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Optimal Climate Policy as If the Transition Matters

Abstract

The optimal transition to a low-carbon economy must account for adjustment costs in switching from dirty to clean capital, technological progress, and economic and climatic shocks. We study the low-carbon transition using a dynamic stochastic general equilibrium model with emissions abatement costs calibrated on a large energy modelling database, solved with recursive methods. We show how capital inertia puts upward pressure on emissions and temperatures in the short run, but that nonetheless it is optimal to actively disinvest from – to 'strand' – a significant share of the dirty capital stock. Conversely, clean technological progress, as well as uncertainty about climatic and economic factors, lead to lower emissions and temperatures in the long run. Putting these factors together, we estimate a net premium of 33% on the optimal carbon price today relative to a 'straw man' model with perfect capital mobility, fixed abatement costs and no uncertainty.

JEL-Codes: C610, E220, H230, O440, Q540, Q550.

Keywords: adjustment costs, carbon price, climate change, low-carbon transition, stranded assets, technological progress, uncertainty.

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1 Introduction

In the long run, greenhouse gas emissions will have to be virtually eliminated for global temperatures to stop rising (IPCC, 2021). Almost all countries have signed up to the long-run temperature goals of the 2015 UN Paris Agreement on climate change, namely "Holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C" (UNFCCC, 2016).[1] Although these are fundamentally political goals, there is also broad agreement in economics research with the basic premise that emissions should fall from their current record levels to very low levels eventually (Howard and Sylvan, 2021).

However, how best to make the *transition* from a high- to low-carbon economy remains open for debate. How high should initial carbon prices/taxes be, and how should they change over time (Golosov et al., 2014; Lemoine and Rudik, 2017b; Gerlagh and Liski, 2018; Dietz and Venmans, 2019; Daniel et al., 2019; Mattauch et al., 2020)? Should we make immediate, deep emissions cuts, or should we postpone our efforts until later, in effect using up the remaining carbon budget more quickly (van der Ploeg, 2018; Gollier, 2021)? Can fossil-fuel-burning assets be used until the end of their economic lifetimes, or do we need to leave them 'stranded'[2] (McGlade and Ekins, 2015; Barnett, 2019; van der Ploeg and Rezai, 2020b; Cahen-Fourot et al., 2021)?

One reason why the debate remains open is that views differ about the optimal long-run temperature that would maximise social welfare (Hänsel et al., 2020). But another reason why these questions endure is that making the transition to a low-carbon economy involves complex dynamics that go beyond basic models of emissions abatement. In this paper, we develop a dynamic stochastic general equilibrium model with the aim of analysing these complex transition dynamics. The model exhibits three key features that, we contend, have not been studied in a single model before.

First, we capture limits to how fast emissions can be reduced through the introduction of two kinds of adjustment cost (Gould, 1968; Lucas Jr, 1967). The first kind are convex

[1]The 2021 Glasgow Climate Pact increased the emphasis on the more ambitious 1.5°C goal (UNFCCC, 2021).

[2]A rapid transition could lead carbon-intensive capital to be decommissioned prematurely, used at a lower capacity rate, or retrofitted or repurposed (e.g., via carbon capture and storage technologies) at a cost, with a monetary loss for capital owners.

investment costs to capital accumulation (clean or dirty). These capture the additional costs faced when rapidly accumulating new capital and lead to smoother investment paths, as large capital investments concentrated in a short period are more expensive than the same capital expansion spread over a longer period. The second kind are convex costs of dirty capital disinvestment. To rapidly reduce emissions, it may be optimal to dispose of dirty capital faster than its natural depreciation rate. It is often assumed that capital is perfectly mobile (as are other production factors), so it can be costlessly shifted from dirty to clean sectors. But some capital has been designed to work in tandem with fossil fuels (e.g., coal and gas power plants, steel mills, ships, etc.) and would be costly to convert to low-carbon uses, perhaps prohibitively so. The costs of a rapid transition would be transmitted by production and financial networks, and it has even been suggested that a rapid transition could trigger economic and financial instability (Semieniuk et al., 2021). We assume *physical* asset stranding (i.e., disinvestment from dirty capital) generates convex *monetary* stranding costs: the more rapid is the disinvestment of dirty physical capital, the larger is the proportion of the monetary value of dirty capital that gets lost.

Second, we incorporate the widely observed phenomenon of falling clean technology costs. Like adjustment costs, these can affect the timing of investment in clean capital. Clean technological progress has diverse sources, including knowledge spillovers from other sectors such as artificial intelligence and nanotechnology, economies of scale and learning-by-doing in the clean technology sector, induced R&D into clean technology, and network effects (e.g., relating to charging electric vehicles). In our model, falling clean technology costs are partly autonomous/exogenous and partly induced, i.e., an endogenous function of cumulative emissions abatement, as implied by evidence on experience/learning curves (Boston Consulting Group, 1970; Weiss et al., 2010). Endogenous clean technological progress creates an incentive to abate earlier, unlike exogenous technological progress. In many basic models of emissions abatement, clean technology costs are either fixed or if they fall then the process is fully exogenous (Gillingham et al., 2008).

Third, charting an optimal transition path is fraught with uncertainties, including about economic growth prospects, technological progress, the physical climate system, and the impacts of climate change (see Heal and Millner, 2014; Cai, 2021, for reviews). The optimal transition should account for current uncertainties but also how they change over time. Policy-makers will figure things out along the way. We incorporate multiple sources of uncertainty, including the business cycle, climate variability, and rare macroeconomic shocks (Barro, 2009) made more likely by climate change. These uncertainties affect the optimal transition, which is formulated as a stochastic welfare maximisation problem and solved using recursive dynamic programming methods so that the planner anticipates and reacts to shocks.[3] In doing so, we employ Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which disentangle risk aversion from intertemporal consumption substitution, consistent with empirical data on asset returns. By contrast, most basic models compute deterministic optimal paths, or at most they are subjected to sensitivity analysis, computing multiple deterministic optimal paths, while varying the values of key exogenous parameters (Lemoine and Rudik, 2017a).

We obtain analytical solutions for optimal clean and dirty capital accumulation, as well as for the optimal carbon price, but our main findings are obtained by solving the model numerically. We calibrate emissions abatement costs by fitting the model on a large database of results from detailed energy models using the generalised method of moments. We find the optimal transition to a low-carbon economy is fast. From 2020 to 2030, emissions are roughly halved, whether we consider a pure economic planning problem of maximising the discounted net benefits of emissions reductions, or adopt the international climate policy approach of meeting an exogenous constraint on warming of 1.5°C at the lowest discounted abatement cost. Either way, a high carbon price is required, as is active 'stranding' of dirty assets, rather than working with the grain of capital depreciation. If the imposition of a carbon price is delayed and its eventual introduction is not anticipated, even more dirty assets are stranded.

Disentangling the effects of capital inertia, clean technological progress, and uncertainty, we show how adjustment costs prevent an implausibly large optimal shift from dirty to clean capital initially, and how capital inertia therefore results in temperature inertia and higher carbon prices. Falling clean technology costs lead to lower optimal emissions and temperatures in the long run, while also allowing for modestly lower optimal carbon prices all along the path. The price effect is the net effect of exogenous technological progress, which allows lower prices, and endogenous technological progress, which requires a carbon price premium to the extent that innovation externalities are not internalised using other policy instruments. Uncertainty leads to lower optimal emissions and temperatures, and higher optimal carbon prices and dirty capital disinvestment, driven by temperature-related macroeconomic disaster risk. Putting these three effects together, we estimate a net premium of 33% on the optimal carbon price today relative to a 'straw man' model with perfect capital mobility, fixed abatement costs and no uncertainty. The premium falls to about 20% by mid-century. Higher prices pay off in terms of lower temperatures, but only in the long run, specifically the second half of this century, because of capital inertia.

Lastly, rather than comparing an unconstrained cost-benefit problem with a temperature-

[3] In other words, optimal emissions are set by a closed-loop or feedback policy, where emissions are a function of future states of the coupled climate-economy system (Lemoine and Rudik, 2017a).

constrained cost-effectiveness problem, as above, we compare two different ways to stay below the exogenous temperature constraint: (i) maximising discounted net benefits of emissions reductions, and (ii) minimising discounted abatement costs. The latter stays closer to how the problem is defined in international climate policy. We make the point that ignoring climate damages leads the planner to put off – to back-load – emissions reductions. We estimate the optimal carbon price is 13% lower initially. Since climate damages are not zero, we conclude the latter approach underestimates optimal carbon prices.

Our paper contributes to the literature studying optimal global climate policies using Integrated Assessment Models or IAMs, which was pioneered by William Nordhaus (Nordhaus, 1991; Nordhaus and Boyer, 2000; Nordhaus, 2008, 2013). The focus on *optimal* policies contrasts with the large applied literatures on the 'social cost of carbon' (i.e., marginal damage cost of CO_2) along arbitrary emissions paths (Tol, 2011), and on the marginal abatement cost of meeting pre-determined climate goals (Clarke et al., 2014). While most of the IAM literature has computed deterministic optimal paths, the relevance of uncertainty has led to the recent development of a class of 'recursive IAMs', which compute an adaptive optimal path in the face of unfolding uncertainties using recursive methods (e.g. Lemoine and Traeger, 2014; Cai and Lontzek, 2019; van den Bremer and van der Ploeg, 2021; Karydas and Xepapadeas, 2022). Our model belongs to this class, which has mostly focused on quantifying the effects of sources of uncertainty (Lemoine and Rudik, 2017a, review the literature). We connect this work with a smaller and more recent literature looking at adjustment costs along the low-carbon transition (Coulomb et al., 2019; Hambel et al., 2020; van der Ploeg and Rezai, 2020a; Vogt-Schilb et al., 2018). This line of work is motivated in part by an interest in whether pricing carbon will leave dirty capital assets stranded (also see Baldwin et al., 2020; Rozenberg et al., 2020), but is mostly deterministic.

The remainder of the paper is structured as follows. Section 2 presents the model and section 3 briefly presents some of the key mechanics of the optimal transition. Section 4 explains our calibration strategy. Section 5 presents and discusses our numerical results. Section 6 offers a discussion of several key issues, including further comparisons with the literature, and Section 7 concludes.

2 The model

For expositional reasons, consider an economy in continuous time with an infinite horizon. We suppress time subscripts unless time-dependence is not obvious. A single type of good

Y exists, used for both consumption and investment. The production function is

$$Y = AL^{1-\alpha}K^{\alpha}\Lambda\Omega, \tag{1}$$

where A is total factor productivity, L is labour, K is capital, and α is the capital share. Labour increases exogenously at constant rate g_L. The multiplier Λ represents the costs of emissions abatement. Similarly, Ω is the climate damage multiplier.

Abatement costs

The abatement cost multiplier is

$$\Lambda = \exp\left(-\frac{\varphi_t}{2}\mu^2\right), \tag{2}$$

where μ stands for abatement, which implies a marginal abatement cost (MAC) function

$$-Y_\mu/Y = \varphi_t\mu, \tag{3}$$

with slope φ_t. Note the slope parameter is time-dependent. Abatement is the difference between business-as-usual (BAU) emissions, which specifically means the level of emissions at a carbon price of zero, and actual emissions:

$$\mu = \phi_t/\varphi_t - E. \tag{4}$$

The parameter ϕ_t represents the MAC at zero emissions and thereby contributes to determining the level of BAU emissions ϕ_t/φ_t. Bounding the zero-emissions MAC at ϕ_t is consistent with evidence on and assumptions about zero-carbon technology costs, e.g., direct air capture of CO_2 (Heutel et al., 2016). By contrast, some model structures imply an infinite MAC at zero emissions, e.g., the commonly used model that differentiates between clean and dirty capital/energy inputs and assumes a constant elasticity of substitution between them.

Clean technological progress

In the model, clean technology costs fall over time. Part of this process is exogenous and part of it depends endogenously on emissions abatement. Exogenous cost decreases could result from innovation spillovers elsewhere in the economy (Popp, 2019; Taalbi, 2020). For example, improvements in lithium-ion batteries in the smartphone industry are now benefiting electric mobility, thin-layer nanotechnology might be useful for photo-voltaic energy on plastics, and chemical membrane technology could be employed to produce “green hydrogen” from water.

They could also result from basic R&D directly into clean technology that is independent of levels of clean technology use. Endogenous cost decreases could come from learning-by-doing, network effects, or other sources of increasing returns, as well as induced R&D that is proportional to the size of the clean technology market (Acemoglu et al., 2012).

The MAC at zero emissions evolves according to

$$\phi_t = \phi_0 \left(\omega \frac{1}{M^\epsilon} + (1-\omega) \frac{1}{1+g_\phi \tau} \right), \tag{5}$$

and similarly the MAC function slope parameter evolves according to

$$\varphi_t = \varphi_0 \left(\omega \frac{1}{M^\epsilon} + (1-\omega) \frac{1}{1+g_\varphi \tau} \right). \tag{6}$$

Thus, the overall decrease in clean technology costs is a weighted average of the exogenous and endogenous processes, where ω is the relative weight on endogenous cost reductions.

To represent exogenous cost decreases, the parameters of the MAC function are a decreasing function of time. The exogenous evolution of the intercept term ϕ_t is given by $\phi_0/(1+g_\phi \tau)$ and the slope term by $\varphi_0/(1+g_\varphi \tau)$. τ is just a monotonic transformation of time, $\tau = 1 - \exp(-g_\tau t)$, $g_\tau > 0$, which is a technical detail necessary to solve the model numerically (Traeger, 2014). One can think of it as artificial time, starting at zero and gradually approaching one in the long term. The important point is that ϕ_t starts at ϕ_0 and decreases towards a long run asymptote $\phi_0/(1+g_\phi)$. The same goes for φ_t. We assume $g_\phi = g_\varphi$, which implies constant BAU emissions given (4), consistent with the latest evidence on future emissions conditional on current policies (IPCC, 2022), where current policies still involve trivially low pricing of global aggregate emissions (World Bank, 2021).

For endogenous technological progress, M represents cumulative emissions abatement relative to time zero[4] and ϵ is the constant elasticity of the MAC function parameters with respect to cumulative emissions. This means that for every doubling of cumulative emissions, MACs are reduced by the same factor $1-2^{-\epsilon}$, i.e., we assume a constant rate of cost reduction.

Damages and warming

The damage multiplier

$$\Omega = \exp\left(-\frac{\gamma}{2} T^2 \right), \tag{7}$$

[4]Cumulative abatement in the model is $M_0 + \int_0^t \mu_s ds$, where M_0 is cumulative abatement at time zero. Hence, we can define cumulative abatement relative to time zero as $M(S,t) = [M_0 + E_{BAU} t - (S_t - S_0)]/M_0$, where S stands for cumulative emissions.

where γ is the damage function coefficient and T is global mean temperature relative to its pre-industrial level.

Climate science has shown that T is approximately linearly proportional to cumulative carbon dioxide emissions (IPCC, 2021), so we write

$$T = \zeta S, \tag{8}$$

where S stands for cumulative greenhouse gas emissions and ζ is the slope coefficient, known as the transient climate response to cumulative carbon emissions or TCRE. Although temperature responds to a CO_2 emission impulse with a delay of a few years, the relationship between temperature and cumulative emissions is still well approximated by Eq. (8) (Dietz and Venmans, 2019). Thus, as Dietz and Venmans (2019) and Dietz et al. (2021a) showed, not only is (8) simple, it mimics the behaviour of complex climate science models much better than most climate modules in IAMs that explicitly model the atmospheric stock of carbon and thereby require more state variables.

