

# BUSINESS CYCLES, UNEMPLOYMENT INSURANCE, AND THE CALIBRATION OF MATCHING MODELS

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# BUSINESS CYCLES, UNEMPLOYMENT INSURANCE, AND CALIBRATION OF MATCHING MODELS

## Abstract

This paper points out an empirical failing of real business cycle models in which unemployment is endogenized through a matching function. One can easily choose a calibration to make the cyclical fluctuation in unemployment as large in the model as it is in the data, or to make the response of unemployment to a change in the unemployment benefit as small in the model as it is in the data. We show with a simple analytical calculation that in the standard job matching model, one cannot do both: improving the fit along one dimension makes it worse along the other. This conclusion is robust to the inclusion of capital, variable search intensity, variable match separation, or efficiency wages. We also propose two possible resolutions of the problem. Both sticky wages and embodied technological progress raise the business cycle variability of unemployment, without greatly changing the effects of policies, because they both make the flow of surplus to the firm more procyclical.

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# 1 Introduction

A model of real business cycles with matching (RBCM) is a natural candidate for exploring many dynamic policy issues. Postulating a job matching function helps us give a coherent analysis of unemployment and its response to labor market policies. Moreover, several authors, starting with Merz (1995) and Andolfatto (1996), have claimed that endogenizing unemployment by means of a matching function improves the fit of real business cycle models. Thus it is tempting to use the RBCM framework to measure the costs of business cycles, to measure the purported benefits of output stabilization, to ask whether unemployment benefits should be constant over time, or to ask whether the government should attempt to limit job loss during recessions.

These questions interest us. But when we tried to build an RBCM model to address them, we quickly encountered problems with the RBCM framework which previous literature has not pointed out. For our purposes, we hoped to calibrate our model to be consistent both with business cycle facts and with the effects of labor market policies. We found it easy to choose parameters to make the cyclical fluctuation in unemployment as large in the model as it is in the data, or to make the response of unemployment to a change in the unemployment insurance (UI) benefit as small in the model as it is in the data. But no parameterization permits the standard RBCM model to reproduce both these features of the data; improving the fit over the business cycle makes the fit worse with respect to labor market policies, and *vice versa*. Similar conclusions hold for the volatilities of employment, vacancies, and the probability of job finding.

In this paper, after briefly discussing stylized facts, we show analytically in a benchmark RBCM model that there is a strict relationship between the volatility of unemployment over the cycle, and the responsiveness of unemployment to UI benefits. This prediction of the model is seriously inconsistent with the data. We then show numerically that this problem remains even when we consider more complicated and realistic versions of the model, and is also present but undiagnosed in previous papers.

Finally, we propose two possible solutions to the problem: sticky wages, or embodied technological progress. Sticky wages obviously make firms' share of surplus more procyclical, and thus allow hiring to vary more at business cycle frequencies without greatly changing the long run effects of policies. Embodied technological change also increases the cyclical volatility of the match surplus relative to trend, especially for the firm, so that we can get a similar effect without arbitrarily imposing wage rigidities.

## 1.1 Stylized facts

Productivity shocks affect the benefits of employment, and unemployment insurance affects its costs, so in an RBCM model these two factors have closely related implications for employment and unemployment. This paper shows that the RBCM framework can be

tested by comparing the business cycle variability of unemployment with its variability in response to changes in UI. We now discuss some stylized facts about these two aspects of unemployment variability.

## Unemployment over the business cycle

While employment varies less than output over the business cycle, the same data imply that unemployment is highly volatile relative to its low mean. In seasonally-adjusted US quarterly data from 1951:1 to 2003:1, we calculate a mean unemployment rate of 0.0567.<sup>1</sup> Detrending with the HP filter (using  $\lambda = 1600$ ), we find that the standard deviation of the unemployment rate is 0.00743; that is, the standard deviation is  $\frac{0.00743}{0.0567} = 13.1\%$  of the mean. By contrast, using the same numbers, the standard deviation of the employment rate is only  $\frac{0.00743}{1-0.0567} = 0.787\%$  of its mean.

Similarly, if we consider the log of the unemployment rate, we find that its standard deviation after HP filtering is  $\sigma_U = 0.135$ . The standard deviation of HP-filtered log GDP in our data is  $\sigma_Q = 0.0165$ , so by this measure, unemployment fluctuates much more than output: the ratio  $\sigma_U/\sigma_Q$  equals 8.18. Other authors roughly agree; Merz (1995) finds that  $\sigma_U/\sigma_Q = 6.11$ ,<sup>2</sup> while Greenwood, Gomes, and Rebelo (2002) find  $\sigma_U/\sigma_Q = 7.68$ .<sup>3</sup> Moreover, HP filtering with  $\lambda = 1600$  removes much of the variation in unemployment that might usually be considered cyclical. Before HP filtering, the standard deviation of the log of the (seasonally adjusted) unemployment rate is 0.282 (more than twice the value after HP filtering). Thus by looking at HP-filtered data we may be understating the cyclicity of unemployment.

Several related series also show high cyclical volatility. We find that the log of median unemployment spell duration has a standard deviation of 0.128 after HP filtering, which is 7.77 times the variability of output; likewise, Greenwood et. al. (2002) state that duration is 6.87 times as variable as output. In our data, the HP-filtered log of vacancies (help-wanted advertising) has standard deviation  $\sigma_V = 0.140$ , so that  $\sigma_V/\sigma_Q = 8.49$ ; Merz (1995) reports  $\sigma_V/\sigma_Q = 7.31$ , while Andolfatto (1996) states that this ratio is greater than 9. The variability of workers' probability of job finding is also similar; Shimer (2003) shows that the coefficients of variation of unemployment, vacancies, and workers' probability of job finding are 0.188, 0.183, and 0.17, respectively.<sup>4</sup>

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<sup>1</sup>Our data source is the Federal Reserve Bank of St. Louis' FRED database, quarterly US data, or monthly US data aggregated to quarterly frequency. We use the series GDPC1 for our measure of real output, UNRATE for the unemployment rate, HELPWANT for vacancies, UEMP MED for median unemployment duration. When HP filtering, we always set  $\lambda = 1600$  for comparability with most related papers.

<sup>2</sup>Merz uses US quarterly data, 1959:1-1988:2, in logs, HP filtered with  $\lambda = 1600$ .

<sup>3</sup>Greenwood et. al. use US quarterly data, 1954:1-1991:2, seasonally adjusted, logged, and HP-filtered with  $\lambda = 1600$ .

<sup>4</sup>US data, 1951-2001, quarterly averages of seasonally adjusted monthly data, expressed as ratio to HP trend. Shimer includes more of the cyclical variation of unemployment by setting the HP parameter at  $\lambda = 100000$ .

In our effort to model the cyclical fluctuations of the labor market, we have found that two other stylized facts help to distinguish between competing models. Cole and Rogerson (1999) report that job creation is four times as volatile as employment, and job destruction is six times as volatile as employment.<sup>5</sup> The negative correlation between vacancies and unemployment (the “Beveridge curve”) is also a decisive feature of the data. After HP filtering, we find that the correlation between the log unemployment rate and log vacancies is -0.933. Merz (1995) finds that this correlation is -0.95, while Shimer (2003) reports -0.90.

### **Labor market policy and unemployment**

A large literature has documented the negative effect of unemployment benefits on employment. Many studies based on labor market micro data have regressed reemployment probabilities or unemployment durations on UI benefits. Layard, Nickell, and Jackman (1991) review this literature and conclude that the consensus range of estimates for the elasticity of unemployment duration with respect to the unemployment benefit is from 0.2 to 0.9. Atkinson and Micklewright (1991) come to similar conclusions.

Studies of other types of data have produced similar results. Meyer (1990) measures the rise in workers’ probability of job finding as UI benefits expire. Meyer (1995) summarizes policy experiments in which quick job finding was rewarded with a lump sum payment. Solon (1985) documents a fall in unemployment duration after the imposition of a tax on UI benefits in the US. All these studies are consistent with the relatively low elasticities found in microeconomic regressions of unemployment on benefits. Roughly summarizing Solon’s study, imposing a 25% tax on a 50% UI benefit decreased durations of one quarter by 1.2 weeks, implying an elasticity of duration with respect to benefits of approximately 0.4.<sup>6</sup>

For our purposes, though, what is really interesting is the general equilibrium effect of UI. Layard, Nickell, and Jackman (1991), in a cross-country regression for the OECD, report that a one percentage point rise in the UI replacement ratio results in a highly significant 0.17 percentage point rise in unemployment. Given an average unemployment rate and replacement ratio of roughly 8% and 60% in their sample, this works out to a semielasticity of 2.1 or an elasticity of 1.3. Similarly, Scarpetta (1996) finds a rise of 0.13 percentage points. More recently, with more data, Layard and Nickell (1999) find that the semielasticity of unemployment with respect to the unemployment insurance replacement ratio is 1.3, with a standard error of 0.5.<sup>7</sup> That is, a rise in

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<sup>5</sup>US quarterly manufacturing data, from LRD database, 1972:2-1988:4, in logs, seasonally adjusted, and HP filtered with  $\lambda = 1600$ .

