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Heterogeneity in Monopo-
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On the Effects of Income Heterogeneity in Monopolistically Competitive Markets

Abstract

This paper studies the market and welfare effects of income heterogeneity in monopolistically competitive product markets in the context of nonhomothetic preferences. In a closed economy, where richer individuals' expenditures are less sensitive to price change compared to poorer ones', a mean-preserving contraction of income distribution entices firms to charge higher markups, reduce output, and fosters creation of new varieties. General equilibrium effects have a negative impact on poorer individuals and, in specific circumstances, on the whole population. In an open economy with free trade, lower income inequality in one country creates price divergence between trading countries. Lower inequality not only further decreases trade volumes and values but also creates a general equilibrium effect that may negatively affect poor individuals. Finally, general equilibrium effects are shown to be quantitatively nonnegligible.

JEL-Codes: D430, L160, F120.

Keywords: monopolistic competition, nonhomothetic additive preferences, income inequality, pricing-to-markets, welfare, trade.

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1 Introduction

The current era of globalization has heralded dramatic increases in income inequality, reaching historic highs in some countries. While the rise in the number of billionaires and workers in the 'gig economy' has increased income spread, many governments have considered implementing a wide variety of redistributive policies to mitigate income inequality. What implications do these trends and policies have for product markets and individuals' well-being? How strong are these effects in closed and open economies?

These recent trends and policies regarding inequalities justify accounting for income heterogeneity as one of the most important features that economists should bring to their research agenda, especially in the context of monopolistic competitive markets. Indeed, income discrepancies are likely to affect product prices and product diversity in such markets. The reason is that income redistribution is expected to alter the elasticities of the product demand addressed to firms, which shall accordingly change their markup, output and entry decisions.

In this paper, we show that income heterogeneity matters for the outcomes of imperfect product markets and consumer welfare. In particular, we discuss the effects of income distribution on prices, markups, product diversity and firm output in the context of closed and open economies. We also investigate how such changes affect the welfare of different income groups and assess the quantitative importance of those effects. To the best of our knowledge, these effects have not been discussed extensively in the literature.¹

A detailed discussion of the above questions is important for the following reasons. First, it highlights the limitations of the representative consumer approach, as market outcomes depend not only on average income but also on the entire income distribution. Second, it shows how firms make their price and entry decisions when income distribution impacts their demand elasticity and market power. Third, it qualifies redistributive policy recommendations because general equilibrium effects may accentuate welfare gaps between income groups.

To address these questions, we rely on a general equilibrium model in which individuals have nonhomothetic additive preferences and heterogeneous incomes. Individuals consume a set of varieties produced by a monopolistically competitive sector in which they work. The effects of income inequality on markups, product diversity, trade structure and individual welfare strongly depend on the properties of such preferences. Therefore, to clarify the direction of general equilibrium effects, we focus on additive preferences such that individual demand elasticity decreases and love for variety rises with consumption. This assumption combines the conditions for demand subconvexity (Mrazova and Neary, 2017) and aligned preferences (Dhingra and Morrow, 2019). Demand subconvexity matches Marshall's Second Law of Demand according to which demand elasticities are high for goods with high prices and low consumption (Marshall 1936, bk. 3, chap. 4, sec. 2), and corresponds to the empirical fact that markups decrease with market size (Syverson, 2007; De Loecker *et al.*, 2016).

¹Tarasov (2009) addresses similar questions but within a very different framework with two income groups and indivisible goods as in Matsuyama (2000).

Increasing love for variety is claimed to be the most plausible consumer behavior by Vives (2001, ch. 6).

We set the stage by studying the effect of income distribution in a closed economy. We first formally show that market prices, markups, and firm sizes are independent of income distribution under preferences described by Pollak (1971). Such preferences encompass commonly used specifications like the generalized constant elasticity of substitution (CES), quadratic, constant absolute risk aversion (CARA) and logarithmic utility functions. Under these preferences, individual demands are locally linear in income so that any mean-preserving income redistribution reshuffles individual consumption in a way that each firm's demand and, therefore, its elasticity remains unchanged. As a consequence, markups, output and entry decisions are unaffected. We, however, show that the combination of Pollak preferences and monopolistic competition extends this result to arbitrary changes in income distribution, including changes in average and total income. Indeed, in the context of Pollak preferences and monopolistic competition, a rise in average income pushes the firms' demands up and, therefore, raises their markups and profits. The latter entices new firms to enter until markups return *exactly* to their initial levels. As a result, markups are unrelated to the characteristics of income distribution. The same conclusion holds for firm output. By contrast, the number of entrants completely absorbs the changes in average and total income.

However, the application of Pollak preferences in economic modeling raises several issues. First, these preferences are hardly supported by the data. Indeed, it is well known that the unit income elasticity of CES preferences is not empirically confirmed for many goods (Houthakker, 1957; Samuelson and Nordhaus, 2010, p. 93). Additionally, Pollak preferences do not support any correlation between income redistribution and product diversity, which contradicts empirical findings (e.g., Falkinger and Zweimüller, 1996). Second, starting from Murphy *et al.* (1989), the macroeconomic literature strongly emphasizes the link between income inequality and total demand for manufacturing products. Such a relationship is absent under Pollak preferences. Finally, the latter relationship is also relevant in the trade context as empirical studies show a dependency between the demand for export goods and countries' levels of income inequality (Choi *et al.*, 2009; Dalgin *et al.*, 2008). This motivates us to study preference classes beyond Pollak.

To this end, we explore the properties of preferences for which a change in income distribution induces variations in prices, markups, product diversity, and individual welfare. Beyond Pollak preferences, income redistribution changes both the level and curvature of each individual's demand in a way that actually alters the demand curvature and elasticity of every firm. As a consequence, firms are enticed to change their markup and output. We show that the direction of the effect hinges on the behavior of the convexity of individuals' (direct) demand function.² When this convexity is an increasing function of consumption, the individual expenditure of low-income consumers is more sensitive to price changes than is that of high-income consumers. Such a case is consistent with Bekkers *et al.*'s (2012) empirical obser-

²The property of demand function convexity is also an important factor determining the welfare gains or losses of price discrimination (Aguirre *et al.*, 2010).

vation that prices decrease with rising income inequality. In this case, as prices and markups move in the same direction, a redistribution policy that implements a mean-preserving contraction of income distribution increases equilibrium markups and fosters creation of new varieties.³ The opposite holds for a decreasing convexity of demand or a mean-preserving spread. While the latter result has been known since Foellmi and Zweimueller (2004), we further contribute to the literature in several broad directions: welfare implications, consequences for trade patterns, and quantification. We provide details on each direction below.

First, we present an intuition for these properties and further investigate the welfare effects of income inequality. For instance, under an increasing convexity of demand, changes in the product market generate negative general equilibrium effects on the welfare of poor individuals and may also harm richer ones. Under these circumstances, a policy targeting the lowest income decile leads to welfare losses for the untargeted poor. We also investigate particular classes of additive utility functions used in the literature. We show that the property of increasing convexity of demand depends not only on each particular class of utility function but also on its parametrization. This is of particular importance for the quantification exercises that would pursue to assess the economic impact of income redistribution.

Second, we discuss the effects of income distribution in open economies and shed light on the consequences of a country's income inequality for trade patterns and foreign markets. Indeed, the questions under consideration are also highly relevant in international trade contexts. Since Jones (1965), researchers have studied the impact of trade patterns on various income groups. However, the literature is limited regarding the reverse effect of consumer heterogeneity on trade patterns.⁴ For instance, income redistribution within one country may affect the markups and entry decisions of firms in other countries and may also raise or reduce individuals' well-being in those countries.

To address these questions, we extend our setting to an open economy where countries freely trade their products. We show that mean-preserving change in one country's income distribution does not affect markups and output for all Pollak preferences. For non-Pollak preferences with an increasing convexity of demand, mean-preserving contraction leads to higher markups and broader product diversity in the local market. In the other country, both local and imported good prices (and markups) decrease while product diversity expands. Therefore, we contribute to the literature by showing that international price divergence can stem from discrepancies in income inequality, rather than from the presence of trade costs and/or home market bias. Due to free trade, price divergence here is solely driven by individual income heterogeneity. Next, each country's export volume and value as well as its total trade volume fall with a reduction in income inequality in one of the countries. The opposite results hold for mean-preserving spread or a decreasing convexity of demand.

Finally, local income redistribution policies have welfare effects on other countries. For in-

³Such a policy typically reflects the effect of progressive tax redistribution.

⁴There exists a small body of literature on vertical differentiation and trade that deals with income heterogeneity (Flam and Helpman (1987) and followers). See Hsu *et al.* (2022) for an analysis of trade patterns with two income levels and heterogeneous firms.

stance, all residents of the foreign country gain from the mean-preserving contraction of the domestic income distribution under an increasing convexity of demand. In the home country, the utility of poor individuals may, however, drop. Again, poorer individuals are more negatively affected by general equilibrium effects than richer individuals. These findings show how variation in a country's income inequality shapes the trade patterns and welfare levels of its trading partners.

We also find that Pollak preferences are no longer sufficient for maintaining markup invariance after a change in average income in one of the countries. In particular, markups do not vary only for CES preferences. This is a consequence of market segmentation, which takes place even under free trade. For all other additive preferences, domestic markups increase with the country's average income, whereas they fall in the other country under increasing demand convexity. This generalizes Simonovska's (2015) finding on price divergence in a framework with the additive logarithmic utility encompassed in our study. In addition, while firms in both countries increase (decrease) sales in the richer (poorer) country, their output does not change. Finally, total trade value increases (decreases) if the average income of a country increases (decreases).⁵

Ultimately, we propose a quantification exercise calibrated to the US economy. In this exercise, we retain the preference classes compatible with empirical estimates of the elasticities of demand and pass-through provided in the literature. Despite these constraints on preferences and parameters, the exercise supports demand functions with both increasing and decreasing demand convexity and, therefore, allows us to present cases with opposite general equilibrium effects. We then study the effect of a redistributive transfer from the top to the bottom income decile. We show that general equilibrium effects are quantitatively nonnegligible in both closed and open economies. For instance, in a closed economy, a transfer involving 1.5% of total US income changes production and entry by approximately 2% while altering markups by 0.3% and changing welfare by up to 0.3% (as measured by equivalent consumption). Furthermore, the exercise underscores the relationship between pass-through elasticity and the direction of general equilibrium effects. For instance, a low (high) pass-through elasticity corresponds to an increasing (decreasing) convexity of demand. Thus, for low pass-through elasticity, the above redistribution policy increases markups, prices and variety but harms the bottom nine income deciles as the latter are more sensitive to price changes. This result is overturned with high pass-through elasticity. Finally, domestic redistribution policies affect other countries through trade. In particular, income redistribution in a country significantly affects markups, outputs, individual welfare and import-export values in both countries. Again, these changes are of similar magnitudes in both countries. Yet, redistribution at home can harm or benefit all deciles in both countries depending on the pass-through elasticity. This discussion shows that within-country redistribution poli-

⁵This property is not aligned with the Linder hypothesis, which postulates that total trade values are diminished by any 'dissimilarity' in income patterns. Accordingly, both an increase and a decrease in average income in one country makes the two countries less similar and should therefore decrease total trade value. This is not the case in our setting.

cies substantially impact firms' choices regarding pricing and production, trade flows, and, ultimately, well-being of both local and trade partners' residents.

Literature review. This paper relates to several strands of literature. First, it is linked to the literature studying product markets in the monopolistic competition framework with additively separable preferences (Spence, 1976; Dixit and Stiglitz, 1977; Kuhn and Vives, 1999; Foellmi and Zweimüller, 2004) and with applications to trading countries (Neary, 2004; Zhelobodko *et al.*, 2012, Kichko *et al.*, 2014, among others). The paper also revisits a subset of the demand structures proposed in Bulow and Pfleiderer (1983), Mrazova and Neary (2017, 2019), Mrazova *et al.* (2021) and Nakamura and Zerom (2010). Among them are demand functions with constant superelasticity, translog, constant proportional pass-through and constant elasticity of marginal revenue. This paper shows that those demand structures yield contrasting properties of the convexity of demand and therefore lead to opposite conclusions about the general equilibrium effects of income heterogeneity on product markets and welfare. Such contrasting effects may even take place within the same class of preferences for different parameterizations. Finally, the convexity of demand plays a key role in third-degree price discrimination (Aguirre *et al.*, 2010; Cowan, 2012; Holmes, 1989). In contrast to our paper, the partial equilibrium literature shows that the properties of this convexity shape the welfare and output effects of market segmentation.

Second, there has been a long discussion on the impact of income inequality on aggregate demand through marginal propensities to consume (see Pigou, 1920; Keynes, 1936). Although a strand of this literature emphasizes the independence of aggregate demand from income distribution (Friedman, 1957, and followers), another finds a negative relationship between demand and income inequality (Dynan *et al.*, 2004). In this paper, we uncover a very different mechanism that relates income inequality to aggregate demand through the entry/exit of firms into/from the market. To be precise, if poor individuals' expenditures are more sensitive to price changes, then an increase in income inequality leads to a higher aggregate demand for each variety. When the income of poor individuals falls, firm revenues become more sensitive to prices, which entices firms to set lower prices. This situation pushes a fraction of firms out of the market. As a result, surviving firms increase their level of production.

Third, this paper relates to the trade literature devoted to income heterogeneity. The first set of papers relies on a monopolistic competition framework to investigate the impact of trade liberalization on within-country income inequality. For example, Yeaple (2005) shows how trade widens the income gap between skilled and unskilled workers. Trade liberalization increases the skill premium paid by exporting firms using "high-tech" technologies in the context of a workforce with heterogeneous skills. Egger and Kreickemeier (2009), Helpman *et al.* (2010) and Felbermayr *et al.* (2011) explain the rise of within-country income inequality after trade liberalization through labor market imperfections and the presence of unemployment. Another set of papers discusses the effect of between-country income heterogeneity on trade patterns. Fieler (2011) encompasses both per-capita income inequality

and size differences in a Ricardian model with CES preferences and discusses their impact on trade flows. Using a Ricardian framework, Matsuyama (2000) studies the impact of income redistribution within a country on the wages and well-being of residents in both countries. Bertolotti *et al.* (2018) study trade patterns in the context of countries with heterogeneous per-capita incomes and preferences with income effects. Behrens and Murata (2012) contribute to both sets of papers, as they show that the impact of trade liberalization on the distribution of individual welfare depends on each country's relative per-capita income. This study is close to our paper, as it assumes within-country income heterogeneity. However, because this paper discusses CARA preferences, which belong to the Pollak class, market and trade properties hinge only on countries' average income and their relative position in the global income distribution. We deviate from these two strands of trade literature by studying the role of within-country income distribution in market and welfare outcomes within both countries.

The rest of the paper is organized as follows. Section 2 develops the baseline model and identifies the equilibrium in a closed economy. Section 3 studies the impact of income redistribution on market outcome and welfare, while Section 4 extends the framework to the case of two countries. Section 5 quantifies the general equilibrium effects for different demand systems. Section 6 concludes.

2 Model

The economy includes a mass L of individuals. Each individual h is endowed with $s_h > 0$ labor units, which are distributed with a cumulative distribution function $G : [s_0, s_1] \rightarrow [0, 1]$, where $0 < s_0 < s_1$ and $G' > 0$. We initially normalize the wage per labor unit to one, so that s_h stands for individual h income. In Section 5, we relax this normalization. When it does not lead to confusion, we denote the integral over individuals' income $\int_{s_0}^{s_1} dG(s_h)$ as $\int dG$; that is, we omit the integration boundaries and references to income s_h . The average individual income is then given by $s = \int s_h dG$. In what follows, a variable without subscript h denotes its average over individual incomes.

2.1 Demands

Individuals consume a set of varieties, $\omega \in [0, n]$, where n denotes the endogenous number of varieties (product diversity). Each individual with income s_h is endowed with an additive-separable utility

$$U = \int_0^n u(x_h(\omega)) d\omega$$

which she maximizes subject to her budget constraint $\int_0^n p(\omega)x_h(\omega)d\omega = s_h$, where $p(\omega)$ and $x_h(\omega)$ are the price and her consumption of variety ω , respectively. The utility function is increasing and concave, $u''(x_h) < 0 < u'(x_h)$. We assume that the lowest income s_0 is large enough to ensure positive equilibrium consumption for each available variety. This assump-

tion ensures that equilibrium prices lie below the demand choke prices.⁶ The first-order condition yields the inverse demand function $p(\omega) = \lambda_h^{-1} u'(x_h(\omega))$, where λ_h is the consumer's budget constraint multiplier. For the sake of clarity, we temporarily drop the reference to ω and write individual demand as

$$x_h \equiv v(\lambda_h p), \quad (1)$$

where v is the inverse function of $u'(x_h)$, which decreases with its argument.

Demand side statistics. This paper highlights the role of three statistics of the demand side. The first is the price elasticity of the individual's demand given by

$$\varepsilon(x_h) \equiv -\frac{d \ln x_h}{d \ln p} = -\frac{\lambda_h p v'(\lambda_h p)}{v(\lambda_h p)} = -\frac{u'(x_h)}{x_h u''(x_h)} > 0, \quad (2)$$

which we refer to as *individual demand elasticity*. For conciseness, we denote its value for an individual with consumption x_h as $\varepsilon_h \equiv \varepsilon(x_h)$.

In this paper, we concentrate on *subconvex* demand functions, which are characterized by decreasing demand elasticity: $\varepsilon'_h \equiv \varepsilon'(x_h) < 0$ (see Mrazova and Neary, 2017). Subconvex demands feature the inverse relationship between average consumption and demand elasticity. As mentioned in the introduction, this assumption is congruent with the empirical literature (Syverson, 2007; De Loecker *et al.*, 2016). It corresponds to Marshall's Second Law of Demand (1936), which states that demand becomes less elastic at higher prices. It finally matches Mion and Jacob's (2020) empirical findings about pass-through.⁷ Differentiating expression (2) reveals that individual demand is subconvex if and only if

$$\varepsilon'_h = -\frac{1}{x_h} (1 + \varepsilon_h - r_h) < 0, \quad (3)$$

where $r_h \equiv r(x_h)$ is the second statistic of interest with

$$r_h \equiv -\frac{d \ln v'(\lambda_h p)}{d \ln p} = -\frac{\lambda_h p v''(\lambda_h p)}{v'(\lambda_h p)} = \frac{u'(x_h) u'''(x_h)}{(u''(x_h))^2}. \quad (4)$$

Many results in this paper hinge on the behavior of this statistic, which measures the *convexity of the individual demand function* (Aguire *et al.*, 2010; Mrazova and Neary, 2017).

