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Abstract

We characterize a measure of social welfare for linear production economies in which individuals differ in productive skills and preferences. The key feature of our measure is that it aggregates fairness gaps, defined as the difference between the money-metric utility that the individual currently obtains and the money-metric utility that the individual should obtain in a fair society. Social welfare depends on two normative parameters: society's aversion to unfairness and the degree to which society wants to compensate individuals for productivity differences. The latter parameter makes it possible to accommodate a whole range of ethical perspectives, from libertarianism to resource-egalitarianism. As an illustration, we use our social welfare measure to evaluate four hypothetical earnings tax reforms for Belgian singles. The degree of compensation for productivity differences turns out to be the most important normative choice for the overall evaluation, while allowing for involuntary unemployment is the most important empirical choice.

JEL-Codes: D300, D600, D700, H200, I300, J200.

Keywords: fairness, money-metric utility, excess burden, unfair inequality, earnings tax reforms, involuntary unemployment.

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1 Introduction

In the most common normative approach to evaluate earnings tax reforms, social welfare is an increasing and concave function of utilities (see, e.g., Boadway, 2012, Kaplow, 2008, Tuomala and Weinzierl, 2022). This welfarist approach weighs the utility gains of the winners of a tax reform against the utility losses of the losers. It requires an assumption of interpersonal comparability of subjective utilities, however, and there is no straightforward method to choose between the many utility functions that represent the ordinal preferences of individuals. Normative choices must be made and these become especially difficult when individual preferences differ.

An alternative approach introduces normative fairness principles based on ordinal preference information only. Or, stated differently, in this approach the choice of a cardinal and interpersonally comparable representation of preference orderings is justified on the basis of explicit normative principles (see, e.g., Fleurbaey and Maniquet, 2011, 2018a, for theoretical overviews). In this fairness approach, a cut is introduced between individual endowments (e.g., productivities) and ambitions (e.g., tastes for working). In a setting with productivity and taste differences, the compensation principle requires to compensate individuals for differences in productivities and the responsibility principle keeps individuals responsible for their tastes. The desired degrees of compensation and of responsibility can vary and in some settings the two principles are incompatible and compromises have to be sought. The idea of compensation has been formalized in different transfer axioms. Strong versions of the transfer axiom, combined with the use of only ordinal preference information, yield the result that the worst-off individual receives absolute priority in the fairness approach. This maximin result is seen as too extreme by many.[1]

In the spirit of Bosmans, Decancq, and Ooghe (2018), we propose a transfer principle that approves of progressive transfers, but only if the allocations before and after the transfer are Pareto efficient. We show that this transfer principle, together with a handful of standard axioms, is satisfied if and only if social welfare is equal to the average transformed fairness gap, i.e., $\frac{1}{n}\sum_{i=1}^{n}\phi(m_i - m_i^*)$, with (i) ϕ a differentiable, strictly increasing, and strictly concave transformation function and (ii) $m_i - m_i^*$ the fairness gap, i.e., the difference between the money-metric utility in the current bundle and in the fair bundle of individual $i = 1, 2, \ldots, n$. Individuals can be treated better than fairly (if the gap is positive), exactly fairly (if zero), or worse than fairly (if the gap is negative). The fair allocation (collecting the fair bundles of the different individuals) is implicitly defined as the allocation that maximizes social welfare over the set of feasible allocations. It can take many forms: the only restrictions are that it must be a Pareto efficient and anonymous allocation.

Our social welfare function has two normative parameters. The first is the degree of curvature of the transformation function ϕ, capturing the degree of unfairness aversion. Our formulation of the

[1] It is perhaps less so in this approach since the measure of individual advantage (the utility function) follows from a sophisticated ethical framework and can take account of endowments, desert, and similar considerations (Fleurbaey and Maniquet, 2018a, p. 1073).

transfer principle does not force us into maximin and allows for a whole range of degrees of aversion. We are not the first to propose a fairness theory that makes it possible to avoid maximin; see, e.g., Fleurbaey and Tadenuma (2014), Fleurbaey and Maniquet (2018b), and Piacquadio (2017) for alternative strategies. The second parameter relates to the determination of the fair allocation and thus also of the fairness gaps $m_i - m_i^*$. Different fairness views correspond to different views about the desirable degree of compensation for differences in productivities. At one extreme, libertarians attach special importance to the laisser-faire market allocation and do not want to compensate individuals for differences in productivities. At the other extreme, resource-egalitarians want to fully compensate individuals for differences in productivities. As suggested by Fleurbaey and Maniquet (2018a), we propose to use a fair allocation with a flexible degree of compensation in between these two extremes. The degree of compensation is our second normative parameter.

The fairness gap is reminiscent of the literature that measures unfair inequality as the divergence between the current and the fair distribution of outcomes; see, e.g., Devooght (2008), Fleurbaey and Schokkaert (2009), Almas et al. (2011), Magdalou and Nock (2011), and Hufe, Kanbur, and Peichl (2022).[2] Of course, contrary to these inequality measures, our social welfare function does satisfy a Pareto criterion. Our approach is therefore more closely related to other papers that impose Pareto efficiency and define social welfare in terms of distances between the actual and an optimal (or equitable) situation. Prominent examples are Weinzierl (2014), and, more recently Berg and Piacquadio (2022). The latter paper is in spirit closest to ours: the authors define social welfare in terms of a concave loss function, where the loss is defined as the distance from a fair distribution.

The flexibility of our approach creates room for many different ethical positions both with respect to unfairness aversion and with respect to the degree of compensation for differences in innate productivity. Recently it has been proposed to specify the welfare weights of the various social groups directly (Saez and Stancheva, 2016).[3] Direct specification of the welfare weights is even more flexible than our approach, but, as mentioned by Fleurbaey and Maniquet (2018a) and worked out further by Sher (2021), the local approach represented by these marginal weights may lead to inconsistencies when extended to the global level. As said, our approach is less flexible, but it has the advantages that it yields a transitive social welfare ordering and that maximizing the corresponding social welfare function is very similar to the current procedures in optimal tax applications. We also show how the concept of the marginal value of public funds (Finkelstein and Hendren, 2020, Hendren and Sprung-Keyser, 2020) can easily be made operational in our framework.

As an illustration, we use EU-SILC (European Union Statistics on Income and Living Conditions) data to estimate the productive skills (hourly gross wage rates) and preferences (tastes

[2]Our approach is also similar to the notion of distributional change, as proposed by Cowell (1985), but he focuses on the comparison of an "old" and a "new" distribution vector in a setting of economic mobility.

[3]See, e.g., Madden and Savage (2020) for an empirical application.

for working) of Belgian singles without children. We use gross earnings to define four groups: a group with zero gross earnings (the unemployed) and three equally sized groups among the working (the working poor, the working middle, and the working rich). We simulate four hypothetical budget-neutral tax reforms. Behavioral responses are taken into account with a model of discrete choice between four labor supply options. We distinguish two cases. In the first one, all labor choices, including unemployment, are assumed to be voluntary. In the second one, individuals may be rationed on the labor market such that their choices, and in particular unemployment, are no longer completely voluntary. The two cases lead to very different results.

If we are assuming a Kolm-Pollak specification for the transformation function ϕ, the changes in social welfare as a result of a tax reform can be additively decomposed in three terms: the effects on government revenue, on excess burden, and on unfair inequality. For the budget-neutral tax reforms that we consider, the changes in inefficiency do not matter much: the changes in welfare are mainly driven by the changes in unfair inequality. For the latter, the most important parameter is the desired degree of compensation for productivity differences. If the desired degree of compensation is low, the working rich are most unfairly treated by the actual tax-benefit system and lowering their tax burden reduces unfair inequality. If the desired degree of compensation is high, the policy evaluation depends on the assumptions about labor choices. If the latter are completely voluntary, then the working poor are most unfairly treated and reforms that redistribute to the working poor reduce unfair inequality, at least if the unfairness aversion is suffciently large. This confirms earlier results in the literature (Fleurbaey and Maniquet, 2006, 2018a). As, under this assumption, the unemployed are considered to be the least unfairly treated in the current tax-benefit system, reforms that further redistribute to the unemployed increase rather than decrease unfair inequality. The results change completely, however, if we allow for involuntary unemployment. In that case the unemployed are most unfairly treated in the actual situation for a large range of the ethical parameter values and increasing transfers to the unemployed now yields the largest welfare improvement.

Our characterization of the social welfare function is explained in section 2. In section 3, we show the implications of assuming a Kolm-Pollak specification for ϕ and introduce the decomposition of the welfare effects of a tax reform, including an expression for the marginal value of public funds. In section 4 we describe our empirical results in case of voluntary and involuntary unemployment. In each of these cases we first discuss the behavioral model and the estimation of skills and preferences, then illustrate the implications of different ethical assumptions with an analysis of the marginal social welfare weights, and finally summarize the evaluation of the four tax reforms. Section 5 concludes.

2 Social welfare as average transformed fairness gaps

A first subsection introduces basic notation. A second subsection discusses four axioms: Representation, Pareto, Anonymity, and Transfer, that are used to characterize our social welfare measure in a third subsection. We finally show how our approach can accommodate different views on compensation for productivity differences ranging from libertarianism to resource-egalitarianism.

2.1 Notation

Let $I = \{1, 2, \ldots, n\}$ be a set of $n \geq 3$ individuals. An allocation $x = (x_1, x_2, \ldots, x_n)$ contains bundles $x_i = (c_i, \ell_i) \in X = \mathbb{R} \times \mathbb{R}_+$ of net income c_i and labor ℓ_i for each individual i in I. A type profile $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ contains a type θ_i for each individual i in I. Each type $\theta_i = (s_i, u_i)$ consists of a productive skill s_i and a utility function u_i. Skill levels $s_i \geq 0$ map labor ℓ_i into gross incomes y_i in a linear way, i.e., $y_i = s_i \ell_i$ for each individual. Utility functions $u_i : X \to \mathbb{R}$ map bundles into ordinal and non-comparable utility levels. The utility functions are continuously differentiable, strictly increasing in consumption, strictly decreasing in labor, and strictly quasi-concave.

A society $\mathscr{S} = (\theta, R_0)$ is fully described by a type profile $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ and an exogenous per capita revenue requirement, denoted R_0. The set of feasible allocations is defined as

$$F(\mathscr{S}) = \{x \in X^n \,|\, \frac{1}{n} \sum_{i \in I} c_i + R_0 \leq \frac{1}{n} \sum_{i \in I} s_i \ell_i\}, \tag{1}$$

and the set of Pareto efficient allocations is

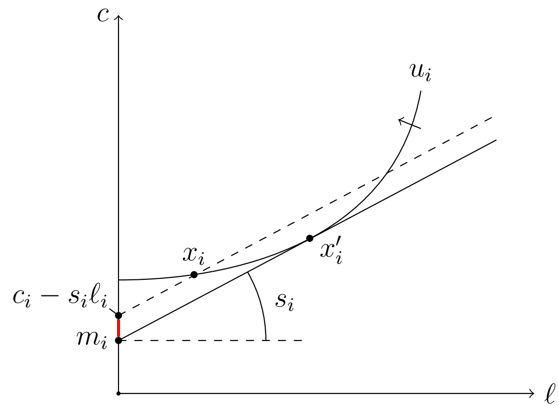
$$\begin{aligned} P(\mathscr{S}) \;=\; & \{x \in F(\mathscr{S}) \,|\, \nexists x' \in F(\mathscr{S}) \text{ s.t.} \\ & \quad \forall i \in I, u_i(x'_i) \geq u_i(x_i) \text{ and } \exists i \in I, u_i(x'_i) > u_i(x_i)\}\,. \end{aligned} \tag{2}$$

The money-metric utility of an individual is defined as the minimal unearned income that is needed to guarantee that this individual, choosing from a budget set based on this unearned income and her own skill level, would not be worse off (according to her own utility function) compared to obtaining her actual bundle (Samuelson and Swamy, 1974; Deaton and Muellbauer, 1980; King, 1983). Formally, money-metric utility—given a bundle $x_i = (c_i, \ell_i)$ and the type θ_i of individual i—is defined as

$$m(x_i, \theta_i) \;=\; \min_{(c,\ell) \in X} (c - s_i \ell) \text{ subject to } u_i(c, \ell) \geq u_i(c_i, \ell_i). \tag{3}$$

Figure 1 illustrates the construction of money-metric utility. Given the bundle $x_i = (c_i, \ell_i)$, individual i earns $s_i \ell_i$ and the unearned income is therefore $c_i - s_i \ell_i$. Yet, individual i could reach the same utility level in bundles that exhibit a lower unearned income. Among these bundles, the bundle x'_i on the indifference curve through x_i provides the same utility level and has the

Figure 1: Money-metric utility and excess burden



lowest unearned income. This is the money-metric utility of individual i in bundle x_i, denoted as $m_i = m(x_i, \theta_i)$.

In equation (3), the expression $\min(c - s_i\ell)$ can be replaced by $-\max(s_i\ell - c)$. So, (minus) money-metric utility can be interpreted as the maximal tax amount that one can hypothetically extract from an individual—or, as in the figure, the minimal subsidy amount one has to give to an individual—without him or her losing utility.[4] The difference between the maximal and actual tax revenue is called (individual) excess burden and is defined as

$$EB(x_i, \theta_i) = -m(x_i, \theta_i) - (s_i\ell_i - c_i) \geq 0. \tag{4}$$

Figure 1 shows the individual excess burden on the vertical axis as the thick (red) line. By construction it is always nonnegative, and zero if the marginal rate of substitution is equal to the productive skill.

2.2 Axioms

A social welfare measure $W(x; \mathscr{S})$ is used to judge the goodness of an allocation x in a given society $\mathscr{S}$. We focus on a class of smooth and additively separable welfare measures, formalized by the next representation axiom.

Representation. For a given society $\mathscr{S}$, for any allocation x in X, social welfare can be represented as

$$W(x; \mathscr{S}) = \frac{1}{n} \sum_{i \in I} v_i(x_i; \mathscr{S}),$$

with $v_1, v_2, \ldots, v_n$ evaluation functions that are continuously differentiable in net earnings and

[4] We deliberately write "hypothetically," as this maximal extraction is possible only with first-best lump-sum taxes. Moreover, to simplify the theoretical reasoning, we assume that the (non-tax) unearned income of the individual is zero. In the empirical exercise, we will take into account the unearned incomes of the individuals in our sample.

labor.

The interpretation of the evaluation functions will become clear later on. The separability assumption underlying the representation axiom is very common in welfare analysis. Its main limitation is that it excludes rank-dependent social welfare measures.[5]

The Pareto principle imposes that higher utility for all is better; and strictly higher utility for some (in addition to higher utility for all) is strictly better.

Pareto. For a given society $\mathscr{S}$, for any two allocations x and x' in X, if $u_i(x_i) \geq u_i(x_i')$ for all i in I, then $W(x;\mathscr{S}) \geq W(x';\mathscr{S})$; if, in addition, $u_i(x_i) > u_i(x_i')$ holds for some individual i in I, then $W(x;\mathscr{S}) > W(x';\mathscr{S})$.

The anonymity principle requires that permuting bundles of individuals with the same type (skills and preferences) does not matter for social welfare. Let $\theta_i = \theta_j$ mean that individuals i and j have the same skills (i.e., $s_i = s_j$) and the same preferences (i.e., $u_i = \varphi(u_j)$ for some strictly increasing transformation function φ).

Anonymity. For a given society $\mathscr{S}$, for any allocation x in X, for any two individuals i, j in I, if $\theta_i = \theta_j$ holds, then $W(\ldots, x_i, \ldots, x_j, \ldots; \mathscr{S}) = W(\ldots, x_j, \ldots, x_i, \ldots; \mathscr{S})$.

The transfer principle requires that a *progressive* and *Pareto-efficiency-preserving* transfer of resources between two individuals improves social welfare. A transfer is called *progressive* if it is directed from a "better-off" to a "worse-off" individual without changing their relative position. The measures that determine who is better off and worse off are the functions $v_1, v_2, \ldots, v_n$ defined in the representation axiom.[6] A transfer is called *Pareto-efficiency-preserving* if the allocations before and after the transfer are Pareto efficient. This additional condition is inspired by Bosmans, Decancq, and Ooghe (2018). As we will see, it allows us to avoid maximin-type results, while still leading to an inequality averse welfare specification.