<sup>6</sup>See Costain (1997) for details of these calculations.

<sup>7</sup>Regression of log unemployment on replacement ratio and other labor market policy variables, for 20 OECD countries, treating averages for 1983-88 and 1989-94 as separate observations.

the replacement ratio by one percentage point increases the log of the unemployment rate by 0.013, an elasticity of approximately 0.78. Thus, as we should expect, the general equilibrium effects of UI on unemployment appear moderately larger than the partial equilibrium effects on workers' unemployment durations. Literature reviews including Layard, Nickell, and Jackman (1991), and Disney (2000) are consistent with the conclusion that the semielasticity of unemployment with respect to the replacement ratio is slightly larger than one. Therefore we will take Layard and Nickell's (1999) semielasticity estimate as our main point of reference.

## 1.2 Related models

Two influential studies, Merz (1995) and Andolfatto (1996), have argued that including a matching function improves the fit of RBC models by increasing the persistence of fluctuations. Obviously, it also allows them to compare the model more closely to labor market data. Andolfatto claims success in matching the volatility of employment, though he does not address the volatility of unemployment and underpredicts the volatility of vacancies. Merz is fairly successful with all three variables; her model generates about 80% of the observed standard deviation of US employment and unemployment (relative to output), and about 60% of that of vacancies.

However, other studies report difficulties with the model. Cole and Rogerson (1999) show that even in a minimalist reduced form of the model it is hard to reproduce the negative correlation and persistence of job creation and job destruction, though they are successful if they assume a high baseline unemployment rate (around 15%). Millard, Scott, and Sensier (1997) find that their RBCM model generates fluctuations of employment and unemployment that are too small and insufficiently persistent. Shimer (2003) finds a similar result, and argues that the problem lies in making unemployment and vacancies sufficiently volatile without generating large counterfactual fluctuations in wages or labor productivity. Most recently, Hall (2003) argues that sticky wages are needed to generate volatile unemployment.

The papers of Shimer and Hall are the ones most closely related to our own work. These papers fail to make unemployment fluctuate significantly because they calibrate a relatively large match surplus. In contrast, we show that when we remove this restriction, our model fits cyclical labor market fluctuations well. But the standard model still fails on another measure. We show that the RBCM model has strong simultaneous implications for business cycle variability and for the effects of labor market policy. This is where we locate the failure of the standard model: the small match surplus needed to reproduce business cycles is inconsistent with the effects of policy. However, we also differ from previous literature in that we spell out a flexible-price, optimal version of the model which does in fact fit quite well: a version with embodied technological change.

Of course, many authors have looked at the effects of UI benefits in the context of matching models; see for example the references in Pissarides (2000), p. 233. One study that uses a matching model to explain cross-country differences in unemployment is Millard and Mortensen (1996). One of the few studies that attempts to model both the cyclical volatility of unemployment and the response of unemployment to unemployment benefits is Greenwood, Gomes, and Rebelo (2002). Their model does not fall into the class consider here, because it has no matching function. Interestingly, though, it suffers from the same failing as the RBCM models we analyze. It does well at business cycle frequencies, but reports a much larger response of unemployment to UI benefits than that found in the data.

## 2 The model

Our general model is a version of the standard RBCM model, as spelled out in Pissarides (2000) and elsewhere. We simplify by leaving out capital; including it would be likely to reinforce our result that RBCM models exaggerate policy effects relative to cyclical volatility, since capital can more easily adjust to long term policy changes than to short term business cycle fluctuations.<sup>8</sup> In hopes of finding a successful version of the model, we generalize in several ways: we allow productivity to vary across matches, and we allow separation rates and bargaining power to vary too.

### 2.1 Values and surpluses

Let  $Z$  be a random shock to the productivity of the economy. Let  $z$  be the value of this shock at the time when a given job was formed. We consider a process for the marginal product of labor  $y$  that allows the output of a match to depend on its vintage:

$$y(z, Z) = 1 + \alpha_Z Z + \zeta(1 - \alpha_Z)z \tag{1}$$

The commonest specification is  $\alpha_Z = 1$ , so that technology shocks affect all matches equally, while  $\alpha_Z = 0$  means that a change in the productivity index only affects new matches. The parameter  $\zeta$  allows us to adjust the productivity impact of the cohort-specific shock  $z$  relative to the aggregate shock  $Z$ .

It is well known that in matching models without a capital stock, surpluses and most decision variables are independent of the unemployment rate. Without mentioning unemployment, we can write transition probabilities in terms of labor market tightness, which in turn depends on productivity. To save on notation, we immediately impose

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<sup>8</sup>We also simplify by ignoring two other generalizations that are unlikely to resolve the dilemma that interests us. One might want to consider procyclical unemployment benefits (since benefits are usually computed as a fraction of the wage) or procyclical hiring costs (since the cost of hiring may consist mostly of labor time). However, these factors would only make firms' hiring expenditure *less* procyclical, so they are not likely to help us.

these restrictions by writing the value and policy functions in terms of their appropriate state variables. Later we point out why these restrictions are valid.

The value for a employed worker,  $W^E(z, Z)$ , satisfies

$$W^E(z, Z) = w(z, Z) + \beta E_{Z'|Z} [(1 - \delta(z, Z))W^E(z, Z') + \delta(z, Z)W^U(Z')] \quad (2)$$

Note that we generalize to allow the separation rate  $\delta$  to depend on productivity. We will see that the probability of finding a job can be written as  $p(S, \theta)$ , where  $S$  is search effort and  $\theta$  is labor market tightness. The value  $W^U(Z)$  of unemployment is:

$$W^U(Z) = \max_S \{b - h(S) + \beta E_{Z'|Z} [p(S, \theta(Z))W^E(Z', Z') + (1 - p(S, \theta(Z)))W^U(Z')]\} \quad (3)$$

Here  $b$  represents the unemployment benefit, though in general it should also be understood to capture other costs of working, such as disutility costs.  $S$  is the intensity of job search and  $h(S)$  are the costs of searching. Most of the time we will fix  $S = 1$  and  $h(S) = 0$ , but we will also investigate the effect of varying search intensity.

The workers' surplus is defined as the difference between the values of employment and unemployment; it satisfies

$$\begin{aligned} \Sigma^W(z, Z) &= W^E(z, Z) - W^U(Z) \\ &= w(z, Z) - b + h(S(Z)) + \beta E_{Z'|Z} [(1 - \delta(z, Z))\Sigma^W(z, Z') - p(S(Z), \theta(Z))\Sigma^W(Z', Z')] \end{aligned} \quad (4)$$

where  $S(Z)$  denotes the optimal search at  $Z$ . If search is endogenous, then it obeys the first-order condition

$$h'(S(Z)) = \beta \frac{\partial p(S(Z), \theta(Z))}{\partial S} E_{Z'|Z} \Sigma^W(Z', Z') \quad (5)$$

The value to the firm of a filled job,  $J(z, Z)$ , satisfies the recursive equation

$$J(z, Z) = \Sigma^F(z, Z) = y(z, Z) - w(z, Z) + \beta(1 - \delta(z, Z))E_{Z'|Z} J(z, Z') \quad (6)$$

Unlike a worker's job acceptance decision, filling a job is assumed (as usual) to have no opportunity cost in terms of lost hiring opportunities. Therefore (6) shows that the surplus  $\Sigma^F(z, Z)$  associated with a filled job is the same as the value of that job. Firms offer new jobs until the expected profits associated with a vacancy are zero. If the probability of a filling a job is  $p^F(S, \theta)$ , then the zero profits condition is:

$$\kappa = p^F(S(Z), \theta(Z))E_{Z'|Z} \Sigma^F(Z', Z') \quad (7)$$

The wage is determined by the Nash bargaining condition

$$\frac{\Sigma^W(z, Z)}{\Sigma^F(z, Z)} = \frac{\mu(z, Z)}{1 - \mu(z, Z)} \quad (8)$$

Here we generalize again, by letting bargaining power  $\mu$  vary with the aggregate state.



## 2.2 The labor market

We assume that total matches  $M$  are given by

$$M = \gamma V^{1-\lambda} U^\lambda S \quad (9)$$

where  $V$  is total vacancies, and  $U$  is unemployment. Tightness is defined as  $\theta \equiv V/U$  so that it depends on unemployment  $U$  rather than effective search  $US$ , which is unobservable. Matching probabilities are thus functions of tightness and search:

$$p(S, \theta) = \frac{M}{U} = \gamma \theta^{1-\lambda} S \quad (10)$$

and

$$p^F(S, \theta) = \frac{M}{V} = \frac{p(S, \theta)}{\theta} \quad (11)$$

Equ. (10) implicitly provides a metric for search effort, saying that the individual probability of finding a job is proportional to search.