Finally, we define the statistic for the love for variety as $1 - \eta_h$ where

$$\eta_h \equiv \eta(x_h) = \frac{x_h u'(x_h)}{u(x_h)} \in (0, 1) \quad (5)$$

is the elasticity of the utility function defined in Dixit and Stiglitz (1977). This represents the degree of preference for variety as the proportion of social surplus not captured by revenues (Vives, 2001). Because u is concave and increasing, η_h lies between 0 and 1. The index $1 - \eta_h$

⁶It also leads to the same properties as those of the Inada condition. However, we do not impose the latter restriction to encompass the broader set of demands studied in the literature (see Section 3).

⁷It can be shown that, for low enough levels of income heterogeneity, subconvex demand functions generate a decreasing elasticity of pass-through. This is confirmed by Mion and Jacob (2020) using French data.

is equal to zero in the absence of love for variety (because utility u is linear) and rises to one as the latter becomes stronger. As explained in Vives (2001), $1 - \eta_h$ measures the preference for variety, namely, the utility gain from increasing variety while holding the quantity fixed. This statistic plays an important role in consumption behavior and welfare assessment. In this paper, we assume that individuals are more sensitive to product diversity when they consume more, i.e., $\eta'(x_h) < 0$. This situation is considered more plausible in economic theory (Vives, 2001). Combined with subconvex demand, this assumption is congruent with Dhingra and Morrow's (2019) definition of "aligned preferences" according to which individual demand elasticity ε_h and the elasticity of utility η_h move in the same direction. Hence, in this paper, individuals become both less sensitive to prices and more sensitive to product diversity when they increase their consumption.⁸

2.2 Firms

Labor is the only production factor. Each firm produces a single variety ω and finds the price $p(\omega)$ that maximizes its profit $\pi(\omega) = L \int (p(\omega) - c)x_h(\omega)dG - f$. In this expression, c and f are the firm's marginal and fixed labor requirements which are common for all firms. Since demands are symmetric across varieties, we omit the reference to ω . Plugging the demand function (1) into profit and differentiating, we obtain the first-order condition for the producer problem:

$$\frac{d\pi}{dp} = (p - c) \int \lambda_h v'(\lambda_h p) dG + \int v(\lambda_h p) dG = 0.$$

After some algebra and using (1), we obtain the profit-maximizing price

$$p = \frac{\varepsilon}{\varepsilon - 1} c, \tag{6}$$

where

$$\varepsilon \equiv \frac{\int x_h \varepsilon_h dG}{\int x_h dG} \tag{7}$$

is *market demand elasticity*.⁹ Firm markup is given by $m \equiv (p - c)/p = 1/\varepsilon$. Therefore, both the price and markup decrease with higher market demand elasticity.

The second-order condition of the producer problem imposes

$$\frac{d^2\pi}{dp^2} = 2 \int \lambda_h v'(\lambda_h p) dG + (p - c) \int \lambda_h^2 v''(\lambda_h p) dG < 0.$$

⁸The literature has focused on benchmark preferences with constant elasticity of substitution (CES), defined by the utility function $u = x^{1-1/\sigma}$ with $\sigma > 1$ and yielding the three statistics $\varepsilon_h = \sigma$, $r_h = \sigma + 1$ and $\eta_h = 1 - 1/\sigma$, where $\sigma > 1$ is a constant. Hence, because $\eta_h \in (0, 1)$, individuals express love for variety. Additionally, because $\varepsilon'_h = 0$, the individual demand functions are neither sub- nor superconvex. As a result, subconvexity can be interpreted in reference to the CES demand functions: a demand function is subconvex at an arbitrary price and quantity if it is less convex at those levels than a CES demand function.

⁹The pricing rule for the monopolistic competition setting (6) has been known since Dixit and Stiglitz (1977). Market demand elasticity (7) is a weighted average on individual elasticities, which is a standard aggregation result under income inequality (e.g., Foellmi and Zweimüller, 2004).

Using the definitions of ε_h and r_h and plugging the optimal prices (6), this condition takes the following form:

$$\int (2\varepsilon - r_h)\varepsilon_h x_h dG > 0. \quad (8)$$

We make two remarks. First, in the absence of individual heterogeneity, $s_h = s$, consumption is homogenous, $x_h = x$, so condition (8) collapses to $r < 2\varepsilon$, as in Zhelobodko *et al.* (2012). Second, condition (8) is always satisfied when $r_h < 0$ for all values of h . When $r_h > 0$, Appendix A shows that (8) holds under $r'_h > 0$. Other configurations must be checked on a case-by-case basis.

2.3 Equilibrium

An equilibrium is defined as the set of consumption x_h , the price p , the number of firms n , and the firm output y that are consistent with the consumers' budget constraints

$$npx_h = s_h, \quad (9)$$

the firm's optimal price

$$p = \frac{\varepsilon}{\varepsilon - 1}c, \quad (10)$$

the zero-profit condition (free entry), the product and labor market clearing conditions

$$p = \frac{f}{y} + c, \quad y = L \int x_h dG, \quad L \int s_h dG = n(f + cy). \quad (11)$$

By the Walras law, one identity is redundant. We prove the following proposition in Appendix B.

Proposition 1. *Under subconvex demands, there exists a unique equilibrium if (8) holds and $\varepsilon(0) > 1$.*

3 Income distribution

The aim of this paper is to investigate the effects of income heterogeneity on product markets and welfare. To this aim, we first consider small changes in the distribution of individual income s_h and then extend the results for arbitrary changes in distribution.

Suppose that every individual with income s_h gets a new income $s_h + ds_h$ where ds_h is an infinitely small change in income. We denote the (relative) individual income changes as the mapping $\widehat{s}_h \equiv d \ln s_h = ds_h/s_h$. Broadly speaking, \widehat{s}_h measures the percentage change in income s_h of each individual. The change in the average income s is given by $\widehat{s} \equiv d \ln s = \frac{1}{s} \int s_h \widehat{s}_h dG$.¹⁰ This notation implies the following small changes in endogenous variables: $\widehat{x}_h = d \ln x_h$, $\widehat{m} = d \ln m$, $\widehat{p} = d \ln p$, $\widehat{y} = d \ln y$ and $\widehat{n} = d \ln n$. Thus, any income redistribution

¹⁰That is, $\widehat{s} \equiv d \ln (\int s_h dG) = [d (\int s_h dG)] / (\int s_h dG) = (\int ds_h dG) / s = \frac{1}{s} [\int s_h (ds_h/s_h) dG] = \frac{1}{s} \int s_h \widehat{s}_h dG$.

may be split into two transformations: (i) a *common proportional change in individual incomes*, with $\widehat{s}_h = \widehat{s}$, and (ii) a *mean-preserving change in individual incomes* where $\widehat{s} = 0$ and $\widehat{s}_h \neq 0$ for some s_h .

The log-linearization of the equilibrium conditions (9)-(11) yields (see Appendix C for details):

Budget	$\widehat{x}_h = \widehat{s}_h - \widehat{p} - \widehat{n}$
Pricing	$\widehat{p} = -\frac{\widehat{\varepsilon}}{\varepsilon-1}, \quad \widehat{\varepsilon} = -\frac{1}{\varepsilon x} \int (1 + \varepsilon - r_h) x_h \widehat{x}_h dG$
Entry	$\widehat{y} = -\varepsilon \widehat{p}$
Product market	$\widehat{y} = \frac{1}{x} \int x_h \widehat{x}_h dG$
Labor market	$\widehat{n} = \widehat{s} - \widehat{\varepsilon}$

Table 1: Changes in consumption, price, output and number of firms.

The first line shows that a rise in an individual's income raises her consumption x_h whereas higher prices and broader product diversity reduce it. The second line shows that changes in individual consumption have heterogeneous effects on firms' pricing through variation in firm demand elasticity. Income redistribution leads to changes in individual consumption \widehat{x}_h which results in a change $\widehat{\varepsilon}$ in each firm demand elasticity. This highlights the role of demand convexity statistics r_h in firm pricing. A rise in average income \widehat{s} inflates the labor supply and triggers the entry of new firms (last line), which then has a negative effect on the individual consumption of each good. Finally, the change in markup is given by $\widehat{m} = (\varepsilon - 1)\widehat{p}$ where $\varepsilon > 1$ by (10). Thus, markup and price vary in the same direction. The latter allows us to report our results in terms of price variation and to provide a discussion on both markup and price changes along the same lines.

Using Table 1, changes in consumption, output and number of firms can be expressed as functions of changes in individual income and price (see details in Appendix C):

$$\widehat{p} = -\frac{1}{\Psi \varepsilon} \int r_h (\widehat{s}_h - \widehat{s}) s_h dG, \quad (12)$$

$$\widehat{x}_h = (\widehat{s}_h - \widehat{s}) - \varepsilon \widehat{p}, \quad \widehat{y} = -\varepsilon \widehat{p}, \quad \widehat{n} = \widehat{s} + (\varepsilon - 1)\widehat{p}, \quad (13)$$

where

$$\Psi \equiv \int (2\varepsilon - r_h) s_h dG \quad (14)$$

is positive under subconvex demands (see (3)).

At equilibrium, the changes in price and markup depend on the change in firm demand elasticity through the difference between individual income \widehat{s}_h and average income \widehat{s} . Expression (12) again makes apparent the role of demand convexity r_h in price formation.

Note that, by (12) and (13), the effect of a common proportional change in income level, $\widehat{s}_h = \widehat{s}$, is given by

$$\widehat{p} = \widehat{m} = \widehat{x}_h = \widehat{y} = 0 \quad \text{and} \quad \widehat{n} = \widehat{s}. \quad (15)$$

In words, a common proportional change in income has no impact on prices and, therefore, on consumption and firm output. However, the number of firms varies proportionally to income change because total labor supply changes.¹¹ Total output, $Y \equiv ny$, and GDP, $G \equiv npy$, then also move in proportion to the change in labor supply \widehat{s} because $\widehat{Y} = \widehat{n} + \widehat{y} = \widehat{s}$ and $\widehat{G} = \widehat{p} + \widehat{n} + \widehat{y} = \widehat{s}$.

3.1 Invariance of prices to income distribution

In this subsection, we discuss the preferences under which prices, markups, and firm output are invariant to changes in individual income distribution, which includes both changes in average income and inequality. While firms' behavior is unaffected, they impact the number of entrants only.

These preferences are such that r_h is independent of each consumption level x_h and therefore income s_h . Indeed, under constant r_h , (12) remains equal to zero since $\int (\widehat{s}_h - \widehat{s}) s_h dG = \int ds_h dG - ds = 0$. That is, these preferences have the following property:

$$r_h = \sigma + 1, \quad (16)$$

where σ is a constant. Using the definition of $r = u'u''' / (u'')^2$, we solve the differential equation (16) to uniquely determine the utility functions that satisfy this condition as:¹²

$$u(x_h) = \begin{cases} x_h(\gamma - x_h) & \text{if } \sigma = -1, \\ 1 - e^{-(x_h - \gamma)} & \text{if } \sigma = 0, \\ \ln(x_h + \gamma) & \text{if } \sigma = 1, \\ \frac{\sigma}{\sigma - 1} (x_h + \gamma)^{\frac{\sigma - 1}{\sigma}} & \text{if } \sigma > 1. \end{cases} \quad (17)$$

where γ is a constant whose positivity ensures subconvexity of demands (see Appendix D for details). Each line denotes the quadratic utility function, the utility with constant absolute risk aversion (CARA), the logarithmic utility and finally the generalized constant elasticity of substitution (CES) utility, which collapses to a standard CES when $\gamma = 0$. Note first that for this class of utility functions, the second-order condition (8) reduces to $r < 2\varepsilon$.¹³

The utility functions (17) correspond to Pollak's (1971) demand functions which are *locally linear in income*. The latter implies that individual demand for each variety is linear in income at equilibrium prices, i.e., $x_h = A(p) + B(p) \cdot s_h$ where $A(p)$ and $B(p)$ are two functions

¹¹A common proportional change in income includes the special case of a change in per-capita income in a context of homogenous income. Hence, our analysis extends Zhelobodko *et al.*'s (2012) result about the absence of impact of per-capita income to a setting with heterogeneous income.

¹²Computing r_h for all utilities in (17) shows that r_h is a constant, i.e., common for all income levels.

¹³The latter holds for quadratic, CARA and logarithmic utility functions because $\varepsilon > 1$ and $r \leq 2$ in these cases. For the generalized CES, it also holds because $r = \sigma + 1 < \sigma = \varepsilon$ and $\sigma > 1$.

of prices p .¹⁴ The firm demand for each variety, $xL = [A(p) + B(p) \cdot s] L$, depends only on the average income and is otherwise independent of the income distribution. Therefore, a mean preservation of individual income distribution reshuffles individual consumption x_h in a way that does not change firm demand and, thus, firms do not change optimal prices.

Nevertheless, we have shown that *arbitrary* changes in distribution yield price invariance for Pollak preferences. This includes the changes that affect income averages. The income linearity property is itself not sufficient to obtain this result. Indeed, an increase in average income $\hat{s} > 0$ increases the average demand for each variety x . However, this rise is precisely compensated by additional entry. Thus, changes in average income are fully shifted onto product diversity so that $\hat{n} = \hat{s}$ (see (13)). This property stems from the balance among pricing, entry and firm output. A positive shock on average income simultaneously raises firms' output and prices so that it triggers entry until the market demand for each variety falls back to precisely its initial level. The zero-profit condition thus ensures price invariance. The property stems from the combination of Pollak preferences and monopolistic competition, in particular, free entry. The proportionality of product diversity to average income is a standard property of CES preferences and is shown here to apply to all Pollak preferences.

Proposition 2. *Markups, prices, and output are not affected by any changes in individual income distribution if and only if consumers are endowed with Pollak preferences, which include the generalized CES, quadratic, CARA, and logarithmic utility functions. Product diversity changes proportionally to average labor supply or average income.*

Note that beyond Pollak preferences, an increase in firms' output and markups is not precisely compensated by additional entry. The validity of Pollak preference has been empirically tested by checking income linearity in demand functions. Empirical works often report that income elasticities of the demand for commodities are significantly different from 1,¹⁵ which is *incompatible* with locally linear demand in income. This leads us to pay attention to other classes of demand functions.

3.2 Mean-preserving redistribution

Income redistribution policies often implement transfers across individuals under a government budget constraint. When transfers sum to zero, progressive income tax policies correspond to mean-preserving contractions in income distribution. In this subsection, we first establish how such policies affect market prices, markups, product diversity, and firm output in terms of the properties of our statistics r_h . We further provide an intuitive explanation for these effects and study properties of several classes of additive preferences used in the literature. Finally, we address the welfare effects of income distribution changes.

¹⁴Under this condition, preferences are homothetic with respect to a specific quantity profile $x_0(\omega) = x_0$ for all ω . See also Mrazova and Neary (2017) for a relationship between utility moments and Pollak preferences.

¹⁵Income elasticities range from 0.15 for urban residential water to 2.9 for cars (McCarthy, 1996). See a recent discussion based on trade data in Hummels and Lee (2018).

3.2.1 Market outcome

Here, we discuss the effect of a mean-preserving change in income distribution. Since the latter keeps average income s constant, we set $\hat{s} = 0$ in (12) and (13) and obtain

$$\hat{p} = -\frac{\int r_h s_h \hat{s}_h dG}{\varepsilon \Psi}, \quad (18)$$

$$\hat{x}_h = \hat{s}_h - \varepsilon \hat{p}, \quad \hat{y} = -\varepsilon \hat{p}, \quad \hat{n} = (\varepsilon - 1) \hat{p}. \quad (19)$$

The price change obviously depends on how r_h covaries with s_h and \hat{s}_h . Firm output and mass of firms also adjust following the price change, while the general equilibrium (GE) effect on consumption is captured by $-\varepsilon \hat{p}$. This is clarified in the following proposition (see Appendix E for the proof, which recasts Foellmi and Zweimüller's (2004) result).

Proposition 3. (Foellmi and Zweimüller, 2004) *Consider a mean-preserving contraction of income distribution. Then, the market outcome is described by the following three patterns:*

Variable	$r'_h > 0$	$r'_h = 0$	$r'_h < 0$
Price p and markup m	rise	constant	fall
Product diversity n	rises	constant	falls
Firm output y	falls	constant	rises
GE effect on consumption x	negative	null	positive

The opposite result holds for a mean-preserving spread.

Income redistribution leads to changes in individual consumption and, therefore, individual demand elasticity. As each firm's demand elasticity is an average of all individual demand elasticities, it also changes. As a consequence, firms alter their decisions on markups and output following profit maximization. Moreover, Proposition 3 shows that the direction of the effect of income redistribution depends on the sign of r'_h , which characterizes the increasing or decreasing pattern of the convexity of individual demand function. It is, however, more intuitive to relate r_h to the price sensitivities of consumer expenditure and firm revenue. Individual h 's expenditure is given by $pv(\lambda_h p)$, and its sensitivity with respect to price by

$$\frac{d}{dp} [pv(\lambda_h p)] = v(\lambda_h p) + p \lambda_h v'(\lambda_h p) = x_h - x_h \varepsilon_h \quad (20)$$

where the second equality stems from (2). The price sensitivity of firm revenue is given by

$$\frac{d}{dp} \int pv(\lambda_h p) dG = \int \frac{d}{dp} [pv(\lambda_h p)] dG = \int (x_h - x_h \varepsilon_h) dG = -x(\varepsilon - 1), \quad (21)$$

which aggregates the effect of prices on consumers' expenditures. The latter is negative at equilibrium. How does the price sensitivity of consumer expenditure vary with redistribution? The effect of an infinitesimally small transfer, Δx_h , on (20) is given by $(x_h - x_h \varepsilon_h)' \cdot \Delta x_h$.