Transfer. For a given society $\mathscr{S}$, for any two Pareto efficient allocations $x, x' \in P(\mathscr{S})$, for any two individuals i, j in I, if the transition from allocation x' to x is based on a progressive transfer of resources from i to j, i.e.,

$$v_i(x_i';\mathscr{S}) > v_i(x_i;\mathscr{S}) \geq v_j(x_j;\mathscr{S}) > v_j(x_j';\mathscr{S}),$$

without affecting other individuals, i.e.,

$$x_k = x_k', \text{ for all } k \neq i, j,$$

then $W(x;\mathscr{S}) > W(x';\mathscr{S})$.

[5]See, e.g., the discussion in Adler (2022).

[6]This "implicit" choice guarantees that the transfer principle is "consistent" as defined in Bosmans, Lauwers, and Ooghe (2009, 2018).

2.3 Social welfare and fairness gaps

Theorem 1 summarizes our main theoretical result: a social welfare measure satisfies all axioms if and only if it can be written as the average transformed fairness gap (with fairness gap defined as the difference in money-metric utility between the actual and the fair bundle of an individual). A proof can be found in Appendix A.

Theorem 1. A social welfare measure $W(x;\mathscr{S})$ satisfies Representation, Pareto, Anonymity, and Transfer if and only if it can be written as

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I}\phi(m_i - m_i^*;\mathscr{S}),$$

with

1. $\phi(\cdot;\mathscr{S})$ satisfying $\phi'(\cdot;\mathscr{S}) > 0$ and $\phi''(\cdot;\mathscr{S}) < 0$,
2. $m_i = m(x_i, \theta_i)$ and $m_i^* = m(x_i^*(\mathscr{S}), \theta_i)$ for each individual i in I, and
3. $x^*(\mathscr{S}) = (x_1^*(\mathscr{S}), x_2^*(\mathscr{S}), \ldots, x_n^*(\mathscr{S}))$ a Pareto efficient and anonymous allocation (i.e., $x^*(\mathscr{S}) \in P(\mathscr{S})$ and $x_i^*(\mathscr{S}) = x_j^*(\mathscr{S})$ for all i, j in I with $\theta_i = \theta_j$).

Several remarks apply.

First, our axioms only impose that $x^*(\mathscr{S})$ is a Pareto efficient and anonymous allocation. This means that Theorem 1 remains open to very different fairness views, implemented through the choice of $x^*(\mathscr{S})$. We shall refer to that allocation as the 'fair' allocation from now on. In the next section we will illustrate the flexibility of our approach and show how it is consistent with the libertarian view (that does not want to compensate for differences in productive skills) and the resource-egalitarian view (that wants to fully compensate for differences in productive skills). In particular, we will introduce a first normative parameter that captures the degree of compensation in between (and including) these extreme views.

Second, the fairness gap $m_i - m_i^*$ measures whether individual i is treated better than fairly (if the gap is positive), exactly fairly (if zero), or worse than fairly (if negative). These gaps do not measure individual well-being. Two individuals with the same preferences can be on the same indifference curve (and can therefore be said to have the same individual well-being), but one of them can still be treated less fairly than the other. This would be the case, e.g., if one adopts a libertarian view in which the laisser-faire market allocation is considered fair. Indeed, in this view, a high-skilled is treated less fairly than a low-skilled if both have the same preferences and reach the same indifference curve.

Third, the transformation function $\phi(\cdot;\mathscr{S})$ is strictly increasing and strictly concave. Increasingness ensures that Pareto holds: a more preferred bundle leads to a higher actual money-metric utility (without changing the fair money-metric utility), which, in turn, implies a higher social welfare, ceteris paribus. Concavity implies that (money-metric utility) transfers that reduce unfairness, i.e., transfers that are directed from more fairly to less fairly treated individuals, are approved of. The degree of curvature of the transformation function is called unfairness aversion, and will be our second normative parameter.

Fourth, the so-called fair social ordering literature also deals with measuring social welfare in production settings in which individuals differ in productivity and preferences; see, e.g., Fleurbaey and Maniquet (2011, 2018a) for overviews. In this literature the resulting individual well-being measures are often (variants of) money-metric utilities too. However, the transfer principles to compensate individuals for productivity differences are often stronger than our transfer axiom, as they do not impose the additional condition that the transfer is between two Pareto efficient allocations. These stronger principles imply that the worst-off should receive absolute priority. As described in the introduction, our use of a weaker transfer principle that preserves Pareto efficiency opens up the possibility of transformation functions with curvature in between utilitarianism and maximin.

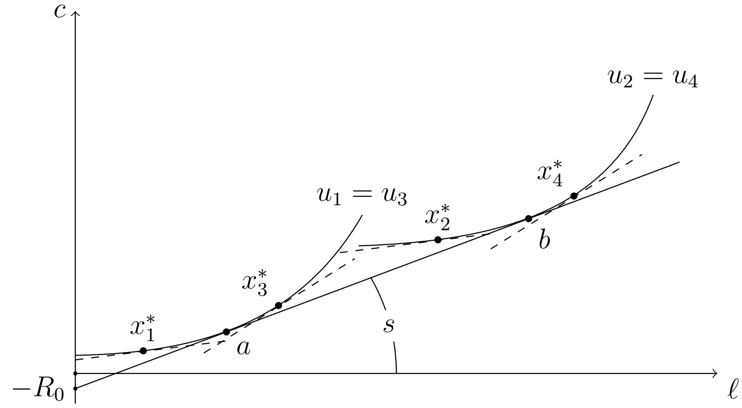
2.4 A flexible fair allocation

As mentioned before, our framework allows for many different fairness views, that will be made operational through the choice of $x^*(\mathscr{S})$. In a setting with differences in both skills and preferences, the most important issue is the degree to which one considers these differences to lead to unfair outcomes. Most economists and lay people alike (see, e.g., Konow, 2003, and Gaertner and Schokkaert, 2012) accept that earnings differences following from differences in preferences, i.e., in the taste for working, are ethically legitimate. Opinions differ, however, about the acceptability of earnings differences reflecting differences in skills. Libertarians and resource-egalitarians are at the extreme sides of that debate. Libertarians state that, in the absence of market failures, the market allocation is fair. This can be interpreted as an assumption that individuals are fully responsible for both skills and preferences. Resource-egalitarians hold individuals fully responsible for differences in outcomes caused by differences in preferences (ambitions), but not responsible at all for differences in outcomes caused by differences in skills (endowments). The latter outcome differences must therefore be fully compensated.

The optimal libertarian solution is the laisser-faire allocation, defined as no intervention, except possibly for a head tax to finance the exogenous per capita revenue requirement R_0. A prominent resource-egalitarian solution is the so-called equal-skill-equivalent allocation rule, proposed by Fleurbaey and Maniquet (1999). This rule selects the Pareto efficient allocation in which everyone is indifferent between her bundle and the bundle she would have chosen if everyone had the same skill in the laisser-faire.[7] Figure 2 illustrates for the case of four individuals differentiated by

[7]As the chosen allocation is Pareto efficient, the common skill is defined by the feasibility constraint. In other

Figure 2: The equal-skill-equivalent allocation

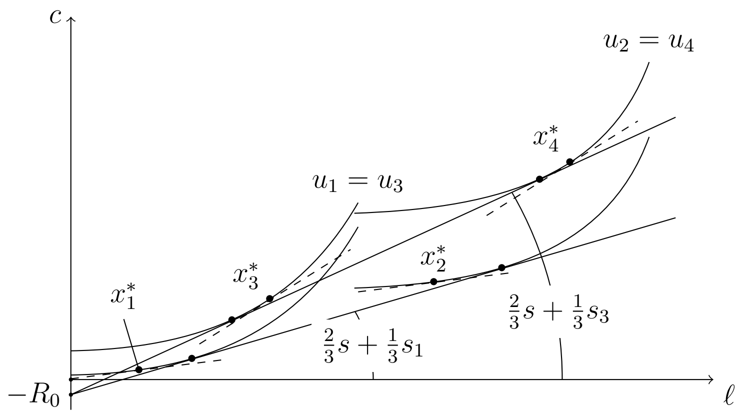


productive skills and preferences. Skill heterogeneity is such that individuals 1 and 2 have the same low skills, while individuals 3 and 4 have the same high skills. Preference heterogeneity is such that individuals 1 and 3 have the same low tastes for working, while 2 and 4 have the same high tastes for working. The fair allocation x^* is indeed Pareto efficient (the dashed lines indicate that the marginal rate of substitution is equal to the skill for each individual). Moreover, each individual is indifferent between her bundle and the bundle she would have chosen in the laisser-faire with a common skill level, denoted s in Figure 2, and a head tax equal to R_0. This equal-skill-equivalent rule offers full compensation for skills: individuals with the same preferences, but different skills end up at the same indifference curve. In terms of responsibility, if an individual has the same skills as another individual, but a stronger taste for working, he/she will end up with a higher consumption level. Indeed, they will end up on the indifference curves through the bundle where consumption is proportional to labor, being the bundles a and b in Figure 2.

Opinions in society differ about the degree of responsibility for skills. We therefore propose to adjust the equal-skill-equivalent allocation rule in the way inspired by Fleurbaey and Maniquet (2018a, p. 1060) to allow for a partial degree of compensation in between the extremes of no compensation (libertarianism) and full compensation (resource-egalitarianism). We introduce a (normative) parameter γ that captures the degree of compensation for skills. The resulting γ-skill-equivalent allocation rule selects the Pareto efficient allocation in which everyone is indifferent between her bundle and the bundle she would have chosen in the laisser-faire if everyone had a skill equal to $\gamma s + (1-\gamma)s_i$, with s a common skill level. The parameter γ lies between no compensation ($\gamma = 0$, consistent with the libertarian view) and full compensation ($\gamma = 1$, the typical resource-egalitarian view), and the γ-skill-equivalent allocation contains the (libertarian) market allocation ($\gamma = 0$) and the (unadjusted) equal-skill-equivalent allocation ($\gamma = 1$) as extreme cases. Figure 3 presents the γ-skill-equivalent allocation with $\gamma = \frac{2}{3}$ in case of the same four individuals. Here

words, the sum of money-metric utilities must be equal to $-R_0$ at the Pareto-optimal allocation.

Figure 3: The γ-skill-equivalent allocation, with $\gamma = \frac{2}{3}$



there is some compensation for skill differences, but it is incomplete in that individuals with the same preferences will no longer end up on the same indifference curve. Varying the parameter γ makes it possible to cover different ethical perspectives. In our empirical application, we will perform a sensitivity analysis with respect to the parameter γ.

3 Towards application: a Kolm-Pollak specification

Until now, we have remained fairly general. To apply our framework in practice, it will be necessary to specify the transformation function ϕ. Given that our social welfare function is defined in terms of absolute gaps, the most natural choice is to make this function invariant to additions. This allows us to handle both positive and negative values for the fairness gaps without any problem. In our application later on, we will use the Kolm-Pollak specification. We show the marginal social welfare weights for this specification in section 3.2. We first present a decomposition of social welfare, that will be useful to interpret the results on tax reforms later on.

3.1 A decomposition and the evaluation of tax reforms

Without loss of generality, we rewrite social welfare in its equally-distributed-equivalent form, i.e., $W(x; \mathscr{S}) = \phi^{-1}\{\frac{1}{n}\sum_{i\in I}\phi(m_i - m_i^*)\}$, with ϕ defined as $\phi(\cdot; \mathscr{S})$. Let $R(x; \mathscr{S}) = \frac{1}{n}\sum_{i\in I}(s_i\ell_i - c_i)$ be the average tax revenues. Making use of the fact that a transformation function ϕ, which is invariant to additions, satisfies $\phi(a + b) = \phi(a)\phi(b)$ for any two scalars a, b, and hence also, as a consequence, $\phi^{-1}(ab) = \phi^{-1}(a) + \phi^{-1}(b)$, we have

$$
\begin{aligned}
W(x;\mathscr{S}) &= \phi^{-1}\{\frac{1}{n}\sum_{i\in I}\phi(m_i - m_i^*)\}, \\
&= \frac{1}{n}\sum_{i\in I} m_i - \underbrace{\frac{1}{n}\sum_{i\in I} m_i^*}_{-R_0} + \phi^{-1}\{\frac{1}{n}\sum_{i\in I}\phi(m_i - m_i^* - \frac{1}{n}\sum_{i\in I}(m_i - m_i^*))\}, \\
&= [R_0 - R(x;\mathscr{S})] - [-\frac{1}{n}\sum_{i\in I} m_i - R(x;\mathscr{S})] - \\
&\qquad [-\phi^{-1}\{\frac{1}{n}\sum_{i\in I}\phi(m_i - m_i^* - \frac{1}{n}\sum_{i\in I}(m_i - m_i^*))\}], \\
&= RS(x;\mathscr{S}) - EB(x;\mathscr{S}) - UI(x;\mathscr{S}). \qquad (5)
\end{aligned}
$$

The first component $RS(x;\mathscr{S})$ measures the revenue shortage as the difference between the amount of taxes that should be raised and the amount that is currently raised. The second component $EB(x;\mathscr{S})$ measures inefficiency as excess burden, i.e., the difference between what revenue can be maximally raised (without losses in utility) and what is currently raised on average. It is the sum of the individual excess burdens that are defined in equation (4). The third component $UI(x;\mathscr{S})$ measures unfair inequality, defined as the inequality in fair treatment between individuals. If all individuals are treated equally fairly or unfairly (i.e., the fairness gap $m_i - m_i^*$ is the same for all individuals), then unfair inequality is zero. This illustrates that zero unfair inequality does not correspond to zero unfairness. All individuals can be treated unfairly, but in an equal way. Only if some individuals are treated differently (i.e., the fairness gap is not the same for all individuals), then unfair inequality becomes positive. To sum up, social welfare is equal to revenue shortage minus excess burden minus unfair inequality.

To assess tax reforms, one usually focuses on the change in welfare before and after a reform. Let x_0 be the allocation before the reform and x_1 the allocation after the reform. Minus the change in welfare is equal to

$$
\begin{aligned}
-\triangle W &= W(x_0;\mathscr{S}) - W(x_1;\mathscr{S}), \\
&= (R(x_1;\mathscr{S}) - R(x_0;\mathscr{S})) + (EB(x_1;\mathscr{S}) - EB(x_0;\mathscr{S})) + (UI(x_1;\mathscr{S}) - UI(x_0;\mathscr{S})), \\
&= \triangle R + \triangle EB + \triangle UI. \qquad (6)
\end{aligned}
$$

Changes in welfare depend negatively on changes in tax revenues, in excess burden, and in unfair inequality. If the tax reform is budget-neutral, then the first term at the right-hand side is equal to zero and the change in welfare depends only on changes in excess burden and in unfair inequality.

$$
-\triangle W = \triangle EB + \triangle UI. \qquad (7)
$$

If the tax reform is not budget-neutral, the decomposition can also be written as

$$-\frac{\triangle W}{\triangle R} = 1 + \frac{\triangle EB}{\triangle R} + \frac{\triangle UI}{\triangle R}. \tag{8}$$

It provides the net welfare cost of public funds (to the left) as the sum of the efficiency cost of public funds (the first two terms to the right, being respectively the direct cost and the indirect cost) and the equity gain of public funds in terms of a reduction in unfair inequality (the last term to the right).

