

# Conversations

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# Conversations

# Abstract

We develop a theory of conversations. Two agents with different interests take turns choosing the topic of the conversation. Talking about a single topic allows them to delve deeper, making the conversation more informative (or enjoyable). To capture this dynamic, we assume that the marginal utility from conversing increases when the agents stay on topic. The equilibrium conversation is extreme: it either maximizes or minimizes welfare. Long conversations are deep and thus efficient. Short ones are often superficial. The topic of a deep conversation depends in subtle ways on who speaks when. Applications range from echo chambers to team production.

JEL-Codes: D830.

Keywords: communication, information acquisition, team production.

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## 1 Introduction

Communication plays a fundamental role in how people generate new insights. Talking to others allows people to structure their thoughts, focus their attention on a particular matter, and, through bilateral exchange, reject or affirm ideas. As Pinker (1994, p.1) puts it, "[s]imply by making noises with our mouths, we can reliably cause precise new combinations of ideas to arise in each other's minds." In conversations people therefore do not only exchange known facts, but also produce new information not known by either person beforehand. And, even if no new information is produced, people enjoy conversations that, for example, bring back pleasant memories in a vivid way. Crucially, an informative, or enjoyable, conversation requires people to delve deeply into a topic. Because different people are interested in different topics, however, conversations can easily be derailed, hindering information production. This raises two questions: When do people manage to have a focused, in-depth conversation? And, if they do, what determines the topic they discuss?

We address these questions by formalizing the idea that conversations can directly generate utility, by creating new information or offering plain enjoyment. So far, economists have modeled communication mainly as a means of transmitting existing or previously acquired information.<sup>1</sup> By instead modeling communication as a process that in and of itself generates utility, we are able to explain why some conversations explore a topic in depth while others remain at the surface.

We develop a stylized model of two agents having a conversation for a finite number of periods. The agents take turns choosing one of two topics. They cannot commit in advance to a given course of the conversation. We model talking as generating and sharing "pieces" of information or enjoyment about the topic of choice.<sup>2</sup> An agent's utility increases linearly in the total number of pieces generated on a topic over the course of the conversation. While the two agents differ in their interests, they are symmetric in that they each attach the same weight to their preferred and less preferred topic.

Our key assumption is that the conversation becomes "more interesting" as the agents continue

<sup>&</sup>lt;sup>1</sup> This includes the literature on communication with hard information (e.g., Grossman and Hart, 1980, Milgrom, 1981) as well as the large literature on cheap talk following Crawford and Sobel (1982). Similarly, classical information theory (e.g., Shannon, 1948) reduces communication to the encoding and decoding of known facts.

 $<sup>^{2}</sup>$  When thinking of information production in conversations, we assume that information is transmitted truthfully.

to talk about the same topic. Focusing for a prolonged period of time on a single topic arguably allows for deeper insights and makes conversations more enjoyable in general. In contrast, frequent switches of topic break the flow of a conversation, making it hard to keep the train of thought or to delve deeply into a pleasant memory. We capture this idea via a simple complementarity in the production of information (or enjoyment). Suppose that the agents have talked about the same topic for k consecutive periods. If the agents stay on topic, and thereby maintain the flow of the conversation, in the next period they produce k + 1 pieces of information on this topic. Once an agent derails the conversation by switching topics, however, they fall back to producing a single piece of information. Thus, a *deep* conversation, in which the agents talk only about one topic, maximizes the total number of pieces produced. Such a deep conversation is efficient in that it also maximizes utilitarian welfare. At the other extreme, a *superficial* conversation, in which the topic of discussion changes every period, minimizes the total number of pieces and thus utilitarian welfare.<sup>3</sup>

We microfound the returns to staying on topic by formalizing Pinker's (1994) observation that communication "reliably cause[s] precise new combinations of ideas." Think, for example, of two researchers who are looking for a "breakthrough." We model breakthroughs as matching the right pieces of a puzzle. This captures the notion that knowledge creation emanates from the right combination of ideas (Weitzman, 1998).<sup>4</sup> Because neither agent has access to all pieces of the puzzle, a conversation is necessary for achieving a breakthrough. In each period, both agents contribute a new piece of the puzzle. These new pieces might not only match each other but may also match pieces contributed earlier in the conversation. By staying on topic, the agents, therefore, generate *increasingly* more potential matches over time. And, as a consequence, a complementarity in information production naturally arises in deep conversations.

We solve the game backward. Each period, an agent chooses a topic based on the history of the conversation and her anticipation of how it will evolve in the future. The preceding conversation

 $<sup>^{3}</sup>$  We do not necessarily think of a superficial conversation as switching back-and-forth between completely unrelated topics (like, say, sports and politics). Instead, the topics can be interpreted as different aspects of the same overarching theme (e.g., different parts of a research project). A superficial conversation may thus simply be shallow in that it does not explore any single aspect in depth.

<sup>&</sup>lt;sup>4</sup> Our microfoundation goes beyond Weitzman (1998) by explicitly modeling the role of communication in combining ideas in order to generate knowledge.

determines the current complementarity in information production and, thus, the additional utility from staying on topic for one more period. Whether an agent stays on topic depends not only on the current complementarity, however; it also depends on the number of remaining periods. Both make the game non-stationary, and backward induction tricky. Nevertheless, because information production resets after any switch of topics, we can decompose the conversation into a series of deep subconversations. This is what makes the model tractable after all.

We fully characterize the equilibrium conversation. The equilibrium conversation is deep (and thus efficient) if and only if the agents' interests are sufficiently aligned. Once their interests diverge enough, however, the conversation immediately becomes superficial, switching topics every period to the one preferred by the current speaker. The equilibrium conversation is, therefore, extreme in the sense that it either maximizes or minimizes utilitarian welfare. Strikingly, whenever the conversation is superficial, *both* agents would be *strictly* better off if they could commit to talking first about one topic and then about the other topic. At the same time, whenever at least one agent prefers a deep conversation. Commitment, therefore, cannot induce deep conversations that would not have taken place otherwise, but it can help the agents in preventing superficial ones. More time, on the other hand, does help in general, as long conversations tend to be efficient. Intuitively, because the returns to staying on topic increase with the length of the conversation, focusing on a single topic — no matter which one — is more attractive in longer conversations.

Why is the equilibrium conversation either deep or superficial? First, the additional utility from staying on topic for one more period increases over time. Second, switching topics early on in the conversation leaves more time for talking about one's preferred topic, which is the reason for switching topics in the first place. Combining both suggest that switching topics is particularly tempting at the beginning of the conversation. Using a revealed-preference argument, we indeed show that if the agents switch topics once, they do so all the time.

The topic of a deep conversation depends in subtle ways on how aligned the agents' interests are. If their interests are closely aligned, the person opening the conversation can steer it toward her preferred topic. Intuitively, if both agents care enough about both topics, they are mostly concerned with maximizing the total number of pieces. Hence, by starting on her preferred topic, the first mover can induce a deep conversation on exactly that topic. Crucially, this "first-mover advantage" is more pronounced in shorter conversations: after a switch of topics, shorter conversations offer less room to delve deeply into the other topic, so staying on topic is more valuable in this case.

This observation has interesting implications for how the topic of a deep conversation changes if interests are less aligned. Suppose for a moment that the first mover starts on her preferred topic. If the second mover immediately switches topics, it is as if a *new, but shorter conversation starts*. In this subconversation, the second mover talks first. Because the first-mover advantage described above is more pronounced in shorter conversations, even for less aligned interests, the remaining conversation following such a switch of topics would be deep *and* about the topic preferred by the second mover. Anticipating that she will talk about her less preferred topic in the third (and any later) period, the first mover actually starts on her less preferred topic. If interests are even more misaligned, it is again the first mover who dictates the topic of the conversation. She can now credibly threaten that, from the third period onward, the conversation will be deep about her preferred topic. In equilibrium, the conversation then indeed "unravels" to be deep about this topic. We can apply the same argument repeatedly to less and less aligned interests until it is the agent speaking in the *second-to-last* period who steers the conversation toward her preferred topic.

