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Mohammed Mardan, Mark J. Tremblay

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Network Effects: Betwixt and Between

Abstract

We challenge the dichotomy of network effects and highlight that they are not an exogenous characteristic of networks, but endogenous to the decisions of network users. When users choose which activities to perform in a network, multi-activity users transform indirect into direct network effects and a network effectively becomes one-sided if merely multi-activity users frequent it. Our work contributes to theory by determining the underlying micro-foundations that produce what the literature calls a two-sided market and by highlighting how the standard two-sided pricing results arises only under very specific conditions. We also contribute to estimation by illustrating how the presence of multi-active users can challenge identification in network industries.

JEL-Codes: L100, L200, D210, D420.

Keywords: platforms, one-sided markets, two-sided markets, multi-siding users.

Mohammed Mardan
Department of Business and Management Science, Norwegian School of Economics
Helleveien 30
Norway - 5045 Bergen
mohammed.mardan@nhh.no

Mark J. Tremblay
Miami University
Farmer School of Business, FSB 2017
USA - Oxford, OH 45056
tremblmj@miamioh.edu

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"(...) in a technical sense, the literature on two-sided markets could be seen as a subset of the literature on network effects."
– Rysman (2009, p. 127)

1 Introduction

Networks are characterized by demand interdependencies between users, usually referred to as network effects. Depending on the number of activities a user can perform in the network, the literature distinguishes two types of network effects: direct and indirect. Direct network effects describe the effect a user has on other users performing the same activity. Instead, indirect network effects capture the effect a user performing one activity has on users performing another activity. To date, these network effects have been largely regarded as distinct concepts.

We challenge this dichotomy and illustrate that network effects become versatile once individuals are allowed to perform multiple activities. In that sense, the literature on two-sided networks should not just be seen as a subset of the literature on network effects as Rysman (2009) suggested, but rather as one and the same. The reason is that a network, in which users can perform several complementary activities, such as selling and buying, treats multi-activity users as if they were on the same side because the sum of the indirect network effects is the same across multi-activity users. Hence, their presence implies a paling need to skew prices between users performing unlike activities.

There are several examples that highlight the pervasiveness of multi-activity users. In the peer-to-peer sharing industry, large events might attract downtown residents to host at a high price and then use a suburb home as a rental. In the ride-sharing market, users might work as drivers during the work week and take a ride during the weekend, while dog owners can use dog sitting platforms to find a host for their dogs but often act as sitters by hosting other dogs. In addition, platforms serving niche markets often observe considerable amounts of multi-activity. For example, parents at local clothing swaps with multiple children are frequently buyers (of new cloths) and sellers (of cloths that are now too small) and trading or sports card owners regularly trade or trade-in cards and are therefore buying and selling simultaneously.

In our model, a user's decision which of two activities to perform is based on idiosyncratic preferences between activities as well as potential economies of activity, that is utility gains or losses when performing both activities. Network effects are by assumption only indirect, and thus between users performing unlike activities. We show that users' possibility to perform both activities transforms indirect into direct network effects whenever economies of activity are non-zero. In the extreme case, when the network is only frequented by multi-activity users, the network is transformed into a one-sided network featuring direct network effects because multi-activity users only care about the total price, as opposed to the price structure, and the overall participation in the network.

Our generalized model allows us to determine the conditions under which the network is converted into a (quasi-) pure two-sided platform that features the classical two-sided pricing found in Armstrong (2006). We first find that a pure two-sided platform with two distinct sides arises if user preferences are negatively correlated and bimodally distributed between activities and economies of activity are not too positive. These two conditions ensure that individuals have a strong preference for performing only one activity and thus no incentive to become a multi-activity user. Second, classical two-sided pricing can also arise with multi-activity users, but only if economies of activity are absent. In this case, the network treats multi-activity users as if they were two separate users performing unlike activities resulting in a quasi-pure two-sided platform. Hence, our results suggest that the previous literature on two-sided networks implicitly assumes one of the aforementioned conditions.

The results of our study have important practical and empirical implications. First, under joint distributions of agents' tastes that are marginal distribution equivalent,[1] the

[1] Marginal distribution equivalence occurs when the marginal distributions from two different joint density functions are equivalent.

network's pricing strategy and its profits are unaffected by the correlation between activity preferences, because the total participation in each activity is unaffected. However, the total number of users performing an activity, as opposed to the participation in the activity, may still be affected by the correlation between activity preferences. This means that the number of active users as a performance metric should be treated with caution in industries with negligible economies of activity such as advertising-financed media and credit cards.

Second, the presence of multi-activity users can possibly bias any estimation related to network users' behavior, such as strength of network effects or demand price elasticities. Concerning the quantification of network effects, Rysman (2019) argues that the reflection problem impedes identification of direct, but not indirect, network effects. Our results suggest that even in the absence of direct network effects in the traditional sense, consistent estimation of indirect networks remains challenging because multi-activity users transform indirect into direct network effects. We argue that richer and more fine-grained data, which are more frequently used in recent studies, allow to investigate to which extent multi-activity may be relevant. The reason is that the likelihood of multi-activity decreases the narrower the observational unit, both in time and space. However, a bias may still arise in markets where users frequently switch roles, which necessitates to consider the endogenous decision by individuals of which activities to perform. Given the observability of detailed user information, the complex user behavior can be incorporated by means of a discrete choice model as done recently by Affeldt et al. (2022) in the context of multi-homing.

Our study relates to the early literature on network effects, which has exclusively focused on direct network effects (Katz and Shapiro (1985)). Starting with Rochet and Tirole (2003), Parker and Van Alstyne (2005), and Armstrong (2006), a separate strand has focused on indirect network effects. Although a few studies have analyzed platform decisions when direct and indirect network effects co-occur (Belleflamme and Toulemonde (2009), Weyl (2010), and Belleflamme and Peitz (2019)), the two types of network effects have so far been treated as distinct concepts. A recent exception is Belleflamme and Peitz (2020) who illustrate that a provider can transform a one-sided network into a two-sided network through price discrimination. Their analysis provides a complementary reason for the versatility of network effects that is based on the network's pricing decisions, while the mechanism of our analysis is rooted in users' participation decisions, precisely the decision to perform several activities in the network.

Two other studies have investigated the implications of users' ability to choose which activity to perform. Gao (2018) explores the incentives of a monopolist platform to bundle the services it provides to its two sides and compares the optimal pricing strategies to those in a context of single-siding individuals, but does not focus on the implications of multi-activity users for the nature of network effects nor the practical and empirical implications of multi-activity users. Choi and Zennyo (2019) analyze platform price competition when users' decide which side to join, but essentially abstract from multi-activity users as their model allows users to join only one side.

2 The Model

2.1 Users

Suppose that a single network connects a unit mass of users that are interested in interacting with each other. In order to accomplish an interaction, two activities need to be performed by two different users. Let $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i]$, $i = \{1, 2\}$, denote a user's type for activity i, which is drawn from a joint density function $g(\theta_1, \theta_2)$. This implies that the distribution of agents is given by $G(\theta_1, \theta_2) = \int_{\underline{\theta}_1}^{\theta_1} \int_{\underline{\theta}_2}^{\theta_2} g(\theta_1', \theta_2') d\theta_2' d\theta_1'$, with marginal distributions given by $G_i(\theta_i) = \int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_j}^{\overline{\theta}_j} g(\theta_i', \theta_j') d\theta_j' d\theta_i'$. We assume there are network benefits across activities such that a user that participates in activity i benefits from greater participation in activity

$j \neq i$.[2] More formally, let the utility from participating exclusively in activity i be given by

$$u_i(\theta_i) = \alpha_i n_j - p_i - \theta_i. \tag{1}$$

The indirect network benefit of performing activity i in a single interaction is given by α_i. Hence, the user's overall network benefit will be determined by how often she performs activity i, which is directly proportional to the number of users performing activity j, n_j. In addition, a user's utility of performing activity i is negatively affected by the price she has to pay to the network to perform activity i, p_i, and the idiosyncratic cost term related to participating in activity i, θ_i. The latter generates an extensive margin for activity i participation. Because users only differ in their idiosyncratic costs of participating in activity i, there exists a threshold (denoted by θ_i^S) such that all users with type $\theta_i \leq \theta_i^S$ will perform activity i. Equation (1) implies that θ_i^S is given by

$$\theta_i^S = \alpha_i n_j - p_i. \tag{2}$$

Instead of performing only one activity, a user may decide to perform both activities, which generates a utility of

$$u_M(\theta_1, \theta_2) = u_1(\theta_1) + u_2(\theta_2) + \gamma, \tag{3}$$

where $\gamma \in (-\infty, \infty)$ captures various aspects of the network or its available activities that affect individuals incentives to perform both activities. When activities are clearly distinct, as for example in the gaming industry where activities can be categorized broadly as playing and developing games, this amounts to the case of $\gamma = 0$. In other industries, activities or their functionality may be more similar, which means that multi-activity users can enjoy efficiency gains due to saving of time or maintaining larger relevance in the network implying

[2] For convenience, we abstract from direct network effects in the analysis to highlight the versatile role indirect network effects play in our setting.

$\gamma > 0$. For instance, networks frequently give consideration to allow multi-activity users to exist under a single account.[3] Examples for $\gamma < 0$ can be linked to networks' rules and policies that regulate user behavior. For example, eBay reserves the right to suspend a user's account in case of possible misbehavior. Importantly, although a user can have multiple accounts, eBay's policies apply equally to all accounts meaning that users need to meet buyer and seller standards across all accounts.[4] This implies that a user may be confronted with a suspension of *both* the seller *and* the buyer account although she may have only violated seller standards. We will refer to these various aspects, captured by the γ parameter, as economies of activity.

2.1.1 Positive Economies of Activity ($\gamma > 0$)

If $\gamma > 0$, then all users which receive a positive utility from performing each activity in isolation will also perform both activities at the same time (the agents with $\theta_i \leq \theta_i^S,\ \forall i$). However, there are two other groups of users that can have an incentive to perform both activities.

First, there are users that derive positive utility only from one activity. Whether these users are also willing to perform both activities will depend on whether their utility is higher when performing both activities, which is only possible because $\gamma > 0$. More specifically, a user deriving a positive utility only from activity j but a negative utility from activity i, so that $u_j = \max\{u_1, u_2, 0\}$, will still perform both activities if

$$u_M > u_j \quad \Leftrightarrow \quad \alpha_i n_j - p_i - \theta_i + \gamma > 0,\ \forall j \neq i. \tag{4}$$

More specifically, Equation (4) illustrates that a user with $\theta_j \in [0, \theta_j^S]$ and $\theta_i \in (\theta_i^S, \theta_i^S + \gamma]$

[3] Airbnb allows a user to manage hosting and booking on one account and eBay maintains user reviews as both a buyer and a seller. Moreover, prior to collecting taxes, Amazon allowed users to buy and sell through the same account.