Note that equations (4), (5), (6), (7), (8), (10), and (11) are seven equations that determine the seven functions  $\Sigma^W(z, Z)$ ,  $S(Z)$ ,  $\Sigma^F(z, Z)$ ,  $\theta(Z)$ ,  $w(z, Z)$ ,  $p(S, \theta)$ , and  $p^F(S, \theta)$ , without reference to unemployment  $U$ . Thus it is reasonable to look for a solution of these equations that is independent of  $U$ .

When we incorporate the dynamics of employment and unemployment in our model, we must note that  $\alpha_Z < 1$  implies a distribution of matches with different productivities. To deal with this effect in the simplest possible way, in Section 4 where we allow  $\alpha_Z < 1$  we will assume that productivity follows a two-state Markov process, taking a low value  $Z^{LO}$  or a high value  $Z^{HI}$ . We then distinguish between the fraction of the labor force in matches with low productivity,  $N_t^{LO}$ , and the fraction matched with high productivity,  $N_t^{HI}$ . Total employment plus unemployment must sum to one:

$$N_t + U_t \equiv N_t^{HI} + N_t^{LO} + U_t = 1 \quad (12)$$

If we write total matches at time  $t$  as  $M_t \equiv \gamma \theta(Z_t)^{1-\lambda} S(Z_t) U_t$ , then the three labor market state variables follow the dynamics

$$N_{t+1}^{HI} = (1 - \delta(Z^{HI}, Z_t)) N_t^{HI} + M_t \mathbf{1}(Z_{t+1} = Z^{HI}) \quad (13)$$

$$N_{t+1}^{LO} = (1 - \delta(Z^{LO}, Z_t)) N_t^{LO} + M_t \mathbf{1}(Z_{t+1} = Z^{LO}) \quad (14)$$

$$U_{t+1} = \delta(Z^{LO}, Z_t) N_t^{LO} + \delta(Z^{HI}, Z_t) N_t^{HI} - M_t + U_t \quad (15)$$

where  $\mathbf{1}(x)$  is an indicator function equalling 1 if statement  $x$  is true, and 0 if  $x$  is false.

Here we see that total job destruction is  $D_t = \delta(Z^{HI}, Z_t) N_t^{HI} + \delta(Z^{LO}, Z_t) N_t^{LO}$ . Finally, given that there is no capital stock, aggregate output  $Q_t$  is:

$$Q_t = (1 - U_t)(1 + \alpha_Z Z_t) + \zeta(1 - \alpha_Z)(N_t^{HI} Z^{HI} + N_t^{LO} Z^{LO}) \quad (16)$$

### 3 Unemployment volatility: cycles and policies

We now consider the simplest and most standard version of this model, in which labor productivity is just  $y = 1 + Z$ , and the separation rate  $\delta$  and bargaining power  $\mu$  are constants.<sup>9</sup> For this case, we can characterize the dynamics explicitly, and demonstrate how the variability of labor market variables over the business cycle is related to their variability in response to UI policy changes.

Define total surplus as  $\Sigma_t \equiv \Sigma_t^F + \Sigma_t^W$ . Summing equations (4) and (6), and using the fact that the worker's share of surplus is  $\mu$ , we see that  $\Sigma$  must satisfy

$$\begin{aligned}\Sigma_t &= y_t - b + h(S_t) + \beta(1 - \delta)E_t\Sigma_{t+1}^F + \beta(1 - \delta - p_t)E_t\Sigma_{t+1}^W \\ &= y_t - b + h(S_t) + \beta(1 - \delta - \mu p_t)E_t\Sigma_{t+1}\end{aligned}\tag{17}$$

where  $h(S) = 0$  if search is exogenous. In addition, we have the zero profit condition

$$\kappa = p_t^F E_t J_{t+1} = p_t^F (1 - \mu) E_t \Sigma_{t+1}\tag{18}$$

In equations (17) and (18),  $p_t = \gamma S_t \theta_t^{1-\lambda}$  and  $p_t^F = \gamma S_t \theta_t^{-\lambda}$  depend only on tightness  $\theta_t$  and search effort  $S_t$ . Thus when search is exogenous, (17) and (18) suffice to determine total surplus  $\Sigma_t$  and tightness  $\theta_t$ .

In the endogenous search case, the first-order condition (5) plus the zero profit condition (18) allow us to eliminate search in favor of tightness:

$$\frac{\kappa \theta_t}{\gamma(1 - \mu)} = \frac{h'(S_t) S_t}{\beta \gamma \mu}\tag{19}$$

Since  $h(S)$  is convex, (18) says that search and tightness are positively related: people search harder when job market conditions are good. We call the relation  $S(\theta)$ , with elasticity  $\eta_\theta^S(\theta) \equiv (1 + h''(S(\theta))S(\theta)/h'(S(\theta)))^{-1}$ . In what follows, we will assume that search costs are small on average, but that  $h(S)$  is very convex, so that job finding is relatively inelastic in response to  $\theta$ . Intuitively, this seems a reasonable calibration; more importantly, these restrictions improve the model's behavior. They suffice for existence of a unique equilibrium, and we will see below repeatedly that large search costs or elastic search effort have counterfactual implications.

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<sup>9</sup>To simplify notation, we now use the time subscript  $t$  to denote dependence on the aggregate state  $Z_t$  (and also on  $U_t$  where appropriate).

## Steady state

In the nonstochastic steady state (indicated by dropping the subscript  $t$ ), equations (17) and (18) give two different expressions for  $\Sigma$ . Substituting for  $p$  and  $p^F$ , we have:

$$\Sigma = \frac{\kappa\theta^\lambda}{\gamma S(1-\mu)} = \frac{y-b+h(S)}{1-\beta(1-\delta-\mu\gamma S\theta^{1-\lambda})} \quad (20)$$

If  $S$  is exogenous, then the left-hand side is increasing in  $\theta$ , and the right-hand side is decreasing in  $\theta$ , so there exists a unique steady state for  $\theta$  and  $\Sigma$ .

In the case of endogenous search, we assume  $S$  is sufficiently inelastic so that

$$\lambda^* \equiv \lambda - \eta_\theta^S(\theta) > 0 \quad (21)$$

This suffices to make the left-hand side of (20) increasing. Furthermore, the right-hand side is decreasing if  $h$  is sufficiently small but  $S$  is sufficiently inelastic; this then suffices for a unique steady state equilibrium.

We can now use (20) to derive comparative statics for  $\theta$  in terms of  $b$ . Let hats represent changes in the log of the steady state. Then we have:

$$\lambda\hat{\theta} - \hat{S} = -\frac{b}{y-b+h(S)}\hat{b} - \left(\frac{\beta\mu p}{1-\beta(1-\delta-\mu p)}\right)[(1-\lambda)\hat{\theta} + \hat{S}] + \frac{h(S)}{y-b+h(S)}\eta_S^h(S)\hat{S} \quad (22)$$

where  $\eta_S^h(S) \equiv h'(S)S/h(S)$ . We now simplify, using the formula (20) for steady state surplus  $\Sigma$ , and we write the equations in terms of  $\hat{p} = (1-\lambda^*)\hat{\theta}$ . We obtain:

$$\eta_b^p \equiv \frac{\hat{p}}{\hat{b}} = -\frac{1-\lambda^*}{\lambda^*} \left(\frac{b}{y-b+h}\right) \left(\frac{1-\beta+\beta\delta+\beta\mu p}{1-\beta+\beta\delta+\beta\mu p/\lambda^* - h\eta_S^h\eta_\theta^S/(\lambda^*\Sigma)}\right) < 0 \quad (23)$$

The steady state effect of  $b$  on unemployment is approximately the opposite of the effect on the job finding probability  $p$ . In steady state,

$$\delta(1-U) = pU \quad (24)$$

which implies

$$\eta_b^U \equiv \frac{\hat{U}}{\hat{b}} = -(1-U)\frac{\hat{p}}{\hat{b}} > 0 \quad (25)$$

Equations (23) and (25) show that  $\lambda^* > 0$  is necessary for the negative effect of UI on unemployment that is observed in the data; this justifies assumption (21).