The *last* period is different in that derailing the conversation has no negative impact on future information production. This implies that, for sufficiently misaligned interests, incentives to switch topics are strongest in the last period. The agent having the last word can thus credibly threaten to switch topics in this last period. We show that, as a consequence, the conversation would turn superficial if, at any point in the conversation, the other agent switched to her preferred topic. Hence, as long as the other agent does not prefer such a superficial conversation, the agent having the last word dictates the topic of the conversation. This completes the equilibrium.

While the first (last) word crucially shapes short (long) conversations, speaking first or last is not all that matters. Rather, the topic of a deep conversation depends in more subtle ways on *who speaks when*. Prolonging the conversation by two periods can result in a change of topic, although it changes neither the first nor the last speaker in the overall conversation. We show that *speaking first*  *in the "right" subconversation* is key for steering the conversation toward one's preferred topic. Our results thus highlight the importance of decision rights over the communication process, thereby adding a novel perspective to the literature on delegation of decision rights versus communication (e.g., Dessein, 2002, Alonso et al., 2008).

While our model is stylized, most assumptions are made mainly for tractability. Most importantly, our qualitative results do neither rely on information production increasing linearly over time nor on utility being linear in the total number of pieces. All we need for our qualitative results to hold is that the additional utility from staying on topic increases over time. This assumption seems particularly plausible when discussing complex matters (in which case learning takes time) or when talking for a limited period of time (e.g., when meeting a co-author for a couple of hours).

Not every conversation has a fixed duration, and how a conversation ends may affect its nature. We show this in two extensions of our baseline model. First, we consider a random ending. Every period the conversation stops with a fixed probability. Focusing on Markov equilibria, we show that the conversation is again either deep, and, thus, efficient, or superficial. Other than in our baseline model, however, a deep conversation is always about the topic preferred by the first mover. Second, we consider an endogenous ending. The agents can talk for a fixed number of periods, but they can also leave the conversation at any time. Every period that the agents talk, they incur some fixed costs. For large enough costs, the conversation is either deep if interests are sufficiently aligned, or otherwise it does not start. This means that the agents would rather not talk to each other than have a superficial conversation. Our theory thus provides a novel rationale for the emergence of "echo chambers" (e.g., Levy and Razin, 2019), which is grounded in a desire for deep conversations

Our model can be applied to contexts beyond conversations. It captures any productive process requiring close collaboration among agents who lack commitment: most prominently, team production in the "knowledge economy" (see Gibbons and Roberts, 2013, for an overview).

**Related Literature** Our paper relates to the literatures on communication, information acquisition and attention, and team production. No previous paper, however, has addressed our main question: allowing conversations to generate information or enjoyment, what is their nature and topic? Communication. Information theory traditionally thinks of communication as the transmission of existing information (e.g., Shannon, 1948). Economic models of communication — with hard information (e.g., Grossman and Hart, 1980, Milgrom, 1981) or soft information (following Crawford and Sobel, 1982)<sup>5</sup> — take a similar view, but add the incentives to transmit information to the picture. Dewatripont and Tirole (2005) model the transmission of hard information as a process that requires costly effort by both parties. And while some papers allow for costly information acquisition (Di Pei, 2015, Argenziano et al., 2016, Deimen and Szalay, 2019, Eső and Szalay, 2020), information acquisition is never part of the communication process itself. By contrast, we focus on the production of information during conversations through the endogenous choice of topics.

Closer to our model, Glazer and Rubinstein (2001, 2006), Stein (2008), and Antic et al. (2022) present explicit models of conversations.<sup>6</sup> Glazer and Rubinstein (2001, 2006) ask how an uninformed outside observer should structure a debate to extract as much information as possible from two informed agents with opposing interests.<sup>7</sup> Different from our model, there is no new information or enjoyment produced during the conversation. Stein (2008) analyzes conversations among competitors who want to extract information from the other party without revealing too much information themselves. Relatedly, Antic et al. (2022) study agents who share a common objective but must disguise their communication so that an outside observer does not learn.

Information Acquisition and Attention. Our paper is related to work on optimal information acquisition and (in)attention (e.g., Wald, 1945, Sims, 2003, Morris and Strack, 2019). One can think of talking about a certain topic rather than another one as devoting attention to this specific topic. In our baseline model, there is no direct cost of acquiring information or paying attention, however. Talking about one topic becomes costly only in one respect: through not talking about the other topic. This is similar to rational inattention models with a fixed "attention budget" (e.g., Sims, 2003). But we differ from this literature in that we study information production in a strategic setting.

 $<sup>^{5}</sup>$  Although conceptually different, our results share similarities with the "cheap-talk" literature. For example, Aumann and Hart (2003) show that longer cheap talk can be more efficient. And Blume et al. (2007) argue that introducing noise (into the sender's signal) can enhance efficiency, just like a random ending does in our model.

<sup>&</sup>lt;sup>6</sup> Weizsäcker (2022) discusses the role of inaccurate expectations and misunderstandings in conversations.

 $<sup>^{7}</sup>$  The idea that their order determines how convincing arguments and counter-arguments are (and, thus, the beliefs they induce) is related to the literature on Bayesian persuasion following Kamenica and Gentzkow (2011).

Wojtowicz et al. (2021) model the "feeling of flow" as a simplification device that helps people allocate their scarce attention to its most valuable use. We, in contrast, assume that maintaining the flow of a conversation is what makes the conversation valuable in the first place. Similarly, Ely et al. (2015) study how to reveal (non-instrumental) information over time to keep a listener engaged. For most of the paper, we abstract from the need to keep others engaged, and instead focus on how the strategic choice of topics affects information production.

Team Production. A large literature studies (optimal) team production in the "knowledge economy" (e.g., Garicano, 2000, Garicano and Rossi-Hansberg, 2004, 2006, Cremer et al., 2007, Gibbons and Roberts, 2013, Blume et al., 2021). As we argue in Section 6, our model can be re-interpreted as a model of team production (with conflict of interest). More straightforwardly, we can also think of our model as capturing team production of information. Closest in this regard is the paper by Liang and Mu (2020), who show that social learning can be suboptimal when agents ignore complementarities in their signal production. While the literature treats communication and information acquisition as separate actions, we focus on information production *through* communication.

#### 2 Framework

#### 2.1 Setup

Two agents, i = 1 and i = 2, have a conversation for  $T \ge 2$  periods. In every period, the agents can talk about one of two topics, A or B. The topics may range from recent political events over (different aspects of) a research or work project to joint memories. The agents take turns in choosing the topic of the conversation. Agent 1 chooses the topic in every odd period  $t \in \{1, 3, ...\}$ while Agent 2 picks the topic in every even period  $t \in \{2, 4, ...\}$ .

We model the conversation as generating and sharing "pieces" of information (or enjoyment) about the topic of choice. This captures the idea that talking is part of the information production process. For example, when talking about a research project, two colleagues might realize a new aspect they had not been aware of before. In this sense, information production naturally arises during conversations. At the same time, "pieces" of enjoyment may refer to anecdotes or memories

— such as a joint vacation — that are pleasant to talk about. A conversation may thus be interesting not only because of the new information that is produced but also because it is enjoyable in itself.

Our key assumption is that the conversation becomes "more interesting" as the agents keep talking about the same topic. Formally, having talked about the same topic for  $k \in \{0, ..., T-1\}$ consecutive periods, the agents generate k + 1 additional pieces (of information or enjoyment) if they stay on topic for one more period. Whenever an agent derails the conversation by switching topics, the production of pieces resets, and only a single piece is produced. This captures the idea that frequent switches of topic break the flow of a conversation. Intuitively, switching topics back and forth makes it hard to keep the train of thought (as when discussing research) or to delve deeply into a memory (as when talking about one's last vacation). In contrast, by focusing on one aspect of a problem, the agents may realize new facets. And delving deeply into a memory may cue other memories (Kahana, 2012, Bordalo et al., 2020), making the conversation more enjoyable.