Temperature is subject to a geometric Brownian motion capturing uncertainty in the temperature impulse response to emissions,

$$\mathrm{d}T = \zeta E \mathrm{d}t + \sigma_T T \mathrm{d}W_T, \tag{9}$$

where emissions times the TCRE parameter ζ is by definition the drift and the temperature volatility is σ_T. The Brownian motion represents both epistemic uncertainty, i.e., the fact that our climate model is imperfect, and aleatoric uncertainty, i.e., genuinely random volatility of the climate system.[5].

Clean and dirty capital

The capital stock is composed of dirty and clean technologies. Dirty capital K_d represents all forms of capital requiring fossil-fuel inputs (e.g., coal- and gas-fired power plants, integrated steel mills, and cement plants).[6] Clean capital K_c represents capital that does not directly require fossil fuels for production. Upstream sectors where fossil fuels are a direct input

[5]This could be explicitly modelled as follows: assume epistemic uncertainty (Bayesian updating) of the TCRE follows the process $d\zeta = \sigma_1 dW_1$ and aleatoric (weather) shocks are proportional to warming, adding an uncorrelated geometric Brownian motion with volatility σ_2. This results in

$$dT = \zeta E \mathrm{d}t + \frac{\sigma_1}{\zeta} T \mathrm{d}W_1 + \sigma_2 T \mathrm{d}W_2, \tag{10}$$

which is identical to our equation with $\sigma_T = \frac{\sigma_1}{\zeta} + \sigma_2$.

[6]Or which otherwise emit greenhouse gases as a by-product, as cement and some chemical plants do, for example.

need to shift towards low-carbon production technologies. Downstream sectors where fossil fuels are not directly used can already be considered clean. They are dirty only insofar as their intermediate inputs come from fossil-fuel-burning upstream sectors, e.g., electricity (Kemp-Benedict, 2018). Therefore, rather than being a small expanding niche in a system dominated by dirty capital, our definition implies most of the capital stock is clean (or at least not dirty *per se*), and we treat as dirty only the capital that directly uses fossil fuels or otherwise directly emits greenhouse gases.

Use of K_d produces emissions E at intensity ψ_t,

$$E = \psi_t K_d. \tag{11}$$

The emissions intensity of dirty capital ψ_t decreases at rate g_ψ, which represents autonomous improvements in energy efficiency that occur without carbon pricing due to incentives to save on energy costs. This is distinct from falling clean technology costs.[7]

Use of K_c does not produce any emissions. Substituting dirty capital with clean capital reduces emissions at a cost to output (and reduces climate damages). The technical rate of substitution is

$$\frac{Y_{K_d}}{Y_{K_c}} = 1 + \frac{(\phi_t\psi_t - \varphi_t\psi_t^2 K_d)(K_d + K_c)}{\alpha}, \tag{12}$$

which is decreasing in the stock of dirty capital assuming $K_d < K_c$, meaning that the less dirty capital there is, the more clean capital is required at the margin to keep output constant. At BAU emissions, $K_d = \phi_t/(\varphi_t\psi_t)$ and both forms of capital are equally productive,[8] whereas at zero emissions dirty capital is several times more productive than clean capital (determined by ϕ_t). Substitutability is further impaired by adjustment costs, to which we now turn.

Capital accumulation, adjustment and disinvestment costs

Clean and dirty capital are expanded by investment, but this is subject to convex increasing adjustment costs:

$$\iota_j = \frac{\chi_j i_j^2}{k},\ j \in \{c, d\}, \tag{13}$$

where we use lower case to denote units of effective labour (see Appendix A), thus i is gross investment per unit of effective labour in clean and dirty capital respectively, $k = k_c + k_d$

[7]More specifically, we define $\psi_t = \psi_0 \frac{1+\tau}{e^{(g+g_L)t}}$.

[8]Although our model has constant BAU emissions, BAU dirty capital is initially *in*creasing at rate g_ψ due to the effect of decreasing emissions intensity. By contrast, BAU dirty capital as a proportion of GDP equals $\frac{K_{d,BAU}}{Y} = \frac{\phi_0}{Y_0\varphi_0\psi_0(1+\tau)}$ and is *de*creasing to half its initial value in the long run. With constant emissions intensity, this proportion of dirty capital would decrease at rate $g + g_L$ and become zero in the steady state.

is total capital per unit of effective labour, and χ_j is the adjustment cost parameter. For any amount i_j of disbursed investment expenditure, the value of newly installed capital will only be a proportion of investment expenditure equal to $i_j - (\chi_j i_j^2)/k$. Adjustment costs are proportional to total capital such that the dynamics of total capital are homogeneous of degree one.[9]

In addition to adjustment costs of positive investment in clean/dirty capital, we also model the costs of negative investment or disinvestment of dirty capital, which may be optimal in order to make a rapid transition, rather than waiting for dirty capital to depreciate.[10] Denoting negative dirty investment $r \equiv i_d$ iff $i_d < 0$,

$$\kappa(r) = \frac{\theta_1 r^{\theta_2}}{k}, \tag{14}$$

where the cost function has coefficient $\theta_1 > 0$ and exponent $\theta_2 > 1$. When dirty assets are disinvested and retrofitted/repurposed, strictly less than 100% of the value of those assets can be recovered. Thus, those dirty assets are devalued and become partly 'stranded'. For low r, a large share of the value of the original dirty capital can be recovered. But as r increases, the share of the dirty capital assets' value that is lost increases more than proportionally.

Both capital stocks are subject to a geometric Brownian motion with correlation coefficient ρ_k. This stochastic process represents random changes to the value of capital and its productivity, similar to changes in the valuation of firms on the stock market, and allows us to capture part of the uncertainty about future growth prospects.

Putting these elements together, the equations of motion of dirty and clean capital are

$$\mathrm{d}k_j = \left(i_j - \frac{\chi_j i_j^2}{k} - (\delta_j + g_L + g)k_j \right) \mathrm{d}t + \sigma_j k_j \mathrm{d}W_j, \tag{15}$$

where δ_j is the depreciation rate and $\mathrm{d}W_c = \rho_k \mathrm{d}W_d + (1 - \rho_k^2)^{1/2} \,\mathrm{d}W_2$ with $\mathrm{d}W_2 \perp \mathrm{d}W_d$.

At zero dirty investment, emissions naturally decrease at rate $\delta_d + g_\psi$. This can be interpreted as the 'speed limit' to decarbonisation, beyond which dirty capital needs to be repurposed at a cost.[11] This penalty on rapid abatement is important in determining the cost of an abrupt transition to a low-carbon economy.

[9]Note that one cannot divide by k_d because the ratio would converge to zero.

[10]Given we start in a high-carbon economy, the opposite case $i_c < 0$ is of limited relevance.

[11]Analytically, we can see this by expressing the (initial) speed of abatement as $\frac{\dot{E}}{E} = g + g_L - g_\psi + \frac{\dot{k}_d}{k_d} = -\delta_d - g_\psi + \frac{i_d - \chi_d i_d^2/k}{k_d}$. The exact formula for all future periods is more convoluted but with the same interpretation.

Disasters and tipping points

TFP follows a geometric Brownian motion capturing normal economic fluctuations. In addition, TFP is subject to rare macroeconomic disasters, such as wars, financial crises and pandemics (Barro, 2009). But, in a twist on the original formulation of these shocks, we specify them as an increasing function of temperature to account for the risk of climate-related shocks, e.g., due to crossing tipping points in the physical climate system (Cai and Lontzek, 2019; Dietz et al., 2021b). TFP therefore evolves according to

$$\mathrm{d}\tilde{A} = \Big(g_{A1}\tilde{A} + g_{A2}(\tilde{A}_0 - \tilde{A})\Big)\,\mathrm{d}t + \sigma_A\tilde{A}\mathrm{d}W_A - \xi_1(1+\xi_2 T)\tilde{A}\mathrm{d}P, \tag{16}$$

where $\tilde{A}$ is TFP adjusted for production per unit of effective labour,[12] The trend growth rate g_{A1} is set so that $g_{A1}\tilde{A}$ corresponds to the expected effect of the shocks (as in Barro, 2009), while g_{A2} governs a very slow return to a balanced growth path. σ_A is 'normal' economic volatility, ξ_1 is the size of the macro-economic shock in the absence of climate change (from Barro, 2009) and ξ_2 parameterises how this increases as a function of temperature. dP is a Poisson process and equals one with probability λ_A.

Consumption and utility

Agents have rational expectations. Consumption per unit of effective labour is

$$c = y - i_d - i_c - \kappa(r). \tag{17}$$

Disinvestment costs reduce the income available from liquidating dirty capital (or, equivalently, retrofitting/repurposing costs must be paid out of income). In order to disentangle risk aversion from intertemporal consumption smoothing, we use the recursive preference formulation of Duffie and Epstein (1992), which is equivalent to the discrete-time recursive preferences of Epstein and Zin (1989), Kreps and Porteus (1978), and Weil (1990). The representative household maximises utility from per-capita consumption, discounted at the pure time preference rate ρ, with standardised aggregator function f and value function V:

$$V(k_d, k_c, S, \tilde{A}, t) = \max_{i_c, i_d} \mathbb{E}\int_t^\infty -f(k_d, k_c, S, \tilde{A}, t, V, i_d, i_c)ds, \tag{18}$$

with

$$f(k_d, k_c, S, \tilde{A}, t, V, i_d, i_c) = \frac{c^{1-\eta}}{1-\eta}\Upsilon - \hat{\rho}V\frac{1-RRA}{1-\eta}, \tag{19}$$

[12] $\tilde{A} = Ae^{(\alpha-1)gt}$ and production per unit of effective labour $y = \tilde{A}k^{\alpha}\Lambda\Omega$.

$$\Upsilon = \hat{\rho}\left((1 - RRA)\, V\right)^{\frac{\eta - RRA}{1-RRA}}, \tag{20}$$

and

$$\hat{\rho} = \rho - g_L + (\eta - 1)g. \tag{21}$$

η is the elasticity of marginal utility of consumption (equal to the inverse of the elasticity of intertemporal substitution) and RRA is the coefficient of relative risk aversion. Setting $\eta = RRA$ would simplify the expression leading back to the standard expected utility case. However, this would be difficult to reconcile with empirical data on risky and risk-free asset returns, and in the context of climate policy it has the property that high levels of risk aversion also increase the discount rate and lead to lower climate ambition, all else being equal.

3 Optimal capital accumulation and carbon pricing

In this short section, we build intuition for our quantitative results below by looking at how adjustment costs affect the optimal accumulation of clean and dirty capital, and how adjustment costs, clean technological progress and uncertainty affect the optimal pricing/taxation of greenhouse gas emissions.

The Hamilton-Jacobi-Bellman (HJB) equation is

$$\max_{i_c, i_d} \left\{ f + \frac{1}{\mathrm{d}t}\mathbb{E}[dV(k_d, k_c, S, \tilde{A}, t)] \right\} = 0. \tag{22}$$

Applying Itô's Lemma (see Appendix A) and maximising the HJB equation with respect to the capital stocks yields expressions for the shadow prices of clean and dirty capital:

$$V_{k_c} = c^{-\eta}\Theta \frac{k}{k - 2\chi_c i_c} \tag{23}$$

and

$$V_{k_d} = c^{-\eta}\Theta \left(\frac{k}{k - 2\chi_d i_d}\right)\left(1 - \frac{\theta_1\theta_2 r^{(\theta_2 - 1)}}{k}\right), \tag{24}$$

where $c^{-\eta}$ is the marginal utility of consumption, adjusted for Duffie/Epstein preferences via

$$\Theta = \hat{\rho}\left((1 - RRA)\, V\right)^{1 - \frac{1-\eta}{1-RRA}}. \tag{25}$$

Combining (23) and (24) into a ratio of the shadow prices of clean and dirty capital helps make the role of adjustment costs clear:

$$\frac{V_{k_c}}{V_{k_d}} = \frac{k - 2\chi_d i_d}{k - 2\chi_c i_c} \left(\frac{1}{1 - \left(\theta_1 \theta_2 r^{(\theta_2 - 1)}\right) / k} \right) \tag{26}$$

In the absence of adjustment costs, the ratio of the shadow prices of clean and dirty capital would be one, as the opportunity cost of investing an extra dollar is the utility of consuming that dollar instead, regardless of where the investment goes. However, adjustment costs change this. The first element on the right-hand side is the ratio of adjustment costs of dirty and clean capital investment. The ratio is increasing in clean capital investment and decreasing in dirty capital investment. Moreover, if we assume $\chi_c = \chi_d$, as we do below, then the ratio just depends on the absolute amount of investment going into each form of capital. Thus, adjustment costs put a brake on how quickly capital accumulation can shift from dirty to clean. The second element of the ratio is the costs of disinvesting dirty capital. The shadow price of clean capital is relatively larger, the larger these dirty capital disinvestment costs are. For high levels of disinvestment, i.e., $r > \left(\frac{\theta_1 \theta_2}{k}\right)^{\frac{1}{1-\theta_2}}$, this factor becomes negative: dirty capital becomes a burden, because the costs of disinvestment exceed its initial value.

Applying the envelope theorem to the HJB equation, we can also obtain an analytic expression for the optimal marginal abatement cost. For simplicity, we call this the optimal 'carbon price', but note that if endogenous clean technological progress is affected by market imperfections other than climate damages themselves, the first best would be to internalise these using complementary policy instruments such as innovation subsidies. The optimal carbon price is:

$$\begin{aligned}
p = \quad & \Phi\{\Psi c^{-\eta} y \gamma S - V_{SS} E && \text{marginal damage cost} \\
& + \Psi c^{-\eta} y \left[\tfrac{\varphi}{2} \mu^2 (1-\omega) \tfrac{\epsilon}{M^{\epsilon+1}}\right] - V_{tS} && \text{technological change effect} \\
& - V_{Sk_d} \left[i_d - \chi_d i_d^2 - (\delta_d + g_L + g) k_d\right] && \text{dirty capital contraction effect} \\
& - V_{Sk_c} \left[i_c - \chi_c i_c^2 - (\delta_c + g_L + g) k_c\right] && \text{clean capital expansion effect} \\
& - V_{SS} S \sigma_S^2 - {}^1\!/_2 V_{SSS} S^2 \sigma_S^2 && \text{temperature risk premium} \\
& - {}^1\!/_2 V_{Sk_d k_d} k_d^2 \sigma_d^2 - {}^1\!/_2 V_{Sk_c k_c} k_c^2 \sigma_c^2 - V_{Sk_c k_d} k_d k_c \rho^k \sigma_c \sigma_d && \text{capital risk premiums} \\
& - {}^1\!/_2 V_{SAA} \tilde{A}^2 \sigma_A^2 && \text{TFP volatility risk premium} \\
& - \lambda_A \left[V_S(\tilde{A} - \Delta\tilde{A}) - V_S(\tilde{A})\right] && \text{disaster risk} \\
& + \lambda_A V_A (\tilde{A} - \Delta\tilde{A}) \xi_1 \xi_2 \zeta \tilde{A} - V_{SA} \left(\tilde{A} g_{A1} + \left(A_0 - \tilde{A}\right) g_{A2}\right)\}, && \text{premium}
\end{aligned} \tag{27}$$

where

$$\Phi = \frac{(1-\eta)\, L_0 \exp\left[(g+g_L)\, t\right]}{\hat{\rho}\left[1 - RRA + (RRA-\eta)\, c^{1-\eta}\left((1-RRA)\, V\right)^{-\frac{1-\eta}{1-RRA}}\right] V_{k_c}} \tag{28}$$

and

$$\Psi(V) = \frac{\hat{\rho}}{\left((1-RRA)\, V\right)^{\frac{1-\eta}{1-RRA}-1}}. \tag{29}$$

To interpret Eq. (27), bear in mind that $-V_S$ is the optimal carbon price in utils and is positive. This helps sign the numerous cross-derivatives.