[4] See `https://www.ebay.com/help/policies/identity-policies/multiple-accounts-policy?id=4232&mkevt=1&mkcid=1&mkrid=711-53200-19255-0&campid=5336728181&customid=&toolid=10001` for more details on eBay's multiple accounts policy.

will perform both activities because the additional benefit of performing both activities compensates for the utility loss of performing activity i.

Second, there are users that perform both activities although they derive a negative utility from performing any activity in isolation. More specifically, users with $\theta_1 > \theta_1^S$ and $\theta_2 > \theta_2^S$, will perform both activities if

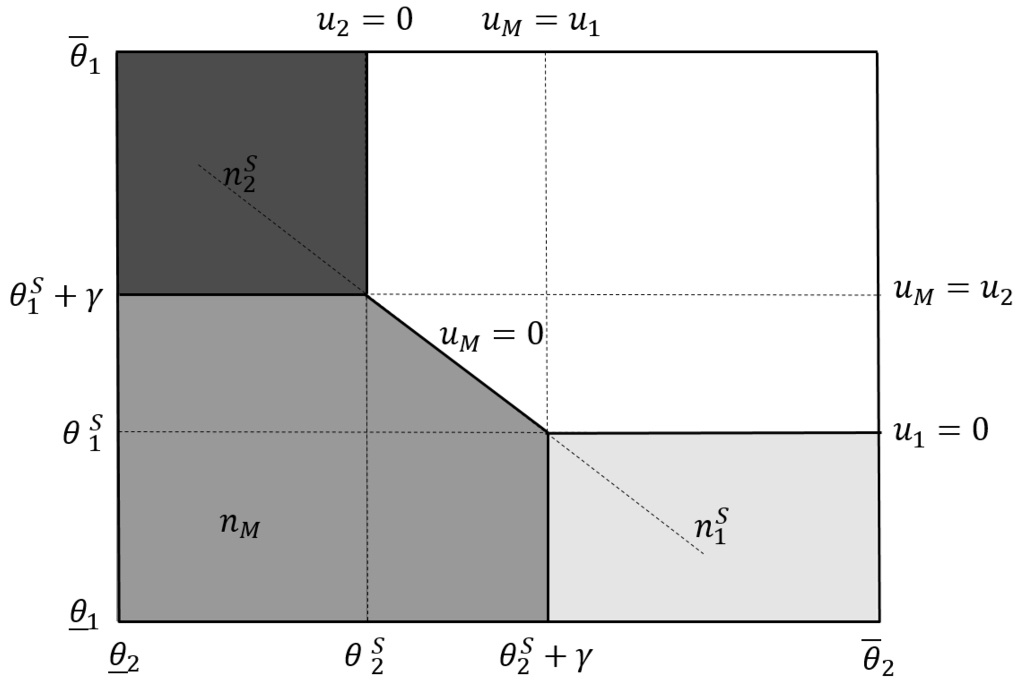
$$u_M > 0 \quad \Leftrightarrow \quad \alpha_1 n_2 - p_1 - \theta_1 + \alpha_2 n_1 - p_2 - \theta_2 + \gamma > 0. \tag{5}$$

Denoting by $\hat{\theta}_1(\theta_2)$ the combinations of θ_1 and θ_2 for which the utility of performing both activities is exactly zero, users with $(\theta_1, \theta_2) \in [\theta_1^S, \hat{\theta}_1(\theta_2)] \times [\theta_2^S, \theta_2^S + \gamma]$ perform both activities although they are not willing to perform any activity in isolation.

We illustrate in Figure 1 how the various trade-offs partition the set of users into the four groups: users not joining the network (white area), users performing activity 1 exclusively (light gray area), users performing activity 2 exclusively (dark gray area), and users performing both activities (gray area). In the figure, utilities of users increase in the south-west direction. The lines $u_i = 0$ indicate that all users with $\theta_i < \theta_i^S$ will at least perform one activity. The lines $u_M = u_i$ partition these two groups into user that perform only one activity ($\theta_i > \theta_i^S + \gamma$) and those that perform two activities ($\theta_i < \theta_i^S + \gamma$). Finally, there is a set of users confined by the lines $u_i = 0$ and $u_M = 0$ that derive negative utility from each single activity, but still join the network and performs both activities because $\gamma > 0$.

Recall that n_i denotes the total number of users performing activity i. This can be decomposed into the number of users performing only activity i, denoted by n_i^S, and the number of users that perform both activities, denoted by n_M, so that $n_i = n_i^S + n_M$. Based on our previous discussion, the number of single- and multi-activity users crucially depends on the level of γ. Single-activity users for activity i only exist if the additional utility of performing both activities is not too high ($\bar{\theta}_i - \theta_i^S \equiv \bar{\gamma}_i > \gamma > 0$). Instead, if $\gamma \geq \bar{\gamma}_i$, activity i will never be performed as a single activity, as every user performing activity j

Figure 1: User classification for $\bar{\gamma}_i > \gamma > 0$



will also perform activity i and therefore act as a multi-activity user. While it depends on the relative magnitudes of $\bar{\gamma}_i$ and $\bar{\gamma}_j$ whether activity i or activity j will be first to feature no single-activity users, the network will be frequented only by multi-activity users if $\sum_i \bar{\gamma}_i \equiv \bar{\gamma}_M > \gamma > \bar{\gamma}_i$, where $\bar{\gamma}_M$ constitutes the threshold above which the universe of individuals join as multi-activity users.

In order to determine the networks optimal pricing strategy, it is necessary to investigate how users' participation choices are impacted by the network's decisions when $\gamma > 0$. The effects are given by[5]

$$\frac{\partial n_i^S}{\partial \theta_i^S} \geq 0, \quad \frac{\partial n_i^S}{\partial \theta_j^S} \leq 0, \quad \frac{\partial n_M}{\partial \theta_i^S} \geq 0.^6 \tag{6}$$

Obviously, whenever $n_i^S > 0$, a rise in the threshold of participating in activity i increases the number of users performing only activity i. This corresponds to a right-ward, respec-

[5]We will analyze how changes in participation thresholds (θ_i^S and θ_j^S) affect participation and provide more detail on why we follow this approach in Section 2.2.

[6]Details about users' participation and their reaction to the network's decisions for the case $\gamma > 0$ can be found in Appendix A.1.

tively up-ward, shift of the $u_i = 0$ lines in Figure 1. Instead, a more lenient threshold for participating in activity j decreases the number of users performing only activity i because some users that have been performing only activity i start to perform both activities. This corresponds to a right-ward, respectively up-ward, shift of the $u_M = u_j$ lines in Figure 1. Moreover, a relaxation of the participation threshold for any activity increases the number of multi-activity users as long as there are individuals that do not connect to the network ($\bar{\gamma}_M > \gamma > 0$). The reason is that either some single-activity users become multi-activity users or individuals that have not joined the network before become multi-activity users.

Our analysis also shows that a more lenient participation threshold for activity i increases total participation in any activity $\left(\frac{\partial n_i}{\partial \theta_i^S}, \frac{\partial n_j}{\partial \theta_i^S} \geq 0\right)$ even though the number of single-activity users for activity j decreases $\left(\frac{\partial n_j^S}{\partial \theta_i^S} < 0\right)$. The reason is that shifts in the $u_M = u_j$ lines do not have quantitative effects on the number of users performing activity i, but merely determine the partitioning into single-activity and multi-activity users. The overall effect is therefore only driven by newcomers to the network.

2.1.2 Non-Positive Economies of Activity ($\gamma \leq 0$)

The analysis of the case when $\gamma \leq 0$ differs slightly from the previous one, as some trade-offs change. First, any user that derives a negative utility from performing activity i ($\theta_i > \theta_i^S$) won't perform that activity. This also implies that only users that derive a non-negative utility from performing either activity ($\theta_i \leq \theta_i^S$, $\forall i$) may perform both activities. However, performing both activities comes with an additional disutility, which means that some users will perform only one activity although each single activity exhibits a positive utility. Thus, if $u_j = \max\{u_1, u_2, 0\}$, then performing both activities is beneficial when

$$u_M \geq u_j \quad \Leftrightarrow \quad \alpha_i n_j - p_i - \theta_i + \gamma \geq 0, \; \forall j \neq i. \tag{7}$$

Since this trade-off is equivalent to Equation (4), performing both activities is only beneficial for users with $\theta_i \leq \theta_i^S + \gamma,\ \forall i$.

All other users that derive a positive utility from performing each activity in isolation do not have an incentive to perform both activities. Hence, these users have to decide, which of the two activities they want to perform. Specifically, users with $\theta_i \in (\theta_i^S + \gamma, \theta_i^S]\ \forall i$, will prefer activity i over activity j if

$$u_i \geq u_j \quad \Leftrightarrow \quad \alpha_i n_j - p_i - \theta_i > \alpha_j n_i - p_j - \theta_j,\ i \neq j. \tag{8}$$

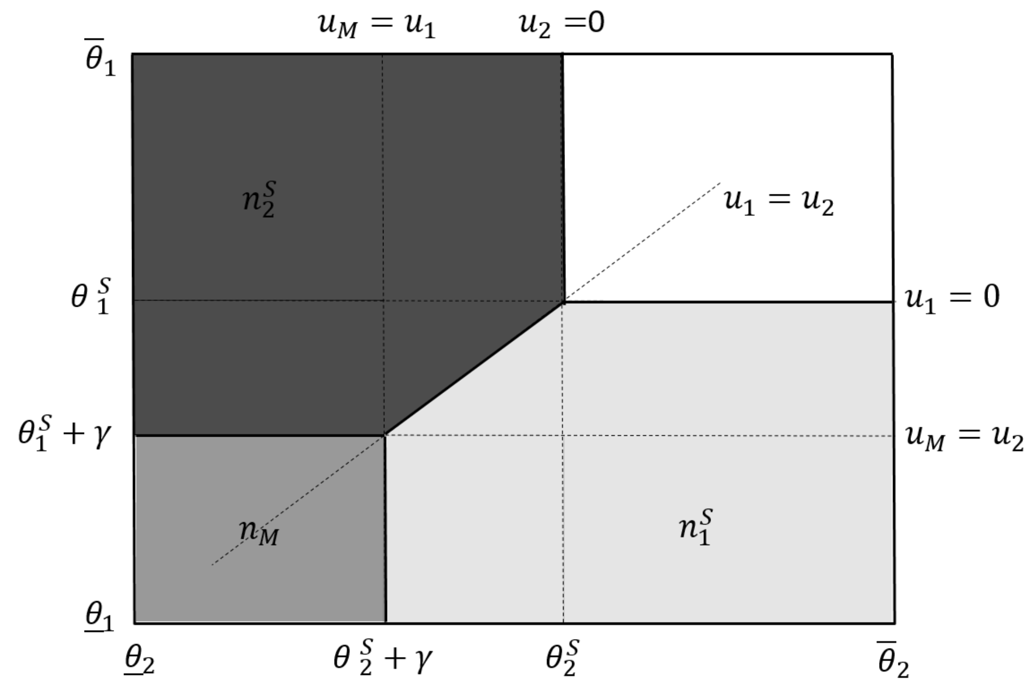
Denoting by $\tilde{\theta}_1(\theta_2)$ the combinations of θ_1 and θ_2 for which the utility of performing either activity is the same ($u_i = u_j$), users with $(\theta_1, \theta_2) \in [\tilde{\theta}_1(\theta_2), \theta_1^S] \times [\theta_2^S + \gamma, \theta_2^S]$ perform activity 1, while the other users, i.e. users with $(\theta_1, \theta_2) \in [\theta_1^S + \gamma, \tilde{\theta}_1(\theta_2)] \times [\theta_2^S + \gamma, \theta_2^S]$ perform activity 2.

