose technologies is sufficiently pronounced and rises otherwise. Thus, the presence of insecurity and conflict has sharply different implications for the payoff effects of increases in α^2 than those that follow from standard economic theory where output security is assumed to be perfect. What's more, while the leader always prefers improvements in security, the laggard need not.

3.2 The Importance of Sector-Specific Technologies

In this section, we study how differences in butter productivities (β^i) across the two countries matter for equilibrium arming and payoffs. Taking a similar approach to that adopted above, we treat the leader's technology parameter β^1 as fixed, and characterize the implications of changes in the laggard's productivity β^2 ($< \beta^1$) and output security σ for arming and payoffs. To highlight the salience of differences in this sort of technology across countries, we suppose $H^i = \frac{1}{2}\bar{H} = 1$ ($i = 1, 2$).²³

3.2.1 Arming

In this setting, the expressions for the best-response functions in (4a) and the unconstrained equilibrium arms in (5) become respectively

$$B^i(G^j) = \min \left\{ -G^j + \sqrt{(1 - \sigma) G^j \left[1 + G^j + \frac{\beta^j}{\beta^i} (1 - G^j) \right]}, 1 \right\} \quad (13a)$$

²²That the leader is made worse off only when the laggard is resource constrained might suggest a weakness of the model. However, as shown below in Section 5, the possible dominance of the adverse strategic payoff effect for the leader derives more generally from the tendency for the laggard to employ intensively its larger effective endowment in appropriative activities. This tendency increases as the initial distance between the countries' dual-purpose technologies rises, and more so as output security falls. Thus, while a binding resource constraint for the laggard is sufficient for the leader to find improvements in the laggard's dual-use technology unappealing, it is not necessary.

²³Clearly, this normalization holds true when $\alpha^i = 1$ and $R^i = 1$ for $i = 1, 2$. However, it also holds true when $H^i = \alpha^i R^i = 1$.

$$\tilde{G}^i = \frac{2(1-\sigma)\beta^j(\beta^i + \beta^j)}{4\beta^i\beta^j + (\beta^i - \beta^j)^2\sigma^2} \left[1 + \frac{\sigma\beta^i + (2-\sigma)\beta^j}{\sqrt{4\beta^i\beta^j + (\beta^i - \beta^j)^2\sigma^2}} \right]^{-1}, \quad (13b)$$

for $i, j \in \{1, 2\}, i \neq j$. Inspection of (13a) reveals immediately that, when country j specializes in appropriation (i.e., $G^j = H^j = 1$), rival i 's best-response function is invariant to changes in β^j . If in addition $B^j(G^i) = H^j = 1$, then neither country's arming depends on country j 's butter productivity β^j . The solutions for the unconstrained equilibrium in (13b) suggest that matters differ otherwise.

Thus, to sharpen our understanding of the importance of differences in the sector-specific technology for equilibrium outcomes, let us first derive the conditions under which the laggard specializes in appropriation. We do so by finding the parameter values for which \tilde{G}^2 in (13b) is greater than $H^2 = 1$. As shown in Appendix A, setting $\tilde{G}^2 = 1$ and solving for β^2 yield the following schedule:

$$\beta_0(\sigma) = \frac{\frac{1}{2} - \sigma}{\frac{3}{2} - \sigma + \sqrt{2}(1-\sigma)}\beta^1, \quad (14)$$

where $\beta_0' < 0$, $\beta_0(\frac{1}{2}) = 0$, $\beta_0(0) = (3 - 2\sqrt{2})\beta^1 \approx 0.172\beta^1$. Based on schedule $\beta_0(\sigma)$, the technology laggard is resource constrained in its arming when output security σ is relatively low and the distance between its productivity in butter production β^2 and the corresponding productivity of the leader β^1 is sufficiently high. That is to say, $G^{2*} = H^2 = 1$ for $\sigma \in [0, \frac{1}{2})$ when $\beta^2 \in (0, \beta_0(\sigma)]$; otherwise, for all $\sigma \in [0, 1)$, when $\beta^2 \in (\max\{0, \beta_0(\sigma)\}, \beta^1)$, the laggard's resource constraint does not bind, allowing it to diversify its production.²⁴

We can now establish the following proposition.

Proposition 4 (Sector-specific technology, arming and power.) *Given any degree of imperfect output security $\sigma \in [0, 1)$ and assuming the two countries have identical effective resource endowments, the unique equilibrium of the arming subgame implies that the laggard in butter production is more powerful than the leader (i.e., $\beta^2 < \beta^1$ implies $\phi^{2*} > \phi^{1*}$).*

(a) *Improvements in the laggard's butter productivity ($\beta^2 \uparrow$) have the following effects on the countries' arming choices and the balance of power depending on the initial value of β^2 , given β^1 , and σ that determine whether the laggard country is resource constrained or not:*

(i) *When the laggard is resource constrained, neither country adjusts its arming choice, thereby leaving the balance of power as well as aggregate arming unchanged.*

²⁴Schedule $\beta_0(\sigma)$ in (14) shows clearly that a sufficient, but not necessary, condition for an interior solution to emerge as the equilibrium of the subgame is that $\sigma > \frac{1}{2}$.

- (ii) *When the laggard is not resource constrained, it decreases its arming, whereas the leader increases its arming, thus eroding the laggard’s power advantage, while reducing the aggregate quantity of guns produced.*
 - (b) *Improvements in output security ($\sigma \uparrow$) always induce the leader to produce fewer guns. Their effects on the laggard’s optimizing choices and the balance of power depend on the initial value of σ that determines, along with β^1 and β^2 , whether the laggard is resource constrained or not:*
 - (i) *When the laggard is resource constrained, it does not adjust its arming.*
 - (ii) *When the laggard is not resource constrained, it produces fewer guns.*
- In both cases, the laggard’s power advantage weakens, while aggregate arming falls.*

We illustrate some of the results of this proposition with the help of Fig. 2(a), which shows the two countries’ best-response functions for various values of β^2 when output is completely insecure (i.e., $\sigma = 0$). Point E on the 45° line depicts the equilibrium when the two countries enjoy equal butter productivities (i.e., $\beta^1 = \beta^2$). At this point, from (13b), the two countries arm identically: $G^{1*} = G^{2*} = \tilde{G} = \frac{1}{2}(1 - \sigma)$. As shown in the figure, regress in the laggard’s butter productivity (i.e., $\beta^2 \downarrow$), which reduces the size of the contested pool and thus the marginal benefit of arming to the leader, decreases its best response to any given arming choice of its rival—i.e., shifts its best-response function $B^1(G^2)$ inward. Such regress also decreases the laggard’s marginal benefit of arming as well as its marginal cost. Nonetheless, as shown in the figure and established in the proposition, the effect on the laggard’s marginal cost dominates such that its best response to any given arming choice by the leader increases.²⁵ Successive reductions in β^2 generate equilibrium adjustments in arms production, illustrated by an upward movement along the negatively-sloped green, dotted-line schedule, inducing the laggard to produce more guns ($G^{2*} \uparrow$) and the leader to produce less guns ($G^{1*} \downarrow$). Once β^2 falls to $\beta_0(\sigma)$, the laggard specializes in appropriation, and further reductions in β^2 are inconsequential for equilibrium arming by both countries.²⁶ Since $G^{1*} < G^{2*}$ holds for all $\beta^2 < \beta^1$, whether or not the laggard is resource constrained, the country with a comparative advantage in butter (i.e., the leader) is always less powerful than its rival (i.e., the laggard). Furthermore, whether or not the laggard is resource constrained, reductions in β^2 never decrease total guns production in equilibrium $\bar{G}^* \equiv G^{1*} + G^{2*}$.

Fig. 2(b) highlights the countries’ arming responses to improvements in security σ for $\sigma \in [0, \frac{1}{2}]$ and, as such, provides more insight. Note that the length of the horizontal

²⁵That the negative effect on the laggard’s own marginal cost dominates can be confirmed by noting the positive effect of a decrease in β^2 (given β^1) on $B^2(G^1)$ in (8b) for $i = 2$.

²⁶The laggard’s best-response function $B^2(G^1)$ with a flat portion shown in Fig. 2(a) for $\sigma = 0$ captures such a possibility that arises when β^2 is sufficiently small relative to β^1 .

segments of these curves, where the laggard is resource constrained and thus where arming does not vary with changes in β^2 , decreases as σ rises within the $[0, \frac{1}{2}]$ interval. The flats disappear when $\sigma \geq \frac{1}{2}$. The figure also shows, given both countries diversify production, the equilibrium adjustments in guns in response to improvements in the laggard's technology ($\beta^2 \uparrow$), with the G^{2*} falling and G^{1*} rising, to erode the laggard's power advantage.²⁷

3.2.2 Payoffs

Based on our analysis above, it should be clear that, as in the case of the general-purpose technology, the country with the better sector-specific technology tends to produce more butter, such that the payoff it derives from its secure output alone is higher than that for the laggard. However, in contrast to the case of the general-purpose technology, it produces fewer guns, implying that the payoff it derives from its share of contested output alone is smaller. Thus, as in Skaperdas (1992) and Skaperdas and Syropoulos (1997) as well as in Gonzalez (2005), we have a variant of Hirshliefer's (1991) paradox of power. In particular, the less productive country tends to have a comparative advantage in arming and thus is more powerful. The implications for the ranking of the two countries' payoffs, however, are not immediately obvious. In general, they depend on the degree of output security as well as on the difference between the two countries' sector-specific technologies.

But, given our primary objective to explore the desirability and feasibility of technology transfers, we turn instead to our analysis of the payoff effects of exogenous improvements in the laggard's butter productivity ($\beta^2 \uparrow$) and output security ($\sigma \uparrow$). From (2) and the definitions of $X^i = \beta^i(1 - G^i)$ and $\bar{X} = \sum_i X^i$, one can see that the direct effects of an increase in β^2 on U^{1*} and U^{2*} are non-negative:

$$U_{\beta^2}^{1*} = (1 - \sigma) \phi^{1*} (1 - G^{2*}) \geq 0 \quad (15a)$$

$$U_{\beta^2}^{2*} = [(1 - \sigma) \phi^{2*} + \sigma] (1 - G^{2*}) \geq 0. \quad (15b)$$

Clearly, these direct effects vanish when the laggard specializes completely in appropriation (i.e., when $G^{2*} = 1 = H^2$). Turning to the indirect effects, we use (2) to calculate

$$U_{G^j}^{i*} = (1 - \sigma)(\phi_{G^j}^{i*} \bar{X}^* - \beta^j \phi^{i*}) < 0, \quad i, j \in \{1, 2\}, i \neq j. \quad (16)$$

We know further, from Proposition 4, that $dG^{2*}/d\beta^2 \leq 0$ and $dG^{1*}/d\beta^2 \geq 0$ (with equality for both if the laggard specializes in appropriation). Thus, combining the direct and indirect effects according to (6), it is clear that an increase in β^2 never makes the technology leader worse off: $dU^{1*}/d\beta^2 \geq 0$ (again with equality for both if the laggard specializes in appropriation). However, from (7), because the direct and indirect payoff effects of an in-

²⁷This erosion corresponds to the equilibrium adjustments in guns illustrated in Fig. 2(a) as a downward movement along the green dotted-line schedule towards point E .

crease in β^2 go in opposite directions for the laggard, whether this country would welcome an improvement in its productivity of butter remains unclear at this point.

To identify the payoff effects of an improvement in output security ($\sigma \uparrow$), we apply the same logic used in the case where countries differ by α^i alone. In particular, equation (12) with $X^{i*} = \beta^i(1 - G^{i*})$ implies that the sum of the direct payoff effects across the two countries is again equal to zero. Accordingly, we can focus on just one country. Focusing on the technology leader, we have $U_\sigma^{1*} = (1 - \phi^{1*})\beta^1(1 - G^{1*}) - \phi^{1*}\beta^2(1 - G^{2*})$, where $\beta^1 \geq \beta^2$ by assumption and $1 - G^{1*} > 0$ since the leader always produces some butter. When the laggard is resource constrained in its arming so that $G^{2*} = 1$, the direct payoff effect for the leader is positive (and thus the corresponding effect for the laggard is negative). Even when the laggard is not resource constrained, however, the result from Proposition 4 that $G^{2*} > G^{1*}$ whenever $\beta^2 < \beta^1$ implies that $\phi^{2*} = 1 - \phi^{1*} > \phi^{1*}$ and, as a result, the direct payoff effect of an increase in σ for the technology leader is again positive (whereas again the direct payoff effect for the laggard is negative). An application of (16) with the result from Proposition 4 that $dG^{2*}/d\sigma \leq 0$ (with strict equality when the laggard is resource constrained in its arming) shows the indirect effect of an increase in σ on the leader's payoff is non-negative. When combined with the direct effect identified above and (6), this result implies that the leader unambiguously benefits from an improvement in output security. But, since $dG^{1*}/d\sigma < 0$, the indirect effect for the laggard is strictly positive to make the total payoff effect for the laggard shown in (7) appear to be ambiguous.

The next proposition characterizes the payoff effects of improvements in the laggard's butter productivity and output security, resolving the ambiguities described above:

Proposition 5 (Sector-specific technology and payoffs.) *Given any degree of imperfect output security $\sigma \in [0, 1)$ and assuming the two countries have identical effective resource endowments, the following characterizes equilibrium payoffs of the arming subgame:*

- (a) *Improvements in the laggard's sector-specific technology ($\beta^2 \uparrow$ given β^1) have no effect on either country's payoff if the laggard specializes in appropriation. Otherwise, the leader's payoff U^{1*} is increasing in β^2 . By contrast, the laggard's payoff U^{2*}*
 - (i) *is J-shaped in $\beta^2 \in [\max\{0, \beta_0(\sigma)\}, \beta^1]$ if $\sigma \in (0, \check{\sigma})$, where $\check{\sigma} = 1/\sqrt{2} \approx 0.707$ with $\lim_{\beta^2 \searrow \max\{0, \beta_0(\sigma)\}} (dU^{2*}/d\beta^2) < 0$ and $\lim_{\beta^2 \searrow \max\{0, \beta_0(\sigma)\}} U^{2*} < \lim_{\beta^2 \rightarrow \beta^1} U^{2*}$;*
 - (ii) *is increasing in $\beta^2 \in (0, \beta^1)$ if either $\sigma = 0$ or $\sigma \in (\check{\sigma}, 1)$.*
- (b) *Improvements in output security ($\sigma \uparrow$) always increase the leader's payoff U^{1*} . These improvements also increase the laggard's payoff U^{2*} if it specializes in appropriation. Otherwise, the security improvements cause U^{2*} to*
 - (i) *fall if output is sufficiently secure to start and $\beta^2 \in (0, \bar{\beta})$, where $\bar{\beta} \equiv \frac{1}{2}(\sqrt{5} - 1)\beta^1 \approx 0.618\beta^1$;*

(ii) rise for all $\sigma \in [0, 1)$ if $\beta^2 \in (\bar{\beta}, \beta^1]$.

This proposition establishes, consistent with the discussion that precedes it, that the leader is never made worse off with improvements in the laggard's sector-specific technology and is strictly better off when the laggard is not resource constrained in its arming choice (i.e., when $\beta^2 \in [\max\{0, \beta_0(\sigma)\}, \beta^1]$). The laggard, however, could be worse off as the direct (and positive) effect of increasing β^2 on the laggard's butter production X^{2*} could be dominated by the indirect (and negative) effect due to the rise in the leader's arming. Such an outcome is more likely when the degree of output security is low, but not zero, and the laggard's initial technology is near but above the threshold ($\max\{0, \beta_0(\sigma)\}$) where it produces both guns and butter.²⁸ These results can be visualized with the help of Fig. 3(a) that identifies the direction of change in U^{2*} as β^2 rises in $(0, \beta^1]$. For points in the grey region to the left of the black dotted-curve that depicts $\sigma = \sigma_0(\beta^2)$, we have $X^2 = 0$ and U^{2*} is invariant to changes in β^2 . For parameter pairs to the right of $\sigma = \sigma_0(\beta^2)$, the laggard diversifies its production. The blue dashed-line curve in this set identifies the pairs of parameters under which U^{2*} attains a minimum.²⁹ Thus, points to the left of this curve and above $\sigma_0(\beta^2)$ (i.e., points in the pink region) imply $dU^{2*}/d\beta^2 < 0$, whereas for points to the right of the blue dotted curve (i.e., in the yellow and green regions) $dU^{2*}/d\beta^2 > 0$.³⁰ The blue solid-line curve is the locus of points that ensure U^{2*} equals $\lim_{\beta^2 \searrow \max\{\beta_0(\sigma), 0\}} U^{2*}$ for $\sigma \in [0, \bar{\sigma})$. As such, U^{2*} in the green region is higher than U^{2*} in all other regions.

Part (b) of the proposition establishes that, while the leader benefits from improvements in output security under all circumstances, the laggard could be made worse off when output security is initially high and, at the same time, its technology is sufficiently inferior relative to that of the leader. The intuition behind this finding, which rises only if the laggard produces both butter and guns, is as follows. When the laggard is not resource constrained, an increase in the initial degree of output security strengthens the negative direct effect of an increase in σ on its payoff U^{2*} (i.e., makes U^{2*} more negative), while weakening the positive indirect effect. In contrast, the direct effect of an increase in σ becomes weaker (less negative) and the indirect positive effect becomes stronger as the value of β^2 rises. For more insight, see Fig. 3(b) that identifies the direction of change in U^{2*} as σ rises from 0 to (approximately) 1. In this figure, U^{2*} attains maximum for parameter pairs on the magenta curve, part of which coincides with $\sigma = \sigma_0(\beta^2)$ that again is shown as the black-dotted

²⁸This dominance of the adverse strategic effect also arises in the dynamic, two-period setting of Gonzalez (2005), where (given the productivity of butter) agents choose the allocation of their initial endowments to current consumption, arming and saving, with the last two allocations supporting future consumption. That it arises in our simpler one-period model without saving strengthens his result.

²⁹As implied by this figure and shown in the proof to this proposition presented in Appendix A, the laggard's payoff is strictly quasi-convex in β^2 .

³⁰Ignore for now the green dotted curve in the pink region.

curve.³¹ Thus, U^{2*} rises with increases in σ for parameter values below the magenta curve (i.e., in the grey and green regions). In contrast, U^{2*} falls with increases in σ for parameters values above the magenta curve (i.e., in the pink region).

3.3 Efficiency

Having discussed the effects of changes in the laggard’s productivity on equilibrium arming and payoffs, we can now address the question of how these changes affect global efficiency. In the rent-seeking and conflict literatures, the cost of socially unproductive activities (an inverse measure of efficiency) is normally proxied by the aggregate quantity of guns produced. Applying this idea to the present setting, it is natural to argue that, insofar as technological progress amplifies the absorption of resources in appropriative/redistributive activities, it could hamper global efficiency. However, the impact of such progress on the total quantity of guns produced is just part of the story. Productivity changes also directly affect the output of butter. To study the overall effect on efficiency, one must account for both effects.

We explore the above ideas, using a simple measure of efficiency—namely, the sum of the countries’ payoffs: $\bar{U} \equiv U^1 + U^2$. From the definition of country i ’s payoff in (2) for countries $i = 1, 2$ and the fact that $\phi^1 + \phi^2 = 1$, one can see that

$$\bar{U} = \bar{X} = \beta^1 (H^1 - G^1) + \beta^2 (H^2 - G^2). \quad (17)$$

Our distinction between (i) “general-purpose” technologies (assuming $\beta^i = 1$ and $H^i = \alpha^i$ for $i = 1, 2$) and (ii) “sector-specific” technologies (assuming $H^i = 1$ for $i = 1, 2$) allows us to identify the direct and indirect effects of changes in α^2 on \bar{U} separately from those due to changes in β^2 . In particular, recalling that $\bar{G} = G^1 + G^2$, we can write the change in equilibrium efficiency arising from an improvement in each type of technology as follows:

$$(i) \quad d\bar{U}^*/d\alpha^2 = 1 - d\bar{G}^*/d\alpha^2 \quad (18a)$$

$$(ii) \quad d\bar{U}^*/d\beta^2 = (1 - G^{2*}) - \sum_{i=1,2} \beta^i (dG^{i*}/d\beta^2). \quad (18b)$$

Under regime (i) where countries differ only with respect to their general-purpose technologies, a necessary condition for productivity improvements to enhance efficiency is that they do not raise aggregate arming by more than they raise the production of butter.³² The corresponding condition under regime (ii), where countries differ only with respect to their sector specific technologies, is slightly different, in that one must compare the change in the individual countries’ arming choices, weighted by their respective productivities in

³¹Note that U^{2*} is not differentiable in σ (i.e., it has a kink) along $\sigma_0(\beta^2)$ for $\beta^2 \in (0, \underline{\beta})$. As shown in the proof to Proposition 5(b), U^{2*} is strictly quasi-concave in σ .

³²Recall from Proposition 2 that $d\bar{G}/d\alpha^2 > 0$ for initial values of $\alpha^2 \in (0, \alpha^1)$ and $\sigma \in [0, 1)$.

butter, with the technology’s direct effect on butter production. Notably, as established in Proposition 4, improvements in the laggard’s butter productivity ($\beta^2 \uparrow$) do not raise the aggregate production of guns (i.e., $d\bar{G}/d\beta^2 \leq 0$ always holds). However, this finding does not necessarily imply that the just noted weighted sum of changes in arming falls. Thus, whether an increase in β^2 necessarily improves efficiency remains an open question.

The next proposition shows precisely how improvements in both general-purpose and sector-specific technologies and in output security matter in this context.

Proposition 6 (Technology, security and efficiency.) *For any given degree of output security $\sigma \in [0, 1)$, efficiency is highest when the distance in the agents’ general-purpose and sector-specific technologies is small. Improvements in the laggard’s*

- (a) *general-purpose technology ($\alpha^2 \uparrow$) raise efficiency unless the laggard specializes in appropriation, in which case such improvements reduce efficiency;*
- (b) *sector-specific productivity ($\beta^2 \uparrow$) raise efficiency unless either:*
 - (i) *the laggard specializes in appropriation, in which case such improvements are inconsequential for efficiency; or,*
 - (ii) *the laggard produces butter and guns, β^2 is sufficiently close to $\max\{0, \beta_0(\sigma)\}$, and $\sigma \in (\underline{\sigma}, \check{\sigma})$, where $\underline{\sigma} = \sqrt{5} - 2 \approx 0.236$ and as previously defined $\check{\sigma} = 1/\sqrt{2} \approx 0.707$, in which case such improvements reduce efficiency.*

For any given technologies, improvements in output security ($\sigma \uparrow$) enhance efficiency.

As this proposition establishes, improvements in the laggard’s general-purpose and sector-specific technologies need not always enhance efficiency. The logic behind this finding, alluded to earlier, is simple and intuitive. In settings where the implementation of enforceable contracts on arming is not feasible—perhaps due to the absence of a supranational authority or weak laws and institutions—technological progress can induce a sufficiently large shift in the allocation of the countries’ resources away from productive activities towards distributive conflicts, and in doing so create additional social costs that outweigh the social benefits driven by productivity gains for the laggard. The proposition suggests that such an efficiency loss is more likely in situations where the laggard’s general-purpose (α^2) and sector-specific (β^2) technologies are further away from those of the leader (respectively, α^1 and β^1) initially. Interestingly, while the parameter space for which marginal improvements in the general-purpose technology reduce global efficiency is precisely the same as the parameter space for which such improvements generate payoff losses for the leader, the parameter space for which marginal improvements in the sector-specific technology reduce efficiency is a subset of the parameter space for which such improvements reduce the laggard’s payoff, shown as the pink region in Fig. 3(a) to the left of the green dotted curve.

Of course, when those conditions are not satisfied, improvements in the laggard’s technology do generate efficiency gains. But, while the presence of such gains is necessary to ensure that *both* countries value such improvements, it is not sufficient, as suggested by Propositions 3 and 5. Indeed, depending on whether countries differ with respect to their general-purpose technologies or with respect to their sector-specific technologies, either the leader or the laggard can find improvements in the laggard’s technology unappealing. Likewise, although Proposition 6 establishes that improvements in output security ($\sigma \uparrow$) always improve efficiency, Propositions 3 and 5 indicate that these improvements can be unappealing to the laggard. This sort of logic provides an economic rationale for the resistance of backward economies to economic reforms related to common security.

4 Technology Transfers

Having studied the effects of exogenous advances in general-purpose and sector-specific technologies in the laggard on equilibrium arming, payoffs and efficiency in the second-stage subgame, we now turn to the first stage to study the endogenous determination of improvements in the laggard’s technology through transfers from the leader. In particular, we seek to understand how the subgame perfect equilibrium of the extended game depends on the nature of the technology under consideration, the technological distance between countries, the laggard’s ability to absorb the state-of-the art technology, and output security. To this end, we suppose that technology transfer decisions are made before arming choices. Each policymaker declares independently in a simultaneous-move stage game whether it accepts (A) or rejects (N) the transfer, taking into account how the transfer will affect arming decisions, the production of butter and its distribution. The transfer materializes (also in that stage) only if both countries choose A .

Following the literature on technology transfers, the analysis admits the possibility that the laggard’s ability to implement the state-of-the art technology is limited by its absorptive capacity (Keller, 2004). For example, there could be a loss in translating the blueprints of the state-of-the-art technology and adjusting them for use by the laggard, similar to the “iceberg” metaphor of trade costs in the trade literature.³³ Let a^2 and b^2 denote respectively the general-purpose and sector-specific technologies that the laggard acquires if both countries agree to the transfer. A simple way to capture the laggard’s possibly limited ability to absorb the technology in question is to suppose that $a^2 = \lambda\alpha^1$ and $b^2 = \mu\beta^1$, where $\lambda, \mu \in (0, 1]$ is the effective rate of absorption. Obviously, $\lambda = 1$ ($\mu = 1$) identifies the case of a costless technology transfer. However, for a transfer to result in a technology upgrade for the laggard, the effective rate of transfer must be sufficiently large: $\lambda \in (\alpha^2/\alpha^1, 1]$

³³Cohen and Levinthal (1989, 1990) provide an extended discussion of the cognitive and organizational aspects of absorptive capacity in the adoption of technology, underscoring the importance of in-house R&D.

in the case of a general-purpose technology transfer and $\mu \in (\beta^2/\beta^1, 1]$ in the case of a sector-specific technology transfer.