Because $(x_h - x_h \varepsilon_h)' = 1 - \varepsilon_h - x_h \varepsilon_h' = 2 - r_h$, this effect takes the form of $(2 - r_h) \cdot \Delta x_h$. Its direction obviously depends on whether r_h rises or falls with income. For instance, if r_h rises with s_h , then the expenditures of individuals with lower income are more reactive to price changes. To keep things simple, consider the transfer from a mass of rich consumers h' to the same mass of poor consumers h : $\Delta s_h = -\Delta s_{h'} > 0$. Due to (9), we have $\Delta x_h = -\Delta x_{h'} > 0$. Then, the aggregate effect on the sensitivity to revenues is augmented by the amount $(r_{h'} - r_h) \cdot \Delta x_h$, which is positive if and only if r_h is an increasing function. In this case, the price sensitivity of revenue becomes less negative so that firm revenue becomes less sensitive to price change. As a consequence, firms raise their prices, as stated in Proposition 3.

Lower sensitivity of firm revenue to price corresponds to lower market demand elasticity, which increases firms' market power. The latter allows firms to charge higher markups and prices. This, in turn, invites new entrants to the product market so that product diversity expands. Finally, the business-stealing effect leads to a decrease in each firm's output.

In the opposite case, the price sensitivity of revenue becomes more negative with mean-preserving contraction, which makes firm revenue more sensitive to price change. This reduces firms' market power and entices them to charge lower markups.

The impact of redistribution on total output is $\widehat{Y} = \widehat{n} + \widehat{y} = -\widehat{p}$, which moves in the opposite direction to prices. However, GDP is not affected because $\widehat{G} = \widehat{p} + \widehat{n} + \widehat{y} = 0$. This is because GDP is the sum of individual labor supplies or incomes and is therefore unaffected by a mean-preserving change in individual incomes.

Proposition 3 applies for non-Pollak preferences, which do not exhibit a constant r_h' . Table 2 presents a set of inverse demand functions discussed in the literature and displaying subconvex demands. It includes (i) the demand with constant superelasticity of demand (CSED), defined by a constant value for $d \ln \varepsilon(x) / d \ln x$ (Gopinath and Itskhoki, 2010); (ii) an additive version of Feenstra's (2003) translog demand functions (TLOG), (iii) the demand function with constant revenue elasticity of marginal revenue (CREMR) (Mrazova *et al.*, 2021); (iv) demand with constant proportional pass-through (CPPT), defined by a constant value for $d \ln p / d \ln c$ (Mrazova *et al.*, 2017); (v) the demand with constant (output) elasticity of marginal revenue (CEMR) demands (Mrazova *et al.*, 2017); and (vi) an inverse "translated" CES demand function (ITCES) (Bulow and Pflaiderer, 1983). We summarize the properties of these demand functions in Table 2 where parameters α and β are positive scalars (see Appendix F for details).

Inverse demand functions		$r'_h > 0$
CSED	$p(x_h) = \frac{1}{\lambda_h} e^{-\frac{1}{\alpha\beta} x_h^\alpha}$	iff $\alpha > 1$
TLOG	$p(x_h) = \frac{1}{\lambda_h} \frac{\alpha + \beta \log x_h}{x_h}$	iff $\varepsilon_h < 3/2$
CREMR	$p(x_h) = \frac{1}{\lambda_h x_h} (x_h - \beta)^{\frac{\alpha}{\alpha+1}}$	no
CPPT	$p(x_h) = \frac{1}{\lambda_h x_h} (x_h^{-\alpha} + \beta)^{-\frac{1}{\alpha}}$	iff $\alpha > 1$
CEMR	$p(x_h) = \frac{1}{\lambda_h x_h} (x_h^{\frac{\alpha}{1+\alpha}} - \beta)$	yes/no
ITCES	$p(x_h) = \frac{1}{\lambda_h} (x_h^{-\frac{\alpha}{1+\alpha}} - \beta)$	no

Table 2: Properties of demand systems.

Table 2 shows that the general equilibrium effect of mean-preserving changes in income distribution depends not only on each preference but also on the values of its parameters.

Although, to the best of our knowledge, there exists no empirical estimations of the shape of the function r_h , we have several reasons to support the plausibility that $r'_h > 0$. First, as mentioned above, this property matches the idea that expenditure of lower income groups are more reactive to price changes. Second, Bekkers *et al.* (2012) show that when consumers purchase all varieties, prices decrease with higher income inequality. This is consistent with $r'_h > 0$ because any increase in income inequality affects prices only through its mean-preserving spread component. In what follows, we discuss mainly the case of $r'_h > 0$ as it appears more consistent with empirical facts. We show in Section 5 that this consistency also holds in the international trade context.

3.2.2 Welfare

We now discuss the welfare impact of income redistribution. Because goods are symmetric, the welfare of an individual with income s_h is given by $U_h = nu(x_h)$. Log-linearization gives the relative welfare change $\widehat{U}_h = \widehat{n} + \eta_h \widehat{x}_h$, which rises with higher product diversity and consumption levels. Under Pollak preferences, prices and product diversity are not affected by changes in individual income distribution so that the welfare implication is trivial: an increase in an individual's income results in welfare gains solely through higher individual consumption.

Beyond Pollak, using (19), welfare changes under mean-preserving income redistribution take the form:

$$\widehat{U}_h = \widehat{s}_h \eta_h + \varepsilon \left(1 - \eta_h - \frac{1}{\varepsilon} \right) \widehat{p}. \quad (22)$$

The first term reflects the direct effect on utility from the change in individual income \widehat{s}_h , while the second term represents the general equilibrium effect. As the mean-preserving contraction of the income distribution raises prices for $r'_h > 0$, the general equilibrium effect depends on the sign of $1 - \eta_h - 1/\varepsilon$. Under $\eta'_h < 0$, love for variety $1 - \eta_h$ increases with consumption. This implies that there exists a consumption level \bar{x} such that $1 - \eta_h \leq 1/\varepsilon$

if and only if $x_h \leq \bar{x}$, where \bar{x} solves $1 - \eta(\bar{x}) = 1/\varepsilon$. In turn, this implies that there exists an income level $\bar{s} \equiv \bar{x}/(np)$ such that $1 - \eta_h \leq 1/\varepsilon$ if and only if $s_h \leq \bar{s}$. Consequently, the general equilibrium effect is negative for individuals with incomes lower than \bar{s} and positive for all others. This effect is negative for all individuals if $\bar{s} > s_1$. However, this effect is never positive for all individuals because $\bar{s} > s_0$. Indeed, some lines of computation show that

$$\frac{d\eta(x_h)}{dx_h} < 0 \iff 1 - \eta(x_h) < \frac{1}{\varepsilon(x_h)}, \quad (23)$$

while $\varepsilon(x_0) > \varepsilon > \varepsilon(x_1)$ since $x_0 < x_1$ and $\varepsilon(x_h)$ is a decreasing function under subconvex demands. The last two sets of conditions imply that $1 - \eta(x_0) < 1/\varepsilon(x_0) < 1/\varepsilon$. Therefore, the poorest individual with consumption x_0 is always harmed by negative general equilibrium effect.

This result has policy implications. If an income redistribution policy targets only a fraction of poor individuals, then it harms those who are not targeted. For example, if a redistribution policy transfers income from the highest to lowest income decile, leaving other deciles unchanged, it leads to losses for middle income deciles due to the negative general equilibrium effect. Similarly, the general equilibrium effect of a redistribution policy may widen the welfare gap between the poorest and richest individuals if the latter are not affected by such transfers.

By contrast, under $r'_h < 0$, the general equilibrium effect is always positive for low-income groups. High-income groups are worse off if $\bar{s} > s_1$ and better off otherwise.

Proposition 4. *For $r'_h > 0$ ($r'_h < 0$), (i) the general equilibrium effect of mean-preserving contraction of income distribution on welfare is negative (positive) at least for the poorest households; (ii) it is negative (positive) for all income groups if $1 - \eta(s_1) < 1/\varepsilon$.*

As mentioned above, for Pollak preferences ($r'_h = 0$), income redistribution does not impact prices and implies no general equilibrium effect on welfare.

3.3 Generic change in income distribution

Consider, now, an arbitrary transformation of income distribution. This is equivalent to a sequence of two transformations: a transformation, a , with a common proportional change in all income levels and a transformation b that preserves its mean. Formally, this is defined as $\hat{s}_h = \hat{s}_h^a + \hat{s}_h^b$ where $\hat{s}_h^a = \hat{s}$ is the common proportional income change and \hat{s}_h^b is a mean-preserving change such that $\int_{s_0}^{s_1} \hat{s}_h^b s_h dG = 0$.

Since transformation a affects only the mass of firms, the total changes in individual consumption, price and variety are as follows:

$$\hat{x}_h = \hat{x}_h^a + \hat{x}_h^b = \hat{x}_h^b, \quad \hat{p} = \hat{p}^a + \hat{p}^b = \hat{p}^b, \quad \hat{n} = \hat{n}^a + \hat{n}^b = \hat{s} + \hat{n}^b. \quad (24)$$

Therefore, the impact on prices, markups, and consumption is driven only by its mean-preserving

change. The impact on product diversity results from both the mean-preserving change and proportional component.

Plugging (19) and (24) into $\widehat{U}_h = \widehat{n} + \eta_h \widehat{x}_h$, we get

$$\widehat{U}_h = \widehat{s}^a + \eta_h \widehat{s}_h^b + \varepsilon(1 - \eta_h - 1/\varepsilon)\widehat{p}^b.$$

The only difference with mean preservation is the first term \widehat{s}^a on the right-hand side. This reflects the positive general equilibrium effect of a higher average income on firm creation and product diversity.

The above analysis can be applied to the assessment of tax reforms. A decomposition of welfare changes shows that the effect of tax reforms must be broken down between the effects of tax revenue and tax progressivity. Suppose, indeed, that the government collects tax revenue $Td\xi$ by applying an average tax rate $\tau_h d\xi$ to individual h where $d\xi > 0$ is an infinitesimally small scalar, $\tau_h \equiv \tau(s_h)$ is the average tax rate and the tax revenue is proportional to $T \equiv \int s_h \tau_h dG > 0$. The tax paid is given by $s_h \tau_h d\xi$ so that the individual's net income is equal to $s_h (1 - \tau_h d\xi)$. The tax is progressive if the average tax rate increases with income, $\tau'_s > 0$, regressive otherwise and neutral on income distribution if $\tau'_s = 0$. In this context, the relative changes in average and individual incomes are given by $\widehat{s} = -(T/s) d\xi$ and $\widehat{s}_h = \tau_h d\xi$. The first transformation a is a common proportional change in income levels given by $\widehat{s}^a = \widehat{s} = (-T/s) d\xi < 0$. It corresponds to a neutral tax policy that raises tax revenues $Td\xi$. This reduces the utility of all individuals proportionally by the same amount. The second transformation b is given by $\widehat{s}_h^b = \widehat{s}_h - \widehat{s}^a = -(\tau_h - T/s) d\xi$, which is a mean-preserving contraction of income distribution if the tax rate is progressive.¹⁶ This second transformation is the general equilibrium effect implied by tax progressivity. By Proposition 4, a progressive marginal tax reform increases the equilibrium price \widehat{p}^b and reduces the welfare of poor groups of households under $\tau'_h > 0$. This shows that the overall general equilibrium effect of this tax policy worsens the welfare of at least the lowest income group. For a sufficiently large common proportional decrease in income levels ($\widehat{s}^a < 0$), these effects are negative for all income groups. This discussion shows that, besides a direct tax effect on income, there is an additional negative general equilibrium effect through the product market. Furthermore, the general equilibrium effect on welfare can be negative despite the progressive tax scheme reducing income inequality.

4 Trade

The monopolistic competition framework is widely applied in trade models, in particular, with a combination of CES preferences. Whereas within-country income heterogeneity is neutral to trade outcomes under CES preferences, it may significantly alter prices, output,

¹⁶Indeed, \widehat{s}_h^b is a mean-preserving contraction of income distribution if $\int_{s_0}^s \widehat{s}_h s_h dG \geq 0$; that is, if $\int_{s_0}^s (\tau_h - T/s) s_h dG \leq 0$. Since the left-hand side of the last expression is nil at $s_h = s_0$ and $s_h = s_1$, it must be negative if and only if $\tau'_h > 0$.

entry and welfare under nonhomothetic additive preferences. Therefore, in this section, we study the impact of changes in within-country income distribution on economic outcomes and welfare in all countries. To capture the effect of income distribution, we focus on two symmetric countries with identical preferences, populations and cost structures and without trade barriers. By doing so, we exclude the effects caused by country asymmetries and trade costs. As shown below, income heterogeneity may break the property of price equalization across countries even in this free trade context. The exercise finally differs from the analysis of a closed economy because of the presence of country-specific markets for each variety and labor force. Thus, a change in income distribution in a country gives rise to asymmetric economic outcomes in two countries.

The population size of each country is denoted by L while the distributions of individual incomes are denoted by G and $G^* : [s_0, s_1] \rightarrow [0, 1]$, where the asterisks refer to the variables of the foreign country. A home country individual consumes a set of home and foreign varieties, $\omega \in [0, n]$ and $\omega^* \in [0, n^*]$, where n and n^* are the masses of varieties produced in each country. She purchases quantities $x_h(\omega)$ and $i_h(\omega^*)$ of the domestically produced and imported varieties at home prices $p(\omega)$ and $p_i(\omega^*)$. She maximizes her utility $\int_0^n u(x_h(\omega))d\omega + \int_0^{n^*} u(i_h(\omega^*))d\omega^*$ subject to her budget constraint $\int_0^n p(\omega)x_h(\omega)d\omega + \int_0^{n^*} p_i(\omega^*)i_h(\omega^*)d\omega^* = s_h w$ where w is the home wage per labor unit. First-order conditions lead to inverse demand functions $p(\omega) = \lambda_h^{-1}u'(x_h(\omega))$ and $p_i(\omega^*) = \lambda_h^{-1}u'(i_h(\omega^*))$, where λ_h is her budget constraint multiplier. As before, by the symmetry of varieties, we can drop variety indices ω and ω^* . A consumer in the foreign country makes a similar choice of local and import consumption (x_h^*, i_h^*) given the prices (p^*, p_i^*) she faces there.

Under monopolistic competition and market segmentation, the home firm chooses its local and export prices, p and p_i^* , that maximize its profit

$$\pi = L \int (p - cw)x_h dG + L \int (p_i^* - cw)i_h^* dG - fw.$$

The optimal prices are given by

$$p = \frac{\varepsilon}{\varepsilon - 1}cw \quad \text{and} \quad p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^* - 1}cw,$$

where

$$\varepsilon = \frac{\int x_h \varepsilon(x_h) dG}{\int x_h dG} \quad \text{and} \quad \varepsilon_i^* = \frac{\int i_h^* \varepsilon(i_h^*) dG}{\int i_h^* dG},$$

while $\varepsilon(x_h)$ is the price elasticity of a home individual's demand for domestic goods and $\varepsilon(i_h^*)$ the one of a foreign individual's demand for her imported goods. Since prices are positive, we have $\varepsilon > 1$ and $\varepsilon_i^* > 1$. Similar definitions and properties hold for foreign producers $(p^*, p_i, \varepsilon^*$ and $\varepsilon_i)$.

Trade equilibrium is defined as the set of variables that are consistent with the consumer choice between local and imported goods, optimal prices set by firms for local and export markets, firms' optimal entry decision and market clearing conditions of product and labor

markets. The equilibrium conditions for the home country are presented in Table 3, and symmetric conditions hold for the foreign country.

Budget	$np_x x_h + n^* p_i i_h = s_h w$ $p/p_i = u'(x_h)/u'(i_h)$
Optimal price	$p = \frac{\varepsilon}{\varepsilon-1} cw$ $p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^*-1} cw$
Entry	$(p - cw)y + (p_i^* - cw)y_i^* = fw$
Product market	$y = L \int x_h dG$ $y_i^* = L \int i_h^* dG$
Labor market	$L \int s_h dG = n(f + c(y + y_i^*))$

Table 3: Domestic trade equilibrium conditions.

Market clearing conditions imply that the trade balance is satisfied, i.e., $p_i^* y_i^* n = p_i y_i n^*$.

When countries are symmetric in their income distribution, the system collapses to equilibrium conditions similar to those obtained for the closed economy. Therefore, a symmetric equilibrium exists under the same equilibrium conditions as in the closed economy (see Appendix G for details).

4.1 Mean-preserving redistribution

We now consider a small mean-preserving change in income distribution in the home country. As before, we denote the individual income change by $\hat{s}_h \equiv d \ln s_h = ds_h/s$, while $\hat{s} \equiv \frac{1}{s} \int \hat{s}_h s_h dG = 0$. We assume no change in individual income distribution in the foreign country and normalize its wage to one so that $\hat{s}_h^* = \hat{s}^* = \hat{w}^* = 0$. Equilibrium conditions can be log-linearized around the symmetric equilibrium with $G = G^*$ (see Appendix H). Denoting $\Upsilon \equiv \Psi + s(\varepsilon - 1)^2 > 0$, we solve them and obtain the following changes in prices, outputs, masses of firms and home wage:

$\hat{p} = \hat{p}_i = -\frac{1}{2\Psi\varepsilon} \left(1 + \frac{\varepsilon\Psi}{\Upsilon}\right) \int r_h s_h \hat{s}_h dG$	$\hat{p}^* = \hat{p}_i^* = \frac{1}{2\Psi} \frac{\varepsilon-1}{\varepsilon} \frac{\Psi-s(\varepsilon-1)}{\Upsilon} \int r_h s_h \hat{s}_h dG$
$\hat{x}_h = \hat{i}_h = \hat{s}_h + \hat{y}$	$\hat{x}_h^* = \hat{i}_h^* = \hat{y}^*$
$\hat{y} = \hat{y}_i = \frac{1}{2\Psi} \left(1 + \frac{\Psi}{\Upsilon}\right) \int r_h s_h \hat{s}_h dG$	$\hat{y}^* = \hat{y}_i^* = \frac{1}{2\Psi} \frac{s(\varepsilon-1)^2}{\Upsilon} \int r_h s_h \hat{s}_h dG$
$\hat{n} = \hat{n}^* = -\frac{1}{2\Psi} \frac{\varepsilon-1}{\varepsilon} \int r_h s_h \hat{s}_h dG$	$\hat{w} = 0$

Table 4: Changes in endogenous variables in a trade equilibrium.

Note that the domestic wage is not affected by income redistribution ($\widehat{w} = 0$) because countries are identical, and therefore, trade flows are initially symmetric. As a result, the terms of trade are not affected by income redistribution. Additionally, as in the closed economy, markups and prices are aligned, i.e., $\widehat{m} = (\varepsilon - 1)\widehat{p}$.

