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# Reflections on “Testing for Unit Roots in Heterogeneous Panels”

## Abstract

This article is our personal perspective on the IPS test and the subsequent developments of unit root and cointegration tests in dynamic panels with and without cross-section dependence. In this note, we discuss the main idea behind the test and the publication process that led to Im, Pesaran and Shin (2003).

JEL-Codes: C010, C230.

Keywords: Dickey and Fuller statistic, stationarity, panel unit root tests, prevalence of unit roots.

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# 1 Context

Following the pioneering contributions of Dickey and Fuller (1979, 1981) and Phillips and Perron (1988) on testing for unit roots, and the work of Engle and Granger (1987) on cointegration, it became increasingly clear that very large time series are required for robust applications of these ideas in empirical economics. As shown by Dickey and Fuller (1981, DF), their DF test statistic converged to a function of Brownian motion (Said and Dickey, 1984), that led to left-skewed distribution functions that depend on the deterministic components such as intercept and trends. As a result, these unit root tests suffered from low power against alternative hypotheses in small samples (see Campbell and Perron, 1991). It was also equally recognized that, even if very long time series data were available, the processes generating them were more likely subject to breaks which considerably complicated the interpretation of the unit root tests applied to individual series (Perron, 1989). The alternative approach proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992, KPSS) to test stationarity did not resolve these shortcomings and the poor power performance of the unit root tests either, especially for alternatives with trends, continued to apply to the KPSS test.

There emerged two strands to the literature. One strand continued to focus on individual series but considered local-to-unity models where the roots only converged to unity with the time series sample size ( $T$ ). There were also concerted efforts to allow for breaks. The other strand of the literature set about exploiting the cross section dimension and, instead of asking if a particular series had a unit root, aimed at investigating the prevalence of the unit root hypothesis across the  $N$  series under consideration. Quah (1992, 1994) and Levin and Lin (1992, LL) were amongst the first to develop such panel based unit root tests. They used pooled and fixed effects (FE) panel data models with common slopes to develop panel unit root tests and thus enhance the power of unit root testing through the cross section dimension. This was achieved at the expense of assuming homogeneity which also gave rise to the misinterpretation of the test as either rejecting or not rejecting the unit root hypotheses for all the  $N$  units in the panel. The LL test was more general than the test proposed by Quah and quickly became popular, but required that  $N/T \rightarrow 0$  as  $N$  and  $T \rightarrow \infty$ . This ruled out many important panel datasets where the cross section dimension was often much larger than the time series dimension (particularly considering the issue of structural breaks alluded to above).

## 2 The main idea

At about the same time Pesaran and Smith (1995) showed that pooled panel estimation methods such as FE that assume homogeneous slopes lead to inconsistent estimates in dynamic heterogeneous panels, and this inconsistency does not vanish even if  $N$  and  $T \rightarrow \infty$ , jointly. This was a surprising result and it took the authors around five years to get the paper published. Previously, it was taken for granted that the heterogeneity of the slopes can be absorbed in the error term with little consequences for the FE estimation. This result was easy to establish for panels with strictly exogenous regressors under the assumption that heterogeneity is purely random and therefore uncorrelated with the regressors. However, in the case of dynamic panels with slope heterogeneity, the regressors (being lagged values of the dependent variable) became correlated with the heterogeneity of the slopes by construction, even under the pure random coefficient model, and thus violated the standard uncorrelated heterogeneity model assumed in the static panel literature. In view of this, Pesaran and Smith proposed estimating the population mean of the slope coefficients by using simple averages of the least squared estimates of the individual estimates. Subsequently, Pesaran, Smith and Im (1996, Section 8.2.2) referred to this estimator as the Mean Group Estimator (MGE). Hsiao, Pesaran and Tahmiscioglu (1999) showed that the MGE is consistent and asymptotically normal if  $\sqrt{N}/T \rightarrow 0$ , as  $N$  and  $T \rightarrow \infty$ , jointly.

Pesaran's research on dynamic heterogeneous panels was supported by a number of research grants which allowed him to recruit Yongcheol Shin in October 1992 and Kyung So Im in 1994 as research officers (post docs) in the Department of Applied Economics (DAE).<sup>1</sup> Both Shin and Im had completed their PhD at Michigan State University under the supervision of Peter Schmidt and Jeff Wooldridge, respectively, and were familiar with the developments in time series and panel data analysis at the time. They added important expertise to an already strong research group comprising Kevin Lee, Richard Smith and Ron Smith.