4.1 Transfers of the General-Purpose Technology

We start with general-purpose (dual-use) technology transfers. Under the conditions of Proposition 3, we know that the laggard's equilibrium payoff U^{2*} is increasing in its technology α^2 . Therefore, accepting a transfer to upgrade its technology (A) is a weakly dominant strategy for the laggard.³⁴ The leader's payoff is also increasing in α^2 , but only when the resource constraint is not binding for the laggard's arming. When the laggard operates as a pure predator, the leader's payoff falls as α^2 rises. For ease of exposition, we discuss the implications with the help of panel (a) of Fig. 4 that depicts this relationship for $\sigma = 0$. As the figure highlights, there exist circumstances under which the leader would favor the transfer (A) and circumstances under which it would oppose it (N). Clearly, in this subgame, assuming the transfer results in an upgrade for the laggard, the leader's preferences over the transfer determine the equilibrium outcome. In particular, if the leader chooses N , then (N, N) is part of the subgame perfect equilibrium. And if the leader chooses A then (A, A) is the stage outcome.³⁵

Turning to the subgame perfect equilibrium, observe from the figure that $\lim_{\alpha^2 \rightarrow 0} U^{1*} > \lim_{\alpha^2 \rightarrow \alpha^1} U^{1*}$. The logic underlying this ranking is as follows. As α^2 approaches 0, both countries' arming become infinitesimal; at the same time, both the laggard's contribution to the contested output in the second stage and its share of that output are also infinitesimal. Thus, as α^2 goes to zero, the leader can realize (approximately) its payoff under perfect security $U_n^1 (= \alpha^1)$. At the other extreme as α^2 approaches α^1 , the two countries produce identical quantities of guns $G^{i*} = \tilde{G} = \frac{1}{2}\alpha^1 > 0$ for $i = 1, 2$, such that $U^{1*}|_{\sigma=0} = \frac{1}{2}\alpha^1 < U_n^1$. These properties together with the ones established in Proposition 3 imply that the functional dependence of U^{1*} on α^2 is V -shaped.

If $\lambda = 1$, the leader's payoff under the transfer is the one associated with point D in Fig. 4(a). The leader's payoff in the absence of a transfer depends on the initial value of α^2 . For any value of α^2 below the level associated with point C , the leader would refrain from offering the transfer. Why? Although the transfer would induce the laggard to contribute to the contested pool, it also causes the laggard to arm more aggressively in the contest over output. Because low values of α^2 (underdevelopment) constrain the laggard's ability to arm, refusing to make the transfer caps the laggard's power in the contest and generates a higher payoff for the leader. By contrast, for values of α^2 in the interval between points C and D , the leader favors a transfer.

³⁴The dominance is "weak" for the laggard because its payoff from declaring either A or N is the same when the leader declares N . One can verify that this is also true for the leader.

³⁵It should be noted that (N, N) is always part of a weakly dominated subgame perfect equilibrium.

Next, consider a value of $\lambda < 1$ that implies $a^2 = \lambda\alpha^1 \in (\alpha_0(\sigma), \alpha^1)$, as indicated by point D' in Fig. 4(a) when $\sigma = 0$. The laggard's limited absorptive capacity implies that, when α^2 is sufficiently large (i.e., associated with points to the right of D'), a transfer would be inconsequential. What about for smaller values of α^2 ? Once again, the leader would find the transfer unappealing if the value of α^2 is below the value associated with point C' . However, it would prefer to make the transfer if α^2 falls within the range associated points C' and D' . Importantly, if $a^2 = \lambda\alpha^1 \leq \alpha_0(0)$ (or equivalently $\lambda \leq \frac{1}{3}$), then the set of initial α^2 values that render the transfer appealing to the leader would be empty.

Two salient findings emerge from the above analysis. First, countries that are at the low end of the technology ladder are more likely to find themselves locked in a “low-technology trap” or “underdevelopment” associated with general-purpose technologies. The reason is that, by refusing the transfer to such rivals the leader can contain their ability to arm, thereby securing a higher payoff for itself. Second, the more limited is the laggard's ability to absorb the state-of-the-art technology ($\lambda \downarrow$), the larger is the range of parameter values for which the laggard remains trapped in a low development state. Put differently, countries with lower absorptive capacity are more likely to be denied access to the superior technology, a sort of self-reinforcing phenomenon.

We can tease out the implications of improvements in the degree of security ($\sigma \uparrow$) here with the help of Fig. 1(c), which shows that the leader's payoff (which continues to be V-shaped) rises at each value α^2 , while the threshold value $\alpha_0(\sigma)$ falls. For any given absorptive capacity λ , the horizontal line associated with the new payoff function rises to intersect it at the same value of $a^2 = \lambda\alpha^1$ (corresponding to point D' in Fig. 4), but the new horizontal line must intersect the new payoff function at a smaller value of α^2 (relative to that corresponding to point C' in Fig. 4). Thus, given any λ , a higher degree of output security ($\sigma \uparrow$) expands the range of α^2 values (given α^1) for which the leader will find a transfer appealing; at the same time, the range of initial α^2 values under which the laggard finds itself in a low-technology trap shrinks. Hence, countries at the low end of the technology ladder are less likely to be locked in a low-technology trap as σ increases.

4.2 Transfers of the Sector-Specific Technology

To examine the possible equilibria that arise when a sector-specific (civilian-use) technology transfer is on the table, observe from Proposition 5 that the leader finds improvements in the laggard's technology appealing provided they imply $b^2 > \beta^2 \in \max\{0, \beta_0(\sigma)\}, \beta^1$. Therefore, the leader would be willing to make a transfer to the laggard under these circumstances.³⁶ The same proposition implies that the laggard's equilibrium payoff is strictly in-

³⁶Otherwise, when the improvement leaves the laggard resource constrained (i.e., $b^2 \leq \max\{\beta_0(\sigma), 0\}, \beta^1$), the leader would be indifferent between A and N .

creasing in β^2 , where initially $\beta^2 > \max\{0, \beta_0(\sigma)\}$, under two sets of circumstances: (i) when output is either perfectly insecure (i.e., $\sigma = 0$) or sufficiently secure (i.e., $\sigma > \check{\sigma} \approx 0.707$), for any initial technological distance between the two countries; or (ii), when output security is low or moderate (i.e., $\sigma \in (0, \check{\sigma})$), for sufficiently small technological distances.³⁷ Applying the logic employed in our analysis of general-purpose technologies, one can infer that a technology transfer will be appealing to both sides under these circumstances, such that (A, A) emerges as the equilibrium of this subgame. However, Proposition 5 also shows that the laggard’s payoff could be J -shaped for low to intermediate degrees of output security when its initial productivity in butter β^2 is sufficiently small relative to the leader’s productivity in butter β^1 .³⁸ For clarity, we explore this possibility and its implications with the help of panel (b) of Fig. 4, which assumes $\sigma = 0.45$.

When the laggard’s absorptive capacity μ is sufficiently high so that a transfer results in a large enough technology upgrade, the laggard is always willing to embrace the state-of-the-art technology as that improves its payoff. To be more precise, the laggard will choose A , if its original technology satisfies $\beta^2 \in (0, b^2)$, while μ delivers a value of b^2 technology to the laggard within the range corresponding to points E and D in Fig. 4(b). However, if μ is low enough as illustrated, for example, by point E' in Fig. 4(b) that delivers technology $b^2 = \mu\beta^1$, the laggard might choose to reject technology b^2 —in particular, when β^2 is less than the technology associated with point C' in Fig. 4(b). This finding relates to our previous result in connection with Propositions 4 and 5 that, when β^2 is sufficiently low to start, the laggard’s adoption of marginally more efficient technologies erodes its power and lowers its payoff. In cases like the one considered here, the implied payoff for the laggard if it were to adopt the better technology is less than the payoff it would obtain without it. This is an interesting and, to the best of our knowledge, previously unidentified low-technology trap. In this trap, it is the laggard who refuses to upgrade its technology. In doing so, it can preempt a dilution of its power and avoid the implied reduction in its payoff.

As in the case of general-purpose technology transfers, a lower absorptive capacity for the laggard ($\mu \downarrow$) expands the range of technology values that generate a trap. The difference here is that the possibility of such a trap arises only when μ falls below a critical threshold level, the one associated with the β^2 value that corresponds to point E in Fig. 4(b).

4.3 Policy Implications of Low-Technology Traps

Summarizing our analysis above, we have

Proposition 7 (Low-technology traps.) *In the presence of output insecurity with differences in either the general-purpose (dual-use) or the sector-specific (civilian-use) technology,*

³⁷See the (combined) green and yellow regions in panel (a) of Fig. 3.

³⁸See the pink region in Fig. 3(a).

the countries that are more likely to be locked in a low-technology trap are those at the lowest end of the technology ladder. The range of parameter values for which a trap emerges expands as the laggards' absorptive capacity (λ or μ) falls. The influence of output security (σ) depends on the type of technology considered:

- (a) In the case of general-purpose technology differences, laggards are more likely to be locked in a low-technology trap when σ is relatively low.
- (b) In the case of sector-specific technology differences, laggards are more likely to be locked in a low-technology trap when σ is low or moderate.

To flesh out the policy implications of the emergence of such traps, observe first that global efficiency, $\bar{U}^* = U^{1*} + U^{2*}$, necessarily rises whenever both the leader and the laggard agree to implement a transfer. That leaves open the question of whether transfers could raise efficiency even when, given the laggard's capacity to absorb the leader's better technology and the degree of output security, the laggard is trapped.

Recall from Proposition 6(a) that global efficiency declines with marginal improvements in the laggard's general-purpose technology ($\alpha^2 \uparrow$) when it is resource constrained in its arming choice. That is not to say, however, that discrete improvements in the laggard's general-purpose technology via transfers necessarily reduce efficiency in such cases. As illustrated in Fig. 4(a), provided that α^2 is not too small relative to α^1 , the leader is willing to make such transfers and global efficiency rises as a result even when the technology laggard is constrained initially. Furthermore, since the threshold value of α^2 , below which the leader refuses to make a transfer, is decreasing in the laggard's absorptive capacity λ , efforts by the leader to increase λ could raise the leader's payoff and hence global efficiency. Nevertheless, since a trap remains even when $\lambda = 1$, such efforts, however extensive, cannot eliminate the trap for all laggards. What's more, these efforts are not costless, and the leader would have to balance the costs against the resulting (gross) payoff gains. Proposition 7(a) suggests an alternative approach to reduce the importance of technology traps and raise global efficiency—namely, by improving output security ($\sigma \uparrow$). But, while the range of α^2 values, given $\lambda \leq 1$, for which a trap emerges falls as σ increases, Proposition 3(b.i) suggests that the laggard could be made worse off. Accounting for the additional costs that would have to be borne by both countries to improve output security further diminishes the possible appeal of this alternative approach.

Although similar issues arise when the laggard is initially trapped at the low end of the sector-specific technology ladder, the possibility of freeing laggards from that trap would appear greater. The reasoning is as follows. Recall from Proposition 5 that improving the laggard's technology ($\beta^2 \uparrow$) always increases the leader's payoff; our discussion in connection to Proposition 6 suggests further that the leader's payoff gains can more than offset the

laggard’s payoff losses (see the part of the pink region of Fig. 3(a) to the right of the green dotted curve). In such cases, the leader would have a stronger incentive to provide technical assistance to enhance the laggard’s capacity to absorb its own state-of-the-art technology ($\mu \uparrow$), thereby making a transfer more appealing to the laggard. Indeed, providing complete assistance that gives $\mu = 1$ would wipe out the low-technology trap for all laggards.

5 Extensions

In this section we sketch out two extensions of the basic model to explore the robustness of some of our findings and, to some extent, sharpen the model’s predictive power. First, we introduce the possible presence of international differences in the technology for arms production and explore the leader’s incentive to grant the laggard access to its superior technology as well as the laggard’s incentive to accept it. Second, allowing for complementarity of the countries’ inputs into butter production, we substantiate the idea that complete specialization by the laggard in arming and appropriation is a sufficient but not a necessary condition for the leader to find general-purpose technology transfers unappealing. Most of the technical details are relegated to Appendix B.

5.1 The Desirability of Military-Use Technology Transfers

Given we have studied the appeal of transfers of the general-purpose (or dual-use) technology and the appeal of transfers of the technology for butter production specifically, one might naturally wonder about differences in the military-use technology (another sort of sector-specific technology) held by the two countries and the possible appeal of transfers between them. Such transfers might be expected to occur between countries that form military alliances or enter into cooperative defense agreements. But, is it also possible that such transfers occur between adversaries? Holding fixed the leader’s resource allocation to conflict and butter production, an improvement in the laggard’s military-use technology decreases the amount of resources the laggard has to employ to establish a given amount of military power and thus, given the leader’s arming choice, increases the size of the prize (i.e., contested output) to benefit both sides. At the same time, however, such an improvement affects the countries’ incentives to arm. Whether the associated strategic payoff effects are positive or negative and, if negative, which effect dominates is not immediately obvious.

To address these issues, we consider a slightly modified version of the model that allows for differences in the military-use technology across countries. Let γ^i denote the (constant) productivity of country i ’s initial resource H^i in the production of guns where, as was the case earlier, country 1 (2) is the technology leader (laggard); that is, $\gamma^2 < \gamma^1$. Then, the amount of the effective resource endowment country i devotes to the conflict to produce military power G^i that enters the conflict technology (1) is given by $g^i \equiv G^i/\gamma^i$. Under this

slightly modified specification, each country i 's payoff continues to be as shown in (2), but where $X^i = \beta^i (H^i - g^i)$ and $\phi^i (G^i, G^j) = \phi^i (\gamma^i g^i, \gamma^j g^j)$. Without loss of generality, we change policy variables. Specifically, for given values of γ^i with $\gamma^2 \in (0, \gamma^1)$, each country i allocates g^i units of its effective resource H^i to produce military power ($G^i = \gamma^i g^i$) so as to maximize its payoff U^i . To stay focused on international differences in military-use technologies, we assume that $\beta^i = 1$ and $H^i = \bar{H}/2$ for $i = 1, 2$. Since $\phi_{g^i}^i = \phi^i \phi^j / g^i$, we can then write country i 's FOC for an interior solution as

$$U_{g^i}^i = (1 - \sigma) \bar{X} \phi^i \phi^j / g^i - (\phi^i + \sigma \phi^j) = 0, \quad i, j \in \{1, 2\}, i \neq j. \quad (3')$$

This expression for the net marginal benefit of g^i is clearly similar to the one for G^i based on possible differences only in the general-purpose and/or sector-specific technologies as shown in (3). The first term represents the marginal benefit of g^i ($MB_{g^i}^i$) and the second represents the marginal cost ($MC_{g^i}^i$). Yet, as confirmed below, for any $\sigma \in [0, 1)$, neither country ever chooses, in equilibrium, to specialize in appropriation—i.e., $X^i > 0$ always holds for $i = 1, 2$.

Combining the FOCs in (3') for both countries, while keeping in mind that the specification of the conflict technology implies $g^j/g^i = (\gamma^i/\gamma^j) (\phi^j/\phi^i)$, yields the following condition

$$\frac{\phi^i}{\phi^j} = \frac{\gamma^i(\phi^j + \sigma\phi^i)}{\gamma^j(\phi^i + \sigma\phi^j)}, \quad (19)$$

which must hold in an interior equilibrium. Since $\phi^i + \phi^j = 1$, this condition implicitly defines the equilibrium distribution of power, $\phi^{i*} = \phi^{i*}(\gamma^i, \gamma^j, \sigma)$, as a function of the marginal productivities in conflict γ^i and γ^j and of the degree of security σ . Manipulation of (19) shows that, for $\sigma \in [0, 1)$ and $\gamma^2 \in (0, \gamma^1)$, we have $\phi^{1*} \in (\frac{1}{2}, 1)$, which readily implies that the leader is more powerful than the laggard. Furthermore, an application of the implicit function theorem to (19) shows that $d\phi^{1*}/d\gamma^2 < 0$. Thus, improvements in the laggard's military technology erode the leader's power.³⁹

To study the underlying drivers of this last result that informs our analysis of the payoff effects of improvements in the laggard's military-use technology, we obtain solutions for the countries' allocations of their respective resources to conflict. By substituting $\bar{X} = \bar{H} - g^i - g^j$ into countries' FOCs and then solving the resulting system of equations for g^i as functions of ϕ^i , ϕ^j and \bar{H} , one can obtain

$$g^i = \frac{(1 - \sigma) \phi^i \phi^j (\sigma \phi^i + \phi^j)}{\sigma + 2(1 - \sigma) \phi^i \phi^j} \bar{H}, \quad i, j \in \{1, 2\}, i \neq j \quad (20a)$$

³⁹One can also confirm that we have $\text{sign}\{d\phi^{i*}/d\sigma\} = \text{sign}\{\gamma^i - \gamma^j\}$, implying that common security improvements ($\sigma \uparrow$) enhance (erode) the leader's (laggard's) power (see Appendix B.2).

$$\bar{g} \equiv g^i + g^j = \frac{(1 - \sigma^2) \phi^i \phi^j}{\sigma + 2(1 - \sigma)\phi^i \phi^j} \bar{H}. \quad (20b)$$

An application of the equilibrium distribution of power ϕ^{i*} implicitly defined by (19) to these expressions then pins down the equilibrium values g^{i*} and \bar{g}^* .⁴⁰ Keeping in mind that $\phi^{i*} \in (0, 1)$ for $i = 1, 2$, it is easy to confirm that the coefficient on \bar{H} in (20a) is less than $\frac{1}{2}$. Accordingly, we have $X^{i*} \in (0, H^i)$ for $i = 1, 2$, which confirms our earlier claim that neither country specializes in the production of guns.

Based on these results along with (19) and (20), we show in Appendix B.2 that improvements in the laggard's military technology ($\gamma^2 \uparrow$) increase the leader's allocation to conflict: $dg^{1*}/d\gamma^2 > 0$. The effect on the laggard's resource allocation to conflict, by contrast, depends on the initial value of γ^2 , with $\lim_{\gamma^2 \rightarrow 0} dg^{2*}/d\gamma^2 > 0$ and $\lim_{\gamma^2 \rightarrow \gamma^1} dg^{2*}/d\gamma^2 < 0$. Nonetheless, for all $\gamma^2 \in (0, \gamma^1)$, such improvements increase the aggregate quantity of global resources diverted into arming ($\bar{g}^* \uparrow$) and reduce the production of butter ($\bar{X}^* \downarrow$).⁴¹

We can now examine the effects of improvements in the laggard's technology on global efficiency and payoffs. Since $\bar{U} = U^i + U^j = \bar{X}$ and $d\bar{X}^*/d\gamma^2 = -d\bar{g}^*/d\gamma^2 < 0$, global efficiency falls with increases in γ^2 .⁴² Turning to the individual payoff effects for each country, we start with the laggard. Differentiation of U^2 with respect to γ^2 , while invoking the envelope theorem, yields

$$\frac{dU^{2*}}{d\gamma^2} = U_{\gamma^2}^{2* (+)} + U_{g^1}^{2* (-)} \left(\frac{dg^{1*}}{d\gamma^2} \right)^{(+)},$$

which shows the positive direct effect and the opposing indirect effect through the induced change in the leader's resource allocation to conflict.⁴³ With an application of our previous findings, we can establish that $dU^{2*}/d\gamma^2 > 0$, such that the direct payoff effect of γ^2 always dominates; thus, the laggard would always welcome transfers in the military-use technology from the leader. Next, turning to the payoff effects for the leader, we have

$$\frac{dU^{1*}}{d\gamma^2} = U_{\gamma^2}^{1* (-)} + U_{g^1}^{2* (-)} \left(\frac{dg^{2*}}{d\gamma^2} \right).$$

It is easy to verify that the direct payoff effect for the leader of an improvement in the laggard's military-use technology is negative, while the indirect strategic effect could be negative or positive depending on whether the initial value of γ^2 is sufficiently small or large

⁴⁰We have omitted the asterisk from variables on the RHS of each solution in (20) to avoid cluttering.

⁴¹Exactly the opposite is true for security enhancements ($\sigma \uparrow$): $d\bar{g}^*/d\sigma < 0$.

⁴²By contrast, as revealed by the calculus shown in Appendix B.2, global efficiency would rise if instead the leader were to experience an improvement in its military technology ($\gamma^1 \uparrow$). Furthermore, improvements in output security ($\sigma \uparrow$) that reduce incentives to allocate resources to the conflict (and thus increase the aggregate quantity of butter produced) would enhance efficiency.

⁴³Note the direct effects of a change in γ^2 on U^{i*} ($i = 1, 2$) are channeled through their impact on ϕ^{i*} .

to induce the laggard to allocate more or less of its resources to the conflict. Nonetheless, since $\bar{U}^* = U^{1*} + U^{2*}$ and $d\bar{U}^*/d\gamma^2 < 0$ as shown earlier, the finding that $dU^{2*}/d\gamma^2 > 0$ readily implies $dU^{1*}/d\gamma^2 < 0$ for all $\gamma^2 \in (0, \gamma^1)$. Thus, the leader would always seek to undermine military-use technology transfers to the laggard.⁴⁴

5.2 Complementary Inputs and Diversification

Our analysis of general-purpose technology transfers unveiled a noteworthy insight: technologically advanced countries might refuse to share their superior know-how to prevent laggards from using it for predatory purposes. However, the linear dependence of the countries' payoffs on butter in our baseline model implies that this refusal arises only when laggards are pure predators, because in this case they direct any improvements in their general-purpose technology solely to predation. In this section, we argue that our model, though simple, captures the essence of the problem at hand and, more generally, that specialization in arming/predation is sufficient but not necessary for the validity of the insight.

To proceed, we modify the baseline model to allow for the presence of a fixed and complementary input in each country's production of butter, an input that gives rise to diminishing returns in the employment of the variable input, human capital, we had considered before. Specifically, we assume that the production function of butter in country i is given by $X^i = (H^i - G^i)^n$, where $n \in (0, 1]$ and $H^i = \alpha^i$ for $i = 1, 2$.⁴⁵ One can view n as the elasticity of butter with respect to human capital. The corresponding elasticity with respect to the complementary input (whose value is normalized to unity for simplicity and can thus be suppressed) equals $1 - n$. To keep the analysis simple and focused, we assume that output is perfectly insecure (i.e., $\sigma = 0$), so that country i 's payoff function can be written as: $U^i(G^i, G^j) = \phi^i \bar{X}^i$. Differentiation of U^i with respect to G^i gives:

$$U_{G^i}^i = \phi_{G^i}^i \left[(\alpha^i - G^i)^n + (\alpha^j - G^j)^n \right] - \phi^i n (\alpha^i - G^i)^{n-1}, \quad (3'')$$

where $\phi_{G^i}^i = \phi^i \phi^j / G^i$ for $i, j \in \{1, 2\}$, $i \neq j$. As in the baseline model (where $n = 1$), the first term shows the marginal benefit of arming (MB_G^i). This term is decreasing in G^i and increasing in α^i . The second term represents the marginal cost of arming (MC_G^i) and is increasing in G^i , with $\lim_{G^i \rightarrow 0} MC_G^i = 0$, again, as in the baseline model; but, in contrast to that model, MC_G^i depends on α^i and negatively so provided $n < 1$. However, there is another important difference here. When $n < 1$, $\lim_{G^i \rightarrow \alpha^i} MC_G^i = \infty$, implying each

⁴⁴We can also show that, while the laggard always benefits from security improvements, the leader's payoff could fall, particularly if the distance between their military technologies and the initial degree of security is sufficiently large.

⁴⁵Observe that our baseline model arises as a special case of this setup when $n = 1$. Also note that it is possible to extend the analysis to consider possible differences in the technologies for butter and guns. We abstract from these possibilities here to highlight the importance of differences in general-purpose technologies.

country i necessarily produces both arms and butter in equilibrium.

We can visualize the above points with the help of Fig. 5 that shows the marginal benefit and marginal cost for country 2 (the laggard). For comparison and contrast, panel (a) depicts MB_G^2 and MC_G^2 in the absence of diminishing returns (i.e., when $n = 1$) as functions of G^2 for several values in $\alpha^2 \in (0, \alpha^1)$ with a fixed value of G^1 . As we saw earlier, when the value of α^2 is sufficiently small ($= \alpha_I^2$ and α_{II}^2 in the figure), $MB_G^2 > MC_G^2$ for any feasible G^2 , implying that the resource constraint on the laggard's arming choice is binding. Allowing for diminishing returns (i.e., $n \in (0, 1)$), panel (b) illustrates the corresponding MB_G^2 and MC_G^2 functions for the same G^1 and the same values of α^2 . Notice especially, from panel (a) that, when $n = 1$, MC_G^2 is independent of α^2 ; by contrast, as shown in panel (b) when $n \in (0, 1)$, MC_G^2 rotates clockwise from the origin as α^2 rises, with MC_G^2 becoming infinitely large as G^2 approaches α^2 . As illustrated in panel (b), the laggard's optimal arming decision now arises at the points where $MB_G^2 = MC_G^2$ even for the smaller values of α^2 , thereby always ensuring the laggard's engagement in both predatory and productive activities.

In Appendix B.3, we establish that, for $n \in (0, 1)$, a unique equilibrium in the arming subgame exists, with $G^{i*} \in (0, \alpha^i)$ for $i = 1, 2$. As before, when $n \in (0, 1)$ an increase in α^2 generates both a positive direct payoff effect and an adverse strategic payoff effect for each side. Furthermore, numerical analysis reveals that, when $n < 1$ (as was the case when $n = 1$), the positive payoff effect always dominates for the laggard. By contrast, the net payoff effect for the technology leader depends on the initial value of α^2 .

To dig a little deeper, suppose that n is in the neighborhood of 1, so that the modified model is an approximation of the baseline model. Even though for very low values of α^2 the laggard will not specialize completely in arming and predation, its resource constraint on arming will nonetheless be very tight. As a consequence, a marginal increase in α^2 for low initial values of α^2 tends to induce the laggard to apply that increase primarily to predation. Accordingly, improvement generates a disproportionately large strategic effect relative to the direct effect on the leader's payoff, implying that $dU^{1*}/d\alpha^2 < 0$. By contrast, when α^2 is sufficiently large to start, the resource constraint on the laggard's arming is not very tight and the direct effect of α^2 on U^{1*} prevails over the indirect effect, such that $dU^{1*}/d\alpha^2 > 0$. Panel (c) of Fig. 5, which illustrates the dependence of U^{1*} on α^2 (and is obtained by solving the model numerically), confirms this finding for large values of n .⁴⁶ This discussion with the figure also supports the idea that our initial analysis of general-purpose technology transfers remains intact and that the leader will once again find such transfers unappealing when the technological distance between countries is sufficiently large

⁴⁶The figure suggests that, while this relationship is V-shaped in the baseline model (as noted earlier), it is U-shaped when n is sufficiently large but sufficiently less than 1.

even though the laggard is not resource constrained.

However, Fig. 5(c) also unveils another interesting (if not striking) result when the elasticity of butter production with respect to human capital n is sufficiently small. In particular, the leader’s payoff U^{1*} could fall with improvements in the laggard’s technology ($\alpha^2 \uparrow$) for all values of α^2 in $(0, \alpha^1)$, not just small values. The intuition here, developed more fully in Appendix B.3, is that, when n is smaller to make the degree of diminishing returns in human capital stronger, the laggard tends to employ more intensively any improvement in its general-purpose technology in guns production.⁴⁷ Thus, a reduction in n tends to amplify the adverse strategic payoff effect relative to the positive direct payoff for the leader, so that $dU^{1*}/d\alpha^2 < 0$ for initially large as well as small values of α^2 .