The term on the first row of (27) is the marginal damage cost of an emission, comprising the current level of marginal damages $\Psi c^{-\eta} y \gamma S$ and the future change in marginal damages $-V_{SS}E = -V_{SS}\dot{T}/\zeta$. On a path of rising temperatures, a convex damage function ensures rising marginal damages, i.e., $-V_{SS} > 0$, which is true of our damage function (7) like most others.[13]

The term on the second row is the technical change effect. The first element is the marginal gain of endogenous technical change, which increases the optimal carbon price because the prospect of reducing the MAC creates an added incentive to abate emissions today. Building on our observation above, part of this incentive may be external to innovating firms. If another policy instrument exists to internalise this externality, then less than the full marginal gain of endogenous technical change should be factored into the *explicit* carbon price levied on the economy. However, there are many drivers of endogenous technology cost reductions (learning-by-doing, network effects, induced R&D, etc.), so in reality the distinction is likely to be less than clear-cut. The second element, V_{tS}, embodies the two exogenous, time-dependent processes that reduce the MAC in the model, i.e., falling clean technology costs and autonomous improvements in energy efficiency. These reduce the optimal carbon price by making emissions abatement exogenously cheaper. Note that V_{tS} is also affected by endogenous technological change, $\partial M/\partial t = E_{BAU} > 0$, i.e., observing the same level of cumulative emissions in a later period implies higher cumulative abatement. This will also reduce the optimal carbon price.

The terms on the third and fourth rows capture, respectively, the effect of shrinking dirty capital on the optimal carbon price, and the effect of expanding clean capital. In both cases, the effect of a marginal emission on the opportunity cost of capital (V_{Sk_d} and V_{Sk_c}) is positive, because the marginal emission increases temperatures along the path, higher temperatures reduce consumption, and lower consumption results in higher marginal utility,

[13] Technological change uncertainty will affect the impact of rising damages V_{SS}. Therefore, we will compare the total effect on the carbon price in the results section

thus a higher opportunity cost. Dirty capital is in decline along the optimal path, so the trend effect $[i_d - \chi_d i_d^2 - (\delta_d + g_L + g)k_d]$ is negative and overall the dirty capital contraction effect reduces the optimal carbon price. Decreasing dirty capital reduces future warming. Clean capital is expanding along the optimal path, with a positive trend effect and thus the optimal carbon price is reduced.

The remaining rows include the risk premiums corresponding to the four stochastic processes in the model: uncertainty about the temperature impulse response to emissions (the TCRE), uncertainty about capital valuations, uncertainty about TFP due to normal economic fluctuations/volatility, and the effect of uncertain disasters. These all increase the optimal carbon price.[14] The disaster risk premium has two components. The first is the difference a disaster makes to marginal damages. On the one hand, marginal damages are smaller in a smaller, post-disaster economy. On the other hand, marginal utility is higher in a smaller economy. The latter effect outweighs the former when $\eta > 1$. The second component of the disaster risk premium takes account of the fact that disasters are larger at higher temperatures.

In Section 5, we quantitatively decompose our optimal carbon price trajectory into these different components.

4 Calibration

We calibrate our model using one-year time steps, with g_τ set equal to 0.015[15]. The initial period of our numerical simulations is 2020. Population is measured in billion people; capital stock and output in trillion 2020US$; GHG emissions in gigatonnes of CO2 equivalent (GtCO2eq); and temperature in degrees Celsius (°C).

We adopt three main strategies to calibrate our parameters. A first subset of parameters is defined by adopting standard values in the literature or recent empirical estimates. A second subset is calibrated to replicate empirical evidence or desired features of starting values. Finally, a third subset of particularly important parameters is estimated by fitting our model to a large database of results from detailed energy systems models using maximum likelihood methods. Table A1 in Appendix B provides an overview of the numerical values used for the parameters and initial conditions.

[14]Notice that since temperature is linearly proportional to cumulative emissions, the Brownian motion on temperature can be rewritten as a Brownian motion on S instead.

[15]This value was chosen so that $g_\tau = g + g_L - g_\psi$, which facilitates the convergence of the model algorithm.

4.1 Preferences

We use the expert survey of Drupp et al. (2018) to set both the elasticity of marginal utility of consumption (i.e., the inverse of the elasticity of intertemporal substitution) $\eta = 1.35$ and the utility discount rate $\rho = 1.1\%$. The coefficient of relative risk aversion RRA is set equal to 4, following Barro (2009).

4.2 Production

We calibrate the starting value of TFP $A_0 = 3.44$ so that, given initial values for labour and the capital stock, the 2020 value of global GDP is US$84.5 trillion (World Bank, 2022). The TFP growth rate compensating disasters is set to $g_{A1} = 0.0037$, with fluctuations following a Brownian motion with standard deviation $\sigma_A = 0.013$ (van den Bremer and van der Ploeg, 2021), and the steady state stabilisation rate (which has a negligible effect) $g_{A2} = 0.01$. These values allow us to obtain a mean economic growth rate of 2% in the steady state. We follow UN (2019) in setting the initial value of population $L_0 = 7.795$ and population growth $g_L = 0.42\%$.[16] The share of capital in production is assumed to be 0.3, as is common in the literature. We set the initial value of the aggregate capital stock, $K_0 =$ US$348 trillion, close to its steady state value,[17] determined by the parameter values and equal to a fixed proportion of output.[18]

To identify the shares of clean and dirty capital, we use the World Input-Output Database (WIOD), containing sectoral capital stock data for 43 countries at a NACE level 2 disaggregation (Timmer et al., 2015). As discussed in Section 2, we interpret dirty capital as the upstream productive assets requiring fossil fuels to operate. We thus label certain WIOD sectors as dirty because of their fossil-dependent technological basis (e.g., mining, coke, chemicals, plastics, metals, electricity, transportation). Where possible, we employ data from the Exiobase multi-regional input-output database (Stadler et al., 2018) to further disaggregate stocks and isolate those directly related to fossil fuels (e.g., the capital stock used for the exploration and extraction of coal, gas and oil) from those with less of a connection to fossil fuels (e.g., mining of non-ferrous metals and other materials). To perform this disaggregation, we assume that the share of fossil-related capital stock of a specific sector is equal to the share of fossil-related output in total sectoral output. With this method, we estimate that approximately 8% of the capital stock is dirty. We thus set $K_{d,0} =$ US$28 trillion and

[16] This is equal to the average growth rate in the UN population medium variant scenario for the 2020-2100 period.

[17] The steady state level of total capital is given by $K^* = L\left(Ae^{-\frac{\gamma}{2}T^{*2}}\frac{\alpha}{\rho+\delta+\eta g}\right)^{\frac{1}{1-\alpha}}$.

[18] In other words, we assume that the steady state capital-output ratio, equal to 3.7 in our central parameter simulations, also applies to the neighbourhood of the steady state.

$K_{c,0} =$ US\$320 trillion.

The depreciation rate of dirty capital δ_d plays a particularly important role in our model as, together with the decline rate of emissions intensity ψ, it defines the maximum speed at which the system can abate emissions without having to disinvest from dirty capital. We assume δ_d is equal to the inverse of the average asset lifetime. Since K_d captures a wide variety of productive stocks with different asset lifetimes,[19] we choose a central value of 4%, corresponding to an average asset lifetime of 25 years. This is roughly in line with depreciation rates adopted by models including long-lived physical assets.[20] For simplicity and to facilitate the dynamics of the model, we also set $\delta_c = \delta_d$. Given our initial value of $K_d =$ US\$28 trillion, and 2020 GHG emissions of 56 GtCO2eq, we set the initial value of emissions intensity ψ_0 equal to 2 GtCO2eq/trillion US\$. We assume emissions intensity declines at a rate approximately equal to its historical decline rate $g_\psi = 0.01$ (World Bank, 2022).[21]

A wide range of estimates exist for the adjustment cost parameter (see for instance Cooper and Haltiwanger, 2006; David and Venkateswaran, 2019). We follow van der Ploeg and Rezai (2020a) in setting a mid-range value of $\chi_d = \chi_c = 0.1$.

Both capital stocks are subject to Brownian motions, with standard deviations $\sigma_d = \sigma_c = 0.013$. The combined volatility of TFP and capital mimics the GDP volatility in van den Bremer and van der Ploeg (2021). In addition to capital volatility, our model also includes the possibility of macroeconomic disasters, which we assume to lead to a loss of 20% of GDP with a 1.7% probability, thus setting $\xi_1 = 0.2$. This is taken from Barro (2009), with a downward adjustment to reflect the fact that Barro estimated the size of regional disasters whereas our model is globally aggregated.

4.3 Climate and emissions

IPCC (2021) estimates the temperature increase in 2020 relative to pre-industrial $T_0 = 1.2°\text{C}$,

[19] Coal plants, for instance, have an average lifetime of 46 years (Cui et al., 2019), with many of them lasting up to 60 or 70 years. Similarly, IEA (2019) uses a reference lifetime of coal plants of 50 years. Other carbon-related assets like transport infrastructure and buildings have even longer life-spans, reaching 100 or 200 years, while manufacturing equipment is estimated to last between 10 and 40 years (Hallegatte, 2009; IEA, 2002).

[20] For instance, van der Ploeg and Rezai (2020a) use a depreciation rate of 5% for fossil-fuel exploration capital; Coulomb et al. (2019) use 3.33% for electricity generation plants; Baldwin et al. (2020) use 5% for the general capital stock and 2.5% for fossil energy capital; Vogt-Schilb et al. (2018) use 4% for industry and 2.5% for energy capital.

[21] We calculate emissions intensity using total GHG emissions (net of agricultural methane and nitrous oxide emissions) and value added in the industry sector (including construction). We then calculate the average emissions intensity decline using data from 1994 to 2007, to avoid an excessive distortionary effect of climate policy on the data).

which we use. IPCC (2021) also offers a central estimate of the TCRE ζ of 0.00045°C/TtCO2. However, this estimate does not take into account the 'Zero Emissions Commitment' to warming,[22] and excludes some known climate feedbacks. Taking these into account, and including the warming effect of non-CO2 emissions, we estimate $\zeta = 0.0006$.[23] The combination of T_0 and ζ also allows us to calibrate the starting value of cumulative emissions $S_0 = 2000$. The parameter γ governing the climate damage function is set equal to 0.0077, based on the meta-analysis of Howard and Sterner (2017).

Temperature is subject to a Brownian motion with a standard deviation $\sigma_T = 0.03$, calibrated to fit the range of simulated temperatures at the end of this century across the IPCC climate models.[24] Thus, it represents uncertainty about the mechanisms linking emissions to temperature change.

Finally, the parameter governing the dependence of macro disasters to temperature ξ_2 is set to 0.07. This assumes an increase of the macro disaster jump from 20% to 21.4% when T is equal to 2°C.

4.4 Abatement, disinvestment and technical change

We estimate a set of key parameters linked to abatement and disinvestment costs (φ_0, θ_1, g_φ and ϵ) using energy systems modelling results, in particular from: (i) the model/scenario ensemble underpinning the IPCC Special Report on Global Warming of 1.5°C (IPCC, 2018); and (ii) the model/scenario ensemble of the Network of Central Banks and Supervisors for Greening the Financial System or NGFS (Bertram et al., 2021).[25] These databases together span 18 leading energy systems models.

Each model or project has a BAU reference scenario, and these scenarios differ depending on assumptions about population growth, GDP growth, openness of economies and other variables. Various emissions abatement scenarios are included, with a resulting range of temperature increases from below 1.5°C to more than 3°C. After excluding scenarios with temperatures too high to be compatible with meaningful emissions abatement,[26] we are left with 134 scenarios, for which we have values for carbon prices, as well as for emissions and

[22] That is, the unrealised warming to which we are committed due to CO2 already emitted to the atmosphere.

[23] The cumulative CO2 emission budget for 0.73°C warming from 2020 onwards is 1000 GtCO2 (IPCC, 2021). We assume that 20% of warming stems from other greenhouse gases.

[24] Specifically, in 2081-2100 for all combinations of Representative Concentration Pathways (RCPs) and Shared Socioeconomic Pathways (SSPs), as reported by IPCC (2021).

[25] The databases are available at https://data.ene.iiasa.ac.at/iamc-1.5c-explorer and https://www.ngfs.net/ngfs-scenarios-portal/, respectively.

[26] We exclude scenarios with cumulative emissions of more than 2166GtCO2eq in 2100, corresponding to a temperature increase of above 2.5°C.

GDP, from which we can calculate the scenarios' total abatement costs. The calibration strategy is to simultaneously choose the values of φ_0, θ_1, g_φ and ϵ that maximise the fit of our model to a combination of the marginal and total abatement costs from the scenario database.

As mentioned in Section 2, BAU emissions are given by $E_{BAU} = \phi_t/\varphi_t$, and since ϕ_t and φ_t decrease at the same rate, BAU emissions are constant. We use this to set $\phi = \varphi E_{BAU}$, with $E_{BAU} = 60$GtCO2eq.[27] From (1), production under BAU emissions is $Y_{BAU} = AL^{1-\alpha}K^{\alpha}\exp\left(-\frac{\gamma}{2}T^2\right)$, thus allowing us to define total abatement costs as a percentage of GDP as $1 - \frac{Y}{Y_{BAU}}$. In turn, we can write

$$\frac{Y}{Y_{BAU}} = \exp\left[-\frac{\varphi_t}{2}\left(E - \frac{\phi_t}{\varphi_t}\right)^2\right] - \theta_1 r^{\theta_2}\frac{L_0 e^{(g+g_L)t}}{k^* Y_{BAU}}, \tag{30}$$

where $r = \frac{-\dot{E}+E\left(-\delta-g-g_L+g_\tau\frac{1-\tau}{1+\tau}\right)}{\psi_0 L_0(1+\tau)}$ is the optimal disinvestment from dirty capital and k^* is the steady state total capital value from the model. Using the MAC function (3), marginal abatement costs are

$$-Y_\mu = Y\left(\phi_t - \varphi_t E\right) \tag{31}$$

where the MAC, $-Y_\mu$, is the optimal carbon price.

Parameters φ_0, θ_1, g_φ and ϵ are estimated by the Generalised Method of Moments using (30) and (31) simultaneously. Fitting to both total and marginal abatement costs allows us to obtain more robust results, in case the MAC functions of the underlying models are nonlinear.[28] We truncate the sample to eliminate outliers, excluding the 5% highest and lowest costs in each period. We put double weight on model results from the NGFS database. This database includes many of the models in the IPCC database, but since it is more recent it incorporates improvements in those models, e.g., calibration on the latest evidence.

Table 1 reports the results. We assume the errors of both equations are normally distributed. We report estimation results assuming technology costs that decrease exogenously, and technology costs that decrease endogenously. We do not have precise available estimates for ω, governing the relative weight of exogenous and endogenous technical change. Taalbi (2020) finds that at least 30% of variations in innovation activity are driven by the stimulus

[27] This is 4GtCO2eq above the 2020 level of GHG emissions of 56GtCO2eq, which itself was negatively impacted by the Covid-19 pandemic, perhaps by up to 2GtCO2eq (IPCC, 2022). Thus, we assume policies actually implemented to price carbon, implicitly or explicitly, reduced GHG emissions by at least 2GtCO2eq below BAU in 2020.

[28] The MAC is the most economically meaningful cost measure, because it determines the first-order conditions of the optimum. Therefore, we use a diagonal weighting matrix with a 60% weight on the marginal abatement cost fit and 40% weight on the total abatement cost fit.