## Dynamics

Now consider the dynamics. Suppose that  $y_t = 1 + Z_t$  is AR1 in logs:

$$\tilde{y}_{t+1} = \rho\tilde{y}_t + \epsilon_{t+1} \quad (26)$$

where  $\epsilon$  is *i.i.d.* with  $E_t \epsilon_{t+1} = 0$ , and  $\rho \in (0, 1)$ . (All variables with tildes signify log deviations from steady state, and unadorned variables are steady state values or constants.) If we linearize the surplus dynamics (17) and the zero profit condition (18) and impose saddle path stability, we find an explicit formula for the dynamics of the job-finding probability, in terms of the productivity shock:

$$\frac{\tilde{p}_t}{\tilde{y}_t} = \frac{1 - \lambda^*}{\lambda^*} \left( \frac{y}{y - b + h} \right) \left( \frac{1 - \beta + \beta\delta + \beta\mu p}{1/\rho - \beta + \beta\delta + \beta\mu p/\lambda^* - h\eta_S^h \eta_\theta^S / (\lambda^* \Sigma)} \right) \quad (27)$$

It is the close resemblance between (23) and (27) that enables us to test the model. For comparability with Layard and Nickell (1999), we will state our results in terms of semielasticities instead of elasticities.<sup>10</sup> To keep our results unit-free, we will calculate semielasticities with respect to the unitless variable  $\xi \equiv b/y$ , the steady state ratio of the unemployment benefit to the marginal product of labor. For ease of expression, we will call  $\xi$  the “replacement ratio”, even though the correct definition is  $b/w$ . In steady state, the difference is small, and we have verified numerically that the quantitative impact of working with  $b/y$  instead of  $b/w$  is trivial. Thus we define  $\epsilon_\xi^p \equiv \eta_\xi^p / \xi \equiv \eta_b^p / \xi$ , which we call the semielasticity of job finding with respect to the replacement ratio. Using (23) and (27), we obtain:

**Proposition 1.** The dynamic elasticity of the probability of job finding with respect to productivity, and the long-run semielasticity of the probability of job finding with respect to the replacement ratio  $\xi$ , have the following ratio in absolute value:

$$\left| \frac{\tilde{p}_t / \tilde{y}_t}{\epsilon_\xi^p} \right| = \left( \frac{1 - \beta + \beta\delta + \beta\mu p / \lambda^* - h\eta_S^h \eta_\theta^S / (\lambda^* \Sigma)}{1/\rho - \beta + \beta\delta + \beta\mu p / \lambda^* - h\eta_S^h \eta_\theta^S / (\lambda^* \Sigma)} \right) \leq 1 \quad (28)$$

This ratio equals one if and only if  $\rho = 1$ , that is, if technology shocks are permanent. For any  $\rho < 1$ , the ratio is strictly less than one. Endogenous search does not alter this ratio if  $\rho = 1$ , and it makes the ratio smaller if  $\rho < 1$ , because the search term  $h\eta_S^h \eta_\theta^S / (\lambda^* \Sigma)$  decreases the numerator proportionally more than the denominator.

Proposition 1 is helpfully simple, but to address familiar data it will be better to focus on the unemployment rate  $U$  instead of the job-finding probability  $p$ . Turning to the dynamics of  $U$ , we have:

$$U_{t+1} = U_t + \delta(1 - U_t) - \gamma S_t \theta_t^{1-\lambda} U_t \quad (29)$$

In the appendix we calculate the ratio of the standard deviations of the logs (the usual business cycle volatility measure) of unemployment and the technology shock, which we can then compare to the semielasticity  $\epsilon_\xi^U \equiv \partial \log U / \partial \xi$  of unemployment with respect to the replacement ratio. Using the notation  $\sigma_x \equiv \sqrt{\text{Var}(\tilde{x}_t)}$ , we obtain:

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<sup>10</sup>The other crucial reason to state our results in terms of the semielasticity is that it is invariant to any unobserved disutility component in  $b$ . In contrast, an estimate of the elasticity with respect to  $b$  or  $\xi$  changes depending on what portion of  $b$  we assume consists of UI benefits rather than work disutility.

**Proposition 2.** The relative standard deviation of log unemployment to log output, and the long-run semielasticity of unemployment with respect to the replacement ratio  $\xi$ , have the following ratio:

$$\begin{aligned} \frac{\sigma_U/\sigma_Q}{\epsilon_\xi^U} &= \frac{(\sigma_y/\sigma_Q)(\sigma_U/\sigma_y)}{\epsilon_\xi^U} \\ &= \left( \frac{1 - \beta + \beta\delta + \beta\mu p/\lambda^* - h\eta_S^h\eta_\theta^S/(\lambda^*\Sigma)}{1/\rho - \beta + \beta\delta + \beta\mu p/\lambda^* - h\eta_S^h\eta_\theta^S/(\lambda^*\Sigma)} \right) \left( \frac{\delta(U + \rho(U - \delta))}{(2U - \delta)(U + \rho(\delta - U))} \right)^{\frac{1}{2}} \frac{\sigma_y}{\sigma_Q} \end{aligned} \quad (30)$$

The first term, as we saw before, is strictly less than one unless technology shocks are permanent. The second term is less than or equal to one if  $U > \delta$ , which is true if and only if  $\delta + p$  is less than one. Thus this restriction is satisfied unless we choose a very long period (a Cobb-Douglas matching model like this is not well behaved if periods are so long that transition probabilities are near one). The last term is less than one in the data, and it cannot exceed one in our model except in the irrelevant case of a large positive correlation between  $y$  and  $U$ . We conclude that for any sensible parameters, the ratio in Proposition 2 is strictly less than one.

Returning to the data, we have seen estimates of the ratio of standard deviations of log unemployment and log output ranging from 6.11 (Merz 1995) to 8.18 (our calculations from the FRED database). Layard and Nickell's (1999) estimate of the semielasticity of unemployment with respect to the replacement ratio is 1.3, with standard error 0.5. Thus the ratio in Proposition 2 is around six in the data, while the model implies that it should be less than one. Even if we interpret Nickell's results more generously, by considering the whole 95% confidence interval for his semielasticity estimate, the ratio is still off by at least a factor of three.

We must emphasize that this rejection of the model is independent of the mean level of unemployment  $U$ , since both the numerator and denominator of the ratio in Proposition 2 state the variability of unemployment as a proportion of its mean. In the numerator,  $\sigma_U$  is approximately the standard deviation of unemployment divided by  $U$ . In the denominator,  $\epsilon_\xi^U$  is approximately  $U^{-1}\partial U/\partial\xi$ . Hence  $U^{-1}$  cancels. Thus we need not be concerned (as in Cole and Rogerson 1999) that the success of our model depends on how we calibrate mean  $U$ .<sup>11</sup>

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<sup>11</sup>Equivalently, this says the model would still be rejected if we studied *levels* of unemployment, instead of logs of unemployment, since when we cancel out  $U^{-1}$  Proposition 2 becomes a statement about the variability of unemployment in *levels*.

## 4 Numerical extensions

Our analytical results show that our simplified model exaggerates UI policy effects relative to cyclical volatility by a factor of at least six (in terms of point estimates) or at least three (allowing for uncertainty about Layard and Nickell’s regression coefficient). This will not be resolved by tinkering with parameters, since the upper bound implied by Proposition 2 is independent of any calibration. However, we must still ask whether some more realistically complicated version of the model might fit better. Therefore we now turn to numerical simulations of the general model from Section 2.

For concreteness, we start by calibrating the model in terms of a conservative interpretation of Layard and Nickell’s (1999) results. Our first calibration is chosen to produce a semielasticity of unemployment with respect to the unemployment benefit of 2, which is in the upper range of their confidence interval, and roughly equals the largest point estimate we have seen (that of Layard et. al., 1991).

### 4.1 Benchmark parameters

The productivity shock  $Z$  follows a two-state Markov process, taking the values  $Z^{LO} = -0.018$  and  $Z^{HI} = 0.018$ . The benchmark parameterization assumes that all firms have equal productivity ( $\alpha_Z = 1$ ). The probability that  $Z$  remains constant from one period to the next is denoted  $\rho_Z$ . We simulate the model at weekly frequency, but we report results aggregated to quarterly frequency. In our benchmark parameterization, we impose an approximate yearly persistence of  $\bar{\rho}_Z \equiv 2/3$  for technology, implying business cycles lasting roughly six years, by assuming that  $Z$  remains unchanged from one week to the next with probability  $\rho_Z \equiv \bar{\rho}_Z^{1/52} \approx 0.9922$ .<sup>12</sup>

The elasticity of total matches to unemployment is set to  $\lambda = 0.5$ , consistent with Blanchard and Diamond (1989). We assume workers’ share of surplus is also  $\mu = 0.5$ , for lack of a better estimate; hence our benchmark equilibrium is efficient (Hosios 1990). We calibrate an annual job loss rate of approximately  $\bar{\delta} \equiv 25\%$  by setting the weekly probability of job loss to  $\delta \equiv \bar{\delta}/52$ . This is reasonable for the US, though job separation rates are higher for the most unstable classes of jobs and workers. To get a discount factor of  $\bar{\beta} \equiv 95\%$  annually, we set the weekly discount factor to  $\beta \equiv \bar{\beta}^{1/52}$ . The matching efficiency and vacancy cost parameters  $\gamma$  and  $\kappa$  are reset in each simulation so that the steady state unemployment rate is always  $U = 0.06$  (again, a US calibration) and so that a vacancy lasts two weeks on average. Note that vacancy duration is just a normalization: doubling it would mean doubling vacancies, reducing  $\kappa$  by half, and adjusting  $\gamma$  to keep total matches unchanged; vacancy costs  $\kappa V$  and workers’ job finding

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<sup>12</sup>Although this is less persistence than many business cycle models assume, we prefer this calibration because longer cycles would make our results more sensitive to the HP filter.

probabilities would be unaffected. In addition to constant  $\delta$  and  $\mu$ , the benchmark specification assumes exogenous search intensity ( $h = 0$  and  $\eta_s^h = \infty$ ).