In summary, complete specialization in arming is unnecessary for the validity of our finding that the leader need not grant general-purpose technology transfers.⁴⁸ Perhaps more alarmingly though, in the presence of sufficiently salient complementary inputs in the production of the consumption good, the leader could find general-purpose technology transfers unappealing for all possible technological distances from the laggard.

6 Concluding Remarks

While it is widely believed that productivity improvements through the diffusion of technology represent a major driver of world economic growth, sanctions recently imposed by the US against the technology sectors of China and Russia, particularly those that can support the respective countries’ military strength, point to the possibility that countries might seek to limit such improvements. This paper develops a one-period, guns-versus-butter model with two countries, where output insecurity is the source of inefficiency; and, it is this inefficiency that could render a technology transfer undesirable to one of the two parties and thereby undermine its feasibility. To be more precise, our simple setting nests the striking benchmark case where output is perfectly secure, such that countries would not arm, and technology transfers would never be blocked. The presence of output insecurity motivates countries to arm, and each country’s arming choice depends on the technology held by both countries. Thus, technology transfers from the leader to the laggard generate not only a positive direct payoff effect, but also possibly an adverse strategic effect.

Our analysis characterizes qualitatively the total payoff effects for both countries, showing the importance of the security of output, the technological distance between countries, the laggard’s absorptive capacity and the type of technology considered. In the case of a general-purpose (dual-use) technology, a transfer always benefits the laggard but not nec-

⁴⁷Appendix B.3 provides a detailed analysis of how complementary inputs shape the effects of improvements in the general-purpose technology on the laggard’s payoff.

⁴⁸This finding also arises when: (i) we introduce risk aversion, (ii) consider a “winner-take-all” contest; and (iii) allow output insecurity to be incomplete (i.e., $\sigma \in (0, 1)$).

essarily the leader. In particular, when the technological distance is large initially and the degree of output security is sufficiently low, the laggard applies intensively any technology improvement to building its military strength. The resulting adverse strategic effect for the leader swamps the positive direct effect and thus reduces the leader's payoff.⁴⁹ By contrast, in the case of sector-specific (civilian-use) technology, the transfer always benefits the leader but not necessarily the laggard. In this case, it is the leader who responds more aggressively with its security policy, while the laggard tends to apply this technology more intensively in its production of butter. When the initial technological distance is sufficiently large and the degree of output security is small to moderate, the resulting adverse strategic payoff effect dominates the direct payoff effect for the laggard.

Regardless of which sort of technology we consider, these findings point to the possible emergence of a low-development or low-technology trap, wherein a sufficiently low initial level of technology for the laggard relative to the leader makes it more likely that a transfer would be blocked by one country—namely, by the leader in the case of the general-purpose technology and by the laggard in the case of the sector-specific technology. Our analysis shows that a country's incentive to block a proposed transfer derives from the effect of the transfer to induce more aggressive arming by the rival that undermines its own power.

Is it possible to alter the strategic interactions between countries in this conflict to eliminate the trap? Since the presence of insecurity is essential for this effect to materialize, it is natural to conjecture that improvements in security could make the emergence of a trap less likely. In the case of the general-purpose technology, it is true that such improvements expand the range of distances in initial technology for which a laggard would not be trapped. However, they also can reduce the laggard's payoff. Moreover, in the case of the sector-specific technology, improved security does not necessarily enhance the appeal of technology transfers to the laggard. A more promising approach would be to improve the laggard's absorptive capacity through investments by the laggard and/or assistance from the leader. In the case of the sector-specific technology, such improvements could eliminate the trap. But, matters differ in the case of the general-purpose technology. Specifically, even if the laggard could fully absorb the leader's superior technology, a trap could remain.⁵⁰ In a more fully articulated model allowing for differences in each type of technology, one could consider transfers of both types of technologies. But, here again, a low-technology trap would not be eliminated if the laggard's absorptive capacity is sufficiently limited.

One interesting and useful extension of our analysis would be to study the implications of

⁴⁹As shown in the previous section, although specialization in predation by the laggard is sufficient for this result, it is not necessary.

⁵⁰Furthermore as suggested by the extension in Section 5.2, this approach would not work if the degree of diminishing returns (with respect to residual human capital) in the production of butter were sufficiently strong.

ex-ante resource transfers for the diffusion of technology, arming and payoffs in the extended game. Focusing on sector-specific technologies, suppose that the degree of output security, the laggard’s absorptive capacity, and the technology gap between the two countries are such that the laggard is locked in a technology trap. As explained earlier, the trap in this case is due to the laggard’s refusal to accept the superior technology. For specificity, suppose the laggard’s initial technology falls short of the level implied by point *C* in Fig. 4(b) and that, if a technology transfer were made, the laggard could only absorb an amount that is marginally below the level associated with point *E* in the same figure. Now consider an *ex-ante* resource transfer from the leader to the laggard. One can show that such a transfer causes the laggard’s payoff function in Fig. 4(b) to rotate counterclockwise around a point whose location depends on the degree of output security.⁵¹ In the special case of a moderately high degree of output security, this rotation point lies southwest of point *E* in Fig. 4(b). Since after the transfer the laggard’s payoff at the technology level implied by point *E* rises above the payoff it would obtain under any technology below point *C*, the laggard will adopt the superior technology. Under additional parameter restrictions, the resultant diffusion of technology, in turn, improves the leader’s payoff relative to its pre-transfer payoff. In other words, it is possible for a resource transfer to improve both countries’ payoffs by changing the laggard’s preference over the superior technology. Extending the analysis along these lines can also shed light on the importance of asymmetries in countries’ initial resource endowments. In particular, an application of the logic of this argument in reverse shows that, under some circumstances, the smaller is the relative endowment base of a technologically backward country, the more likely it is that such a country will be locked in a technology trap.

The analysis could be extended in a number of other directions. Consider, for example, an extension with an additional country—say, the rest of the world (ROW)—that is not directly involved in the conflict but might have an interest in promoting the diffusion of the leader’s superior technology towards the laggard or perhaps a broader interest in limiting conflict between the two adversaries. Alternatively, ROW might be allied with one of the two adversaries. One could ask, depending on ROW’s objectives and constraints, how might

⁵¹The transfer under consideration induces both countries to reduce their guns—provided, of course, neither specializes in arming—because it reduces the value of the contested output, thus giving rise to a non-negative strategic effect on payoffs. For the leader, the direct and adverse effects of the transfer on the contested and secure parts of its payoff outweigh the just noted strategic effect, causing its payoff to fall. The effect of the transfer on the laggard’s payoff depends on the technological distance between the two countries. If this distance is large, the increase in the laggard’s secure output due to the transfer (which is small when β^2 small) and the potentially positive strategic effect noted above are overwhelmed by the reduction in the value of the contested prize (which is large when β^2 is small relative to β^1). By contrast, if the technological distance is small, the favorable effects described above prevail, causing the laggard’s payoff to rise. This explains the rotation in the laggard’s payoff discussed in the text. Importantly, a resource transfer from the leader to the laggard alone always reduces global efficiency.

it intervene and what effects its intervention would have. Of course, the answer would also depend on the structure of technology across the three countries. But, with the appropriate modifications, the model could be used to understand why a third (friendly) country might provide assistance to one of the adversaries and the form that this assistance takes—for example, a transfer of resources or technology.

One could also extend the model to consider the importance of international trade (e.g., in intermediate inputs) between the two adversaries. Two distinct questions emerge in such contexts. First, for any given difference in technologies, does trade benefit both sides? In standard settings—where, there is typically no arming or arms are kept fixed at predefined levels—the answer to this question is a resounding yes. Because arming is endogenous in our setting, however, this answer could be incorrect. Indeed, Garfinkel et al. (2022) have shown that, when countries can direct their (income) gains from trade into productive and predatory investments, sufficiently affluent countries find trade unappealing.⁵² But trade (and possibly trade agreements) could take place after countries have made their arming decisions. This alternative timing of trade relative to arming choices raises a host of new possibilities. Our preliminary analysis of such a setting reveals increased economic interdependence that manifests itself in higher *ex-post* gains from trade could induce technologically advanced countries to pursue their security interests more aggressively relative to their rivals. Thus, increased interdependence could prove disadvantageous to technologically backward countries. In sum, the sequence of countries’ arming and trading decisions and the way countries use their gains from trade—which are especially relevant in dynamic environments—matter.

The second question is this: How does trade that occurs after countries have armed affect the two countries’ attitudes toward technology transfers? Perhaps unsurprisingly, our preliminary analysis suggests that—in addition to the degree of insecurity, the type of technology considered, and the technological distance between countries—the magnitude of their gains from trade plays a prominent role in this context. Interestingly, even though trade can amplify countries’ incentives to arm, trade can also enhance the appeal of technology transfers to both sides even more.⁵³ We plan to pursue these questions more vigorously in future research.

⁵²The finding is driven by the tendency of relatively poorer countries to channel more of their relatively larger *ex-ante* gains from trade into arming as compared with their rivals.

⁵³This appears to happen in models when the trade elasticity (Arkolakis et al., 2012) is sufficiently low, so that the gains from trade are large.

References

- Acemoglu, Daron, Simon Johnson and James A. Robinson (2001). “The Colonial Origins of Comparative Development: An Empirical Investigation,” *American Economic Review* 91(5), 1369–1401.
- Anbarci, Nejat, Stergios Skaperdas and Constantinos Syropoulos (2002). “Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects,” *Journal of Economic Theory* 106(1), 1–16.
- Arkolakis, Costas, Arnaud Costinot and André Rodríguez-Clare (2012). “New Trade Models, Same Old Gains?” *American Economic Review* 102(1), 94–130.
- Anderson, James E., Mario Larch and Yoto V. Yotov (2019). “Trade and Investment in the Global Economy: A Multi-Country Dynamic Analysis,” *European Economic Review* 120, 103311.
- Atkin, David, Azam Chaudhry, Shamyra Chaudry, Amit Khandelwal, and Eric Verhoogen (2017). “Organizational Barriers to Technology Adoption: Evidence from Soccer-Ball Producers in Pakistan,” *Quarterly Journal of Economics* 132(3), 1101–1164.
- Azariadis, Costas and John Stachurski (2005). “Poverty Traps,” in Philippe Aghion and Steven Durlauf (eds.), *Handbook of Economic Growth* 1(A), Netherlands: Elsevier, 295–384.
- Beviá, Carmen and Luis Corchón (2010). “Peace Agreements without Commitment,” *Games and Economic Behavior* 68(2), 469–487.
- Bhagwati, Jagdish (1958). “Immiserizing Growth: A Geometrical Note,” *Review of Economic Studies* 25(3), 201–205.
- Caselli, Francesco and Daniel J. Wilson (2004). “Importing Technology,” *Journal of Monetary Economics* 51(1), 1–32.
- Cohen, Wesley M. and Daniel A. Levinthal (1989). “Innovation and Learning: The Two Faces of R&D,” *Economic Journal* 99(397), pp. 569–596.
- Cohen, Wesley M. and Daniel A. Levinthal (1990). “Absorptive Capacity: A New Perspective on Learning and Innovation,” *Administrative Science Quarterly* 35(1), pp. 128–152.
- Dinopoulos, Elias and Constantinos Syropoulos (1998). “International Diffusion and Appropriability of Technological Expertise,” in M.R. Baye (ed.), *Volume 7: Advances in Applied Microeconomics*, Stamford CT: JAI Press, pp. 115–137.
- Dinopoulos, Elias and Constantinos Syropoulos (2007). “Rent Protection as a Barrier to Innovation and Growth,” *Economic Theory* 32(2), 309–332.

- Desmet, Klaus and Stephen Parente (2014). “Resistance to Technology Adoption: The Rise and Decline of Guilds,” *Review of Economic Dynamics* 17(4), 437–458.
- Garfinkel, Michelle R. and Constantinos Syropoulos, (2021). “Self-Enforcing Peace Agreements that Preserve the Status Quo,” *Games and Economic Behavior* 130, 148–178.
- Garfinkel, Michelle R. and Constantinos Syropoulos (2020). “Inequality and Conflict: Burning Resources to Support Peace,” *Economics Letters* 197, 109601.
- Garfinkel, Michelle R., Constantinos Syropoulos and Thomas Zylkin (2022). “Prudence versus Predation and the Gains from Trade,” *Journal of Economic Theory* 201, 105434.
- Gonzalez, Francisco M. (2012). “The Use of Coercion in Society: Insecure Property Rights, Conflict, and Economic Backwardness,” in Michelle R. Garfinkel, and Stergios Skaperdas (eds.), *The Oxford Handbook of the Economics of Peace and Conflict*, NY: Oxford University Press, 633–675.
- Gonzalez, Francisco M. (2005). “Insecure Property and Technological Backwardness,” *Economic Journal* 115(505), 703–721.
- Hall, Robert E. and Charles I. Jones (1999). “Why Do Some Countries Produce So Much More Output Per Worker Than Others?” *Quarterly Journal of Economics* 114(1), 83–116.
- Hirshleifer, Jack (1991). “The Paradox of Power,” *Economics & Politics* 3(3), 177–200.
- Johnson, Harry (1967). “The Possibility of Income Losses from Increased Efficiency or Factor Accumulation in the Presence of Tariffs,” *Economic Journal* 77(305), 151–154.
- Jones, Ronald and Roy J. Ruffin (2008). “The Technology Transfer Paradox,” *Journal of International Economics*, 75(2), 321–328.
- Keller, Wolfgang (2004). “International Technology Diffusion,” *Journal of Economic Literature* 42(3), 752–782.
- Krusell, Per and José-Víctor Ríos-Rull (1996). “Vested Interests in a Positive Theory of Stagnation and Growth,” *The Review of Economic Studies* 63, 301–329.
- Milner, Helen V. and Sondre Ulvund Solstad (2021). “Technological Change and the International System,” *World Politics* 73(3), 545–589.
- Olson, Stephen (2019). “Trade Tensions Rise on Dual Use Technologies, Protectionism and Geopolitics,” *Heinrich Foundation*. <https://www.heinrichfoundation.com/research/article/tech/dual-use-technology-trade-tensions/> (October 30).

- Parente, Stephen L. and Edward C. Prescott (1994). “Barriers to Technology Adoption and Development,” *Journal of Political Economy* 102(21), 298–321.
- Parente, Stephen L. and Edward C. Prescott (1999). “Monopoly Rights: A Barrier to Riches?,” *American Economic Review* 89(5), 1216–1233.
- Perla, Jesse, Christopher Tonetti and Michael E. Waugh (2021). “Equilibrium Technology Diffusion, Trade, and Growth,” *American Economic Review* 111(1), 73–128.
- Rodrik, Dani, Arvind Subramanian and Francesco Trebbi (2004). “Institutions Rule: The Primacy of Institutions Over Geography and Integration in Economic Development,” *Journal of Economic Growth* 9, 131–165.
- Samuelson, Paul, A. (2004). “Where Ricardo and Mill Rebut and Confirm Arguments of Mainstream Economists Supporting Globalization,” *Journal of Economic Perspectives* 18(3), 135–146.
- Skaperdas, Stergios (1992). “Cooperation, Conflict, and Power in the Absence of Property Rights,” *American Economic Review* 82(4), 720–739.
- Skaperdas, Stergios (1996). “Contest Success Functions,” *Economic Theory* 7, 283–290.
- Skaperdas, Stergios and Constantinos Syropoulos (1997). “The Distribution of Income in the Presence of Appropriative Activities,” *Economica* 64, 101–117.
- Sykes, Alan O. (2021). “The Law and Economics of ‘Forced’ Technology Transfer and Its Implications for Trade and Investment Policy (and the U.S.-China Trade War),” *Journal of Legal Analysis* 13(1), 127–171.

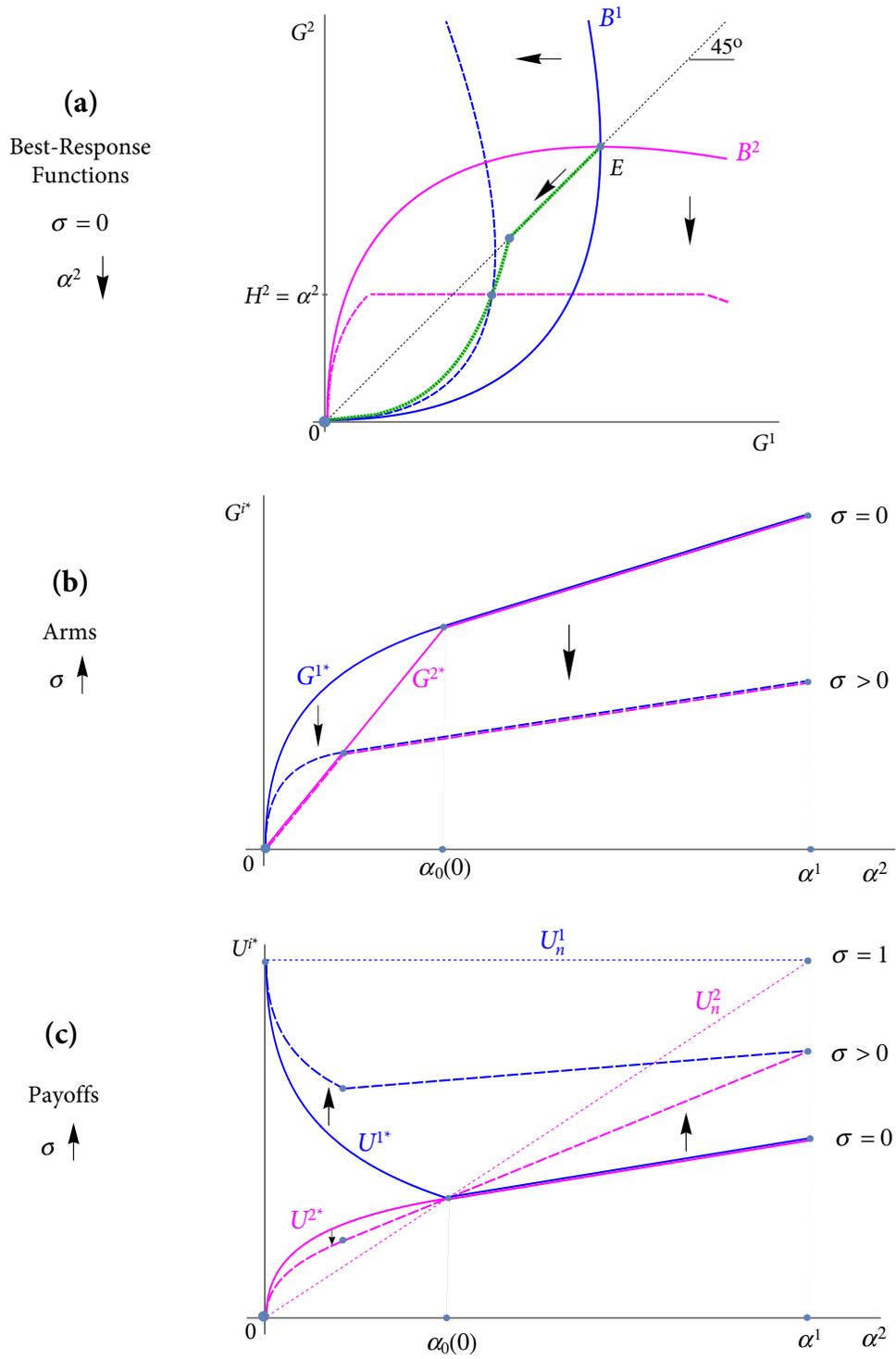


Figure 1: The Importance of General-Purpose Technology Differences and Security for Arming and Payoffs

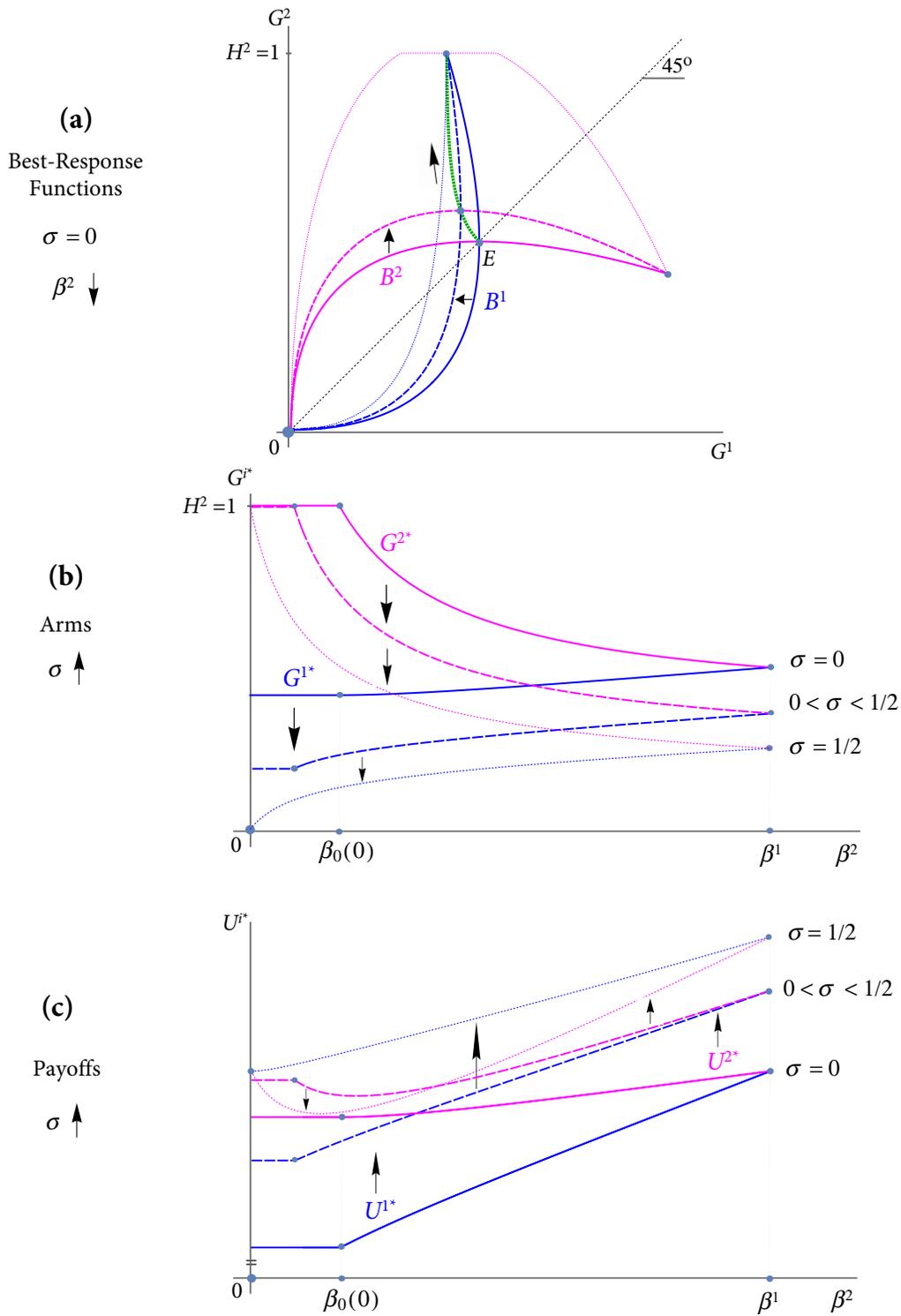


Figure 2: The Importance of Sector-Specific Technology Differences and Security for Arming and Payoffs

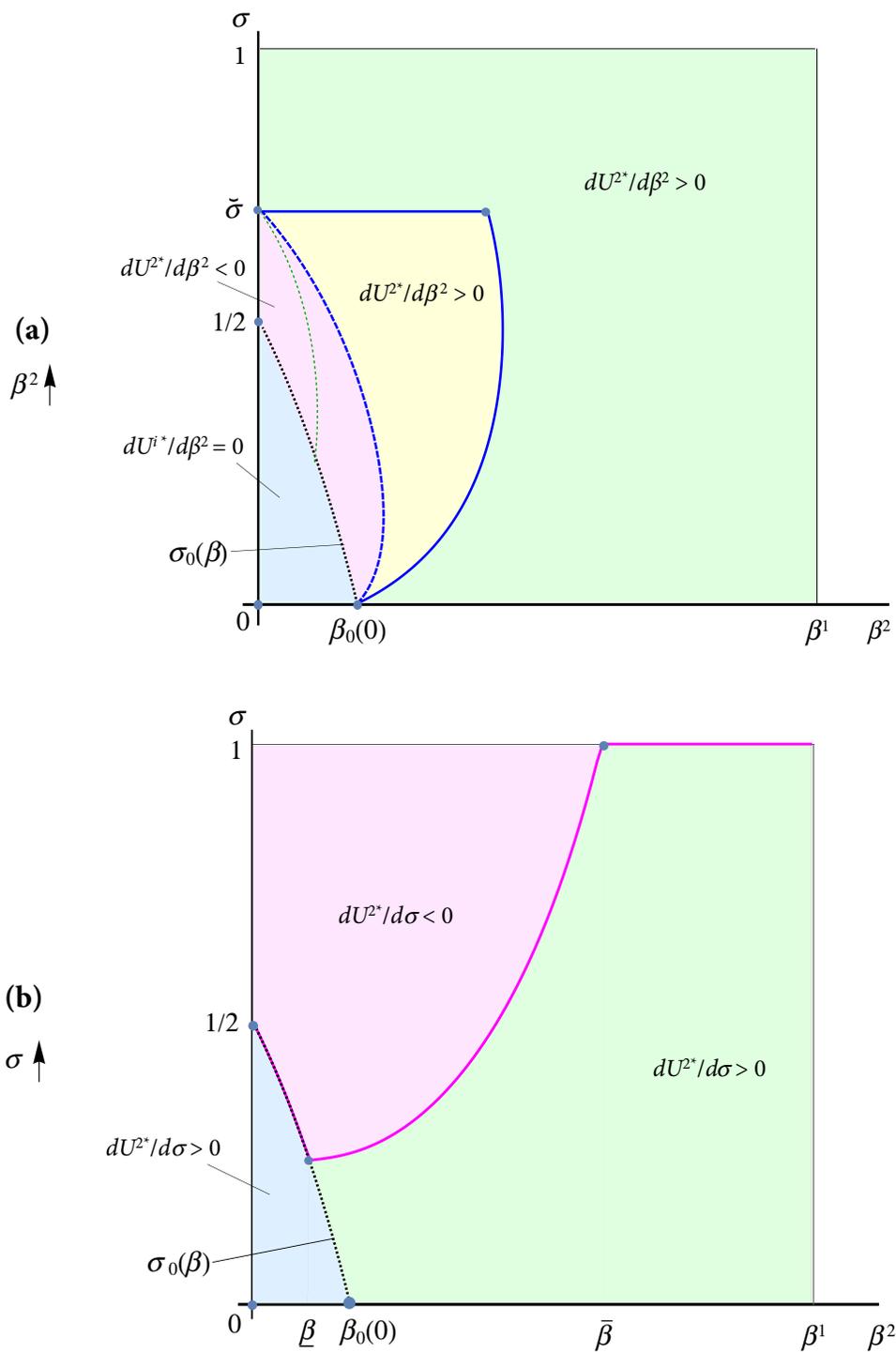


Figure 3: The Effects of Changes in Sector-Specific Technology and Security on the Laggard's Payoff