Table 1: GMM estimates of total abatement costs (equation 30) and marginal abatement costs (equation 31) simultaneously. The database consists of 122 scenarios of the IPCC Special Report on 1.5°C (excluding scenarios with cumulative emissions beyond 2500GtCO2) and 12 scenarios from the NGFS database. BIC: Bayesian Information Criterion. AIC: Akaike Information Criterion.

	Exogenous technology	Endogenous technology
φ_0	4.2e-05***	4.2e-05***
θ_1	7.0e+02**	7.1e+02**
g_φ	0.74***	
ϵ		0.10***
N	1605	1605
Log likelihood	6664	6660
BIC	-13305	-13297
AIC	-13321	-13313

* p<.1; ** p<.05; *** p<.01

coming from the network of other productive activities. We opt for a mid-value of 0.5.

g_ϕ is by design set to be equal to g_φ. The 2020 cumulative abatement value M_0 is set at an approximate value of 100 GtCO2, building on the information provided by UNEP (2021).[29] Finally, θ_2 is set to 2.1 to ensure our convex function has a continuous second derivative.

5 Results

In this section, we present our main results. We show the optimal transition to a low-carbon economy in different versions of the model, focusing on four key variables: (i) GHG emissions (E); (ii) global mean temperatures (T); (iii) carbon prices (p); and (iv) disinvestment from dirty capital assets (R). For (i) and (ii), we take a century-long perspective, whereas for the economic variables (iii) and (iv) we only look out as far as 2050, so that important near-term differences are easier to see.

We consider two different policy/planning objectives. The first is the economist's natural objective of maximising welfare as per Eq. (18), i.e., maximising the discounted net benefits of reducing GHG emissions through shifting from dirty to clean capital, trading off avoided climate damages with the costs of reducing emissions. We call this the 'cost-benefit' problem for short. The second objective places a constraint on expected warming of 1.5°C and minimises only discounted emissions abatement costs, rather than minimising the sum of

[29]We slightly increase the values from UNEP (2021) to improve the log likelihood fit of the model.

discounted emissions abatement costs and climate damages.[30] We call this the '1.5°C cost-effectiveness' problem for short and include it because it is more true to the international climate negotiations in reality.

Technical details of how we solve the model can be found in Appendix C.

5.1 The optimal transition

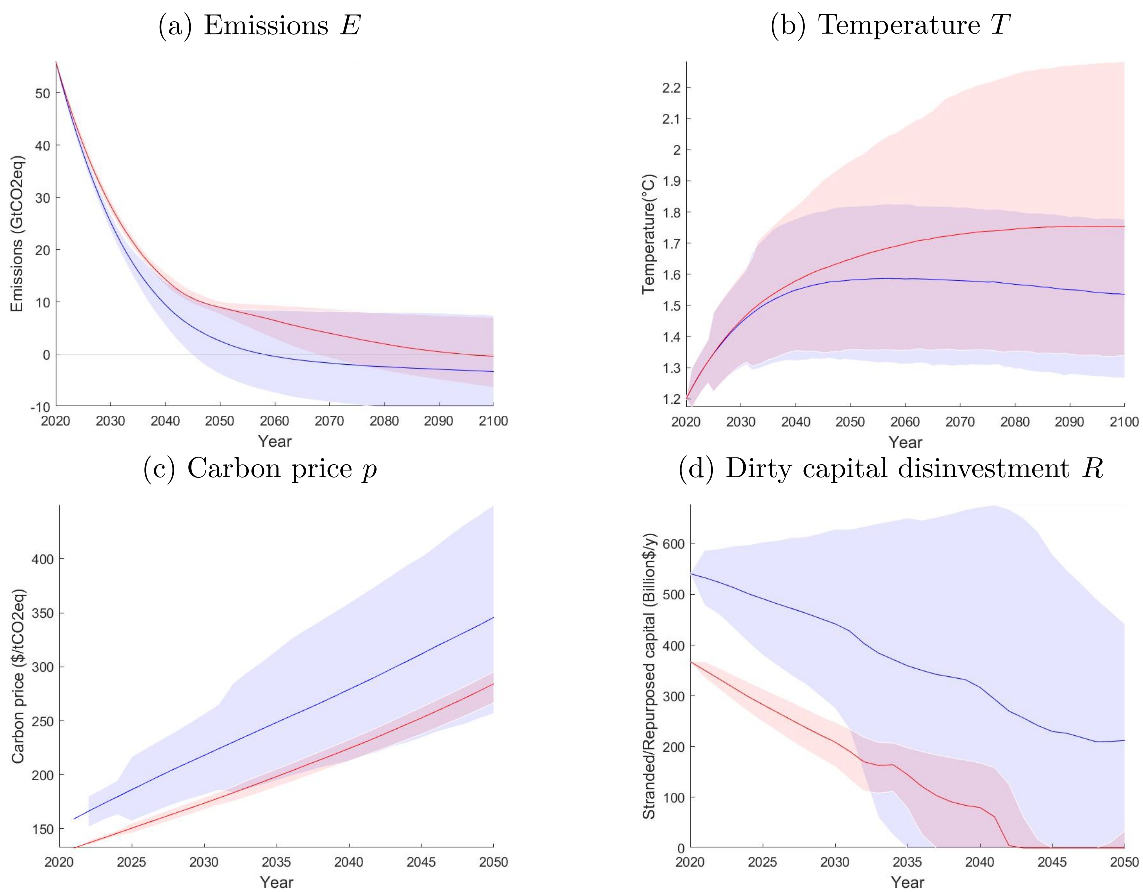
Figure 1 shows the optimal transition to a low-carbon economy in the full model, with the cost-benefit solution in red and the 1.5°C cost-effectiveness solution in blue. The cost-benefit solution involves immediate, steep GHG emissions reductions. Optimal emissions are halved in each of the 2020s and the 2030s. Thereafter, the rate of emissions decrease slows. As a consequence of the steep initial drop in emissions, expected warming is limited to 1.65°C above pre-industrial in 2050 and 1.76°C in 2100, which is approximately peak warming.

To deliver steep optimal emissions reductions, the planner levies a high initial carbon price of US$133/tCO2eq, rising to US$283/tCO2eq in 2050. Pricing carbon at this level triggers an initial US$375 billion/year in disinvestment from dirty capital assets, decreasing to US$213 billion/year in 2030 and US$80bn/year in 2040. Cumulatively, US$4.8 trillion of disinvestment from dirty capital is made, which equates to 17% of the dirty capital stock at the start. Thus, the pace of the optimal transition requires not only an immediate halt to dirty capital investment but also active decommissioning or repurposing of dirty assets, rather than using them until the end of their erstwhile economic lifetimes. In Section 2, we characterised an emissions reduction speed limit of $\delta_d + g_\psi$, equal to 5%/year according to our calibration. This is how fast emissions would decrease if investment in dirty capital were zero, there were no dirty capital disinvestment, and reductions were thus left to a combination of capital depreciation and autonomous improvements in energy efficiency. The optimal transition involves breaking this speed limit, reducing emissions by 6.5-7%/year for the next two decades. This additional disinvestment creates cumulative stranding costs of US$1.6 trillion, i.e., roughly 1/3 of the monetary value of dirty capital that is disinvested is lost.

Despite rapidly reducing emissions, the welfare-maximising path in Figure 1 does not limit warming to 1.5°C above pre-industrial. Therefore, keeping warming to no more than 1.5°C requires even steeper emissions reductions, not so much in the 2020s but increasingly thereafter. Indeed, emissions are slightly negative for most of the second half of the

[30] Technically, this is achieved by maximising (18) with a damage function (7) that is almost discontinuous, i.e., a ninth-order polynomial, giving negligible damages below 1.5°C and very high marginal damages above that level.

Figure 1: Optimal transition paths under unconstrained welfare maximisation (cost-benefit; red) and when minimising emissions reduction costs to limit warming to 1.5°C above pre-industrial (1.5°C cost-effectiveness; blue). The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



century.[31] The carbon price is accordingly higher. We estimate an initial carbon price of US$159/tCO2eq, rising to US$346/tCO2eq in 2050. Both the initial level and growth rate are higher, so the price paths diverge. The substantially higher carbon price has a large effect on dirty capital disinvestment, which starts at US$541 billion/year, is US$442 billion/year in 2030 and remains positive beyond 2050. Cumulatively, US$11.4 trillion in dirty capital disinvestment is made by mid-century, with US$5.8 trillion in stranding costs (i.e., 51% of the monetary value of the disinvested dirty capital stock is lost).

The 1.5°C cost-effectiveness solution has wide confidence intervals around the carbon price and dirty capital disinvestment, while the cost-benefit solution has a wide confidence

[31] Technically, the model achieves negative emissions through negative dirty capital, but this should be seen as an abstraction. The MAC of negative emissions comes from the calibration on energy systems modelling results. Negative emissions are cost-effective in many of these models.

interval around temperatures. This reflects the different ways in which the planner responds to shocks, especially temperature shocks. The cost-benefit planner is much more willing to accept uncertainty about temperature. The 1.5°C cost-effectiveness planner is determined to keep temperatures below the limit and thus responds to temperature shocks with potentially large adjustments to carbon prices, with knock-on effects on dirty capital disinvestment.

5.2 Optimal carbon price decomposition

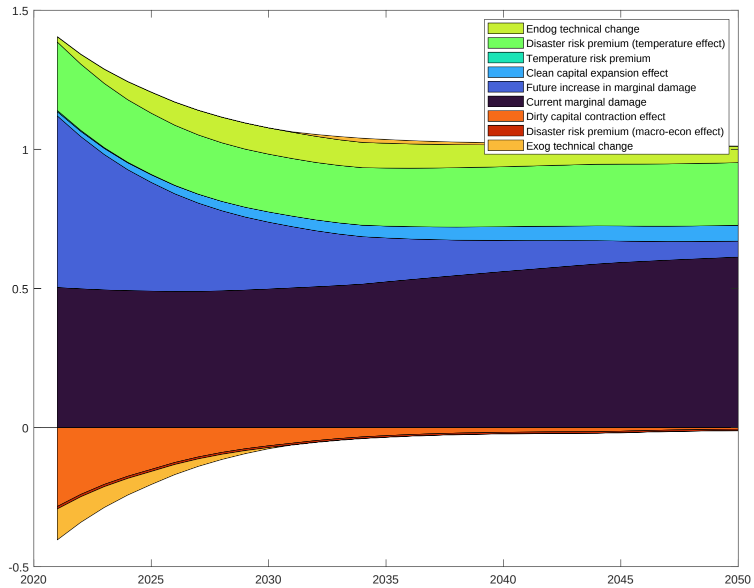
Figure 2 decomposes the optimal carbon price from the cost-benefit problem into its various elements as set out in Section 3 (Eq. 27). The y-axis is scaled so that a value of one corresponds to 100% of the optimal price in any given year, thus the figure plots shares.

The present value of (deterministic) marginal climate damages accounts for 112% of the optimal carbon price in 2021. This is divided into a contribution of 50 percentage points from the present value of marginal damages at the current temperature (i.e., 'current marginal damage', $\Psi c^{-\eta} y \gamma S$ in Eq. 27), and 62% from the future change in marginal damages (i.e., $-V_{SS}E$). The optimal temperature path is rapidly increasing initially, because capital inertia leads to high emissions at the start, but temperature growth slows down (Figure 1b). This explains both the large share of the future change in marginal damages initially, and the growing share of current marginal damage over time.

The disaster risk premium contributes 24% of the optimal carbon price in 2021 and this comes from the effect of temperature on disasters (we discuss this further in Section 6). The share remains relatively constant over time. Thus, avoiding climate-related macro-economic shocks is a significant motivation for reducing emissions, which persists. The contributions of the other risk premiums on temperature, capital and TFP respectively are very small.

The technical change effect starts negative overall, because the negative effect of exogenous clean technological progress (-11%) outweighs the positive effect of endogenous clean technological progress (+2%), however the situation is reversed as soon as 2024, and by 2030 technical change contributes +9% to the optimal carbon price. The dirty capital contraction effect reduces the optimal carbon price by 28% initially. This effect is inflated by dirty capital disinvestment costs, which are initially large, but it diminishes over time as the dirty capital stock settles down to a much lower and more slowly decreasing level. Opposing the dirty capital contraction effect is a small positive clean capital expansion effect (2% in 2021), which is increased by adjustment costs on clean capital investment. In the long run, the clean capital expansion effect is larger because investment flows into clean capital are larger.

Figure 2: Decomposition of the optimal carbon price. Y-axis is scaled in percent of the absolute price.



5.3 Disentangling the effects of capital inertia, clean technological progress, and uncertainty

The analysis just above decomposed the optimal carbon price into a set of constituents including effects attributable to capital accumulation/decumulation, clean technological progress, and uncertainty. However, we need a different kind of analysis if we want to show the *overall* effect on optimal emissions, temperatures, etc. of modelling our three key features of the transition: capital inertia, clean technological progress, and uncertainty. This is because, in general, modifying the model to omit one of these transition dynamics affects all the elements of the optimal carbon price laid out above. For example, ignoring capital inertia not only changes the clean capital expansion and dirty capital contraction effects. Rather, by affecting the optimal temperature trajectory, it also changes current marginal damage, the future increase in marginal damage, the disaster risk premium, and so on. Therefore, to analyse what difference capital inertia, clean technological progress and uncertainty make to the optimal trajectories, in this section we compare results from the full model with a series of model versions omitting each feature in turn. Again we focus on the cost-benefit problem.

Capital inertia Figure 3 compares the full model with a version that omits adjustment costs of capital investment, and dirty capital disinvestment/stranding costs. This version is made by setting the cost parameters $\chi_c = \chi_d = \theta_1 = 0$. Adjustment/disinvestment costs make it expensive to reduce emissions quickly. Without them, it is optimal to immediately disinvest from a large portion of the dirty capital stock (US$13 trillion of disinvestment in the first year, equal to 46% of the dirty capital stock). This leads to a huge drop in emissions of 29.5 GtCO2eq in the first year, and a flatter emissions path thereafter. Dirty capital disinvestment costs (Eq. 14) are the critical factor here. Were it possible to disinvest from dirty capital without "taking a hair cut", i.e., to recover the full value of the dirty assets, the imperative of avoiding climate damages would optimally drive nearly instantaneous disinvestment from many dirty assets. The emissions paths cross later in the 2030s, but the additional cumulative emissions in the next decade and a half that capital inertia brings about are not fully compensated this century, so optimal temperature in the scenario with adjustment costs is higher throughout.[32] The difference is consistently of the order of 0.1°C. The optimal carbon price in the absence of adjustment/disinvestment costs is initially US$129/tCO2eq, rising to US$277/tCO2eq in 2050. Thus, the initial carbon price premium from capital inertia is 3.4%, falling to 2.2% in 2050.