In applied work, equation (8) is often computed at the margin. This was already proposed in Ahmad and Stern (1984). More recently, Hendren and Sprung-Keyser (2020) and Finkelstein and Hendren (2020) introduced the "marginal value of public funds," i.e., the welfare gain per euro invested:

$$MVPF = \frac{\triangle W}{\triangle R} = -\left(1 + \frac{\triangle EB}{\triangle R}\right) - \frac{\triangle UI}{\triangle R}. \tag{9}$$

3.2 The marginal social welfare weights

To interpret the distributive implications of earnings tax reforms, it is useful to introduce the notion of the marginal social welfare weight of individual i. This is often defined as the effect on social welfare of a small increase in the consumption (or the net income) of individual i (see Saez and Stantcheva, 2016). In our framework it is more convenient to focus on the effect on social welfare of a marginal increase in the money-metric utility of individual i. Using a Kolm-Pollak specification $\phi : x \mapsto \exp(-rx)$, this marginal social welfare weight of individual i is given by

$$msww_i = \frac{\exp(-r(m_i - m_i^*))}{\sum_{k=1}^{n} \exp(-r(m_k - m_k^*))}, \tag{10}$$

where $r > 0$ measures the degree of unfairness aversion. Higher values of the marginal social welfare weight indicate a higher social priority. The marginal social welfare weight depends on the fairness gap $m_i - m_i^*$, which in its turn depends on the degree of compensation γ, ranging from 0 (libertarianism, no compensation) to 1 (resource-egalitarianism, full compensation). A larger fairness gap leads to a larger marginal social weight, at least when $r > 0$. Differences in fairness gaps will lead to larger differences in the marginal social welfare weights if the social planner cares more about unfairness, i.e., if the degree of unfairness aversion r gets larger. In the absence of efficiency considerations, a small transfer from an individual with a lower priority to an individual with a higher priority will increase social welfare.

4 Fair earnings tax reforms

We illustrate the use of our social welfare measure to evaluate tax reforms. We focus on the taxation of singles without children in the EU-SILC data for Belgium. We use gross earnings to distinguish four groups of individuals: the unemployed with zero gross earnings (13.6% of the sample) and three equally sized groups among the working, called the working poor (gross earnings below 2409 euro), the working middle (gross earnings between 2409 and 3493 euro), and the working rich (gross earnings above 3493 euro). We consider four hypothetical earnings tax reforms. Each reform assigns 75 euro extra to the monthly net disposable income of one of the four income groups (step 1 of the reform) and reduces the net disposable incomes in the other groups with an equal amount so as to ensure budget neutrality (step 2).

A first subsection introduces the data and discusses the estimation of the gross hourly wages. The second and third subsections analyze the cases of voluntary and involuntary unemployment. In each of these two cases, we first describe the estimation of individual preferences and interpret the marginal social welfare weights. We then discuss the evaluation of the four tax reforms.

4.1 Data, sample selection, and estimation of the gross wages

Our empirical illustration is based on the cross-sectional EU-SILC data of Belgium for 2016 (wave 2017). We select all singles without children between 18 and 65 years old, who are active on the labor market, i.e., either working (but not self-employed) or unemployed (but searching for work) in 2017. Some further details of the data selection can be found in appendix B. Table 1 provides descriptive statistics for our sample of 861 individuals (about 6% of the total EU-SILC sample of 13974 individuals for Belgium).

Gross earnings are in theory equal to the product of skill and labor. Defining labor as labor hours, skill corresponds to the gross hourly wage rate. For singles with non-zero gross earnings in 2016, the wage rate is computed as the ratio of yearly gross earnings and (an estimate of) yearly labor hours in 2016. For singles with zero gross earnings, we predict hourly wages based on a simple OLS regression model.[8] Details on the computation and prediction of hourly gross wages can be found in appendix C.

Labor hours are discretized in four groups. The set of possible labor hours is $L = \{0, 24, 38, 51\}$ and the set of bundles reduces to $X = \mathbb{R} \times L$.[9] Individuals are assigned to 0 hours if their reported labor hours are equal to 0 (13.3% of the sample), 24 hours if reported hours lie in $]0, 30[$ (16.8%), 38 hours if reported hours lie in $[30, 45[$ (63.3%), and 51 hours if reported hours lie in $[45, 61[$ (6.6%). We compute gross earnings based on (computed or predicted) wages and (re-assigned) labor hours

[8] We also experimented with Heckman selection models. However, the inverse Mills ratio was never statistically significant. Output is available upon request.

[9] The classification is based on a visual inspection of the (local peaks of the) labor hours distribution.

Table 1: Descriptive statistics

gender (%)	male	43.5
	female	56.5
age (years)	mean	44.0
nationality (%)	Belgian	85.8
	EU (& not Belgian)	9.4
	not EU	4.8
area (%)	urban	43.1
	middle	44.6
	rural	12.3
highest degree (%)	$<$ secondary	22.1
	$=$ secondary	34.7
	tertiary	43.2
experience (years)	mean	19.7
labor market status (%)	unemployed	13.3
	working	86.7

Notes: "= secondary" also includes individuals with a highest degree in non-tertiary higher education. Area is based on Eurostat's degree of urbanization (DEGURBA) classification.

and use EUROMOD (version 3.3.8) to simulate the corresponding net disposable incomes.[10] Table 2 provides descriptive statistics for the hourly wage rate, labor hours, gross earnings, and net incomes.

4.2 Voluntary unemployment

The evaluation of earnings tax reforms depends on how much society cares about the resulting changes in the money-metric utilities, as expressed by the marginal social welfare weights, and on how people react to tax changes. A necessary first step is the estimation of preferences, as these are essential to compute money-metric utilities and behavioral reactions.

4.2.1 Estimating preferences and simulating behavioral changes

We estimate preferences using a discrete choice model. Each individual i chooses labor hours ℓ from the discrete choice set L to maximize utility, specified as

$$u(c_i(\ell), \ell; z_i) + \epsilon_i(\ell), \tag{11}$$

with (i) $u(c_i(\ell), \ell; z_i)$ the deterministic utility as a function of net income $c_i(\ell)$, labor hours ℓ, and a vector of individual characteristics z_i (gender, age, nationality, area, and highest educational degree) and (ii) $\epsilon_i(\ell)$ a random utility term (independent and identically distributed over individuals

[10] In these simulations, unearned incomes are kept constant.

Table 2: Gross wage rates, labor hours, gross earnings, and net incomes

		p25	p50	p75
gross hourly wages (euro)	all	14.48	17.82	23.26
	working	14.38	17.89	23.75
	unemployed	14.72	17.66	20.75
actual weekly labour hours	all	29	38	40
	working	34	38	40
assigned weekly labour hours	all	24	38	38
	working	38	38	38
monthly gross earnings (euro)	all	1624.20	2692.38	3599.75
	working	2160.58	2893.21	3756.00
monthly net income (euro)	all	1498.10	1837.81	2213.45
	working	1659.71	1923.31	2290.10
	unemployed	1433.83	1466.18	1469.12

Notes: gross hourly wages of the unemployed are predicted using a simple OLS regression. Assigned weekly labor hours are recomputed after assigning individuals to one of the four possible labor hours choices that we use in the discrete choice model. Gross earnings are computed as the product of the (computed or predicted) hourly gross wage rate and the assigned weekly labor hours, multiplied with 52/12. Monthly net incomes are simulated using EUROMOD.

and choices according to an extreme value type I distribution).

The deterministic utility term is specified as the sum of (i) the log of (augmented) net income, (ii) labor dummies, and (iii) a taste-for-work shifter, i.e.,

$$u(c_i(\ell), \ell; z_i) = \alpha \log(c_i(\ell) + \kappa) + 1[\ell = 24]\beta + 1[\ell = 38]\gamma + 1[\ell = 51]\delta + 1[\ell \neq 0]t(z_i), \qquad (12)$$

with κ a prespecified positive constant (introduced to avoid negative numbers for the logarithmic function), $1[\cdot]$ a dummy variable that is equal to one if the expression between brackets is true and zero otherwise, α, β, γ, and δ preference parameters (to be estimated), and t a function of the covariates (as further specified in appendix D.1).[11] Preference heterogeneity enters in two ways: the taste function t captures observed preference heterogeneity and the random utility terms reflect unobserved preference heterogeneity.[12] Note that the function t is specified to be the same for the different labor choices. While somewhat restrictive, this assumption allows us to unambiguously define someone's taste for working: a higher value for t corresponds to more ambitious types with flatter indifference curves in labor-consumption space.

The estimation results can be found in appendix D.1.[13] The pseudo-R^2 is equal to 0.31, which

[11] We also experimented with polynomial utility-of-consumption functions, but these were outperformed in terms of fit (using Akaike's information criterion) by the simple log-specification with κ set to 5000.

[12] This interpretation of the random utility terms as unobserved preference heterogeneity is only one possibility. An alternative interpretation would be to see them as optimization errors. This would of course change the welfare evaluation. See, e.g., Creedy, Hérault, and Kalb (2011) for a discussion.

[13] We follow the estimation procedure proposed by Van Soest (1995).

Table 3: Actual and predicted probabilities (voluntary unemployment)

all	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	13.3	16.8	63.3	6.6
predicted (%)	13.2	16.8	63.3	6.6
predicted - actual	-0.1	0.0	0.1	0.0
men	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	14.5	10.1	66.8	8.7
predicted (%)	14.4	16.4	62.5	6.7
predicted - actual	-0.1	6.3	-4.3	-2.0
women	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	11.8	25.7	58.6	3.9
predicted (%)	11.7	17.4	64.3	6.5
predicted - actual	-0.1	-8.2	5.7	2.6

is reasonable.[14] Table 3 presents the actual and predicted probabilities for our sample. The model predicts almost exactly the overall probabilities by definition. Yet, the model overpredicts working half-time for men and underpredicts it for women. The opposite is true for working full-time. These biases follows from (deliberately) specifying the taste function t to be the same for the different labor choices.

We can now use the estimated parameters to simulate the behavioral effects of the different reforms. These simulations are based on an artificial sample in which we replace each individual by 809 artificial individuals with the same observable characteristics, gross hourly wages, and deterministic utility functions, but with a randomly drawn vector of unobserved utility terms to create unobserved preference heterogeneity.[15] Table 4 reports the behavioral reactions in terms of the fractions of individuals in the different income groups before and after each reform. We also report the fraction of individuals in the different income groups after step 1 of each reform (which assigns extra net income without budget neutrality). This fraction allows us to compute the corresponding (net income) elasticities, defined as the percentage point change in the fraction of individuals in an income group divided by the percentage change in (average) net income.

The own-elasticities (on the diagonal in bold) indicate that the behavioral reactions in the different income groups are fairly similar, except for the working middle. If the net income of a certain income group increases with 1%, then the fraction of individuals in that group increases with 0.22 (the unemployed), 0.25 (the working rich), 0.25 (the working poor), and 0.34 (the working middle) percentage points. The cross-elasticities (off the diagonal) indicate where these increases come from. If the net income of a certain income group increases with 1%, then the fraction of individuals in the other groups decreases with between 0.03 and 0.15 percentage points. As

[14]Compare, e.g., with Bargain, Orsini, and Peichl (2014), who obtain 0.28 for singles (on average across a selection of countries).

[15]This replication number is equal to the ratio of the Belgian population (11303528) and the EU-SILC sample size for Belgium (13974) in 2016.

Table 4: Behavioral reactions caused by each reform (voluntary unemployment)

	unemployed	working poor	working middle	working rich
monthly gross income (€)	$y = 0$	$0 < y \leq 2409$	$2409 < y \leq 3493$	$3493 < y$
average monthly net income (€)	1415	1513	1920	2925
fraction before reform (%)	13.5	28.8	28.8	28.8
fraction after reform 1 (%)	15.8	27.8	28.0	28.4
fraction after step 1 only (%)	14.6	28.4	28.4	28.6
percentage point change	+1.2	-0.5	-0.4	-0.2
elasticity (change / 5.3)	+**0.22**	-0.09	-0.08	-0.04
fraction after reform 1 (%)	12.8	30.5	28.0	28.7
fraction after step 1 only (%)	13.0	30.1	28.2	28.7
percentage point change	-0.5	+1.2	-0.6	-0.1
elasticity (change / 5.0)	-0.09	+**0.25**	-0.13	-0.03
fraction after reform 1 (%)	13.0	28.2	30.4	28.4
fraction after step 1 only (%)	13.1	28.3	30.2	28.5
percentage point change	-0.4	-0.6	+1.3	-0.4
elasticity (change / 3.9)	-0.10	-0.15	+**0.34**	-0.09
fraction after reform 1 (%)	13.3	28.7	28.5	29.6
fraction after step 1 only (%)	13.3	28.7	28.5	29.5
percentage point change	-0.2	-0.1	-0.3	+0.6
elasticity (change / 2.6)	-0.07	-0.05	-0.12	+**0.25**

Notes: the denominators of the elasticities are reported in the diagonal of Table 5.

expected, the cross-elasticities are usually stronger if the income group where the change occurs is 'closer'. For example, assigning extra net income to the unemployed, leads to the strongest behavioral reactions among the working poor, then among the working middle, and finally among the working rich.

Table 5 presents the income changes that are required to make each tax reform budget-neutral, while taking behavioral changes into account. Especially the first reform (assigning 75 euro extra to those unemployed) is costly.

4.2.2 Marginal social welfare weights before reform

We can now compute the labor hours, gross earnings, and net disposable incomes for each artificial individual in the actual and the fair situation. Based on this information we can compute money-metric utilities. Let $\theta_i = (s_i, u_i, \epsilon_i)$ denote the type of an (artificial) individual i, consisting of a wage rate s_i, a deterministic utility function $u_i(\cdot) = u(\cdot; z_i)$, and a vector of random utility terms $\epsilon_i = (\epsilon_i(\ell))_{\ell \in L}$. Given a bundle $x_i = (c_i, \ell_i)$—e.g., the actual or fair bundle—and the type θ_i of

Table 5: Four hypothetical earnings tax reforms (voluntary unemployment)

	unemployed	working poor	working middle	working rich
with monthly gross income	$y = 0$	$0 < y \leq 2409$	$2409 < y \leq 3493$	$3493 < y$
with average monthly net income	1415	1513	1920	2925
reform 1 (€ net extra per month)	75	-75	-75	-75
reform 1 (as % of net income)	5.3	-5.0	-3.9	-2.6
reform 2 (€ net extra per month)	-28	75	-28	-28
reform 2 (as % of net income)	-2.0	5.0	-1.4	-1.0
reform 3 (€ net extra per month)	-14	-14	75	-14
reform 3 (as % of net income)	-1.0	-1.0	3.9	-0.5
reform 4 (€ net extra per month)	-14	-14	-14	75
reform 4 (as % of net income)	-1.0	-0.9	-0.7	2.6

individual i, money-metric utility is equal to[16]

$$m(x_i, \theta_i) = \min_{(c,\ell)\in X} (c - s_i\ell) \text{ subject to } u_i(c, \ell) + \epsilon_i(\ell) \geq u_i(c_i, \ell_i) + \epsilon_i(\ell_i). \tag{13}$$

In order to better understand the normative foundations of our analysis, we first show the fairness gaps and then the marginal social welfare weights.