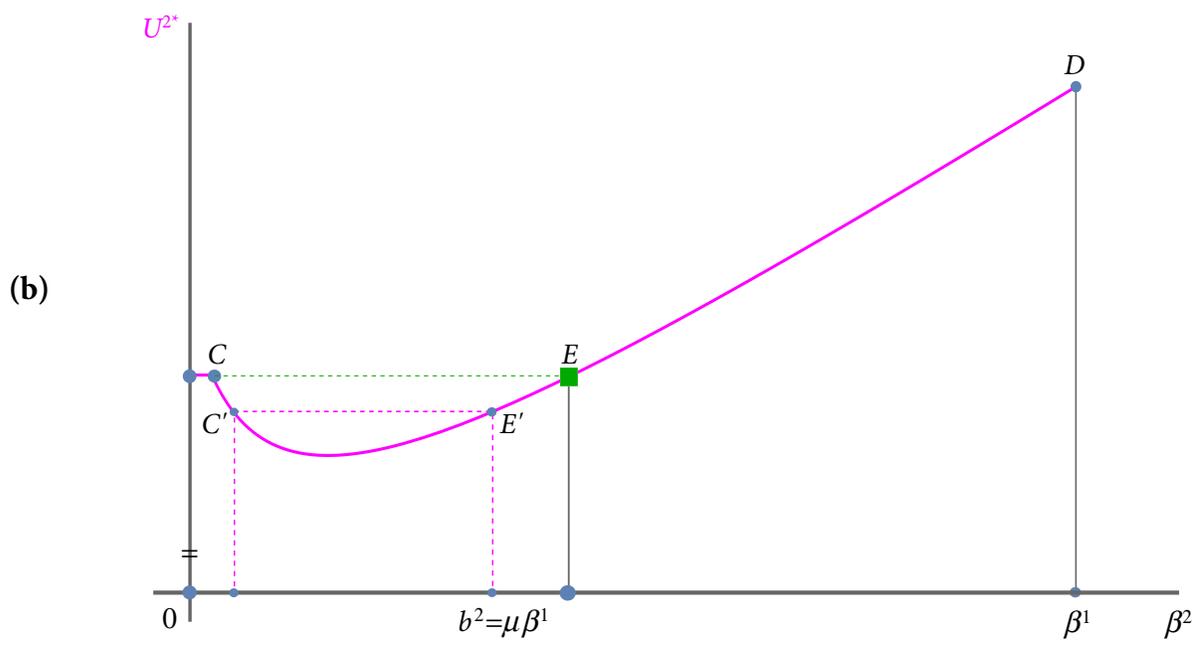
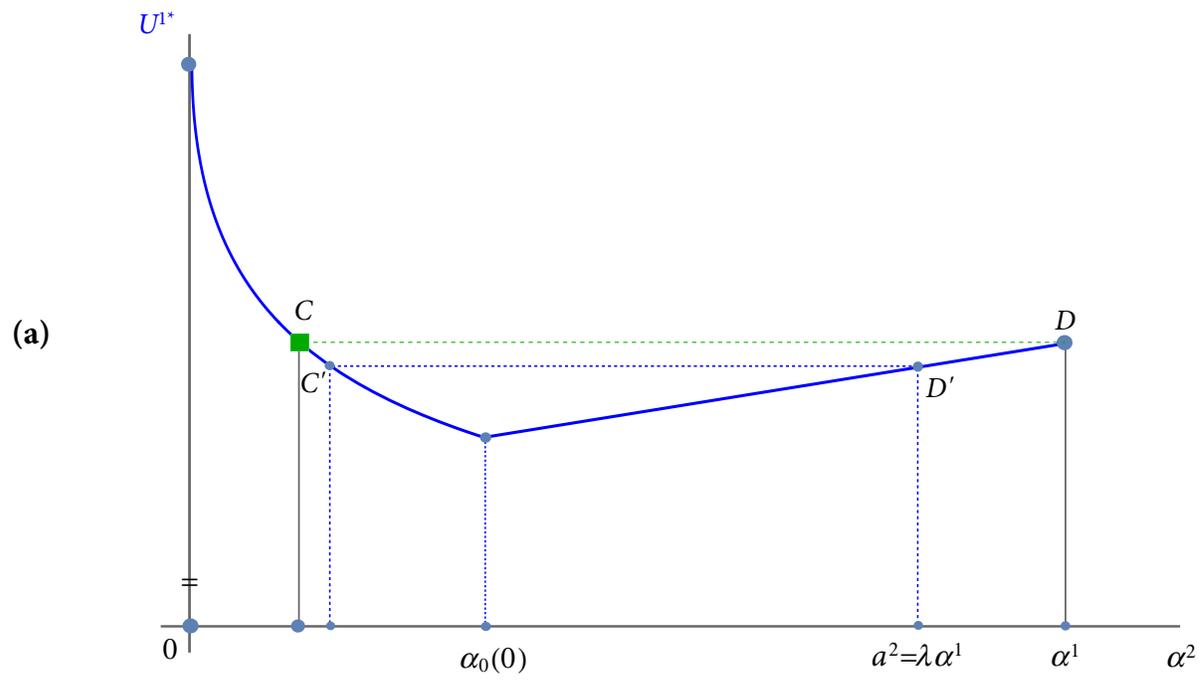


Figure 4: Payoff Effects of Transfers of General-Purpose and Sector-Specific Technologies

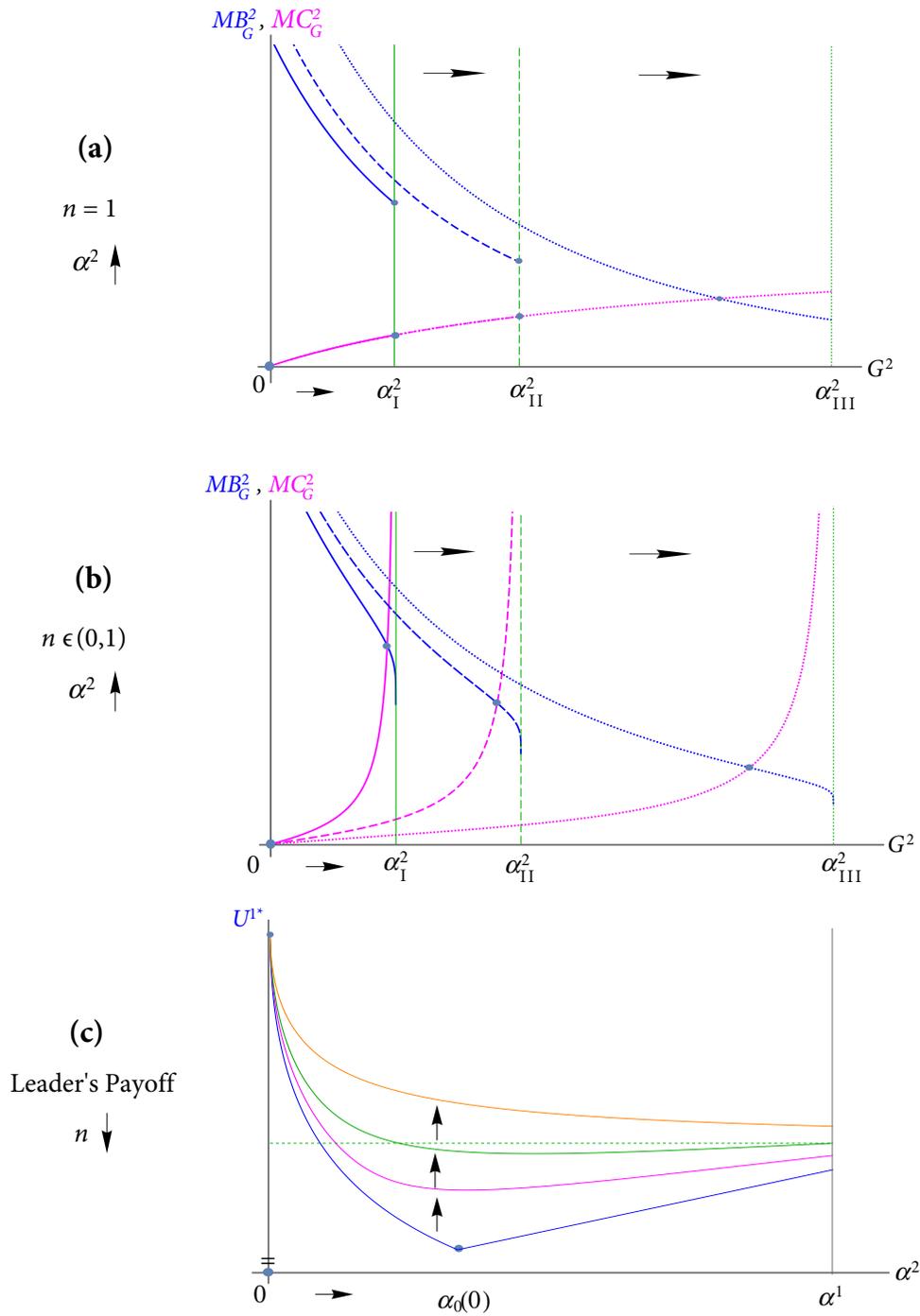


Figure 5: Complementary Inputs and the Payoff Effects of Transfers of the General-Purpose Technology

A Appendix

This appendix presents proofs of the propositions in the main text. Additional technical details, including those related to the extension on differences in military technologies across countries considered in Section 5.1 and those related to the presence of complementary inputs and diversification in Section 5.2, can be found in Appendix B.

Proof of Proposition 1. This proposition follows from the analysis in the text. $\quad ||$

For greater insight, let us describe some important properties of the unconstrained best-response function $\tilde{B}^i(G^j)$ in (4b). First, notice that $\tilde{B}^i(G^j)$ approaches 0 as $G^j \rightarrow 0$. Next, observe that the expression for $\tilde{B}^i(G^j)$ in (4b) equals zero for some value of the rival's arming choice G^j . That value, which we denote by G_0^j , is given by

$$G_0^j \equiv \frac{(1-\sigma)(\beta^i H^i + \beta^j H^j)}{\beta^i \sigma + \beta^j (1-\sigma)}, \quad i, j \in \{1, 2\}, i \neq j. \quad (\text{A.1})$$

Thus, we have $\lim_{G^j \rightarrow 0} \tilde{B}^i(G^j) = 0$, $\tilde{B}^i(G^j) > 0$ for $G^j \in (0, G_0^j)$, and $\tilde{B}^i(G^j) = 0$ for $G^j \geq G_0^j$. Additionally, one can confirm that $\tilde{B}^i(G^j)$ is strictly concave in its argument G^j for all $G^j \in (0, G_0^j]$, reaching a unique maximum at a value of the rival's arming $G_{\max}^j \in (0, G_0^j)$:

$$G_{\max}^j = \begin{cases} \left(\frac{\beta^i H^i + \beta^j H^j}{2(\beta^i - \beta^j)} \right) \left(\sqrt{\frac{\beta^i}{\sigma \beta^i + (1-\sigma)\beta^j}} - 1 \right) & \text{if } \beta^j \neq \beta^i \\ \frac{1}{4}(1-\sigma)\bar{H} & \text{if } \beta^j = \beta^i \end{cases}$$

such that

$$\tilde{B}^i(G_{\max}^j) = \begin{cases} \left(\frac{\beta^i H^i + \beta^j H^j}{2(\beta^i - \beta^j)} \right) \left(1 - \sqrt{\frac{\sigma \beta^i + (1-\sigma)\beta^j}{\beta^i}} \right) & \text{if } \beta^j \neq \beta^i \\ \frac{1}{4}(1-\sigma)\bar{H} & \text{if } \beta^j = \beta^i, \end{cases}$$

for $i, j \in \{1, 2\}$, $i \neq j$, where $H^i = \alpha^i R^i$ and $\bar{H} = H^1 + H^2$. Finally, country i 's unconstrained best-response function crosses the 45° degree line at a unique point, labeled $G_s^i (< G_0^j)$, which is given by

$$G_s^i \equiv \frac{(1-\sigma)(\beta^i H^i + \beta^j H^j)}{(3+\sigma)\beta^i + (1-\sigma)\beta^j}, \quad i, j \in \{1, 2\}, i \neq j.$$

One can easily confirm that $\text{sign}\{\beta^i - \beta^j\}$ determines the sign of the slope of country i 's unconstrained best-response function, when evaluated at $G^i = G^j = G_s^i$.⁵⁴ Accordingly, our assumption that $\beta^2 \leq \beta^1$ implies $\partial \tilde{B}^1/dG^2|_{G^2=G^1} \geq 0$ and $d\tilde{B}^2/dG^1|_{G^1=G^2} \leq 0$.⁵⁵ It

⁵⁴To see this result, note that the slope of $\tilde{B}^i(G^j)$ generally equals $-U_{G^i G^j}^i / U_{G^i G^i}^i$. Since $U_{G^i G^i}^i < 0$ holds as noted above, the sign of $d\tilde{B}^i/dG^j$ is determined by the sign of $U_{G^i G^j}^i$; when evaluated at $G^i = G^j = G_s^i$, $U_{G^i G^j}^i = (1-\sigma)(\beta^i - \beta^j)/4G_s^i$.

⁵⁵Below, we also show that if (i) countries differ only in their butter productivities, and (ii) G^i is a strate-

also implies that $G_s^1 \leq G_s^2$ holds regardless of the ranking of the countries' effective resource endowments.

From this analysis, we can conclude that $\tilde{B}^1(G^2)$ and $\tilde{B}^2(G^1)$ will intersect at a unique interior point: $G^{1*} = \tilde{G}^1$ and $G^{2*} = \tilde{G}^2$, shown in (5). However, as argued in the main text, it is possible that the technology laggard ($i = 2$) is resource constrained, implying that the unique equilibrium is where $G^{2*} = H^2$ and $G^{1*} = \tilde{B}^1(H^2)$.

A.1 Differences in the General-Purpose Technology

For the proofs of Propositions 2 and 3 to follow, we study the effects of changes in $\theta \in \{\alpha^2, \sigma\}$, distinguishing between the two cases depending on whether the laggard ($i = 2$) is resource constrained in its arming choice and thus specializes in appropriation or not. This partition of the parameter space is determined by the value of α^2 in relation to the threshold derived in the main text, $\alpha_0(\sigma) \equiv \frac{1-\sigma}{3+\sigma}\alpha^1$ for $\sigma \in [0, 1)$:

Case 1: If $\alpha^2 \in (0, \alpha_0(\sigma))$, the laggard country ($i = 2$) is resource constrained, in which case $G^{2*} = \alpha^2$ and $G^{1*} = B^1(\alpha^2) = -\alpha^2 + \sqrt{(1-\sigma)\alpha^2(\alpha^1 + \alpha^2)}$.

Case 2: If $\alpha^2 \in [\alpha_0(\sigma), \alpha^1]$, neither country is resource constrained. Then, $G^{1*} = G^{2*} = \tilde{G}(\alpha^2, \sigma) = \frac{1}{4}(1-\sigma)(\alpha^1 + \alpha^2)$.

For future reference, observe that $\alpha_0(\sigma)$ for $\sigma \in [0, 1)$ is equivalent to

$$\sigma \underset{\geq}{\leq} \sigma_0(\alpha^2) \equiv \max \left\{ \frac{3}{\alpha^1 + \alpha^2} (\alpha_0(0) - \alpha^2), 0 \right\}$$

where $\alpha_0(0) = \alpha^1/3$. Accordingly, case 1 arises when $\sigma < \sigma_0(\alpha^2)$ for $\alpha^2 < \alpha_0(0)$, and case 2 arises when $\sigma \geq \sigma_0(\alpha^2)$ for $\alpha^2 \in (0, \alpha^1]$.

Proof of Proposition 2. Let us define $m^2 \equiv \frac{\alpha^2}{\alpha^1 + \alpha^2}$, and note that $\alpha^2 < \alpha_0(\sigma)$ implies $m^2 < \frac{1}{4}(1-\sigma)$. With that relationship and recalling that $\alpha'_0 < 0$ where $\alpha_0(0) = \alpha^1/3$ and $\alpha_0(1) = 0$, we proceed to consider the two cases just described:

Case 1, $\theta = \alpha^2$: Using the definition of m^2 , we can rewrite G^{1*} shown above in this case as

$$G^{1*} = \alpha^2 \left(-1 + \sqrt{\frac{1-\sigma}{m^2}} \right). \quad (\text{A.3})$$

When combined with the arming solution for country 2 ($G^{2*} = \alpha^2$), (A.3) implies

$$\frac{G^{1*}}{G^{2*}} = -1 + \sqrt{\frac{1-\sigma}{m^2}} > 1, \quad (\text{A.4})$$

gic substitute (complement) for G^j in the noncooperative equilibrium, then G^j is a strategic complement (substitute) for G^i .

where the inequality follows from the requirement in this case, $\alpha^2 < \alpha_0(\sigma)$, that implies $m^2 < \frac{1}{4}(1 - \sigma)$. Thus, by the conflict technology in (1), the leader is the more powerful country when the laggard specializes in appropriation.

Next, we differentiate the expression for G^{1*} in (A.3) with respect to α^2 , while taking into account the definition of $m^2 \equiv \frac{\alpha^2}{\alpha^1 + \alpha^2}$, to obtain

$$\frac{dG^{1*}}{d\alpha^2} = -1 + \frac{1 + m^2}{2} \sqrt{\frac{1 - \sigma}{m^2}}. \quad (\text{A.5})$$

One can verify that $d^2G^{1*}/(d\alpha^2)^2 < 0$, such that $dG^{1*}/d\alpha^2$ attains a minimum at $\alpha^2 = \alpha_0$. Since $dG^{1*}/d\alpha^2|_{\alpha^2=\alpha_0(\sigma)} = \frac{1}{4}(1 - \sigma) > 0$, $dG^{1*}/d\alpha^2 > 0$ for all $\alpha^2 \in (0, \alpha_0(\sigma)]$. Furthermore, it follows from (A.4) that, since m^2 is increasing in α^2 , G^{1*}/G^{2*} is decreasing in α^2 . It then follows from (1) that the leader's power ϕ^{1*} is also decreasing in α^2 .

Case 2, $\theta = \alpha^2$: In this case, the two countries arm identically $G^{1*} = G^{2*} = \tilde{G}$, meaning that they share power equally. In addition, from the solution for \tilde{G} shown above, we have $dG^{1*}/d\alpha^2 = dG^{2*}/d\alpha^2 = d\tilde{G}/d\alpha^2 = \frac{1}{4}(1 - \sigma) > 0$, such that increases in α^2 induce more arming, while leaving the balance of power unchanged, as needed.

Cases 1 and 2, $\theta = \sigma$: Finally, we characterize the dependence of G^{i*} on output security σ in the two cases. In case 1, where G^{1*} satisfies (A.3) and $G^{2*} = \alpha^2$, it follows immediately that $dG^{1*}/d\sigma < 0$ and $dG^{2*}/d\sigma = 0$, which imply from (1) that security improvements erode the leader's power advantage. In case 2, because $G^{1*} = G^{2*} = \tilde{G}$ shown in (8b), we have $dG^{i*}/d\sigma = d\tilde{G}/d\sigma = -\frac{1}{4}(\alpha^1 + \alpha^2) < 0$ for $i = 1, 2$, implying less arming by both countries with no change in the balance of power and thereby completing the proof. \parallel

Proof of Proposition 3. Again, distinguishing between cases 1 and 2, we now use equations (6) and (7) to calculate the total payoff effects of an increase in $\theta \in \{\alpha^2, \sigma\}$ for the leader ($i = 1$) and laggard ($i = 2$) respectively.

Case 1, $\theta = \alpha^2$: Starting with the resource-constrained country (i.e., the technology laggard, $i = 2$), we proceed to fill in the three components of (7) for $\theta = \alpha^2$. Note first that (11a) shows $U_{\alpha^2}^{2*} > 0$, the direct effect of an increase in laggard's general-purpose technology on its own payoff. Furthermore, we can use (11b) with our assumptions that imply $\bar{X}^* = X^{1*} + X^{2*} = X^{1*} = (\alpha^1 - G^{1*})$ and the specification for conflict in (1) that implies $\phi_{G^1}^2 = -\phi^1\phi^2/G^1$ to find the indirect payoff effect that works through the leader's arming response:

$$U_{G^1}^{2*} \frac{dG^{1*}}{d\alpha^2} = - \left[(1 - \sigma) \phi^{1*} \phi^{2*} \frac{G^{1*}}{\alpha^1 - G^{1*}} + (1 - \sigma) \phi^{2*} \right] \frac{dG^{1*}}{d\alpha^2} \quad (\text{A.6a})$$

Next, we use (3) for $i = 2$ with the conflict technology in (1) that implies $\phi_{G^2}^2 = -\phi^1\phi^2/G^2$ to identify the indirect payoff effect that arises as the (resource-constrained) laggard adjusts

its own guns according to $dG^{2*}/d\alpha^2 = 1$:

$$U_{G^2}^{2*} \frac{dG^{2*}}{d\alpha^2} = (1 - \sigma) \phi^{1*} \phi^{2*} \frac{(\alpha^1 - G^{1*})}{\alpha^2} - [\sigma + (1 - \sigma) \phi^{2*}]. \quad (\text{A.6b})$$

Since the leader is not resource constrained, $U_{G^1}^{1*} = 0$ always hold. Using (3) for $i = 1$ shows

$$U_{G^1}^{1*} = (1 - \sigma) \phi^{1*} \phi^{2*} \frac{(\alpha^1 - G^{1*})}{G^{1*}} - [\sigma + (1 - \sigma) \phi^{1*}] = 0,$$

which we apply to simplify (A.6a) and (A.6b) respectively as

$$U_{G^1}^{2*} \frac{dG^{1*}}{d\alpha^2} = -\frac{dG^{1*}}{d\alpha^2} \quad \text{and} \quad U_{G^2}^{2*} \frac{dG^{2*}}{d\alpha^2} = [\sigma + (1 - \sigma) \phi^{2*}] \frac{G^{1*}}{\alpha^2} - [\sigma + (1 - \sigma) \phi^{2*}].$$

Then, adding the expressions above with that for $U_{\alpha^2}^{2*}$ in (11a) yields:

$$\frac{dU^{2*}}{d\alpha^2} = U_{\alpha^2}^{2*} + U_{G^1}^{2*} \frac{dG^{1*}}{d\alpha^2} + U_{G^2}^{2*} \frac{dG^{2*}}{d\alpha^2} = [\sigma + (1 - \sigma) \phi^{1*}] \frac{G^{1*}}{\alpha^2} - \frac{dG^{1*}}{d\alpha^2}.$$

We now substitute G^{1*} shown in (A.3) and $dG^{1*}/d\alpha^2$ shown in (A.5) into the expression above, while using (1) that implies $\phi^{1*} = 1 - \sqrt{m^2/(1 - \sigma)}$ in this case, to find

$$\frac{dU^{2*}}{d\alpha^2} = (1 - \sigma) \left[\frac{1 + m^2}{2\sqrt{(1 - \sigma)m^2}} - 1 \right]. \quad (\text{A.7})$$

To see that $dU^{2*}/d\alpha^2 > 0$ holds for $\alpha^2 \in (0, \alpha_0(\sigma)]$, observe that $d^2U^{2*}/(d\alpha^2)^2 < 0$, which implies that $dU^{2*}/d\alpha^2$ attains a minimum at $\alpha^2 = \alpha_0(\sigma)$. But, since m^2 evaluated at $\alpha^2 = \alpha_0(\sigma)$ equals $\frac{1}{4}(1 - \sigma)$, we have $\lim_{\alpha^2 \rightarrow \alpha_0(\sigma)} (dU^{2*}/d\alpha^2) = \frac{1}{4}(1 + 3\sigma) > 0$. It follows that $dU^{2*}/d\alpha^2 > 0$ for all $\alpha^2 \in (0, \alpha_0(\sigma)]$ and $\sigma \in [0, 1)$.

Turning to the technology leader, we apply the same logic as above but based on (6) for $\theta = \alpha^2$. Using (10a) and (10b) with $dG^{2*}/d\alpha^2 = 1$ gives

$$\frac{dU^{1*}}{d\alpha^2} = U_{\alpha^2}^{1*} + U_{G^2}^{1*} \frac{dG^{2*}}{d\alpha^2} = -[\sigma + (1 - \sigma) \phi^{1*}] \frac{G^{1*}}{\alpha^2} < 0, \quad (\text{A.8})$$

as claimed in the proposition.

Case 2, $\theta = \alpha^2$: Recalling that $G^{i*} = \tilde{G} = \frac{1}{4}(1 - \sigma)(\alpha^1 + \alpha^2)$ for $i = 1, 2$ such that $\phi^{i*} = \frac{1}{2}$ for $i = 1, 2$ when neither country is resource constrained, the easiest way to establish this part of the proposition is to substitute these values in $U^i(G^i, G^j)$ for $i, j \in \{1, 2\}$, $i \neq j$ shown in (2) with $X^{i*} = \alpha^i - \tilde{G}$; by differentiating the resulting expressions with respect to α^2 , one can confirm that $dU^{1*}/d\alpha^2 = \frac{1}{4}(1 - \sigma) > 0$ and $dU^{2*}/d\alpha^2 = \frac{1}{4}(1 + 3\sigma) > 0$.