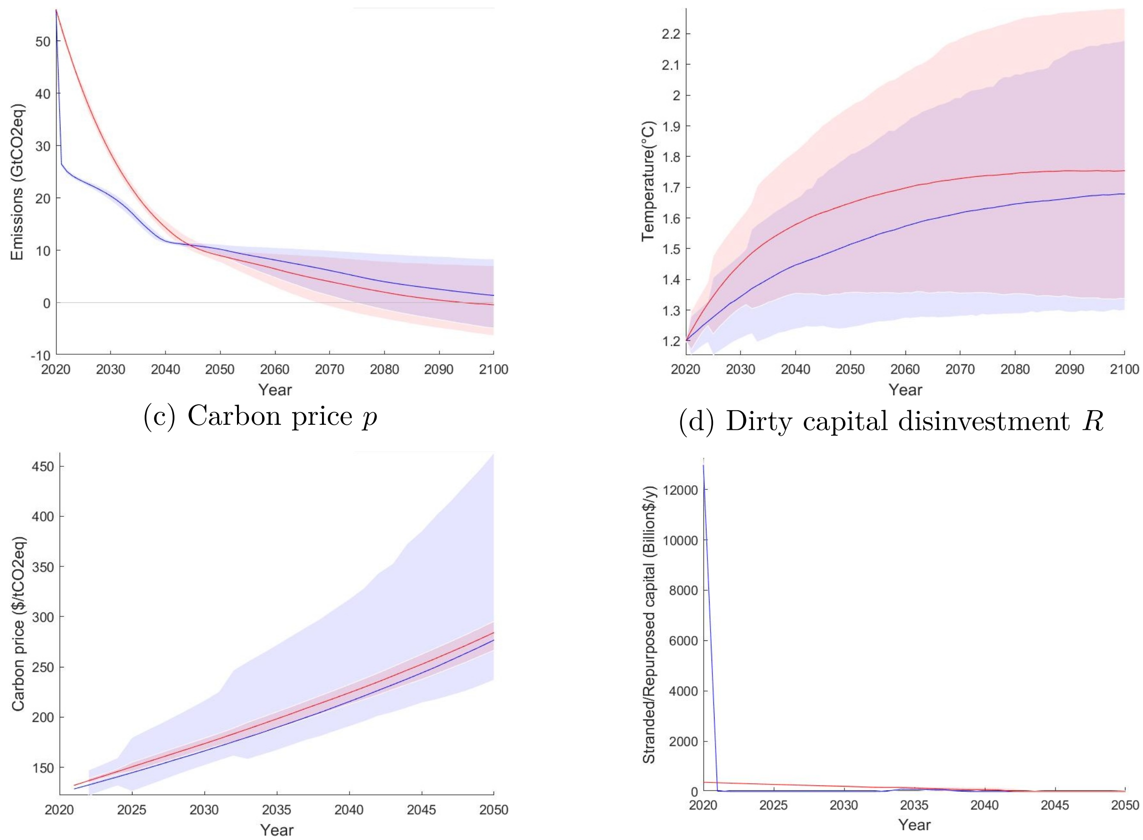
Clean technological progress Figure 4 compares the full model with a model version in which clean technology costs do not fall, either exogenously or endogenously. MACs are fixed at their 2020 level, i.e., $\phi_t = \phi_0$ and $\varphi_t = \varphi_0$, $\forall t$. We find that the initial steep drop in emissions is virtually unaffected. The difference between the paths in 2030 is just 0.5GtCO2eq. It is only after 2030 that the emissions paths begin to noticeably diverge, with emissions higher in the absence of clean technological progress. This reflects the fact that clean technological progress takes time to make a big difference to MACs. Optimal emissions reductions in the early years are more strongly driven by dirty capital disinvestment costs. Higher optimal emissions lead to higher temperatures, with the optimal temperature in 2100 reaching 2.1°C. The carbon price starts at US$138/tCO2eq with fixed MACs, thus clean technological progress provides a carbon price discount of 3.7% initially, rising to 5% in 2050. Disinvestment from dirty capital is similar in the 2020s, consistent with the similar emissions paths, but in the absence of clean technological progress there is no more dirty capital disinvestment from the early 2030s, and hence cumulative dirty capital disinvestment is lower at US$3 trillion. Without clean technological progress, it becomes less economical to convert dirty capital to clean capital, and instead emissions/temperatures are allowed to

[32]Steady-state temperature is the same, but adjustment/disinvestment costs make it optimal to close the gap slowly.

Figure 3: Optimal transition paths with adjustment/disinvestment costs (red) and without (blue). The planner's objective is to maximise welfare. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.

(a) Emissions E (b) Temperature T

(c) Carbon price p (d) Dirty capital disinvestment R

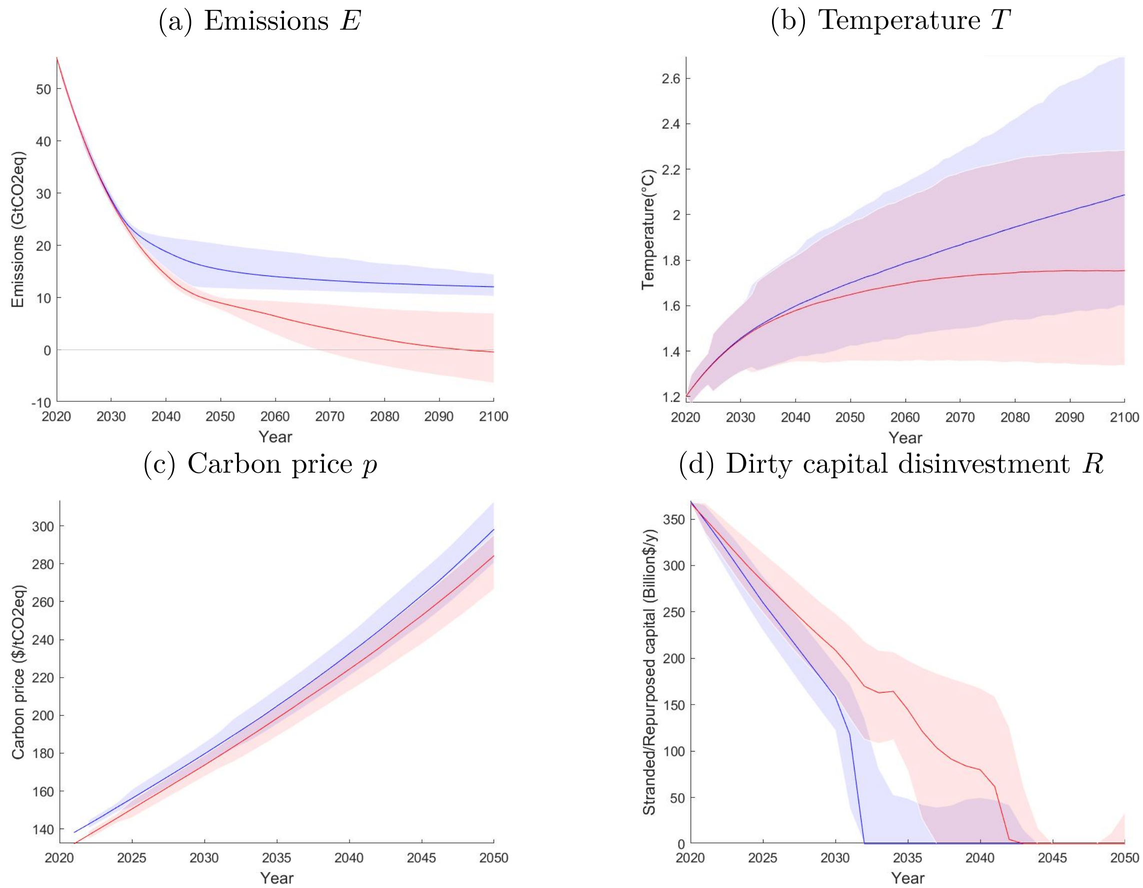


rise.

Uncertainty Figure 5 compares the full model with a model version that has no uncertainty. This is achieved by setting the volatility terms to zero in the temperature, clean/dirty capital and TFP state equations, and eliminating TFP shocks.[33] Uncertainty coupled with risk aversion leads to lower optimal emissions and temperatures in expectations than when the climate problem is treated as deterministic. Without uncertainty, the optimal temperature in 2100 is 2.1°C. The effect of uncertainty on the optimal carbon price is particularly marked – it adds 1/3 to the optimal carbon price in 2021, which is US$100/tCO2eq without uncertainty. In 2050, it adds 23%. Accordingly, dirty capital disinvestment is substantially lower without uncertainty, at only US$224 billion/year initially and US$1.3 trillion cumulatively.

[33]The TFP drift term is adjusted to obtain the same long-term growth path.

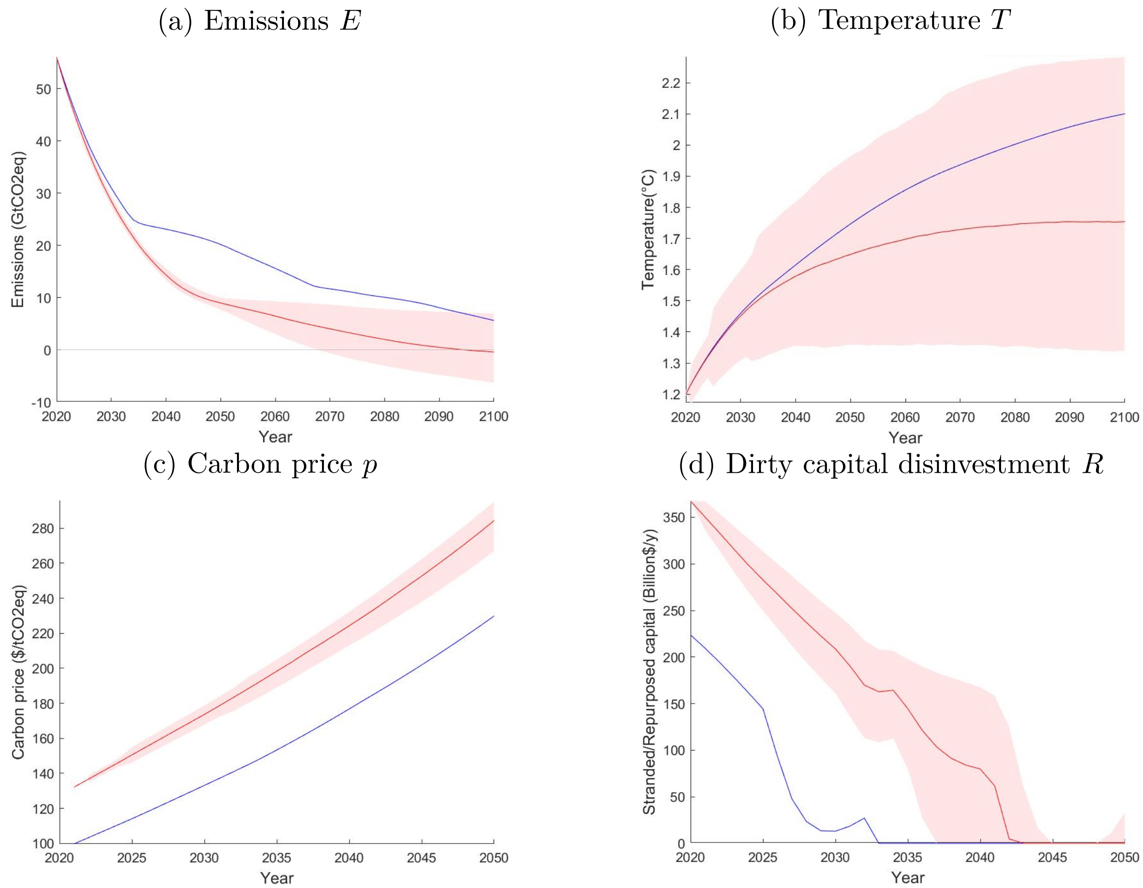
Figure 4: Optimal transition paths with falling clean technology costs (red) and fixed clean technology costs (blue). The planner's objective is to maximise welfare. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



5.4 Comparison with 'straw man' model

In this section, we compare the full model with a 'straw man' model that omits capital inertia, clean technological progress, *and* uncertainty. We intentionally favour the term straw man over 'standard' as we do not want to create a misleading impression of the literature – the idea behind this model variant is that it should fairly represent a basic model of optimal management of the GHG externality. Most models allow capital to be costlessly shifted from clean sectors to dirty sectors, and many are deterministic. The treatment of clean technological progress is more nuanced, however. Some models allow clean technology costs to fall, but in most cases they fall exogenously (Gillingham et al., 2008; Nikas et al., 2018). Some models have fixed technology costs, but few if any would fix the cost parameters on 2020 data, which is what we did above in order to make the effect of falling costs clear. Therefore, for the straw man model we fix MACs at their estimated 2030 level, rather than

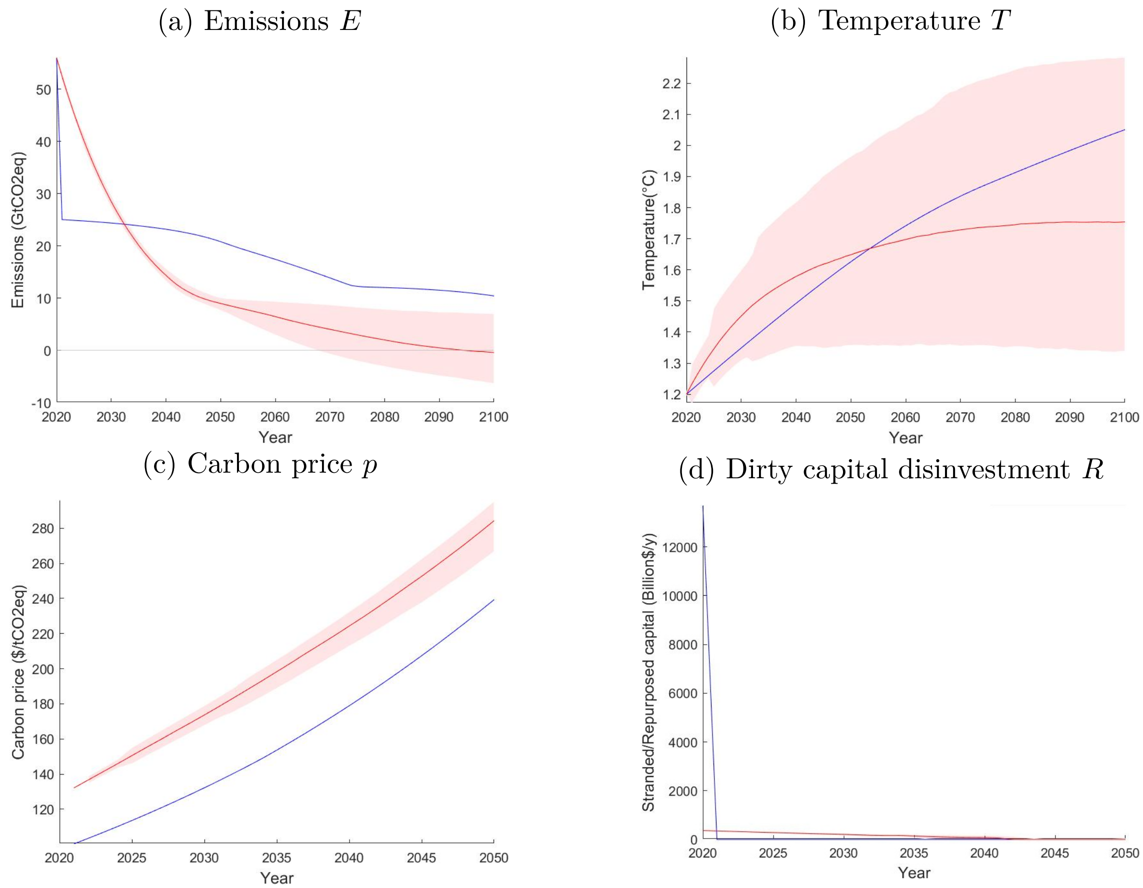
Figure 5: Optimal transition paths with uncertainty (red) and without uncertainty (blue). The planner's objective is to maximise welfare. The optimal path under uncertainty depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



2020. This mimics the famous McKinsey MAC curves, for example (Enkvist et al., 2007).

Figure 6 presents the comparison, with the full model in red and the straw man in blue. The straw man emissions trajectory is initially dominated by the effect of ignoring capital inertia, resulting in an immediate drop of 31GtCO2eq, which is in fact a slightly larger drop than in the variant that just omits capital inertia. Thereafter, the emissions trajectory is relatively flat, with emissions in 2100 still as high as 10GtCO2eq. This persistently high level reflects the combined effect of fixed clean technology costs and no uncertainty. The shape of the straw man emissions path translates into a more linear warming path, so temperature is initially lower than the full model optimum (due to the rapid conversion of dirty capital into clean capital), but it crosses over in the early 2050s and by 2100 the straw man optimal temperature is 2.1°C above pre-industrial. Again, this reflects the combined effect of fixed clean technology costs and no uncertainty, both of which result in a higher steady-state temperature, whereas capital inertia has no effect on the steady state. In the straw man

Figure 6: Optimal transition paths for full model (red) and in straw man model without capital inertia, falling clean technology costs, or uncertainty (blue). The planner's objective is to maximise welfare. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



model, the optimal carbon price starts at US$100/tCO2eq. Thus, the carbon price premium in the full model relative to the straw man model is US$18/tCO2eq or 1/3 initially, falling to 18% in 2050. There is no dirty capital disinvestment in the straw man model beyond the initial, huge adjustment, and no monetary stranding costs. Table 2 summarises the results of this section and the last. Appendix D decomposes the optimal carbon price in these different model variants.