The Markov process spends equal time, on average, in good and bad productivity states, so mean productivity  $y$  is 1. We set  $b = 0.745$  for the benchmark parameterization; that is, the cost of working is 74.5% of the mean marginal product of labor. This parameter is crucial, because a larger  $b$  implies a smaller match surplus, which makes unemployment and vacancies more volatile. Intuitively, if  $b$  is large, then firms own a highly leveraged claim on the productivity process  $y$ ; a small percentage change in  $y$  implies a large percentage change in the surplus, motivating a big change in hiring. In fact, (27) shows that as  $b$  approaches  $y + h$ , the variance of job finding goes to infinity: clearly, the RBCM model cannot be rejected solely on grounds of insufficient unemployment volatility. Our parameterization  $b = 0.745$  is picked to set the semielasticity  $\epsilon_b^U$  to two (our conservative interpretation of Layard and Nickell's (1999) results). This  $b$  may seem high; in Shimer (2003) the ratio of the cost of working to the wage is less than 0.4. However, we must remember that  $b$  includes more than unemployment benefits (which average 44% of the wage in the US for newly unemployed workers, according to Engen and Gruber (1994)). In the structure of our model,  $b$  also includes any utility costs (or any other costs) of working. These costs are presumably nontrivial. Table 1 shows the results for this parameterization, together with the rest of our simulations.

### Results of the benchmark parameterization

The first two lines of Table 1 show the benchmark results. All relative standard deviations and correlations refer to data aggregated to quarterly frequency. Results are reported with (line 1) and without (line 2) HP filtering; the filter has little impact.

In line 1, the long run semielasticity of unemployment with respect to the unemployment benefit is  $\epsilon_\xi^U = 2.00$  by construction. But this parameterization yields insufficient variability of log unemployment over the business cycle, with  $\sigma_U/\sigma_Q = 1.40$ , when this ratio is over six in the data. The punchline is that  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  equals 0.70, far too low for consistency with the data, and also well below our analytical upper bound of one. Similar results hold for the probability of job finding  $p$ : the business cycle variability is  $\sigma_p/\sigma_Q = 1.61$  (too low), while the semielasticity  $\epsilon_\xi^p$  is  $-2.13$  (approximately correct; not shown in table). The cyclical variability of vacancies  $\sigma_V/\sigma_Q = 3.23$  is also too low.

As we mentioned above, the way to make unemployment more variable is to impose a larger cost of working  $b$ , so that the surplus is smaller and more volatile. In the baseline version with  $b = 0.745$ , the total surplus  $\Sigma$  equals 45.2% of the mean quarterly marginal product of labor. In line 3 we raise the cost of working to 90% of the mean marginal product of labor, that is,  $b = 0.90$ . This shrinks the joint surplus  $\Sigma$  to just 17.7% of the mean quarterly marginal product of labor. This raises  $\sigma_U/\sigma_Q$  to 3.16, an improvement but still less than in US data (to actually match the data we need  $b = 0.95$ ). However,

unemployment also becomes more responsive to the UI benefit, so that  $\epsilon_\xi^U = 5.41$  now far exceeds the estimates in the literature. Actually, (23) and (27) show that raising  $b$  increases the response to UI even more than it increases cyclical volatility; so raising  $b$  to 0.9, lowers the key ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  to 0.58. Thus we see the main tradeoff of the benchmark model. We can make the model more volatile to better match cyclical data, or less volatile to better match labor market data, but the two goals are at odds with each other; and in relative terms the tradeoff is worse when  $b$  is large.

In line 4, we go in the opposite direction and decrease  $b$  to 0.4, which is similar to Shimer's (2003) calibration. Total surplus  $\Sigma$  is now 106.3% of the mean quarterly marginal product of labor. The unemployment semielasticity  $\epsilon_\xi^U$  falls to 0.82, and the cyclical volatility of unemployment falls to  $\sigma_U/\sigma_Q = 0.62$ . Thus this parameterization not only produces insufficient cyclical volatility: it is slightly too inelastic to match even the (small) observed effects of the UI benefit.

Before moving to other versions of the model, we check robustness to several parameter changes. In line 5, we set  $\bar{\rho}_Z = 75\%$ , so that cycles are more persistent, lasting roughly eight years. In line 6, we increase the separation rate to  $\bar{\delta} = 40\%$  annually; this would be a reasonable calibration for the US if we chose to focus on relatively unstable jobs and workers. In line 7, we lower the elasticity of matching with respect to unemployment to  $\lambda = 0.3$ , with worker bargaining power  $\mu = 0.5$ , while in line 8 we lower  $\mu$  to 0.3, with  $\lambda = 0.5$ . Though there are mild changes in some statistics, the ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  remains close to 0.7 in all these experiments.

## 4.2 Variable separation and variable search

In line 9, we ask whether variable separation changes the results. This is important, since most data show that destruction varies more than creation over the business cycle. The usual way to endogenize separation, following Mortensen and Pissarides (1994), is to assume that productivity has a match-specific component, so that workers and firms both prefer to separate when their joint surplus becomes negative. Here, for simplicity, we just exogenously impose a variable separation rate depending negatively on the aggregate technology shock, which is essentially what the model of Mortensen and Pissarides implies.

Thus we now have two possible separation rates, depending on the aggregate shock. We set  $\bar{\delta}(Z^{LO}) = 0.25 * 1.15 = 0.2875$  and  $\bar{\delta}(Z^{HI}) = 0.25/1.15 \approx 0.2174$ , so that  $\bar{\delta}$  varies by  $\pm 15\%$ , depending on  $Z$ . This amount of variation in separation suffices to produce business cycle variability of unemployment close to that in the data:  $\sigma_U/\sigma_Q$  rises to 5.89. The semielasticity of unemployment with respect to  $\xi$  changes only slightly, so that the ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  rises to 2.79.

The problem is that this way of resolving the conflict destroys the Beveridge curve: the correlation between unemployment and vacancies switches sign to  $\rho_{U,V} = 0.95$ . The



fact that variable separation helps increase unemployment volatility, but eliminates the Beveridge curve, has also been noted by Cole and Rogerson (1999). Moreover, while unemployment becomes more variable, the probability of job finding varies less in this case: the ratio  $\sigma_p/\sigma_Q$  falls from 1.61 with the baseline parameters to 1.40 with variable separation. This contradicts the data of Section 1, which showed that job finding has roughly the same percentage variability as unemployment. Also, the amount of variation in the separation probability needed here is too large. The relative standard deviation of job destruction to employment,  $\sigma_D/\sigma_N$ , is now 13.51, well above Cole and Rogerson's (1999) figure of six. (In the benchmark, it is exactly one by construction.)

Lines 10 and 11 allow for variable search effort, first considering the relatively inelastic case  $\eta_S^h = 4$  and then the more elastic case  $\eta_S^h = 2$ . Variable search effort makes unemployment more cyclical because (as we saw in Section 2) search rises when productivity is high. With  $\eta_S^h = 4$ , we have  $\sigma_U/\sigma_Q = 2.75$ , while with  $\eta_S^h = 2$ , we match cyclical data quite well, reaching  $\sigma_U/\sigma_Q = 5.31$ . However, the semielasticity of unemployment with respect to the replacement rate  $\xi$  rises even more, so that the ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  falls to 0.66 when  $\eta_S^h = 4$ , and to 0.59 when  $\eta_S^h = 2$ . As our analytical calculations indicated, endogenous search only makes the tradeoff worse. Also, sufficiently elastic search effort again destroys the Beveridge curve: with  $\eta_S^h = 2$ , we have  $\rho(U, V) = -0.17$ .<sup>13</sup>

### 4.3 Sticky wages

We have seen that higher  $b$  means higher percentage variation in the firm's surplus over the cycle, increasing the variability of hiring and unemployment. Another obvious way to make the firm's surplus volatile would be to impose some form of wage stickiness, as has been emphasized recently by Hall (2003). Furthermore, it seems natural to assume that sticky wages are only a short run phenomenon, so that they should have less influence on the long run impact of the UI benefit.

Again, we choose an easy *ad hoc* way of making wages sticky. We assume that workers' bargaining power varies negatively with the technology shock, so that workers get a larger share of surplus in recessions. This stabilizes the wage over the cycle, and thus destabilizes the firm's hiring incentives. In line 12 we assume that the worker's bargaining power increases (decreases) by 15% when the aggregate technology shock is low (high). This amount of variation in bargaining power suffices to raise  $\sigma_U/\sigma_Q$  to 5.67, roughly consistent with the data. The semielasticity  $\epsilon_\xi^U$  hardly changes, so that  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  increases to 2.73.