Case 1, $\theta = \sigma$: Let us start with the technology leader. We know that $\alpha^2 < \alpha_0(\sigma)$ means that $G^{2*} = \alpha^2$, which implies $dG^{2*}/d\sigma = 0$. With no strategic effect for the leader, the

second term in (6) vanishes, leaving only the direct effect U_{σ}^{1*} , which from (12) for $i = 1$, simplifies as:

$$\frac{dU^{1*}}{d\sigma} = (\alpha^1 - G^{1*}) (1 - \phi^{1*}) > 0. \quad (\text{A.9})$$

Turning to the laggard, the result that $dG^{2*}/d\sigma = 0$ also implies that the indirect payoff effect for the laggard through adjustments in its own arming equals zero as well. Thus, the third term in (7) vanishes, leaving only the first two terms. Using (12) for $i = 2$ and (11b) with the FOC for the leader $U_{G^1}^1 = 0$, we can write the total payoff effect for the laggard from an increase in σ as:

$$\frac{dU^{2*}}{d\sigma} = U_{\sigma}^{2*} + U_{G^1}^{2*} \frac{dG^{1*}}{d\sigma} = -(\alpha^1 - G^{1*}) \phi^{2*} - \frac{dG^{1*}}{d\sigma}.$$

To sign that expression, we use the solution for G^{1*} in (A.3), to find

$$\frac{dG^{1*}}{d\sigma} = -\frac{\alpha^2}{2\sqrt{m^2(1-\sigma)}} < 0.$$

Now observe that the solutions for G^{i*} in this case, as shown above, with (1) imply $1 - \phi^{1*} = \phi^{2*} = \sqrt{m^2/(1-\sigma)}$. Combining (A.3) with the two expressions above and this result, after simplifying, yields:

$$\frac{dU^{2*}}{d\sigma} = \frac{\alpha^2}{\sqrt{m^2(1-\sigma)}} \left[\sqrt{m^2(1-\sigma)} - \frac{1}{2} \right]. \quad (\text{A.10})$$

Since $m^2 < \frac{1}{4}(1-\sigma)$ when $\alpha^2 < \alpha_0(\sigma)$, the expression inside the brackets in (A.10) is necessarily negative, implying that $dU^{2*}/d\sigma < 0$ holds in this case as needed.

Case 2, $\theta = \sigma$: The most straightforward way to study this case is to apply the assumptions that $H^i = \alpha^i$ and $\beta^i = 1$ for $i = 1, 2$ with our previous finding that $G^{i*} = \tilde{G} = \frac{1}{4}(1-\sigma)(\alpha^1 + \alpha^2)$ for $i = 1, 2$ to $U^i(G^i, G^2)$ shown in (2). Then, differentiating the resulting expressions with respect to σ gives $dU^{1*}/d\sigma = \frac{1}{4}[3\alpha^1 - \alpha^2] > 0$ and $dU^{2*}/d\sigma = \frac{3}{4}[\alpha^2 - \alpha_0(0)] > 0$, thereby completing the proof. $\quad ||$

A.2 Differences in the Sector-Specific Technology

First, we derive the $\beta_0(\sigma)$ schedule in (14). To do so, let us define

$$\Lambda \equiv 4\beta^i\beta^j + (\beta^i - \beta^j)^2\sigma^2 > 0. \quad (\text{A.11})$$

Then, using the expression for \tilde{G}^{2*} in (13b), we can write the condition that must be satisfied for the laggard to be constrained $\tilde{G}^2 \geq 1$ as

$$\frac{2(1-\sigma)\beta^1(\beta^1+\beta^2)}{\Lambda\left[1+\frac{\beta^1\sigma+\beta^2(2-\sigma)}{\sqrt{\Lambda}}\right]}-1\geq 0,$$

which implies

$$[2(1-\sigma)\beta^1(\beta^1+\beta^2)-\Lambda]^2-\Lambda[\beta^1\sigma+\beta^2(2-\sigma)]^2\geq 0.$$

Then, using the expression for Λ allows us, after some tedious algebra, to rewrite the inequality above as

$$8[\beta^1(\beta^1-\beta^2)]^2(1-\sigma^2)\left[\frac{1}{2}-\frac{2\beta^1\beta^2}{(\beta^1-\beta^2)^2}-\sigma\right]\geq 0. \quad (\text{A.12})$$

Equation (A.12) gives us the following condition for the laggard to specialize in appropriation:

$$\sigma\leq\sigma_0(\beta^2), \quad \text{where } \sigma_0(\beta^2)\equiv\frac{1}{2}-\frac{2\beta^1\beta^2}{(\beta^1-\beta^2)^2} \text{ with } \sigma'_0<0, \quad (\text{A.13})$$

which implies $\sigma < \frac{1}{2}$ is a necessary but not sufficient to make the laggard resource constrained. The expression for $\beta_0(\sigma)$ shown in (14) can be obtained by solving for the value of β^2 that makes the expression in the square brackets in (A.12) equal to zero.⁵⁶

Based on the $\beta_0(\sigma)$ schedule in (14), which is relevant assuming the two countries differ with respect to their sector-specific technologies only, we can again distinguish between the case where the laggard ($i = 2$) is resource constrained in its arming choice and thus specializes in appropriation and the case where the laggard is not resource constrained and thus produces both guns and butter as follows:

Case 1: For $\sigma \in [0, \frac{1}{2})$, when $\beta^2 \in (0, \beta_0(\sigma)]$, the laggard is resource constrained such that $G^{2*} = H^2 = 1$ and $G^{1*} = \tilde{B}^1(1) = \sqrt{2(1-\sigma)} - 1 > 0$, where the inequality follows from the fact that $\sigma < 1/2$ is a necessary (though not sufficient) for this case to hold.

Case 2: For any $\sigma \in [0, 1)$, when $\beta^2 \in (\max\{0, \beta_0(\sigma)\}, \beta^1]$, both countries diversify their production, in which case the FOCs satisfy $U_{G^i}^i = 0$ shown in (3), so that $G^{i*} = \tilde{G}^i$, as shown in (13b), for $i = 1, 2$.

Of course, this partition can be written equivalently in terms of σ as a function of β^2

⁵⁶There are two possible solutions, but the one which is relevant for us is that which subtracts the square root of the discriminant as it implies (in contrast to the other solution but consistent with the schedule $\sigma_0(\beta^2)$) a negative relationship between β^2 and σ along $\beta_0(\sigma)$.

(see (A.13)). More specifically, the laggard specializes in arming when $\sigma \in (0, \sigma_0(\beta^2)]$ and $\beta^2 \in (0, \beta_0(0)]$, where $\sigma_0(0) = \frac{1}{2}$, $\beta_0(0) \approx (3 - 2\sqrt{2})\beta^1$ and $\sigma_0(\beta_0(0)) = 0$; and, it diversifies its production when $\sigma \in (\max\{\sigma_0(\beta^2), 0\}, 1)$ for any $\beta^2 \in (0, \beta^1)$.

Proof of Proposition 4. We characterize equilibrium arming and the implications for the balance of power as they depend on $\theta \in \{\beta^2, \sigma\}$ in the two cases defined above.

Case 1, $\theta = \beta^2$: The solutions for arming in this case as shown above, $G^{1*} = \sqrt{2(1 - \sigma)} - 1$ and $G^{2*} = 1$, imply that both countries' equilibrium arming choices and thus aggregate arming \bar{G}^* and power ϕ^{i*} for $i = 1, 2$ are invariant to changes in β^2 .

Case 1, $\theta = \sigma$: The solution for the laggard's arming in this case trivially implies $dG^{2*}/d\sigma = 0$, whereas the solution for the leader's arming (see above) clearly shows $dG^{1*}/d\sigma < 0$. Thus, we must have $d\phi^{2*}/d\sigma > 0$, as stated in the proposition.

Case 2, $\theta = \beta^2$: Assuming both countries diversify their production, we can rewrite the system of equations that consists of the 2 FOCs ($U_{G^i}^i = 0$ for $i = 1, 2$) using (3), as $\Omega^i \equiv \frac{MC^i/MC^j}{MB^i/MB^j} = 1$ ($i \neq j$). Then, applying the fact, from (1), that $\phi_{G^i}^i = \phi^i \phi^j / G^i$ implies $\phi_{G^i}^i / \phi_{G^j}^j = G^j / G^i = \phi^j / \phi^i$, we can rewrite Ω^i as

$$\Omega^i(\phi^i, \beta^j, \sigma) = \frac{\beta^i \phi^i [\sigma + (1 - \sigma) \phi^i]}{\beta^j \phi^j [\sigma + (1 - \sigma) \phi^j]} = 1. \quad (\text{A.14})$$

Since $\phi^i + \phi^j = 1$, the above expression defines the equilibrium solution ϕ^{i*} implicitly as a function of β^j and σ , with $\beta^j = \beta^i$ implying $\phi^i = \frac{1}{2}$. Appropriate differentiation of Ω^i gives $\Omega_{\phi^i}^i > 0$, $\Omega_{\beta^j}^i < 0$, and $\text{sign}\{\Omega_{\sigma}^i\} = -\text{sign}\{\phi^{i*} - \phi^{j*}\}$. Hence, $d\phi^{i*}/d\beta^j = -\Omega_{\beta^j}^i / \Omega_{\phi^i}^i > 0$, which confirms the notion in our setting that an agent's power is increasing in its rival's butter productivity.⁵⁷

These observations allow us to draw two conclusions. First, $\beta^2 < \beta^1 \Rightarrow \phi^{2*} > \phi^{1*}$.⁵⁸ Second, $\text{sign}\{d\phi^{i*}/d\sigma\} = -\text{sign}\{\beta^i - \beta^j\}$ for $i \neq j$, which implies that an increase in the output security σ erodes the power of the more productive country. One can confirm these findings by differentiating appropriately the explicit solution to the equation in (A.14) which can be shown to be given by

$$\phi^{i*} = \frac{2\beta^j}{(2 - \sigma)\beta^j + \sigma\beta^i + \sqrt{\Lambda}}, \quad i, j \in \{1, 2\}, i \neq j, \quad (\text{A.15})$$

⁵⁷By the same reasoning, since $\Omega_{\beta^i}^i > 0$, we also have that country i 's power is decreasing in its own productivity: $\phi_{\beta^i}^i = -\Omega_{\beta^i}^i / \Omega_{\phi^i}^i < 0$.

⁵⁸While successive reductions in β^2 with $\beta^2 = \beta^1$ initially (technological regress) imply \tilde{G}^2/\tilde{G}^1 starts at 1 and rises continuously above that level, it is unclear how \tilde{G}^1 and \tilde{G}^2 themselves respond in levels to changes in β^2 . Nonetheless, one thing is clear: if, as noted earlier, only one agent eventually specializes completely in appropriation that agent must be the laggard.

where Λ was previously defined in (A.11).

To study the dependence of arming on β^2 in case 2 for $\beta^2 < \beta^1$, we differentiate appropriately the FOCs ($U_{G^i}^i = 0$) based on (3) for $i = 1, 2$, and use those conditions in the resulting expressions to find⁵⁹

$$U_{G^1 G^1}^1 = -\frac{2\beta^1 \phi^1}{G^1} < 0; \quad U_{G^1 G^2}^1 = -\frac{(\beta^1 - \beta^2) \sigma \phi^1}{G^1} < 0; \quad (\text{A.16a})$$

$$U_{G^1 \beta^2}^1 = \frac{(1 - \sigma) \phi^1 \phi^2 X^2}{\beta^2 G^1} > 0; \quad U_{G^1 \sigma}^1 = -\frac{\beta^1}{1 - \sigma} < 0; \quad (\text{A.16b})$$

$$U_{G^2 G^2}^2 = -\frac{2\beta^2 \phi^2}{G^2} < 0; \quad U_{G^1 G^2}^2 = \frac{(\beta^1 - \beta^2) \sigma \phi^2}{G^2} > 0; \quad (\text{A.16c})$$

$$U_{G^2 \beta^2}^2 = -\frac{(1 - \sigma) \phi^1 \phi^2 X^1}{\beta^2 G^2} < 0; \quad U_{G^1 \sigma}^2 = -\frac{\beta^2}{1 - \sigma} < 0. \quad (\text{A.16d})$$

Based on the expressions above, we make three observations. First, the finding that $U_{G^i G^i}^i < 0$ ($i = 1, 2$) is consistent with our earlier argument that agent i 's payoff is strictly concave in its arming. Second, the expressions for $U_{G^i G^j}^i$ ($i = 1, 2$) show neatly how the qualitative effect of arming by a country's rival on its own net marginal benefit of arming depends on their relative productivities in butter; in equilibrium, the leader's arming is a strategic substitute for its rival's arming (i.e., $U_{G^1 G^2}^1 < 0$), whereas the laggard's arming is a strategic complement for its rival's arming (i.e., $U_{G^1 G^2}^2 > 0$). Third, the direct effect of an increase in β^2 on the leader's (laggard's) net marginal benefit to arming is positive (negative). This last observation explains why the best-response functions studied in Fig. 2(a) shift in response to technical regress as shown there.

It is also easy to show that

$$\Delta \equiv U_{G^1 G^1}^1 U_{G^2 G^2}^2 - U_{G^1 G^2}^1 U_{G^2 G^1}^2 = \Lambda \left(\frac{\phi^1 \phi^2}{G^1 G^2} \right) > 0, \quad (\text{A.17})$$

which is a standard condition for (static) stability of equilibrium that also ensures uniqueness of equilibrium. By applying the implicit function theorem to the agents' FOCs and solving for the associated changes in equilibrium arms when neither country is resource constrained, one can find

$$\begin{aligned} \frac{d\tilde{G}^1}{d\beta^2} &= \frac{1}{\Delta} \left[-U_{G^2 G^2}^2 U_{G^1 \beta^2}^1 + U_{G^1 G^2}^1 U_{G^2 \beta^2}^2 \right] \\ &= \frac{1 - \sigma}{\beta^2 \Lambda} [2\beta^2 \phi^2 X^2 + \sigma (\beta^1 - \beta^2) \phi^1 X^1] > 0 \end{aligned} \quad (\text{A.18a})$$

⁵⁹In what follows for the remainder of this proof, we suppress the asterisk superscript to avoid cluttering of notation. We recognize that a more direct approach here would be to simply differentiate the solutions \tilde{G}^i shown in (13b); however, that type of analysis is cumbersome and obscures the channels through which β^2 influences equilibrium arming choices.

$$\begin{aligned}
\frac{d\tilde{G}^2}{d\beta^2} &= \frac{1}{\Delta} \left[U_{G^2G^1}^{2(+)} U_{G^1\beta^2}^{1(+)} - U_{G^1G^1}^{1(-)} U_{G^2\beta^2}^{2(-)} \right] \\
&= -\frac{1-\sigma}{\beta^2\Lambda} [2\beta^1\phi^1X^1 - \sigma(\beta^1 - \beta^2)\phi^2X^2] < 0,
\end{aligned} \tag{A.18b}$$

where $\Lambda > 0$ was previously defined in (A.11). Inspection of (A.18a) reveals that the leader's equilibrium arming is increasing in the laggard's butter productivity β^2 due to its positive direct effect on $U_{G^1}^1$ noted earlier ($U_{G^1\beta^2}^1 > 0$) and the reinforcing positive effect in the presence of strategic substitutability ($U_{G^1G^2}^1 < 0$) that lowers the net marginal benefit of arming for the laggard ($U_{G^2\beta^2}^2 < 0$). Turning to the laggard, inspection of (A.18b) reveals that the presence of strategic complementarity ($U_{G^2G^1}^2 > 0$) works against the direct and negative effect of β^2 on $U_{G^2}^2$ ($U_{G^2\beta^2}^2 < 0$). Thus, the claim in (A.18b) that $d\tilde{G}^2/d\beta^2 < 0$ is not obvious and requires proof.

To establish the dominance of the direct effect of β^2 that effectively requires the expression inside the square brackets in the second line of (A.18b) to be positive, we first observe that $2\beta^1 > \sigma(\beta^1 - \beta^2)$. Thus, to prove our claim, it is sufficient to demonstrate that $\phi^1X^1 > \phi^2X^2$. To proceed, let us define $Z^i \equiv 1 - G^i$, such that $X^i \equiv \beta^i Z^i$. By forming the ratio ϕ^1X^1/ϕ^2X^2 and multiplying it by $\Omega^2(\cdot)$ which equals 1 by (A.14), we can find

$$\frac{\phi^1X^1}{\phi^2X^2} = \left(\frac{\phi^1\beta^1Z^1}{\phi^2\beta^2Z^2} \right) \frac{\beta^2\phi^2[\sigma + (1-\sigma)\phi^2]}{\beta^1\phi^1[\sigma + (1-\sigma)\phi^1]} = \frac{Z^1[\sigma + (1-\sigma)\phi^2]}{Z^2[\sigma + (1-\sigma)\phi^1]}.$$

Since $\phi^2 > \phi^1$ implies $\frac{[\sigma + (1-\sigma)\phi^2]}{[\sigma + (1-\sigma)\phi^1]} > 1$, and $\tilde{G}^1 < \tilde{G}^2$ implies $Z^1/Z^2 > 1$, the expression in the far right above exceeds 1; therefore, $\phi^1X^1 > \phi^2X^2$ and $d\tilde{G}^2/d\beta^2 < 0$ as needed.

To study the effects of an increase in β^2 on aggregate arming \bar{G} , let us define $x^i \equiv \phi^iX^i/(\phi^1X^1 + \phi^2X^2)$ ($i = 1, 2$) and note that $\phi^1X^1 > \phi^2X^2$ implies $x^1 > \frac{1}{2} > x^2$. Also, define $n^i \equiv \beta^i/(\beta^1 + \beta^2)$ and note that $\beta^2 < \beta^1$ implies $n^2 < \frac{1}{2} < n^1$. Using these definitions and Λ in (A.11) with the expressions for $d\tilde{G}^1/d\beta^2$ and $d\tilde{G}^2/d\beta^2$ from (A.18a) and (A.18b) respectively, one can find that the change in aggregate spending $d\bar{G}/d\beta^2$ is given by

$$\begin{aligned}
\frac{d\bar{G}}{d\beta^2} &= -\frac{2(1-\sigma)}{\beta^2\Lambda} [\beta^1\phi^1X^1 - \beta^2\phi^2X^2 - \frac{1}{2}\sigma(\beta^1 - \beta^2)(\phi^1X^1 + \phi^2X^2)] \\
&= -\frac{2(1-\sigma)}{n^2\Lambda} (\phi^1X^1 + \phi^2X^2) [n^1x^1 - n^2x^2 - \frac{1}{2}\sigma(n^1 - n^2)] \\
&= -\frac{2(1-\sigma)}{n^2\Lambda} (\phi^1X^1 + \phi^2X^2) (x^1 - [\frac{1}{2}\sigma n^1 + (1 - \frac{1}{2}\sigma)n^2])
\end{aligned} \tag{A.19}$$

Now, observe that the expression inside the square brackets in the last line of (A.19) is a weighted sum of n^1 and n^2 . As σ rises from 0 to (approximately) 1, the value of this weighted sum rises from n^2 to (approximately) $\frac{1}{2}$. But, because $x^1 > \frac{1}{2}$, the expression inside the last set of parentheses in the last line of (A.19) is positive. Therefore, $d\bar{G}/d\beta^2 < 0$ in

this case, as stated in the proposition.

Case 2, $\theta = \sigma$: Turning to the effects of σ on arming under case 2, one can use the expressions in (A.16), to find:

$$\frac{d\tilde{G}^i}{d\sigma} = \frac{1}{\Delta} \left[-U_{G^j G^j}^j U_{G^i \sigma}^i + U_{G^i G^j}^i U_{G^j \sigma}^j \right] = -\frac{\beta^j \phi^i [(2 - \sigma) \beta^i + \sigma \beta^j]}{\tilde{G}^i (1 - \sigma) \Delta} < 0. \quad (\text{A.20})$$

for $i \in \{1, 2\}, i \neq j$. Since $d\tilde{G}^i/d\sigma < 0$ holds for $i = 1, 2$, it follows immediately that $d\bar{G}/d\sigma < 0$. Furthermore, although arming by both countries falls, our previous analysis, in connection with (A.14) already established that $\text{sign}\{d\phi^1/d\sigma\} = -\text{sign}\{\beta^1 - \beta^2\}$, which is negative given $\beta^1 > \beta^2$. $\quad \parallel$

Proof of Proposition 5. To study the payoff effects of changes in the laggard's butter productivity ($\theta = \beta^2$) and output security ($\theta = \sigma$), we again distinguish between cases 1 and 2 defined above that capture, respectively, the scenario where the laggard specializes in appropriation and the scenario where both agents' production is diversified.

Case 1, $\theta = \beta^2$: Since $X^{2*} = 0$ in this case, equation (2) implies the two countries' payoffs are $U^{1*} = [\phi^{1*} (1 - \sigma) + \sigma] X^{1*}$ and $U^{2*} = \phi^{2*} (1 - \sigma) X^{1*}$. From Proposition 4, we know that neither country's arming depends on β^2 . Thus, changes in β^2 have no influence on X^{1*} or ϕ^{i*} for $i = 1, 2$. As a result, when the laggard specializes in appropriation, neither country's payoff depends on β^2 .⁶⁰

Case 1, $\theta = \sigma$: Because X^{2*} remains equal to 0 for small changes in σ and $dG^{2*}/d\sigma = 0$, equation (6) implies, by the envelope theorem, that the total impact on the leader's payoff equals the direct effect: $dU^{1*}/d\sigma = \phi^{2*} X^{1*} > 0$. From equation (7), the effect on the laggard's payoff is $dU^{2*}/d\sigma = -\phi^{2*} X^{1*} - \beta^1 (dG^{1*}/d\sigma)$.⁶¹ The arming solutions G^{i*} in this case (shown in the beginning of the proof to Proposition 4) imply $\phi^{2*} = 1/\sqrt{2(1 - \sigma)}$, $X^{1*} = \beta^1 [2 - 1/\phi^{2*}]$ and $dG^{1*}/d\sigma = -1/\sqrt{2(1 - \sigma)} = -\phi^{2*} < 0$. Combining these expressions shows $dU^{2*}/d\sigma = \beta^1 \phi^{1*} > 0$, thereby substantiating the claim that the positive indirect of an increase in σ outweighs the negative direct effect, as needed in this case.

Case 2, $\theta = \beta^2$: We now turn to the case where the laggard diversifies its production, starting with the leader ($i = 1$). Equation (15a) shows the direct payoff effect is positive ($U_{\beta^2}^1 > 0$); since $U_{G^2}^1 < 0$ (by (16)) and $dG^{2*}/d\beta^2 < 0$ (by Proposition 4), the indirect effect is $U_{G^2}^1 (dG^{2*}/d\beta^2) > 0$. Hence, by (6), we have $dU^{1*}/d\beta^2 > 0$.

For the laggard, equation (7) with (15b) implies

$$\frac{dU^{2*}}{d\beta^2} = [(1 - \sigma) \phi^2 + \sigma] Z^{2*} - \beta^1 \left(\frac{dG^{1*}}{d\beta^2} \right), \quad (\text{A.21})$$

⁶⁰Their payoffs do depend, however, on β^1 .

⁶¹The third term in (7) vanishes since $dG^{2*}/d\sigma = 0$.

where $Z^{2*} = 1 - G^{2*} > 0$ and $dG^{1*}/d\beta^2$ is given by (A.18a) with $X^{2*} = \beta^2 Z^{2*}$.⁶² To obtain the coefficient on $dG^{1*}/d\beta^2$ in (A.21), we used the leader's FOC, $U_{G^1}^1 = 0$ based on (3), in the expression for $U_{G^1}^2$ shown in (16) with the implication of (1) that $\phi_{G^1}^2 = -\phi_{G^1}^1$, and simplified the resulting expression. Clearly, the direct effect shown in the first term is positive; in contrast, by Proposition 4 that establishes $dG^{1*}/d\beta^2 > 0$, the indirect effect shown in the second term is negative. To prove the claim made in the proposition that the laggard's payoff can be J -shaped in β^2 , we begin by considering the impact for β^2 close to $\max\{0, \beta_0(\sigma)\}$. It helps here to divide the set of values of σ into the following subsets: $[0, \frac{1}{2}]$ and $\sigma \in (\frac{1}{2}, 1)$.

Starting with the first subset $\sigma \in [0, \frac{1}{2}]$, observe that as β^2 approaches $\beta_0(\sigma) \geq 0$ from above, Z^{2*} approaches 0 while $Z^{1*} > 0$. Equation (A.21) then implies that the sign of $dU^{2*}/d\beta^2$ in this limit is given by minus the sign of $\lim_{\beta^2 \searrow \beta_0(\sigma)} dG^{1*}/d\beta^2$. Notice further that the first term inside the brackets of the second line of the expression for $dG^{1*}/d\beta^2$ in (A.18a) vanishes. Keeping mind that $X^{i*} = \beta^i Z^{i*}$ for $i = 1, 2$, we have

$$\lim_{\beta^2 \searrow \beta_0(\sigma)} \frac{dG^{1*}}{d\beta^2} = \sigma(1 - \sigma) \times \lim_{\beta^2 \searrow \beta_0(\sigma)} \left[\frac{(\beta^1)^2 \phi^{1*} Z^{1*}}{\beta^2 \Lambda} - \frac{\beta^1 \phi^{1*} Z^{1*}}{\Lambda} \right] \quad \text{for } \sigma \in [0, \frac{1}{2}],$$

where $\Lambda > 0$ is shown in (A.11) and ϕ^{1*} is shown in (A.15). It follows immediately from the expression above that, when $\sigma = 0$ which implies $\beta_0(\sigma) > 0$, $\lim_{\beta^2 \searrow \beta_0(0)} (dG^{1*}/d\beta^2) = 0$, and thus $\lim_{\beta^2 \searrow \beta_0(0)} (dU^{2*}/d\beta^2) = 0$. Since $\beta^1 > \beta^2$, the expression also shows that, when $\sigma \in (0, \frac{1}{2})$, $\lim_{\beta^2 \searrow \beta_0(\sigma)} dG^{1*}/d\beta^2 > 0$ and hence $\lim_{\beta^2 \searrow \beta_0(\sigma)} (dU^{2*}/d\beta^2) < 0$. Finally, for $\sigma = \frac{1}{2}$, we consider the limit of $dG^{1*}/d\beta^2$ as β^2 approaches $\beta_0(\frac{1}{2}) = 0$. As one can easily verify from (13b), $\lim_{\beta^2 \rightarrow 0} G^{1*} = 0$, which implies $\lim_{\beta^2 \rightarrow 0} \phi^{1*} = 0$ and $\lim_{\beta^2 \rightarrow 0} \phi^{2*} = 1$.⁶³ Accordingly, the limit of the second term in brackets in the expression immediately above vanishes. Using the solution for ϕ^{1*} in (A.15), one can confirm that the limit of the first term is strictly positive. Thus, for any $\sigma \in [0, \frac{1}{2}]$, $\lim_{\beta^2 \searrow \beta_0(\sigma)} dG^{1*}/d\beta^2 \geq 0$ (with equality for $\sigma = 0$), which implies from (A.21) that $\lim_{\beta^2 \searrow \beta_0(\sigma)} dU^{2*}/d\beta^2 \leq 0$ (with equality for $\sigma = 0$).