Table 2: Summary of effects of omitting capital inertia, clean technological progress, and uncertainty respectively. Differences are expressed as (full model - model version) / model version. Note that MACs are fixed at their 2030 level in the straw man model, not 2020 as in the version with no learning.

	Cumulative emissions 2020-2100	Temperature in 2100	CO_2 price in 2021	Cumulative dirty capital disinvestment 2020-2050
Δ with no inertia	17.6%	0.08°C	3.4%	-63.9%
Δ with no learning	-36.2%	-0.33°C	-3.7%	60.5%
Δ with no uncertainty	-36.5%	-0.34°C	33.3%	257.5%
Δ with straw man	-34.4%	-0.29°C	32.6%	-64.7%

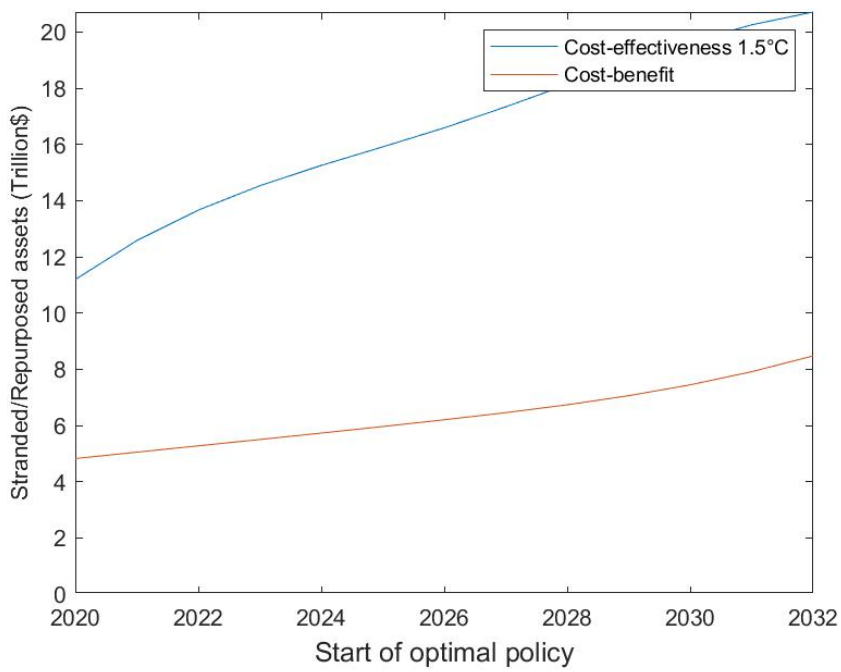
5.5 Effect of delaying carbon pricing on dirty capital disinvestment

One of our main findings is that the optimal transition involves significant disinvestment from dirty assets, as the optimal speed of emissions reductions is beyond what can be achieved by just letting dirty capital depreciate and relying on autonomous improvements in energy efficiency.

Figure 7 plots cumulative disinvestment from dirty capital assets as a function of how long optimal climate policy is delayed. That is, all transitions analysed so far have optimally restricted GHG emissions from 2020 onwards. Capital inertia inhibits the capacity of the economy to make a large jump from dirty to clean capital, but the planner still imposes the optimal carbon price at the outset. Instead, here the internalisation of the GHG externality is delayed by increments of one year, as a representation of policy failures at the international and national levels. During the period of policy delay, we assume there is no anticipation of the introduction of carbon pricing, thus it is a policy shock.

The figure shows that total disinvestment from dirty capital is increasing in the length of the delay. The rate of increase is particularly fast in the case of limiting warming to 1.5°C at minimum abatement cost, because delaying emissions reductions just uses up the available carbon budget, given that temperature is a linear function of cumulative emissions (Eq. 8). Total disinvestment thus rises from US$11.4 trillion if optimal policy starts in 2020 to US$25.6 trillion if it is delayed by 10 years. Even a delay of one year to 2021 increases total disinvestment by US$1.6 trillion. Dirty capital disinvestment increases more slowly when the policy objective is to maximise welfare without a temperature constraint, because the planner re-evaluates optimal long-run temperatures. Thus, despite not facing a temperature constraint, the planner still makes up for lost time with faster rates of emissions reduction.

Figure 7: Total cumulative (undiscounted) disinvestment from dirty capital as a function of how long the imposition of optimal emissions reductions is delayed. Unconstrained welfare maximisation in red; limiting warming to 1.5°C at minimum abatement cost in blue.



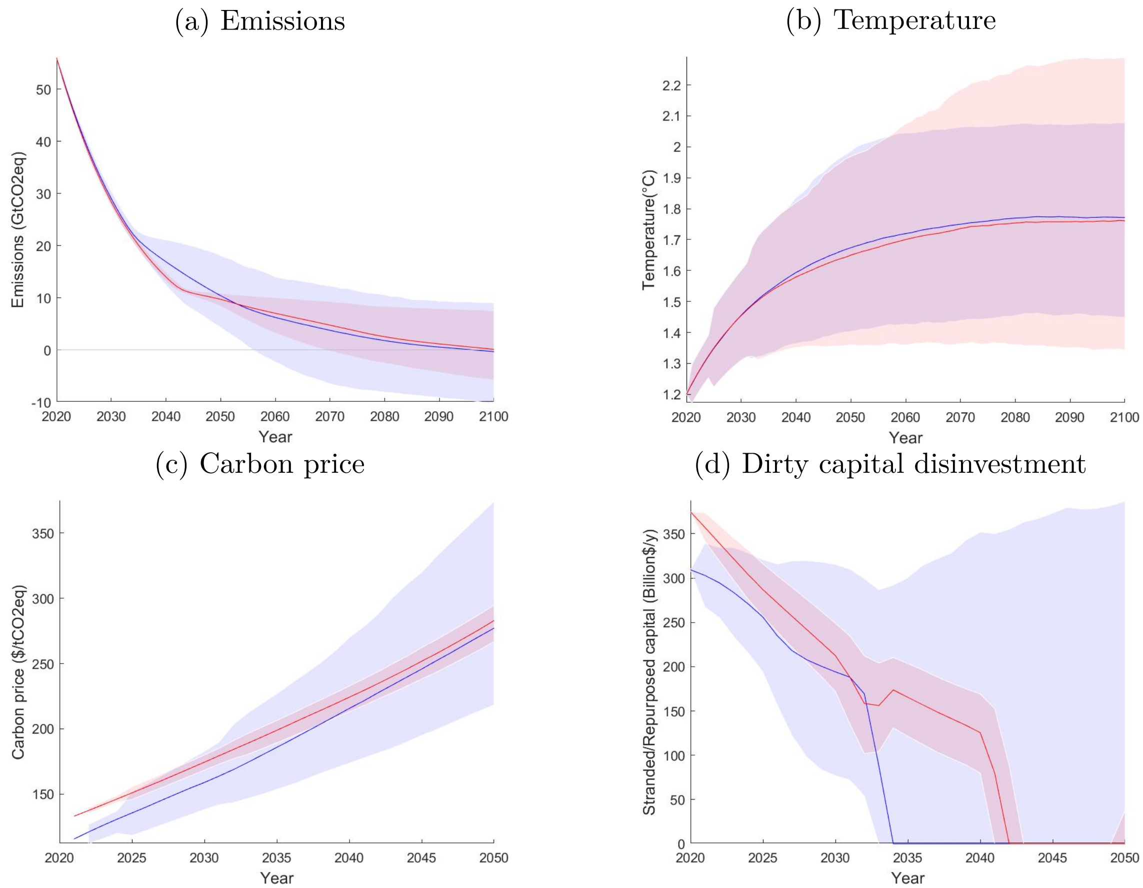
5.6 Cost-benefit versus cost effectiveness

Above, we compared two different formulations of the climate planning problem, unconstrained welfare maximisation (cost-benefit), and limiting expected warming to 1.5°C at minimum discounted abatement cost (1.5°C cost-effectiveness). Thus, in switching from the former to the latter, two things change: (i) a temperature constraint is imposed, and (ii) damages are effectively ignored. In this section, we isolate the effect of switching from a cost-benefit to a cost-effectiveness problem, i.e., we focus on the implications of (ii) for the optimal path. We do this by relaxing the temperature constraint in the cost-effectiveness problem so that it is just equal to the 2100 temperature that solves the cost-benefit problem.

Figure 8 shows that, since the cost-effectiveness solution is insensitive to the timing of damages, emissions are higher in the next few decades than they are in the cost-benefit solution. As a result, temperature increases slightly faster. In the second half of the century, the ordering of the emissions paths is reversed, as the cost-effectiveness solution must compensate for the initially higher emissions, which eat up the available carbon budget faster.

These differences are not large, however the differences in the optimal carbon price and dirty capital disinvestment paths are more so. The cost-effectiveness carbon price starts at US$116/tCO2eq, which is 13% lower than the cost-benefit carbon price. It grows more quickly, so the difference between the price paths narrows to just 2.1% in 2050. The lower

Figure 8: Cost-benefit (red) and cost-effectiveness (blue) paths to achieving a common temperature in 2100. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



carbon price results in 17% less dirty capital disinvestment initially, and 33% less cumulatively.

Comparing these results with Figure 1, the implication is that the substantially higher initial carbon price and dirty capital disinvestment required to limit warming to 1.5°C at minimum abatement cost is entirely due to the imposition of a very strict temperature constraint. Conversely, casting the policy problem as one of limiting temperature at lowest abatement cost in itself requires a lower initial carbon price and less dirty capital disinvestment (the latter in expectations, at least). Without needing to worry about damages, the planner back-loads effort until later in the century.

6 Discussion

In this section, we provide further context to and discussion of four key issues/results: (i) the level and growth rate of the optimal carbon price; (ii) clean technological progress; (iii) the risk premium on the optimal carbon price; (iv) stranded assets.

Carbon prices We estimate an optimal carbon price of US\$133/tCO2eq in 2021 to solve the cost-benefit problem. This is consistent with a recent trend towards higher optimal carbon prices in the literature. There are various reasons for this, especially lower discounting and higher climate damages, but also updating IAMs' climate dynamics (Dietz et al., 2021a). The recent study by Hänsel et al. (2020) is a good illustration of the trend. They implement various changes to DICE (the 2016 version; Nordhaus, 2017), which has an optimal carbon price in 2020 of US\$37/tCO2 (in 2010 US dollars). Reducing the pure rate of time preference from 1.5% to 0.5% and reducing the elasticity of marginal utility from 1.45 to 1 raises the optimal carbon price to US\$199/tCO2 in 2020. Our discount rate parameters are in between (1.1% and 1.35 respectively), as is our optimal carbon price. Retaining Nordhaus' preferred discount rate parameters but *inter alia* updating the DICE model's climate dynamics and increasing assumed damages, Hänsel et al. (2020) estimate a 2020 optimal carbon price of US\$82/tCO2. We use similar climate dynamics and a similar damage function calibration, but add adjustment costs, falling clean technology costs, and risk.

Besides initial carbon prices, the carbon price growth rate is also an important variable. Our optimal carbon price grows at an average rate of 3% between 2021 and 2030, falling thereafter and averaging 2.5% over the whole period 2021-2100. How this relates to GDP growth has been of particular interest ever since Golosov et al. (2014) showed that, under restrictive assumptions, carbon price and GDP growth are the same. Since GDP growth in our model is c. 2%, our carbon price grows faster. Dietz and Venmans (2019) showed in a simple analytical model that the carbon price grows faster than GDP if marginal damages are increasing in cumulative emissions, but temperature is linear in cumulative emissions. The former assumption is standard and the latter is a foundational result from climate science (IPCC, 2021). Our analysis above shows that capital inertia further contributes to the wedge between carbon price and GDP growth in the near term.

The optimal carbon price that solves the 1.5°C cost-effectiveness problem is higher at US\$159/tCO2eq in 2021, since the 1.5°C temperature constraint binds, especially under risk. It tends to grow a bit more slowly, at an average rate of 2.4% between 2021 and 2100, although in the first decade the growth rate is 3.4%. The 1.5°C cost-effectiveness problem is evaluated by energy systems models participating in the IPCC and NGFS inter-comparison exercises. We fit our abatement cost parameters on these model results, so clearly they cannot be used to validate our carbon price estimates. However, we fit on the relationship between emissions and carbon prices, which leaves room for a different relationship between prices and time in the IPCC/NGFS models. Indeed, what is interesting about these energy systems models is that they yield lower initial carbon prices, but a much steeper trajectory, resulting in much higher prices by mid-century and beyond (Gollier, 2021). This is accompanied by higher initial emissions, and lower emissions later on, often significantly negative. Two likely reasons for this difference are that: (a) the IPCC/NGFS models tend to be solved to meet a temperature constraint in 2100 and allow overshoot in the meantime, whereas we impose the temperature constraint throughout, and (b) energy systems models tend to use higher discount rates calibrated on the market opportunity cost of capital, which leads to postponing emissions reductions.

Clean technological progress In our model, the cost of clean technology falls along the transition. Part of this process is exogenous and just depends on time. Part of it is endogenous, depending on cumulative past abatement. This dual structure is intended to capture in reduced form the diverse sources of falling clean technology costs, which include knowledge spillovers from other sectors, economies of scale and learning-by-doing in the clean technology sector, induced R&D into clean technology, and network effects. Some of these are associated with market imperfections and can be addressed by a wide range of policies including R&D subsidies, both economy-wide and specific to clean technology. The literature on clean technological progress has differentiated between R&D models and learning-by-doing models (Goulder and Mathai, 2000; Gerlagh et al., 2009). The latter models assume technologies need to be used for their costs to fall, the former need not. A pure R&D model would map on to exogenous clean technological progress in our model, with any existing policies to stimulate R&D also exogenous. A pure learning-by-doing model would map on to endogenous clean technological progress in our model. However, the dichotomy is somewhat misleading given the diverse sources of technological progress as mentioned above. Economies of scale also create a linkage between past abatement and current abatement costs, without necessarily involving an externality.

The carbon price we report corresponds to the optimal marginal abatement cost. Along the transition, it is optimal to price carbon because of (i) climate damages, which are an external cost of carbon emissions, and (ii) future abatement cost reductions, which result from current abatement. Insofar as the second component results from a different market imperfection, the first best would be to address it with a separate policy instrument such as an innovation/deployment subsidy, rather than being priced into a carbon tax (or indirectly into the emissions quota in a system of tradeable emissions permits). In this case, our optimal carbon price would be second best, specifically it would be larger than the explicit carbon price that one would want to levy in the real world. The results of our decomposition of the optimal carbon price can be used to identify the difference. Figure 2 shows that the endogenous technical change effect is about 2% initially and reaches a maximum of about 9% in the early 2030s.