Under subconvex demands, we have $\Psi > 0$ and $\Psi - s(\varepsilon - 1) = -\int \varepsilon'_h x_h s_h dG > 0$. Given that $\varepsilon > 1$, all coefficients in Table 4 are positive so that the direction of changes is governed by the sign of $\int r_h s_h \widehat{s}_h dG$. As mentioned above, we focus our exposition on the case of $r'_h > 0$ so that $\int r_h s_h \widehat{s}_h dG < 0$ for a mean-preserving contraction of home income distribution. Table 4 shows that a mean-preserving contraction of home income raises all prices and markups in the home country, while diminishing all prices and markups in foreign country. This leads to a divergence in home and foreign market prices: in particular, *prices become relatively higher in the country with lower income inequality*. This point is remarkable, as *price differences between countries are caused by differences in income distribution* and not by the presence of trade costs and/or home bias as emphasized in the literature. Such effects on prices are also consistent with the above cited empirical evidence (Bekkers *et al.*, 2012). Furthermore, the number of produced goods increases in each country, while firms in both countries produce less (lower y , y_i^* , y^* and y_i^*). Hence, a reduction in home income inequality fosters the creation of new varieties worldwide at the expense of their production. In other words, *a reduction in a country's income inequality stimulates extensive margins and mitigates intensive margins of trade*. Finally, the fall in foreign prices entices foreigners to increase their spending on wider ranges of goods, n and n^* , but to consume smaller quantities, x_h^* and i_h^* .

We finally explain the effect of redistribution on trade patterns. While each firm's export volume y_i and y_i^* fall, its export value also diminishes because

$$\widehat{p}_i + \widehat{y}_i = \widehat{p}_i^* + \widehat{y}_i^* = \frac{\varepsilon - 1}{2\Psi\varepsilon} \int r_h s_h \widehat{s}_h dG < 0.$$

However, as shown in Table 2, the number of varieties increases by the same amount. As a result, the value of aggregate trade flows is unaffected by income redistribution, that is, $\widehat{p}_i + \widehat{y}_i + \widehat{n}^* = \widehat{p}_i^* + \widehat{y}_i^* + \widehat{n} = 0$. Import volumes $y_i n^*$ in the home country fall because, by the last statement, $\widehat{y}_i + \widehat{n}^* = -\widehat{p}_i < 0$. By the same argument, the opposite takes place for export volumes, $\widehat{y}_i^* + \widehat{n} = -\widehat{p}_i^* > 0$. Overall, total trade volume $y_i n^* + y_i^* n$ changes by $(\widehat{y}_i + \widehat{n}^* + \widehat{y}_i^* + \widehat{n})/2$, which can be shown to be negative.¹⁷ Hence, total trade volume diminishes.

We summarize this discussion in the following proposition:

Proposition 5. *Assume subconvex demands and two initially symmetric countries. Then, for $r'_h > 0$, a mean-preserving contraction of domestic income distribution raises all product prices and markups in the home country and diminishes all prices and markups in foreign country. This fosters the creation of new varieties and reduces firm production in each country. Domestic export volumes increase and import volumes fall. Total trade volume diminishes. The opposite holds for $r'_h < 0$ or the mean-preserving*

¹⁷One can show that $(\widehat{y}_i + \widehat{n}^* + \widehat{y}_i^* + \widehat{n})/2$ is equal to $-(\widehat{p}_i + \widehat{p}_i^*)/2$. Some lines of algebra lead to $\widehat{p}_i + \widehat{p}_i^* = -\frac{1}{\Psi\varepsilon} \frac{1}{\varepsilon} \left[\Psi + s(\varepsilon - 1) \right]^2 \int r_h s_h \widehat{s}_h dG > 0$.

spread.

Regarding welfare, a home individual has an equilibrium utility $U_h = nu(x_h) + n^*u(i_h)$, which yields a relative welfare change equal to $\widehat{U}_h \equiv \frac{1}{2}(\widehat{n} + \eta_h \widehat{x}_h) + \frac{1}{2}(\widehat{n}^* + \eta_h \widehat{i}_h)$, where weights 1/2 reflect the symmetric contributions of local and imported varieties to her utility. Applying the result in Table 4 leads to

$$\widehat{U}_h = \eta_h \widehat{s}_h + \varepsilon \left[1 - \eta_h - \frac{1}{\varepsilon} \right] \widehat{p}, \quad (25)$$

which is the same as welfare changes in the closed economy (22) up to differences in prices, \widehat{p} . An individual is directly affected by the change in her own income \widehat{s}_h (first term) and indirectly through the general equilibrium effect (second term). Under $r'_h > 0$, individuals with weaker love for variety (higher η_h) face a more negative general equilibrium effect on their welfare. Under increasing love for variety, this negative general equilibrium effect harms poorer individuals, as is the case in the closed economy. We then conclude that the general equilibrium effects are negative for at least the poorest groups of individuals.

By contrast, foreign residents are better off because both domestic and imported prices decrease in their market, while product diversity expands. Their gains are, however, distributed unequally. To be precise, the change in the welfare of a foreign individual is given by

$$\widehat{U}_h^* = \widehat{n}^* + \eta_h^* \widehat{x}_h^*.$$

The first term on the right-hand side is positive, while the second one is negative. Under $\eta'_h < 0$, poorer individuals have smaller love for variety (higher η_h) and, therefore, get lower welfare gains.

Proposition 6 summarizes this discussion.

Proposition 6. *Assume subconvex demands with $r'_h > 0$ and two initially symmetric countries. Then, a mean-preserving contraction of domestic income distribution benefits all residents in the foreign country. Under $\eta'_h < 0$, the general equilibrium effect of domestic redistribution reduces the welfare of at least the poorest individuals. The opposite holds for $r'_h < 0$.*

4.2 Changes in average income

We now consider a common proportional increase in income levels in the home country, $\widehat{s}_h = \widehat{s} > 0$.¹⁸ As shown in Appendix H, we still have $\widehat{w} = 0$, while the changes in other variables are presented in Table 5:

¹⁸All results are opposite for $\widehat{s} < 0$.

$\widehat{p} = \widehat{p}_i = \frac{\widehat{s}}{2\Upsilon} \int (1 + \varepsilon - r_h) s_h dG$	$\widehat{p}^* = \widehat{p}_i^* = -\frac{\widehat{s}}{2\Upsilon} \int (1 + \varepsilon - r_h) s_h dG$
$\widehat{x}_h = \widehat{i}_h = \widehat{y}$	$\widehat{x}_h^* = \widehat{i}_h^* = \widehat{y}^*$
$\widehat{y} = \widehat{y}_i = \frac{\varepsilon(\varepsilon-1)s\widehat{s}}{2\Upsilon}$	$\widehat{y}^* = \widehat{y}_i^* = -\frac{\varepsilon(\varepsilon-1)s\widehat{s}}{2\Upsilon}$
$\widehat{n} = \widehat{s}$	$\widehat{n}^* = 0$

Table 5: Changes in endogenous variables in a trade equilibrium.

Table 5 shows that prices and markups diverge between countries after the change in domestic average income. Indeed, $\int (1 + \varepsilon - r_h) s_h dG$ is equal to $\int (1 + \varepsilon_h - r_h) s_h dG$ and is positive under subconvex demands since $1 + \varepsilon_h - r_h = -x_h \varepsilon'_h > 0$ by (3). Hence, prices increase in the richer (home) country whereas they decrease in the poorer (foreign) country. This stems from firms' price discrimination between the two markets. With subconvex preferences, the individual demands of richer people are less elastic and imply less elastic aggregate demand in the richer country. In the presence of market segmentation, firms are able to charge strictly higher markups in the richer market.¹⁹ This argument deserves two remarks. First, the effect of market segmentation is not mitigated by terms of trade ($\widehat{w} = 0$) when countries are close to symmetry. Second, CARA, logarithmic and quadratic utility functions also imply price and markup divergence. To clarify, note that Pollak functions imply a constant parameter $r_h = r$ so that the price changes in Table 5 simplify to

$$\widehat{p} = \widehat{p}_i = -\widehat{p}^* = -\widehat{p}_i^* = \frac{1}{2} \frac{1 + \varepsilon - r}{1 + \varepsilon^2 - r} \widehat{s}.$$

It can be shown that quadratic, logarithmic and CARA utility functions demonstrates $1 + \varepsilon - r > 0$ (whereas CES utility functions imply an equality; see Appendix D). This remark concurs with the finding of Simonovska (2015), who theoretically and empirically shows that prices are higher in countries with higher average income. She uses a theoretical framework with homogenous income consumers and logarithmic utility. Moreover, the result on price and markup divergence contradicts the one obtained for a closed economy where income changes do not affect prices for all Pollak utility functions.

By contrast, a change in average income has the same effects on firm output and the mass of domestic firms as in the closed economy. On one hand, firm output in both countries does not vary with this income shock as $\widehat{y} + \widehat{y}_i^* = \widehat{y}_i + \widehat{y}^* = 0$. However, domestic and foreign firms increase both their prices and output in the richer (home) market (in particular, $\widehat{y} = \widehat{y}_i = -\widehat{y}^* = -\widehat{y}_i^* > 0$). While they increase their sales in the richer country, they equally reduce them in the poorer foreign country. On the other hand, the increase in the number of domestic firms is proportional to the increase in home average income, $\widehat{n} = \widehat{s}$. The number of varieties produced in the home country rises while it remains constant in the foreign

¹⁹The only exception is when individuals are endowed with CES preferences that give them identical and constant elasticities.

country.²⁰ In other words, for both closed and open economies, each firm labor force remains unchanged, while changes in labor endowments are fully absorbed by firm entry. The last two properties have been discussed in international trade frameworks with monopolistic competition and CES preferences. The present paper extends them to arbitrary additive preferences.

Regarding the welfare effect, we show in Appendix I that it is given by

$$\hat{U}_h = \left(1 + \frac{\varepsilon(\varepsilon - 1)s}{\Upsilon} \eta_h\right) \frac{\hat{s}}{2}, \quad \text{and} \quad \hat{U}_h^* = \left(1 - \frac{\varepsilon(\varepsilon - 1)s}{\Upsilon} \eta_h\right) \frac{\hat{s}}{2}$$

in home and foreign countries, respectively. For $\hat{s} > 0$, the welfare effect is positive and greater than $\hat{s}/2$ in the home country. As η_h is a decreasing function, poor domestic income groups experience larger gains. Furthermore, we also show in Appendix I that $\hat{U}_h^* > 0$ under subconvex demands. In words, individuals in foreign country also gain from an increase in the average income in the other country; this effect is milder than in the home country ($\hat{U}_h^* < \hat{s}/2$), while rich individuals gain more. To sum up, an increase in average income in a country leads to welfare gains in both countries, with a larger effect occurring in home country. These effects are asymmetric across income groups in both countries.

Proposition 7. *Assume subconvex demands and two initially symmetric countries. Then, an increase in domestic average income raises domestic prices and markups while decreasing foreign ones (except for CES preferences). While all individuals in both countries gain, poor domestic income groups experience the largest gains, whereas rich foreign groups have the lowest gains.*

5 Quantification

In the previous sections, we have shown that the general equilibrium effects of income distribution depend on the properties of r_h . While our theoretical study helps determine the existence and direction of such effects, it does not shed light on their amplitude. The main purpose of this section is therefore to quantify the general equilibrium effects of income redistribution on the product market and individuals' welfare.

Towards this aim, we calibrate our model to the US industry and income distribution. We use a total employment of 148 million workers and a total of 2,22 million firms with more than 5 employees and compute the average employment per firm of 66 workers (US census data, 2015). The average income is 56,516 USD (in 2018). We normalize the quantities of goods such that the variable cost is equal to one, while we set the fixed cost consistent with the above calibration values and equilibrium conditions (9)-(11).²¹ The worker population

²⁰Finally, an increase in a country's average income may be split into a common proportional increase in each income level and a mean-preserving contraction of the income distribution. In this model, both components increase prices in the home country and decrease them in the foreign country if $r'_h > 0$. Therefore, any additive preferences with $r'_h > 0$ are consistent with empirical evidence about the higher prices in countries with the higher income, as mentioned in Subsection 3.2.1.

²¹By solving (9)-(11), one obtains $p = \varepsilon/(\varepsilon - 1)$, $px = (\text{employment per firm} * \text{average income}) / (\text{total employ-}$

is divided into deciles of after-tax income (such that the distribution $G(s_h)$ is a discontinuous function with 10 levels). The lowest and highest deciles' incomes are 2,832 and 172,358 USD, respectively.

5.1 Calibration and selection of demand functions

We first explore how the demand systems in Table 2 match existing market statistics. Each demand system includes two parameters (α, β) to match with two empirical statistics.

The first obvious statistic to match is market demand elasticity ε . Under monopolistic competition, demand elasticity coincides with the elasticity of substitution among goods.²² The latter has been estimated mainly with two approaches. The first approach identifies its value through the effect on the gravity equations of long-term changes in trade policies and geographical factors such as distance (Head and Ries, 2001; Head and Mayer, 2004; Bergstrand *et al.*, 2013). The estimations range from 6 to 11, and there seems to be a consensus among researchers of an estimate approximately 7. The second approach identifies the same elasticity using an estimation of demand functions through short-run price variations and reports a wide dispersion of elasticities across goods or sectors with median values between 1 and 3 (Reinert and Roland-Holst, 1992; Broda and Weinstein 2006). As mentioned by Ruhl (2008), the latter approach more likely reflects the short-run evolution of demands with rigidities in firms' entry, whereas the former is more likely to measure long-run changes with free entry. Feenstra *et al.* (2018) reconsider the discrepancies between the macro- and microelasticities of substitution and report strong differences only for a subset of goods. Since our monopolistic competition model emphasizes the effect of firm entry, we concentrate our exposition on the case of $\varepsilon = 7$. The analysis for lower elasticities reports general equilibrium effects of similar magnitudes (see Appendix J).

The second statistic that we propose to match is pass-through elasticity, defined as $\mathcal{E}_{pt} \equiv d \log p / d \log c$. Using (6), we obtain

$$\mathcal{E}_{pt} = 1 + \frac{d \log}{d \log c} \left(\frac{\varepsilon}{\varepsilon - 1} \right).$$

In our context of income heterogeneity, we differentiate (7) and obtain

$$\mathcal{E}_{pt} = \frac{\varepsilon(\varepsilon - 1)x}{\int (2\varepsilon - r_h)\varepsilon_h x_h dG}, \quad (26)$$

which is positive due to the second-order condition (8). Pass-through elasticity has been estimated in the range of 0.3 to 0.8. For instance, using trade macro data and exchange rate

ment), $n = (\text{total employment}) / (\text{employment per firm})$, and $f = (\text{employment per firm} * \text{average income}) / \varepsilon$. These values are consistently adjusted for elasticity ε , which is determined by demand parameters (α, β) .

²²Under additive preferences and symmetric goods, the elasticity of substitution between two goods ω and ω' for a consumer with income s_h , defined as $d \ln(x_h(\omega)/x_h(\omega')) / d \ln(p(\omega)/p(\omega'))$, can be shown to be equal to $\varepsilon(x_h)$. At the aggregate level, the demand for a good, ω , is given by $x(\omega) \equiv \int x_h(\omega) dG$ and the elasticity of substitution between goods is defined as $d \ln(x(\omega)/x(\omega')) / d \ln(p(\omega)/p(\omega'))$. It can be shown that the latter is equal to ε .

shocks, Campa and Golberg (2005) suggest average values of 0.46 and 0.64 for the short and long terms. Amiti *et al.* (2019) also suggest 0.6 based on Belgian micro-level manufacturing data. Using Indian firm-level production data, De Loecker *et al.* (2016) find a range of $[0.3, 0.4]$, while Mion and Jacob (2020) find a value of approximately 0.8 using French manufacturing firm data. To reflect this disparity, we match two target pairs of values $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.4)$ and $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.6)$.

To match the target elasticities, we use equations (10) to (11) to compute the equilibrium price, number of firms and fixed costs as a function of the market demand elasticity ε . Using equation (9), we compute the consumption of each decile x_h as a function of ε . From (7), $\varepsilon = \int x_h \varepsilon_h dG / \int x_h dG$ is itself a function of individual elasticities ε_h weighted by equilibrium consumption x_h . We solve for the fixed point to recover the equilibrium market demand elasticity ε , which is then used to obtain equilibrium price p and consumption levels x_h . We ensure that the equilibrium exists by checking condition (8).

The preferences proposed in Table 2 add two restrictions to the calibration process. Some utility functions are indeed defined on supports that do not include zero consumption and/or are not concave functions everywhere on their supports. In the context of income heterogeneity, this implies that strong income discrepancies might not be possible for calibration because the consumption levels of the lowest-income individuals would lie below the support at which utility is defined and concave. Furthermore, the absence of concavity implies that the lowest-income individuals may not express love for variety. In particular, condition (5) may not be maintained so that low-income individuals refrain from consuming all varieties, and the fixed-point computation then would not lead to an equilibrium.

We first take an extensive set of random draws for the parameter pairs (α, β) and apply them to each demand class in Table 2. We then search for the parameter values that match the target $(\varepsilon, \mathcal{E}_{\text{pt}})$. Figure 1 summarizes the sets of elasticity pairs $(\varepsilon, \mathcal{E}_{\text{pt}}) \in (1, 8) \times (0, 1)$ that are supported by parameters (α, β) for each of the six preference classes presented in Table 2. We briefly discuss each one. First, constant superelasticity demands (CSED) are displayed in the background in white. Figure 1 shows that they support all elasticity pairs such that they also match the target elasticity values.

Second, inverse translog demands (TLOG) are displayed by the (one-dimensional) red curve. They yield demand elasticities lower than 2 and cannot match the target pairs of elasticity values. The reason is that the number of US firms implies high product diversity and, consequently, low consumption levels, while inverse translog demands have low individual elasticity at low consumption levels. In what follows, we exclude this demand system from our quantification exercise.

Third, demands with constant revenue elasticity of marginal revenue (CREMR) are displayed by the black area. They are supported by parameters only for pass-through elasticities close to 1 and cannot support the target pairs of elasticity values. These utility functions are not concave everywhere and therefore do not guarantee that lower-income individuals consume all available goods. We also exclude these from our quantification exercise.