Turning to the second subset $\sigma \in (\frac{1}{2}, 1)$, we consider the impact of β^2 as it approaches 0. Interestingly, one can confirm from (13b) that $\lim_{\beta^2 \rightarrow 0} G^{2*} = (\sigma - 1)/\sigma$, whereas

⁶²The third term in (7) vanishes since $U_{G^2}^{2*} = 0$ in this case. While in case 2 $G^{i*} = \tilde{G}^i$ for $i = 1, 2$ as shown (13b), we use an asterisk to indicate equilibrium solutions for arming to avoid confusion.

⁶³While the laggard could choose a smaller quantity of guns that would continue to imply $\phi^2 = 1$ given $G^1 = 0$, equation (A.1) implies $G^{2*} = 1 = \lim_{\beta^2 \rightarrow 0} G_0^1$ is the smallest quantity of G^2 that induces the leader to specialize in butter production in this limit when $\sigma = \frac{1}{2}$. If the laggard were to choose a smaller quantity of guns, it would induce the leader to shift its production away from butter towards guns, implying not only less butter production by the leader but also a smaller share $\phi^2 < 1$ of the contested pool for itself; in addition, since $\beta^2 = 0$ in this limit, the laggard would not gain from allocating any of its resource to butter production. Hence, $G^{2*} = 1$ is a best response to $G^1 = 0$ in this limit as β^2 approaches 0 for $\sigma = \frac{1}{2}$.

$\lim_{\beta^2 \rightarrow 0} G^{1*} = 0$.⁶⁴ Thus, in this limit with $\sigma \in (\frac{1}{2}, 1)$, the laggard allocates some of its resource to the production of butter, $\lim_{\beta \rightarrow 0} Z^{2*} = (2\sigma - 1)/\sigma > 0$, and the direct payoff effect of an increase in β^2 is positive and given by $\lim_{\beta^2 \rightarrow 0} [(1 - \sigma)\phi^{2*} + \sigma] Z^{2*} = 2 - 1/\sigma$. But, as in the previous cases considered when $\sigma > 0$, $\lim_{\beta^2 \searrow \beta_0(\sigma)} dG^{1*}/d\beta^2 > 0$, such that the indirect effect will be negative. To see how these two effects compare, we find an explicit expression for the indirect effect in that limit. To proceed, note the following: even though $\lim_{\beta \rightarrow 0} Z^{2*} > 0$, we have $\lim_{\beta \rightarrow 0} X^{2*} = \lim_{\beta \rightarrow 0} \beta^2 Z^{2*} = 0$. Thus, the first term in the brackets of the second line of (A.18a) divided by β^2 again approaches 0 as $\beta^2 \rightarrow 0$. Since in addition we have $\lim_{\beta \rightarrow 0} \phi^{1*} = 0$ and $\lim_{\beta^2 \rightarrow 0} \Lambda = (\beta^1 \sigma)^2$, we can use the solution for ϕ^{1*} in (A.15) to write the relevant limit of $dG^{1*}/d\beta^2$ as

$$\begin{aligned} \lim_{\beta^2 \rightarrow 0} \left(\frac{dG^{1*}}{d\beta^2} \right) &= \lim_{\beta^2 \rightarrow 0} \left[\frac{(\beta^1)^2 (1 - \sigma) \sigma Z^{1*} \phi^{1*}}{\beta^2 \Lambda} \right] \\ &= \lim_{\beta^2 \rightarrow 0} \left[\frac{(\beta^1)^2 (1 - \sigma) \sigma Z^{1*}}{\Lambda} \right] \times \lim_{\beta^2 \rightarrow 0} \left[\frac{2}{(2 - \sigma)\beta^2 + \beta^1 \sigma + \sqrt{\Lambda}} \right] \\ &= \left[\frac{1 - \sigma}{\sigma} \right] \times \lim_{\beta^2 \rightarrow 0} \left[\frac{2}{2(\beta^1 \sigma)} \right] = \frac{1 - \sigma}{\beta^1 \sigma^2}. \end{aligned}$$

Thus, the indirect effect is $-\lim_{\beta^2 \rightarrow 0} \beta^1 dG^{1*}/d\beta^2 = -(1 - \sigma)/\sigma^2$. Bringing this result and the result above for direct effect in this limit together in (A.21), we have

$$\text{sign} \left\{ \lim_{\beta^2 \rightarrow 0} \left(\frac{dU^{2*}}{d\beta^2} \right) \right\} = \text{sign} \{ \sigma - \check{\sigma} \},$$

where $\check{\sigma} = 1/\sqrt{2}$.

In summary, $dU^{2*}/d\beta^2 = 0$ at points $(\beta_0(0), 0)$ and $(0, \check{\sigma})$; therefore, $\beta^2 = \beta_0(0)$ and $\beta^2 = 0$ represent extreme points of U^{2*} respectively for $\sigma = 0$ and $\sigma = \check{\sigma}$. Additionally, because $\lim_{\beta^2 \searrow \max\{0, \beta_0(\sigma)\}} dU^{2*}/d\beta^2 < 0$ for $\sigma \in (0, \check{\sigma})$, the value of β^2 that solves $dU^{2*}/d\beta^2 = 0$ will exceed $\max\{0, \beta_0(\sigma)\}$. Lastly, because $\lim_{\beta^2 \searrow \beta_0(\sigma)} dU^{2*}/d\beta^2 > 0$ for $\sigma \in (\check{\sigma}, 1)$, $\beta^2 = 0$ is an extreme (though not necessarily global) point.

To see that the above extremes are local minima, we now compare the laggard's payoff levels, first, when β^2 approaches $\max\{0, \beta_0(\sigma)\}$ from above and, second, when $\beta^2 \rightarrow \beta^1$ for $\sigma \in [0, 1)$. Focusing on this second limit, note that (13b) implies $\lim_{\beta^2 \rightarrow \beta^1} G^{i*} = \frac{1}{2}(1 - \sigma)$, such that $\lim_{\beta^2 \rightarrow \beta^1} Z^{i*} = \frac{1}{2}(1 + \sigma)$ and (from (1)) $\lim_{\beta^2 \rightarrow \beta^1} \phi^{i*} = \frac{1}{2}$ for $i = 1, 2$. With these findings and our previous analysis, we use (2) and the definition of $\beta_0(\sigma)$ in (14), to find

$$\lim_{\beta^2 \searrow \beta_0(\sigma)} U^{2*} = \beta^1 \sqrt{1 - \sigma} \left(\sqrt{2} - \sqrt{1 - \sigma} \right), \text{ for } \sigma \in [0, \frac{1}{2}) \quad (\text{A.22a})$$

⁶⁴The logic spelled out in footnote 63, based on (A.1), applies here as well: $G^{2*} = (\sigma - 1)/\sigma = \lim_{\beta^2 \rightarrow 0} G^1$ is the smallest quantity of G^2 that induces the leader to specialize in butter production in this limit for any $\sigma \in [\frac{1}{2}, 1)$.

$$\lim_{\beta^2 \rightarrow 0} U^{2*} = \beta^1 (1 - \sigma), \text{ for } \sigma \in [\frac{1}{2}, 1) \quad (\text{A.22b})$$

$$\lim_{\beta^2 \rightarrow \beta^1} U^{2*} = \frac{1}{2}\beta^1(1 + \sigma), \text{ for any } \sigma \in [0, 1). \quad (\text{A.22c})$$

Comparing these limits reveals that $\lim_{\beta^2 \rightarrow \beta^1} U^{2*}$ shown in (A.22c) exceeds the other two limits in (A.22a) for $\sigma \in [0, \frac{1}{2})$ and (A.22b) for $\sigma \in [\frac{1}{2}, 1)$.

On the basis of the preceding analysis we may now conclude that $\beta_{\min} \equiv \arg \min_{\beta^2} U^{2*}$ always exists. In particular, (i) $\beta_{\min} = \beta_0(\sigma)$ for $\sigma = 0$, (ii) $\beta_{\min} \in (\beta_0(\sigma), 1)$ for $\sigma \in (0, \check{\sigma})$, and (iii) $\beta_{\min} = 0$ for $\sigma \in [\check{\sigma}, 1)$ are local minima. What remains unclear, however, is whether these minima are unique. In Appendix B, we establish that $d^2 U^{2*} / d(\beta^2)^2 |_{\beta^2 = \beta'} > 0$ at any β' that solves $dU^{2*} / d\beta^2 = 0$, which is a sufficient condition to ensure that the local minima identified above are unique. In effect, this inequality implies U^{2*} is strictly quasi-convex in β^2 and, consequently, for any given $\sigma \in [0, 1)$, $\beta_{\min} \equiv \arg \min_{\beta^2} U^{2*}$ is global.

Case 2, $\theta = \sigma$: We have already established in the main text that the direct payoff effects of an increase in σ satisfy $U_{\sigma}^{i*} = X^{i*} - \phi^{i*} \bar{X}^*$ for $i = 1, 2$, so that $U_{\sigma}^{1*} = -U_{\sigma}^{2*} > 0$. The indirect effect of σ on U^i is given by $U_{G^j}^{i*} (dG^j / d\sigma)$, where $U_{G^j}^{i*} = -\beta^j < 0$, as one can verify using (3) to form agent j 's FOC $U_{G^j}^j = 0$, substituting that into the expression for $U_{G^j}^{i*}$ that is shown in (16) with $\phi_{G^j}^i = -\phi_{G^j}^j$, and then simplifying. The earlier result that $d\tilde{G}^j / d\sigma < 0$ (Proposition 4), therefore, implies that the indirect payoff effect is positive for both agents. Clearly, then, the leader always benefits from improvements in security.

Whether the laggard benefits from such improvements or not depends on how the two offsetting effects compare. In particular, we have

$$\frac{dU^{2*}}{d\sigma} = (X^{2*} - \phi^{2*} \bar{X}^*) - \beta^1 \frac{dG^{1*}}{d\sigma}, \quad (\text{A.23})$$

where using (A.20) with (A.17) we rewrite $dG^{1*} / d\sigma$ as

$$\frac{dG^{i*}}{d\sigma} = -\frac{\beta^j \bar{G}^* [(2 - \sigma) \beta^i + \sigma \beta^j]}{(1 - \sigma) \Lambda} < 0, \quad i \in \{1, 2\}, i \neq j. \quad (\text{A.24})$$

Although the text focuses on the case where $\beta^2 \in (0, \beta^1]$, in what follows we also consider, for completeness, the possibility that the laggard does not have the know-how to produce butter (i.e., $\beta^2 = 0$), in which case (A.24) does not apply directly. In this case, we have $dG^{1*} / d\sigma \leq 0$ that holds as a strictly inequality for $\sigma < \frac{1}{2}$ and as an equality otherwise.⁶⁵ Thus, from (A.23), the indirect payoff effect for the laggard is non-negative.

To characterize the total impact of changes in σ on the laggard's payoff in this case where its production is diversified, we proceed in three steps:

⁶⁵To be more precise, the solutions for arming derived earlier show that, when $\sigma < \frac{1}{2}$, $\beta^2 = 0$ implies $G^{2*} = 1$ and $G^{1*} = \sqrt{2(1 - \sigma)} - 1 > 0$ so that $dG^{1*} / d\sigma < 0$ continues to hold; however, when $\sigma \geq \frac{1}{2}$, $\beta^2 = 0$ implies that $G^{1*} = 0$, such that $dG^{1*} / d\sigma = 0$ holds.

Step 1: Considering the sign of the limit of $dU^{2^*}/d\sigma$ as $\sigma \rightarrow \max\{\sigma_0(\beta^2), 0\}$ from above, we prove that there exists a value of $\beta^2 \in (0, \beta_0(0))$, denoted by $\underline{\beta}$ ($\approx 0.099\beta^1$), such that $\lim_{\sigma \searrow \sigma_0(\beta^2)} (dU^{2^*}/d\sigma) \begin{matrix} \leq \\ \geq \end{matrix} 0$ for $\beta^2 \begin{matrix} \leq \\ \geq \end{matrix} \underline{\beta}$. A salient implication of this characterization is that $\sigma_0(\beta^2)$ is a local maximizer of U^{2^*} for $\beta^2 \in [0, \underline{\beta}]$.

Step 2: Then, considering the sign of the limit of $dU^{2^*}/d\sigma$ as $\sigma \rightarrow 1$, we show that there exists a value of $\beta^2 \in (\underline{\beta}, \beta^1)$, denoted by $\bar{\beta}$ ($= \frac{1}{2}(\sqrt{5} - 1)\beta^1 \approx 0.618\beta^1$), such that $dU^{2^*}/d\sigma \begin{matrix} \leq \\ \geq \end{matrix} 0$ as $\beta^2 \begin{matrix} \leq \\ \geq \end{matrix} \bar{\beta}$. An important implication of this step and the first one combined is that U^{2^*} attains a maximum at some $\sigma' \in (\max\{\sigma_0(\beta^2), 0\}, 1)$ for all $\beta^2 \in (\underline{\beta}, \bar{\beta})$.

Step 3: Finally, we prove that, for any given $\beta^2 \in [0, \beta^1]$, U^{2^*} is strictly quasi-concave in $\sigma \in [0, 1)$. The key implication of this property is that $\sigma_{\max} \equiv \arg \max_{\sigma} U^{2^*}$ is unique. More specifically, we show that: (i) $\sigma_{\max} = \sigma_0(\beta^2)$ when $\beta^2 \in [0, \underline{\beta}]$; (ii) $\sigma_{\max} \in (\sigma_0(\underline{\beta}), 1)$ when $\beta^2 \in (\underline{\beta}, \bar{\beta})$ where $\bar{\beta} \approx 0.618\beta^1$; and (iii) $dU^{2^*}/d\sigma > 0$ for all $\sigma \in [0, 1)$ when $\beta^2 \in [\bar{\beta}, \beta^1]$.

Step 1: Starting with the limit of $dU^{2^*}/d\sigma$ as $\sigma \rightarrow \max\{\sigma_0(\beta^2), 0\}$, let us suppose that $\beta^2 = 0$. Since $dG^{1^*}/d\sigma = 0$ for $\sigma \geq \frac{1}{2}$, $dU^{2^*}/d\sigma < 0$ in this limit. Given $dU^{2^*}/d\sigma > 0$ for $\sigma < \frac{1}{2}$ in this case, we have $\sigma_{\max} = \sigma_0(0) = \frac{1}{2}$. Let us now evaluate the limit of $dU^{2^*}/d\sigma$ in (A.23) as $\sigma \rightarrow \sigma_0 \equiv \sigma_0(\beta^2)$ from above for any $\beta^2 \in (0, \beta_0(0)]$. Importantly, we now have: $G^{2^*} \rightarrow 1$, $X^{2^*} \rightarrow 0$; $G^{1^*} \rightarrow B^1(1) = \sqrt{2(1 - \sigma_0)} - 1$ and $\bar{X}^* = X^{1^*} \rightarrow \beta^1[2 - \sqrt{2(1 - \sigma_0)}]$; and, $\phi^{2^*} \rightarrow 1/\sqrt{2(1 - \sigma_0)}$. We then substitute (A.24) into (A.23) and apply the above limits in the resulting expression for $dU^{2^*}/d\sigma$ to transform it (after some manipulation) as follows:

$$\lim_{\sigma \searrow \sigma_0} \left(\frac{dU^{2^*}}{d\sigma} \right) = \frac{\beta^1}{\sqrt{2}\sqrt{1 - \sigma_0}} \Upsilon(\beta^2, \sigma_0),$$

where

$$\Upsilon(\beta^2, \sigma_0) \equiv -2 + \sqrt{2}\sqrt{1 - \sigma_0} + \frac{2\beta^2 [2\beta^1 - (\beta^1 - \beta^2)\sigma_0]}{\Lambda_0}$$

and $\Lambda_0 \equiv \Lambda|_{\sigma=\sigma_0} = 4\beta^1\beta^2 + (\beta^1 - \beta^2)^2\sigma_0^2$.

Since $\text{sign}\{\lim_{\sigma \searrow \sigma_0} (dU^{2^*}/d\sigma)\} = \text{sign}\{\Upsilon\}$, we focus on identifying the sign of $\Upsilon(\cdot)$. To this end, first note that, when we use the definitions of σ_0 and Λ_0 in $\Upsilon(\cdot)$, we find $\Upsilon|_{\beta^2=0} = -1 < 0$ and $\Upsilon|_{\beta^2=\beta_0(0)} = \sqrt{2} - 1 > 0$. Clearly, then, $\Upsilon(\cdot)$ is negative when $\beta^2 \rightarrow 0$ and becomes positive when $\beta^2 \rightarrow \beta_0(0)$. What we do not know is how $\Upsilon(\cdot)$ behaves as β^2 rises in $(0, \beta_0(0)]$.

We now prove that $d\Upsilon/d\beta^2 = \Upsilon_{\sigma_0}\sigma'_0(\beta^2) + \Upsilon_{\beta^2} > 0$ which, by the continuity of Υ in β^2 , ensures the existence of a unique value of β^2 , labeled $\underline{\beta}$, such that $\Upsilon \begin{matrix} \leq \\ \geq \end{matrix} 0$ as $\beta^2 \begin{matrix} \leq \\ \geq \end{matrix} \underline{\beta}$.

Differentiation of Υ gives

$$\begin{aligned}\Upsilon_{\sigma_0} &= - \left[\frac{1}{\sqrt{2}\sqrt{1-\sigma_0}} + \frac{2\beta^2(\beta^1 - \beta^2)}{\Lambda_0} v_1 \right] < 0 \\ \Upsilon_{\beta^2} &= \frac{2\beta^2\sigma_0}{\Lambda_0} + \frac{2[2\beta^1 - (\beta^1 - \beta^2)\sigma_0]}{\Lambda_0} v_2,\end{aligned}$$

where

$$\begin{aligned}v_1 &\equiv 1 + \frac{2\sigma(\beta^1 - \beta^2)[2\beta^1 - (\beta^1 - \beta^2)\sigma_0]}{\Lambda_0} > 0 \\ v_2 &\equiv 1 - \frac{2\beta^2[2\beta^1 - (\beta^1 - \beta^2)\sigma_0^2]}{\Lambda_0}.\end{aligned}$$

Since $\sigma'_0(\beta^2) = -\frac{2\beta^1(\beta^1 + \beta^2)}{(\beta^1 - \beta^2)^2} < 0$, we have $\Upsilon_{\sigma_0}\sigma'_0(\beta^2) > 0$. By contrast, the sign of Υ_{β^2} is not immediately obvious from inspection of the expression shown above, because the sign of v_2 inside it appears to be ambiguous. Using the value of σ_0 in Λ_0 and then in the definition of v_2 , however, enables us to rewrite v_2 (after some algebra) as follows:

$$v_2 = \frac{16(\beta^1 - \beta^2)^4(3\beta^1 - \beta^2)\sigma_0^2}{(\beta^1 + \beta^2)^6} > 0.$$

We thus have $d\Upsilon/d\beta^2 > 0$ which confirms the claim that $\underline{\beta}$ is unique. Using numerical methods, we find $\underline{\beta}$ to be given by $\underline{\beta} \approx 0.099\beta^1$.

Next, we examine the sign of $\lim_{\sigma \rightarrow 0} (dU^{2*}/d\sigma)$ when $\beta^2 \in (\beta_0(0), \beta^1]$. From (A.23) and (A.24) one can easily verify that $\lim_{\sigma \rightarrow 0} dU^{2*}/d\sigma = \beta^1(1 - \sqrt{2}) + \beta^1(1/\sqrt{2}) = \beta^1(1 - 1/\sqrt{2}) > 0$. This completes step 1.

Since $dU^{2*}/d\sigma > 0$ for parameter values that imply the laggard specializes in appropriation whereas, as we have just shown, $dU^{2*}/d\sigma|_{\sigma=\sigma_0(\beta^2)} \leq 0$ as $\beta^2 \leq \underline{\beta}$, it follows that $\sigma_0(\beta^2)$ is a local (though not necessarily a global) maximum of U^{2*} for $\beta^2 < \underline{\beta}$. Furthermore, because $dU^{2*}/d\sigma|_{\sigma=\max\{\sigma_0(\beta^2), 0\}} > 0$ when $\beta^2 > \underline{\beta}$, it is possible that U^{2*} attains a maximum at some $\sigma \in (\max\{\sigma_0(\beta^2), 0\}, 1)$ or, alternatively, that $dU^{2*}/d\sigma > 0$ for all relevant parameter values. The next step will shed some light on this issue.

Step 2: Let us now examine the sign of $dU^{2*}/d\sigma$ as $\sigma \rightarrow 1$ for any $\beta^2 \in (0, \beta^1]$. Using (A.23) and (A.24) under diversification, it is easy to show that in this limit the direct effect of σ converges to $-(\beta^1 - \beta^2)$ (< 0) whereas the indirect effect converges to $\frac{\beta^1\beta^2}{\beta^1 + \beta^2}$ (> 0). Putting these effects together gives

$$\lim_{\sigma \rightarrow 1} (dU^{2*}/d\sigma) = \frac{1}{\beta^1 + \beta^2} [(\beta^2)^2 + \beta^1\beta^2 - (\beta^1)^2].$$

Clearly, the sign of the above expression depends on the value of $\beta^2 \in (0, \beta^1]$. However, it is

straightforward for one to verify that $\lim_{\sigma \rightarrow 1} dU^{2*}/d\sigma \leq 0$ as $\beta^2 \leq \bar{\beta}$, where $\bar{\beta} = \frac{1}{2}(\sqrt{5}-1)\beta^1$ ($\approx 0.618\beta^1$). What's more, we have $\underline{\beta} < \bar{\beta}$.

Step 3: Since in step 1 we showed that $dU^{2*}/d\sigma|_{\sigma=\max\{\sigma_0(\beta^2),0\}} > 0$ for $\beta^2 > \underline{\beta}$ and we have just shown in step 2 that $\lim_{\sigma \rightarrow 1} dU^{2*}/d\sigma < 0$ for $\beta^2 < \bar{\beta}$ with $\underline{\beta} < \bar{\beta}$, there must exist an $\arg \max_{\sigma} U^{2*} \in (\max\{\sigma_0(\beta^2), 0\}, 1)$. Because $dU^{2*}/d\sigma > 0$ in the neighborhoods of $\sigma \rightarrow 0$ and $\sigma \rightarrow 1$ when $\beta^2 \in (\bar{\beta}, \beta^1]$, it is conceivable that U^{2*} is increasing in σ for all $\sigma \in [0, 1)$. However, it is also conceivable that the maxima identified above are not unique. In Appendix B.1, we establish that U^{2*} is strictly quasi-concave in $\sigma \in (\max\{\sigma_0(\beta^2), 0\}, 1)$. In addition, for parameter values that imply the laggard is resource constrained (i.e., $\sigma \in [0, \sigma_0(\beta^2)]$ and $\beta^2 \in [0, \beta_0(0)]$ where $\beta^2(0) = (3-2\sqrt{2})\beta^1 \approx 0.172\beta^1$), we have $U^{2*} = \beta^1[\sqrt{2(1-\sigma)} - (1-\sigma)]$ which is increasing in σ at a decreasing rate. We have, thus, established that $\sigma_{\max} \equiv \arg \max_{\sigma} U^{2*}$ is global for all $\sigma \in [0, 1)$.

For additional insight, we compare the laggard's payoffs at several extremes. It is trivial to show the following:

$$\begin{aligned} \lim_{\sigma \rightarrow 0} U^{2*} &= \begin{cases} \beta^1(\sqrt{2}-1) & \text{if } \beta^2 \in [0, \beta_0(0)] \\ \frac{\sqrt{\beta^1}}{\sqrt{\beta^1}+\sqrt{\beta^2}} \frac{\beta^1+\beta^2}{2} & \text{if } \beta^2 \in [\beta_0(0), \beta^1] \end{cases} \\ \lim_{\sigma \searrow \sigma_0} U^{2*} &= \frac{\beta^1(\beta^1 - 3\beta^2)(\beta^1 + \beta^2)}{2(\beta^1 - \beta^2)^2}, \text{ for } \beta^2 \in [0, \beta_0(0)] \\ \lim_{\sigma \rightarrow 1} U^{2*} &= U_n^2 = \beta^2, \quad \text{for } \beta^2 \in [0, \beta^1]. \end{aligned}$$

A comparison of the above levels reveals that $\lim_{\sigma \rightarrow 0} U^{2*} \geq U_n^2$ as $\beta^2 \leq 0.432\beta^1$.⁶⁶ ||

A.3 Efficiency of Improvements in Technology and Security

Proof of Proposition 6. Let us first consider the effects of improvements in the degree of output security, σ . From (17), under regime (i) we have $d\bar{U}^*/d\sigma = -d\bar{G}^*/d\sigma$, whereas under regime (ii) we have $d\bar{U}^*/d\sigma = -\sum_{i=1,2} \beta^i (dG^{i*}/d\sigma)$. But, Propositions 2 and 4 establish that, under both regimes, $dG^{1*}/d\sigma < 0$ and $dG^{2*}/d\sigma \leq 0$ always hold (the second as a strict inequality provided the laggard diversifies its production). Thus, $d\bar{U}^*/d\sigma > 0$ always holds.

To examine the effects of improvements in technology, we break down each of the two parts of the proposition ((a) and (b)) into cases 1 and 2: as before, case 1 is when the laggard specializes in appropriation and case 2 is when the laggard diversifies its production.

Part (a): General-purpose technologies ($\beta^1 = \beta^2 = 1$ and $H^1 = \alpha^1 > H^2 = \alpha^2$).

Case 1. Recall from the proof to Proposition 2 that the condition for agent 2 to specialize in

⁶⁶Keep in mind that $\beta_0(0) = (3-2\sqrt{2})\beta^1 \approx 0.172\beta^1$.

appropriation is $m^2 \leq \frac{1}{4}(1 - \sigma)$ (where $m^2 \equiv \frac{\alpha^2}{\alpha^1 + \alpha^2}$) or, equivalently, $\sqrt{(1 - \sigma)/4m^2} \geq 1$. Then, our previous results from the proof to Proposition 2, $dG^{2*}/d\alpha^2 = 1$ and the expression for $dG^{1*}/d\alpha^2$ shown in (A.5), imply $d\bar{G}^*/d\alpha^2 = (1 + m^2)\sqrt{(1 - \sigma)/4m^2} \geq 1 + m^2 > 1$. Hence, from (18a), we have $d\bar{U}^*/d\alpha^2 = 1 - d\bar{G}^*/d\alpha^2 < 0$. One can also show, using (17), that $\bar{U}^* = (\alpha^1 + \alpha^2)[1 - \sqrt{(1 - \sigma)m^2}]$, which implies $\lim_{\alpha^2 \rightarrow 0} \bar{U}^* = \alpha^1$.