We define utilities over the total number of pieces generated and shared during the conversation. To simplify the exposition, we refer to pieces of *information* for the rest of the paper. Agent 1 cares more about topic A, while Agent 2 cares more about topic B. Otherwise, the two agents are symmetric. More specifically, for some  $\alpha \in (1/2, 1)$ , the agents' utility functions are given by

$$U_1 = \alpha \cdot (\text{total } \# \text{ of pieces on A}) + (1 - \alpha) \cdot (\text{total } \# \text{ of pieces on B}),$$

and

$$U_2 = (1 - \alpha) \cdot (\text{total } \# \text{ of pieces on A}) + \alpha \cdot (\text{total } \# \text{ of pieces on B}).$$

The additional utility from receiving *one* more piece of information on a given topic is constant and, therefore, independent of the course of the conversation. Combined with our assumption on information production, this implies that the additional utility from staying on topic (for one more period) increases linearly over time. This captures the benefits of a focused conversation in the simplest possible form, constituting a natural starting point (see Section 2.2 for a microfoundation). As we discuss in Section 4, however, our qualitative results only rely on the assumption that the additional utility from staying on topic (for one more period) increases over time.

An efficient conversation – that maximizes utilitarian welfare – maximizes the total number of pieces generated and shared. An efficient conversation is therefore deep, in that the agents talk

only about a single topic. At the other extreme, the conversation is *superficial* if and only if the agents switch topics every period. Such a superficial conversation minimizes utilitarian welfare.

We solve for the set of (pure-strategy) subgame-perfect Nash equilibria (SPNE), and we resolve multiplicity of equilibria in favor of efficiency. Because the game is finite, an SPNE in pure strategies exists (e.g., Osborne, 2009, Proposition 173.1). And, as we show in our main result, for almost any parameter value  $\alpha \in (1/2, 1)$ , this pure strategy SPNE is also unique.

#### 2.2 Microfoundation: Putting together a Puzzle

We now provide a microfoundation for our reduced-form model. This microfoundation takes seriously the proposition that conversations "cause precise new combinations of ideas" (Pinker, 1994). We can think, for example, of two researchers who are talking towards a "breakthrough" (e.g., on how to prove a theorem, how to fix a software, how to save on costs, or how to cure a disease).

We model breakthroughs as the result of combining existing ideas in the right way (Weitzman, 1998); or, in other words, putting together the pieces of a puzzle. Because neither of the agents has access to all pieces of the puzzle, any breakthrough requires a conversation. Every period *both* agents contribute a new piece of information on the topic of choice. As before, the topic is chosen by *one* of the agents every period, and the agents take turns in choosing the topic. If two pieces "match," the agents have a breakthrough. A breakthrough on an agent's preferred topic generates utility  $\overline{\beta}$ ; a breakthrough on her less preferred topic gives utility  $\underline{\beta} \in (0, \overline{\beta})$ . As in Stein (2008), the agents learn about a breakthrough only after the conversation has ended.

The value of staying on topic derives from the additional matches that become possible as a result. Suppose, for instance, that the agents start talking about topic A. In t = 1, Agent 1 then contributes piece  $a_1$ , and Agent 2 contributes piece  $\tilde{a}_1$ . We assume that these two pieces match with probability  $p_0 \in (0, 1)$ . If the agents keep talking about A in t = 2, they add pieces  $a_2$  and  $\tilde{a}_2$  to the picture. Pieces  $a_2$  and  $\tilde{a}_2$  again match with probability  $p_0$ . Crucially,  $a_2$  might also match with  $\tilde{a}_1$ , and  $\tilde{a}_2$  might also match with  $a_1$ . All matches are independent.<sup>8</sup> Because of the temporal distance between  $a_2$  and  $\tilde{a}_1$  (as well as  $\tilde{a}_2$  and  $a_1$ ), however, these pieces match only with

<sup>&</sup>lt;sup>8</sup> This also means that a single conversation can result in multiple breakthroughs on the same or different topics.

probability  $p_1 \leq p_0$ . Similarly, when still talking about A in t = 3, the agents contribute pieces  $a_3$  and  $\tilde{a}_3$ , adding another five potential matches (of varying likelihood).<sup>9</sup> As illustrated in Figure 1, staying on topic, therefore, has the benefit of generating increasingly more potential matches over time.

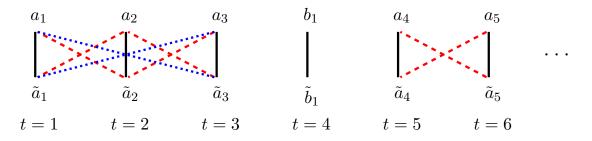


Figure 1: The figure illustrates the potential matches in a conversation. The pieces connected by the solid, black lines match with probability  $p_0$ . The pieces connected by the dashed, red lines match with probability  $p_1$ . And the pieces connected by the dotted, blue lines match with probability  $p_2$ .

Any switch of topics creates (topical) distance, however. This, in turn, prevents agents from drawing connections to the pieces of the puzzle that were contributed earlier in the conversation. In particular, suppose the agents switch to topic B in t = 4 before coming back to topic A in the next period. In t = 5, the agents contribute two new pieces,  $a_4$  and  $\tilde{a}_4$ , which again match with probability  $p_0$ . But, because their conversation on A was interrupted by the switch to B, the agents cannot link these new pieces to the ones drawn in the first three periods. As a consequence, talking about A in t = 5 here generates only one potential match — that is, any switch of topics resets information production.

Setting (i)  $p_0 = p$  and  $p_l = p/2$  for  $l \in \{1, ..., T-1\}$  with  $p \in (0, 1)$  and (ii)  $\overline{\beta}p = \alpha$  and  $\underline{\beta}p = 1 - \alpha$  yields precisely our reduced-form model. By (i), when having talked about, say, A for k consecutive periods, staying on topic in the next period yields an additional k + 1 potential matches. By (ii), the additional (expected) utility from staying on topic for one more period then is  $\alpha(k+1)$  for Agent 1 and  $(1-\alpha)(k+1)$  for Agent 2. As we argue in Section 4, our qualitative

<sup>&</sup>lt;sup>9</sup> The pieces  $a_3$  and  $\tilde{a}_3$  match with probability  $p_0$ ; the pieces  $a_3$  and  $\tilde{a}_2$  as well as  $\tilde{a}_3$  and  $a_2$  are one period apart and, thus, match with probability  $p_1$ ; the pieces  $a_3$  and  $\tilde{a}_1$  as well as  $\tilde{a}_3$  and  $a_1$  are two periods apart and, thus, match with probability  $p_2 \leq p_1$ .

results generalize to any decreasing sequence of match probabilities  $p_0 \ge p_1 \ge p_2 \ge \ldots \ge p_{T-1}$ .<sup>10</sup>

# 3 The Equilibrium Conversation

#### 3.1 Statement of the Theorem

Our main result completely characterizes the equilibrium conversation.

Theorem (The Equilibrium Conversation).