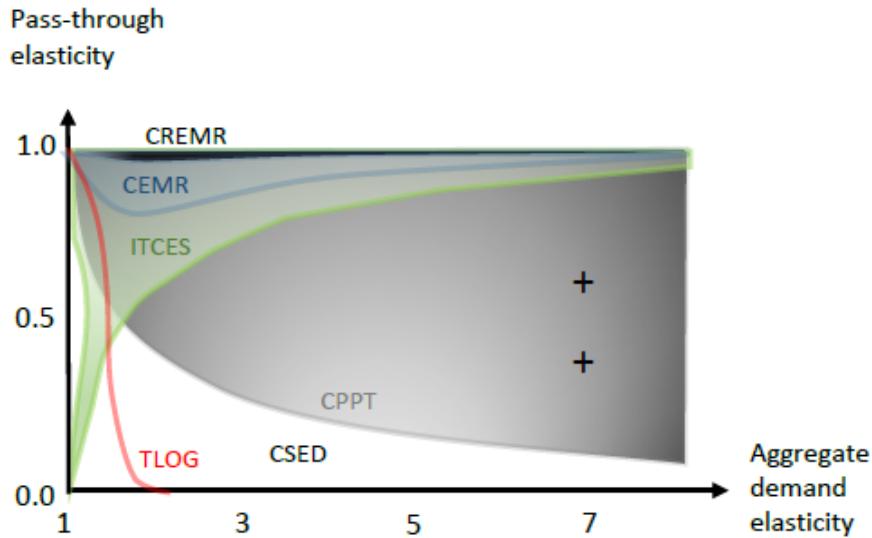


Figure 1: Feasible demand and pass-through elasticities

Fourth, demands with constant proportional pass-through (CPPT) are presented in gray. They support pairs of sufficiently large elasticities ε and ε_{pt} , and, in particular, the target pairs of elasticities. In general, they are suited to reproduce economies with demand elasticity ε greater than 3 and elasticity of pass-through higher than 0.4, which is consistent with empirical studies.

Fifth, demands with constant elasticity of marginal revenue (CEMR) support a set of elasticities displayed by the blue area. They only support pass-through elasticities greater than 0.8 and therefore do not encompass the target pairs of elasticities. As the CREMR utility, these functions hardly guarantee that lower-income individuals consume all available goods. Finally, demands with inverse translated CES (ITCES) are presented in green. They support low-demand elasticities and high pass-through elasticity. Figure 1 shows that they do not support the target pairs of elasticities and are unsuited to the calibration exercise.

To sum up, only the CSED and CPPT are well suited to reproduce our target values of demand and pass-through elasticities in the context of a production economy and income distribution similar to those in the US. As Figure 1 shows, these demand systems are robust to reasonable changes in target values. The other demand systems produce either insufficient demand elasticities or excessive pass-through elasticities, or they may be incompatible with the assumption that all consumers purchase all available varieties. Note that they might be better suited to replicate economies with lower income inequality than the US'.

5.2 Income redistribution

We now examine the effect of income redistribution on market outcome and individual welfare. To keep things simple, we simulate the redistribution from the top to the bottom decile that raises the latter by 300%. This represents a mean-preserving contraction of the income distribution and increases the average income of the bottom decile to 11 328 USD, which is slightly lower than that of the second decile. The total transfer involves approximately 1.5% of total income. We make demand systems comparable by fixing the elasticities of market demand and pass-through to the target values $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.4)$ and $(7, 0.6)$.

The effects of this redistribution are presented in Table 5 for the CPPT and CSED demand systems. The two top rows present demand parameters α and β , which match the target elasticities before income redistribution, while the third and fourth rows report the target elasticity values. The next three rows give the percent change in price, number of firms and firm output, compared to the initial situation. To preserve consistency among different demand systems, we report the welfare changes as ‘consumption equivalent’ for each decile. The consumption equivalent x_h^{eq} is defined as the consumption level that gives the same utility as that obtained at the initial price and number of goods. In other words, x_h^{eq} is such that $n_a u(x_h^{\text{eq}}) = n_b u(x_h^b)$ where subscripts a and b refer to the initial and final allocations, respectively.

The first column in Table 5 indicates the direct effect of redistribution; that is, the changes when prices and entry do not adjust to the redistribution. The direct effect causes the bottom decile to gain 300% and the top decile to lose 4.95% of the consumption equivalent. Other columns indicate the general equilibrium effects, net of the direct redistribution effect from the top to the bottom decile and for each set of preferences and parameter values. Magnitudes are reported in percentage points (%).

The second column reports the effects of the above redistribution with CPPT preferences matching $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.4)$. These elasticities are reached with demand parameters $\alpha = 1.11$ and $\beta = 13.62$. As shown in Table 2, $\alpha > 1$ implies that r_h is an increasing function. The mean-preserving contraction of income distribution entices firms to increase their prices by 0.30%, decrease their production by 2.17% and, in the end, enter the market with an additional 1.85% of firms, as predicted by Proposition 3. Therefore, the general equilibrium effect leads to a reduction in the consumption equivalent between 0.31% and 0.05% from the first to the ninth decile and to a rise in the consumption equivalent for the top decile. Lower deciles are more negatively affected by the general equilibrium effect. This is because, by (22), welfare weight $1 - \eta(s_h) - 1/\varepsilon$ takes less negative values as income rises and reverts to a positive value for top income individuals (see Proposition 4). This calibrated example confirms that the general equilibrium effect may work in opposite directions for different income groups. Finally, recall that this income redistribution involves a transfer of 1.5% of the total US income. The changes in prices, production and product diversity have the same order of magnitude. The changes in consumption equivalent are slightly lower but still significant. Thus, *general equilibrium effects cannot be considered negligible*.

	Direct effect	General equilibrium effects			
		CPPT		CSED	
α		1.11	0.82	1.06	0.76
β		13.61	3.88	0.10	0.39
ε		7	7	7	7
\mathcal{E}_{pt}		0.4	0.6	0.4	0.6
$\widehat{p}(\%)$	0	0.30	-0.26	0.18	-0.33
$\widehat{n}(\%)$	0	1.85	-1.52	1.08	-1.95
$\widehat{y}(\%)$	0	-2.17	1.76	-1.27	2.26
Deciles					
$\widehat{x}_1^{\text{eq}}(\%)$	300	-0.31	0.24	-0.18	0.30
$\widehat{x}_2^{\text{eq}}(\%)$	0	-0.29	0.21	-0.17	0.27
$\widehat{x}_3^{\text{eq}}(\%)$	0	-0.26	0.19	-0.16	0.25
$\widehat{x}_4^{\text{eq}}(\%)$	0	-0.24	0.17	-0.15	0.23
$\widehat{x}_5^{\text{eq}}(\%)$	0	-0.22	0.16	-0.13	0.21
$\widehat{x}_6^{\text{eq}}(\%)$	0	-0.20	0.14	-0.12	0.19
$\widehat{x}_7^{\text{eq}}(\%)$	0	-0.16	0.12	-0.10	0.16
$\widehat{x}_8^{\text{eq}}(\%)$	0	-0.12	0.09	-0.08	0.13
$\widehat{x}_9^{\text{eq}}(\%)$	0	-0.05	0.05	-0.03	0.07
$\widehat{x}_{10}^{\text{eq}}(\%)$	-4.95	0.16	-0.06	0.13	-0.09

Table 6: Effects of income redistribution in a closed economy.

The third column reports the effect with CPPT preferences and equilibrium elasticities $(\varepsilon, \mathcal{E}_{\text{pt}}) = (7, 0.6)$. With a value of $\alpha = 0.82 < 1$, r_h is a decreasing function. In this case, the mean-preserving contraction of income has exactly the opposite effect. As stated by Proposition 3, income redistribution entices firms to reduce their prices and raise their production, while entry falls. The general equilibrium effect of redistribution increases the consumption equivalent in all deciles except for the top decile. Appendix K shows that this result also applies for lower values of elasticities ε .

The effects of income redistribution under CSED preferences are reported in Columns 4 and 5. They have the same directions and similar amplitudes as CPPT preferences. Again, these demands feature opposite behaviors of market aggregates and individual welfare according to each value of pass-through elasticity $\mathcal{E}_{\text{pt}} \in \{0.4, 0.6\}$. We provide a formal link between r_h and pass-through elasticity in Appendix K. For instance, in the case of CPPT, we show that $\mathcal{E}_{\text{pt}} < 0.5$ if and only if $r'_h > 0$. Additionally, for the CSED, $r'_h < 0$ if $\mathcal{E}_{\text{pt}} > 0.5$. Thus, at least for these two demand classes, there is a link between the directions of general equilibrium effects and the value of pass-through elasticity being higher or lower than 0.5. Since

both values are supported by the empirical literature, this exercise highlights the importance of an accurate empirical assessment of pass-through elasticity for the welfare impact of income redistribution.

To sum up, CPPT and CSED preferences yield similar and nonnegligible effects of income redistribution on prices, consumption and welfare. The direction of these effects crucially depends on pass-through elasticity.

5.3 Trade

Finally, we study the quantitative impact of income redistribution in the presence of trade. Towards this aim, we equally divide the population of the above closed economy and create two trading symmetric countries. We then apply the same mean-preserving contraction of income redistribution in the home country only. This division strategy makes the open economy comparable to the above closed economy because it yields the same demand and pass-through elasticities and the same pattern of r_h for identical parameter values. Then, we study the effect of the division of a unique labor and product market into symmetric independent markets. Table 5 presents the prices, product diversity, firm output and individual welfare for the CPPT and CSED preferences calibrated for the target elasticities $(\varepsilon, \mathcal{E}_{pt}) = (7, 0.4)$ and $(7, 0.6)$. Rows and columns are organized as in the previous subsection.

For conciseness, we focus on CPPT preferences with $(\varepsilon, \mathcal{E}_{pt}) = (7, 0.4)$, which implies that $r'_h > 0$ (second column of Table 6). As predicted by theory, the mean-preserving contraction of the home income distribution raises all home prices and diminishes foreign prices. It also fosters the creation of new varieties and the reduction in firm production scales in each country. Compared to the closed economy, home income redistribution raises home prices by 0.43% in the trade economy, whereas it increases them only by 0.30% in the closed economy. Therefore, the effect on home prices is about half as strong as under trade. Foreign prices move with a milder amplitude by 0.13% in the opposite direction. Hence, home income redistribution leads to a price difference of 0.56% between the two countries. The home price hike allows home firms to dampen their output responses by a decline of 1.37% in production instead of 2.17% in the closed economy. By contrast, local product diversity rises by the same amount in both countries, and global product diversity reaches the same value as in the closed economy.

	Direct effect	General equilibrium effect							
		CPPT				CSED			
α		1.11	0.82	1.06	0.76				
β		13.61	3.88	0.10	0.39				
ε		7	7	7	7				
\mathcal{E}_{pt}		0.4	0.6	0.4	0.6				
	home	home	foreign	home	foreign	home	foreign	home	foreign
$\hat{p}(\%)$	0.	0.43	-0.13	-0.34	0.07	0.26	-0.09	-0.44	0.10
$\hat{n}(\%)$	0.	0.92	0.92	-0.76	-0.76	0.54	0.54	-0.98	-0.98
$\hat{y}(\%)$	0.	-1.37	-0.81	1.08	0.67	-0.81	-0.46	1.40	0.86
Deciles									
$\hat{x}_1^{eq}(\%)$	300.	-0.44	0.13	0.32	-0.09	-0.27	0.08	0.42	-0.12
$\hat{x}_2^{eq}(\%)$	0.	-0.43	0.14	0.31	-0.10	-0.26	0.08	0.4	-0.15
$\hat{x}_3^{eq}(\%)$	0.	-0.42	0.15	0.3	-0.11	-0.25	0.09	0.39	-0.15
$\hat{x}_4^{eq}(\%)$	0.	-0.41	0.16	0.28	-0.12	-0.25	0.10	0.38	-0.16
$\hat{x}_5^{eq}(\%)$	0.	-0.4	0.17	0.28	-0.13	-0.24	0.10	0.37	-0.17
$\hat{x}_6^{eq}(\%)$	0.	-0.38	0.18	0.27	-0.15	-0.23	0.11	0.36	-0.18
$\hat{x}_7^{eq}(\%)$	0.	-0.37	0.2	0.26	-0.15	-0.23	0.12	0.35	-0.19
$\hat{x}_8^{eq}(\%)$	0.	-0.35	0.22	0.25	-0.16	-0.21	0.13	0.33	-0.21
$\hat{x}_9^{eq}(\%)$	0.	-0.31	0.26	0.23	-0.18	-0.19	0.15	0.31	-0.24
$\hat{x}_{10}^{eq}(\%)$	-4.95	-0.21	0.36	0.18	-0.24	-0.11	0.24	0.23	-0.32

Table 7: Effects of home income redistribution in an open economy.

Since domestic consumers face higher home prices, the general equilibrium effect reduces their welfare. Table 6 shows that domestic workers in the second-lowest decile reduce their consumption equivalent by 0.43% in the open economy instead of 0.29% in an integrated market. In the trade economy, however, the richest home individual does not benefit from a positive general equilibrium effect as in the closed economy. Because foreigners face lower prices, their welfare increases. It is apparent that welfare effects are greater for poorer home and richer foreign individuals. Interestingly, the relative consumption-equivalent loss of the poorest home individuals has the same magnitude as the gain of the richest foreigners. Finally, changes in firm trade values are given by $\hat{p}_i + \hat{y}_i = 0.43 - 1.37 = -0.94\%$. This is a significant change with regard to the transfer of 1.5% of total income in the home country. Similar effects can be observed for the CSED preference, yielding the same elasticities. Opposite effects occur in economic contexts with pass-through elasticities \mathcal{E}_{pt} equal to 0.6. To sum up, in an open economy, income redistribution in a country significantly affects prices, output, individual welfare and import-export values in both countries.

6 Conclusion

In this paper, we study the effect of income distribution on product markets, welfare, and trade patterns in a framework of monopolistic competition and nonhomothetic additive preferences. We show that the property of individual demand convexity is the key driver of the effects of income distributions. If individual demands display increasing convexity, then a mean-preserving contraction of the income distribution in the home country leads to a rise in its prices, an increase in its export volumes and a decline in of its import volumes, ultimately implying a reduction in total trade volumes. Home individuals can be harmed, and even more so if they are poor. The lower level of domestic inequality has welfare effects on other countries as all foreign consumers gain. These results are reversed not only if the income distribution spreads but also if product demands display decreasing convexity. These findings show that within-country income inequality shapes trade patterns and the distribution of gains from trade across countries and individuals. By contrast, the general equilibrium effects of the changes in average income do not depend on the above property of individual demand convexity. Yet, we show that prices and markups diverge across countries as average income increases in the home country, for all additive preferences except the CES. While all individuals in both countries gain from such a change, poor income groups in the home country experience the largest gains, whereas rich foreign residents have the lowest gains.

Beyond theoretical results, our quantitative exercise suggests that redistributive policies from the rich to the poor have impacts on prices, production, entry and individual welfare, with orders of magnitude similar to the sizes of transfers. Thus, the general equilibrium effects of income redistribution are not negligible.

The present analysis makes clear that more empirical work is needed to uncover the properties of individual demand convexity. However, estimations of such properties at the individual level are a challenging task. Nevertheless, researchers may seek and policy makers may use indirect evidence. First, existing empirical studies suggest that prices are higher in richer countries, which supports the case of the increasing convexity of the demand function. Second, empirical estimations of the relationship between prices and income inequality could also allow to quantify the general equilibrium effects of income inequality. To the best of our knowledge, most studies focus on the differences in countries' average incomes.²³ However, we highlight the importance of income heterogeneity, a dimension which is missing in those studies. Income heterogeneity is shown to play a crucial role in imperfect product markets.

Finally, our numerical exercise suggests that the property of increasing demand convexity relates to pass-through elasticity. Thus, the latter may be a good predictor of the direction of the general equilibrium effect and, therefore, the economic and welfare effects of redistributive policies. Such a conclusion provides an undiscovered relationship between the trade literature on pass-through and the welfare literature on income inequality. Hence, another empirical approach would be to investigate and use the relationship between pass-through elasticity and general equilibrium effects. If empirical research devotes additional efforts to

²³One exception is Bekkers *et al.* (2012).

measuring pass-through elasticities, then this model can allow policy makers to adjust their redistributive policies, while taking into account the general equilibrium consequences for each income group.

References

- [1] Aguirre, Inaki, Simon Cowan, and John Vickers. 2010. "Monopoly price discrimination and demand curvature." *American Economic Review*, Vol. 100(4): 1601-15.
- [2] Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2019. "International shocks, variable markups, and domestic prices." *The Review of Economic Studies*, Vol. 86(6): 2356-2402.
- [3] Behrens, Kristian, and Yasusada Murata. 2012. "Globalization and individual gains from trade." *Journal of Monetary Economics*, Vol. 59(8): 703-720.
- [4] Bekkers, Eddy, Joseph Francois, and Miriam Manchin. 2012. "Import prices, income, and inequality." *European Economic Review*, Vol. 56(4): 848-869.
- [5] Bergstrand, Jeffrey H., Peter Egger, and Mario Larch. 2013. "Gravity redux: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs." *Journal of International Economics*, Vol. 89(1): 110-121.
- [6] Bertolotti, Paolo, Federico Etro, and Ina Simonovska. 2018. "International trade with indirect additivity." *American Economic Journal: Microeconomics*, Vol. 10(2): 1-57.
- [7] Broda, Christian, and David E. Weinstein. 2006. "Globalization and the gains from variety." *The Quarterly journal of economics*, Vol. 121(2): 541-585.
- [8] Bulow, Jeremy I., and Paul Pfleiderer. 1983. "A note on the effect of cost changes on prices." *Journal of Political Economy*, Vol. 91(1): 182-185.
- [9] Campa, Jose Manuel, and Linda S. Goldberg. 2005. "Exchange rate pass-through into import prices." *Review of Economics and Statistics*, Vol. 87(4): 679-690.
- [10] Choi, Yo Chul, David Hummels, and Chong Xiang. 2009. "Explaining import quality: The role of the income distribution." *Journal of International Economics*, Vol. 77(2): 265-275.
- [11] Cowan, Simon. 2012. "Third-degree price discrimination and consumer surplus." *The Journal of Industrial Economics*, Vol. 60(2): 333-345.
- [12] Dalgin, Muhammed, Vitor Trindade, and Devashish Mitra. 2008. "Inequality, Nonhomothetic Preferences, and Trade: A Gravity Approach." *Southern Economic Journal*: 747-774.