Case 2. From the solution in this case where neither country is resource constrained (8b), we have $\bar{G}^* = 2\tilde{G} = \frac{1}{2}(1 - \sigma)(\alpha^1 + \alpha^2)$. Thus, (17) readily implies $\bar{U}^* = \frac{1}{2}(1 + \sigma)(\alpha^1 + \alpha^2)$ and, therefore, $d\bar{U}^*/d\alpha^2 = \frac{1}{2}(1 + \sigma) > 0$, such that \bar{U}^* reaches a maximum (for $\alpha^2 \in [\alpha_0(\sigma), \alpha^1]$) at $\alpha^2 = \alpha^1$, where $\bar{U}^*|_{\alpha^2 = \alpha^1} = (1 + \sigma)\alpha^1$. As shown in case 1 directly above, $\lim_{\alpha^2 \rightarrow 0} \bar{U}^* = \alpha^1$, a maximum value of $\bar{U}^*(\alpha^2)$ for $\alpha^2 \in [0, \alpha_0(\sigma)]$. Thus, $\bar{U}^*(\alpha^2)$ reaches a maximum for all $\alpha^2 \in (0, \alpha^1]$ at $\alpha^2 = \alpha^1$.⁶⁷

Part (b): Sector-specific technology ($H^1 = H^2 = 1$ and $\beta^1 > \beta^2$).

Case 1. Proposition 5(a) has already shown that each country's payoff, in this case, is independent of β^2 . It follows immediately that the sum of their payoffs, $\bar{U}^* = \beta^1[2 - \sqrt{2(1 - \sigma)}]$, is also independent of β^2 .

Case 2. Recall from Proposition 5 that the technology leader always benefits from such progress in this case (i.e., $dU^{1*}/d\beta^2 > 0$). Also recall that, while the laggard benefits from increases in β^2 under most circumstances, the possibility of immiserizing growth—i.e., $dU^{2*}/d\beta^2 < 0$ —cannot be ruled out. In fact, this is what happens for parameter pairs (β^2, σ) in the magenta-colored region of panel (a) of Fig. 3. Hence, if efficiency falls at all with increases in β^2 , it can do so only for a subset of parameters within this region.

We now prove that this is indeed the case when β^2 is sufficiently small and the degree in security is moderate. Noting that $Z^i = H^i - G^i$, we use (A.18) in (18b) to rewrite this effect as

$$\begin{aligned} \frac{d\bar{U}^*}{d\beta^2} &= Z^2 - \frac{\beta^1(1 - \sigma)}{\beta^2\Lambda} [2\beta^2\phi^2X^2 + \sigma(\beta^1 - \beta^2)\phi^1X^1] \\ &\quad + \frac{(1 - \sigma)}{\Lambda} [2\beta^1\phi^1X^1 - \sigma(\beta^1 - \beta^2)\phi^2X^2]. \end{aligned}$$

For $\sigma \in [0, \frac{1}{2})$, we find that

$$\lim_{\beta^2 \searrow \beta_0(\sigma)} \frac{d\bar{U}^*}{d\beta^2} = \lim_{\beta^2 \searrow \beta_0(\sigma)} \frac{\beta^1(1 - \sigma)\phi^1X^1}{\beta^2\Lambda} \times \lim_{\beta^2 \searrow \beta_0(\sigma)} [\beta^2(2 + \sigma) - \beta^1\sigma].$$

Clearly, the first limit in the RHS of the above expression is strictly positive. Thus, the sign

⁶⁷Observe further that there exists a value of α^2 , $\underline{\alpha} \equiv \frac{1 - \sigma}{1 + \sigma}\alpha^1 (> \alpha_0(\sigma) = \frac{1 - \sigma}{3 + \sigma}\alpha^1)$, such that $\bar{U}^*(\alpha^2) > \lim_{\alpha^2 \rightarrow 0} \bar{U}^*$ for all $\alpha^2 \in (\underline{\alpha}, \alpha^1]$. The range of α^2 values for which this last inequality holds expands as σ rises (i.e., $d\underline{\alpha}/d\sigma < 0$).

of $\lim_{\beta^2 \searrow \beta_0} d\bar{U}^*/d\beta^2$ is determined by the sign of the second limit. Using the expression for $\beta_0(\sigma)$ in (14), one can confirm that

$$\lim_{\beta^2 \searrow \beta_0(\sigma)} [\beta^2 (2 + \sigma) - \beta^1 \sigma] = \frac{\beta^1}{\frac{3}{2} - \sigma + \sqrt{2(1 - \sigma)}} \left[1 - \left(3 + \sqrt{2(1 - \sigma)} \right) \sigma \right].$$

The sign of the expression inside the brackets depends on the value of σ . One can show that this expression equals 0 if $\sigma = \underline{\sigma}$, previously defined as being equal to $\sqrt{5} - 2 \approx 0.236$, and $\lim_{\beta^2 \searrow \beta_0} d\bar{U}^*/d\beta^2 \leq 0$ as $\sigma \geq \underline{\sigma}$ for $\sigma \in [0, \frac{1}{2})$. Consequently, efficiency falls with increases in β^2 if the initial value of β^2 is sufficiently close to $\beta_0(\sigma)$ and the value of σ is moderate—or more precisely, $\sigma \in (\underline{\sigma}, \frac{1}{2})$.

Consider now values of $\sigma > \frac{1}{2}$. After some tedious algebra, one can show that

$$\text{sign} \left\{ \lim_{\beta^2 \rightarrow 0} \frac{d\bar{U}^*}{d\beta^2} \right\} = \text{sign} \left\{ 2 - \frac{1}{\sigma^2} \right\},$$

which is negative if $\sigma \in (\frac{1}{2}, \check{\sigma})$, where as previously defined $\check{\sigma} = 1/\sqrt{2} \approx .707$. Bringing together these last two sets of results establishes that $d\bar{U}^*/d\beta^2 < 0$ when β^2 is initially close to $\max\{0, \beta_0(\sigma)\}$ and $\sigma \in (\underline{\sigma}, \check{\sigma})$, and thereby completes the proof. \parallel

B Additional Technical Details (Intended for Online Publication Only)

B.1 Proof of Proposition 5 Continued

Case 2, $\theta = \beta^2$: We have shown that, when $\sigma \in [0, \check{\sigma}]$, there exists at least one $\beta_{\min} \in [0, \beta^1)$ such that $dU^{2*}/d\beta^2|_{\beta^2=\beta_{\min}} = 0$. To prove that β_{\min} is unique in this case, it suffices to show that $d^2U^{2*}/(d\beta^2)^2|_{\beta^2=\beta_{\min}} > 0$. For $\sigma \in (\check{\sigma}, 1)$, we have $dU^{2*}/d\beta^2|_{\beta^2 \rightarrow 0} > 0$ and $U^{2*}|_{\beta^2 \rightarrow 0} < U^{2*}|_{\beta^2 \rightarrow \beta^1}$. Thus, it is conceivable that $dU^{2*}/d\beta^2 < 0$ for some values of $\beta^2 \in [0, \beta^1)$. But, if this inequality were true, then, by the continuity of U^{2*} in β^2 , there would exist a value of β^2 , labeled β' , that implies $dU^{2*}/d\beta^2|_{\beta^2=\beta'} = 0$ and $d^2U^{2*}/d(\beta^2)^2|_{\beta^2=\beta'} < 0$. Clearly, then, for any $\sigma \in [0, 1)$, to prove that U^{2*} is strictly quasi-convex in β^2 all we need to do is prove that $d^2U^{2*}/d(\beta^2)^2|_{\beta^2=\beta'} > 0$ at any β' that solves

$$\frac{dU^{2*}}{d\beta^2} = [(1 - \sigma)\phi^2 + \sigma]Z^{2*} - \beta^1 \left(\frac{dG^{1*}}{d\beta^2} \right) = 0. \quad (\text{B.1})$$

As this expression shows, $dU^{2*}/d\beta^2$ depends on $dG^{1*}/d\beta^2$. It also depends on $G^{2*}/d\beta^2$ and the balance of power $\phi^{i*}/d\beta^2$. Hence, we reproduce (A.18) here for convenience, and also calculate the influence of β^2 on ϕ^{1*} using (A.15):

$$\frac{dG^{1*}}{d\beta^2} = \frac{(1 - \sigma) \left[\beta^1 (\beta^1 - \beta^2) \sigma \phi^{1*} Z^{1*} + 2 (\beta^2)^2 \phi^{2*} Z^{2*} \right]}{\beta^2 \Lambda} > 0 \quad (\text{B.2a})$$

$$\frac{dG^{2*}}{d\beta^2} = - \frac{(1 - \sigma) \left[2 (\beta^1)^2 \phi^{1*} Z^{1*} - \beta^2 (\beta^1 - \beta^2) \sigma \phi^{2*} Z^{2*} \right]}{\beta^2 \Lambda} < 0 \quad (\text{B.2b})$$

$$\frac{d\phi^{1*}}{d\beta^2} = \frac{\phi^{1*} \phi^{2*} [\sigma + (1 - \sigma) \phi^{1*}] [\sigma + (1 - \sigma) \phi^{2*}]}{\beta^2 [\sigma + 2(1 - \sigma) \phi^{1*} \phi^{2*}]} > 0. \quad (\text{B.2c})$$

Clearly, $dG^{1*}/d\beta^2$ in (B.2a) depends on β^2 directly and indirectly through Λ (since $\Lambda \equiv 4\beta^1\beta^2 + (\beta^1 - \beta^2)^2\sigma^2$). It also depends indirectly on β^2 through G^{i*} (because $Z^{i*} = 1 - G^{i*}$) and through ϕ^{i*} for $i = 1, 2$.

With these observations in mind, we can differentiate $dU^{2*}/d\beta^2$ with respect to β^2 and rearrange terms appropriately to obtain

$$\begin{aligned} \frac{d^2U^{2*}}{d(\beta^2)^2} \Big|_{\beta^2=\beta'} &= \overset{(+)}{\Phi_0} + \overset{(-)}{\Phi_1} \left(\frac{d\phi^{1*}}{d\beta^2} \right) + \overset{(+)}{\Phi_2} \left(\frac{d\phi^{2*}}{d\beta^2} \right) \\ &\quad + \overset{(+)}{\Gamma_1} \left(\frac{dG^{1*}}{d\beta^2} \right) + \overset{(-)}{\Gamma_2} \left(\frac{dG^{2*}}{d\beta^2} \right), \end{aligned} \quad (\text{B.3})$$

where defining

$$\pi_0 \equiv \frac{1}{\beta^2} Z^{2*} [\sigma + (1 - \sigma) \phi^{2*}] > 0$$

$$\begin{aligned}
\pi_1 &\equiv \frac{1}{\beta^2 \Lambda} (1 - \sigma) \sigma (\beta^1)^2 \phi^{1*} Z^{1*} > 0 \\
\pi_2 &\equiv \frac{1}{\Lambda} 2\sigma Z^{2*} (\beta^2 \sigma [\sigma + (1 - \sigma) \phi^{2*}] + \beta^1 [2 - \sigma [\sigma + (1 - \sigma) \phi^{2*}]]) > 0 \\
\rho &\equiv 1 - \frac{1}{\Lambda} 2\beta^1 \beta^2 \in [\frac{1}{2}, 1],
\end{aligned}$$

the coefficients Φ_0 , Φ_1 , Φ_2 , Γ_1 and Γ_2 that appear in (B.3) can be shown to be given by

$$\Phi_0 = \pi_0 + \pi_1 + \pi_2 > 0 \quad (\text{B.4a})$$

$$\Phi_1 = -\frac{(1 - \sigma) \sigma (\beta^1)^2 (\beta^1 - \beta^2) Z^{1*}}{\beta^2 \Lambda} < 0 \quad (\text{B.4b})$$

$$\Phi_2 = (1 - \sigma) (1 - \rho) Z^{2*} > 0 \quad (\text{B.4c})$$

$$\Gamma_1 = \frac{(1 - \sigma) \sigma (\beta^1)^2 (\beta^1 - \beta^2) \phi^{1*}}{\beta^2 \Lambda} > 0 \quad (\text{B.4d})$$

$$\Gamma_2 = -[\sigma + (1 - \sigma) \phi^{2*} (1 - \rho)] < 0. \quad (\text{B.4e})$$

To ensure that (B.3) is evaluated at $\beta^2 = \beta'$, we have relied on the following transformations of $dU^{2*}/d\beta^2 = 0$ in (B.1), using (B.2a):

$$1 = \frac{\beta^2 \Lambda [\sigma + (1 - \sigma) \phi^{2*}] Z^{2*}}{\beta^1 (1 - \sigma) [\beta^1 (\beta^1 - \beta^2) \sigma \phi^{1*} Z^{1*} + 2 (\beta^2)^2 \phi^{2*} Z^{2*}]} \quad (\text{B.5a})$$

$$Z^{2*} = \frac{(\beta^1)^2 (\beta^1 - \beta^2) (1 - \sigma) \sigma \phi^{1*} Z^{1*}}{\beta^2 \Lambda [\sigma + (1 - \sigma) \phi^{2*} \rho]}. \quad (\text{B.5b})$$

In particular, we relied on the above relationships to obtain Φ_0 in (B.4a).

Inspection of (B.3) reveals that the last two terms on the first line are negative, while all other terms are positive. Noting that $d\phi^2 = -d\phi^1$, we can use the definitions of Φ_1 and Φ_2 in (B.4), (B.5b), and the value of $d\phi^{1*}/d\beta^2$ in (B.2c) to find

$$\begin{aligned}
(\Phi_1 - \Phi_2) \left(\frac{d\phi^{1*}}{d\beta^2} \right) \Big|_{\beta^2 = \beta'} &= -\frac{Z^{2*} [\sigma + (1 - \sigma) \rho]}{\phi^{1*}} \left(\frac{d\phi^{1*}}{d\beta^2} \right) \\
&= -\frac{\phi^{2*} Z^{2*} [\sigma + (1 - \sigma) \phi^{1*}] [\sigma + (1 - \sigma) \phi^{2*}] [\sigma + (1 - \sigma) \rho]}{\beta^2 [\sigma + 2(1 - \sigma) \phi^{1*} \phi^{2*}]}.
\end{aligned}$$

Since π_1 and π_2 in Φ_0 are positive, to establish $d^2 U^{2*}/d(\beta^2)^2 \Big|_{\beta^2 = \beta'} > 0$ it suffices to prove

$$\pi_0 + (\Phi_1 - \Phi_2) \left(\frac{d\phi^{1*}}{d\beta^2} \Big|_{\beta^2 = \beta'} \right) > 0.$$

After some straightforward algebra that uses the definition of π_0 , one can show that the

above inequality holds true if the following inequality is satisfied:

$$1 - \frac{\phi^{2*} [\sigma + (1 - \sigma) \phi^{1*}] [\sigma + (1 - \sigma) m]}{[\sigma + 2(1 - \sigma) \phi^{1*} \phi^{2*}]} > 0.$$

But the LHS of the above inequality can be rewritten as

$$\frac{\sigma [1 - \phi^{2*} [\sigma + (1 - \sigma) m \phi^{2*}]] + (1 - \sigma) (2 - \sigma - m) \phi^{1*} \phi^{2*}}{[\sigma + 2(1 - \sigma) \phi^{1*} \phi^{2*}]} > 0,$$

which is clearly positive, as needed. $\quad ||$.

Case 2, $\theta = \sigma$: To establish that U^{2*} is strictly quasi-concave in $\sigma \in (\max\{\sigma_0(\beta^2), 0\}, 1)$, we follow the procedure employed above in establishing the strict quasi-convexity of U^{2*} in β^2 and prove that $d^2U^{2*}/d\sigma^2|_{\sigma=\sigma'} \leq 0$ at any σ' in that range that solves $dU^{2*}/d\sigma = 0$.

Recalling the definition of $\bar{X} = \sum_{i=1,2} \beta^i (1 - G^i)$ and from (1) that $\phi_{G^2}^2 = \phi^1 \phi^2 / G^2$ and $\phi_{G^1}^2 = -\phi^1 \phi^2 / G^1$, differentiation of (A.23) with respect to σ yields

$$\frac{d^2U^{2*}}{d\sigma^2} = \left[\beta^1 \phi^{2*} + \frac{\phi^{1*} \phi^{2*} \bar{X}^*}{G^{1*}} \right] \frac{dG^{1*}}{d\sigma} - \left[\beta^2 \phi^{1*} + \frac{\phi^{1*} \phi^{2*} \bar{X}^*}{G^{2*}} \right] \frac{dG^{2*}}{d\sigma} - \beta^1 \frac{d}{d\sigma} \left(\frac{dG^{1*}}{d\sigma} \right).$$

Using the countries' FOCs with the expressions for $\phi_{G^k}^2$ for $k = 1, 2$ shown above, which imply

$$\frac{\beta^i G^{i*} [\sigma + (1 - \sigma) \phi^{i*}]}{(1 - \sigma) \phi^{i*} \phi^{j*} \bar{X}^*} = 1,$$

we can simplify the expressions inside the square brackets of $d^2U^{2*}/d\sigma^2$ as

$$\beta^i \phi^{j*} + \frac{\phi^{i*} \phi^{j*} \bar{X}^*}{G^{i*}} = \beta^i \phi^{j*} + \frac{\phi^{i*} \phi^{j*} \bar{X}^*}{G^{i*}} \left[\frac{\beta^i G^{i*} [\sigma + (1 - \sigma) \phi^{i*}]}{(1 - \sigma) \phi^{i*} \phi^{j*} \bar{X}^*} \right] = \frac{\beta^i}{1 - \sigma}.$$

This allows us to rewrite $d^2U^{2*}/d\sigma^2$ as

$$\frac{d^2U^{2*}}{d\sigma^2} = \frac{\beta^1}{1 - \sigma} \frac{dG^{1*}}{d\sigma} - \frac{\beta^2}{1 - \sigma} \frac{dG^{2*}}{d\sigma} - \beta^1 \frac{d}{d\sigma} \left(\frac{dG^{1*}}{d\sigma} \right). \quad (\text{B.6})$$

where $dG^{i*}/d\sigma$ ($i = 1, 2$ at a fully interior optimum) is shown in (A.24). Note from that expression that $dG^{i*}/d\sigma$ depends on σ directly and indirectly through the impact of σ on Λ . It also depends indirectly on σ through $\bar{G}^* = \sum_{i=1,2} G^{i*}$. Accordingly, to find an expression for $\frac{d}{d\sigma} (dG^{1*}/d\sigma)$ (i.e., the last term of the RHS of (B.6)), we decompose it as follows:

$$\frac{d}{d\sigma} \left(\frac{dG^{1*}}{d\sigma} \right) = \frac{\partial}{\partial \sigma} \left(\frac{dG^{1*}}{d\sigma} \right) \Big|_{d\bar{G}^*=0} + \frac{dG^{1*}/d\sigma}{\bar{G}^*} \left(\frac{d\bar{G}^*}{d\sigma} \right). \quad (\text{B.7})$$

To obtain the direct effect (the first term in (B.7)), we partially differentiate (A.24):

$$\frac{\partial}{\partial \sigma} \left(\frac{dG_{\sigma}^{1*}}{d\sigma} \right) \Big|_{d\bar{G}^*=0} = \frac{1}{1-\sigma} \left[1 - \frac{(\beta^1 - \beta^2)(1-\sigma)}{2\beta^1 - (\beta^1 - \beta^2)\sigma} - \frac{2(\beta^1 - \beta^2)^2 \sigma(1-\sigma)}{\Lambda} \right] \frac{dG^{1*}}{d\sigma}.$$

The indirect effect (i.e., the second term in (B.7)) can be found by noting that $d\bar{G}^*/d\sigma = \sum_i dG^{i*}/d\sigma$. Then, tedious algebra using (A.24) with (B.7) and the expression for the direct effect allows us write (B.6) as

$$\frac{d^2 U^{2*}}{d\sigma^2} = -\frac{6\beta^1(\beta^1 - \beta^2)(1-\sigma)}{\bar{G}^*[2\beta^1 - (\beta^1 - \beta^2)\sigma]} \left(\frac{dG^{1*}}{d\sigma} \frac{dG^{2*}}{d\sigma} \right) \leq 0.$$

It should now be easy for one to confirm that $\sigma_{\max} = \arg \max_{\sigma} U^{2*}$ is unique and that: (i) $\sigma_{\max} = \sigma_0(\beta^2)$ when $\beta^2 \in [0, \underline{\beta}]$; (ii) $\sigma_{\max} \in (\sigma_0(\underline{\beta}), 1)$ when $\beta \in (\underline{\beta}, \bar{\beta})$ where $\bar{\beta} \approx 0.618\beta^1$; and (iii) $dU^{2*}/d\sigma > 0$ for all $\sigma \in [0, 1)$ when $\beta^2 \in [\bar{\beta}, \beta^1]$. $\quad \square$

B.2 Differences in the Military-Use Technology

In what follows, we fill in some of the technical details concerning the extension that supposes the two countries possess different military technologies as described in Section 5.1. Applying the implicit function theorem to (19), we can identify the influence of γ^j and σ on the equilibrium distribution of power:

$$d\phi^{i*} = \frac{\phi^i \phi^j}{\sigma + 2(1-\sigma)\phi^i \phi^j} \left[-(\phi^i + \sigma\phi^j)(\sigma\phi^i + \phi^j) d\gamma^j + (\phi^i - \phi^j) d\sigma \right], \quad (\text{B.8})$$

for $i, j \in \{1, 2\}$, $i \neq j$.⁶⁸ Consistent with our claims in the main text, this expression establishes that $d\phi^{i*}/d\gamma^j < 0$ and $\text{sign}\{d\phi^{i*}/d\sigma\} = \text{sign}\{\phi^i - \phi^j\}$ for $i \neq j$. Thus, an improvement in the rival's military technology reduces a country's power, and an improvement in the degree of output security raises (reduces) the power of the leader (laggard).

Turning to the effects of γ^j and σ on \bar{g}^* , we differentiate (20b) to show

$$\frac{d\bar{g}^*}{d\gamma^j} = -\frac{\bar{H}\sigma(1-\sigma^2)(\phi^i - \phi^j)(d\phi^{i*}/d\gamma^j)^{(-)}}{[\sigma + 2(1-\sigma)\phi^i \phi^j]^2} \quad (\text{B.9a})$$

$$\frac{d\bar{g}^*}{d\sigma} = -\frac{\bar{H} \left(\phi^i \phi^j [1 + \sigma^2 - 2(1-\sigma)^2 \phi^i \phi^j] + \sigma(1-\sigma^2) [(\phi^i - \phi^j)(d\phi^{i*}/d\sigma)]^{(+)} \right)}{[\sigma + 2(1-\sigma)\phi^i \phi^j]^2}. \quad (\text{B.9b})$$

Since $d\phi^{i*}/d\gamma^j < 0$, equation (B.9a) implies $\text{sign}\{d\bar{g}^*/d\gamma^j\} = -\text{sign}\{\phi^i - \phi^j\}$. Thus, the aggregate quantity of resources diverted from butter production rises when the laggard's

⁶⁸We have omitted the asterisk from the variables on the RHS of (B.8) to avoid clutter.

military technology improves, but falls when the leader's military technology improves. As for the effects of an increase in σ shown in (B.9b), observe that the first bracketed term in the numerator is positive, because $1 + \sigma^2 - 2(1 - \sigma)^2 \phi^i \phi^j \geq 1 - 2\phi^i \phi^j > 0$. In addition, since $\text{sign}\{d\phi^{i*}/d\sigma\} = \text{sign}\{\phi^i - \phi^j\}$, the second bracketed term of the numerator on the RHS is positive as well, implying that $d\bar{g}^*/d\sigma < 0$. Accordingly, security enhancements induce greater aggregate butter production.

Our analysis above in connection with (B.8) showed that an increase in γ^j causes the ratio G^{i*}/G^{j*} ($= (\gamma^i g^{i*})/(\gamma^j g^{j*})$) to fall. To understand the drivers of this change, we now turn to study the dependence of g^{i*} and g^{j*} on γ^j . Differentiating g^{i*} and g^{j*} in (20a) with respect to ϕ^i , while using (B.8), gives

$$\frac{dg^{i*}/d\gamma^j}{g^{i*}/\gamma^j} = \frac{(\phi^i + \sigma\phi^j)(\sigma\phi^i + \phi^j)}{\sigma + 2(1 - \sigma)\phi^i\phi^j} \left[\frac{(1 - \sigma)\phi^i\phi^j}{\sigma\phi^i + \phi^j} + \frac{\sigma(\phi^i - \phi^j)}{\sigma + 2(1 - \sigma)\phi^i\phi^j} \right], \quad (\text{B.10a})$$

$$\frac{dg^{j*}/d\gamma^j}{g^{j*}/\gamma^j} = \frac{(\phi^i + \sigma\phi^j)(\sigma\phi^i + \phi^j)}{\sigma + 2(1 - \sigma)\phi^i\phi^j} \left[-\frac{(1 - \sigma)\phi^i\phi^j}{\sigma\phi^i + \phi^j} + \frac{\sigma(\phi^i - \phi^j)}{\sigma + 2(1 - \sigma)\phi^i\phi^j} \right]. \quad (\text{B.10b})$$

respectively. To fix ideas, let us focus on $i = 1$ as the leader and $j = 2$ as the laggard. Then, from (B.10a), the result that the leader is more powerful in equilibrium (i.e., $\phi^{1*} > \phi^{2*}$) implies that $dg^{1*}/d\gamma^2 > 0$. Turning to the effect on the laggard's allocation to conflict, it suffices to consider the expression inside the brackets on the RHS of (B.10b). First, observe that ϕ^{2*} is in the neighborhood of 0 and thus ϕ^{1*} is in the neighborhood of 1, when γ^2 is sufficiently small. Therefore, the first term of the bracketed expression approaches 0, whereas the second term approaches 1. Clearly, then, $dg^{2*}/d\gamma^2 > 0$ in this limit; but, this allocation rises less than proportionately in relation to g^{1*} to cause ϕ^{1*} to rise. Second, note that, when $\gamma^2 \rightarrow \gamma^1$, ϕ^{2*} approaches $\frac{1}{2}$ from below, while ϕ^{1*} approaches $\frac{1}{2}$ from above. Thus, the first term inside the brackets on the RHS of (B.10b) converges to $-\frac{1-\sigma}{2(1+\sigma)} < 0$, whereas the second expression converges to 0; therefore, $\lim_{\gamma^2 \rightarrow \gamma^1} (dg^{2*}/d\gamma^2) < 0$, again causing ϕ^{1*} to rise.⁶⁹

With our previous findings as well as the expressions shown in (B.10), we can now examine the effects of improvements in the laggard's technology on global efficiency and payoffs. Since $\bar{U} = U^1 + U^2 = \bar{X}$ and $d\bar{X}^*/d\gamma^2 = -d\bar{g}^*/d\gamma^2 < 0$, global efficiency falls with increases in γ^j .⁷⁰ To identify the effect of γ^2 on the laggard's payoff, we differentiate U^2 appropriately and invoke the implicit function theorem to find

$$\frac{dU^{2*}}{d\gamma^2} = U_{\gamma^2}^{2*} + U_{g^1}^{2*} \left(\frac{dg^{1*}}{d\gamma^2} \right)^{+}$$

⁶⁹We can study the dependence of g^{2*} on γ^2 in more detail but this is unnecessary for our purposes here.