In Figure 2, we illustrate the case of $\gamma \leq 0$ and show the trade-offs that partition users into the four groups: users not joining the network (white area), users performing activity 1 (light gray area), users performing activity 2 (dark gray area), and users performing both activities (gray area). The lines $u_i = 0$ indicate that all users with $\theta_i < \theta_i^S$ will at least perform one activity, the lines $u_M = u_i$ confine the users performing two activities ($\theta_i < \theta_i^S + \gamma$), and the line $u_1 = u_2$ separates users into those performing either activity 1 or activity 2.

Similar to before, the level of γ affects the number of single- and multi-activity users. If economies of activity are sufficiently negative, i.e. $\gamma \leq \max\{\underline{\gamma}_1, \underline{\gamma}_2\}$, with $\underline{\gamma}_i \equiv \underline{\theta}_i - \theta_i^S < 0$, no user has an incentive to perform both activities so that the network is frequented only by single-activity users. Instead, some individuals decide to become multi-activity users as long as $\max\{\underline{\gamma}_1, \underline{\gamma}_2\} < \gamma \leq 0$.

Again, to determine the networks optimal pricing strategy, we investigate how users' participation choices are impacted by the network's decisions for the case $\gamma \leq 0$. Irrespective

Figure 2: User classification for $\gamma < 0$



of how negative γ is, we can establish the following effects on participations:[7]

$$\frac{\partial n_i^S}{\partial \theta_i^S} > 0, \quad \frac{\partial n_i^S}{\partial \theta_j^S} < 0, \quad \frac{\partial n_M}{\partial \theta_i^S} \geq 0. \tag{9}$$

The intuition for the effects is similar to before. A rise in the threshold of participating in activity i increases the number of users performing only activity i (right-ward, respectively up-ward, shift of the $u_i = 0$ lines in Figure 2), but decreases the number of users performing only activity j because some users that have been performing only activity j start to perform activity i (shift of the $u_1 = u_2$ line). Furthermore, as long as some users perform both activities, a relaxation of the participation threshold for any activity increases the number of multi-activity users as some single-activity users become multi-activity users.

Moreover, and in contrast to before, a more lenient participation threshold for activity i decreases total participation for activity j $\left(\frac{\partial n_j}{\partial \theta_i^S} < 0\right)$ even though the number of multi-

[7]Details about users' participation and their reaction to the network's decisions for the case $\gamma \leq 0$ can be found in Appendix A.2.

activity users for activity j increases $\left(\frac{\partial n_j^M}{\partial \theta_i^S} > 0\right)$. Again, shifts in the $u_M = u_j$ lines do not have quantitative effects on the number of users performing activity j, but merely determine the partitioning into single-activity and multi-activity users. Thus, the overall effect is only driven by single-activity users that switch activities, which are determined by shifts in the $u_1 = u_2$ line.

2.2 The Network's Problem

In this section, we analyze the network's maximization problem.[8] Profits of the network are given by

$$\Pi = (p_1 - c_1)\left(n_1^S + n_M\right) + (p_2 - c_2)\left(n_2^S + n_M\right), \tag{10}$$

where c_i is the marginal cost the network incurs when a user participates in activity i.

While the natural decision of the network to maximize profits would be to set the prices, p_1 and p_2, related to the two activities, it is analytically equivalent, but more convenient, if the network maximizes profits by optimally setting the participation thresholds θ_1^S and θ_2^S (see Weyl (2010) for details). Deriving the network's profits with respect to θ_i^S yields

$$\begin{aligned} \frac{\partial \Pi}{\partial \theta_i^S} &= (p_i - c_i)\left(\frac{\partial n_i^S}{\partial \theta_i^S} + \frac{\partial n_M}{\partial \theta_i^S}\right) + (p_j - c_j)\left(\frac{\partial n_j^S}{\partial \theta_i^S} + \frac{\partial n_M}{\partial \theta_i^S}\right) - \left(n_i^S + n_M\right) \\ &+ \alpha_i\left(\frac{\partial n_j^S}{\partial \theta_i^S} + \frac{\partial n_M}{\partial \theta_i^S}\right)\left(n_i^S + n_M\right) + \alpha_j\left(\frac{\partial n_i^S}{\partial \theta_i^S} + \frac{\partial n_M}{\partial \theta_i^S}\right)\left(n_j^S + n_M\right) = 0. \end{aligned} \tag{11}$$

Using $n_i = n_i^S + n_M$ to solve the network's pricing problem, we derive the following general result:

[8]Naturally, for two variable maximization problems like this where implicit functions are used, second-order conditions that ensure profit maximization provide little insight toward restrictions on the underlying microfoundations (the distributions of θ_1 and θ_2 relative to γ in this case). However, we can state that the second-order conditions mirror traditional two-sided networks under linear distributions for θ_1 and θ_2 and with γ sufficiently close to zero.

Proposition 1. *The network's optimal prices are given by*

$$p_i^* = c_i - \alpha_j n_j^* + \Delta_{ij}^*,^{9}$$

where $\Delta_{ij}^* := \frac{n_i \frac{\partial n_j}{\partial \theta_j^S} - n_j \frac{\partial n_j}{\partial \theta_i^S}}{\frac{\partial n_i}{\partial \theta_i^S}\frac{\partial n_j}{\partial \theta_j^S} - \frac{\partial n_i}{\partial \theta_j^S}\frac{\partial n_j}{\partial \theta_i^S}} > 0$ *as* $\left|\frac{\partial n_i}{\partial \theta_i^S}\right| \geq \left|\frac{\partial n_i}{\partial \theta_j^S}\right| \forall i \neq j$ *captures the markup term.*

Proof. See Appendix A.3.1. □

Proposition 1 shows that the network's pricing strategy is given by three terms: (i) the marginal cost c_i, (ii) the markdown due to indirect network effects $\alpha_j n_j^*$, and (iii) a markup Δ_{ij}^* related to the sensitivity of users' participation in the activity. As the total number of users performing a specific activity is composed of single-activity and multi-activity users, i.e. $n_i = n_i^S + n_M$, multi-activity users affect pricing via both the indirect network effect and the markup. For example, note that users are exogenously assigned to a specific side resulting in $\frac{\partial n_j}{\partial \theta_i^S} = 0$ in Armstrong (2006). This case implies that $\Delta_{ij} = {n_i}/{\frac{\partial n_i}{\partial \theta_i^S}}$, which mirrors the term ${\phi_i(u_i)}/{\phi\prime_i(u_i)}$ in Armstrong (2006). However, $\frac{\partial n_j}{\partial \theta_i^S} = 0$ can only arise in our model if economies of activity are not too positive ($\gamma < \bar{\gamma}_i, \forall i$) in combination with a bimodal distribution where users' cost parameters are negatively correlated. Hence, multi-activity will naturally impact pricing in the majority of cases and we explore this topic explicitly in the next section.

3 The Transformability of Network Effects

In this section, we dive into the two extremes that are often considered in the literature: Two-sided platforms with indirect network effects and traditional networks with direct network effects. In the following subsections, we consider how each of these extremes can arise with

[9] We use the $*$ superscript to denote the equilibrium. As we show in the proof (see Appendix A.3.1), the platform chooses θ_1^S and θ_2^S when solving the maximization problem so that, technically, the equilibrium pricing solutions should be written as $p_i(\theta_1^{S*}, \theta_2^{S*}) = c_i - \alpha_j n_j(\theta_1^{S*}, \theta_2^{S*}) + \Delta_{ij}(\theta_1^{S*}, \theta_2^{S*})$. However, we drop the θ_i^{S*} terms to simplify the exposition.

multi-activity users and discuss the resulting implications for estimating indirect network effects.

3.1 Two-Sided Networks

Our model allows us to analyze under which conditions traditional two-sided pricing as found in the two-sided platform literature can arise. The most natural way to establish two-sided pricing arises if users are truly distinct so that no multi-activity exists. A second way to attain two-sided pricing is when the presence of multi-activity users is immaterial to the network's pricing decision. We summarize the conditions under which traditional two-sided pricing arises in:

Proposition 2. *Traditional two-sided pricing will only arise if either*

(i) economies of activity are sufficiently negative ($\gamma < \underline{\gamma}_i, \forall i$), or preferences across activities are bimodally distributed and negatively correlated while economies of activity are not too positive ($\gamma < \bar{\gamma}_i \ \forall i$) — pure two-sided networks (with $n_M = 0$),

(ii) or economies of activity are absent ($\gamma = 0$) — quasi-pure two-sided networks (with $n_M > 0$).

Proof. See Appendix A.3.2 □

Proposition 2 effectively captures the implicit assumptions of the two-sided platform literature. To better understand these two settings, we consider each in more detail.

3.1.1 Pure Two-Sided Networks

To understand how multi-activity users affect two-sided network pricing, we first relate our model to the traditional studies on platform pricing presented by literature in which users decide to perform only one activity ($n_M = 0$) and we call this setting pure two-sided networks. This allows us to interpret network activities as sides, and our results from Proposition 2(i) imply that a pure two-sided network occurs either if economies of activity are sufficiently negative or users' activity preferences are bimodal and negatively correlated in combination with economies of activity that are not too positive.