- I. The equilibrium conversation is either deep or superficial.
- II. The equilibrium conversation is deep if and only if  $\alpha \in (\frac{1}{2}, \bar{\alpha}(T)]$ , with the cutoff satisfying

$$\bar{\alpha}(T) = \begin{cases} \frac{T+1}{T+2} & \text{if } T \text{ is odd,} \\ \\ \frac{T}{T+1} & \text{if } T \text{ is even.} \end{cases}$$

III. Let  $T \geq 3$  be odd. The equilibrium conversation is deep about A if

$$\alpha \in \left(\frac{1}{2}, \frac{T+2}{2T+2}\right) \cup \left\{\bigcup_{k \in \{2, 4, \dots, T-3\}} \left(\frac{T+2-(k-1)}{2T+2-2(k-1)}, \frac{T+2-k}{2T+2-2k}\right)\right\} \cup \left(\frac{2}{3}, \frac{T+1}{T+2}\right].$$

The equilibrium conversation is deep about B if

$$\alpha \in \bigcup_{k \in \{1,3,\dots,T-2\}} \left( \frac{T+2-(k-1)}{2T+2-2(k-1)}, \frac{T+2-k}{2T+2-2k} \right).$$

IV. Let  $T \ge 4$  be even.<sup>11</sup> The equilibrium conversation is deep about A if

$$\alpha \in \left(\frac{1}{2}, \frac{T+2}{2T+2}\right) \cup \left\{\bigcup_{k \in \{2,4,\dots,T-2\}} \left(\frac{T+2-(k-1)}{2T+2-2(k-1)}, \frac{T+2-k}{2T+2-2k}\right)\right\}.$$

The equilibrium conversation is deep about B if

$$\alpha \in \left\{ \bigcup_{k \in \{1,3,\dots,T-3\}} \left( \frac{T+2-(k-1)}{2T+2-2(k-1)}, \frac{T+2-k}{2T+2-2k} \right) \right\} \cup \left( \frac{2}{3}, \frac{T}{T+1} \right]$$

<sup>&</sup>lt;sup>10</sup> This also includes the case of restricting the agents to have at most one breakthrough per conversation.

<sup>&</sup>lt;sup>11</sup> If T = 2, for any  $\alpha \in (1/2, 2/3]$ , the conversation is deep and about A, while for any  $\alpha \in (2/3, 1)$  it is superficial.

Part I of the result says that the conversation is either deep — that is, only about one topic — or superficial — that is, the topic switches every period. A deep conversation maximally exploits the complementarity in producing information, thereby maximizing welfare. By contrast, a superficial conversation does not generate any complementarity and, thus, minimizes welfare. The equilibrium conversation is therefore extreme in that it either maximizes or minimizes welfare.

Whether the equilibrium conversation is deep, and thus efficient, depends on how misaligned the agents' interests are; this is captured by the parameter  $\alpha \in (1/2, 1)$ . By Part II, whenever  $\alpha$ exceeds the threshold  $\bar{\alpha}(T)$ , the conversation becomes superficial. This threshold (weakly) increases in the length of the conversation T. Longer conversations, thus, tend to be more efficient.

Parts III and IV characterize the topic of a deep conversation, as a function of  $\alpha$  and T. For any T, if the misalignment of interests is small ( $\alpha$  is close to 1/2), the conversation is deep about A. This implies a "first-mover advantage" for closely aligned interests, which is more pronounced in shorter conversations and vanishes as T grows arbitrarily large. As interests become less and less aligned, the topic of a deep conversation switches back and forth. Once  $\alpha$  exceeds 2/3, however, the topic of a deep conversation is determined by the agent having the last word. This implies a "last-mover advantage" for relatively misaligned interests. The longer the conversation is, the more pronounced this last-mover advantage becomes. As  $T \to \infty$ , it applies for any  $\alpha \in (2/3, 1)$ .

#### 3.2 The Conversation as the Sum of Deep Subconversations

Because switching topics resets information production, after any switch, it is as if a new, but shorter conversation starts. Consider the following conversation as an example:

$$\underbrace{A}_{\text{in }t=1}, \underbrace{A}_{\text{in }t=2}, \underbrace{B}_{\text{in }t=3}, \underbrace{B}_{\text{in }t=4}, \ldots, \underbrace{B}_{\text{in }t=T}.$$

Because, after switching topics, the agents initially produce one piece of information on topic B, the subconversation starting in t = 3 yields precisely the same utility to both agents as a deep conversation about B of length T-2. This has an important implication for how to solve the model: we can think of the conversation as the "sum of subconversations," each of which is deep about one of the topics.<sup>12</sup> The conversation above, for example, can be divided into two deep subconversations

<sup>&</sup>lt;sup>12</sup> A superficial conversation of length T can be thought of as the sum of T deep subconversations of length one.

(marked in blue and red), and an agent's utility adds up to that of a deep conversation about A of length 2 and a deep conversation about B of length T-2. This substantially simplifies the analysis.

#### 3.3 Deep or Superficial

By Part I of our main result, the equilibrium conversation is either deep or superficial. Notice that any other conversation ends either (i) on a superficial subconversation followed by a deep one, (ii) on consecutive deep subconversations on the two different topics, or (iii) on a deep subconversation followed by a superficial one. Because any conversation can be thought of as the sum of deep subconversations (see Section 3.2), to prove Part I, it suffices to rule out that (i), (ii), or (iii) can be part of an equilibrium conversation.

At an intuitive level, this follows from the fact that switching topics is particularly tempting early on in the conversation. First, switching topics early on is less costly in that the existing complementarity is low. Second, an early switch of topics allows the agents to build up a (weakly) larger complementarity on the other topic. The agents, thus, either never switch and have a deep conversation or, if they switch topics once, they do so every period, leading to a superficial conversation.

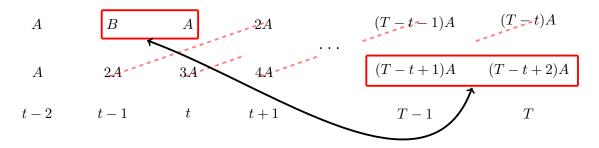


Figure 2: Information production when switching to A in t + 1 (top) or t - 1 (bottom) in Case (i). The fact that Agent 2 prefers talking about A in t + 1 reveals that she prefers (T - t - 1) + (T - t)pieces on A over one piece on A and one piece on B. This, in turn, implies that the red box in the bottom row yields higher utility than the red box in the top row.

The proof, which we provide in Appendix A, uses a revealed-preference argument to rule out all three cases. Cases (i) and (ii) have in common that, at some point t, the conversation turns deep

about, say, A. Suppose that Agent 2, who is more interested in B, is the last one to talk about B. In Case (i), Agent 2 then already has an incentive to switch to A in t-1. This makes talking about A even more attractive later on. As illustrated in Figure 2, this thus allows to further exploit the complementarity in information production. Hence, if Agent 2 prefers talking about A in t + 1, which reveals that  $\alpha$  cannot be too large, she has to prefer talking about A already in t - 1. In Case (ii), it is Agent 1 who would like to switch to A in t - 2. When doing so, the subconversation starting in t - 2 falls into Case (i) again; the subconversation is superficial before it becomes deep about A. By the same argument as in Case (i), then also Agent 2 wants to talk about A one period earlier. Thus, by switching earlier, Agent 1 can induce a longer deep subconversation on her preferred topic, which makes the deviation profitable. In Case (ii), the agents switch from a deep subconversation would be superficial. The incentive to deviate then derives from the fact that switching earlier — at a lower existing complementarity — is less costly. Hence, one of the agents would switch earlier.