Risk premium Our results show that economic uncertainties, i.e., shocks on clean capital, dirty capital and TFP, have a very small effect on the optimal carbon price, and even macroeconomic risk has a modest effect. To intuit this result, it is helpful to discuss the climate beta (Dietz et al., 2018), i.e., the elasticity of marginal climate damages with respect to consumption. The climate beta indicates whether high marginal damages – high marginal benefits of abatement – tend to accrue in future states with high (low) consumption, in which case abatement increases (decreases) consumption risk. Our climate beta is close to one throughout the 21st century (Appendix E). It starts at just above one and decreases, non-monotonically, to about 0.85 in 2100. There are three drivers of this result. First, we assume damages are proportional to production. It is well known that in such models with 'multiplicative' damages, the climate beta will be attracted to one, as shocks on clean capital and TFP will affect consumption and marginal damages proportionally (abstracting from feedbacks via the savings rate). Second, dirty capital shocks create a positive correlation between temperature and consumption, increasing beta. Third, temperature shocks create a negative correlation between temperature and consumption, decreasing beta. The third effect dominates the second when temperature is high and the dirty capital stock is low.

In our model, intertemporal inequality aversion ($\eta = 1.35$) is hence close to the climate beta. In a model where $\eta = \beta$, a negative shock to GDP will increase marginal utility and decrease damages in equal proportion (Dietz et al., 2018; van den Bremer and van der Ploeg, 2021). As a result, the carbon price will not be affected by growth/macroeconomic uncertainty, as is the case in Golosov et al. (2014). Some authors have set $\eta < 1$ in line with the observation in capital markets that the asset/dividend ratio decreases with uncertainty (Olijslagers et al., 2021; Cai and Lontzek, 2019; Bansal et al., 2017). Others use $\eta > 1$ (Lemoine and Traeger, 2014; Cai and Lontzek, 2019; Nordhaus, 2017; van den Bremer and van der Ploeg, 2021). This is more in line with the literature on the social discount rate, and gives the more intuitive result that economic risk increases the carbon price.

Unlike economic uncertainty, the effect of climate disasters is large, increasing the optimal carbon price by 24% in 2021. However, this is mainly due to expected damage from disasters: the risk premium on the mean-preserving volatility of the damages is very small. The effect of the Brownian motion on temperature is similarly very small. Appendix E analyses the sensitivity of our model to risk aversion. Our default assumption is that $RRA = 4$. In the Appendix, we test alternative values of 6 and 1.35 (the latter gives the expected utility model). The optimal carbon price varies little. Other studies investigating climate uncertainties and tipping points find similar results (Cai and Lontzek, 2019; Hambel et al., 2021; Crost and Traeger, 2014; van den Bremer and van der Ploeg, 2021).[34] One explanation is that consumption volatility caused by climate damages is modest compared to stock-market volatility. van den Bremer and van der Ploeg (2021) show that the social cost of carbon more than doubles when RRA is increased from 4.32 to 6, if consumption volatility is calibrated on asset volatility. Conversely, they find almost no effect when consumption volatility is calibrated on GDP volatility. Another explanation is that our model, in keeping with the literature using recursive IAMs, assumes an optimal and immediate policy reaction to new information regarding damages and climate. A shock on temperature is followed by a rapid emissions reduction. Although adjustment costs limit how rapid it can be, the political decision comes without delay. In reality, policy comes with a delay at best and the absence of climate policy can lead to much higher risk premiums. Future research could usefully explore models of policy reaction in between a closed-loop solution with immediate policy adaptation and an open-loop solution that never adjusts the policy.

Stranded assets Our model finds it is optimal to reduce the dirty capital stock faster than its natural rate of depreciation. This can be interpreted as repurposing carbon-intensive capital for low-carbon uses. We find optimal cumulative dirty capital disinvestment of US$4.8 trillion in the cost-benefit case and US$11.4 trillion in the 1.5°C cost-effectiveness case. These values represent 17-41% of the total dirty capital stock in the initial period ($K_{d,0}$ = US$28 trillion), respectively. The associated cumulative monetary stranding costs are equal to US$1.6 trillion in the cost-benefit case and US$5.8 trillion in the 1.5°C cost-effectiveness case. This means that 33-51% of the repurposed capital stock gets lost, respectively. As Figure 8 shows, delaying the introduction of carbon pricing would increase optimal disinvestment (and associated costs), involving almost the entire stock of dirty capital if carbon pricing is postponed to the 2030s and its imposition is not anticipated.

These results are broadly consistent with literature based on technology-rich energy models and asset-level data (see for instance Pfeiffer et al., 2018; Fofrich et al., 2020; Tong et al., 2019). These studies, often focused on specific sectors (e.g., coal- and gas-fueled electricity generation), consistently find that imposing a 1.5-2°C temperature constraint makes it

[34] Cai and Lontzek (2019) find that increasing the RRA from 2 to 5 increases the SCC by a mere 3% (Fig. 7 therein). In Hambel et al. (2021), the effect is 3% for the Dell et al. (2012) damages calibration and $\eta = 1$ (Table 6 therein). The effect is barely visible in Crost and Traeger (2014), SM Fig 5.

optimal to strand a proportion of existing and planned carbon-intensive physical capital. Comparing their numerical results to ours is not trivial. Most of this applied literature provides stranding results in physical rather than monetary values (e.g., gigawatts of stranded power capacity). In addition, their sector-specific results cannot be easily translated into economy-wide stranding estimates (see Cahen-Fourot et al., 2021, for a discussion of stranding network effects). However, our results appear to be roughly in line with the results from Johnson et al. (2015) and on the lower end of the results from Edwards et al. (2022).[35]

In addition to physical capital, reserves of fossil fuels could also be left stranded in low-carbon scenarios. Estimates of fossil fuel reserves/resources at risk of stranding vary quite widely, from $3 trillion to more than $180 trillion (see Hansen, 2022, for a review), but the fact of 'unburnable carbon', i.e., the disparity between, on the one hand, the carbon embodied in available fossil fuels and, on the other hand, the 1.5-2°C cumulative carbon emissions budget, is well documented (see for instance Welsby et al., 2021). This is beyond the scope of our model, which does not include fossil fuel reserves. Given the misalignment between available fossil fuels and allowable cumulative emissions, the scarcity rent on fossil fuels in our model would be zero and would not impact on the optimal transition.

7 Conclusion

In this paper, we have studied the optimal transition to a low-carbon economy, recognising that the transition is not going to be seamless and smooth. It must be prepared for adjustment costs to the reallocation of capital from dirty to clean sectors, and shocks to the economy and climate. It also needs to account for clean technological progress, part of which is exogenous and incentivises waiting, but part of which is endogenous and incentivises acting early.

We find the optimal transition is fast, requiring a high carbon price and active stranding of dirty assets, especially if the imposition of a carbon price is delayed and unanticipated. Adjustment costs create dirty capital inertia, which in turn creates temperature inertia. Clean technological progress allows for lower optimal carbon prices, compared to a counterfactual of no progress beyond today's technology, but conversely endogenous technological progress justifies a carbon price premium that may be rather small, especially if other policy instruments are available. Uncertainty leads to lower optimal emissions and temperatures, and higher optimal carbon prices and stranded assets. However, the driver of this is expected

[35] Johnson et al. (2015), using the MESSAGE-MACRO model, find optimal stranding of coal plants in a 2°C scenario to be between $200 billion and $600 billion, depending on how rapidly policies are implemented. Edwards et al. (2022), using the GCAM model, find stranded physical capital in the coal sector in the range of $1-1.4 trillion.

damages from temperature-related macroeconomic disasters, rather than risk per se. Putting these effects together, we estimate a net premium of one third on the optimal carbon price today relative to a ‘straw man’ model with perfect capital mobility, fixed abatement costs and no uncertainty. Another reason to price carbon at a high level today is to avoid climate damages in the near to medium term, a basic point that is ignored if the planning problem is construed as one of meeting a temperature constraint at minimum discounted abatement cost.

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Appendix A HJB equation, Itô's lemma and optimality conditions

Instantaneous utility in our model is

$$L\frac{\left(\frac{C}{L}\right)^{1-\eta}}{1-\eta}, \tag{32}$$

where L is total population, growing at constant rate g_L and C is total consumption. Substituting in consumption per unit of effective labour $c = \frac{C}{L_0 e^{(g+g_L)t}}$, we obtain

$$e^{(g_L-(\eta-1)g)t}L_0\frac{c^{1-\eta}}{1-\eta}. \tag{33}$$

The exponent of the first factor is added to the discount rate. This transformation has two advantages. First, consumption per unit of effective labour has a steady state, and second, the transformed, larger discount rate will make the HJB equation easier to estimate. Consumption per unit of effective labour is a function of five state variables $\{k_d, k_c, S, \tilde{A}, t\}$ and two decision variables $\{i_d, i_c\}$,

$$c\left(k_d, k_c, S, \tilde{A}, t, i_d, i_c\right) = \tilde{A}\left(k_d + k_c\right)^{\alpha} e^{-\frac{\varphi(t,S)}{2}\mu(t,k_d)^2 - \frac{\gamma}{2}\zeta^2 S^2} - i_d - i_c - \kappa\left(r\right), \tag{34}$$

with abatement $\mu\left(t, k_d\right) = \frac{\phi_0}{\varphi_0} - \psi_0 L_0\left(1+\tau(t)\right)k_d$. This justifies the value function of Equation (18) which depends on the five above mentioned states and the associated aggregator function in Equation (19).

The HJB equation is

$$max_{i_d,i_c}\left\{f + \frac{1}{dt}\mathbb{E}[dV(k_d, k_c, S, \tilde{A}, t)]\right\} = 0.$$

Applying Itô's Lemma gives

$$
\begin{aligned}
0 \;=\; & max_{i_d,i_c}\{f \\
& +V_{k_d}\left(i_d-\chi i_d^2-(\delta+g_L+g)k_d\right) \\
& +V_{k_c}\left(i_c-\chi i_c^2-(\delta+g_L+g)k_c\right) \\
& +{}^1\!/\!_2 V_{k_dk_d}k_d^2\sigma_d^2+{}^1\!/\!_2 V_{k_ck_c}k_c^2\sigma_c^2+V_{k_ck_d}k_dk_c\rho\sigma_c\sigma_d \\
& +V_S E+{}^1\!/\!_2 V_{SS}S^2\sigma_S^2 \\
& +V_A\left(A_0-\tilde{A}\right)g_A+0.5V_{AA}\tilde{A}^2\sigma_A^2 \\
& +\lambda_A\left[V\left(\tilde{A}-\Delta\tilde{A}\right)-V\left(\tilde{A}\right)\right] \\
& +V_t\}
\end{aligned}
$$

with $\Delta\tilde{A}=\xi_1\left(1+\xi_2\zeta S\right)\tilde{A}$ and $V\left(\tilde{A}-\Delta\tilde{A}\right)$ shorthand notation for

$$V\left(k_d,k_c,S,\left(\tilde{A}-\Delta\tilde{A}\right),t\right). \tag{35}$$

We now turn to the explicit formula for the shadow price of capital. The first order condition for the maximum of the HJB equation with respect to dirty capital gives

$$f_c c_{i_d}+V_{k_d}\left(1-2\chi i_d\right)=0. \tag{36}$$

The derivative of the aggregator function with respect to consumption is

$$f_c=\hat{\rho}c^{-\eta}\left[\left(1-RRA\right)V\right]^{\frac{\eta-RRA}{1-RRA}}. \tag{37}$$

From Equation (34) we have

$$c_{i_d}=-1+\theta_1\theta_2 r^{\theta_2-1}. \tag{38}$$

Combining Equations (36), (37) and (38) results in Equation (24) in the main text. The FOC for clean capital is the same, except that there will be no stranding/repurposing costs.

Next, we derive the formula for the carbon price. The shadow price of cumulative emissions (expressed in utils) is V_S. It is the decrease in welfare for an optimal emissions trajectory starting with an extra unit of cumulative emissions. Similarly, the shadow price of a unit of capital (in dollars) expressed in utils is V_{K_c}. To convert to units of effective labour, we can write $V_{K_c}=V_{k_c}\frac{\partial k_c}{\partial K_c}=\frac{V_{k_c}}{L_0e^{(g+g_L)t}}$. Hence the carbon price, expressed in units of capital (dollars) is $p=\frac{-V_S}{V_{k_c}}L_0e^{(g+g_L)t}$. Applying the envelope theorem with respect to S to the HJB equation results in our formula for the carbon price in Equation (27). Note that the derivative of the aggregator function is tedious but straightforward:

$$\begin{aligned} f_S = & \hat{\rho} c^{-\eta} c_S \left((1 - RRA) V \right)^{\frac{\eta - RRA}{1 - RRA}} \\ & + \hat{\rho} V_S \left[\frac{c^{1-\eta}}{1-\eta} (\eta - RRA) \left((1 - RRA) V \right)^{-\frac{1-\eta}{1-RRA}} - \frac{1 - RRA}{1-\eta} \right]. \end{aligned} \tag{39}$$

Appendix B Parameters and initial values

Table A1: Parameters and initial values in the central scenario

Parameter	Symbol	Value	Source
Preferences			
Elasticity of marginal utility of consumption	η	1.35	Drupp et al. (2018)
Relative risk aversion	RRA	4	Barro (2009)
Utility discount rate	ρ	0.011	Drupp et al. (2018)
Production			
Initial TFP value	A_0	3.44	Calibration
TFP growth rate compensating disasters	g_{A1}	0.0037	Barro (2009)
Steady state stabilisation rate	g_{A2}	0.01	Calibration
Std. dev. of TFP Brownian motion	σ_A	0.013	van den Bremer and van der Ploeg (2021)
Initial population	L_0	7.794	UN (2019)
Population growth rate	g_L	0.0042	UN (2019)
Output elasticity of capital	α	0.3	Standard
Initial dirty capital	$K_{d,0}$	28	Calibration
Initial clean capital	$K_{c,0}$	320	Calibration
Depreciation rate	δ_d,δ_c	0.04	Calibration
Initial carbon intensity	ψ_0	2	Calibration
ψ decline rate	g_ψ	0.01	Calibration
Adjustment cost parameter	χ_d,χ_c	0.1	van der Ploeg and Rezai (2020a)
Std. dev. of k_d and k_c Brownian motions	σ_d,σ_c	0.013	van den Bremer and van der Ploeg (2021)
Correlation between k_d and k_c Brownian motions	ρ_k	0.5	By assumption
Size of macro disasters without climate	ξ_1	0.2	Calibration
Likelihood of a macro disaster	λ_A	0.017	Barro (2009)
Climate and emissions			
Temperature increase in 2020	T_0	1.2°C	IPCC (2021)
TCRE	ζ	0.0006	IPCC (2021)
Climate damage function parameter	γ	0.0077	Howard and Sterner (2017)
Std. dev. of T Brownian motion	σ_T	0.03	IPCC (2021)
Dependence of macro disasters on T	ξ_2	0.07	Calibration
Abatement, disinvestment and learning			
ϕ initial value	ϕ_0	0.00252	Calibration
ϕ decline rate	g_ϕ	0.69	Calibration
φ initial value	φ_0	0.00004	Estimation
φ decline rate	g_φ	0.69	Estimation
Disinvestment cost function parameter	θ_1	705	Estimation
Disinvestment cost function exponent	θ_2	2.1	By assumption
Exogenous learning weight parameter	ω	0.5	Calibration
Cumulative abatement at t_0	M_0	100	UNEP (2021)
Elasticity of MAC to cumulative abatement	ϵ	0.1	Estimation

Appendix C Variable transformations and equations in the CompEcon toolbox

The value function is estimated over a five-dimensional 'rectangular' grid of 1600 points or nodes. The value function is approximated by a complete Chebyshev polynomial, containing 232 basis functions.[36]

We use a transformation of variables to obtain a smoother function and a more stable solution. Instead of time, we use 'synthetic' time $\tau = 1 - e^{-g_\tau t}$, a transformed variable with a steady state. The transformation gives more precision for the initial decades, and less for the centuries after 2100.