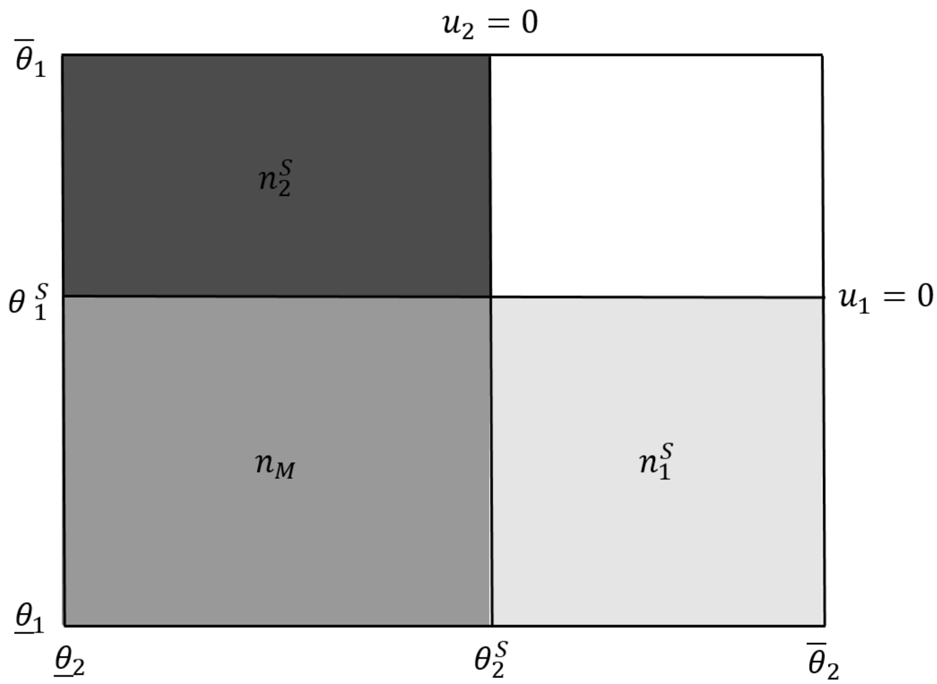
The first case in Proposition 2(i) under which a pure two-sided network can arise is if $\gamma < \underline{\gamma}_i, \forall i$. The only relevant decision for users will be which activity to perform if they decide to join the network. In Figure 2, active users will only be partitioned by the $u_1 = u_2$ line, as the trade-offs $u_M = u_i, \forall i$, become irrelevant when γ is sufficiently negative. This structure matches the framework analyzed in Choi and Zennyo (2019) who extend Armstrong (2006) by allowing users to endogenously choose one (and only one) side of a platform to join.

For the second case in Proposition 2(i), note that users' ability to choose which activity to perform is the reason why the optimal pricing in Proposition 1 marginally differs from the optimal pricing of a monopolist platform in Armstrong (2006). In Armstrong (2006) users are exogenously assigned to a specific side resulting in $\frac{\partial n_j}{\partial \theta_i^S} = 0$. Applied to the network's optimal pricing simplifies the last term to $\Delta_{ij}^* = n_i / \frac{\partial n_i}{\partial \theta_i^S}$, which mirrors the term $\phi_i(u_i) / \phi\prime_i(u_i)$ in Armstrong (2006). In our model, $\frac{\partial n_j}{\partial \theta_i^S} = 0$ can only arise if economies of activity are not too positive ($\gamma < \bar{\gamma}_i, \forall i$) in combination with a bimodal distribution where users' cost parameters are negatively correlated. Graphically, this means that the set of multi-activity users is empty and that users are located either in the upper left or the lower right corner in the figures.

3.1.2 Quasi-Pure Two-Sided Networks

Proposition 2 also reveals another scenario in which the network's optimal pricing resembles the pricing in Armstrong (2006) despite the presence of multi-activity users ($n_M > 0$). This is the case when $\gamma = 0$, which implies that the set of multi-activity users joining the network only to perform both activities (the triangle under $u_M = 0$ in Figure 1) no longer exists. From Figure 3, it becomes clear that for any given number of users performing activity i, the network's decision about θ_j^S has no effect on the number of users performing activity i, but merely determines the partitioning into single- and multi-activity users. As a consequence, a user's decision whether to perform an activity is an isolated choice (cf. Equation (3)).

Figure 3: User classification for $\gamma = 0$



The independence of users' decisions about performing activities implies that the network treats multi-activity users as if they were two separate single-activity users performing unlike activities. From an analytical point of view, it can be easily seen from Figure 3 that $\frac{\partial n_M}{\partial \theta_i^S} = -\frac{\partial n_j^S}{\partial \theta_i^S}$ if $\gamma = 0$. When we use this relationship in the first-order condition (Equation (11)) in conjunction with $n_i^S + n_M = n_i$, and solve for the optimal price, we get

$$p_i^* = c_i - \alpha_j n_j + \frac{n_i}{\frac{\partial n_i}{\partial \theta_i^S}}, \tag{12}$$

which again resembles the optimal pricing of a monopolist platform in (Armstrong, 2006). Hence, if $\gamma = 0$, the network is a quasi-pure two-sided network as the presence of multi-activity users has no important implication for the pricing policy. It is important to note that Proposition 2(ii) has already been highlighted by Gao (2018) Corollary 1. While this specific result is not new per se, we develop important practical implications relating to the case where $\gamma = 0$ that only arise because of multi-activity users.

3.2 One-Sided Networks

In this section, we illustrate how the presence of multi-activity users can effectively generate a network with direct network effects akin to the traditional one-sided network. Consider the case in which economies of activity are so large that all users which decide to join the network are interested in performing both activities ($\bar{\gamma}_M > \gamma \geq \bar{\gamma}_i, \forall i$). If the network is merely frequented by multi-activity users, that is $n_i^S = 0, \forall i$, and thus $n_i = n_j = n_M$, Equation (11) reduces to

$$\frac{\partial \Pi}{\partial \theta_i^S} = (p_i - c_i + p_j - c_j + n_M(\alpha_i + \alpha_j)) \frac{\partial n_M}{\partial \theta_i^S} - n_M = 0. \tag{13}$$

The qualitative difference of the network's pricing if there are only multi-activity users emerges because, in this case, $\frac{\partial n_M}{\partial \theta_i^S} = \frac{\partial n_M}{\partial \theta_j^S}$. The reason is that multi-activity users only decide whether or not to join the network taking into account the total price level (cf. Equation (5)). This user behavior implies that the first-order conditions for θ_1^S and θ_2^S become identical with infinite possibilities for the network to satisfy Equation (13).

The relevance of the total price level as opposed to the price structure highlights that a network frequented only by multi-activity users faces the same trade-off as a network featuring direct network effects even though our model only exhibits indirect network effects. However, as multi-activity users benefit from both indirect network effects, the sum of the indirect network effects is the same across multi-activity users. This implies that indirect network effects are transformed into direct network effects, at least as long as $\gamma \neq 0$ as illustrated in the previous section. Hence, we summarize in:

Proposition 3. *The presence of multi-activity users creates direct network effects if* $\gamma \neq 0$*.*

If economies of activity are large ($\bar{\gamma}_M > \gamma \geq \bar{\gamma}_i, \forall i$), all active users perform both activities transforming a potentially two-sided network into a one-sided network.

Proof. See Appendix A.3.3 □

Proposition 3 illustrates that the dichotomy of direct and indirect network effects traditionally used in the literature needs to be seen in a more nuanced light as network effects become versatile in the presence of multi-activity users. The transformability of indirect network effects can be most saliently illustrated by telecommunication systems like telephony or instant messaging. While telecommunication systems are frequently used in the literature to describe direct network effects, they can be characterized as networks with two activities, sending and receiving. Hence, in principle, the network can price each of these activities separately. Indeed, in the early years of cellular markets, network operators traditionally priced either the party that made the call (calling party pays) or the party that received the call (receiving party pays) — a two-sided pricing strategy.[10] However, in many countries nowadays, virtually all cellphone users are frequent performers of both activities and therefore conclude postpaid contracts that typically specify a limit or "allowance" of minutes or text messages that disregard what kind of activities a user performs. Effectively, such contracts resemble a one-sided network pricing strategy.

4 Practical and Empirical Implications of Multi-activity Users

4.1 Practical Implications

In this section, we focus on how the correlation between θ_1 and θ_2 impact the extent of multi-activity and the resulting equilibrium outcome. To simplify our analysis we assume that $\gamma = 0$ in everything that follows and this implies that $u_M = u_1 + u_2$. While we draw

[10]Similarly, early texting contracts charged different prices for sending and receiving text messages.

our attention to the practical implications in this section, we show that following results also generate novel caveats to empirical settings considered in the platform literature. We elaborate on the empirical implications in the next section.

In the following, we illustrate that there are preference distributions that generate identical equilibria except for the number of multi-activity users. This occurs when the marginal distributions of two different joint density functions are equivalent, which we refer to in the following as *marginal distribution equivalence.* In contrast to the conditional distribution, which informs about the density of θ_i conditional on the values of θ_j, the marginal distribution of θ_i is the distribution of θ_i without taking into account the values of θ_j (that is, by integrating over all values of θ_j). More formally, two joint density functions, $\hat{g}(\theta_i, \theta_j)$ and $\tilde{g}(\theta_i, \theta_j)$, are *marginal distribution equivalent* (denoted by $\hat{g} \sim \tilde{g}$) if $\hat{G}_i(\theta_i) = \int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_j}^{\overline{\theta}_j} \hat{g}(\theta_i', \theta_j') d\theta_j' d\theta_i' = \int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_j}^{\overline{\theta}_j} \tilde{g}(\theta_i', \theta_j') d\theta_j' d\theta_i' = \tilde{G}_i(\theta_i)$ for all θ_i and for $i = 1, 2$.[11] We summarize the implications of marginal distribution equivalence as follows:

Proposition 4. *In the absence of economies of activity ($\gamma = 0$), if the marginal distributions of their users' preferences are equivalent, then two networks only differ in the number of multi-activity users and thus feature the same prices and participation levels on each side of the market.*

Proof. See Appendix A.3.4 □

There are two points of note regarding Proposition 4. First, relating this result to optimal pricing given in Proposition 1 we see that marginal distribution equivalence ensures that both the Δ_{ij} and the n_i are constant so that pricing strategies will be quantitatively equivalent. Second, the proposition implies that networks whose users have vastly different preferences

[11]There is a longstanding mathematical literature on the optimization over these marginal equivalent joint densities. In transportation theory, joint distribution $g(\theta_1, \theta_2)$ with marginal distributions $G_1(\theta_1)$ and $G_2(\theta_2)$ corresponds to a "transport plan" in $[\underline{\theta}_1, \overline{\theta_1}] \times [\underline{\theta}_2, \overline{\theta_2}]$ between two "marginals" in $[\underline{\theta}_1, \overline{\theta_1}]$ and $[\underline{\theta}_2, \overline{\theta_2}]$ respectively. And in the commonly studied Monge-Kantorovich problem, one determines the transportation cost minimizing transport plan, g, from the set of transport plans between G_1 and G_2. Page 119 in Ambrosio et al. (2008) provides more information on transport plans (i.e., distributions that are marginal distribution equivalent).

may produce similar outcomes in terms of pricing and participation in each activity, but not in terms of multi-activity.[12]

The second point pertains to how "number of users" is defined as this will impact whether or not it serves as a reliable performance measure. Specifically, if the number of users is defined by the number of users performing activity 1 plus the number of users performing activity 2 (that is, the total number of users), then $N = n_1 + n_2$. Instead, if each user profile is considered individually, regardless of which activities are pursued through a user's profile, then the total number of active users is given by $N = n_1 + n_2 - n_M$ (that is, the unique number of users in the network). To elaborate on the practical implications, consider the following example.