#### 3.4 The First Word

If the agents' interests are sufficiently aligned (i.e., if  $\alpha$  is close to 1/2), the conversation is deep and about A. To see why, suppose that, coming into period  $t \in \{2, 4, 6, ...\}$ , the agents have only talked about topic A. Agent 2 then values a deep conversation about A at

$$(1-\alpha)\Big(\underbrace{t}_{\text{in }t} + \underbrace{t+1}_{\text{in }t+1} + \ldots + \underbrace{T}_{\text{in }T}\Big) = (1-\alpha)\frac{(T+t)(T-t+1)}{2}.$$
(1)

Agent 2's incentive to switch topics depends on how the ensuing conversation following such a "deviation" would look like. The incentive to switch topics is maximized if the remaining conversation would be deep and about B. In this case, Agent 2's utility from deviating would be

$$\alpha \Big(\underbrace{1}_{\text{in }t} + \underbrace{2}_{\text{in }t+1} + \ldots + \underbrace{T - t + 1}_{\text{in }T}\Big) = \alpha \ \frac{(T - t + 1)(T - t + 2)}{2}.$$
(2)

By comparing (1) and (2), we conclude that, for any

$$\alpha < \frac{T+t}{2T+2},\tag{3}$$

Agent 2 prefers a deep conversation about A over establishing a deep subconversation about her preferred topic B. Since (2) constitutes an upper bound on Agent 2's utility in case of switching topics, for any such  $\alpha$  she prefers a deep conversation about A over *any* other subconversation.

The right-hand side of (3) is clearly increasing in t, suggesting that switching topics is particularly tempting early on in the conversation. Intuitively, if Agent 2 was able to get the maximal utility from switching topics (as in Eq. (2)) by having a deep subconversation on her preferred topic, her incentive to deviate is stronger the more periods remain. Hence, if  $\alpha < (T+2)/(2T+2)$ , Agent 2 does not want to switch topics in t = 2 or at any later stage in the conversation. Clearly, Agent 1 has no incentive to deviate either.

# **Lemma 1** (The First Word). If $\alpha \in (\frac{1}{2}, \frac{T+2}{2T+2})$ , the equilibrium conversation is deep about A.

Lemma 1 shows that when the agents' interests are sufficiently aligned, it is Agent 1 who determines the topic of the conversation. Intuitively, for sufficiently aligned interests, the agents' objective is to exploit the complementarity in information production as much as possible. Hence, the second mover will stay on topic in any case, and the agent who has the first word can steer the conversation toward her preferred topic.

This first-mover advantage becomes less important in longer conversations. In fact, since (T+2)/(2T+2) approaches 1/2 as  $T \to \infty$ , it vanishes completely as the conversation becomes arbitrarily long. This accords well with basic intuition: in longer conversations it is less important for the second mover to stay on topic, since there is more room to delve deeply into the other topic.

#### 3.5 Having the First Word: A Matter of Perspective

For more misaligned interests, a deep conversation is not necessarily about A. In fact, if the conversation starts on A, Agent 2 might switch topics later on. As a result, Agent 1 might have an incentive not to start on A in the first place. Whether Agent 2 has an incentive to switch, depends on the subconversation that she believes would materialize afterward. Hence, we first have to understand what happens *after* such a switch of topics. Lemma 1 will be useful in this first step. In a second step, we can then ask whether one of the agents indeed wants to switch topics at some point in the conversation. And if they do, we can finally study how, in equilibrium, the

conversation "unravels," and becomes deep. Starting the conversation is not what matters in the end; instead what matters is talking first in the "right" subconversation.

First, suppose that the conversation starts on A, and that Agent 2 switches topics in t = 2. Recall that after any switch of topics it is as if a new but shorter conversation starts. We can thus treat the subconversation starting in t = 2 as a new conversation. Then, by Lemma 1, for any

$$\alpha \in \left(\frac{1}{2}, \frac{(T-1)+2}{2(T-1)+2}\right) = \left(\frac{1}{2}, \frac{T+1}{2T}\right),\tag{4}$$

this subconversation of length T-1 is deep about B. This follows from the fact that Agent 2 has the first word in this subconversation, which allows her to steer it toward her preferred topic.

Second, we can ask whether, for  $\alpha$  in (4), Agent 2 indeed has an incentive to switch topics in t = 2. Since the subconversation following such a switch would be deep about B, by Eq. (2), she does so for any  $\alpha$  larger than (T+2)/(2T+2). Combining this with Eq. (4), we conclude that, for any

$$\alpha \in \left(\frac{T+2}{2T+2}, \frac{T+1}{2T}\right),\tag{5}$$

a conversation that starts on A would immediately become deep about B in t = 2. Intuitively, because the first-mover advantage is more pronounced in shorter (sub)conversations, Agent 1 stays on topic in t = 3 even if interests are so misaligned that Agent 2 prefers to switch topics in t = 2.

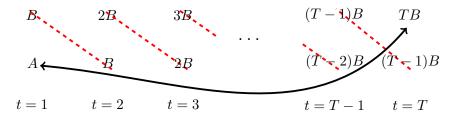


Figure 3: Information production in conversations starting on B (top) or A (bottom) for  $\alpha$  in (5).

Third, because Agent 2 would otherwise switch to B in t = 2, Agent 1 already begins to discuss B in t = 1, as follows by revealed preferences. As illustrated in Figure 3, from the perspective of t = 1, Agent 1 trades off T pieces of information on B against one piece of information on A. We know that, even when starting out by discussing A, Agent 1 talks about B in t = 3. If Agent 1 had instead switched to A in t = 3, by Lemma 1, the remaining conversation would have been deep

about A. As illustrated in Figure 4, this reveals that Agent 1 prefers T-1 pieces of information on B over one piece of information on A. But then she must also prefer T pieces of information on B over one piece of information on A. The equilibrium conversation, therefore, unravels and is deep about B. This is another manifestation of the fact that switching topics early in a conversation is particularly tempting.

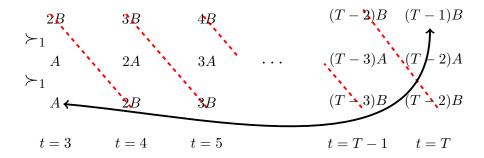


Figure 4: Information production in a conversation that starts on A (in t = 1), switches to B (in t = 2), and continues either on B (top) or A (middle) in t = 3 for any  $\alpha$  in (5). By Lemma 1, Agent 1 prefers to talk about B in t = 3 (as indicated by the first  $\succ_1$ ). Since Agent 1's preferred topic is A, she prefers the sequence in the middle row to that in the bottom row (as indicated by the second  $\succ_1$ ). This implies, by transitivity, that Agent 1 also prefers the top to the bottom row.

If interests are even more misaligned, Agent 2 can no longer induce a deep subconversation about B by switching topics in t = 2. Instead, it is Agent 1 who can, by Lemma 1, make sure that from the third period onward, the conversation will be deep about A. And because interests are sufficiently misaligned, she actually has an incentive to do so. By similar arguments as above (that we spell out in the proof), the conversation then unravels becoming deep about A. We can repeat the same arguments until the agent talking in t = T - 1 induces a deep (sub)conversation (of length 2) on her preferred topic, which then unravels in equilibrium. As illustrated in Figure 5, when gradually increasing  $\alpha$  from 1/2 to 2/3, the topic of a deep conversation switches back and forth (exactly T - 2 times) between A and B.<sup>13</sup>

<sup>&</sup>lt;sup>13</sup> If  $\alpha > 2/3$ , however, a (sub)conversation of length 2 (starting in t = T - 1) is superficial. Hence, we need a different argument to characterize the equilibrium conversation in this case.

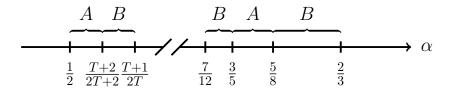


Figure 5: Illustration of the equilibrium conversation as a function of  $\alpha \in (\frac{1}{2}, \frac{2}{3})$  for odd  $T \geq 7$ .

#### 3.6 The Last Word

If interests are misaligned enough, the agent having the last word sets the topic of a deep conversation. As we show below, this follows from the fact that in the last period the agent does not have to worry about the future anymore. As a consequence, for sufficiently misaligned interests, she can credibly threaten to talk about her preferred topic in this last period, irrespective of the course of the preceding subconversation. Hence, whenever the other agent switches topics, the remaining conversation will be superficial.<sup>14</sup> As long as interests are not too different, in order to avoid a superficial conversation, the other agent then talks about the topic preferred by the last mover.