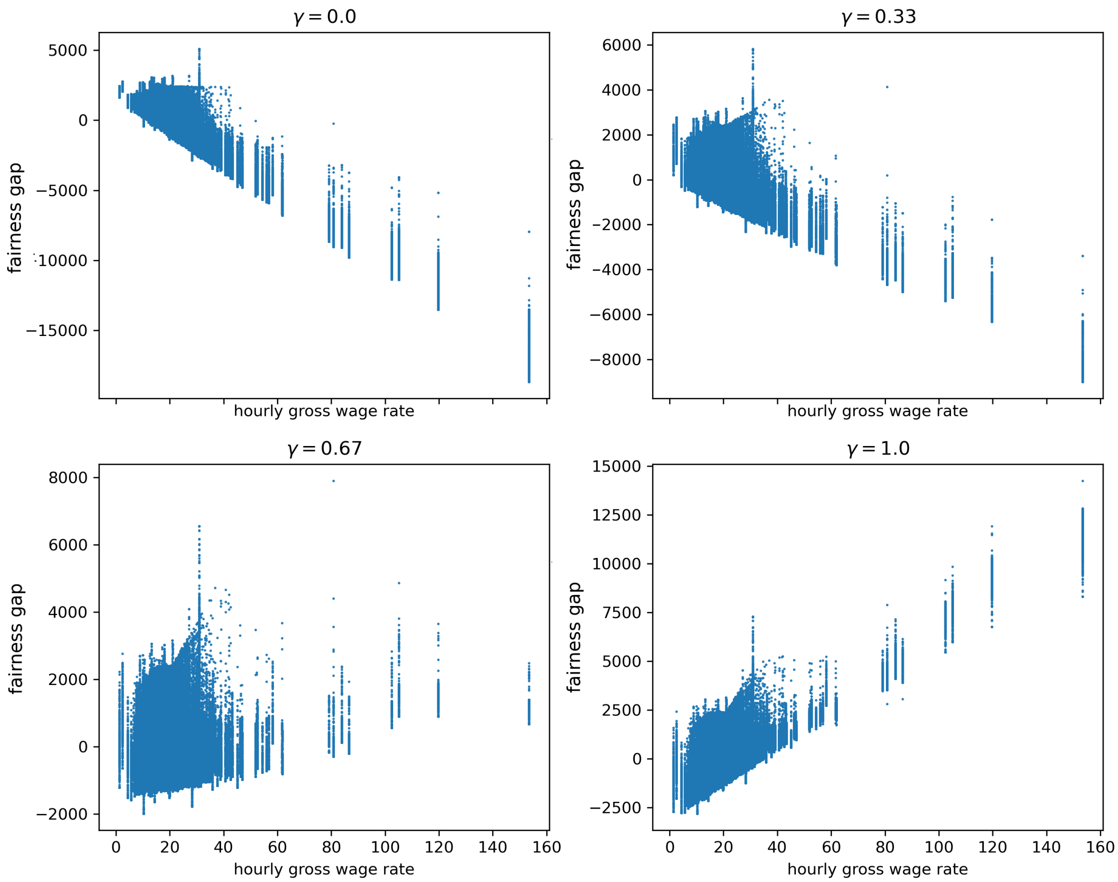
Fairness gaps Figure 4 plots the fairness gaps before reform as a function of the hourly gross wage rate for different degrees of compensation γ. In the libertarian case $\gamma = 0$, the fairness gap decreases with the hourly wage rate. The reason is that the actual Belgian tax system is redistributive from the rich to the poor (having, respectively, high and low wage rates on average). Since according to libertarians, the laisser-faire outcome is fair, the poor (rich) currently receive more (less) than their fair share. In the resource-egalitarian case $\gamma = 1$, the fairness gap increases with the hourly wage rate. Indeed, the present level of redistribution in the Belgian tax system is not sufficient to reach the ideal of full compensation. The poor (with low wage rates) receive less than their fair share, while the rich receive more than their fair share. For intermediate levels of compensation we see the same decreasing or increasing pattern, but to a lesser extent.

Figure 5 plots the fairness gaps before reform as a function of the taste for working for different values of γ. Fairness gaps never correlate with the tastes for working in a substantial way. This stands to reason: our fairness view aims at (partial) compensation for skill differences, while remaining neutral towards taste differences.

The marginal social welfare weights As equation (10) shows, the marginal social welfare weights do not only depend on the fairness gaps, but also on society's aversion to unfairness r. For

[16] We keep non-labor income in our computations, both for the actual and the fair money-metric utilities. From a normative point of view it means that we hold individuals responsible for these non-labor incomes.

Figure 4: Fairness gap before reform as a function of the hourly gross wage rate (voluntary unemployment)

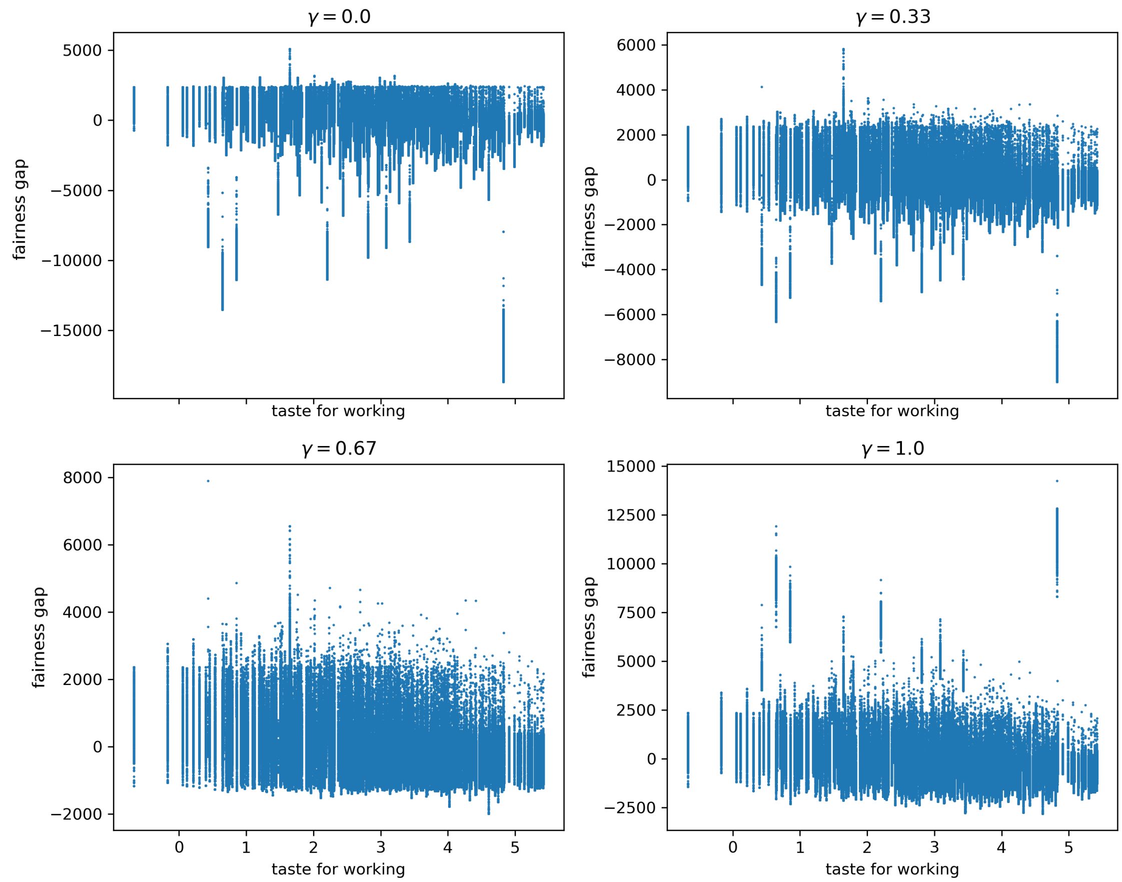


ease of interpretation, we re-express this unfairness aversion r as the maximal leak ρ (a fraction of the transferred amount) that society is willing to accept in case of progressive transfers. As soon as $r > 0$, the social planner is inequality averse, and she will approve of a shift of resources of someone with high money-metric utility to someone with low money-metric utility, even if a part of that resource "leaks away" during the transfer. The maximal leak is determined by the value of r.[17] If r approaches zero, there is minimal aversion to unfairness and ρ becomes close to zero. If r approaches infinity, there is maximal aversion to unfairness and ρ becomes close to 1.

Figure 6 plots the average marginal social welfare weights for the four groups in our tax reform analysis (unemployed, working poor, working middle, working rich) as a function of the degree of compensation γ. Each figure corresponds to a different value of the maximal leak ρ. As the tastes for working hardly influence the fairness gap, the pattern of the (average) marginal welfare weights for each specific value of ρ is driven by the relationship between the fairness gap and the hourly

[17] In fact, the maximal leak ρ also depends on the difference in the fairness gaps of the donor and the receiver of the transfer. We choose 500 units (roughly equal to the average difference in fairness gap in our sample for $\gamma = 0.5$) such that the maximal leak is defined as $\rho = 1 - \exp(-500r)$.

Figure 5: Fairness gap before reform as a function of the taste for working (voluntary unemployment)

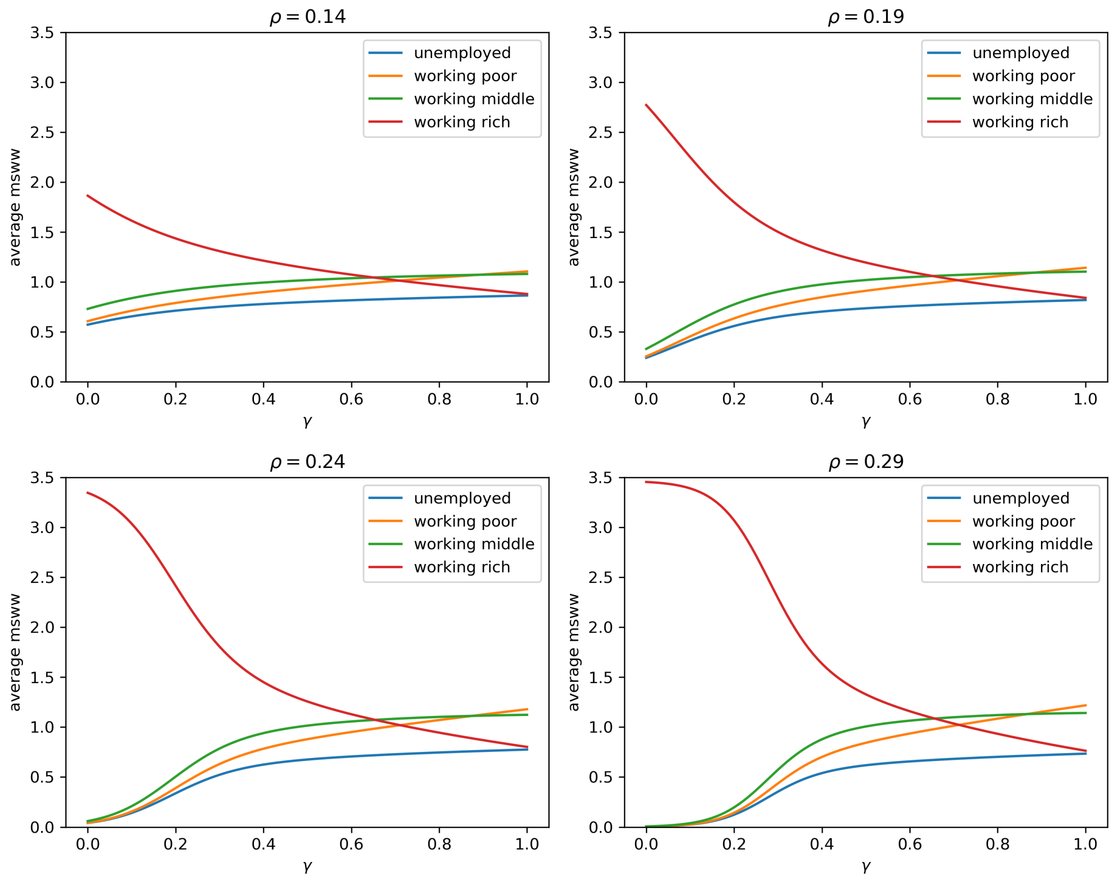


wage rate, shown in Figure 4. The degree of unfairness aversion magnifies the differences between the different income groups, but does not change the social priority ranking. For low to middle degrees of compensation (roughly between 0 and 0.65), the working rich have the highest social priority, followed by the working middle, the working poor, and the unemployed. Over a small range (roughly between 0.65 and 0.9), the working middle get the highest social priority. For high degrees of compensation (between 0.9 and 1), the working poor have the highest social priority, followed by the working middle, and either the working rich or the unemployed. The unemployed have the lowest social priority.

4.2.3 Evaluation of the tax reforms

We can now turn to the evaluation of the four tax reforms. The decomposition in equation (7) shows that the overall welfare evaluation of budget-neutral tax reforms depends only on the change in excess burden ($\triangle EB$) and the change in unfair inequality ($\triangle UI$). We look at these two components in turn.

Figure 6: The average marginal social welfare weight before reform of the different income groups (voluntary unemployment)



Behavioral reactions and excess burden Table 6 shows the changes in the average individual excess burden caused by the four reforms. Given that these reforms are all budget-neutral, the change in the total excess burden boils down to minus the change in the sum of money-metric utilities (see equation (5)). The pattern is striking, but not surprising. An increase in the income of the unemployed corresponds to an increase of social transfers, paid for by the working individuals. It distorts labor choices and increases the excess burden considerably. Note the importance of the second step, which basically captures the effects of the behavioral changes that are required from the groups that are not directly affected by the reform to restore budget neutrality. The other three reforms are decreases in the tax burden of a working group, financed by increases in the taxes of the other working individuals and a decrease in the transfer to the unemployed. As a result, some of the latter start working. In each of these three cases, the overall excess burden decreases as a result of the reform, and the effect is most outspoken in the case of tax relief for the working middle and the working rich.

Table 6: Changes in the average individual excess burdens (voluntary unemployment)

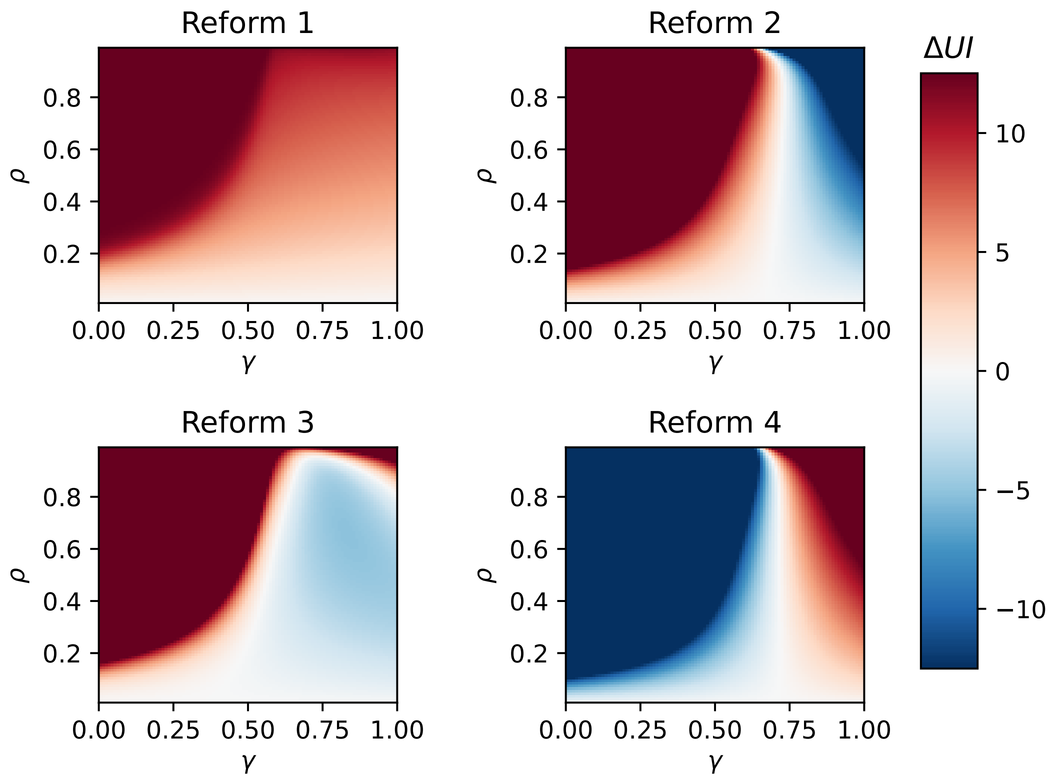
	unemployed	working poor	working middle	working rich	all
Excess burden before reform	1528.41	93.68	60.96	115.75	283.96
Reform 1: ΔEB after step 1	-14.21	28.50	38.85	27.81	25.53
Reform 1: ΔEB after step 2	-14.21	62.11	80.83	56.28	55.54
Reform 2: ΔEB after step 1	-55.33	-2.04	15.54	5.32	-2.03
Reform 2: ΔEB after step 2	-75.57	-2.04	21.82	7.55	-2.30
Reform 3: ΔEB after step 1	-72.69	-12.54	-1.44	11.76	-10.43
Reform 3: ΔEB after step 2	-86.39	-14.64	-1.44	14.37	-12.13
Reform 4: ΔEB after step 1	-46.21	-4.53	-9.43	-1.00	-10.54
Reform 4: ΔEB after step 2	-53.70	-5.12	-11.05	-1.00	-12.18

Unfair inequality Figure 7 shows the change in unfair inequality as a function of the degree of compensation and unfairness aversion (re-expressed as maximal leak). The unfair inequality reflects the combined effect of the income changes in Table 5 and the marginal social welfare weights in Figure 6. The upper-left panel confirms that unfair inequality always increases in case of the first reform, that favors the unemployed who, for all values of the parameters, get the lowest priority. The valuation is most negative for low values of γ, i.e., for ethical views that are close to libertarian. The results for reform 4 (lower-right panel) are almost the mirror image: for low values of γ, an increase of the income of the working rich lowers unfair inequality. In fact, this is true for almost all values of the parameters. Tax reform 4 increases unfair inequality only for values of γ and ρ close to 1 (at the same time). Reforms 2 and 3 are in between: as shown in the upper-right and the lower-left panel of Figure 7, extra net income reduces unfair inequality unless the degree of compensation is small and the degree of unfairness aversion high. In fact, the working poor and the working middle have very similar marginal social welfare weights in Figure 6. For high values of γ and ρ, i.e., for highly unfairness averse resource-egalitarians, increasing the net incomes of the working poor (reform 2) decreases unfair inequality.