With the research group in place, we started to explore the application of the MG approach to a number of problems, such as output growth convergence, analysis of great ratios, and estimation of long run effects in panel data

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models. The idea of using simple averages of individual estimates seemed particularly convenient for panel unit root testing and had the added advantage over the LL's pooled approach as the averages of individual DF statistics (t-ratios) were robust to slope and error variance heterogeneity. By contrast, the pooled t-statistic used by LL was subject to the Nickell (1981) type bias in the presence of fixed effects and its validity required the much stronger condition on the relative expansion rates of  $N$  and  $T$ . As shown in Theorem 4 of Levin, Lin and Chu (2002, LLC), the LLC test required  $N/T \rightarrow 0$  when the panel included fixed effects, which is the standard model used in the empirical literature. LLC also acknowledged that *"the assumption that all individuals are identical with respect to the presence or absence of a unit root is restrictive. Readers are referred to Im et al. (1995) for a panel unit root test without the assumption of identical first order correlation under the alternative."*

The reason why a simple average of t-ratios works well in panel data contexts stems from two important facts. First, t-ratios are self-normalized statistics and in standard DF regressions their distribution does not depend on error variances. Second, under cross-sectional independence the average of the t-ratios tends to a normal distribution so long as the individual t-ratios have finite second-order moments. The conditions under which t-ratios have second-order moments clearly depend on the underlying error distributions. For example, when the errors are normally distributed we were able to show that t-ratios from a simple DF regression (without lagged changes) have second order moments when  $T > 5$  in the case of DF regressions with an intercept, and  $T > 6$  in the case of regressions with linear trends. Due to the self-normalized nature of t-ratios we also conjectured that in general there exists a fixed  $T_0$  such that t-ratios have second-order moments for all  $T > T_0$ . The issue of the existence of moments is also addressed in a number of other papers that follow the MG approach. Recent examples are Pesaran and Yamagata (2017) and Yang (2022).

### 3 Publication

We first released the DAE working paper version of the panel unit root paper in 1995 and then submitted a revised version to *Econometrica*, which was assigned to Peter Robinson as the editor in charge of our paper. We received detailed comments from three reviewers. It was acknowledged that

"unit roots in panel data are certainly of importance, and a rather fresh and interesting issue", but one of the referees raised concerns regarding the novelty of the paper and another found issues with some of our technical derivations. One reviewer even requested us to extend our test to allow for error cross-sectional dependence! Eventually, we submitted a third revision in 1996, hoping it would be acceptable to the referees. It was agreed that the paper makes a novel contribution but one of the referees insisted on formal proofs regarding the existence of the moments mentioned above. This coincided with Peter Robinson's last year as the Co-Editor of *Econometrica* in 1996, and the start of his term as the Co-Editor of *Journal of Econometrics* in 1997. Realizing the impasse, he suggested that we might wish to re-submit our paper to the *Journal of Econometrics*, which we did. The paper was finally accepted on 12 January 2003, at which time our proposed test, labelled as the IPS test, had already become established as a viable alternative to the LLC test.

## 4 Interpretation

Another important feature of the IPS test, as compared to the LLC test, was the heterogeneous nature of the alternative hypotheses. IPS tested the null of the unit root for all cross-section units under consideration but allowed one or more of the units to be stationary under the alternative. The LLC test implicitly considered the alternative hypothesis to be the same across all units. This led to possible misinterpretation of panel unit root test outcomes, interpreting the rejection of the null of unit root as the rejection of the unit root hypothesis for all cross section units. Westerlund and Breitung (2013) showed that the LLC test could have power under a broader class of alternatives, where not all individual series are stationary. This issue does not arise for the IPS test which explicitly considers and allows for heterogeneous alternatives. In effect panel unit root tests do not consider if a particular individual series is stationary but whether which of the two scenarios "unit roots" or "stationary" best characterizes the time series properties of the units in the panel. Suppose  $M_N$  out of the  $N$  individual time series have unit roots. Then rejection of the panel unit root test (using either IPS or LLC tests) implies that  $M_N/N$  does not tend to zero. The power of the panel unit root test depends on the magnitude of  $\pi = \lim_{N \rightarrow \infty} (M_N/N)$ , but panel unit root tests are not informative about the magnitude of  $\pi$ , which is clearly

of interest in empirical research. Pesaran (2012) provides further discussion.