- [13] De Loecker, Jan, Goldberg, Pinelopi K., Khandelwal, Amit K., and Pavcnik Nina. 2016. "Prices, markups, and trade reform." *Econometrica*, Vol. 84(2), 445-510.
- [14] Dhingra, Swati, and John Morrow. 2019. "Monopolistic competition and optimum product diversity under firm heterogeneity." *Journal of Political Economy*, Vol. 127(1): 196-232.
- [15] Dixit, Avinash K., and Joseph E. Stiglitz. 1977. "Monopolistic competition and optimum product diversity." *The American Economic Review*, Vol. 67(3): 297-308.
- [16] Dynan, Karen E., Jonathan Skinner, and Stephen P. Zeldes. 2004. "Do the rich save more?" *Journal of Political Economy*, Vol. 112(2): 397-444.
- [17] Egger, Hartmut, and Udo Kreickemeier. 2009. "Firm heterogeneity and the labor market effects of trade liberalization." *International Economic Review*, Vol. 50(1): 187-216.
- [18] Falkinger, Josef, and Josef Zweimüller 1996. "The cross-country Engel curve for product diversification." *Structural Change and Economic Dynamics*, Vol. 7(1): 79-97.
- [19] Feenstra, Robert C. 2003. "A homothetic utility function for monopolistic competition models, without constant price elasticity." *Economics Letters*, Vol. 78(1): 79-86.
- [20] Feenstra, Robert C., Philip Luck, Maurice Obstfeld, and Katheryn N. Russ. 2018. "In search of the Armington elasticity." *Review of Economics and Statistics*, Vol. 100(1): 135-150.
- [21] Felbermayr, Gabriel, Julien Prat, and Hans-Jorg Schmerer. 2011. "Globalization and labor market outcomes: Wage bargaining, search frictions, and firm heterogeneity." *Journal of Economic Theory*, Vol. 146(1): 39-73.
- [22] Fieler, Ana Cecilia. 2011. "Nonhomotheticity and bilateral trade: Evidence and a quantitative explanation." *Econometrica*, Vol. 79(4): 1069-1101.
- [23] Flam, Harry, and Elhanan Helpman. 1987. "Vertical product differentiation and North-South trade." *The American Economic Review*: 810-822.
- [24] Foellmi, Reto, and Josef Zweimüller. 2004. "Inequality, market power, and product diversity." *Economics Letters*, Vol. 82 (1): 139-145.
- [25] Friedman, Milton. 1957. "A Theory of the Consumption Function." Princeton, NJ: Princeton University Press.
- [26] Gopinath, Gita, and Oleg Itskhoki. 2010. "Frequency of price adjustment and pass-through." *The Quarterly Journal of Economics*, Vol. 125(2): 675-727.
- [27] Head, Keith and Thierry Mayer. 2004. "The empirics of agglomeration and trade." *Handbook of Regional and Urban Economics*, Vol. 4, pp.2609-2669.

- [28] Head, Keith and John Ries. 2001. "Increasing returns versus national product differentiation as an explanation for the pattern of US-Canada trade." *American Economic Review*, Vol. 91(4): 858–876.
- [29] Helpman, Elhanan, Oleg Itskhoki, and Stephen Redding. 2010. "Inequality and unemployment in a global economy." *Econometrica*, Vol. 78(4): 1239-1283.
- [30] Holmes, Thomas J. 1989. "The effects of third-degree price discrimination in oligopoly." *The American Economic Review*, Vol. 79(1): 244-250.
- [31] Houthakker, Hendrik Samuel. 1957 "An international comparison of household expenditure patterns, commemorating the centenary of Engel's law." *Econometrica*: 532-551.
- [32] Hummels, David, and Kwan Yong Lee. 2018. "The income elasticity of import demand: Micro evidence and an application." *Journal of International Economics*, Vol. 113: 20-34.
- [33] Jones, Ronald W. 1965. "The structure of simple general equilibrium models." *Journal of Political Economy*, Vol. 73(6): 557-572.
- [34] Keynes, John Maynard. 1936. "The general theory of employment, interest, and money", New York: Harcourt, Brace.
- [35] Kichko, Sergey, Sergey Kokovin, and Evgeny Zhelobodko. 2014. "Trade patterns and export pricing under non-CES preferences." *Journal of International Economics*, Vol. 94(1): 129-142.
- [36] Kuhn, Kai-Uwe, and Xavier Vives. 1999. "Excess entry, vertical integration, and welfare." *RAND Journal of Economics*, Vol. 30: 575–603.
- [37] Hsu, Wen-Tai, Lu, Lin, and Picard, Pierre M. 2022. "Income inequality, productivity, and international trade." *Economic Theory*, 1-47.
- [38] Marshall, Alfred, *Principles of Economics*, 8th ed. (London: Macmillan, 1936).
- [39] Matsuyama, Kiminori. 2000. "A ricardian model with a continuum of goods under non-homothetic preferences: Demand complementarities, income distribution, and north-south trade." *Journal of Political Economy*, Vol. 108(6): 1093-1120.
- [40] McCarthy, Patrick S. 1996. "Market price and income elasticities of new vehicle demands." *The Review of Economics and Statistics*, 543-547.
- [41] Mion, Giordano, and Nicholas Jacob. 2020. "On the productivity advantage of cities." CEPR DP 14644.
- [42] Mrazova, Monika, and J. Peter Neary. 2017. "Not so demanding: Demand structure and firm behavior." *American Economic Review*, Vol. 107 (12): 3835-74.

- [43] Mrazova, Monika, and J. Peter Neary. 2019. "IO for exports (s)." *International Journal of Industrial Organization*: 102561.
- [44] Mrazova, Monika, J. Peter Neary, and Mathieu Parenti. 2021. "Sales and markup dispersion: Theory and empirics.", *Econometrica*, Vol. 89(4): 1753-1788.
- [45] Murphy, Kevin M., Andrei Shleifer, and Robert Vishny. 1989. Income distribution, market size, and industrialization. *The Quarterly Journal of Economics*, 104(3), 537-564.
- [46] Nakamura, Emi, and Dawit Zerom. 2010. "Accounting for incomplete pass-through." *The Review of Economic Studies*, Vol. 77(3): 1192-1230.
- [47] Neary, J. Peter. 2004. "Monopolistic competition and international trade theory," in *The Monopolistic Competition Revolution in Retrospect*, ed. by S. Brakman and B. J. Heijdra. Cambridge: Cambridge University Press, 159–184.
- [48] Pigou, Arthur C. 1920. "The Economics of Welfare," Palgrave Macmillan.
- [49] Pollak, Robert A. 1971. "Additive utility functions and linear Engel curves." *The Review of Economic Studies*, Vol. 38(4): 401-414.
- [50] Reinert, Kenneth A., and David W. Roland-Holst. 1992. "Disaggregated Armington elasticities for the mining and manufacturing sectors of the United States." *Journal of Policy Modeling*, Vol. 14(5): 1-28.
- [51] Ruhl, Kim J. (2008). The international elasticity puzzle. unpublished paper, NYU.
- [52] Samuelson, Paul A., and William D. Nordhaus. 2010. Nordhaus. "Economics (19. Aufl.)."
- [53] Simonovska, Ina. 2015. "Income differences and prices of tradables: Insights from an online retailer." *The Review of Economic Studies*, Vol. 82(4): 1612-1656.
- [54] Spence, Michael. 1976. "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies*, Vol. 43: 217–235.
- [55] Syverson, Chad. 2007. "Prices, spatial competition and heterogeneous producers: An empirical test." *Journal of Industrial Economics*, Vol. 55(2): 197–222.
- [56] Tarasov, Alexander. 2009. "Income distribution, market structure, and individual welfare." *The BE Journal of Theoretical Economics*, 9(1).
- [57] Vives, Xavier. 2001. Oligopoly pricing: old ideas and new tools. MIT press.
- [58] Yeaple, Stephen Ross. 2005. "A simple model of firm heterogeneity, international trade, and wages." *Journal of International Economics*, Vol. 65(1): 1-20.
- [59] Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-Francois Thisse. 2012. "Monopolistic competition: Beyond the constant elasticity of substitution." *Econometrica*, Vol. 80(6): 2765-2784.

Appendices for Online Publication

Appendix A. Second order condition

In this Appendix, we discuss the second order condition (8), $\Lambda \equiv \int (2\varepsilon - r_h)\varepsilon_h x_h dG > 0$. This clearly holds for $r_h < 0$. We now show that this condition holds for subconvex demands $\varepsilon'_h < 0$ with $r_h > 0$ and $r'_h > 0$.

Consider first the behavior of the function $\Psi \equiv \int (2\varepsilon_h - r_h) x_h dG$ when $\varepsilon'_h < 0$. Using (3), we have $\Psi > 0$ because

$$\Psi = \int (\varepsilon_h - 1 - \varepsilon'_h x_h) x_h dG = (\varepsilon - 1)x - \int \varepsilon'_h x_h^2 dG > 0.$$

Furthermore, since $\varepsilon > 0$, we have $\varepsilon\Psi > 0$. Then,

$$\begin{aligned} \varepsilon\Psi &= \int (2\varepsilon\varepsilon_h - \varepsilon r_h) x_h dG = \int (2\varepsilon - r_h)\varepsilon_h x_h dG - \int (\varepsilon - \varepsilon_h) r_h x_h dG \\ &= \Lambda - \int (\varepsilon - \varepsilon_h) r_h x_h dG > 0, \end{aligned}$$

implies that

$$\Lambda > \int (\varepsilon - \varepsilon_h) r_h x_h dG.$$

We now prove that the right-hand side of the latter condition, $\int (\varepsilon - \varepsilon_h) r_h x_h dG > 0$, holds under the conditions $\varepsilon'_h < 0$ and $r'_h > 0$. We can rewrite this condition as $\int r(x_h) f(x_h) dG > 0$ where $f(x_h) \equiv (\varepsilon - \varepsilon(x_h))x_h$. The function $f(x_h)$ is a continuous and increasing since $\varepsilon' < 0$. Note that $x_h = x(s_h)$ is continuous and increasing in s_h : $x'(s_h) > 0$. Integrating by parts gives

$$\begin{aligned} \int_{s_0}^{s_1} r(x(s_h)) f(x(s_h)) dG(s_h) &= \left[r(x(s_h)) \int_{s_0}^{s_h} f(x(\xi)) dG(\xi) \right]_{s_h=s_0}^{s_h=s_1} \\ &\quad - \int_{s_0}^{s_1} r'(x(s_h)) x'(s_h) \left(\int_{s_0}^{s_h} f(x(\xi)) dG(\xi) \right) dG(s_h). \end{aligned}$$

The first term vanishes because

$$\int_{s_0}^{s_1} f(x(\xi)) dG(\xi) = \int_{s_0}^{s_1} (\varepsilon - \varepsilon(x_h)) x_h dG = \varepsilon x - \int \varepsilon_h x_h dG = 0 \quad (27)$$

since $\varepsilon = \int \varepsilon_h x_h dG / x$ by (7). The second term is positive because the integral $\int_{s_0}^{s_h} f(x(\xi)) dG(\xi)$ is negative. Indeed, because $f(x)$ is increasing in x and $x(\xi)$ is increasing in ξ , this integral is a convex function of s_h . Since it furthermore has zeroes at $s_h = s_0$ and $s_h = s_1$ by (27), this integral is strictly negative on the interval (s_0, s_1) .

Appendix B. Fixed point

The fixed point can be shown as it follows. Note that, using (9), the market demand elasticity (7) at the equilibrium can be expressed as

$$\varepsilon = \frac{\int \varepsilon_h s_h dG}{\int s_h dG}. \quad (28)$$

The optimal price (10), entry and product market (11) conditions imply the following condition for the existence of an equilibrium:

$$\frac{\int x_h \varepsilon_h dG}{\int x_h dG} = \frac{cL}{f} \int x_h dG + 1. \quad (29)$$

Using $z \equiv 1/(np)$ so that $x_h = s_h z$ and using $s = \int s_h dG$, the equilibrium condition writes as

$$\frac{1}{s} \int s_h \varepsilon(s_h z) dG = \frac{cLs}{f} z + 1,$$

The right-hand side is a function of z that increases and lies above one. The left-hand side decreases in z under subconvex demands ($\varepsilon' < 0$). It lies above one at $z = 0$ if and only if $\varepsilon(0) > 1$. Therefore, there exists a fixed point if and only if $\varepsilon(0) > 1$.

Appendix C. Log-linearization of closed economy equilibrium

We first log-linearize the FOC (10): $(p - c)/p = 1/\varepsilon$. Using the definition of ε , we write the latter as

$$(p - c) \int x_h \varepsilon(x_h) dG = p \int x_h dG$$

and totally differentiate it as

$$dp \int x_h \varepsilon_h dG + (p - c) \int (x_h \varepsilon(x_h))' dx_h dG = dp \int x_h dG + p \int dx_h dG,$$

Note that $(x_h \varepsilon(x_h))' = -1 + r_h$ by (3) and (4). Using $(p - c) = p/\varepsilon$ by (10) and $\hat{x}_h = d \ln x_h = dx_h/x_h$, this yields

$$\hat{p} = \frac{dp}{p} = \frac{\int (1 + \varepsilon - r_h) x_h \hat{x}_h dG}{\varepsilon(\varepsilon - 1)x}$$

Other conditions (9) and (11) are log-linearized in the same way and yield Table 1. Finally, we can replace \hat{x}_h by its value in Table 1 and simplify the expression of \hat{p} as

$$\hat{p} = \frac{\int (1 + \varepsilon - r_h) x_h (\hat{s}_h - \hat{s}) dG}{\varepsilon \int (2\varepsilon - r_h) x_h dG}.$$

In the closed economy, the budget constraint $pnx_h = s_h$ gives the consumption levels and changes as $x_h = s_h/pn$ and $\hat{x}_h = d \ln x_h = d \ln s_h = \hat{s}_h$. Note also that $\int (\hat{s}_h - \hat{s}) s_h dG = \int \hat{s}_h s_h dG - \hat{s} \int s_h dG = \int ds_h dG - \hat{s} s = 0$. Therefore, $\int (1 + \varepsilon) x_h (\hat{s}_h - \hat{s}) dG = (1 + \varepsilon)$

$\int (\widehat{s}_h - \widehat{s}) s_h dG = 0$. The price change simplifies to

$$\widehat{p} = -\frac{1}{\varepsilon \Psi} \int r_h (\widehat{s}_h - \widehat{s}) s_h dG,$$

where $\Psi \equiv \int (2\varepsilon - r_h) s_h dG$. This gives (12). Note that Ψ is positive under subconvexity of demand.

Appendix D. Pollak preferences

We characterize the class of utility functions $u(x)$ that solve the differential equation $r(x) = u'(x)u'''(x)/(u''(x))^2 = 1 + \sigma$ with $u'' < 0 < u'$. This identity is equivalent to

$$g'/g^2 = \sigma \quad \text{and} \quad u''/u' = g \tag{30}$$

where $g < 0$. We can sequentially solve the first differential equation for g and then the second one for u' . Since utility u is defined up an affine transformation, we report its simplest form.

Consider first $\sigma = 0$. Then, (30) is equivalent to $g' = 0$ and $u''/u' = -\alpha$ where $\alpha > 0$ is a first integration constant. This solves as $u' = \alpha e^{-\alpha(x-\gamma)}$ for $x > \gamma$ where $\gamma \in \mathbb{R}$ is another integration constant. Since u is defined up an affine transformation, we report the subutility function $u(x) = 1 - e^{-\alpha(x-\gamma)}$. The utility function u is the integral of the last expression. Then, $\varepsilon(x) = 1/(\alpha x)$, which decreases in x . So, $1 - \varepsilon + r = -x\varepsilon'(x) > 0$.

Consider then $\sigma = 1$. Then, (30) accepts the class of solutions $g = -(x - \gamma)^{-1}$ and $u' = k_1 (x - c)^{-1}$ for $x > \gamma$ and the integration constant $k_1 > 0$. The utility function u is the integral of the last expression. Since u is defined up an affine transformation, we can report utility function $u(x) = \ln(x + \gamma)$ for $x > \gamma \in \mathbb{R}$. Then, $\varepsilon(x) = 1 + \gamma/x$, which is a decreasing function if and only if $\gamma > 0$. Under this last condition, $1 - \varepsilon + r = -x\varepsilon'(x) > 0$.

Consider finally $\sigma > 1$, (30) accepts the class of solutions $g = -(x - \gamma)^{-\frac{1}{\sigma}}$ and $u' = k_1 (x - \gamma)^{1-\frac{1}{\sigma}}$ for $x > c$. The utility function u is the integral of the last expression. Since u is defined up an affine transformation and u must be an increasing function, we propose $u(x) = (\sigma - 1) \cdot (x + \gamma)^{1-\frac{1}{\sigma}}$ for $x > \gamma \in \mathbb{R}$. Then, $\varepsilon(x) = \sigma(1 + \gamma/x)$, which is a decreasing function if and only if $\gamma > 0$. Under this condition, $1 - \varepsilon + r = -x\varepsilon'(x) > 0$.

Note that, for $\sigma = -1$, we obtain an affine transformation of the quadratic utility function $u(x) = x(x - \gamma)$ for $x > \gamma \in \mathbb{R}$. Then, $\varepsilon(x) = \gamma/(2x) - 1$, which is a decreasing function since $\gamma > 0$. Under this condition, $1 - \varepsilon + r = -x\varepsilon'(x) > 0$.