This does not seem like an unreasonable degree of wage stickiness: the ratio of the standard deviations of log wages and log output is now  $\sigma_w/\sigma_Q = 0.59$ .<sup>14</sup> This is better

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<sup>13</sup>Merz (1995) also finds that variable search effort acts against the Beveridge curve.

<sup>14</sup>However, note that this suffices to make the worker's surplus (i.e. not only his share of surplus) countercyclical. With higher unemployment in recessions, but only a relatively small fall in the wage, workers' surplus is larger in recessions for these parameters.

than the figure of 0.91 in the baseline model, though still not as low as in the data; for example, Merz (1995) reports  $\sigma_w/\sigma_Q = 0.37$  for the US. Therefore, sticky wages seem a potentially promising way of improving the model’s fit. But obviously they are controversial, and debate goes on about possible justifications for wage stickiness.

One possible microfoundation for wage stickiness is an “efficiency wage”. Here, if we follow Shapiro and Stiglitz (1984) by assuming a constant probability of observing shirking behavior, firms should offer a contract giving workers a constant surplus just sufficient to prevent shirking. Thus in line 13 we report a version of our model where the Nash bargaining condition (8) is replaced by an equation that fixes a constant surplus for the worker at all times (it is set equal to the average surplus in the benchmark model of line 1). The cyclical variability of unemployment increases relative to the baseline, though less than it did with variable  $\mu$ . However, the semielasticity of  $U$  with respect to  $\xi$  is greatly increased, at 4.00, and  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  falls to 0.64. Imposing a constant surplus for the worker means that hiring incentives fall sharply as the replacement ratio rises, so our efficiency wage model fits less well than the *ad hoc* sticky wage version, which permits flexible adjustment to long run changes in UI.

#### 4.4 Embodied technological change

Finally, we argue that embodied technological change offers another possible resolution to the problem that concerns us, which may seem preferable to those skeptical of wage rigidity. If technological progress is embodied in new capital, and requires the hiring of a new worker with different skills, then technology shocks should affect new matches without changing the productivity of old ones. So now we set  $\alpha_Z = 0$ , making the productivity of each match specific to the time of its creation. Since shocks no longer affect all matches equally, the persistence of aggregate output increases, and we therefore decrease the persistence  $\bar{\rho}_Z$  of the shock from 0.67 to 0.6 annually. We also initially set  $\zeta = 1$ , so that the cohort-specific shock has the same impact as the aggregate shock did; and we lower  $b$  slightly to 0.7, to keep  $\epsilon_\xi^U$  near its target level of two.

This simple change of specification, which we call the “cohort-specific benchmark” in the table, more than suffices to solve the problem. In line 14, with HP filtering, we find that  $\sigma_U/\sigma_Q = 9.66$ , higher than in the data, while  $\epsilon_\xi^U = 1.79$  is slightly decreased, so that the ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$  rises to 5.40. Since the variables are more persistent, the results are now more sensitive to the HP filter: in line 15, without filtering, we have  $\sigma_U/\sigma_Q = 7.86$ . Either way, the cyclical variability of unemployment is no longer problematic. The job finding probability also varies more: with filtering,  $\sigma_p/\sigma_Q = 11.32$ .

When technology shocks are disembodied ( $\alpha_Z = 1$ ) and thus immediately affect all matches, workers and firms know that a high match productivity  $Z$  may fall before separation, while a low  $Z$  may rise before separation. In contrast, with  $\alpha_Z = 0$ , the match productivity  $z$  will be unchanged until separation; other things equal, this increases

the difference in value between high and low productivity matches. Furthermore, even though  $\alpha_Z = 0$  means that a high  $z$  remains high until separation, the wage in a high- $z$  match could nonetheless fall before separation, since it depends on the outside option and thus on the aggregate  $Z$  rather than the cohort-specific  $z$ . Therefore when  $\alpha_Z = 0$ , firms know that the high wage associated with a good match could fall, while the low wage of a bad match could rise. This second effect also increases the difference between the payoffs to hiring in good and bad times. Both these effects amplify the volatility of hiring for the embodied technological change case  $\alpha_Z = 0$ . Since employment now varies more relative to output, we also find that the average productivity across matches varies less compared with output than it does in the baseline model. That is,  $\sigma_y/\sigma_Q$  falls from 0.92 in the benchmark case of line 1 to 0.54 in the cohort-specific benchmark of line 14. This also improves the model's fit, since the relative standard deviation of labor productivity compared with output is only 0.68, according to Merz' (1995) data.

A disadvantage of this new specification is that the standard deviation of log output is now too low, falling to  $\sigma_Q = 0.85$ . But this can be fixed by reparameterization. A more serious problem is that although labor productivity varies less relative to output, the wage becomes more variable. The ratio  $\sigma_w/\sigma_Q$  more than triples from its benchmark value in line 1, which is already too high. The reason is that as we mentioned above, even though a technological improvement does not change the productivity of existing matches, it does raise all workers' outside options, and thereby the wage.<sup>15</sup> As we have emphasized, theory does not tie down a unique optimal form of wage contracting, so we may not want to reject this model on the basis of its wage implications. However, those who wish to take wage data literally may prefer the sticky wage model of line 12.

Since output varies across matches, it now seems especially important to allow for variable separation, depending on the match-specific productivity shock. Thus in line 16 we assume that separation rises or falls by 6% when  $z = Z^{LO}$  or  $z = Z^{HI}$ , respectively. The variability of unemployment increases again, to  $\sigma_U/\sigma_Q = 10.02$ , and there is little change in the semielasticity  $\epsilon_\xi^U$ . The relative standard deviation of job destruction compared with employment rises from 1.00 to  $\sigma_D/\sigma_N = 2.08$ . Furthermore, since productivity now varies across matches, imposing a relation between productivity and separation does less damage to the Beveridge curve than it did in line 9: there remains a strong negative correlation  $\rho_{U,V} = -0.68$  between unemployment and vacancies.

To improve the fit further, we next raise  $\zeta$  to 1.6. This increases the impact of cohort-specific shocks and thus helps raise the standard deviation of log output, which is too low in the  $\alpha_Z = 0$  case. This is likely to increase even more the ratio  $\sigma_U/\sigma_Q$ ,

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<sup>15</sup>Our parameterizations ensure that the outside option never rises enough to make separation optimal. But if we allowed a wider range of productivities, and endogenized separation like Mortensen and Pissarides (1994), then old matches might sometimes separate in response to a positive technology shock. This would raise the volatility of job destruction and vacancies, while making unemployment and job finding probabilities somewhat less volatile.

which is by now too high, so at this point we can afford to go to the intermediate case  $\alpha_Z = 0.5$  where technology shocks have both aggregate and cohort-specific effects. This parameterization (without variable separation) is shown in line 17, and both  $\sigma_Q = 1.39$  and  $\sigma_U/\sigma_Q = 5.36$  fit quite well.

Finally, in line 18 we allow the separation rate to vary by  $\pm 10\%$ , which gives our most successful simulation. The ratio  $\sigma_U/\sigma_Q$  rises to 6.43, while the semielasticity of unemployment with respect to the replacement ratio  $\xi$  remains nearly unchanged at  $\epsilon_\xi^U = 1.80$ , and the ratio  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U = 3.58$  is consistent with the data. Again, variable separation combined with embodied technological change has little adverse impact on the Beveridge curve: the correlation between unemployment and vacancies is now -0.60. Also, as in the data, the variability of job finding is similar to that of  $U$ , at  $\sigma_p/\sigma_Q = 6.79$ . The relative standard deviation of job destruction compared with employment is now  $\sigma_D/\sigma_N = 3.36$ , lower than the figure of six reported by Cole and Rogerson (1999), but a big improvement relative to the case of constant  $\delta$  where it is exactly one. Furthermore, the relative standard deviation of labor productivity  $\sigma_y/\sigma_Q = 0.60$  now fits well, and the biggest problem is again the high variability of the wage,  $\sigma_w/\sigma_Q = 2.43$ .

## 5 Matching in business cycle models with capital

Up to now we have simplified our calculations by forgetting about physical capital. We have argued that this is probably unimportant for the issue at hand, but to be sure, we now take a closer look at Merz (1995) and Andolfatto (1996), where capital is included. While these papers reported some success in modeling the cyclical behavior of the labor market, we find that they suffer from the same problem as our benchmark model: insufficient cyclical volatility compared with policy effects.

### 5.1 The model of Andolfatto (1996)

To understand both models it is helpful to start by looking at the surplus. In Andolfatto's case, we calculate that the match surplus is equal to only 17.3% of mean quarterly labor productivity—close to the lowest surplus we considered, in line 3 of Table 1.<sup>16</sup> This suggests that the labor market in his model should be quite volatile.