The states k_d and k_c have two computational disadvantages. First, they interact strongly in determining their contribution to the value function because the marginal rate of substitution depends largely on the level of dirty capital. These strong interaction effects make the functional form of the value function more complicated. Second, k_d interacts strongly with the time state due to the time-dependent emissions intensity of the economy. For example the node on the grid corresponding to high k_d far in the future will have emissions above BAU (although E/K_d decreases over time, E/k_d increases over time). This node will have a strong curvature, while being useless to estimate an optimal path. Therefore, we use emissions $E = k_d L_0 \psi_0 (1+\tau)$ and total capital $k = k_d + k_c$ as transformed states in a value function $\tilde{V}(E, k, S, \tilde{A}, \tau)$.

Differentiating the new variables gives

$$dE = dk_d L_0 \psi_0 (1+\tau) + k_d L_0 \psi_0 d\tau, \tag{40}$$

$$dk = dk_d + dk_c. \tag{41}$$

To sum up, consumption per unit of effective labour is now

$$c\left(E, k, S, \tilde{A}, \tau\right) = \tilde{A} k^\alpha e^{-\frac{\varphi(\tau,S)}{2}(\phi_0/\phi_0 - E)^2 - \frac{\gamma}{2}\zeta^2 S^2} - i_d - i_c - \kappa(i_d), \tag{42}$$

with the slope of the marginal abatement curve decreasing over time as follows:

$$\varphi(\tau, S) = \varphi_0 \left[\omega M^{-\epsilon} + (1-\omega)\frac{1}{1+g_\varphi \tau}\right], \tag{43}$$

[36] Chebyshev polynomials are linear combinations of cosinus functions with different frequencies, which can also be written as power functions. Our Chebyshev polynomial can be thought of as a polynomial containing 232 combinations of states where the sum of all powers is maximum 5 (for example $S_1^2 * S_2^2 * S_3$).

and

$$M = \frac{M_0 + \phi_0/\phi_0 \left(-g_\tau^{-1} \ln(1-\tau)\right) - (S - S_0)}{M_0}, \tag{44}$$

subject to the following equations of motion:

$$\begin{aligned}
dE &= \left[\left(i_d - \frac{\chi i_d^2}{k}\right) L_0\psi_0 \left(1+\tau\right) - \left(\delta + g_L + g\right) E\right] dt + E\sigma_d dW_d + \frac{E}{1+\tau}\left(1-\tau\right) g_\tau dt, \\
dk &= \left[i_d + i_c - \chi \frac{i_d^2 + i_c^2}{k} - (\delta + g_L + g)k\right] dt, \\
&\quad + \underbrace{\frac{E}{L_0\psi_0\left(1+\tau\right)}}_{k_d} \sigma_d dW_d + \left(\underbrace{k - \frac{E}{L_0\psi_0\left(1+\tau\right)}}_{k_c}\right) \sigma_c dW_c, \\
dS &= Edt + S\sigma_S dW_S, \\
d\tilde{A} &= \left(g_{A1}\tilde{A} + g_{A2}\left(A_0 - \tilde{A}\right)\right) dt + \sigma_A \tilde{A} dW_A - \xi_1 \left(1 + \xi_2 \zeta S\right) \tilde{A} dP, \\
d\tau &= \left(1-\tau\right) g_\tau dt.
\end{aligned}$$

To convert the derivatives for the carbon price decomposition we have

$$V_{Sk_d} = \tilde{V}_{Sk} \underbrace{1}_{\partial k/\partial k_d} + \tilde{V}_{SE} \underbrace{L_0\psi_0\left(1-\tau\right)}_{\partial E/\partial k_d}, \tag{45}$$

$$V_{Sk_c} = \tilde{V}_{Sk}, \tag{46}$$

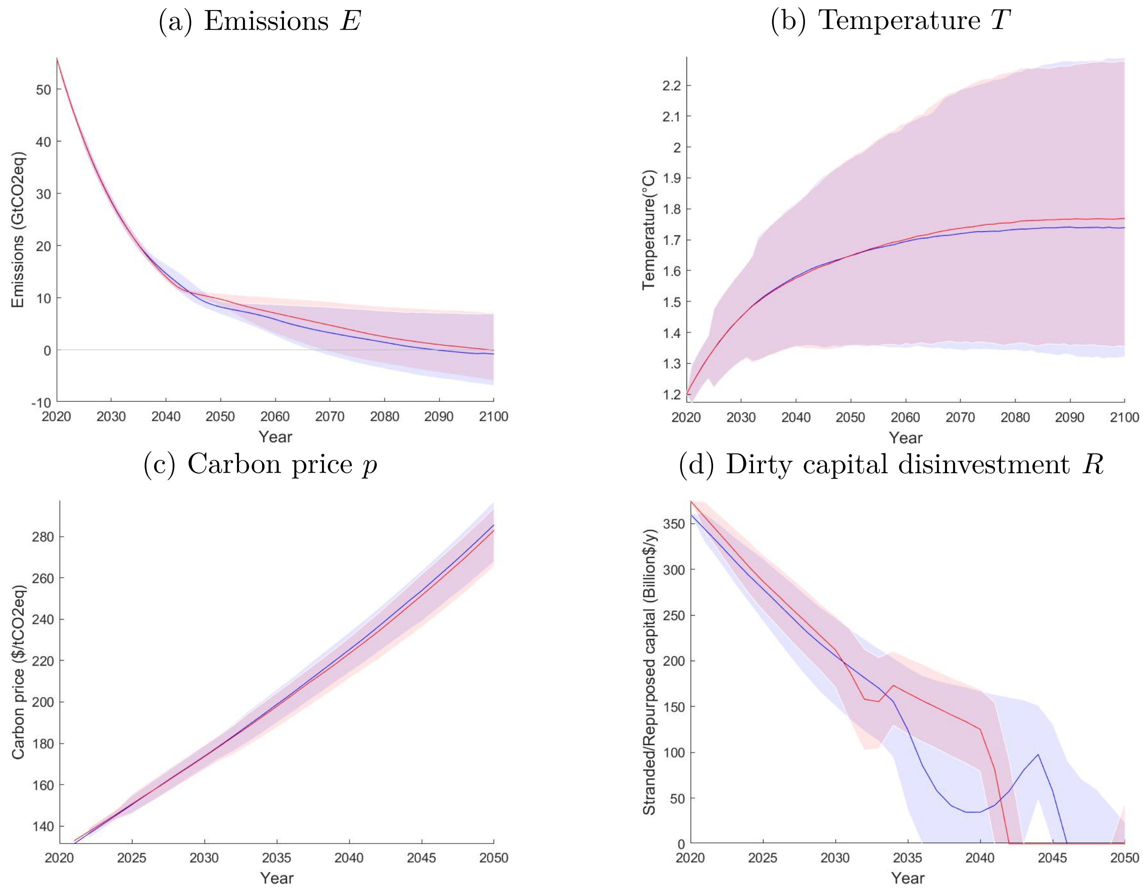
$$V_{St} = \left(\tilde{V}_{S\tau} + \tilde{V}_{SE} \underbrace{\frac{E}{1+\tau}}_{\partial E/\partial kd}\right) \left(1-\tau\right) g_{S\tau}. \tag{47}$$

The HJB equation is solved using a discrete time step of one year. The differential equations are approximated by $dX \cong X_t - X_{t-1}$, except for synthetic time where an exact solution is available, $\tau_{t+1} = 1 - \left(1-\tau_t\right) e^{-g_\tau}$, and for emissions where the relationship between emissions and dirty capital is linear, $E_{t+1} = k_{d_{t+1}} L_0\psi_0 \left(1 + \tau_{t+1}\right)$.

We use the CompEcon Toolbox (Miranda and Fackler, 2002) to solve the model, and adapt the dpsolve command to include Epstein-Zin preferences. The CompEcon Toolbox applies a projection method where the value function is approximated by a polynomial, collocated on Chebyshev nodes. We adapt the toolbox to introduce complete Chebychev polynomials (Cai, 2019). Complete Chebyshev polynomials perform better than the tensor-product Chebyshev polynomials, because higher order terms oscillate too much and lead to non-convergence of the algorithm. The polynomial is fitted to have a minimal squared deviation from the estimated value function on our 1600 grid points.

In our main specification, we use a 5-dimensional grid with 5, 4, 4, 4 and 5 Chebyshev nodes, respectively. This results in a grid with 1600 nodes. Our Chebyshev polynomial has 232 terms, which is less than the number of nodes because we leave out terms with a degree (sum of all powers) higher than 5. Figure A1 shows very similar results for a solution with 7776 nodes compared to our baseline solution with 1600 nodes.

Figure A1: Optimal transition paths comparing our baseline solution on a grid with 1600 nodes (red) with a solution using a finer grid with 7776 nodes, i.e. 6 nodes in each dimension (blue). The planner's objective is to maximise welfare. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.



The initial guess for the value function is based on a linear-quadratic approximation of the problem. Next, we use value function iteration to fit the polynomial to the HJB equation. We do this for our model versions that converge most easily (cost-benefit, expected utility, only exogenous technical change). Finally, we start with the outcome of this model and gradually change parameters to solve more complicated models (cost-effectiveness, Epstein-Zin, endogenous technical change, etc.). These gradual parameter updates are possible with the Newton algorithm in the Toolbox, which converges substantially faster, but requires a better initial guess.

The most difficult aspect for convergence of the algorithm is the fact that the shocks can bring the value of the state variable in the next period outside the grid, where polynomials give notoriously bad approximations. For example, with a 1.7% probability of a disaster in any given year, there is a 0.03% probability that a disaster is followed by another disaster, which brings TFP to 64% of its steady state value. We shrink the size of repeated shocks such that TFP does not decrease below this level of 64%. Similarly, we shrink the size of brownian motions at the highest node of emissions and cumulative emissions. These nodes are far from the optimal path. We also introduce bounds on both decision variables in case repeated shocks in the Monte Carlo simulation bring one of the state variables out of the grid.[37] These bounds, which are not binding within optimal regions of the grid also help for convergence of the HJB equation.

The Compecon Toolbox discretises the multidimensional probability density function of the Brownian Motions and jumps (our shocks on dirty and clean capital are correlated). The right hand side of the HJB equation is therefore a sum of stochastic realisations. We adapted the Compecon Toolbox to calculate each realisation in parallel using the multi-core 'parfor' loop in Matlab.

[37] We adapted the CompEcon command dpsimul to that purpose.

Appendix D Optimal carbon price decomposition in different model variants

We repeated the optimal carbon price decomposition for the model versions that omit capital inertia, clean technological progress, and uncertainty, as well as the 'straw man' model. Table A2 reports differences in the share of the optimal carbon price attributable to each element, relative to the full model. A value of -10 in the table means the share of the optimal carbon price is 10 percentage points lower, for example. We compare the difference in shares in 2021 and 2050.

In the model version that omits adjustment costs, the temperature path is flatter, because emissions can instantaneously jump downwards. This intuitively results in a much smaller contribution to the optimal carbon price in 2021 from the future increase in marginal damages. Conversely, the technological change effect is much larger because the large initial abatement leads to rapid technological change. These effects are short-lived, as the 2050 comparison shows. The optimal carbon price with MACs fixed at their 2020 level naturally has a smaller technological change effect, but larger contributions from marginal damages, particularly future, given the higher temperature path. The model version with no uncertainty has no disaster risk premium by definition, while the straw man model shares features of the versions omitting adjustment costs and uncertainty, namely a much smaller contribution from the future increase in marginal damage initially, and no disaster risk premium.

Table A2: Differences in the share of the optimal carbon price attributable to each element, relative to the full model. Units are percentage points.

	No capital inertia	Fixed clean tech. costs	No uncertainty	Straw man
		2021		
Current marginal damage	-4.9	6.5	15.1	19.5
Future increase marginal damage	-22.7	17.1	27.9	-0.3
Technological change effect	19.3	-8.5	-3.8	-10.2
Clean capital expansion effect	0.4	0.8	0.3	1.0
Dirty capital contraction effect	10.0	-22.1	-21.5	8.0
Disaster risk premium	-1.8	5.8	-16.9	-16.9
		2050		
Current marginal damage	-2.6	-5.0	3.6	5.0
Future increase marginal damage	2.7	7.4	20.4	21.8
Technological change effect	-0.6	-5.9	-2.2	-5.2
Clean capital expansion effect	-0.1	0.8	-0.2	-0.6
Dirty capital contraction effect	-0.4	-1.1	-2.1	-1.5
Disaster risk premium	0.6	1.3	-20.1	-20.7

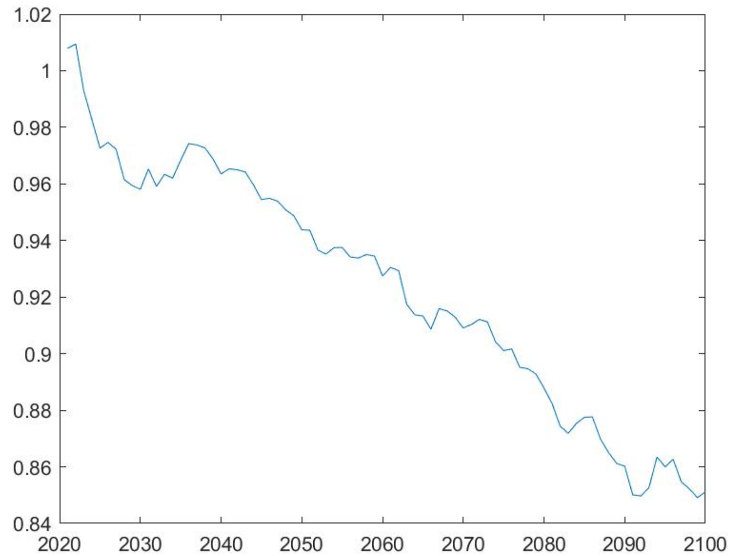
Appendix E The climate beta and the effect of risk aversion

Figure A2 plots the climate beta over this century, as estimated by our model. The climate beta (Dietz et al., 2018) may be calculated as

$$\beta_t = \frac{\mathrm{Cov}\left(\ln c_t, \ln D_{T_t}\right)}{\mathrm{Var}\left(\ln c_t\right)},$$

where marginal damage in our model $D_{T_t} = Y_t \zeta^2 \gamma T_t$. The climate beta is close to one throughout.

Figure A2: The climate beta



To investigate the effect of risk aversion we compare two models with relative risk aversion (RRA) of 1.35 and 6 respectively. The low value of RRA equals the intertemporal inequality aversion and corresponds therefore to expected utility. Increasing the risk averseness gives an insight in the effect of mean-preserving uncertainty. In the limit case where RRA converges to zero, uncertainty is not priced, only expected damages matter. RRA equals 4 in our main analysis. Figure A3 shows that increasing risk aversion has only a mild effect on carbon prices and emission trajectories.

Figure A3: Optimal transition paths comparing a low relative risk aversion of 1.35 (red) with a high relative risk aversion of 6 (blue). The planner's objective is to maximise welfare. The optimal path depends on the stochastic realisations along the path. The solid lines represent the mean of 5000 Monte Carlo runs. The shaded areas represent the 10-90% confidence interval.