Appendix E. Proof of Proposition 3

The numerator of the right-hand side of (18) can be integrated by parts as

$$\int_{s_0}^{s_1} r(x_h) \widehat{s}_h s_h dG(s_h) = \left[r(x_h) \int_{s_0}^{s_h} \widehat{s}_l s_l dG(s_l) \right]_{s_0}^{s_1} - \int_{s_0}^{s_1} r'(x_h) \frac{\partial x_h}{\partial s_h} \left(\int_{s_0}^{s_h} \widehat{s}_l s_l dG(s_l) \right) ds_h,$$

where we temporarily make explicit the variable of income distribution function $G(s_h)$ for the sake of clarity. The first term is zero because of mean preservation. Thus, (18) takes the form

$$\widehat{p} = \frac{1}{\varepsilon\Psi} \int_{s_0}^{s_1} r'(x_h) \frac{\partial x_h}{\partial s_h} \left(\int_{s_0}^{s_h} \widehat{s}_l s_l dG(s_l) \right) ds_h$$

where $\partial x_h / \partial s_h > 0$ due to (9). A mean-preserving contraction implies the second-order stochastic dominance of the final distribution of income s_h . In terms of relative income changes \widehat{s}_h , it implies that $\int_{s_0}^{s_h} s_l \widehat{s}_l dG(s_l) \geq 0$ for all s_h . To show this, consider an initial and final distribution $G^A(s_h)$ and $G^B(s_h)$. B is a mean-preserving contraction of A if and only if A is second-order stochastically dominated by B ; that is, iff $\int_{s_0}^{s_h} [G^B(z) - G^A(z)] dz \leq 0$ for all s_h . Consider an income mapping $m(s_h)$ such that $s_h^B = s_h^A + m(s_h^A)$, with $1 + m' > 0$ and m close to zero. We have $G^A(s_h) = G^B(s_h + m(s_h))$. So, $\int_{s_0}^{s_h} [G^B(z) - G^A(z)] dz = \int_{s_0}^{s_h} [G^B(z) - G^B(z + m(z))] dz \simeq - \int_{s_0}^{s_h} m(z) dG^B(z)$. Hence, the income mapping $m(s_h)$ gives a change in the distribution such that resulting distribution B is a mean-preserving contraction of initial distribution A if and only if $\int_{s_0}^{s_h} m(z) dG(z) \geq 0$ for all s_h . In the previous analysis, $m(s_h)$ is equal to $s_h^B - s_h^A$, which is equivalent to $ds_h = \widehat{s}_h s_h$. So, the change in individual income \widehat{s}_h is associated with a mean-preserving contraction of income distribution if and only if $\int_{s_0}^{s_h} \widehat{s}_h s_h dG \geq 0$ for all s_h .

Similarly, the condition $\int_{s_0}^{s_h} s_l \widehat{s}_l dG(s_l) \leq 0$ holds for mean-preserving spread. Therefore, the equilibrium price increases if $r'(x_h)$ is positive for all consumption levels x_h . Finally, this conclusion holds if we integrate over a set of infinitesimally small changes \widehat{s}_h and therefore for any finite change in the income distribution.

Appendix F. Demand properties

In this appendix we characterize the demand properties of the demand functions proposed in Table 2.

Demands with **constant super-elasticity** are given by $p(x_h) = e^{-\frac{1}{\alpha\beta}x_h^\alpha} / \lambda_h$ with $x \in \mathbb{R}^+$ and $\alpha, \beta > 0$. Note that, for $\alpha = 1$, this matches the demand function under CARA preferences. This implies that $\varepsilon(x_h) = \beta x_h^{-\alpha} > 0$ and $\varepsilon'(x_h) = -\alpha\beta x_h^{-\alpha-1} < 0$, i.e., individual demand is subconvex. Using (3), $r(x_h) = 1 + \varepsilon(x_h) + x_h \varepsilon'(x_h) = 1 + (1 - \alpha)\beta x_h^{-\alpha}$ so that $r(x_h)$ increases if and only if $\alpha > 1$. One computes $u(z) = \int_0^z e^{-z^\alpha} dx - u(0)$. One can numerically check that $\eta(x) = xu'(x)/u(x)$ is a decreasing function of x for all $x, \alpha > 0$.

Translog functions are given by $p(x_h) = (\alpha + \beta \log x_h) / (\lambda_h x_h)$ with $x \in (\exp(-\alpha/\beta), \infty)$ and $\alpha, \beta > 0$. This yields $p'(x_h) = -(\alpha + \beta \log x_h - \beta) / (\lambda_h x_h^2)$, which is negative for $x_h > \underline{x} \equiv \exp(1 - \alpha/\beta)$. Hence the domain of definition and concavity of $u(x_h)$ is (\underline{x}, ∞) . Furthermore, one computes $\varepsilon(x_h) = 1 + \beta / (\alpha + \beta \log x_h - \beta) > 1$ and $\varepsilon'(x_h) = -\beta^2 / [x_h(\alpha + \beta \log x_h - \beta)^2] < 0$. Individual demand is therefore subconvex. Using (3), it can be checked that $r(x_h) = 1 + x_h \varepsilon'(x_h) + \varepsilon(x_h) = \varepsilon(x_h)(3 - \varepsilon(x_h))$ so that $r'(x_h) = (3 - 2\varepsilon(x_h))\varepsilon'(x_h)$, which is positive if and only if $\varepsilon(x_h) > 3/2$. Using the definition of $p(x_h)$, we have $u'(x_h) = (\alpha + \beta \log x_h) / x_h$, which integrates to $u(x_h) = \alpha \log x_h + (\beta/2) \log^2 x_h$. Thus, $\eta(x_h) = x_h u'(x_h) / u(x_h) = (\log x_h)^{-1} +$

$(2\alpha/\beta + \log x_h)^{-1}$ is a decreasing function since it is a sum of two decreasing functions.

Consider the **CREMR** inverse demand function: $p(x_h) = (x_h - \beta)^{\frac{\alpha}{\alpha+1}} / (\lambda_h x_h)$, defined for $x_h \in (\beta, \infty)$ and $\alpha, \beta > 0$. Thus, $p'(x_h) = -(x_h - \beta)^{-\frac{1}{\alpha+1}} (x_h - \underline{x}) / (\lambda_h x_h^2 (\alpha + 1))$, which is negative if $x_h > \underline{x} \equiv (\alpha + 1)\beta > \beta$. Hence the domain of definition and concavity of $u(x_h)$ is (\underline{x}, ∞) . The elasticity of demand is given by $\varepsilon(x_h) = 1 + \alpha x_h (x_h - \underline{x})^{-1} > 1$ and $\varepsilon'(x_h) = -\alpha(\alpha + 1)\beta(x_h - \underline{x})^{-2} < 0$. Individual demand is therefore subconvex. Furthermore, one computes $r(x_h) = 2 + \alpha x_h (x_h - 2\underline{x})(x_h - \underline{x})^{-2}$ and $r'(x_h) = 2\alpha(\alpha + 1)^2 \beta^2 (x_h - \underline{x})^{-3} > 0$. Our simulations also show that $\eta(x_h)$ may decrease or increase depending on the parameters of demand.

Consider **constant proportional pass-through** (CPPT) demand with $p(x_h) = (x_h^{-\alpha} + \beta)^{-\frac{1}{\alpha}} / (\lambda_h x_h)$ for $x \in \mathbb{R}^+$ and $\alpha, \beta > 0$. Its derivative is given by $p'(x_h) = -\beta (x_h^{-\alpha} + \beta)^{-\frac{1+\alpha}{\alpha}} / (\lambda_h x_h^2) < 0$. Elasticity of individual demand takes the form $\varepsilon(x_h) = 1 + x_h^{-\alpha} / \beta > 1$ and $\varepsilon'(x_h) = -\alpha x_h^{-\alpha-1} / \beta < 0$. Individual demand is therefore subconvex. Furthermore, $r(x_h) = 2 - (\alpha - 1)x_h^{-\alpha} / \beta$ and $r'(x_h) = (\alpha - 1)\alpha x_h^{-\alpha-1} / \beta$. Thus, $r'(x_h) > 0$ if and only if $\alpha > 1$ while $r'(x_h) < 0$ otherwise.

Consider the **CEMR** demand functions: $p(x_h) = (x_h^{\frac{\alpha}{\alpha+1}} - \beta) / (\lambda_h x_h)$ for $x \in (\beta, \infty)$ and $\alpha, \beta > 0$. Thus, $p'(x_h) = -(x_h^{\frac{\alpha}{\alpha+1}} - \underline{x}^{\frac{\alpha}{\alpha+1}}) / [\lambda_h x_h^2 (\alpha + 1)] < 0$ if $x_h > \underline{x} \equiv [(\alpha + 1)\beta]^{\frac{\alpha+1}{\alpha}} > \beta^{\frac{\alpha+1}{\alpha}}$. Hence, those demands are defined and decreasing over the support (\underline{x}, ∞) . The elasticity of individual demand is given by $\varepsilon(x_h) = (\alpha + 1)(x_h^{\frac{\alpha}{\alpha+1}} - \beta) / (x_h^{\frac{\alpha}{\alpha+1}} - (\alpha + 1)\beta) > 1$ while $\varepsilon'(x_h) = -\alpha^2 \beta x_h^{-\frac{1}{\alpha+1}} (x_h^{\frac{\alpha}{\alpha+1}} - (\alpha + 1)\beta)^{-2} < 0$. This demand system is therefore subconvex. Furthermore, taking derivative of $r(x_h) = 1 + x_h \varepsilon'(x_h) + \varepsilon(x_h)$ shows that $r'(x_h) \geq 0$ if and only if $x_h \leq \bar{x}$ where $\bar{x} \equiv [(2\alpha + 1)(\alpha + 1)\beta]^{\frac{\alpha+1}{\alpha}} > \underline{x}$. Integrating $u'(x_h) = (x_h^{\frac{\alpha}{\alpha+1}} - \beta) / x_h$, we get $u(x_h) = (\alpha + 1)x_h^{\frac{\alpha}{\alpha+1}} / \alpha - \beta \log x_h$. Thus, $\eta(x_h) = (x_h^{\frac{\alpha}{\alpha+1}} - \beta) / (\frac{\alpha+1}{\alpha} x_h^{\frac{\alpha}{\alpha+1}} - \beta \log x_h)$ which decreases for large values of x_h while it might increase for low enough x_h . One can show that $\eta(x_h)$ decreases for all x_h if $\beta > (\alpha + 1)^{-1} \exp[(2\alpha + 1) / (\alpha + 1)]$ while it can increase for low values of x_h otherwise. Using (26), the elasticity of pass-through \mathcal{E}_{pt} is larger than 0.5 if and only if α and/or β are small enough.

Consider finally the **inverse translated CES**, $p(x_h) = (x_h^{-\frac{\alpha}{\alpha+1}} - \beta) / \lambda_h$ for $x \in (\beta^{-\frac{\alpha+1}{\alpha}}, \infty)$ and $\alpha, \beta > 0$. This implies $p'(x_h) = -\frac{\alpha}{\alpha+1} \frac{1}{\lambda_h} x_h^{-\frac{2\alpha+1}{\alpha+1}} < 0$. The elasticity of individual demand is given by $\varepsilon(x_h) = \frac{\alpha+1}{\alpha} (1 - \beta x_h^{\frac{\alpha}{\alpha+1}})$ and $\varepsilon'(x_h) = -\frac{\alpha+1}{\alpha} \beta x_h^{-\frac{1}{\alpha+1}} < 0$. This demand is subconvex. We also have $r(x_h) = \frac{2\alpha+1}{\alpha} (1 - \beta x_h^{\frac{\alpha}{\alpha+1}})$ and $r'(x_h) = -\frac{2\alpha+1}{\alpha} \beta x_h^{-\frac{1}{\alpha+1}} < 0$. Using $u'(x_h) = (x_h^{-\frac{\alpha}{\alpha+1}} - \beta)$, we integrate so that $u(x_h) = (\alpha + 1)x_h^{\frac{1}{\alpha+1}} - \beta x_h$. Thus, $\eta(x_h) = \frac{x_h u'(x_h)}{u(x_h)} = (x_h^{\frac{1}{\alpha+1}} - \beta x_h) / [(\alpha + 1)x_h^{\frac{1}{\alpha+1}} - \beta x_h]$ and $\eta'(x_h) = -\frac{\alpha^2}{\alpha+1} \beta x_h^{\frac{1}{\alpha+1}} [(\alpha + 1)x_h^{\frac{1}{\alpha+1}} - \beta x_h]^{-2} < 0$. Therefore, $\eta(x)$ decreases for all values of x_h . Using (26), the elasticity of pass-through $\mathcal{E}_{pt} < 1/2$, when x_h is high enough, otherwise $\mathcal{E}_{pt} > 1/2$.

Appendix G. Trade equilibrium

The monopolistic competitive equilibrium is defined as the set of variables $\{x_h, x_h^*, i_h, i_h^*, p, p^*, p_i, p_i^*, y, y^*, y_i, y_i^*, w, w^*, n, n^*\}$ that are consistent with the following relationships:

Consumer	$np_x + n^*p_i i_h = s_h w$	$n^*p^*x_h^* + np_i^*i_h^* = s_h w^*$
	$p/p_i = u'(x_h)/u'(i_h)$	$p^*/p_i^* = u'(x_h^*)/u'(i_h^*)$
FOC	$p = \frac{\varepsilon}{\varepsilon-1}cw$	$p^* = \frac{\varepsilon^*}{\varepsilon^*-1}cw^*$
	$p_i^* = \frac{\varepsilon_i^*}{\varepsilon_i^*-1}cw$	$p_i = \frac{\varepsilon_i}{\varepsilon_i-1}cw^*$
Entry	$(p - cw)y + (p_i^* - cw)y_i^* = fw$	$(p^* - cw^*)y^* + (p_i - cw^*)y_i = w^*f$
Product	$y = L \int x_h dG$	$y^* = L^* \int x_h^* dG$
	$y_i^* = L^* \int i_h^* dG$	$y_i = L \int i_h dG$
Labor	$L \int s_h dG = n(f + c(y + y_i^*))$	$L^* \int s_h dG = n^*(f + c(y^* + y_i))$

Table F1: Trade equilibrium conditions

Under symmetry, we have $L = L^*, x_h = x_h^* = i_h = i_h^*, p = p^* = p_i = p_i^*, y = y^* = y_i = y_i^*, w = w^*$ and $n = n^*$. So, we can simplify the above conditions as

Consumer	$2np_x = s_h w$
FOC	$p = \frac{\varepsilon}{\varepsilon-1}cw$
Entry	$2(p - cw)y = fw$
Product market	$y = L \int x_h dG$
Labor market	$L \int s_h dG = n(f + 2cy)$

Table F2: Symmetric trade equilibrium conditions

Those conditions yield the same equilibrium conditions as in the closed economy if we divide each country population by two; in particular, $(L^o, y^o, n^o) = (L^c/2, y^c/2, n^c/2)$ where the superscripts o and c stand for the open and closed economies. Therefore, the symmetric equilibrium exists under the same equilibrium conditions as in closed economy. In this case, revenues, costs and elasticities are related in the following way:

$$\frac{p - cw}{p} = \frac{1}{\varepsilon}, \quad \frac{cwy}{py} = 1 - \frac{1}{\varepsilon}, \quad \frac{2cwy}{f} = \varepsilon - 1, \quad \text{and} \quad \frac{f}{2py} = \frac{1}{\varepsilon}.$$

Also symmetry guarantees that, as in closed economy, the following condition holds $x_h/x = s_h/s$.

Appendix H. Trade and income redistribution

Equilibrium conditions can be log-linearized about the symmetric equilibrium as follows:

Consumer	$\frac{1}{2}(\hat{n} + \hat{p} + \hat{x}_h) + \frac{1}{2}(\hat{n}^* + \hat{p}_i + \hat{i}_h) = \hat{s}_h + \hat{w}$	$\frac{1}{2}(\hat{n}^* + \hat{p}^* + \hat{x}_h^*) + \frac{1}{2}(\hat{n} + \hat{p}_i^* + \hat{i}_h^*) = 0$
	$\hat{i}_h - \hat{x}_h = \varepsilon_h(\hat{p} - \hat{p}_i)$	$\hat{i}_h^* - \hat{x}_h^* = \varepsilon_h(\hat{p}^* - \hat{p}_i^*)$
FOC	$\hat{p} - \hat{w} = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h) x_h \hat{x}_h dG$	$\hat{p}^* = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h) x_h \hat{x}_h^* dG$
	$\hat{p}_i^* - \hat{w} = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h) x_h \hat{i}_h^* dG$	$\hat{p}_i = \frac{1}{x\varepsilon(\varepsilon-1)} \int (1 + \varepsilon - r_h) x_h \hat{i}_h dG$
Entry	$\frac{1}{2}\varepsilon(\hat{p} + \hat{p}_i^*) + \frac{1}{2}(\hat{y} + \hat{y}_i^*) = \varepsilon\hat{w}$	$\frac{1}{2}\varepsilon(\hat{p}^* + \hat{p}_i) + \frac{1}{2}(\hat{y}^* + \hat{y}_i) = 0$
Product	$\hat{y} = \frac{1}{x} \int x_h \hat{x}_h dG$	$\hat{y}^* = \frac{1}{x} \int x_h \hat{x}_h^* dG$
	$\hat{y}_i^* = \frac{1}{x} \int x_h \hat{i}_h^* dG$	$\hat{y}_i = \frac{1}{x} \int x_h \hat{i}_h dG$
Labor	$\hat{s} = \hat{n} + \frac{1}{2} \frac{\varepsilon-1}{\varepsilon} (\hat{y} + \hat{y}_i^*)$	$0 = \hat{n}^* + \frac{1}{2} \frac{\varepsilon-1}{\varepsilon} (\hat{y}^* + \hat{y}_i)$

Table I.1: Log-linearization around symmetric trade equilibrium

We proceed in two steps.