Evaluation: the change in welfare caused by the four reforms We now combine the results for the changes in the excess burden and unfair inequality. Figure 8 presents the welfare change of each reform as a function of the degree of compensation and unfairness aversion (re-expressed as maximal leak). Comparing figures 7 and 8, it is immediately clear that the unfairness component dominates the results. The increase in excess burden for reform 1 and the strong decrease in the excess burden for reforms 3 and 4 only strengthen the findings of Figure 7.

The upper-left panel of Figure 8 shows that the first reform (extra net income for the unemployed) always decreases welfare. This is not surprising, given that it strongly increases the excess burden and for most values of the parameters also increases unfair inequality. As before, the results for reform 4 (lower-right panel of the Figure) are in some sense the opposite. Except for very high degrees of compensation and unfairness aversion, increasing the net income of the working rich

Figure 7: The change in unfair inequality caused by the four reforms (voluntary unemployment)

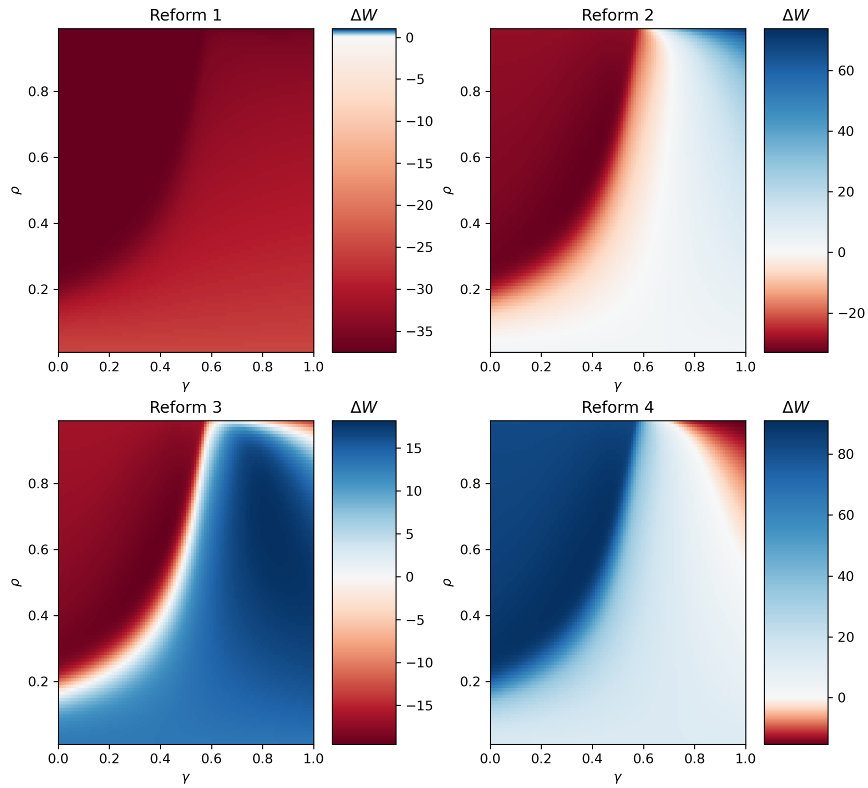


increases welfare. It decreases the excess burden and it also decreases unfair inequality over a large range of the possible values for γ and ρ.

The welfare evaluation of reforms 2 and 3 almost mimic the resulting change in unfair inequality. Of special interest are the results of reform 2 in the upper-right panel of Figure 8. A transfer to the working poor is in line with the recommendation by Fleurbaey and Maniquet (2006), whose normative setting is resource-egalitarian ($\gamma = 1$) with absolute priority to the worst-off ($\rho = 1$). Under these assumptions, our approach also shows that a transfer to the working poor can increase welfare. Yet, this recommendation no longer holds if one adopts less extreme levels of compensation and unfairness aversion. This finding illustrates the added insights generated by a more flexible ethical approach.

A marginal approach: marginal value of public funds A more direct comparison of the welfare effects of the different reforms becomes possible on the basis of their marginal values of public funds. We compute these values for the first step (without budget neutrality) of the four tax reforms, i.e., we compute the welfare gain per euro, given an increase of the monthly net disposable income of each individual whose gross earnings lies in one of the four gross income groups with 75 euro. Figure 9 shows the reform with the lowest and the highest marginal value of public funds

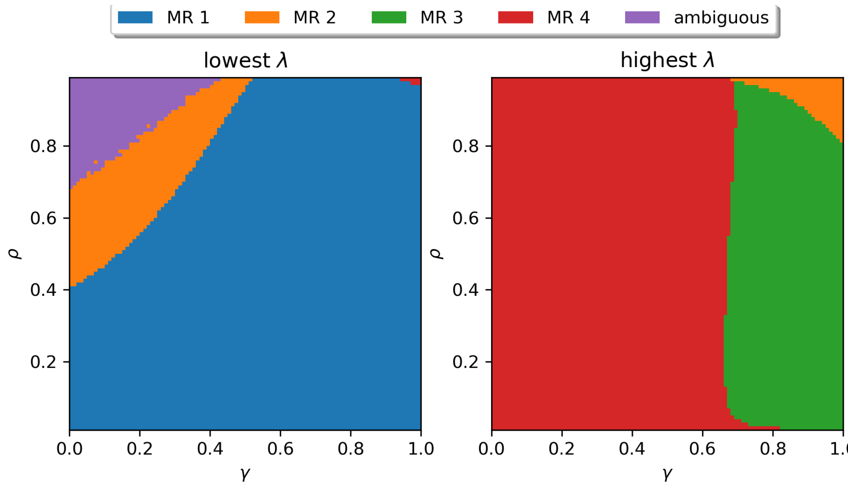
Figure 8: The change in welfare caused by the four reforms (voluntary unemployment)



for different values of ρ and γ. The results are in line with what we observed before. We start with the right-hand panel. If γ is not close to 1, the social planner prefers the reform in favor of the working rich, whatever the degree of unfairness aversion. As seen already before, the social planner will favor the working poor only if she adopts a high degree of both unfairness aversion and compensation. Reforms in favor of the unemployed never yield the highest marginal value of public funds. In fact, the left-hand panel indicates that helping the unemployed yields the lowest marginal value of public funds for a wide range of normative positions.

To sum up, reforms that take money from the unemployed (left-panel of Figure 9) and give it to the working (right-panel) will increase social welfare according to a wide range of unfairness aversion and compensation degrees. At first glance, this may seem surprising. Yet, note that we modeled unemployment as entirely voluntary. This implies that the unemployed are often considered more fairly treated than the working. In the next section we make the alternative assumption that individuals may be rationed on the labor market, so that a fraction of the observed unemployment is involuntary.

Figure 9: Lowest and highest marginal value of public funds (voluntary unemployment)



Notes: the term 'ambiguous' refers to the fact that due to machine precision we are not able to rank reforms 2 and 3 in terms of the lowest λ.

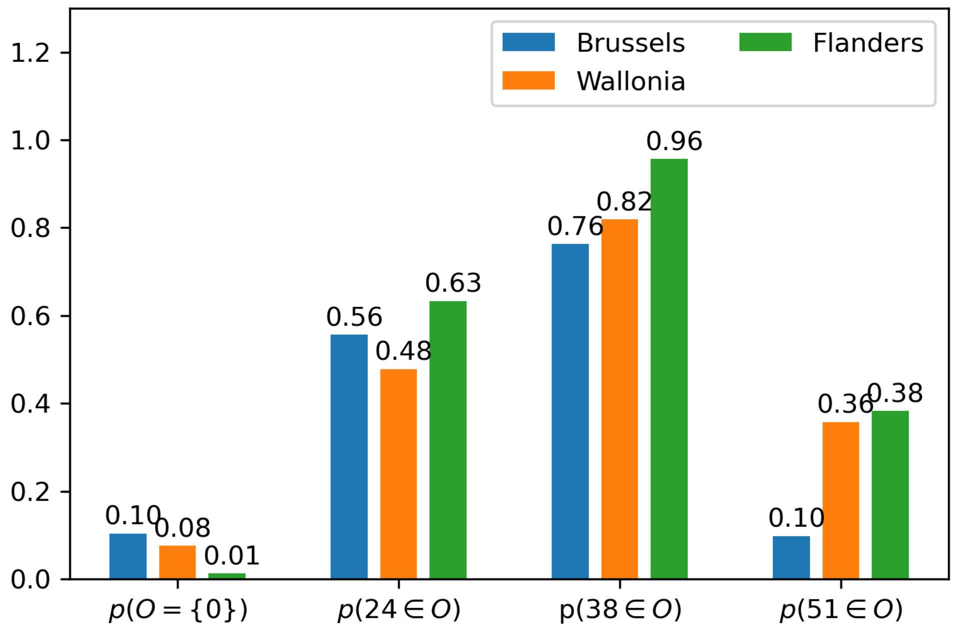
4.3 Involuntary unemployment

We model rationing on the labor market by assuming that an individual cannot necessarily choose from a full opportunity set, i.e., that not all elements in the discrete choice set $L = \{0, 24, 38, 51\}$ are available for everyone. For example, if not working is the only available option, then unemployment is involuntary. Clearly note that rationing goes much further than allowing for involuntary unemployment because, e.g., also working half-time can be involuntary. However, we often use involuntary unemployment to refer to rationing more generally. We analyze the welfare effects of the same four reforms as before. The assumption that not all labor choices are available to everyone, has implications for all the elements in the analysis, starting with the estimation of preferences. We will follow the same structure as in the previous section, but focus on the differences between the two cases.

4.3.1 Estimating preferences, opportunities, and behavioral reactions

To model rationing of labor hours, we assume that the opportunity sets of individuals, denoted $O_i \subseteq L$, are probabilistic. We also assume that not working is available in every opportunity set. Let $p(\ell \in O_i; z_i) \in [0, 1]$ be the probability that individual i can choose to work ℓ hours. These probabilities depend on the observable individual characteristics in z_i. This raises a difficult identification problem. Observed choices will reflect both the opportunities that are available to the individuals and their preferences — and the same individual characteristics (e.g., gender) may affect both. For our illustration, we assume that preferences vary with gender and age and that opportunities vary with region, population density, gender, education, and nationality. This is of course only an illustration. Further details and estimation results can be found in appendix D.2.

Figure 10: Regional heterogeneity with respect to opportunity sets



In the model of voluntary unemployment, heterogeneity in labor choices was captured by observed and unobserved preference heterogeneity. Probabilistic opportunity sets now take up a substantive amount of this heterogeneity. As an illustration, Figure 10 displays the average probabilities of (not) having certain labor hour choices available for individuals living in different regions in Belgium. The probability of involuntary unemployment ranges between 0.01 in Flanders and 0.10 in Brussels. Overall job opportunities are best in Flanders, followed by Wallonia, and Brussels. In the model of section 4.2, the smaller number of unemployed in Flanders was ascribed to differences in preferences, now it will be related to regional differences in opportunities. It stands to reason that this will have serious consequences for the welfare evaluation of social policies, especially when the social planner is unfairness averse and the desired degree of compensation γ is large enough.

Table 7 shows the probabilities of the different job choices, as predicted by the model. Comparing it to Table 3, it is clear that the approach with rationing on the labor market gives a (much) better fit. More specifically, the overprediction of working half-time for men has almost completely disappeared.

For the simulations, we again replicate every individual 809 times, but we now also simulate the opportunity sets using the estimated probabilities. As individuals will typically choose among a lower number of labor options, the behavioral responses to the tax reforms are in general weaker (see Table 8). To illustrate, the elasticity with respect to an increase in the unemployment benefit

Table 7: Actual and predicted probabilities (involuntary unemployment)

all	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	13.3	16.8	63.3	6.6
predicted (%)	13.3	16.8	63.3	6.6
predicted - actual	0.0	0.0	0.0	0.0
men	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	14.5	10.0	66.8	8.7
predicted (%)	14.7	10.3	66.2	8.8
predicted - actual	0.2	0.3	-0.6	0.1
women	$\ell = 0$	$\ell = 24$	$\ell = 38$	$\ell = 51$
actual (%)	11.8	25.7	58.6	3.9
predicted (%)	11.5	25.3	59.5	3.7
predicted - actual	-0.2	-0.4	0.8	-0.2

is 0.07 in the model with involuntary unemployment, compared to 0.22 in the model of voluntary unemployment. This is also true for the other elasticities on the diagonal, but to a somewhat lesser extent. As people get richer, they are typically less constrained in their labor choices.

Table 9 shows that the differences in the behavioral reactions have also an impact on the size of the income changes imposed to reach budget neutrality on the groups that are not directly affected by the reform. As the increase in the number of unemployed as a result of the first tax reform now is much smaller, the increase in the transfers to the unemployed can be financed with a smaller increase in the tax paid by the other groups. On the other hand, to compensate the tax decrease of the working groups, the smaller behavioral reactions imply that larger increases in the taxes of the other groups are necessary to reach budget neutrality.

4.3.2 Marginal social welfare weights

The possibility of involuntary unemployment also has a first-order effect on the money-metric utilities and, hence, on the fairness gaps and the marginal social welfare weights.

Fairness gaps Figure 11 confirms the pattern that was already found in Figure 4 for the case of voluntary unemployment. For small values of γ, the fairness gap decreases with the hourly wage rate, for large values of γ the opposite holds. Since individuals can now less freely choose their number of hours, the actual money-metric utilities, and thus also the fairness gaps, tend to be lower. This holds for all values of γ, but it is most striking when the desired degree of compensation increases. We do not show the results for the relationship between the fairness gaps and the taste for working, as they just confirm the finding of Figure 5 that this relationship is weak.[18]

[18]Results are available from the authors on request.