## 5 Extensions

The LLC and IPS tests were developed for panels without error cross-sectional dependence. This proved to be a restrictive assumption, particularly for cross-country data sets being used in the empirical growth and finance literature. Some researchers resorted to cross-sectional de-meaning before applying the LLC or IPS tests, but Monte Carlo studies showed that cross-sectional de-meaning did not work in general. Thus emerged a sizeable literature on panel unit root tests that could handle cross-sectional dependence. These so called “second generation” panel unit root tests are reviewed in Breitung and Pesaran (2008) and Choi (2015, Ch. 7). Some used bootstrapping techniques to correct for cross-sectional dependence, but many employed latent factor models to model dependence whilst testing for unit roots. Phillips and Sul (2003) used an orthogonalization procedure to eliminate the latent factors, whilst Bai and Ng (2004) and Moon and Perron (2004) consider a more general multi-factor set up. Moon and Perron (2004) followed LLC and proposed a pooled panel unit root test using de-factored observations. Bai and Ng (2004) considered a similar multi-factor model but allowed for the possibility of unit roots in the latent factors as well as in the unit-specific components. These tests require  $N/T \rightarrow 0$ , as  $N, T \rightarrow \infty$ . To relax this restriction Pesaran (2007) extends the IPS test by augmenting the individual DF regressions of  $\Delta y_{it}$  with cross section averages  $\bar{y}_{t-1} = N^{-1} \sum_{i=1}^N y_{j,t-1}$  and  $\Delta \bar{y}_t$ , to take account of error cross section dependence. These cross-sectionally augmented DF regressions can be further augmented with lagged changes  $\Delta y_{i,t-s}$ ,  $\Delta \bar{y}_{t-s}$ , for  $s = 1, 2, \dots$ , to deal with possible residual serial correlation. These doubly augmented DF regressions are referred to as CADF regressions. The panel unit root test statistic, called CIPS, is then computed as the scaled version of the average of the CADF statistics. Individual CADF statistics are further truncated to ensure the existence of second-order moments which is shown to be important when  $T$  is quite small (in the region of 10 – 20). The limiting distribution of the CIPS test is not Gaussian but tends to a distribution which is free of nuisance parameters. Most importantly, the test is valid so long as  $\sqrt{N}/T \rightarrow 0$ , as  $N$  and  $T \rightarrow \infty$ . Monte Carlo experiments show that the test has desirable small sample properties in the presence of a single latent common factor while showing size distortions



if the number of common factors is larger than one.

Pesaran, Smith and Yamagata (2013) extend Pesaran's CIPS test to the case of a multifactor error structure. The basic idea is to utilize information contained in a number of  $k$  additional covariates,  $\mathbf{x}_{it}$ , that are assumed to share the common factors of the series of interest,  $y_{it}$ . The ADF regression for  $y_{it}$  is then augmented with cross-sectional averages of  $y_{it}$  and  $\mathbf{x}_{it}$  in order to eliminate the effects of multiple unobserved common factors on panel unit root tests.

The LLC and IPS tests and their various extensions have also played an important role in development of panel cointegration tests and estimation of long run effects in heterogeneous dynamic panel data models. Important pioneering contributions were made by Pedroni (2004) and Westerlund (2007). Pesaran, Shin and Smith (1999) propose the idea of Pooled Mean Group (PMG) estimator where long run effects are assumed to be the same across units, with short run dynamics and error variances allowed to be heterogeneous. There are often good reasons to believe that equilibrium relations should be common across units but that the same is not true of short-run dynamics.

## 6 A retrospective

The IPS test turned out to be highly influential, both with academic and professional economists, and provided a means for researchers to identify differing degrees of persistence in time series data and thereby to avoid pitfalls in the estimation of spurious regressions, a phenomenon that arises repeatedly in macroeconomics and finance. The IPS test and its extensions to models with error cross-sectional dependence are now included in all of the leading statistical software packages and in major textbooks. The attention that our paper has received is far greater than we had expected or imagined, particularly given the initial lukewarm reviews we received. Over the years we have come to realize that in academic publishing new ideas are likely to take longer than papers that contribute at the margin, and applied econometricians are more likely to welcome simple ideas that work than theoretical econometricians that are more impressed with the technical innovations and complexity. It is important not to lose heart and try and try again to explain the novelty of one's research.

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