We now flesh out this logic in more detail. Consider an odd T, so that Agent 1 has the last word, and suppose that the conversation starts on A. Then, for any  $\alpha > (T-1)/T$ , Agent 1 will talk about A in the last period even if the entire conversation in between (t = 2, ..., T - 1) has been about B. Thus, by Part I of our main result, if Agent 2 switches to B in some period t, the subconversation following this switch, which we can treat as a new conversation, will be superficial.

Based on this, we can ask whether Agent 2 has an incentive to switch topics at any point. As illustrated in Figure 6, if Agent 2 switches topics, then she does so early on in the conversation (namely, in t = 2). Hence, the conversation is deep about A if and only if Agent 2, from the perspective of t = 2, prefers such a deep conversation over a superficial one. Because T is odd, this is indeed the case if and only if

$$(1-\alpha)\left(\frac{T(T+1)}{2}-1\right) \ge \alpha \frac{T-1}{2} + (1-\alpha)\frac{T-1}{2} \quad \text{or, equivalently,} \quad \alpha \le \frac{T+1}{T+2} = \bar{\alpha}(T).$$

Part I of Lemma 2 summarizes this "last-mover advantage" of Agent 1. A similar argument

<sup>&</sup>lt;sup>14</sup> Recall that, with less misaligned interests (as in Section 3.5), the conversation after a switch of topics turned deep. As a consequence, the incentives to switch topics and the equilibrium conversation are different in this case.

(t-2)A	(t-1)A	tA	(t+1)A	۱	В	A	
(t-2)A	(t-1)A $(t-1)A$ $A$	В	A		В	A	
В	A	В	A		В	A	
	t-1						

Figure 6: Information production in conversations that switch to B in t + 2 (top), in t (middle), and in t - 2 (bottom) for odd T and any  $\alpha > T/(T+1)$ . If Agent 2 would like to switch to B in t, then she has to prefer switching to B in t - 2 over switching in t.

(that we spell out in the Appendix) implies a last-mover advantage for Agent 2 when T is even.

#### Lemma 2 (The Last Word).

I. Let  $T \ge 3$  be odd. Then, if  $\alpha \in (\frac{T-1}{T}, \frac{T+1}{T+2})$ , the equilibrium conversation is deep about A. II. Let  $T \ge 4$  be even. Then, if  $\alpha \in (\frac{T-2}{T-1}, \frac{T}{T+1})$ , the equilibrium conversation is deep about B.

By repeatedly applying Lemma 2, we can complete the characterization of the equilibrium conversation. Consider an odd  $T \ge 5$ , so that it is again Agent 1 who talks last. (The argument for even  $T \ge 6$  is analogous.)<sup>15</sup> Lemma 2 implies that, for any

$$\alpha \in \left(\frac{(T-2)-1}{(T-2)}, \frac{(T-2)+1}{(T-2)+2}\right) = \left(\frac{T-3}{T-2}, \frac{T-1}{T}\right),\tag{6}$$

Agent 1 can ensure that a deep conversation about A takes place from t = 3 onward. To see why, suppose that Agent 2 talks about B in t = 2, which makes talking about A in  $t \ge 3$  less attractive. We can then treat the subconversation of length T - 2 starting on A in t = 3 as a new conversation. Because Agent 1 still has the last word in this subconversation, we can directly apply Lemma 2 to conclude that, for any  $\alpha$  in (6), this subconversation has to be deep about A. A revealed-preference argument, similar to that illustrated in Figure 4, then implies that Agent 2 best responds by talking about A already in t = 2. In equilibrium, the conversation unravels and, thus, is deep about A.

<sup>&</sup>lt;sup>15</sup> For T = 3 and T = 4, lemmas 1 and 2 together fully characterize the topic of a deep conversation.

With an odd T, the second-to-last time that Agent 1 talks is in period t = T - 2. The full proof simply applies the same argument as above T - 2 times in total. By doing so, we establish that for any odd T and any  $\alpha \in (2/3, \bar{\alpha}(T))$ , the conversation unravels to be deep about A. For even T and any such  $\alpha$ , by the exact same arguments, the conversation unravels to be deep about B. In sum, before the conversation turns superficial, it is the last mover who can steer it toward her preferred topic.

#### 3.7 Longer Conversations Are More Efficient

Taking a closer look at the threshold value  $\bar{\alpha}(T)$  reveals some interesting insights into the structure of the equilibrium conversation. First,  $\bar{\alpha}(T)$  weakly increases in T, which implies that longer conversations are more efficient. Moreover, the sequence of equilibrium cutoffs is given by

$$\underbrace{\frac{2}{3}}_{T=2}, \underbrace{\frac{4}{5}}_{T=3}, \underbrace{\frac{4}{5}}_{T=4}, \underbrace{\frac{6}{7}}_{T=5}, \underbrace{\frac{6}{7}}_{T=6}, \ldots, \underbrace{1}_{T=\infty}$$

This means that the cutoff  $\bar{\alpha}(T)$  increases when prolonging the conversation by one period if and only if it is Agent 1 who gets another opportunity to talk. In contrast, simply handing over the last word to Agent 2 does not increase efficiency. This follows from the fact that for any odd number of periods, Agent 1 values a superficial conversation (by  $2\alpha - 1$ ) more than does Agent 2, while for any even number of periods both agents assign the same value to a superficial conversation. And as we have seen in Section 3.6, whether the equilibrium conversation is deep or superficial depends on how much the agent talking in the second-to-last period values a superficial conversation.

Second, handing over the last word to Agent 2 affects the topic of a deep conversation, and even more so in longer conversations. In fact, since  $\bar{\alpha}(T)$  approaches one as  $T \to \infty$ , having the last word becomes more and more important in longer conversations. In the limit of T approaching infinity, the agent having the last word determines the topic of the conversation for any  $\alpha \in (2/3, 1)$ .

Third, when increasing the number of periods T, the topic of a deep conversation switches more often as interests become successively more misaligned (see Figure 7 for an illustration). Zooming in to the interval (1/2, 2/3), the topic of a deep conversation shifts exactly T - 2 times, with the partition becoming finer and finer when moving closer toward  $\alpha = 1/2$ . For any  $T \ge 3$ , there is an

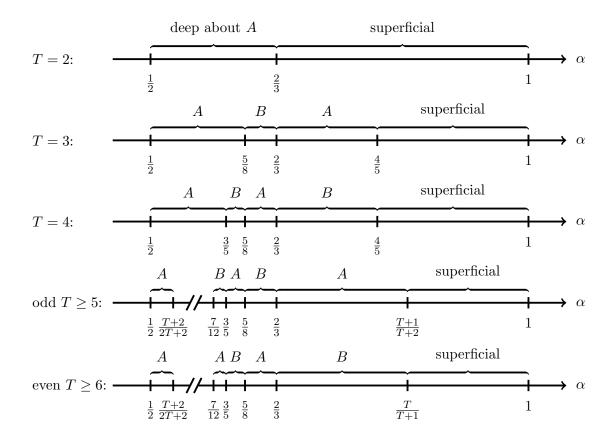


Figure 7: Illustration of the equilibrium conversation as a function of  $\alpha$  for different lengths.

additional change of topics on the interval  $(2/3, \bar{\alpha}(T))$ , making it a total of T-1 topic switches.

Fourth, still focusing on  $\alpha \in (1/2, 2/3)$ , we observe that even when prolonging the conversation by two periods, the topic may shift, although this does not change either who speaks first or who speaks last. This comparative static highlights the subtleties of "managing" the topic of a conversation; it is not about having the first word overall, but about speaking first in the right subconversation.