Table 8: Behavioral reactions caused by each reform (involuntary unemployment)

	unemployed	working poor	working middle	working rich
monthly gross income (€)	$y = 0$	$y \leq 2404$	$2404 < y \leq 3463$	$3463 < y$
average monthly net income (€)	1415	1509	1907	2913
fraction before reform (%)	13.4	28.9	28.9	28.8
fraction after reform 1 (%)	13.9	28.7	28.8	28.7
fraction after step 1 only (%)	13.8	28.7	28.8	28.7
percentage point change	+0.4	-0.1	-0.1	-0.1
elasticity (pp change / 5.3)	+**0.07**	-0.03	-0.02	-0.02
fraction after reform 1 (%)	13.2	29.5	28.6	28.7
fraction after step 1 only (%)	13.2	29.4	28.7	28.7
percentage point change	-0.1	+0.5	-0.3	-0.1
elasticity (pp change / 5.0)	-0.03	+**0.09**	-0.05	-0.01
fraction after reform 1 (%)	13.2	28.5	29.6	28.6
fraction after step 1 only (%)	13.3	28.6	29.5	28.6
percentage point change	-0.1	-0.3	+0.5	-0.2
elasticity (pp change / 3.9)	-0.03	-0.06	+**0.14**	-0.04
fraction after reform 1 (%)	13.2	28.8	28.7	29.2
fraction after step 1 only (%)	13.3	28.8	28.8	29.1
percentage point change	-0.1	-0.1	-0.2	+0.3
elasticity (pp change / 2.6)	-0.04	-0.03	-0.07	+**0.13**

Table 9: Hypothetical tax reforms (involuntary unemployment)

	unemployed	working poor	working middle	working rich
monthly gross income (€)	$y = 0$	$0 < y \leq 2404$	$2404 < y \leq 3463$	$3463 < y$
average monthly net income (€)	1415	1509	1907	2913
reform 1 (€ net extra per month)	75	-27	-27	-27
reform 1 (as % of net income)	5.3	-1.8	-1.4	-0.9
reform 2 (€ net extra per month)	-31	75	-31	-31
reform 2 (as % of net income)	-2.2	5.0	-1.6	-1.1
reform 3 (€ net extra per month)	-25	-25	75	-25
reform 3 (as % of net income)	-1.8	-1.7	3.9	-0.9
reform 4 (€ net extra per month)	-21	-21	-21	75
reform 4 (as % of net income)	-1.5	-1.4	-1.1	2.6

Figure 11: Fairness gap as function of hourly wage rate (involuntary unemployment)

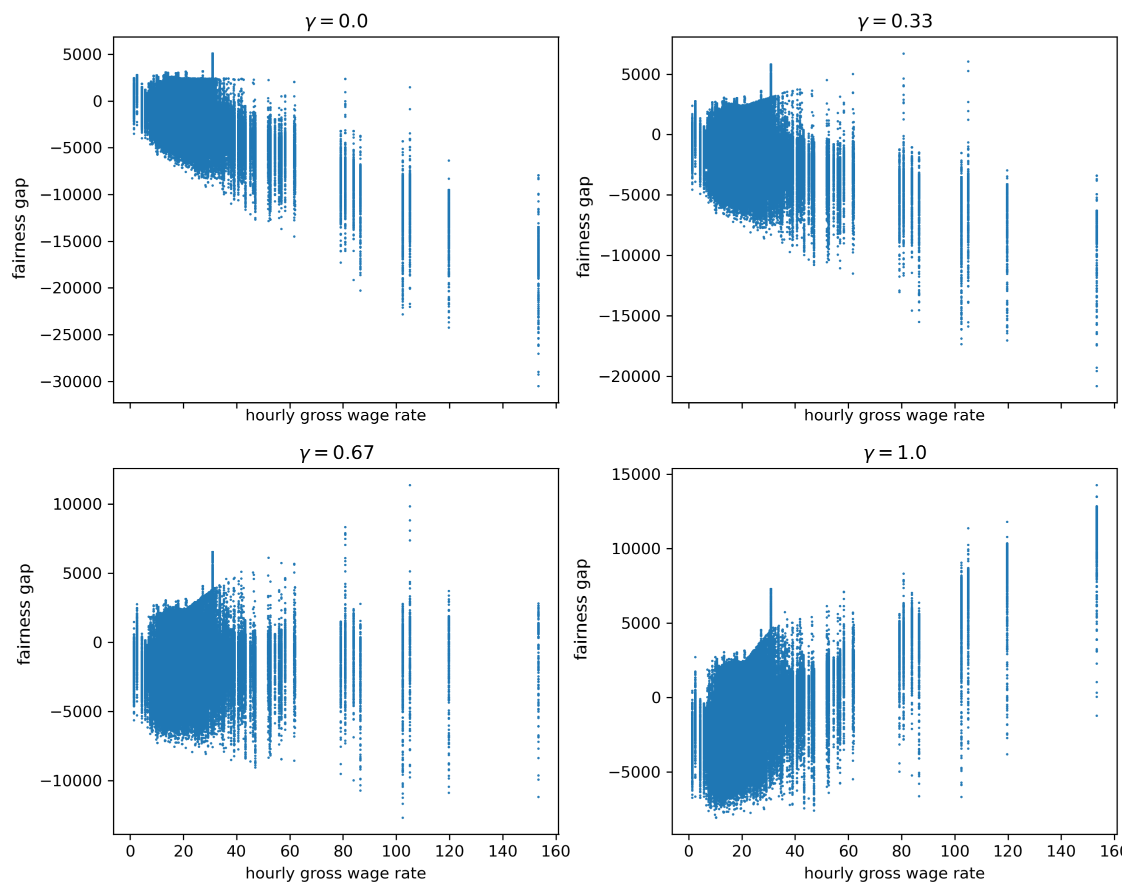
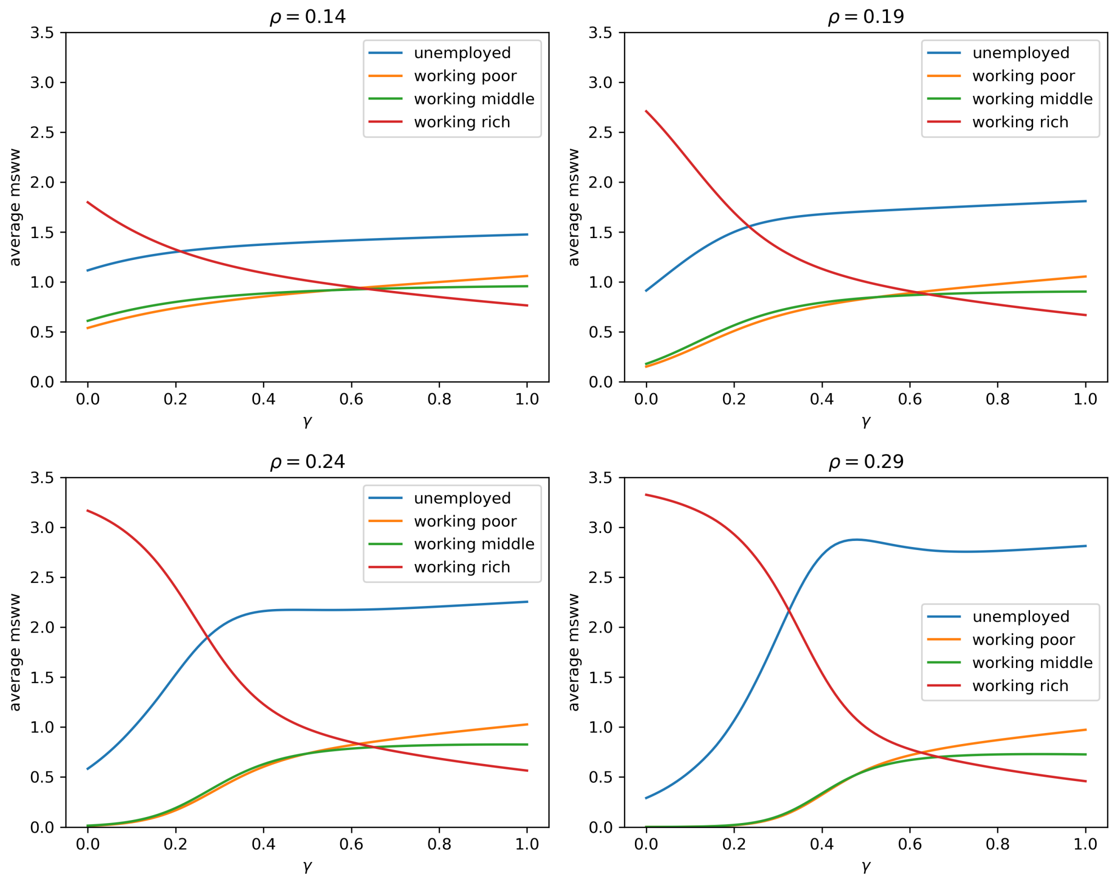


Figure 12: The average marginal social welfare weight for the different income groups (involuntary unemployment)



Marginal social welfare weights The assumptions about the nature of unemployment have huge consequences for the average marginal social welfare weight of the unemployed, as is clear from a comparison of Figure 12 with Figure 6. While the results for the other social groups remain rather stable, the marginal social welfare weights of the unemployed, a large fraction of which is now rationed, increase substantially. While they had the lowest social priority for all values of γ in the case of voluntary unemployment, they now have the highest, except when the social planner is close to libertarian (γ smaller than 0.2-0.3, depending on the degree of unfairness aversion ρ).

4.3.3 Evaluation of the tax reforms

Excess burden Table 10 summarizes the changes in the excess burden for the four tax reforms. It can be usefully compared to the corresponding Table 6 for the case of voluntary unemployment. Since the restrictions on the choice sets limit the freedom of choice of the agents, the current situation is much further removed from the first best and the excess burden in the current situation is therefore much larger than in the case of voluntary unemployment. On the other hand, the changes in excess burden as a result of the tax reform are minor. This is in line with the rather

Table 10: Changes in the average individual excess burdens (involuntary unemployment)

	unemployed	working poor	working middle	working rich	total
Excess burden before reform	3528.22	564.10	382.73	437.19	872.01
Reform 1: ΔEB after step 1	4.69	8.83	11.75	15.33	10.99
Reform 1: ΔEB after step 2	4.69	11.21	15.70	20.31	14.26
Reform 2: ΔEB after step 1	-17.44	1.54	6.38	2.66	0.72
Reform 2: ΔEB after step 2	-28.84	1.54	8.90	3.75	0.23
Reform 3: ΔEB after step 1	-21.59	-5.47	0.86	5.67	-2.58
Reform 3: ΔEB after step 2	-32.62	-7.73	0.86	7.59	-4.16
Reform 4: ΔEB after step 1	-26.02	-2.18	-4.97	0.11	-5.51
Reform 4: ΔEB after step 2	-36.60	-3.37	-6.46	0.11	-7.71

weak behavioral reactions in Table 8. It will imply that the evaluation of the tax reforms will depend even more on the ethical stance that is taken.

Unfair inequality The changes in unfair inequality as a result of the tax reforms (Figure 13) reflect the pattern of the marginal social welfare weights. Both reforms 2 and 3 increase unfair inequality, except for small values of ρ and large values of γ. The difference with the results under the assumption of voluntary unemployment is especially striking for reform 2, an increase in the net income of the working poor. Indeed, the marginal social welfare weights in Figure 12 are much smaller for the working poor (and the working middle) than for the unemployed, and the differences are increasing with ρ. Moreover, for small values of γ, the former are also much smaller than those for the working rich. Tax reforms, benefiting the working poor or the working middle, financed by tax increases for the unemployed and the working rich will therefore increase unfair inequality. The results for reforms 1 and 4 also immediately follow from the marginal social welfare weights. Reform 1 (an increase of the transfers to the unemployed) decreases unfair inequality for almost all values of ρ and γ. Reform 4 (an increase in the net income of the working rich) increases unfair inequality for γ large enough, and decreases unfair inequality for small values of γ.

Evaluation Combing the findings for ΔEB and ΔUI, we get the results in Figure 14. As in the case of voluntary employment, the results are mainly influenced by the effects on unfair inequality, and, hence, by the ethical parameters γ and ρ. Reforms 2 and 3 have a negative effect on social welfare for almost all values of the parameters. The only exception is when ρ is rather low. Again, this may seem surprising for reform 2, which is an increase in the net income of the working poor. For sufficiently large values of γ and ρ, increasing the social transfers to the unemployed improves social welfare. For the complementary values of the parameters, increasing the net income of the working rich improves welfare.

Figure 13: Changes in unfair inequality (involuntary unemployment)

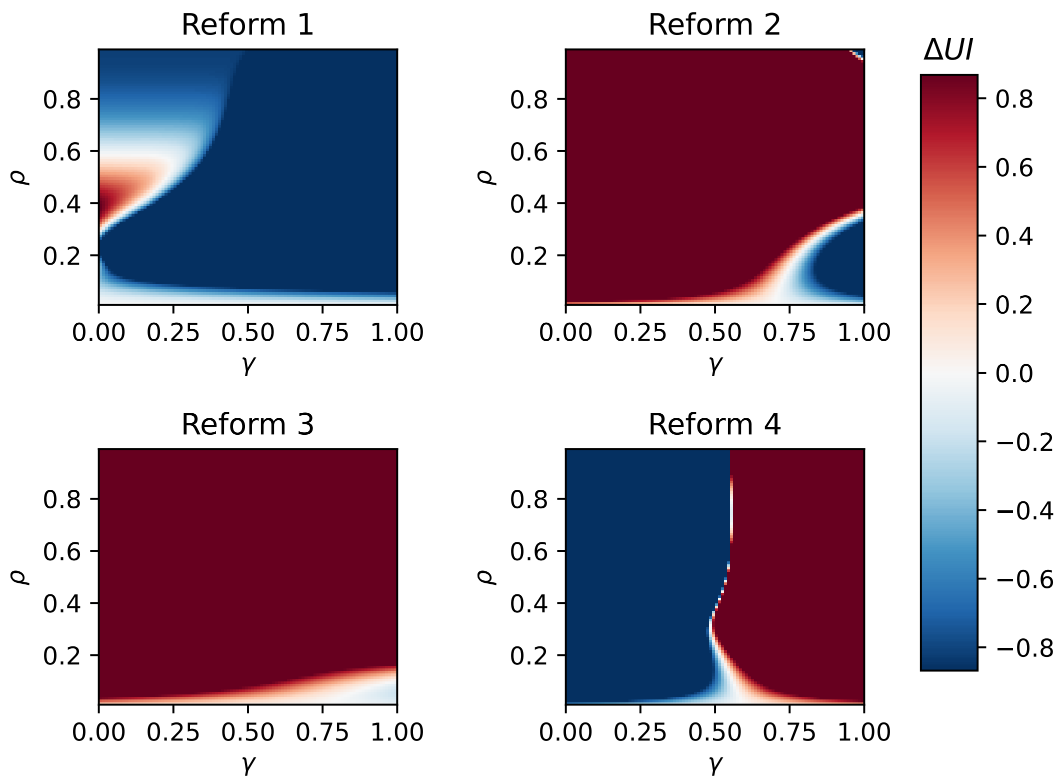


Figure 14: Changes in welfare (involuntary unemployment)

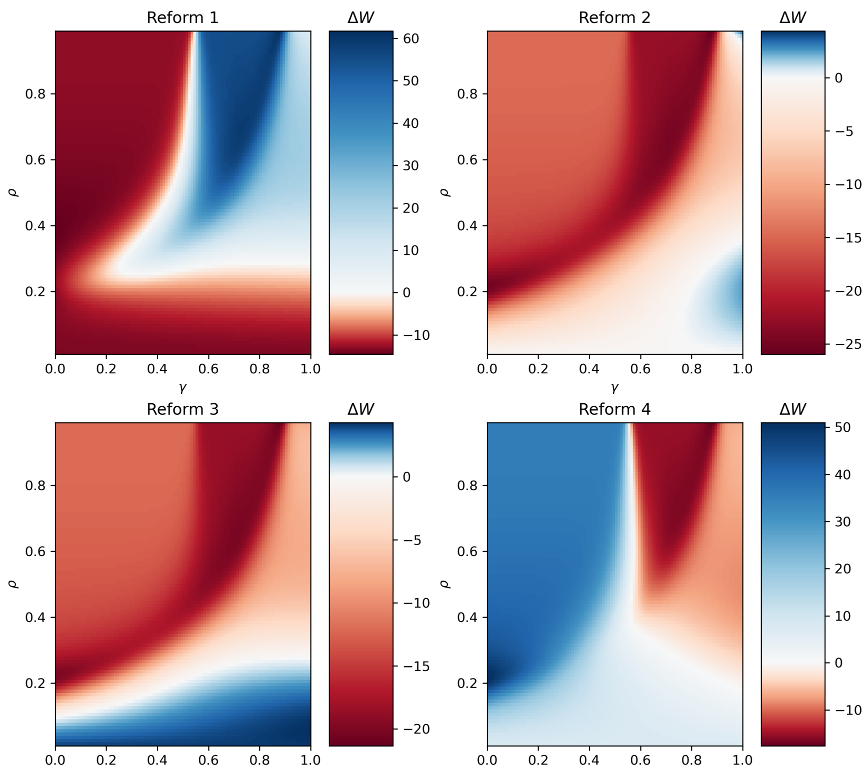
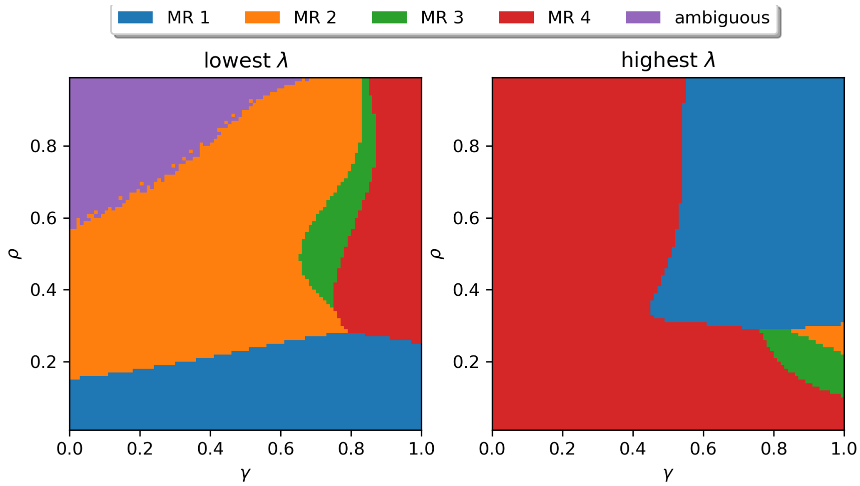


Figure 15: Lowest and highest marginal value of public funds (involuntary unemployment)



Notes: the term 'ambiguous' refers to the fact that due to machine precision we are not able to rank reforms 2 and 3 in terms of the lowest λ.