On a first glance, Andolfatto's labor market appears to work well. His Table 1 shows that the percentage variability of employment is 0.51 times that of output, compared with 0.67 in his data. However, this hides a surprising failure to explain unemployment, because of an unusual calibration. Andolfatto sets the mean employment rate to 57%,

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<sup>16</sup>In Andolfatto's notation, from  $qJ = \kappa$  and  $J = \alpha\Sigma$  we get the total surplus as  $\Sigma = \kappa/(q\alpha) = 0.105/(0.9 * 0.6) = 0.194$  in units of quarterly output. (This is equal to  $\mu$ , the shadow value of a match, divided by the marginal utility of consumption.) The marginal productivity of labor is  $(1 - \theta)y/n = 0.64/0.57 = 1.123$ , so that match surplus is equal to 17.3 percent of quarterly marginal productivity.

so that the mean unemployment rate is 43%. Unlike many matching papers (including this one) which ignore the “out of the labor force” state, Andolfatto treats any person over 16 years of age who is not working as unemployed. This goes far beyond some authors, such as Cole and Rogerson (1999) or den Haan et. al. (2000), who have claimed that it is helpful to work with a broader definition of unemployment.

Thus while many papers understate the number of people looking for a job, Andolfatto’s grossly overstates it, by including all pensioners, students, and homemakers in  $U$ , and thus making them inputs to the matching function. Any given standard deviation of log employment therefore corresponds to a smaller standard deviation of log unemployment in Andolfatto’s calibration than it would if baseline unemployment were lower. With his numbers, we find:

$$\frac{\sigma_U}{\sigma_Q} = \frac{1-U}{U} \frac{\sigma_N}{\sigma_Q} = \left( \frac{0.57}{0.43} \right) 0.51 = 0.68 \quad (31)$$

roughly ten times lower than the US data, based on the usual definition of unemployment. Moreover, the fact that his unemployment rate is less volatile than in the data makes hiring incentives more volatile than his model would otherwise imply: unemployment falls less in a boom, and therefore the payoff to hiring expands more. Without this effect, employment and unemployment would both fluctuate less in his simulations.

Even if we choose to ignore the low variability of unemployment in Andolfatto’s model, it also implies insufficient variation in other labor market variables. The standard deviation of log vacancies, divided by the standard deviation of log output, is 3.2 in Andolfatto’s model, compared to 9 in his data. This means that the percentage variability of vacancies is about 4.4. The variability of tightness is only slightly higher (4.6 percent) since unemployment hardly varies. Using  $1 - \lambda = 0.6$  in his parameterization, this means the variability of workers’ job finding probability is  $0.6 * 4.6 = 2.8$  percent. This is about twice the variability of output, while in the data the probability of job finding varies about seven times as much as output.

Andolfatto’s model has no unemployment benefits, but since they are equivalent to work disutility in these models, their effect can be calculated. We mimic a one percentage point increase in the UI replacement ratio by raising the utility of the non-employed by one percent of the mean marginal product of labor, scaled by the marginal utility of consumption. We find that the semielasticity of unemployment with respect to the replacement ratio is 2.41 in Andolfatto’s model, around twice Nickell’s point estimate. However, here a one percent increase in  $\log U$  means a 0.43 *percentage point* increase in unemployment; that is, a one percentage point rise in the replacement ratio increases unemployment by  $2.41 * 0.43 = 1.04$  percentage points, about six times higher than the estimate of Layard, Nickell, and Jackman (1991).<sup>17</sup> Seen in this way, Andolfatto’s labor market is both insufficiently volatile over the business cycle, and excessively volatile in response to UI; the punchline for his paper is  $(\sigma_U/\sigma_Q)/\epsilon_\xi^U = \frac{0.68}{2.41} = 0.28$ .

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<sup>17</sup>Here, for comparability, we refer back to a slope estimate rather than a semielasticity estimate.

## 5.2 The model of Merz (1995)

Merz (1995) comes close to fitting the variability of unemployment and job finding probability in US data. With her benchmark specification, the standard deviation of log unemployment over that of log output is 4.77,<sup>18</sup> and for the job finding probability it is 5.41. However, if we back out the effect of the unemployment benefit in the same way we did for Andolfatto, we see that the model exaggerates the sensitivity of unemployment to benefits. For her model, the semielasticity of unemployment with respect to the replacement ratio is 6.54. The statistic  $\frac{\sigma_U}{\sigma_Q} \frac{1}{\epsilon_U^\xi}$  is therefore 0.73, so Merz' model fails by roughly the same factor as our benchmark model in Section 4.1.

When we calculate the joint match surplus in Merz' model, it turns out to be only 1.69 percent of mean quarterly marginal product of labor—ten times smaller than anything we have seen before. Thus Merz achieves sufficient volatility to match business cycles only by assuming an almost negligible surplus, and in doing so exaggerates the response to the unemployment benefit.

If anything, the surprising aspect of Merz' results is how little fluctuation she obtains, given the tiny surplus she assumes. The explanation lies in the fact that she defines the surplus differently from all the other papers we have discussed. Most matching models assume that the marginal disutility of work is constant along the extensive margin (increases in employment) even if it is increasing along the intensive margin (increasing marginal disutility as hours per job increase, like Andolfatto assumes). In contrast, Merz assumes that surplus accrues to a family with a continuum of members, and that marginal disutility of work is increasing as more family members find jobs. At the margin in her equilibrium, the disutility from one more job almost equals the wage income from that job, so the surplus is extremely small. To us, the usual formulation seems more appropriate, since typical households contain only one or two earners, each of whom may have a large inframarginal gain when they find a job.

## 5.3 Other models with capital

Den Haan et. al. (2000) study an RBCM model with an endogenous separation decision. They are successful in explaining variations in job creation and destruction, and find that the interaction between job destruction and investment helps amplify shocks. Their results are consistent with our finding that we can make matching volatile by varying separation over the cycle. However, our calculations suggest that their model will fail to generate a Beveridge curve. They do not report the correlation between vacancies and unemployment in their paper.<sup>19</sup>

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<sup>18</sup>This is the result of our own calculation and differs slightly from the number in Merz' Table 2.

<sup>19</sup>Fujita (2003) explores extensions of an RBCM model with endogenous separation to better match the Beveridge curve.

Gomes et. al. (2001) simulate a business cycle model in which individuals search for jobs. It is not entirely comparable with the models we are analyzing, because there is no matching function. Instead, the distribution of job offers is exogenous, making their model a dynamic extension of McCall's (1970) partial equilibrium search model. They successfully reproduce the cyclical fluctuations of unemployment. However, they state that a rise in the replacement ratio from 0.5 to 0.7 makes unemployment increase from 6.1% to 13.9%, which is a semielasticity of 6.49, at least three times too large to be consistent with the data. Thus their model suffers from the same problem as the RBCM models we have discussed here.

## 6 Conclusions

A model of real business cycles and matching implies that match formation depends on the surplus available to the matched pair. Procyclical employment fluctuations occur if match productivity rises in booms, and increased unemployment benefits diminish employment by decreasing match surplus. The standard RBCM model implies a close relationship between these two aspects of employment variability, which is strongly at odds with data. To reproduce business cycle fluctuations, matching must be quite elastic with respect to the surplus; but to reproduce the observed effects of policies, matching must be more inelastic. We have shown analytically that these two requirements cannot be reconciled in a baseline version of the model. We have shown numerically that this result is robust to endogenous search, endogenous separation, or efficiency wages, and we have also argued that capital, variable benefits, variable hiring costs, and alternatives to the HP filter are unlikely to solve the problem.

Embodied technological change can help reconcile these two implications of the model, because it makes the surplus accruing to the firm substantially more procyclical, so that hiring, unemployment, and the worker's job-finding probability all fluctuate more. Sticky wages have a similar impact on the firm's surplus, so they also help increase cyclical variability without affecting the response to labor market policy.

Future research will have to determine whether the problem lies mainly in the RBC mechanism, in the matching function, or in the empirical estimates of the effects of labor market policy. Perhaps one of the variants of the model which we have proposed will prove to be a satisfactory framework for labor market analysis. But for now the most important conclusion is that we must be very skeptical about using models calibrated to reproduce business cycles as laboratories for labor market policy experiments.