# 4 Discussion of Modeling Assumptions

#### 4.1 Returns to Staying on Topic

We make two assumptions that together determine the returns to staying on topic. First, the production of pieces increases linearly as the agents keep talking about the same topic. Second, the additional utility from receiving *one* more piece of information is constant and thus independent of the history of the conversation. We can substantially relax either assumption. All we need for our qualitative results to hold is that the additional utility from staying on topic increases over time. This seems particularly plausible when discussing complex matters (in which case learning takes time) or when time is limited (e.g., when talking to a co-author for a couple of hours).

Production function. The assumption that the production of pieces increases linearly as the agents stay on topic is mainly made for tractability; it allows us to obtain closed-form solutions. Qualitatively, however, our results hold for any production function of pieces, f(k), that increases strictly in the number of periods k that the agents stayed on topic.<sup>16,17</sup> The reason is that (a) the additional utility from staying on topic still strictly increases over time, and (b) switching topics still derails the conversation by resetting information production. This implies that the incentives to switch topics are strongest early on in the conversation and that after any change of topics it is as if a new, shorter conversation starts. These implications are the driving forces in our main theorem.

Constant additional utility. The constant extra utility from receiving one additional piece of information has bite. Clearly, if the additional utility from receiving one more piece of information is increasing, the incentives to stay on topic become even stronger, which only strengthens our qualitative results. More interestingly, even when allowing for a decreasing additional utility, the basic trade-offs remain the same as before. In the end, our qualitative results only require that the additional utility from one more piece of information does not decrease too fast (see Appendix C for details).

#### 4.2 Symmetry

The fact that the equilibrium conversation is either deep or superficial does not rely on the agents being symmetric. It holds for any combination of weights  $\alpha_i \in (0, 1)$  that Agent *i* assigns to topic *A*. Asymmetric interests can change the topic of a deep conversation, however. Consider the extreme

<sup>&</sup>lt;sup>16</sup> The case of a decreasing production function  $f(\cdot)$  is not interesting in that the agents would always agree on a (in this case efficient) superficial conversation.

<sup>&</sup>lt;sup>17</sup> In terms of the microfoundation provided in Section 2.2, this implies that our qualitative results generalize to any decreasing sequence of match probabilities  $p_0 \ge p_1 \ge p_2 \ge \ldots \ge p_{T-1}$ .

case where Agent 1 cares only about topic A (i.e.,  $\alpha_1 = 1$ ). The equilibrium conversation is thus either deep about A — namely, if and only if Agent 2 prefers it over a superficial conversation — or superficial. Strong interests (i.e.,  $\alpha_i$  close to 0 or 1) thus serve as a "commitment device" for talking about one's preferred topic and, therefore, help in steering the conversation in this direction.

#### 4.3 Commitment

In our model, agents cannot commit ex ante to a specific course of the conversation. If instead the agents could fully commit to an agenda in advance, they should be able to avoid superficial conversations. A lack of commitment is pervasive, however. Clearly, when bumping into a colleague in the hallway, people just start talking without a specific agenda in mind. In general, many conversations are not planned. And even if they are planned, people often do not specify an agenda. Moreover, in conversations that do have an agenda (e.g., a faculty meeting) people are essentially never committed to sticking to it because derailing the meeting is basically never punished.

#### 4.4 Strategic Sophistication

The exact equilibrium structure relies on the agents being strategically sophisticated. We conceptualize an agent's strategic sophistication via the number of periods she thinks ahead before deciding on a topic. In period  $t \in \{1, ..., T\}$ , a fully sophisticated agent takes all T - t future periods into account before choosing a topic. An  $\eta$ -naive agent only contemplates  $\eta$  periods. For  $\eta = 1$ , the agent is fully naive, or myopic, in that she only thinks about the current period when deciding on a topic. If both agents are completely naive, the equilibrium conversation is superficial. More generally, a somewhat naive agent behaves as if the conversation is shorter than it actually is, and is thus less willing to "invest" into a deep conversation. The interests of less sophisticated agents thus need to be more aligned for the conversation to be deep. Ignorance can actually be bliss in that naivete acts as a commitment device for only speaking about one's preferred topic. If the other agent is sufficiently sophisticated, any deep conversation is about the myopic agent's preferred topic.

#### 4.5 More Agents, More Topics

Our results do not hinge on *two* agents discussing *two* topics. Consider a conversation among n agents who take turns talking. There are n topics, and each agent prefers a different topic. The agents are symmetric in that they assign the same weight,  $\alpha \in (1/2, 1)$ , to their preferred topic. They are indifferent between all other topics, assigning weight  $1-\alpha$  to every piece generated on such a topic. Switching topics is still particularly tempting early on. Because the agents are symmetric, the conversation is superficial whenever the agent speaking in the second period switches topics. Hence, just like in our baseline model, the equilibrium conversation is either deep or superficial.

### 5 The End

#### 5.1 Random End

We adjust our baseline model by making the duration of the conversation random. After every period the conversation stops with probability  $1 - \delta \in (0, 1)$ . There is no restriction on the maximum number of periods, however. Otherwise the game is identical to our baseline model. We focus on Markovian equilibria in which the agents can condition their strategies only on the current complementarity. The following proposition, which we prove in Appendix B, summarizes the equilibrium.

**Proposition 1** (Conversations with a Random Ending).

- I. If  $\alpha \in (\frac{1}{2}, \frac{2}{3-\delta}]$ , the Markovian equilibrium conversation is deep about A.
- II. If  $\alpha \in (\frac{2}{3-\delta}, 1)$ , the Markovian equilibrium conversation is superficial.

Again the equilibrium conversation is either deep, and thus efficient, or superficial. Compared to conversations with a fixed duration, however, a deep conversation is always about the topic preferred by the first mover. Because the expected duration *conditional* on reaching a given period is constant (and because we consider Markovian strategies), the incentive to switch topics only depends on the current complementarity. Hence, whenever the second mover would switch to her preferred topic in the second period, the first mover would do so in the third period as well. The second mover thus cannot credibly threaten with a deep subconversation on her preferred topic, which in turn implies that any deep conversation is about the topic preferred by the first mover.

Going further, fixing an expected duration T, randomness enhances efficiency. Intuitively, conditional on reaching period  $t \ge 2$ , the remaining conversation with a random ending has an expected length of T. With a fixed number of periods T, however, the remaining conversation from the perspective of period  $t \ge 2$  is of length T - (t - 1) and thus shorter. As we have argued in Section 3, the returns to staying on topic are larger in longer conversations. Hence, conditional on reaching period  $t \ge 2$ , the random ending reduces Agent 2's incentive to switch topics. Formally:

**Corollary 1** (Randomness Enhances Efficiency). Let  $\delta = \frac{n-1}{n}$  for some  $n \in \mathbb{N}_{\geq 2}$ , and denote the corresponding expected duration by T. Then, for any  $\alpha \in (\bar{\alpha}(T), \frac{2}{3-\delta})$ , the conversation with a continuation probability  $\delta$  is more efficient than the conversation with fixed duration T.

#### 5.2 Endogenous End

Next, we consider an opportunity cost that arises when agents converse. Given that conversing is costly, we also allow the agents to leave the conversation at any time. More specifically, the agents can talk for at most T periods, but before any period starts, each agent can unilaterally end the conversation. For every period in which the agents talk, *both* agents incur a fixed cost of  $c > 0.^{18}$ 

If costs are small (i.e.,  $c \leq 1 - \alpha$ ), the agents never end the conversation prematurely. Even a single piece of information on one's less preferred topic gives positive utility net of cost, so there is no reason to leave the conversation. The agents can thus also never credibly threaten to end the conversation prematurely. So, the analysis is the same as in our baseline model.