Let $g^X(\theta_1, \theta_2)$, $X = \{+, -, 0\}$ denote the case of perfect positive, perfect negative, and zero correlation between cost parameters, respectively. More specifically, let $g^+(\theta_1, \theta_2)$ be captured by $\theta \sim U[0,1]$ with $\theta_i = \underline{\theta}_i + (\overline{\theta}_i - \underline{\theta}_i) \cdot \theta$, for $i = 1, 2$, $g^-(\theta_1, \theta_2)$ by $\theta \sim U[0,1]$ with $\theta_1 = \underline{\theta}_1 + (\overline{\theta}_1 - \underline{\theta}_1) \cdot \theta$ and $\theta_2 = \overline{\theta}_2 - (\overline{\theta}_2 - \underline{\theta}_2) \cdot \theta$, and $g^0(\theta_1, \theta_2)$ by $\theta_i \sim U[\underline{\theta}_i, \overline{\theta}_i]$ for $i = 1, 2$. Although all three distributions imply uniform marginal distributions, the underlying joint distributions differ.[13] From Figure 3, it can be easily seen that $n_i = \int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_j}^{\overline{\theta}_j} g(\theta_i', \theta_j') d\theta_j' d\theta_i'$, which is simply the marginal distribution $G_i(\theta_i)$. It is then straightforward to show that the g^+, g^-, and g^0 are marginal distribution equivalent as $G_i^X(\theta_i) = U[\underline{\theta}_i, \overline{\theta}_i]$ for $i = 1, 2$ and for all $X = \{+, -, 0\}$. Because marginal distribution equivalence implies that participation in activity i, n_i, is equivalent across the three distributions, network pricing must also be equivalent by Equation (12).[14] However, it is important to note that the three distributions cause different numbers of multi-activity users, which implies that the total number of active

[12] That is, if $\gamma = 0$ and $\hat{g} \sim \tilde{g}$, then $p_i^*(\hat{g}) = p_i^*(\tilde{g})$ and $n_i^*(\hat{g}) = n_i^*(\tilde{g})$ for $i = 1, 2$ with $n_M^*(\hat{g}) \neq n_M^*(\tilde{g})$ except by chance.

[13] While g^0 occupies the entire $[\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$ space, g^+ only occupies the 45 degree line between $(\underline{\theta}_1, \underline{\theta}_2)$ and $(\overline{\theta}_1, \overline{\theta}_2)$ uniformly, whereas g^- only occupies the 45 degree line between $(\underline{\theta}_1, \overline{\theta}_2)$ and $(\overline{\theta}_1, \underline{\theta}_2)$ uniformly.

[14] Maximizing Equation (10) under any of the three specifications results in first-order conditions given by: $0 = -\underline{\theta}_i + \alpha_i n_j - 2(\overline{\theta}_i - \underline{\theta}_i) n_i - c_i + \alpha_j n_j$. The second-order conditions of profit maximization require that $4(\overline{\theta}_1 - \underline{\theta}_1)(\overline{\theta}_2 - \underline{\theta}_2) > (\alpha_1 + \alpha_2)^2$. Solving the two equations for n_1 and n_2, we have that $n_i^* = \frac{(\alpha_1+\alpha_2)(-\underline{\theta}_i - c_i) + 2(\overline{\theta}_i - \underline{\theta}_i)(-\underline{\theta}_j - c_j)}{4(\overline{\theta}_1 - \underline{\theta}_1)(\overline{\theta}_2 - \underline{\theta}_2) - (\alpha_1 + \alpha_2)^2}$.

users, given by $N = n_1 + n_2 - n_M$, can differ drastically across networks that maintain the same prices, in this example, $N^*(g^-) > N^*(g^0) > N^*(g^+)$ despite $p^*(g^-) = p^*(g^0) = p^*(g^+)$. To illustrate this point, if for example $\underline{\theta}_1 = \underline{\theta}_2 = 0$ and $\overline{\theta}_1 = \overline{\theta}_2 = 1$, then $n_M^*(g^+) = \min\{n_1^*, n_2^*\}$, $n_M^*(g^-) = \max\{n_1^* + n_2^* - 1, 0\}$, and $n_M^*(g^0) = n_1^* \cdot n_2^*$.[15]

This result has important practical implications, especially for those interested in investing in platforms. The number of active users is not only a key performance indicator, but frequently used as the benchmark indicator to inform investors about a platform's profit and growth potential. While the number of active users as a performance indicator has been widely criticized mainly due to missing industry standards for measurement and thus a lack of its comparability across firms, our findings suggest that the measure should be taken with caution even if measurement is standardized. Specifically, Proposition 4 illustrates that the number of active users may not contain any informational content to evaluate two platforms' potential profitability, especially in industries with negligible economies of activity, as is arguably the case for credit cards, video gaming and advertising-financed media, such as, social media platforms.

4.2 Empirical Implications

4.2.1 The Reflection Problem and Indirect Network Effects

To highlight the empirical implications on estimating network effects, we revisit the empirical identification of network effects in light of the reflection problem. The reflection problem described by Manski (1993) characterizes a setting in which a researcher tries to predict whether the behavior of an individual within a group is affected by the average behavior in the group. According to Manski (1993), identification is impossible in the context of local

[15]Under g^+, the first agent to engage in activity i for $i = 1, 2$ is the agent identified by $\theta_i = 0$. Hence, the activity with fewer agents is comprised entirely of multi-activity users, i.e., $n_M^*(g^+) = \min\{n_1^*, n_2^*\}$. Under g^-, the first agent to engage in activity 1, respectively activity 2, is the agent identified by $\theta_1 = 0$, respectively $\theta_1 = 1$. Hence, if at all, the only agents that are multi-activity users are those that overlap in the middle of the interval, i.e., $n_M^*(g^-) = \max\{n_1^* + n_2^* - 1, 0\}$. Lastly under g^0, the only agents that are multi-activity users are those that draw sufficiently low θ_i, for each activity, i.e., $n_M^*(g^0) = n_1^* \cdot n_2^*$.

spillovers between individuals because causal neighborhood spillovers cannot be separated from local correlation driven by unobserved neighborhood effects.

Recently, Rysman (2019) stated that the identification of direct network effects suffers from the same problem as the set-up of a standard model of direct network effects resembles the set-up in the reflection problem. Specifically, an individual's valuation of a product depends on how many other individuals use the product meaning that individuals' base their product choice on the product choices made by other individuals in their market. However, Rysman (2019) argues that, while the reflection problem exists when estimating direct network effects, indirect network effects offer a natural way of addressing the reflection problem. The reason is that indirect network effects correspond to the unobserved correlated effect and can be identified under the typical assumption in the platform literature that direct network effects are absent.

However, as we show in Proposition 3, abstracting from direct network effects in the traditional sense is a necessary, but not a sufficient condition to ensure identification, because the presence of multi-activity users also generates direct network effects whenever economies of activity exist. Hence, identification requires the additional restriction that either economies of activity are absent or multi-activity users play a negligible role in the overall activity on the platform.

The early literature on quantifying indirect network effects often focused on industries in which the provision of physical goods or services played a prominent role, such as banking (Ackerberg and Gowrisankaran (2006)), video gaming (Clements and Ohashi (2005)) and advertising-financed media (Rysman (2004); Kaiser and Wright (2006); Wilbur (2008)). These industries are usually characterized by a quite substantial dissimilarity of activities and a necessity for significant investments to scale for at least one activity, which likely implies that either economies of activity are absent or multi-activity users play a negligible role in the overall activity in the network. Hence, the quantification of indirect network effects that rely on such industries is arguably not affected by a potential bias caused by multi-activity users.

However, nowadays, many parts of the economy, such as e-commerce, ride-hailing or accommodation, are no longer characterized by a sharp distinction between consumers and producers. This is not exclusively, but especially the case in niche markets and early stage platforms where multi-active users, so-called prosumers, might make up a considerable mass of the platform's users. For example, many parents with children are simultaneously buying new cloths that fit and selling old ones that are too small at local clothing swaps. In its early years, Amazon was an online book store that specialized in selling rare books. At that time, buying and selling occurred under a single account (much like eBay today) which likely encouraged greater amounts of multi-activity. For example, rare book collectors buying and selling and students buying new textbooks while selling old ones. Trading card platforms is another example where the means of being a buyer and being a seller occurs simultaneously when users exchange cards.

When analyzing the magnitude of network effects in these markets, the presence of multi-activity users becomes an important concern for identification. For example, Chu and Manchanda (2016) estimate the strength of direct and indirect network effects on Taobao.com, a consumer-to-consumer online shopping platform, and how they contribute to platform growth over the life cycle. While they find large and positive contributions of indirect network effects on both sides to platform growth, direct network effects do not seem to play an important role. However, their analysis is based on the assumption that the decision of a buyer (seller) to join the platform is independent from the decision of a seller (buyer) to join. This assumption is only plausible in a situation where either $\gamma = 0$ and θ draws are independent, which implies independence of the decision to join any side, or multi-activity is negligible. Since their data begins from the platform's inception, these assumptions are unlikely to hold which means that, at least in the early years of Taobao.com, the magnitude of indirect network effects may be overstated because part of it may actually be a direct network effect according to our model.