Step 1. First, we show that $\hat{w} = 0$. To this end, we take the difference of price changes in country 1 and get

$$\hat{p} - \hat{p}_i = \hat{w} + \frac{\int (1 + \varepsilon - r_h) x_h (\hat{x}_h - \hat{i}_h) dG}{(\varepsilon - 1)\varepsilon x}.$$

Combining it with the second line of Table I.1 leads to

$$\hat{p} - \hat{p}_i = \hat{w} - \frac{\int (1 + \varepsilon - r_h) x_h \varepsilon_h (\hat{p} - \hat{p}_i) dG}{(\varepsilon - 1)\varepsilon x},$$

or, after simplifications,

$$\hat{p} - \hat{p}_i = \frac{\hat{w}}{a},$$

where $a = \frac{\int (2\varepsilon - r_h) \varepsilon_h x_h dG}{(\varepsilon - 1)\varepsilon x} > 0$ by the second order condition (8). By analogue, in country 2

$$\hat{p}^* - \hat{p}_i^* = -\frac{\hat{w}}{a}$$

Therefore,

$$\hat{i}_h - \hat{x}_h = (\hat{p} - \hat{p}_i)\varepsilon_h = \frac{\hat{w}\varepsilon_h}{a}, \quad \hat{i}_h^* - \hat{x}_h^* = (\hat{p}^* - \hat{p}_i^*)\varepsilon_h = -\frac{\hat{w}\varepsilon_h}{a}.$$

Plugging $\hat{i}_s - \hat{x}_s$ into difference of firm outputs

$$\hat{y} - \hat{y}_i = \frac{\int x_h (\hat{x}_h - \hat{i}_h) dG}{x}$$

we obtain

$$\hat{y} - \hat{y}_i = -\frac{\hat{w}\varepsilon}{a},$$

while similar equations for country 2 yields

$$\hat{y}^* - \hat{y}_i^* = \frac{\hat{w}\varepsilon}{a}.$$

Combining entry conditions for both countries

$$\varepsilon\hat{p} + \varepsilon\hat{p}_i^* + \hat{y} + \hat{y}_i^* = 2\varepsilon\hat{w}, \quad \varepsilon\hat{p}_i + \varepsilon\hat{p}^* + \hat{y}_i + \hat{y}^* = 0$$

leads to

$$\varepsilon(\hat{p} - \hat{p}_i) + \varepsilon(\hat{p}_i^* - \hat{p}^*) + \hat{y} - \hat{y}_i + \hat{y}_i^* - \hat{y}^* = 2\varepsilon\hat{w}.$$

Plugging the differences for price and output changes into the last equation, we get

$$\frac{\hat{w}}{a}\varepsilon + \frac{\hat{w}}{a}\varepsilon - \frac{\hat{w}\varepsilon}{a} - \frac{\hat{w}\varepsilon}{a} = 2\varepsilon\hat{w},$$

thus, $\hat{w} = 0$ which yield

$$\hat{p} = \hat{p}_i, \quad \hat{p}_i^* = \hat{p}^*, \quad \hat{l}_h = \hat{x}_h, \quad \hat{l}_h^* = \hat{x}_h^*, \quad \hat{y} = \hat{y}_i, \quad \hat{y}^* = \hat{y}_i^*, \quad \hat{n} = \hat{n}^* + \hat{s}.$$

Step 2. The first two lines of Table I.1 take the form

$$2\hat{x}_h = 2\hat{s}_h + \hat{s} - 2\hat{n} - 2\hat{p},$$

$$2\hat{x}_h^* = \hat{s} - 2\hat{n} - 2\hat{p}^*.$$

By plugging product market clearing conditions into entry and labor market clearing conditions, we obtain

$$\int x_h(\hat{x}_h + \hat{x}_h^*)dG = -\varepsilon x(\hat{p} + \hat{p}^*),$$

$$\hat{n} = \hat{s} - \frac{1}{2} \frac{\varepsilon - 1}{\varepsilon x} \int x_h(\hat{x}_h + \hat{x}_h^*)dG.$$

Combining these two equations results in

$$\hat{n} = \hat{s} + \frac{1}{2}(\varepsilon - 1)(\hat{p} + \hat{p}^*)$$

Replacing \hat{n} in equations for \hat{x}_h^* and \hat{x}_h yields

$$2\hat{x}_h = 2\hat{s}_h - \hat{s} - (\varepsilon - 1)(\hat{p} + \hat{p}^*) - 2\hat{p},$$

$$2\hat{x}_h^* = -\hat{s} - (\varepsilon - 1)(\hat{p} + \hat{p}^*) - 2\hat{p}^*.$$

Plugging it into FOC, we get

$$2\hat{p} = \frac{1}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h (2\hat{s}_h - \hat{s} - (\varepsilon - 1)(\hat{p} + \hat{p}^*) - 2\hat{p}) dG,$$

$$2\widehat{p}^* = \frac{1}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h (-\widehat{s} - (\varepsilon - 1)(\widehat{p} + \widehat{p}^*) - 2\widehat{p}^*) dG.$$

Taking the sum of the two, we obtain

$$\widehat{p} + \widehat{p}^* = \frac{\int (1 + \varepsilon - r_h) x_h (\widehat{s}_h - \widehat{s}) dG}{\varepsilon \int (2\varepsilon - r_h) x_h dG}.$$

Plugging it back to \widehat{p} and \widehat{p}^* yields

$$2\widehat{p} = \frac{1}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h (2\widehat{s}_h - \widehat{s} - (\varepsilon - 1)(\widehat{p} + \widehat{p}^*)) dG - \frac{2\widehat{p}}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h dG,$$

$$2\widehat{p}^* = \frac{1}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h (-\widehat{s} - (\varepsilon - 1)(\widehat{p} + \widehat{p}^*)) dG - \frac{2\widehat{p}^*}{x\varepsilon(\varepsilon - 1)} \int (1 + \varepsilon - r_h) x_h dG.$$

After simplifications, we obtain

$$\widehat{p} = \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) x_h (\widehat{s}_h - \widehat{s}) dG}{\varepsilon \int (2\varepsilon - r_h) x_h dG} + \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) x_h \widehat{s}_h dG}{x\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) x_h dG}$$

$$\widehat{p}^* = \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) x_h (\widehat{s}_h - \widehat{s}) dG}{\varepsilon \int (2\varepsilon - r_h) x_h dG} - \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) x_h \widehat{s}_h dG}{x\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) x_h dG}$$

Finally, using $x_h = ws_h/2np$ and $\int s_h (\widehat{s}_h - \widehat{s}) dG = 0$ yields

$$\widehat{p} = -\frac{1}{2} \frac{\int r_h (\widehat{s}_h - \widehat{s}) s_h dG}{\varepsilon \Psi} + \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) s_h \widehat{s}_h dG}{s\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) s_h dG}$$

$$\widehat{p}^* = -\frac{1}{2} \frac{\int r_h (\widehat{s}_h - \widehat{s}) s_h dG}{\varepsilon \Psi} - \frac{1}{2} \frac{\int (1 + \varepsilon - r_h) s_h \widehat{s}_h dG}{s\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) s_h dG}$$

Under a common and porportinal change in each income, we have $\widehat{s}_h = \widehat{s}$, which leads to expressions in Table 5.

Under mean-preserving changes, we have $\widehat{s} = 0$ and $\int s_h \widehat{s}_h dG = 0$, which leads to

$$\widehat{p} = -\frac{1}{2\varepsilon} \frac{\int r_h \widehat{s}_h s_h dG}{\int (2\varepsilon - r_h) s_h dG} - \frac{1}{2} \frac{\int r_h s_h \widehat{s}_h dG}{s\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) s_h dG},$$

$$\widehat{p}^* = -\frac{1}{2\varepsilon} \frac{\int r_h \widehat{s}_h s_h dG}{\int (2\varepsilon - r_h) s_h dG} + \frac{1}{2} \frac{\int r_h s_h \widehat{s}_h dG}{s\varepsilon(\varepsilon - 1) + \int (1 + \varepsilon - r_h) s_h dG}.$$

After simplification, we have

$$\widehat{p} = -\frac{1}{2\varepsilon\Psi} \left(\frac{\varepsilon\Psi}{s(\varepsilon - 1)^2 + \Psi} + 1 \right) \int r_h s_h \widehat{s}_h dG$$

$$\widehat{p}^* = \frac{1}{2\varepsilon\Psi} \left(\frac{\varepsilon\Psi}{s(\varepsilon - 1)^2 + \Psi} - 1 \right) \int r_h s_h \widehat{s}_h dG$$

Plugging those values in the other equations in Table I.1 and solving a linear system in

the aggregate variable, yields closed-form solutions for the changes in prices, output, and product diversity as reported in Table 4.

Appendix I. Welfare and trade

An individual in home country with income s_h gets an equilibrium utility $U_h = nu(x_h) + n^*u(i_h)$. Log-linearization of the utility yields

$$\hat{U}_h = \frac{1}{U_h} \left(u(x_h)n\hat{n} + nu'(x_h)x_h\hat{x}_h + u(i_h)n^*\hat{n}^* + n^*u'(i_h)i_h\hat{i}_h \right).$$

Using $U_h = 2nu(x_h)$ and $x_h = i_h$, we get

$$\hat{U}_h = \frac{\hat{n}}{2} + \frac{n^*\hat{n}^*}{2n} + \eta_h\hat{x}_h.$$

Plugging $\hat{n} = \hat{s}$ and $\hat{n}^* = 0$ results in

$$\hat{U}_h = \frac{\hat{s}}{2} + \eta_h\hat{x}_h.$$

Finally, make use of expression for \hat{x}_h in Table 5 gives us

$$\hat{U}_h = \left(1 + \frac{\varepsilon(\varepsilon - 1)s}{\Upsilon} \eta_h \right) \frac{\hat{s}}{2}.$$

We proceed in the same way for equilibrium welfare $U_h^* = n^*u(x_h^*) + nu(i_h^*)$ of an individual in foreign country. Log-linearization implies

$$\hat{U}_h^* = \frac{1}{U_h^*} \left(u(x_h^*)n^*\hat{n}^* + n^*u'(x_h^*)x_h^*\hat{x}_h^* + u(i_h^*)n\hat{n} + nu'(i_h^*)i_h^*\hat{i}_h^* \right).$$

Using $U_h^* = 2nu(x_h^*)$, $x_h^* = i_h^* = x_h$, $\hat{n} = \hat{s}$, $\hat{n}^* = 0$, and expression for \hat{x}_h in Table 5 yields

$$\hat{U}_h^* = \left(1 - \frac{\varepsilon(\varepsilon - 1)s}{\Upsilon} \eta_h \right) \frac{\hat{s}}{2}.$$

Now we replace $\Psi + s(\varepsilon - 1)^2$ to obtain

$$\hat{U}_h^* = \left(1 - \frac{\varepsilon(\varepsilon - 1)s}{\Psi + s(\varepsilon - 1)^2} \eta_h \right) \frac{\hat{s}}{2}.$$

Foreign residents gain from an increase in the average income in home if and only if

$$1 - \frac{\varepsilon(\varepsilon - 1)s}{\Psi + s(\varepsilon - 1)^2} \eta_h > 0.$$

Using $\Psi = \int (2\varepsilon_h - r_h)s_h dG$, it can be written as

$$(\varepsilon - 1)s(1 - \varepsilon(1 - \eta_h)) < \int (2\varepsilon_h - r_h)s_h dG.$$

Further simplifications leads to

$$-(\varepsilon - 1)\varepsilon s(1 - \eta_h) < \int (1 + \varepsilon_h - r_h)s_h dG.$$

The left-hand side is negative as $\varepsilon > 1$ and $\eta_h < 1$ while the right-hand side is positive under subconvex demands due to (3). Thus, under subconvex demands foreign residents always gain from an increase in the average income in home.

Appendix J. Calibration and simulation for lower elasticities of substitution

In this appendix we replicate the calibration and simulation exercises for the target elasticities $\varepsilon = 2.5$ and $\mathcal{E}_{pt} \in \{0.4, 0.6\}$ under CPPT and CSED preferences. It can be seen from Figure 1 that those target elasticities are feasible in the sense that they satisfy the restrictions on non-zero consumption and utility concavity in the calibration process. We can then estimate demand parameters α and β that match those elasticities and study the effect of income distribution and trade.

Table K.1 presents the direct and general equilibrium effects of income redistribution in a closed economy. The redistribution raises the bottom decile by 300% and is paid by the top decile. The table should be compared with Table 6 for the case with $\varepsilon = 7$. As it can be seen, the changes in the endogenous variables keep the same order of magnitude, that is slightly below the percentage point. Under CPPT, the changes in prices are slightly larger with the lower demand elasticity while the changes in the number of firms and firm scales are slightly lower. This is explained by the higher markups and profits that allow firms to more easily survive in their markets. The general equilibrium effects of income redistribution are roughly doubled. This suggests that a lower demand elasticity impacts more the income groups that are not directly concerned by the redistribution policy. Under CSED, the lower demand elasticity has effects of the same order of magnitude. However, in the third column when the pass-through elasticity is equal to $\mathcal{E}_{pt} = 0.4$, the direction of the effect is opposite to the one with higher demand elasticities. This is because the specific calibration to US data imposes a value of $\alpha = 0.96672$, which is lower than one and implies $r'_h < 0$ (see Appendix H). Therefore the general effects have same direction for $\mathcal{E}_{pt} = 0.4$ and 0.6.

	Direct effect	General equilibrium effects			
		CPPT		CSED	
α		1.14107	0.78097	0.96672	0.51598
β		91.3304	17.0354	0.04193	0.30921
ε		2.5	2.5	2.5	2.5
\mathcal{E}_{pt}		0.4	0.6	0.4	0.6
\hat{p} (%)	0.	0.46	-0.41	-0.11	-0.73
\hat{n} (%)	0.	0.69	-0.62	-0.16	-1.1
\hat{y} (%)	0.	-1.14	1.04	0.27	1.85
\hat{x}_1^{eq} (%)	300.	-1.69	1.4	0.4	2.41
\hat{x}_2^{eq} (%)	0.	-0.42	0.35	0.1	0.6
\hat{x}_3^{eq} (%)	0.	-0.39	0.32	0.09	0.55
\hat{x}_4^{eq} (%)	0.	-0.36	0.29	0.09	0.51
\hat{x}_5^{eq} (%)	0.	-0.34	0.27	0.08	0.48
\hat{x}_6^{eq} (%)	0.	-0.31	0.25	0.08	0.45
\hat{x}_7^{eq} (%)	0.	-0.27	0.22	0.07	0.41
\hat{x}_8^{eq} (%)	0.	-0.22	0.19	0.06	0.35
\hat{x}_9^{eq} (%)	0.	-0.15	0.14	0.03	0.27
\hat{x}_{10}^{eq} (%)	-4.93	0.02	0.04	-0.04	0.06

Table J.1: Effects of income redistribution in a closed economy: $\varepsilon = 2.5$.

Table K.2 presents the direct and general equilibrium effects of income redistribution in the open economy. It compares with Table 7. We firstly remark that the changes in the endogenous variables keep the same order of magnitude as for larger demand elasticities. However, the effect on prices, output and number of firms depends on the chosen specification. Under CPPT, the changes in price, output, number of firms and general equilibrium effect on welfare have same directions for all elasticity specifications. The changes in prices and output are more pronounced but the general equilibrium effects are however weaker in this lower elasticity scenario. Under CSED, effects have same directions because, as above, $r'_h < 0$ in both specifications of pass-through elasticities.

	Direct effect		General equilibrium effect						
			CPPT		CSED				
α	0.39		0.6		0.96		0.51		
β	1.14		0.78		0.04		0.3		
ε	2.5		2.5		2.5		2.5		
\mathcal{E}_{pt}	0.4		0.6		0.4		0.6		
	home	home	foreign	home	foreign	home	foreign	home	foreign
\hat{p} (%)	0.	1.35	-0.26	-0.95	0.14	-0.3	0.06	-1.46	0.23
\hat{n} (%)	0.	0.82	0.82	-0.61	-0.61	-0.18	-0.18	-0.92	-0.92
\hat{y} (%)	0.	-2.18	-0.58	1.54	0.45	0.47	0.1	2.35	0.68
\hat{x}_1^{eq} (%)	300.	-1.35	0.26	0.92	-0.17	0.28	-0.08	1.4	-0.29
\hat{x}_2^{eq} (%)	0.	-1.31	0.3	0.88	-0.21	0.28	-0.08	1.34	-0.35
\hat{x}_3^{eq} (%)	0.	-1.27	0.34	0.85	-0.24	0.28	-0.09	1.3	-0.39
\hat{x}_4^{eq} (%)	0.	-1.24	0.37	0.82	-0.26	0.27	-0.09	1.27	-0.42
\hat{x}_5^{eq} (%)	0.	-1.21	0.4	0.8	-0.29	0.27	-0.1	1.24	-0.44
\hat{x}_6^{eq} (%)	0.	-1.17	0.44	0.78	-0.3	0.26	-0.1	1.21	-0.47
\hat{x}_7^{eq} (%)	0.	-1.14	0.48	0.76	-0.33	0.25	-0.11	1.17	-0.51
\hat{x}_8^{eq} (%)	0.	-1.07	0.54	0.73	-0.36	0.24	-0.13	1.14	-0.55
\hat{x}_9^{eq} (%)	0.	-0.98	0.63	0.68	-0.41	0.22	-0.15	1.07	-0.62
\hat{x}_{10}^{eq} (%)	-4.95	-0.76	0.86	0.57	-0.52	0.12	-0.24	0.88	-0.81

Table J.2: Effects of home income redistribution in an open economy: $\varepsilon = 2.5$.

Appendix K. Pass through elasticity and the direction of general equilibrium effect

Here we provide a formal link between pass-through elasticity and the direction of general equilibrium effect for two demand classes, CSED and CPPT. We use expressions for x_h , ε_h , and r_h computed for these two demands in Appendix F and plug them into (26) to get pass-through elasticities. For CSED, we obtain

$$\mathcal{E}_{\text{pt}} = \frac{\varepsilon(\varepsilon - 1)x}{\int (2\varepsilon - r_h)x_h \varepsilon_h dG} = \frac{\varepsilon(\varepsilon - 1)x}{(2\varepsilon - 1)x\varepsilon - (1 - \alpha) \int \varepsilon_h^2 x_h dG}.$$

Then, $\mathcal{E}_{\text{pt}} < 1/2$ if

$$(1 - \alpha) \int \varepsilon_h^2 x_h dG < \int \varepsilon_h x_h dG.$$

Since in equilibrium

$$\varepsilon \equiv \frac{\int x_h \varepsilon_h dG}{\int x_h dG} > 1$$

which implies $\int x_h \varepsilon_h dG > \int x_h dG$, $\mathcal{E}_{pt} < 1/2$ if $\alpha > 1$. As reported in Table 2, $r'_h > 0$ if and only if $\alpha > 1$. Therefore, if $\mathcal{E}_{pt} > 1/2$ then $r'_h < 0$ which is the case when $\alpha < 1$. Note that $\mathcal{E}_{pt} < 1/2$ does not necessary imply $r'_h > 0$. However, in our quantification exercises the latter holds.

As to CPPT, the same procedure yields

$$\mathcal{E}_{pt} = \frac{\varepsilon x(\varepsilon - 1)}{2\varepsilon x(\varepsilon - 1) + \frac{\alpha-1}{\beta} \int x_h^{1-\alpha} \varepsilon_h dG}.$$

Therefore, $\mathcal{E}_{pt} \leq 1/2$ if and only if $\alpha \geq 1$. As $r'_h > 0$ if and only if $\alpha > 1$ for CPPT, we conclude that $r'_h > 0$ if and only if $\mathcal{E}_{pt} \leq 1/2$. This establishes a one-to-one correspondence between general equilibrium effect and the value of pass through elasticity for CPPT which opens a room for estimations of general equilibrium effect.