The marginal value of public funds Finally, we can look at the marginal value of public funds in Figure 15, which can be compared with Figure 9. The right-hand panel shows that the social planner will mainly favor marginal reforms in favor of the working rich as long as the inequality aversion is low. With higher inequality aversion ($\rho > 0.3$), the preferred policy will depend on γ: for ethical positions close to libertarianism, increasing the income of the working rich is the most desirable policy. For larger values of γ (about $\gamma > 0.5$), however, social welfare is most increased by increasing the transfers to the unemployed. This is in stark contrast to the results in the case of voluntary unemployment. Increasing the income of the working poor is almost never the best policy. On the contrary, as shown in the left-hand panel, it has the lowest value of public funds for a large range of parameter values.

Figure 15 illustrates the advantages of our flexible ethical framework. If we were to accept maximin and resource-egalitarianism, we would unambiguously advocate to increase the transfers to the unemployed. However, even when we stick to maximin, but give a smaller weight to compensation for skill differences, i.e., become "more libertarian," then increasing the income of the working rich yields the largest marginal value of public funds. A similar conclusion is found when we remain resource-egalitarian, but with a lower degree of unfairness aversion.

5 Conclusion

Traditional welfarist social welfare functions run into difficulties with interpersonal utility comparisons under the (obviously realistic) assumption that preferences differ between individuals. One then needs a notion of well-being that can reflect such preference differences. Moreover, differences in outcomes will not only reflect differences in innate productivities (as in traditional optimal tax theory), but also differences in preferences, i.e., in the motivation to work. As there are different views in society about the ethical status of differences in innate productivities and in preferences, this raises the challenge of working out a formal optimal tax framework that is sufficiently flexible to accommodate these different views.

We characterize a social welfare framework in which money-metric utilities are justified as the measures of well-being. These are integrated into a Paretian social welfare function defined in terms of increasing and concave transformations of the gaps between actual money-metric utilities and some definition of "optimal" money-metric utilities, where the optimality refers to an equity or fairness concept. The latter can be chosen (almost) freely and we propose a parameterized range of possibilities going from libertarianism at one extreme to resource-egalitarianism at the other. Moreover, the use of a concave transformation allows us to implement different degrees of inequality aversion, with maximin only one possible extreme.

The flexibility of our social welfare function offers even more opportunities than the ones we have worked out in this paper, as the "optimal" money-metric utilities can be defined in many ways. Of course, it will never be as flexible as the direct specification of the welfare weights proposed by Saez and Stantcheva (2016), but, since it is the representation of a transitive social welfare ordering, it will never lead to inconsistencies when going beyond local approximations. Moreover, maximizing a well-defined social welfare function is perfectly in line with the public economics tradition on optimal taxation.

The applicability of the approach is illustrated with the empirical analysis of four budget-neutral tax reforms for Belgian singles. Our framework allows us to decompose the effects of these reforms in terms of efficiency (excess burden) and unfair inequality. In our specific empirical setting, the latter, and more specifically, the view on whether individuals should be held responsible for their innate opportunities or not, turns out to be the most important normative choice. Not surprisingly, we find that for libertarians the present tax burden of the rich is too large and should be lowered. On the other hand, for resource-egalitarians the results essentially depend on whether we interpret unemployment as voluntary or involuntary. In the case of voluntary unemployment, the unemployed reach a relatively high level of well-being and for a sufficiently high degree of unfairness aversion the working poor are the worst-off group before the tax reform. This is in line with earlier findings in the literature (basically starting with Fleurbaey and Maniquet, 2006). However, if unemployment is involuntary and the degree of unfairness aversion is sufficiently high, then the group of unemployed are worst off and increasing their transfers is the best policy.

Of course, our empirical analysis is only an illustration. It does not make much sense to consider tax reforms and imposing budget neutrality for the singles in isolation. Moreover, the econometric analysis can be further refined. As it stands however, this empirical analysis shows the relevance of a social welfare framework that goes beyond welfarism and can integrate ethical perspectives (such as libertarianism) that are often taken in society, but almost never found in traditional optimal tax analysis. Moreover, our specification of the social welfare criterion in terms of fairness gaps creates scope to introduce still other normative views.

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A Proof of theorem 1

First, Representation allows us to represent social welfare as

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I} v_i(x_i;\mathscr{S}),$$

with $v_1, v_2, \ldots, v_n$ evaluation functions that are continuously differentiable in resources (earnings and labor).

Second, Pareto requires that the evaluation function v_i is a strictly increasing transformation of money-metric utility, i.e., for each individual $i = 1, 2, \ldots, n$, we have $v_i(x_i;\mathscr{S}) = \phi_i(m(x_i,\theta_i);\mathscr{S})$, with $\phi_i'(\cdot;\mathscr{S}) > 0$. Social welfare can thus be represented as

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I} \phi_i(m(x_i,\theta_i);\mathscr{S}), \tag{14}$$

where the transformation functions satisfy $\phi_i'(\cdot;\mathscr{S}) > 0$ for all $i = 1, 2, \ldots, n$.[19]

Third, the (Pareto-efficiency-preserving) transfer principle can be stated in terms of regressive transfers as follows. A regressive transfers of resources between two individuals i and j, i.e.,

$$v_i(x_i';\mathscr{S}) > v_i(x_i;\mathscr{S}) \geq v_j(x_j;\mathscr{S}) > v_j(x_j';\mathscr{S}),$$

that does not affect other individuals, i.e.,

$$x_k = x_k', k \neq i, j,$$

and satisfying $x, x' \in P(\mathscr{S})$ should be disapproved of, i.e., $W(x;\mathscr{S}) > W(x';\mathscr{S})$. We proceed in several steps.

1. Because x, x' are Pareto efficient allocations for the same society $\mathscr{S}$, we must have

$$\frac{1}{n}\sum_{i\in I} m(x_i,\theta_i) = -R_0 = \frac{1}{n}\sum_{i\in I} m(x_i',\theta_i),$$

and thus, given $x_k = x_k'$, for all $k \neq i, j$, we get $m(x_i,\theta_i) + m(x_j,\theta_j) = m(x_i',\theta_i) + m(x_j',\theta_j)$. In other words, any transfer of resources between two individuals that preserves Pareto efficiency corresponds with a mean-preserving transfer of money-metric utilities between

[19] Note that it is not restrictive to use money-metric utility computed at own skills and tastes because the transformation functions $\phi_i(\cdot;\mathscr{S})$ can depend on society. In other words, the current money-metric utility function can be transformed into any other possible function that cardinalizes preferences from knowledge of the money-metric utility, the skill, and the taste of an individual.

these two individuals. For later use, define this transfer in money-metric utility as

$$T = m(x'_i, \theta_i) - m(x_i, \theta_i) = m(x_j, \theta_j) - m(x'_j, \theta_j) > 0. \tag{15}$$

2. Using Representation and given $x_k = x'_k, k \neq i, j$, the requirement $W(x; \mathscr{S}) > W(x'; \mathscr{S})$ can be rewritten as

$$v_i(x_i; \mathscr{S}) + v_j(x_j; \mathscr{S}) > v_i(x'_i; \mathscr{S}) + v_j(x'_j; \mathscr{S}). \tag{16}$$

3. Using $v_i(x_i; \mathscr{S}) = \phi_i(m(x_i, \theta_i); \mathscr{S})$ for each individual i, as derived before, and using equations (15) and (16), the (regressive) transfer principle can now be rewritten as follows. Suppose $x \in P(\mathscr{S})$ holds. A regressive transfer $T > 0$ of money-metric utility from individual j and i (ceteris paribus), i.e.,

$$\phi_i(m(x_i, \theta_i) + T; \mathscr{S}) > \phi_i(m(x_i, \theta_i); \mathscr{S}) \geq \phi_j(m(x_j, \theta_j); \mathscr{S}) > \phi_j(m(x_j, \theta_j) - T; \mathscr{S}),$$

should be disapproved of, i.e.,

$$\phi_i(m(x_i, \theta_i); \mathscr{S}) + \phi_j(m(x_j, \theta_j); \mathscr{S}) > \phi_i(m(x_i, \theta_i) + T; \mathscr{S}) + \phi_j(m(x_j, \theta_j) - T; \mathscr{S}).$$

4. In the limit $T \to 0$, Transfer requires that $\phi_i(m(x_i, \theta_i); \mathscr{S}) \geq \phi_j(m(x_j, \theta_j); \mathscr{S})$ implies $\phi'_i(m(x_i, \theta_i); \mathscr{S}) \leq \phi'_j(m(x_j, \theta_j); \mathscr{S})$ for any two individuals i, j and for any Pareto efficient allocation x. As the opposite is true as well, we get that $\phi_i(m(x_i, \theta_i); \mathscr{S}) = \phi_j(m(x_j, \theta_j); \mathscr{S})$ implies $\phi'_i(m(x_i, \theta_i); \mathscr{S}) = \phi'_j(m(x_j, \theta_j); \mathscr{S})$ for any two individuals i, j and for any Pareto efficient allocation x. As it must hold for any pair of individuals and for any Pareto efficient allocation x, we can reformulate the condition as follows: for each individual i, the derivative $\phi'_i(\cdot; \mathscr{S})$ must be a common, continuous, and strictly positive function, say $f(\cdot; \mathscr{S}) > 0$, of the level function $\phi_i(\cdot; \mathscr{S})$, i.e., $\phi'_i(m; \mathscr{S}) = f(\phi_i(m; \mathscr{S}); \mathscr{S})$ must hold for each individual i and each money-metric utility level m.

5. This differential equation can be solved as follows.[20] Define $\varphi'(\cdot; \mathscr{S}) = 1/f(\cdot; \mathscr{S}) > 0$. We can now rewrite the differential equation as

$$\frac{\phi'_i(m; \mathscr{S})}{f(\phi_i(m; \mathscr{S}); \mathscr{S})} = \varphi'(\phi_i(m); \mathscr{S})\phi'_i(m; \mathscr{S}) = 1.$$

Integrating both sides with respect to m, we get $\varphi(\phi_i(m; \mathscr{S}); \mathscr{S}) = m + k_i(\mathscr{S})$ for some constant $k_i(; \mathscr{S})$, leading to $\phi_i(m; \mathscr{S}) = \varphi^{-1}(m + k_i(\mathscr{S}); \mathscr{S})$ for each i and m. Define a

[20] The proof is based on Bosmans, Lauwers, and Ooghe (2009).

common function $\phi(\cdot;\mathscr{S}) = \varphi^{-1}(\cdot;\mathscr{S})$ to obtain that social welfare can be represented as

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I}\phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S}), \tag{17}$$

with $\phi'(\cdot;\mathscr{S}) > 0$ and $k_1(\mathscr{S}), k_2(\mathscr{S}), \ldots, k_n(\mathscr{S})$ arbitrary constants.[21]

6. Finally, note that $\phi''(\cdot;\mathscr{S}) < 0$ must hold as well to ensure that the transfer principle also works beyond the limiting case $T \to 0$ that we considered before. To see this, we can use $\phi_i(m(x_i,\theta_i);\mathscr{S}) = \phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S})$ to rewrite the transfer principle as follows: a regressive transfer $T > 0$ of money-metric utility from individual j and i (ceteris paribus), i.e.,

$$\phi(m(x_i,\theta_i)+T+k_i(\mathscr{S});\mathscr{S}) > \phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S}) \geq$$
$$\phi(m(x_j,\theta_j)+k_j(\mathscr{S});\mathscr{S}) > \phi(m(x_j,\theta_j)-T+k_j(\mathscr{S});\mathscr{S}),$$

should be disapproved of, i.e.,

$$\phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S}) + \phi(m(x_j,\theta_j)+k_j(\mathscr{S});\mathscr{S}) >$$
$$\phi(m(x_i,\theta_i)+T+k_i(\mathscr{S});\mathscr{S}) + \phi(m(x_j,\theta_j)-T+k_j(\mathscr{S});\mathscr{S}).$$

As we are allowed to choose $m(x_i,\theta_i)+k_i(\mathscr{S}) = m(x_j,\theta_j)+k_j(\mathscr{S}) \equiv \mu$, the transfer principle requires

$$\phi(\mu;\mathscr{S}) + \phi(\mu;\mathscr{S}) > \phi(\mu+T;\mathscr{S}) + \phi(\mu-T;\mathscr{S}),$$

for any μ, which requires $\phi(\cdot;\mathscr{S})$ to be strictly concave and thus, given differentiability, leads to $\phi''(\cdot;\mathscr{S}) < 0$.

Fourth, Anonymity requires $W(\ldots,x_i,\ldots,x_j,\ldots;\mathscr{S}) = W(\ldots,x_j,\ldots,x_i,\ldots;\mathscr{S})$ if $\theta_i = \theta_j$. Using equation (17), this implication can be spelled out as

$$\phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S}) + \phi(m(x_j,\theta_j)+k_j(\mathscr{S});\mathscr{S}) =$$
$$\phi(m(x_j,\theta_i)+k_i(\mathscr{S});\mathscr{S}) + \phi(m(x_i,\theta_j)+k_j(\mathscr{S});\mathscr{S}).$$

As $\theta_i = \theta_j$ implies $m(x_j,\theta_i) = m(x_j,\theta_j) \equiv \mu_j$ and $m(x_i,\theta_j) = m(x_i,\theta_i) \equiv \mu_i$, the implication is

$$\phi(\mu_i+k_i(\mathscr{S});\mathscr{S}) + \phi(\mu_j+k_j(\mathscr{S});\mathscr{S}) = \phi(\mu_j+k_i(\mathscr{S});\mathscr{S}) + \phi(\mu_i+k_j(\mathscr{S});\mathscr{S}),$$

which is, given strict concavity (and thus non-linearity) of ϕ, only possible if $k_i(\mathscr{S}) = k_j(\mathscr{S})$. To sum up, if $\theta_i = \theta_j$, then $k_i(\mathscr{S}) = k_j(\mathscr{S})$ must hold.

[21]Note indeed that $\phi'(m;\mathscr{S}) = 1/\varphi'(\varphi^{-1}(m;\mathscr{S});\mathscr{S}) = f(\varphi^{-1}(m;\mathscr{S});\mathscr{S}) > 0$ for all m.