4.2.2 Identification and the Granularity of Observations

The potential identification problem arises because the strength of indirect network effects is often quantified by analyzing how adoption of one user group is affected by changes in the participation of another user group. While such an approach allows to conclude whether indirect network effects exist, it may fail to provide reliable estimates on the strength whenever economies of activity are non-zero ($\gamma \neq 0$) because it abstracts from user's endogenous decision to perform more than one activity.

However, more recent studies draw on much richer and more fine-grained data that allow to observe transactions at the user level. In the following, we discuss to which extent multi-activity may be relevant and elaborate on the potential empirical implications by highlighting the importance of the granularity of observations. To be more specific, there are two important dimensions: (1) the size of an observation's location and (2) the time length of an observation. These two factors, in addition to the industries characteristics, provide the context by which multi-activity can exist.

There is a growing empirical literature on Airbnb that utilizes data from InsideAirbnb, AirDNA, or Airbnb (e.g., Li and Srinivasan (2019), Barron et al. (2021), Bibler et al. (2021) and Farronato and Fradkin (2022)), and considers both reduced form and structural techniques. In most cases, observations are defined within a metro-month. Such a defined observation may include multi-active users when there are major local events within a metro. For example, Oxford, Ohio (where Miami University is located) lies within the Cincinnati metro so that the majority of Oxford hosts stay as guests in downtown Cincinnati during graduation weekend at Miami University and are therefore multi-active users at the metro-day level.[16] This implies that the welfare effects during major events that generate large amounts of multi-active users might be underestimated, since the marginal cost of some hosts (hosts in Oxford) depends on the price of others (hosts in Cincinnati). For example, the welfare

[16] Similar arguments could be made for downtown areas that host festivals, concerts, and other large events. Locals are hosts within the downtown area but also guests in the suburbs so that they would be considered multi-active users within the metro-day.

benefits are likely underestimated in Farronato and Fradkin (2022) who consider periods of significant demand where hotels reach capacity. A similar issue arises in Yao et al. (2022), who analyze owners' acceptance decisions on a car sharing platform, but abstract from the possibility that owners themselves have the opportunity to rent a car in order to accept an incoming request.

The magnitude of multi-activity users can also be an issue in the literature on ride-hailing. While there is considerable variation in the length of time the observational unit is often defined at the week level.[17] Many drivers on ride-sharing platforms may also be riders within their city for a given week, for example, when driving during the work week and riding during the weekend. Angrist et al. (2021) consider driver-city observations to investigate Uber drivers' willingness to lease a virtual taxi medallion that eliminates the Uber fee. Hence, a driver's decision is based on their expected earnings opportunity. A reason why multi-activity could influence their result is that multi-activity users may have access to more information about market conditions because they are active on both sides and hence may find it easier to make predictions about their earnings opportunities. As Angrist et al. (2021) only consider drivers with average weekly driving hours between 5 and 25 hours, this sample is the most likely to be influenced by multi-activity users as these drivers are the most likely to have time to also be active as a rider.

Hall et al. (2021) consider weekly-city observations and find that, following fare increases set by Uber, ride demand decreases and drivers work more hours as they earn more per ride. Multi-activity users can play an important role because they affect both sides simultaneously. For example, instead of going out and using Uber's services as a rider, they could drive on Friday nights due to higher earnings opportunities following a fare increase. In this case, the substitutability between the two sides of the market by multi-activity users may blur the effects of how an increase in the fare affects demand and supply. More specifically, an

[17] An exception is Cohen et al. (2016), who use session level observations. Unless a rider observes surge pricing, decides to stop searching for a ride, and immediately become a driver, the appearance of multi-activity users is unlikely at this level.

increase of the fare increases drivers' incentives to participate on Uber, while it decreases the incentives of riders to use Uber. Users active merely as riders only have the option to opt out of Uber, while multi-activity users can decide to become a driver instead of opting out and the incentive becomes stronger the more the fare increases. Hence, the price elasticity of demand is a function of the composition of riders, with a higher share of multi-activity users leading to a more elastic demand.

While the mass of multi-activity users certainly decreases the narrower the observational unit, both in time and space, the potential of bias may still be prevalent in markets where users can frequently and easily switch roles. In order to address the potential bias, empirical studies have to take into account the endogenous decision of individuals which activities to perform, which is, in principle, possible given the observability of detailed user information. This decision can be incorporated by means of a discrete choice model as done recently by Affeldt et al. (2022) who investigate the bias in price elasticities and indirect network effects that arises when neglecting users decision of multi-*homing*.

5 Conclusion

We develop a model in which users decide which of two activities to perform in a network based on idiosyncratic preferences and economies of activity. While the network features only indirect network effects between users performing unlike activities, we show that the presence of multi-activity users transforms indirect into direct network effects whenever economies of activity are non-zero. Thus, our analysis suggests that the dichotomy of network effects emerges from an incomplete understanding of network user behavior.

Traditional two-sided network pricing only arises if user preferences across activities are negatively correlated and bimodally distributed and economies of activity are not too positive, which implies that individuals perform at most one activity. However, we highlight that traditional two-sided network pricing also arises in the presence of multi-activity users if economies of activity are absent. In this case, the network treats multi-activity users as if they were two separate users performing unlike activities. Moreover, in the extreme case where the network is frequented only by multi-activity users, the network effectively becomes one-sided.

We show that multi-activity users have important practical implications especially for those interested in investing in platforms that feature negligible economies of activity. Specifically, while the network's pricing strategy and its profits are unaffected by the correlation between activities under marginal distribution equivalence, multi-activity users have an effect on the total number of users performing an activity, as opposed to the participation in the activity. Thus, despite the practical importance of the number of active users as a performance metric, we highlight that it may not contain any informational content to evaluate a platform's potential profitability.

In addition to our contribution to the theoretical literature on network effects, we also highlight meaningful implications to the corresponding empirical literature which largely ignores the existence of multi-activity users. We highlight how the reflection problem, thought to be an issue only when estimating direct network effects, can remain an issue when estimating indirect network effects whenever economies of activity are non-zero as multi-activity users turn indirect into direct network effects. Our work also reveals how the granularity of observation impacts the emergence of multi-activity and the underlying identification strategy using reduced form techniques. These findings suggest that multi-activity will be an important feature of network markets moving forward.

Our analysis provides interesting avenues for future research. For example, a particular challenge for entrepreneurs of platform-based start-ups is the so-called chicken-and-egg problem, that is, a specific group of users will only join a platform, if it expects users of other groups to also join. This coordination problem forces entrepreneurs to secure enough users and in the right proportion in order to thrive. Evans and Schmalensee (2010) have carved out conditions under which a platform clears the hurdle of reaching critical user mass absent users' possibility to join both sides. However, a start-up platform can reduce or in the extreme case eliminate the coordination problem if it is able to incentivize users to join more than one side. This is because such a user generates a network benefit to the platform ecosystem that is equivalent to two individual users joining different sides. Whether eliminating the chicken-and-egg problem by transforming into a one-sided network is economically viable and how to best incentive users when they are heterogeneous both in their willingness to participate and to transact is an interesting issue for further study.

A Appendix

A.1 Participation and responses for $\gamma > 0$

The number of single- and multi-activity users critically depends on the level of γ. For the case $\gamma > 0$, the number of users performing only activity i is given by

$$n_i^S = \begin{cases} \int_{\underline{\theta}_i}^{\theta_i^S} \int_{\theta_j^S+\gamma}^{\bar{\theta}_j} dG(\theta_i, \theta_j), \ i \neq j & \text{if} \quad \bar{\gamma}_j \geq \gamma > 0, \\ 0 & \text{if} \quad \gamma > \bar{\gamma}_j, \end{cases} \tag{A.1}$$

where $\bar{\gamma}_i = \bar{\theta}_i - \theta_i^S \geq 0$. Equation (A.1) shows that the number of single-activity users crucially depends on both the level of γ and the distribution of users' activity types.

In a similar manner, we can determine the number of multi-activity agents for the case $\gamma > 0$, which reads

$$n_M = \begin{cases} \int_{\underline{\theta}_2}^{\theta_2^S} \int_{\underline{\theta}_1}^{\theta_1^S+\gamma} dG(\theta_1, \theta_2) + \int_{\theta_2^S}^{\theta_2^S+\gamma} \int_{\underline{\theta}_1}^{\hat{\theta}_1(\theta_2)} dG(\theta_1, \theta_2) & \text{if} \quad \bar{\gamma}_i \geq \gamma > 0 \ \forall i \\ \int_{\underline{\theta}_2}^{\theta_2^S} \int_{\underline{\theta}_1}^{\theta_1^S+\gamma} dG(\theta_1, \theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2} \int_{\underline{\theta}_1}^{\hat{\theta}_1(\theta_2)} dG(\theta_1, \theta_2) & \text{if} \quad \bar{\gamma}_1 > \gamma \geq \bar{\gamma}_2 \\ \int_{\underline{\theta}_2}^{\theta_2^S} \int_{\underline{\theta}_1}^{\bar{\theta}_1} dG(\theta_1, \theta_2) + \int_{\theta_2^S}^{\theta_2^S+\gamma} \int_{\underline{\theta}_1}^{\hat{\theta}_1(\theta_2)} dG(\theta_1, \theta_2) & \text{if} \quad \bar{\gamma}_2 > \gamma \geq \bar{\gamma}_1 \\ \int_{\underline{\theta}_2}^{\theta_2^S} \int_{\underline{\theta}_1}^{\bar{\theta}_1} dG(\theta_1, \theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2} \int_{\underline{\theta}_1}^{\hat{\theta}_1(\theta_2)} dG(\theta_1, \theta_2) & \text{if} \quad \bar{\gamma}_M > \gamma \geq \bar{\gamma}_i \ \forall i \\ \int_{\underline{\theta}_2}^{\bar{\theta}_2} \int_{\underline{\theta}_1}^{\bar{\theta}_1} dG(\theta_1, \theta_2) & \text{if} \quad \gamma \geq \bar{\gamma}_M, \end{cases} \tag{A.2}$$

where $\bar{\gamma}_M = \sum_i \bar{\theta}_i - \theta_i^S \geq 0$ indicating the threshold, for which the highest type $(\bar{\theta}_1, \bar{\theta}_2)$ derives a zero utility from performing both activities. Equation (A.2) shows that the number of multi-activity users depends on whether there are (i) single-activity users for both activities (first line), (ii) single-activity users only for activity 1 (second line), (iii) single-activity users only for activity 2 (line three), (iv) no single-activity users, where only a subset of users connects to the network (line four), or (v) no single-activity users, where all users connect to the network (line five).