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## Appendix: Linearized dynamics

First we linearize the zero profit condition (18):

$$\lambda\tilde{\theta}_t - \tilde{S}_t = \lambda^*\tilde{\theta}_t = E_t\tilde{\Sigma}_{t+1} \quad (32)$$

and the dynamics (17) of the surplus:

$$\tilde{\Sigma}_t = \frac{y}{\Sigma}\tilde{y}_t + \beta(1 - \delta - \mu p)E_t\tilde{\Sigma}_{t+1} - \beta\mu p\tilde{p}_t + \frac{h}{\Sigma}\eta_S^h\tilde{S}_t \quad (33)$$

These equations can be simplified by writing  $\tilde{p}_t$  and  $\tilde{S}_t$  in terms of  $\tilde{\theta}_t$  and  $E_t\tilde{\Sigma}_{t+1}$ , as follows:  $\tilde{p}_t = (1 - \lambda^*)\tilde{\theta}_t$ , and  $\tilde{S}_t = \eta_\theta^S\tilde{\theta}_t$ , and  $\tilde{\theta}_t = \frac{1}{\lambda^*}E_t\tilde{\Sigma}_{t+1}$ . The following matrix system summarizes the dynamics:

$$\begin{pmatrix} E_t\tilde{y}_{t+1} \\ E_t\tilde{\Sigma}_{t+1} \end{pmatrix} = \begin{pmatrix} \rho & 0 \\ -\frac{y}{\Sigma}\left[\beta(1 - \delta - \frac{\mu p}{\lambda^*}) + \frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1} & \left[\beta(1 - \delta - \frac{\mu p}{\lambda^*}) + \frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1} \end{pmatrix} \begin{pmatrix} \tilde{y}_t \\ \tilde{\Sigma}_t \end{pmatrix} \quad (34)$$

The eigenvalues are  $0 < \rho < 1$  and  $\left[\beta(1 - \delta - \frac{\mu p}{\lambda^*}) + \frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1}$ . The second eigenvalue is greater than one with exogenous search.<sup>20</sup> We restrict our analysis of endogenous search to the case of sufficiently inelastic search so that this eigenvalue remains greater than one; thus the system is saddle-path stable, and has a unique equilibrium. The eigenvector associated with the stable eigenvalue can be written as  $(1 \ x)'$ , where

$$x \equiv \frac{y}{\Sigma\left(1 - \rho\left[\beta(1 - \delta - \frac{\mu p}{\lambda^*}) + \frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]\right)} \quad (35)$$

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<sup>20</sup>We assume periods are short enough that  $p$  is small, so that this eigenvalue is positive.

Using the steady state surplus equation (20), this can be written as

$$x = \left( \frac{y}{y - b + h} \right) \left[ \frac{1 - \beta + \beta\delta + \beta\mu\rho}{1 - \rho\beta + \rho\beta\delta + \rho\beta\mu\rho/\lambda^* - \rho h\eta_S^h\eta_\theta^S/(\Sigma\lambda^*)} \right] \quad (36)$$

Saddle path stability implies that  $x$  is the elasticity  $\tilde{\Sigma}_t/\tilde{y}_t$ . Thus, in terms of the observable variable  $\tilde{p}$ , we have:

$$\tilde{p}_t = (1 - \lambda^*)\tilde{\theta}_t = \frac{1 - \lambda^*}{\lambda^*} E_t \tilde{\Sigma}_{t+1} = \frac{1 - \lambda^*}{\lambda^*} \rho x \tilde{y}_t \quad (37)$$

Again we see that our assumption (21) of sufficiently inelastic search so that  $\lambda^* > 0$  is essential to make the model consistent with data: (37) shows that job finding is negatively related to labor productivity if  $\lambda^* < 0$ .

Now using formula (36) for  $x$ , we obtain equation (27), which is used to derive Proposition 1.

For Proposition 2, we linearize the dynamics (29) of unemployment, to obtain:

$$\tilde{U}_{t+1} = (1 - \delta)\tilde{U}_t - \delta \left( \frac{1 - U}{U} \right) (1 - \lambda^*)\tilde{\theta}_t - \delta \left( \frac{1 - U}{U} \right) \tilde{U}_t \quad (38)$$

On the saddle path, we have:

$$\tilde{\theta}_t = \frac{1}{\lambda^*} E_t \tilde{\Sigma}_{t+1} = \frac{1}{\lambda^*} \rho \tilde{\Sigma}_t = \frac{1}{\lambda^*} \rho x \tilde{y}_t \quad (39)$$

so the dynamics of  $U$  become  $\tilde{U}_{t+1} = A\tilde{U}_t - B\tilde{y}_t$ , where we define  $A \equiv (U - \delta)/U$  and  $B \equiv \delta((1 - U)(1 - \lambda^*)/(U\lambda^*))\rho x$ . This implies:

$$\text{Var}(\tilde{U}_t) = \frac{B^2(1 + \rho A)}{(1 - A^2)(1 - \rho A)} \text{Var}(\tilde{y}_t) \quad (40)$$

which simplifies to:

$$\frac{\sigma_U}{\sigma_y} \equiv \sqrt{\frac{\text{Var}(\tilde{U}_t)}{\text{Var}(\tilde{y}_t)}} = \rho x (1 - U) \left( \frac{1 - \lambda^*}{\lambda^*} \right) \sqrt{\frac{\delta(U + \rho(U - \delta))}{(2U - \delta)(U + \rho(\delta - U))}} \quad (41)$$

This equation, together with the formula (36) for  $x$ , and the formula (23) for the steady state comparative statics, gives us Proposition 2.

	Parameters	Results								
		$\sigma_Q$	$\rho_{Q-1}$	$\frac{\sigma_y}{\sigma_Q}$	$\frac{\sigma_w}{\sigma_Q}$	$\frac{\sigma_p}{\sigma_Q}$	$\frac{\sigma_w}{\sigma_Q}$	$\rho_{U,V}$	$\epsilon_\xi^U$	$\frac{\sigma_U}{\sigma_Q} \frac{1}{\epsilon_\xi^U}$
1)	Benchmark	1.62	0.84	0.92	0.91	1.61	1.40	-0.80	2.00	0.70
2)	Benchmark, without HP filter	1.86	0.88	0.91	0.91	1.61	1.42	-0.84	2.00	0.71
3)	$b = 0.9$	1.84	0.86	0.81	0.91	3.65	3.16	-0.80	5.41	0.58
4)	$b = 0.4$	1.54	0.84	0.96	0.93	0.72	0.62	-0.80	0.82	0.76
5)	$\bar{\rho}_Z = 0.75$	1.55	0.87	0.91	0.93	1.68	1.48	-0.83	2.07	0.71
6)	$\bar{\delta} = 0.4$	1.64	0.84	0.91	0.94	1.69	1.52	-0.89	2.00	0.76
7)	$\lambda = 0.3$	1.70	0.85	0.88	0.95	2.37	2.06	-0.66	2.89	0.71
8)	$\mu = 0.3$	1.61	0.84	0.92	0.82	1.54	1.33	-0.80	2.04	0.65
9)	$\delta$ varies with $Z$ by $\pm 15\%$	2.31	0.88	0.64	1.00	1.40	5.89	0.95	2.11	2.79
10)	$\eta_S^h = 4$	1.78	0.85	0.83	0.95	3.18	2.75	-0.60	4.14	0.66
11)	$\eta_S^b = 2$	2.21	0.87	0.67	1.01	6.14	5.31	-0.17	8.98	0.59
12)	$\mu$ varies with $Z$ by $\pm 15\%$	2.27	0.87	0.65	0.59	6.55	5.67	-0.80	2.08	2.73
13)	Efficiency wages	1.76	0.85	0.84	0.82	2.97	2.58	-0.80	4.00	0.64
14)	Cohort-specific benchmark	0.85	0.93	0.54	3.64	11.32	9.66	-0.77	1.79	5.40
15)	Cohort-specific benchmark, no HP	1.19	0.97	0.62	3.00	9.03	7.86	-0.81	1.79	4.40
16)	Cohort-specific, $\delta$ varies with $z$ by $\pm 6\%$	0.97	0.94	0.47	3.56	11.07	10.02	-0.68	1.80	5.56
17)	Cohort-specific, $\zeta = 1.6$ , $\alpha_Z = 0.5$	1.39	0.88	0.67	2.29	6.28	5.36	-0.77	1.77	3.02
18)	Cohort-specific, $\zeta = 1.6$ , $\alpha_Z = 0.5$ , $\delta$ varies with $z$ by $\pm 10\%$	1.55	0.89	0.60	2.43	6.79	6.43	-0.60	1.80	3.58

Notes:

Benchmark:  $\alpha_Z = 1$ ,  $Z = \pm 0.018$ ,  $\bar{\rho}_Z = 2/3$ ,  $\bar{\beta} = 0.95$ ,  $\bar{\delta} = 0.25$ ,  $\lambda = \mu = 0.5$ ,  $b = 0.745$ ,  $\eta_S^b = \infty$   
Cohort-specific benchmark:  $\alpha_Z = 0$ ,  $Z = \pm 0.018$ ,  $\zeta = 1$ ,  $\bar{\rho}_Z = 0.6$ ,  $\bar{\beta} = 0.95$ ,  $\bar{\delta} = 0.25$ ,  $\lambda = \mu = 0.5$ ,  $b = 0.7$ ,  $\eta_S^b = \infty$

$\sigma_x$ : standard deviation of  $\log x$  (quarterly)

$\rho_{x,y}$ : correlation between  $\log x$  and  $\log y$  (quarterly)

$\rho_{Q-1}$ : annual first order serial correlation of  $\log Q$

$\eta_x^y$ : elasticity of  $y$  w.r.t  $x$

$\epsilon_x^y$ : semielasticity of  $y$  w.r.t  $x$

Table 1: Numerical results

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