Fifth, to sum up, a social welfare measure $W(x;\mathscr{S})$ satisfies Representation, Pareto, Anonymity, and Transfer if and only if it can be written as

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I}\phi(m(x_i,\theta_i)+k_i(\mathscr{S});\mathscr{S}),$$

where the transformation function $\phi(\cdot;\mathscr{S})$ satisfies $\phi'(\cdot;\mathscr{S})>0$ and $\phi''(\cdot;\mathscr{S})<0$ and $k_i(\mathscr{S})$ are individual-specific constants satisfying $k_i(\mathscr{S})=k_j(\mathscr{S})$ if $\theta_i=\theta_j$. Let $x^*(\mathscr{S})$ be an allocation that maximizes welfare $W(x;\mathscr{S})$ subject to the feasibility constraint $x\in F(\mathscr{S})$. Given the properties of the welfare function (implied by Pareto and anonymity), the allocation $x^*(\mathscr{S})$ must be a Pareto efficient and anonymous allocation (i.e., $x^*(\mathscr{S})\in P(\mathscr{S})$ and $x_i^*(\mathscr{S})=x_j^*(\mathscr{S})$ for all i,j in I with $\theta_i=\theta_j$). Using $\lambda>0$ as the Lagrange multiplier, the first-order conditions of the social planner, evaluated at allocation $x^*(\mathscr{S})$, are

$$\phi'(m(x_i^*(\mathscr{S}),\theta_i)+k_i(\mathscr{S});\mathscr{S})\frac{\partial m(x_i^*(\mathscr{S}),\theta_i)}{\partial c}=\lambda,$$

$$\phi'(m(x_i^*(\mathscr{S}),\theta_i)+k_i(\mathscr{S});\mathscr{S})\frac{\partial m(x_i^*(\mathscr{S}),\theta_i)}{\partial \ell}=-\lambda s_i,$$

for each individual. Because the optimal allocation is Pareto efficient, we have $\frac{\partial m(x_i^*(\mathscr{S}),\theta_i)}{\partial c}=1$ and $\frac{\partial m(x_i^*(\mathscr{S}),\theta_i)}{\partial \ell}=-s_i$ for each individual. The system of first-order conditions reduces therefore to

$$\phi'(m(x_i^*(\mathscr{S}),\theta_i)+k_i(\mathscr{S});\mathscr{S})=\lambda,$$

for each individual. These conditions can be satisfied only if there is a constant $k(\mathscr{S})$ such that $k_i(\mathscr{S})=-m(x_i^*(\mathscr{S}),\theta_i)+k(\mathscr{S})$ for each individual i. If we take up the common constant $k(\mathscr{S})$ in the transformation function $\phi(\cdot;\mathscr{S})$, welfare becomes

$$W(x;\mathscr{S}) = \frac{1}{n}\sum_{i\in I}\phi(m(x_i,\theta_i)-m(x_i^*(\mathscr{S}),\theta_i);\mathscr{S}),$$

as required.

B Data selection

This section describes the data selection for both the hourly gross wage estimation and the subsequent preference estimation. For the latter, we work with our basic sample of singles without children, between 18 and 65 years old and not living with their parents, who are either unemployed (job-seeking) or employed (but not self-employed) with a wage higher than the 2016 minimum wage (9.11 euro), and who do not receive any disability allowance. For the hourly gross wage estimation, we extended the sample to individuals with a partner (and possibly children).

Table 11: OLS estimates of the (log of the) hourly gross wage rate.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.3312	0.0249	93.47	0.0000
gender-male	0.1002	0.0187	5.35	0.0000
educ-secondary	0.1399	0.0171	8.16	0.0000
educ-tertiary	0.4688	0.0166	28.28	0.0000
area-middle-density	-0.0159	0.0120	-1.32	0.1855
area-rural	-0.0382	0.0161	-2.38	0.0176
has-partner	-0.0140	0.0158	-0.89	0.3736
gender-male × has-partner	0.0797	0.0225	3.55	0.0004
experience	0.0259	0.0017	14.81	0.0000
experience^2	-0.0004	0.0000	-8.50	0.0000
nat-EU	0.0541	0.0192	2.82	0.0048
nat-not-EU	-0.1401	0.0311	-4.50	0.0000
RMSE	0.3239			
adjusted R^2	0.3422			

Notes: because of the extended sample (see Appendix B), we included a dummy 'has-partner' as a covariate.

C Hourly gross wages

For individuals who worked in 2016, we observe their yearly gross earnings and the numbers of months worked part-time and full-time. We compute their hourly gross wage rate as

$$\text{hourly gross wage} = \frac{\text{yearly gross earnings}}{24 \times \frac{52}{12} \times \#\text{part-time months} + 40 \times \frac{52}{12} \times \#\text{full-time months}},$$

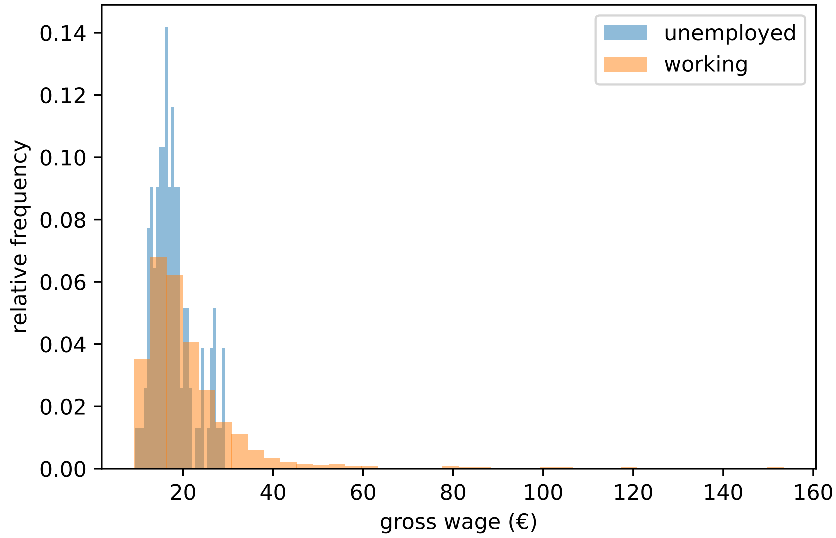
where the numbers 24 and 40 correspond with the median number of hours worked per week as reported by part-time and full-time working individuals in 2017. We exclude individuals with a gross wage rate below the minimum wage (9.11 euro).

We estimate a standard OLS regression model to predict the hourly gross wage rate of individuals who did not work in 2016. The estimation results are presented in Table 11.

Ceteris paribus, males have higher wages (+10%), higher education levels lead to higher wages (+14% for a secondary degree and +47% for a tertiary degree as highest degree relative to lower than secondary degrees), wages in rural areas are lower than in urban areas (-4%), having a partner leads to higher wages for males (+7%), but not for females, work experience leads to higher wages at a decreasing rate, and (non-Belgian) EU-citizens earn higher wages (+5%), while non-EU citizens earn lower wages (-14%) compared to Belgians.

The distribution of hourly gross wages—computed for the working and predicted for the unemployed—is displayed in Figure 16 for the limited sample of singles without children. Predictions below the minimum wage lead to exclusion, so both distributions are truncated to the left

Figure 16: Distribution of the hourly gross wage rate for working and unemployed singles without children.



at the minimum wage (9.11 euro). The mean hourly gross wage rate is 21.16 euro for the working and 17.70 euro for the unemployed. Moreover, the wage dispersion is larger among the working.

D Estimation of preferences

D.1 The case of voluntary unemployment

Equations (11) and (12) lead to the following utility specification:

$$\alpha \log(c_i(\ell) + \kappa) + 1[\ell = 24]\beta + 1[\ell = 38]\gamma + 1[\ell = 51]\delta + 1[\ell \neq 0]t(z_i) + \epsilon_i(\ell).$$

The prespecified parameter κ is set equal to 5000. The taste-for-working function t is specified as

$$\begin{aligned} t(z_i) \quad = \quad & \kappa_{10}1[\text{gender} = \text{male}] + \kappa_{11}1[\text{educ} = \text{secondary}] + \kappa_{12}1[\text{educ} = \text{tertiary}] \\ & +\kappa_{13}1[\text{gender} = \text{male}] \times 1[\text{educ} = \text{secondary}] + \kappa_{14}1[\text{gender} = \text{male}] \times 1[\text{educ} = \text{tertiary}] \\ & +\kappa_2 \frac{\text{age}}{100} + \kappa_3 \left(\frac{\text{age}}{100}\right)^2 \\ & +\kappa_{40}1[\text{area} = \text{middle}] + \kappa_{41}1[\text{area} = \text{rural}] \\ & +\kappa_{50}1[\text{nationality} = \text{EU}] + \kappa_{51}1[\text{nationality} = \text{not EU}]. \end{aligned}$$

As in Van Soest (1995), we use a simulated maximum likelihood procedure to account for the

Table 12: Estimation results for the utility specification (voluntary unemployment)

Parameter	Estimate	Std. Error	Pr(>|z|)
α	13.6492	3.1620	0.0000
β	-3.0194	0.3215	0.0000
γ	-2.6509	0.1827	0.0000
δ	-5.7496	0.6862	0.0000
κ_{10}	-0.9917	0.4716	0.0355
κ_{11}	0.1847	0.5829	0.7513
κ_{12}	0.8075	1.1006	0.4632
κ_{13}	0.2643	0.7702	0.7315
κ_{14}	0.4001	1.4910	0.7884
κ_{2}	23.9396	0.6811	0.0000
κ_{3}	-36.0606	1.1982	0.0000
κ_{40}	0.7697	0.6227	0.2164
κ_{41}	0.5692	0.9502	0.5492
κ_{50}	-0.1663	1.0318	0.8719
κ_{51}	0.2612	1.6762	0.8762

uncertainty in the prediction of the wage rates of the unemployed.[22] The estimation results are presented in Table 12.

The coefficients α, β, γ, and δ are significant and have the expected sign. In particular, the negative coefficients for β, γ, and δ indicate a disutility for working (for the reference type). Among the other covariates only gender (κ_{13}) and age (κ_2,κ_3) are significant. Males (κ_{10}) have a lower taste for working. Getting older (κ_2,κ_3) initially increases the taste for working (up to the age of 33 years), but decreases it afterwards.

D.2 The case of involuntary unemployment

Let $O \subseteq L$ denote an opportunity set and let $\mathscr{O}$ collect all subsets of L that contain the unemployment alternative 0.[23] The probability that an individual chooses an amount of labor $\ell \in L$ is now defined as

$$P(\ell|z_i) = \sum_{O\in\mathscr{O}|\ell\in O} P(O|z_i) \times \frac{\exp(u(c_i(\ell), \ell; z_i))}{\sum_{\ell'\in O}\exp(u(c_i(\ell'), \ell'; z_i))}, \tag{18}$$

where the first factor to the right-hand side of equation (18) is the probability that individual i faces opportunity set O and the second factor is the probability that individual i chooses ℓ from

[22]For the unemployed, we use the OLS regression estimates of Table 11 and add 50 i.i.d. errors terms from a normal distribution $N(0,\sigma_\epsilon^2)$ to simulate their log-likelihood contribution. The variance σ_ϵ^2 is approximated by the variance of the error terms of the OLS regression.

[23]Given four elements in $L = \{0, 24, 38, 51\}$ and given the assumption that not working is available in each opportunity set, we are left with eight possible subsets in $\mathscr{O}$.

this opportunity set. We further impose that the probability that an opportunity ℓ is available in someone's opportunity set, is independent of other opportunities.[24] The probability that an individual faces opportunity set O is therefore defined as

$$P(O|z_i) = \Pi_{\ell \in O} p(\ell|z_i) \Pi_{\ell \notin O}(1 - p(\ell|z_i)),$$

with $p(\ell|z_i)$ the probability that ℓ is in the opportunity set of individual i. We assume $p(0|z_i) = 1$ for all individuals. In order to estimate the other probabilities, we use a logit specification of the form:

$$p(\ell|z_i) = \frac{\exp(f(\ell|z_i))}{1 + \exp(f(\ell|z_i))},$$

with

$$\begin{aligned} f(\ell|z_i) =& \rho_0^\ell + \rho_1^\ell 1[educ = \text{secondary}] + \rho_2^\ell 1[educ = \text{tertiary}] + \rho_3^\ell 1[area = \text{middle-density}] \\ &+ \rho_4^\ell 1[area = \text{rural}] + \rho_5^\ell 1[region = \text{Flanders}] + \rho_6^\ell 1[region = \text{Wallonia}] \\ &+ \rho_7^\ell 1[nationality = \text{EU}] + \rho_8^\ell 1[nationality = \text{Non-EU}] + \rho_9^\ell 1[gender = \text{male}]. \end{aligned}$$

We retain a similar preference specification as described in appendix D.1, with the difference that the deterministic part is now only a function of age and gender, i.e.,

$$t(z_i) = \kappa_{10} 1[gender = \text{male}] + \kappa_2 \frac{age}{100} + \kappa_3 (\frac{age}{100})^2.$$

The results of the estimation are displayed in Table 13.

The coefficients α, β, γ, and δ are again significant, but now the coefficients β, γ, and δ are positive, indicating that the reference type likes working, ceteris paribus. Gender (κ_{13}) and age (κ_2,κ_3) are again significant and their sign is the same as in the case without rationing (Table 12). For the estimation of opportunities, better education (ρ_1, ρ_2) improves work opportunities (for working full-time or more), living in an urban area (the reference group for ρ_3, ρ_4) worsens work opportunities, living in Flanders (ρ_5) improves work opportunities (except for working more than full-time) relative to living in Brussels, living in Wallonia (ρ_6) worsens work opportunities (except for working half-time) relative to living in Brussels, nationality (ρ_7,ρ_8) does not seem to play a significant role, and being male (ρ_9) worsens opportunities for working half-time, but improves opportunities for working full-time and more.

[24] We also estimated a version where these probabilities were not independent, but information criteria such as the AIC criterion favor the current specification.

Table 13: Estimation results for the utility and opportunity specification

Parameter	Estimate	Std. Error	Pr(>|z|)
α	1.4894	0.1286	0.0000
β	2.0619	0.1054	0.0000
γ	0.4682	0.1787	0.0088
δ	6.8668	0.9995	0.0000
κ_{10}	-1.3599	0.2251	0.0000
κ_2	23.4902	0.3165	0.0000
κ_3	-42.3195	0.5441	0.0000
ρ_0^{24}	0.8625	0.1764	0.0000
ρ_1^{24}	-0.5701	0.2714	0.0356
ρ_2^{24}	0.5554	0.3411	0.1034
ρ_3^{24}	0.0117	0.3237	0.9711
ρ_4^{24}	0.8407	0.5336	0.1151
ρ_5^{24}	0.7297	0.3556	0.0402
ρ_6^{24}	-0.0213	0.2959	0.9426
ρ_7^{24}	-0.8578	0.5473	0.1170
ρ_8^{24}	0.5671	0.6988	0.4171
ρ_9^{24}	-1.7220	0.2245	0.0000
ρ_0^{38}	0.0563	0.1508	0.7091
ρ_1^{38}	0.4703	0.2196	0.0322
ρ_2^{38}	2.1472	0.4492	0.0000
ρ_3^{38}	0.7135	0.2990	0.0170
ρ_4^{38}	0.6452	0.4264	0.1302
ρ_5^{38}	1.3774	0.5135	0.0073
ρ_6^{38}	-0.6296	0.2108	0.0028
ρ_7^{38}	-0.2480	0.4001	0.5354
ρ_8^{38}	0.1679	0.6092	0.7828
ρ_9^{38}	0.6895	0.2132	0.0012

Parameter	Estimate	Std. Error	Pr(>|z|)
ρ_0^{51}	-3.4837	0.2303	0.0000
ρ_1^{51}	0.7679	0.3724	0.0392
ρ_2^{51}	2.7129	0.3413	0.0000
ρ_3^{51}	0.7506	0.3657	0.0401
ρ_4^{51}	2.5794	0.7876	0.0011
ρ_5^{51}	-0.1865	0.3420	0.5855
ρ_6^{51}	-2.8667	0.5696	0.0000
ρ_7^{51}	-0.1934	0.6968	0.7814
ρ_8^{51}	0.1355	1.0673	0.8989
ρ_9^{51}	1.4420	0.2969	0.0000