To arrive at the results of our propositions, we need to determine how the network's decision impact single-activity, multi-activity, and gross participation levels. From Equation (A.1), the effect on single-activity users are given by

$$\frac{\partial n_i^S}{\partial \theta_i^S} = \begin{cases} \int_{\theta_j^S+\gamma}^{\bar{\theta}_j} g(\theta_i^S, \theta_j) d\theta_j, \quad i \neq j & \text{if} \quad \bar{\gamma}_j \geq \gamma > 0 \\ 0 & \text{if} \quad \gamma > \bar{\gamma}_j \end{cases} \tag{A.3}$$

$$\frac{\partial n_i^S}{\partial \theta_j^S} = \begin{cases} -\int_{\underline{\theta}_i}^{\theta_i^S} g(\theta_i, \theta_j^S + \gamma) d\theta_i, \quad i \neq j & \text{if} \quad \bar{\gamma}_j \geq \gamma > 0 \\ 0 & \text{if} \quad \gamma > \bar{\gamma}_j \end{cases} \tag{A.4}$$

From Equation (A.2), the effect of changes in θ_1^S and θ_2^S on the number of multi-activity users is respectively given by

$$\frac{\partial n_M}{\partial \theta_1^S} = \begin{cases} \int_{\underline{\theta}_2}^{\theta_2^S} g(\theta_1^S + \gamma, \theta_2) d\theta_2 + \int_{\theta_2^S}^{\theta_2^S+\gamma} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_i \geq \gamma > 0 \ \forall i \\ \int_{\underline{\theta}_2}^{\theta_2^S} g(\theta_1^S + \gamma, \theta_2) d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_1 > \gamma \geq \bar{\gamma}_2 \\ \int_{\theta_2^S}^{\theta_2^S+\gamma} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_2 > \gamma \geq \bar{\gamma}_1 \\ \int_{\theta_2^S}^{\bar{\theta}_2} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_M \geq \gamma \geq \bar{\gamma}_i \ \forall i \\ 0 & \text{if} \quad \gamma \geq \bar{\gamma}_M, \end{cases} \tag{A.5}$$

$$\frac{\partial n_M}{\partial \theta_2^S} = \begin{cases} \int_{\underline{\theta}_1}^{\theta_1^S} g(\theta_1, \theta_2^S + \gamma) d\theta_1 + \int_{\theta_2^S}^{\theta_2^S+\gamma} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_i \geq \gamma \geq 0 \ \forall i \\ \int_{\theta_2^S}^{\bar{\theta}_2} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_1 > \gamma \geq \bar{\gamma}_2 \\ \int_{\underline{\theta}_1}^{\theta_1^S} g(\theta_1, \theta_2^S + \gamma) d\theta_1 + \int_{\theta_2^S}^{\theta_2^S+\gamma} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_2 > \gamma \geq \bar{\gamma}_1 \\ \int_{\theta_2^S}^{\bar{\theta}_2} g(\hat{\theta}_1(\theta_2), \theta_2) d\theta_2 & \text{if} \quad \bar{\gamma}_M \geq \gamma \geq \bar{\gamma}_i \ \forall i \\ 0 & \text{if} \quad \gamma \geq \bar{\gamma}_M. \end{cases} \tag{A.6}$$

Equations (A.5) and (A.6) show that a relaxation of the participation threshold for any activity weakly increases the number of users that perform both activities. In combination with Equation (A.4), it follows that $\frac{\partial n_i}{\partial \theta_j^S} = \frac{\partial n_i^S}{\partial \theta_j^S} + \frac{\partial n_M}{\partial \theta_j^S} \geq 0$, which means that the total participation for activity i increases when the network raises θ_j^S. Finally, because $\frac{\partial n_i}{\partial \theta_i^S} \geq 0 \geq \frac{\partial n_i}{\partial \theta_j^S}$, it also follows that $\left|\frac{\partial n_i}{\partial \theta_i^S}\right| \geq \left|\frac{\partial n_i}{\partial \theta_j^S}\right|$.

A.2 Participation and responses for $\gamma \leq 0$

For the case $\gamma \leq 0$, the number of users performing only activity 1 or activity 2 is respectively given by

$$n_1^S = \begin{cases} \int_{\theta_2^S+\gamma}^{\theta_2^S}\int_{\underline{\theta}_1}^{\tilde{\theta}_1(\theta_2)} dG(\theta_1,\theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2}\int_{\underline{\theta}_1}^{\theta_1^S} dG(\theta_1,\theta_2), & \text{if } 0 \geq \gamma > \underline{\gamma}_i,\ \forall i \\ \int_{\underline{\theta}_2}^{\theta_2^S}\int_{\underline{\theta}_1}^{\tilde{\theta}_1(\theta_2)} dG(\theta_1,\theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2}\int_{\underline{\theta}_1}^{\theta_1^S} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1, \\ \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S}\int_{\underline{\theta}_1}^{\tilde{\theta}_1(\theta_2)} dG(\theta_1,\theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2}\int_{\underline{\theta}_1}^{\theta_1^S} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2 \\ \int_{\underline{\theta}_2}^{\theta_2^S}\int_{\underline{\theta}_1}^{\tilde{\theta}_1(\theta_2)} dG(\theta_1,\theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2}\int_{\underline{\theta}_1}^{\theta_1^S} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma, \\ \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S}\int_{\underline{\theta}_1}^{\tilde{\theta}_1(\theta_2)} dG(\theta_1,\theta_2) + \int_{\theta_2^S}^{\bar{\theta}_2}\int_{\underline{\theta}_1}^{\theta_1^S} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma, \end{cases} \tag{A.7}$$

$$n_2^S = \begin{cases} \int_{\underline{\theta}_2}^{\theta_2^S+\gamma}\int_{\theta_1^S+\gamma}^{\bar{\theta}_1} dG(\theta_1,\theta_2) + \int_{\theta_2^S+\gamma}^{\theta_2^S}\int_{\tilde{\theta}_1(\theta_2)}^{\bar{\theta}_1} dG(\theta_1,\theta_2), & \text{if } 0 \geq \gamma > \underline{\gamma}_i, \forall i \\ \int_{\underline{\theta}_2}^{\theta_2^S}\int_{\tilde{\theta}_1(\theta_2)}^{\bar{\theta}_1} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1 \\ \int_{\underline{\theta}_2}^{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}\int_{\underline{\theta}_1}^{\bar{\theta}_1} dG(\theta_1,\theta_2) + \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S}\int_{\tilde{\theta}_1(\theta_2)}^{\bar{\theta}_1} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2, \\ \int_{\underline{\theta}_2}^{\theta_2^S}\int_{\tilde{\theta}_1(\theta_2)}^{\bar{\theta}_1} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma \\ \int_{\underline{\theta}_2}^{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}\int_{\underline{\theta}_1}^{\bar{\theta}_1} dG(\theta_1,\theta_2) + \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S}\int_{\tilde{\theta}_1(\theta_2)}^{\bar{\theta}_1} dG(\theta_1,\theta_2), & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma. \end{cases} \tag{A.8}$$

Equations (A.7) and (A.8) show that the number of single-activity users depends on whether there are some multi-activity users (first lines), or no multi-activity users (second to fifth lines). The four cases without multi-activity users just differ depending on whether the $u_1 = u_2$ line crosses the θ_2-axis to the left of $\underline{\theta}_1$ (second and fourth lines in (A.7) and (A.8)) or to the right (third and fifth lines).

The number of users performing both activities, n_M, reads

$$n_M = \begin{cases} \int_{\underline{\theta}_i}^{\theta_i^S+\gamma}\int_{\underline{\theta}_j}^{\theta_j^S+\gamma} dG(\theta_i,\theta_j), & \text{if} \quad \gamma \geq \underline{\gamma}_i, \forall i \\ 0, & \text{if} \quad \underline{\gamma}_i > \gamma, \forall i, \end{cases} \tag{A.9}$$

where $\underline{\gamma}_i = \underline{\theta}_i - \theta_i^S \leq 0$ indicates the threshold for γ at which not even the lowest type would perform both activities.

From equations (A.7), (A.8) and (A.9), we can derive how users' participation decisions are affected by the network's pricing strategies. The effects on single-activity users for activity 1 read

$$\frac{\partial n_1^S}{\partial \theta_1^S} = \begin{cases} \int_{\theta_2^S+\gamma}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\theta_1^S,\theta_2)d\theta_2, & \text{if } 0 \geq \gamma > \underline{\gamma}_i, \ \forall i \\ \int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\theta_1^S,\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1, \\ \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\theta_1^S,\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2 \\ \int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\theta_1^S,\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma, \\ \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2 + \int_{\theta_2^S}^{\bar{\theta}_2} g(\theta_1^S,\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma, \end{cases} \tag{A.10}$$

$$\frac{\partial n_1^S}{\partial \theta_2^S} = \begin{cases} -\int_{\underline{\theta}_1}^{\theta_1^S+\gamma} g(\theta_1,\theta_2^S+\gamma)d\theta_1 - \int_{\theta_2^S+\gamma}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } 0 \geq \gamma > \underline{\gamma}_i, \ \forall i \\ -\int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1, \\ -\int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2 \\ -\int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma, \\ -\int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma, \end{cases} \tag{A.11}$$

In a similar manner, we can determine the effects on single-activity users for activity 2

$$\frac{\partial n_2^S}{\partial \theta_1^S} = \begin{cases} -\int_{\underline{\theta}_2}^{\theta_2^S+\gamma} g(\theta_1^S+\gamma,\theta_2)d\theta_2 - \int_{\theta_2^S+\gamma}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } 0 \geq \gamma > \underline{\gamma}_i, \forall i \\ -\int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1 \\ -\int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2, \\ -\int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma \\ -\int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma. \end{cases} \tag{A.12}$$

$$\frac{\partial n_2^S}{\partial \theta_2^S} = \begin{cases} \int_{\theta_1^S}^{\bar{\theta}_1} g(\theta_1,\theta_2^S)d\theta_1 + \int_{\theta_2^S+\gamma}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } 0 \geq \gamma > \underline{\gamma}_i, \forall i \\ \int_{\theta_1^S}^{\bar{\theta}_1} g(\theta_1,\theta_2^S)d\theta_1 + \int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \gamma > \underline{\gamma}_1 \\ \int_{\theta_1^S}^{\bar{\theta}_1} g(\theta_1,\theta_2^S)d\theta_1 + \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \gamma > \underline{\gamma}_2, \\ \int_{\theta_1^S}^{\bar{\theta}_1} g(\theta_1,\theta_2^S)d\theta_1 + \int_{\underline{\theta}_2}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_2 > \underline{\gamma}_1 > \gamma \\ \int_{\theta_1^S}^{\bar{\theta}_1} g(\theta_1,\theta_2^S)d\theta_1 + \int_{\tilde{\theta}_1^{-1}(\underline{\theta}_1)}^{\theta_2^S} g(\tilde{\theta}_1(\theta_2),\theta_2)d\theta_2, & \text{if } \underline{\gamma}_1 > \underline{\gamma}_2 > \gamma. \end{cases} \tag{A.13}$$