⁷⁰The implication of (B.9a) that $d\bar{g}^*/d\gamma^1 < 0$ implies, as noted in the text, that global efficiency would rise if the leader experienced a improvement in its military technology ($\gamma^1 \uparrow$).

$$\begin{aligned}
&= \left(\frac{g^1}{\gamma^2}\right) \left\{ \frac{(1-\sigma)\bar{X}\phi^1\phi^2}{g^1} - \left[(1-\sigma)\phi^2 + \frac{(1-\sigma)\bar{X}\phi^1\phi^2}{g^1} \right] \left(\frac{dg^{1^*}/d\gamma^2}{g^{1^*}/\gamma^2} \right) \right\} \\
&= \left(\frac{g^1}{\gamma^2}\right) \left[\phi^1 + \sigma\phi^2 - \left(\frac{dg^{1^*}/d\gamma^2}{g^{1^*}/\gamma^2} \right) \right].
\end{aligned}$$

The third line in the above equation was obtained by using country 1's FOC in the expressions in the second line and simplifying. Now, define $\Xi \equiv \frac{g^1(\phi^1 + \sigma\phi^2)}{\gamma^2[\sigma + 2(1-\sigma)\phi^1\phi^2]}$. Then, one can apply (B.10a) for $i = 1$ (and $j = 2$) to the above expression and simplify to find:

$$\begin{aligned}
\frac{dU^{2^*}}{d\gamma^2} &= \Xi \left[(1-\sigma)\phi^1\phi^2 + \sigma - \frac{\sigma(\phi^1 - \phi^2)(\sigma\phi^1 + \phi^2)}{\sigma + 2(1-\sigma)\phi^1\phi^2} \right] \\
&= \Xi \left[(1-\sigma)\phi^1\phi^2 + \frac{\sigma(1+\sigma)\phi^2}{\sigma + 2(1-\sigma)\phi^1\phi^2} \right] > 0.
\end{aligned}$$

Clearly, then, the direct (and positive) effect of γ^2 on the laggard's payoff dominates the strategic (and negative) effect, such that laggard would always be willing to accept a military technology transfer. However, as argued in the text, since $\bar{U}^* = U^{1^*} + U^{2^*}$ and $d\bar{U}^*/d\gamma^2 < 0$, the finding that $dU^{2^*}/d\gamma^2 > 0$ implies $dU^{1^*}/d\gamma^2 < 0$; thus, the leader would never offer the laggard access to its superior military technology.

Let us briefly discuss the effects of security improvements. First, our previous results that $d\bar{U}^*/d\sigma = d\bar{X}^*/d\sigma$ and $d\bar{X}^*/d\sigma = -d\bar{g}^*/d\sigma < 0$ imply $d\bar{U}^*/d\sigma > 0$. Second, it is very easy to show, using the FOC for country 2 at an interior solution, that $dU^{1^*}/d\sigma = X^1 - \phi^{1^*}\bar{X}^* - dg^{2^*}/d\sigma$ for $i, j \in \{1, 2\}, i \neq j$. Interestingly, we can show that the laggard's payoff ($i = 2$) is increasing in security σ for all $\gamma^2 \in [0, \gamma^1)$. The leader's payoff ($i = 1$) is also increasing in σ if the technological distance in military technologies between the two countries is sufficiently small; however, if this distance is large enough, then the payoff of the leader $i = 1$ falls at sufficiently high levels of σ .

B.3 Complementary Inputs in Butter Production

In this section, we provide some technical details underlying the extension in Section 5.2, where we considered the presence of a complementary (and fixed) input in the production of butter, that gives rise to diminishing returns in human capital ($H^i = \alpha^i$, with $\beta^1 = \beta^2 = 1$). In what follows, we take an alternative approach to study the equilibrium in the arming subgame. This approach involves a transformation of the system of equations, $U_{G^i}^i = 0$ for $i = 1, 2$ using (3''), and turns the focus to the equilibrium values of the countries' appropriative share of contested butter and their contributive shares to the production of contested butter. As before, ϕ^i identifies country i 's appropriative share. Country i 's contributive share is given by $\psi^i \equiv X^i/\bar{X}$, where as previously defined $X^i = (\alpha^i - G^i)^n$ for $i = 1, 2$, $\bar{X} = X^1 + X^2$, and n denotes the elasticity butter output with respect to human capital.

Applying this definition of ψ^i while recalling that $\phi_{G^i}^i = \phi^i \phi^j / G^i$, we can rewrite (3'') in the text as

$$U_{G^i}^i = 0 \implies \frac{\phi^i \phi^j}{G^i} - \frac{n \phi^i \psi^i}{\alpha^i - G^i} = 0, \quad i, j \in \{1, 2\}, i \neq j. \quad (\text{B.11})$$

Now observe the following: First, the definitions of ψ^i , MB_G^i , and MC_G^i allow us to focus on the laggard's *relative* marginal benefit and its *relative* marginal cost as functions of the contributive and appropriative shares: $MB_G^2/MB_G^1 = G^1/G^2 = \phi^1/\phi^2$ and $MC_G^2/MC_G^1 = (\phi^2/\phi^1) (\psi^1/\psi^2)^{(1-n)/n}$, which lead to

$$S(\psi^2, \phi^2, n) \equiv \frac{MB_G^2}{MB_G^1} - \frac{MC_G^2}{MC_G^1} = \frac{\phi^1}{\phi^2} - \left(\frac{\phi^2}{\phi^1}\right) \left(\frac{\psi^1}{\psi^2}\right)^{\frac{1-n}{n}} = 0 \quad (\text{B.12})$$

and which, of course, holds true in equilibrium. Second, we can solve for G^i from (B.11) to obtain $G^i = \frac{\alpha^i \phi^j}{\phi^j + n \psi^i} < \alpha^i$ ($i, j \in \{1, 2\}, i \neq j$). Then, using the fact that $G^i/G^j = \phi^i/\phi^j$ with the just derived solutions for guns enables us to obtain a second relationship,

$$T(\psi^2, \phi^2, \alpha^2, n) \equiv \frac{\phi^1}{\phi^2} - \frac{\alpha^1 \phi^2 (\phi^1 + n \psi^2)}{\alpha^2 \phi^1 (\phi^2 + n \psi^1)} = 0, \quad (\text{B.13})$$

which also holds true in equilibrium for $n \in (0, 1)$. Since $\psi^1 = 1 - \psi^2$ and $\phi^1 = 1 - \phi^2$, the system of equations in (B.12) and (B.13) defines the equilibrium values of ψ^2 and ϕ^2 implicitly as functions of countries' general-purpose technologies and the elasticity of butter production with respect to human capital and helps us to characterize the equilibrium of the arming subgame, which is shown below to be unique.⁷¹ To proceed, we study the properties of $S(\cdot) = 0$ and $T(\cdot) = 0$. Henceforth, we refer to these relationships as schedules S and T , respectively.

Starting with schedule S in (B.12) that defines ψ^2 implicitly as a function of ϕ^2 , one can verify $\lim_{\phi^2 \rightarrow 0} \psi^2(\cdot)|_{S=0} = 0$ while $\lim_{\phi^2 \rightarrow \frac{1}{2}} \psi^2(\cdot)|_{S=0} = \frac{1}{2}$ for any $n \in (0, 1)$.⁷² Furthermore, since MB_G^2/MB_G^1 is decreasing in ϕ^2 and MC_G^2/MC_G^1 is increasing in ϕ^2 , $S_{\phi^2} < 0$ holds. Similarly, it is easy to confirm that MC_G^2/MC_G^1 is decreasing in ψ^2 while MB_G^2/MB_G^1 is independent of ψ^2 , such that $S_{\psi^2} > 0$ holds. Bringing these two results together, we have that, for any given $n \in (0, 1)$, $d\psi^2/d\phi^2|_{S=0} = -S_{\phi^2}/S_{\psi^2} > 0$, which is akin to a form of complementarity. Provided $\phi^2 \in (0, \frac{1}{2})$, the range of $\psi^2(\cdot)|_{S=0}$ equals $(0, \frac{1}{2})$.

⁷¹With the solutions for G^i for $i = 1, 2$ shown above, we can characterize equilibrium arming. For our purposes, however, it suffices to focus on the equilibrium shares.

⁷²One can also show that $\phi^2 \rightarrow 1$ would imply $\psi^2 \rightarrow 1$ along schedule S . In fact, since the only source of asymmetry in the model is due to differences in general-purpose technologies (α^1 and α^2) and these technologies do not appear in $S(\cdot) = 0$, this schedule is symmetric across countries. Here, we confine our attention to values of ϕ^2 and ψ^2 in $(0, \frac{1}{2})$ because, as we will see shortly, that is the range of equilibrium values when $\alpha^2 \in (0, \alpha^1)$.

Next we explore how the laggard's contributive and appropriative shares compare along schedule S . Here is where the value of elasticity n comes into play. We will show that $\psi^2(\cdot)|_{S=0} \underset{\geq}{\leq} \phi^2$ as $n \underset{\geq}{\leq} \frac{1}{3}$, for any $\phi^2 \in (0, \frac{1}{2})$. Thus, in the special case of $n = \frac{1}{3}$, $\psi^2(\cdot)|_{S=0}$ is linear in ϕ^2 . We will also show that $n > \frac{1}{3}$ (resp., $n < \frac{1}{3}$) implies $\psi^2(\cdot)|_{S=0}$ is strictly convex (resp., concave) in ϕ^2 . These properties prove helpful in characterizing the leader's willingness to offer a general-purpose technology transfers to the laggard.

Maintaining a focus on values of $\phi^2 \in (0, \frac{1}{2})$ (which ensures $\phi^1 > \phi^2$ and $\psi^1 > \psi^2$ along schedule S), one can easily confirm from the expression for MC_G^2/MC_G^1 ($i = 1, 2$) that a decrease in n raises the laggard's marginal cost of arming relative to the technology leader's corresponding marginal cost:

$$\frac{\partial (MC_G^2/MC_G^1) / \partial n}{MC_G^2/MC_G^1} = -\frac{\ln(\psi^1/\psi^2)}{n^2} < 0.$$

In turn, it follows from schedule S in (B.12) that $S_n > 0$. Because $S_{\psi^2} > 0$ as already established, we have $d\psi^2/dn|_{S=0} = -S_n/S_{\psi^2} < 0$. In short, a reduction in the elasticity of human capital in butter production n brings about an increase in the laggard's contributive share ψ^2 , for any given $\phi^2 \in (0, \frac{1}{2})$, along schedule S .

Next, define $z \equiv (\phi^1/\phi^2)^{(1-3n)/(1-n)}$ and note that, because $\phi^1/\phi^2 > 1$ for any $\phi^2 \in (0, \frac{1}{2})$, $n \underset{\geq}{\leq} \frac{1}{3}$ implies $z \underset{\leq}{\geq} 1$. Now observe from (B.12) that

$$S(\cdot) = 0 \implies \frac{\psi^2}{\psi^1} = \left(\frac{\phi^1}{\phi^2}\right)^{-\frac{2n}{1-n}} \implies \frac{\psi^2/\phi^2}{\psi^1/\phi^1} = z.$$

Using the fact that $\psi^1 = 1 - \psi^2$ allows us to transform the last equality shown above as $1 - \psi^2/\phi^2 = (1 - z)/(1 + z\phi^2/\phi^1)$. Hence, $n \underset{\geq}{\leq} \frac{1}{3}$ implies $\phi^2 \underset{\geq}{\leq} \psi^2|_{S=0}$. In addition, we can again use the fact that $\psi^1 = 1 - \psi^2$ with the second equality above to find an explicit solution for $\psi^2|_{S=0}$: $\psi^2|_{S=0} = [1 + (\phi^1/\phi^2)^{\frac{2n}{1-n}}]^{-1}$. Keeping in mind that $\phi^1 = 1 - \phi^2$, we differentiate $\psi^2|_{S=0}$ twice with respect to ϕ^2 to arrive at

$$\text{sign} \left\{ \frac{d^2\psi^2}{(d\phi^2)^2} \Big|_{S=0} \right\} = \text{sign} \left\{ \phi^2 - \psi^2 + \frac{3}{1-n} \left(n - \frac{1}{3}\right) \left(\frac{1}{2} - \psi^2\right)^{(+)} \right\}.$$

With our focus on values of $\psi^2 \in (0, \frac{1}{2})$, an application of the finding that $\phi^2 \underset{\geq}{\leq} \psi^2|_{S=0}$ as $n \underset{\geq}{\leq} \frac{1}{3}$ to the RHS of the above expression allows us to infer that $n \underset{\geq}{\leq} \frac{1}{3}$ implies $d^2\psi^2/(d\phi^2)^2|_{S=0} \underset{\geq}{\leq} 0$, as claimed above.⁷³ The important insight here is that the elasticity of butter with respect to human capital n shapes the equilibrium relationship between the contributive and appropriative shares governed by schedule S .

⁷³Inspection of the above equation also reveals that $\lim_{\phi^2 \rightarrow 0} (d^2\psi^2/(d\phi^2)^2|_{S=0}) = 0$, which signals the presence of an inflection point.

For clarity, we illustrate the functions $\psi^2(\phi^2, n)|_{S=0}$ associated with schedule S in panel (a) of Fig. B.1 for three values of n : $n = \frac{1}{3}$, $n' > \frac{1}{3}$, and $n'' < \frac{1}{3}$. This panel also illustrates the following noteworthy features of schedule S :

- (a) $\lim_{\phi^2 \rightarrow 0} (\psi^2(\cdot)|_{S=0}) / \phi^2 = 0$ when $n > \frac{1}{3}$.
- (b) $\lim_{\phi^2 \rightarrow 0} (\psi^2(\cdot)|_{S=0}) / \phi^2 = \infty$ when $n < \frac{1}{3}$.
- (c) $\lim_{\phi^2 \rightarrow 0} (\psi^2(\cdot)|_{S=0}) / \phi^2 = 1$ when $n = \frac{1}{3}$.

Once again, these features have useful implications for the payoff effects of the laggard's general-purpose technology. However, before getting to those implications, we need to characterize the properties of schedule T .

Schedule T shown in (B.13) implicitly defines the second relationship between ψ^2 and ϕ^2 that also arises in equilibrium. Partial differentiation of $T(\cdot)$ gives

$$\begin{aligned} T_{\phi^2} &= -(\phi^2)^{-2} \left(1 + \phi^1 \frac{n\psi^1}{\phi^2 + n\psi^1} + \phi^2 \frac{n\psi^2}{\phi^1 + n\psi^2} \right) < 0 \\ T_{\psi^2} &= -n(\phi^1/\phi^2) \left(\frac{1}{\phi^2 + n\psi^1} + \frac{1}{\phi^1 + n\psi^2} \right) < 0 \\ T_n &= (\phi^1/\phi^2) \left[\frac{\phi^1\psi^1 - \phi^2\psi^2}{(\phi^2 + n\psi^1)(\phi^1 + n\psi^2)} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ as } \phi^1\psi^1 \begin{matrix} \geq \\ \leq \end{matrix} \phi^2\psi^2 \\ T_{\alpha^2} &= \frac{\phi^1/\phi^2}{\alpha^2} > 0. \end{aligned}$$

From the above, one can see that $d\psi^2/d\phi^2|_{T=0} = -T_{\phi^2}/T_{\psi^2} < 0$; therefore, schedule T is downward sloping as shown in the panels of Fig. B.1. In view of the symmetric structure of schedule T , it should be clear that $\psi^2 = \phi^2 = \frac{1}{2}$ is a point on the schedule when $\alpha^2 = \alpha^1$. Given that $T_{\alpha^2} > 0$ and our focus on $\alpha^2 \in (0, \alpha^1)$, it should also be clear that schedule T cuts the upper horizontal axis (i.e., where $\psi^2 = \frac{1}{2}$) at some point $\phi^2 < \frac{1}{2}$.⁷⁴ These properties of schedule T together with the fact that schedule S is upward sloping imply that, for any given $\alpha^2 \in (0, \alpha^1)$, these schedules will cross each other at a unique point, the equilibrium shares $(\phi^{2*}, \psi^{2*}) < (\frac{1}{2}, \frac{1}{2})$. Since $\phi^{1*}\psi^{1*} > \phi^{2*}\psi^{2*}$ at this equilibrium, we will have $T_n > 0$; therefore, reductions in n shift schedule T leftward. Panel (a) in Fig. B.1 illustrates the equilibria that are associated with alternative values of n .

Let us now study the effects of improvements in the laggard's general-purpose technology ($\alpha^2 \uparrow$) perhaps due to a technology transfer. Observe that, while schedule S is independent of α^2 , $T_{\alpha^2} > 0$. Since an increase in α^2 effectively reduces the laggard's relative marginal cost of producing guns, an improvement in the laggard's general-purpose technology implies a larger value of ϕ^2 for each value of ψ^2 along that curve. This effect is illustrated in panels (b) and (c) of Fig. B.1. In panel (b) assuming $n > \frac{1}{3}$, the associated equilibria are depicted

⁷⁴On a more technical note, observe that the partial derivatives of $T(\cdot)$ shown above make economic sense only for $\phi^2 \in (\underline{\phi}^2, \bar{\phi}^2)$, where $\underline{\phi}^2 = \phi^2(1)|_{T=0} < \bar{\phi}^2 = \phi^2(0)|_{T=0}$ along schedule T .

by points B , B' and B'' ; and in panel (c) where $n < \frac{1}{3}$, the associated equilibria are depicted by points C , C' and C'' . Importantly, in both cases, these productivity improvements induce the laggard to increase both its equilibrium appropriative (ϕ^{2*}) and its contributive (ψ^{2*}) shares. However, there is an important difference. When $n > \frac{1}{3}$ (panel (b)), the laggard's contributive share rises relative to its appropriative share (i.e., $\psi^{2*}/\phi^{2*} \uparrow$) due to the strict convexity of $\psi^2|_{S=0}$ in ϕ^2 (i.e., schedule S) discussed earlier. In contrast, when $n < \frac{1}{3}$ (panel (c)), the ratio ψ^{2*}/ϕ^{2*} falls due to the strict concavity of schedule S . These findings play key roles in the welfare analysis below.

As in the baseline model where $n = 1$, an improvement in the laggard's general-purpose technology ($\alpha^2 \uparrow$) generates a positive direct payoff effect and an adverse strategic payoff effect for each side. Focusing on the leader, differentiation of its payoff with respect to α^2 , while using the laggard's FOC for arming, delivers

$$\frac{dU^{1*}/d\alpha^2}{U^{1*}} = \frac{1}{G^{2*}} [\phi^{1*} - dG^{2*}/d\alpha^2]. \quad (\text{B.14})$$

The first term inside the brackets is associated with the direct effect and the second term with the indirect effect noted above. It is straightforward to show that

$$\frac{dG^{2*}}{d\alpha^2} = \frac{\phi^{1*} (1 - n + n\psi^{2*})}{(1 - n)\phi^{1*} + 2n\psi^{2*}} > 0, \quad (\text{B.15})$$

thereby confirming the point that the strategic effect of increasing α^2 on U^{1*} is negative. The key question here is how these conflicting payoff effects compare to each other. Substituting (B.15) into (B.14) and simplifying the resulting expression allows us to recalculate the net effect as follows:

$$\begin{aligned} \frac{dU^{1*}/d\alpha^2}{U^{1*}} &= \frac{\phi^{1*}}{G^{2*}} \left[1 - \frac{1 - n + n\psi^{2*}}{(1 - n)\phi^{1*} + 2n\psi^{2*}} \right] \\ &= \frac{n\phi^{1*}\phi^{2*}}{G^{2*} [(1 - n)\phi^{1*} + 2n\psi^{2*}]} \left[\frac{\psi^{2*}}{\phi^{2*}} - \frac{1 - n}{n} \right]. \end{aligned} \quad (\text{B.16})$$

Clearly, the sign of the net effect of α^2 on the leader's payoff depends on how the ratio of its contributive share over its appropriative share (i.e., ψ^{2*}/ϕ^{2*}) compares with the ratio of elasticities in butter associated with the complementary input and human capital (i.e., $(1 - n)/n$). *Ceteris paribus*, the more extensive the laggard's use of the general-purpose technology and its resources in the production of butter, as compared to the production of guns, the more likely that the leader will find a technology transfer appealing. This is so because increases in ψ^{2*}/ϕ^{2*} tend to reduce the intensity of the adverse strategic effect of arming. But, there is another side to this. The lower is the value of n , the stronger is the laggard's arming response to increases in α^2 .

To dig deeper, suppose $n = \frac{1}{3}$ (so that $\frac{1-n}{n} = 2$ in (B.16)) and recall from our analysis of schedule S that $\psi^{2*} = \phi^{2*}$ in this case. Since $dU^{1*}/d\alpha^2 < 0$ for all $\alpha^2 \in (0, \alpha^1)$, the technology leader now finds general-purpose technology transfers unappealing. Note that this stands in sharp contrast to the related result in the baseline model.

Next, fix n at some level in $[\frac{1}{2}, 1)$, so that $\frac{1-n}{n} \in (0, 1]$, as indicated by the slope of the green dashed-line ray from the origin in panel (b) of Fig. B.1. Now let α^2 rise gradually from very low levels all the way to α^1 , so that the equilibrium moves from point O , to points B, B', B'' and eventually to point O' on schedule S . At low levels of α^2 the sign of $dU^{1*}/d\alpha^2$ is negative. However, when α^2 becomes sufficiently large, $dU^{1*}/d\alpha^2$ becomes positive. This suggests that U^{1*} is U -shaped in α^2 when $n \in [\frac{1}{2}, 1)$. Interestingly, if we considered values of $n \in (\frac{1}{3}, \frac{1}{2}]$, so that $\frac{1-n}{n} \in (1, 2]$, the slope of the green ray would exceed 1, and U^{1*} would unambiguously decrease with increases in α^2 throughout its domain.

Turning to $n \in (0, \frac{1}{3})$, which implies $\psi^2(\phi^2, n)|_{S=0}$ is strictly concave in ϕ^2 as shown in panel (c) of Fig. B.1, successive increases in α^2 now shift the equilibrium from point O to points C, C', C'' and eventually to point O' on schedule S . In this case, at very low levels of α^2 , the ratio ψ^{2*}/ϕ^{2*} starts at high values ($> \frac{1-n}{n}$), crosses $\frac{1-n}{n}$, and eventually falls to 1. Thus, U^{1*} falls for most values of α^2 , as illustrated in panel (c) of Fig. 5 in the text. It is conceivable that U^{1*} rises with increases in α^2 at sufficiently low levels of α^2 . In this case (not shown in Fig. 5(c)), U^{1*} would attain a maximum. However, numerical analysis of the model reveals that this possibility arises only when n is extremely low.

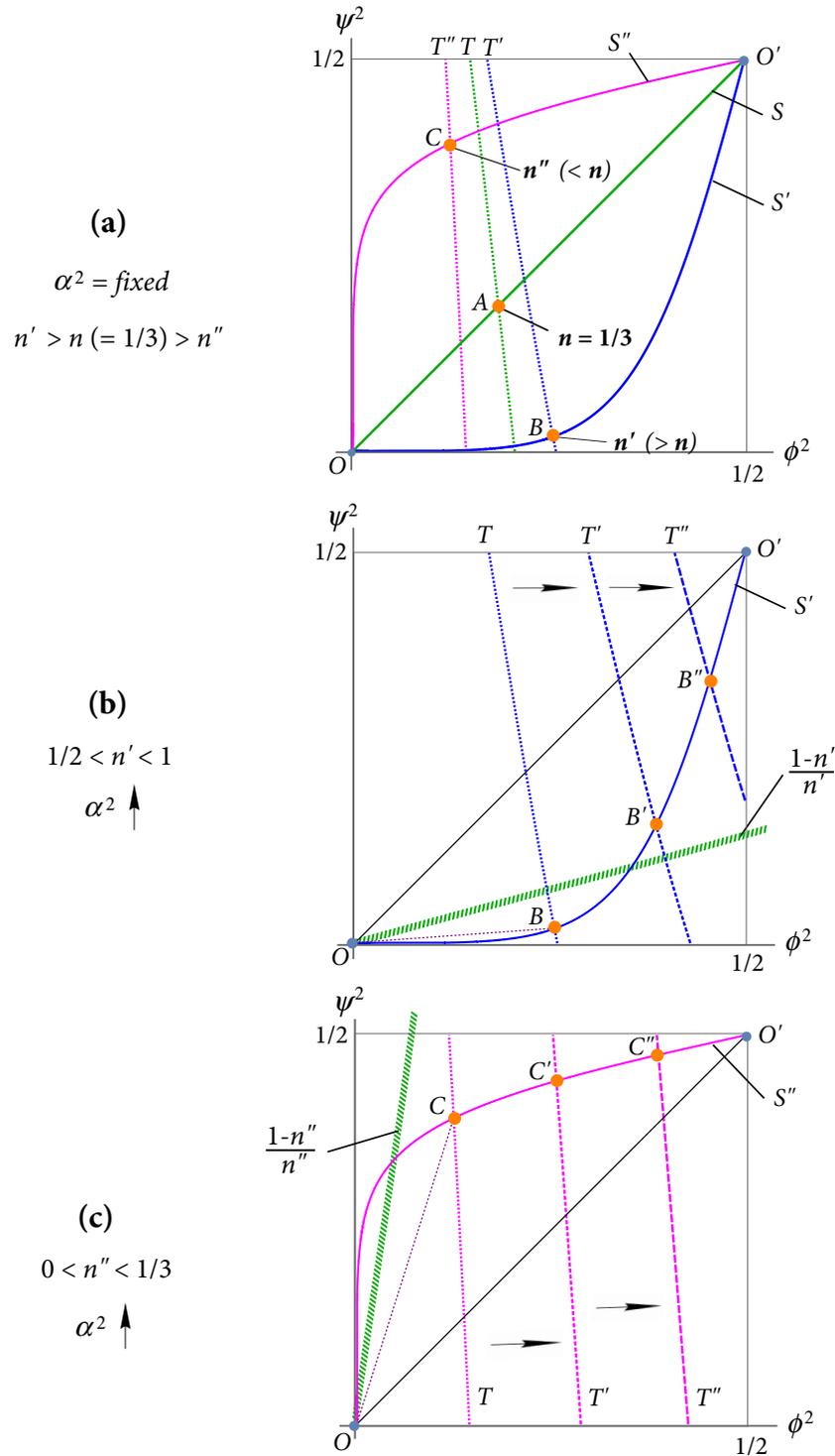


Figure B.1: The Effects of Changes in General-Purpose Technology and the Elasticity of Human Capital in Butter Production on Equilibrium Shares.