Finally, the effects on multi-activity users are given by

$$\frac{\partial n_M}{\partial \theta_i^S} = \begin{cases} \int_{\underline{\theta}_j}^{\theta_j^S+\gamma} g(\theta_i^S+\gamma,\theta_j)d\theta_j, & \text{if } \gamma \geq \underline{\gamma}_i, \forall i \\ 0, & \text{if } \underline{\gamma}_i > \gamma, \forall i, \end{cases} \tag{A.14}$$

which shows that a rise in any threshold θ_1^S and θ_2^S increases the number of multi-activity users conditional on a non-empty set. In combination with equations (A.11) and (A.12), it becomes clear that $\frac{\partial n_i}{\partial \theta_j^S} = \frac{\partial n_i^S}{\partial \theta_j^S} + \frac{\partial n_M}{\partial \theta_j^S} < 0$, which means that the total participation for activity i decreases when the network raises θ_j^S. Finally, it is straightforward to see that $\left|\frac{\partial n_i}{\partial \theta_i^S}\right| > \left|\frac{\partial n_i}{\partial \theta_j^S}\right|$.

A.3 Appendix of Proofs

A.3.1 Proof of Proposition 1

From the analysis above, it is obvious that a more lenient participation threshold for activity i leads to an unambiguously rise in total participation in activity i, that is, $\frac{\partial n_i}{\partial \theta_i^S} = \frac{\partial n_i^S}{\partial \theta_i^S} + \frac{\partial n_M}{\partial \theta_i^S} > 0$. Using this result, we are now able to maximize the network's profit by deriving Equation (10) with respect to θ_i^S. Using $p_i = \alpha_i n_j - \theta_i^S$, this yields

$$\frac{\partial \Pi}{\partial \theta_i^S} = (p_i - c_i + \alpha_j n_j)\frac{\partial n_i}{\partial \theta_i^S} + (p_j - c_j + \alpha_i n_i)\frac{\partial n_j}{\partial \theta_i^S} - n_i = 0. \tag{A.15}$$

This implies that, from $\frac{\partial \Pi}{\partial \theta_j^S}$, we have that

$$(p_j - c_j + \alpha_i n_i) = -\left[(p_i - c_i + \alpha_j n_j)\frac{\partial n_i}{\partial \theta_j^S} - n_j\right]\left(\frac{\partial n_j}{\partial \theta_j^S}\right)^{-1}.$$

Substituting for $(p_j - c_j + \alpha_i n_i)$ in Equation (11) implies that

$$(p_i - c_i + \alpha_j n_j)\cdot\left[\frac{\partial n_i}{\partial \theta_i^S}\cdot\frac{\partial n_j}{\partial \theta_j^S} - \frac{\partial n_i}{\partial \theta_j^S}\cdot\frac{\partial n_j}{\partial \theta_i^S}\right] + n_j\cdot\frac{\partial n_j}{\partial \theta_i^S} - n_i\cdot\frac{\partial n_j}{\partial \theta_j^S} = 0.$$

Solving for p_i implies that

$$p_i = c_i - \alpha_j n_j + \Delta,$$

where $\Delta := \frac{n_i \frac{\partial n_j}{\partial \theta_j^S} - n_j \frac{\partial n_j}{\partial \theta_i^S}}{\frac{\partial n_i}{\partial \theta_i^S}\frac{\partial n_j}{\partial \theta_j^S} - \frac{\partial n_i}{\partial \theta_j^S}\frac{\partial n_j}{\partial \theta_i^S}} > 0$, as $\left|\frac{\partial n_i}{\partial \theta_i^S}\right| \geq \left|\frac{\partial n_i}{\partial \theta_j^S}\right|, \forall i \neq j$. □

A.3.2 Proof of Proposition 2

Optimal pricing in Proposition 1 matches traditional two-sided pricing whenever $\Delta_{ij} := \frac{n_i \frac{\partial n_j}{\partial \theta_j^S} - n_j \frac{\partial n_j}{\partial \theta_i^S}}{\frac{\partial n_i}{\partial \theta_i^S}\frac{\partial n_j}{\partial \theta_j^S} - \frac{\partial n_i}{\partial \theta_j^S}\frac{\partial n_j}{\partial \theta_i^S}}$ equals $n_i / \frac{\partial n_i}{\partial \theta_i^S}$ to mirror the term $\phi_i(u_i)/\phi\prime_i(u_i)$ in Armstrong (2006). For statement (i), this occurs whenever $\frac{\partial n_j}{\partial \theta_i^S} = 0$. First consider the case where $\gamma > 0$. In this case, Equation (A.4) implies that $\frac{\partial n_i^S}{\partial \theta_j^S}$ equal zero only when $g(\theta_i, \theta_j^S + \gamma) = 0$ for all $\theta_i \in [\underline{\theta}_i, \theta_i^S]$.

Similarly, equations (A.5) and (A.6) imply that $\frac{\partial n_M}{\partial \theta_1^S}$ and $\frac{\partial n_M}{\partial \theta_2^S}$ both equal zero only when $g(\theta_i^S + \gamma, \theta_j) = 0$ for $\theta_j \in [\underline{\theta}_j, \theta_j^S]$. Combined, these conditions all require (θ_1, θ_2) to be bimodal with a sufficiently high degree of negative correlation, and together they imply that $\frac{\partial n_j}{\partial \theta_i^S} = 0$ so that traditional pricing occurs. For the case where $\gamma \leq 0$, similar arguments follow using equations (A.11), (A.12), and (A.14).

Statement (*ii*) arises because for $\gamma = 0$, we see from equations (A.11), (A.12), and (A.14) that $\frac{\partial n_M}{\partial \theta_i^S} = -\frac{\partial n_j^S}{\partial \theta_i^S}$ so that $\frac{\partial n_j}{\partial \theta_i^S}$ equals zero. Statement (*ii*) then directly follows from the first-order condition (11). □

A.3.3 Proof of Proposition 3

If $\gamma \neq 0$, then equations (A.3), (A.4), (A.5), and (A.6), and equations (A.11), (A.12), and (A.14) imply that $\frac{\partial n_M}{\partial \theta_i^S} \neq -\frac{\partial n_j^S}{\partial \theta_i^S}$ across all (θ_1, θ_2). Because it is also true that $\frac{\partial n_j}{\partial \theta_i^S} \neq 0$ across all (θ_1, θ_2), the presence of multi-activity users creates direct network effects if $\gamma \neq 0$.

In addition, if $(\bar{\gamma}_M > \gamma \geq \bar{\gamma}_i, \forall i)$, then equations (A.3), (A.4), (A.5), and (A.6), imply that $\frac{\partial n_M}{\partial \theta_i^S} \neq 0$ and $\frac{\partial n_j^S}{\partial \theta_i^S}, \frac{\partial n_i^S}{\partial \theta_i^S} = 0$ so that all active users perform both activities. In this case, Equation (13) implies that the combination of prices must satisfy the following:

$$p_1 + p_2 = c_1 + c_2 - n_M(\alpha_1 + \alpha_2) + {}^{n_M}\!/_{\frac{\partial n_M}{\partial \theta_1^S}}.$$ [18]

Thus, a continuum of activity specific prices must satisfy the unique one-sided single price that is given by $p_1 + p_2$. □

A.3.4 Proof of Proposition 4

It is important to note that $\gamma = 0$ implies that $n_i = \int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_j}^{\overline{\theta}_j} g(\theta_i', \theta_j') d\theta_j' d\theta_i'$ which is simply the marginal distribution $G_i(\theta_i)$ by combining the first line of Equation (A.9) with the first line of Equation (A.7), respectively Equation (A.8). Then, Equation (10) and $p_i = \alpha_i n_j - \theta_i^S$

[18]Note that Equations (A.5) and (A.6) with $\bar{\gamma}_M > \gamma \geq \bar{\gamma}_i, \forall i$ imply that $\frac{\partial n_M}{\partial \theta_1^S} = \frac{\partial n_M}{\partial \theta_2^S}$.

imply that

$$\Pi = (\alpha_i G_j(\theta_j^S) - \theta_i^S - c_i)G_i(\theta_i^S) + (\alpha_j G_i(\theta_i^S) - \theta_j^S - c_j)G_j(\theta_j^S).$$

This generates first-order conditions given by

$$0 = \frac{d\Pi}{d\theta_i^S} = (\alpha_i G_j(\theta_j^S) - \theta_i^S - c_i)\frac{dG_i(\theta_i^S)}{d\theta_i^S} - G_i(\theta_i^S) + \alpha_j \frac{dG_i(\theta_i^S)}{d\theta_i^S} G_j(\theta_j^S). \tag{A.16}$$

Thus if $\hat{g}, \tilde{g} \in \mathcal{G}$ we have that $\hat{G}_i(\theta_i^S) = \tilde{G}_i(\theta_i^S)$ and $\frac{d\hat{G}_i(\theta_i^S)}{d\theta_i^S} = \frac{d\tilde{G}_i(\theta_i^S)}{d\theta_i^S}$ for $i = 1, 2$ so that the first-order conditions (Equation (A.16)) under $\hat{g}$ are identical to those under $\tilde{g}$. This implies that $\hat{\theta}_i^S = \tilde{\theta}_i^S$ for $i = 1, 2$ so that $n_i^*(\hat{g}) = \hat{G}_i(\theta_i^S) = \tilde{G}_i(\theta_i^S) = n_i^*(\tilde{g})$ and $p_i^*(\hat{g}) = \alpha_i \hat{G}_j(\theta_j^S) - \theta_i^S = \alpha_i \tilde{G}_j(\theta_j^S) - \theta_i^S = p_i^*(\tilde{g})$ for $i = 1, 2$ for all $\hat{g}, \tilde{g} \in \mathcal{G}$. □

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