If costs are larger (i.e.,  $c > 1 - \alpha$ ), however, the equilibrium conversation changes. A superficial conversation can no longer be an equilibrium; this is because the agent talking in the second-to-last period would have an incentive to end the conversation right before the last period starts. Going further, the other agent — anticipating this behavior — would actually already want to end the conversation before the second-to-last period. Iterating this argument backwards, if the agents

<sup>&</sup>lt;sup>18</sup> Talking and listening both require effort for a conversation to be productive (see Dewatripont and Tirole, 2005, for an in-depth discussion of this assumption). This is also consistent with our microfoundation in Section 2.2, where every period *both* agents contribute a piece of the puzzle.

anticipate a superficial conversation, they will not even start talking to each other. A conversation therefore requires *some* focus on one of the two topics. By the same arguments as in our baseline model, the incentive to switch topics is strongest early on in the conversation. This implies that, if the agents talk to each other, the conversation has to be deep. And a deep conversation indeed constitutes an equilibrium for sufficiently aligned interests. Specifically, an agent prefers a deep conversation about her less preferred topic over not talking at all if and only if

$$(1-\alpha)\frac{T(T+1)}{2} \ge Tc$$
 or, equivalently,  $\alpha \le 1 - \frac{2c}{T+1}$ .

This necessary condition for having a conversation is also sufficient because, by our main theorem, for any  $\alpha \leq 1 - \frac{2c}{T+1}$ , the agents are able to "coordinate" on a topic and have a deep conversation.

Proposition 2 (Conversations with an Endogenous Ending).

I. If  $c \leq 1 - \alpha$ , the equilibrium conversation is identical to that in our main theorem.

II. If  $c > 1 - \alpha$ , the equilibrium conversation is either deep or it does not start at all. The conversation does not start at all in equilibrium if and only if  $\alpha > 1 - \frac{2c}{T+1}$ .

This result has interesting implications for the kind of conversations that do and do not take place. First, if conversing is sufficiently costly (e.g., because people could use their time differently), some conversations break down completely. If costs are low, on the other hand, people do engage in superficial conversations (e.g., when two colleagues chit-chat to kill time between two meetings). Second, conversations break down if people have very different interests, suggesting that people will rather talk to like-minded people. Third, in shorter conversations interests have to be more aligned for a conversation to take place. Hence, if people are time-constrained (and opportunity costs are non-negligible), they are particularly prone to avoid engaging with people who hold very different views. Our theory thus provides a novel rationale for the emergence of "echo chambers," which is grounded in a desire for deep conversations and is amplified by ever more pressing timeconstraints.<sup>19</sup>

<sup>&</sup>lt;sup>19</sup> Levy and Razin (2019) survey the economics literature on echo chambers, summarizing existing explanations.

## 6 Concluding Remarks

Through specialization – even within narrow sub-fields – knowledge and ideas are dispersed in the economy (Garicano, 2000). Because creating new knowledge requires the combination of these dispersed ideas, we study the role of communication — or *conversations* — in the production of new information. We show that even if existing knowledge is transmitted truthfully, as is the case in many conversations, an inefficiency lurks in the process itself: people with different interests might be unwilling to focus on a topic, hindering information production.

Our model can also be applied to team production more generally. Consider, for example, a firm choosing between two mutually exclusive R&D projects. Assessing and implementing the projects requires the collaboration of two divisions with diverging interests. For example, Division 1 may have a larger stake in Project A, whereas Division 2 may care more about Project B. Because the divisions have different prior knowledge and skills, they need to work together. Concentrating efforts in a single project arguably has positive returns, while going back and forth between the projects kills the work flow. The production process — to which both divisions contribute — thus resembles the conversation in our model.

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# Appendix A: Proof of the Main Result

**Part I.** We proceed in three steps. First, we argue that the conversation cannot end with a deep subconversation that has been preceded by a superficial one. Second, we show that the conversation cannot end with consecutive deep subconversations on the two different topics. Because the conversation can be thought of as the sum of deep subconversations, the first and second step together imply that, if the conversation ends with a deep subconversation, the conversation has to be deep

overall. Third, we argue that, if the conversation does not end with a deep subconversation, it has to be superficial. In combination with the first and second step, this implies that the equilibrium conversation is either deep or superficial.

<u>1. Step</u>: Suppose that, in equilibrium, the conversation ends with a deep subconversation on, say, A of an arbitrary length  $k \ge 2$ , which is preceded by a superficial subconversation. Starting two periods before this deep subconversation on A begins, the sequence of pieces then is

$$\underbrace{A}_{\text{in }t=T-k-1}, \underbrace{B}_{\text{in }t=T-k}, \underbrace{A}_{\text{in }t=T-k+1}, \underbrace{2A}_{\text{in }t=T-k+2}, \dots, \underbrace{kA}_{\text{in }t=T}$$

Without loss of generality, we can assume that Agent 2 — who is more interested in B — talks last about B. For the above to occur in equilibrium, it has to be true that Agent 2 weakly prefers talking about B in t = T - k. If she talked about A instead, then in any future period, talking about A would become more attractive. For this to be an equilibrium, Agents 1 and 2 must thus prefer talking about A following such a deviation. The alternative sequence after a deviation then is

$$\underbrace{A}_{\text{in }t=T-k-1}, \underbrace{2A}_{\text{in }t=T-k}, \underbrace{3A}_{\text{in }t=T-k+1}, \underbrace{4A}_{\text{in }t=T-k+2}, \dots, \underbrace{(k+2)A}_{\text{in }t=T}.$$

Hence, Agent 2 weakly prefers talking about B in period t = T - k if and only if

$$\alpha + (1 - \alpha) \ge (k + 1 + k + 2)(1 - \alpha)$$
 or, equivalently,  $\alpha \ge \frac{2k + 2}{2k + 3}$ 

From now on, we assume  $\alpha \geq (2k+2)/(2k+3)$ ; that is, Agent 2 indeed prefers to talk about B in t = T - k. The conjectured equilibrium further requires Agent 2 to weakly prefer talking about A in period t = T - k + 2. Specifically, she has to weakly prefer the sequence

$$\underbrace{2A}_{\text{in }t=T-k+2},\ldots,\underbrace{kA}_{\text{in }t=T}$$

to any feasible sequence that starts with B in period t = T - k + 2. To complete the proof of <u>Step 1</u>, we derive a lower bound on the utility that Agent 2 could get when deviating to B in t = T - k + 2. If even this lower bound is larger than Agent 2's utility in the conjectured equilibrium, she will have an incentive to deviate and thus the above sequence cannot be part of an equilibrium conversation.

In particular, Agent 2 could unilaterally decide that from now on she will only talk about her preferred topic B. To obtain a lower bound on Agent 2's utility from such a deviation, we assume that Agent 1 would respond by talking about A in every future period. So, we compare our candidate equilibrium to the sequence of pieces

$$\underbrace{B}_{\text{in }t=T-k+2}, \underbrace{A}_{\text{in }t=T-k+3}, \underbrace{B}_{\text{in }t=T-k+4}, \underbrace{A}_{\text{in }t=T-k+5}, \ldots$$

We distinguish two cases. If k is odd, Agent 2 does not want to deviate only if

$$(1-\alpha)\cdot\left(\frac{k(k+1)}{2}-1\right) \ge \alpha\frac{k-1}{2} + (1-\alpha)\frac{k-1}{2} \quad \text{or, equivalently,} \quad \alpha \le \frac{k+1}{k+2}.$$

If k is even, she does not deviate only if

$$(1-\alpha)\cdot\left(\frac{k(k+1)}{2}-1\right) \ge \alpha\frac{k}{2} + (1-\alpha)\frac{k-1}{2} \quad \text{or, equivalently,} \quad \alpha \le \frac{k^2+1}{k^2+k+1}.$$

Because  $\alpha \geq (2k+2)/(2k+3)$  by revealed preferences (i.e., talking about *B* in t = T - k), neither of the above can hold; a contradiction. Hence, an equilibrium conversation ending with a deep subconversation on *A* cannot have a superficial one before. Similarly, we cannot have an equilibrium conversation that ends on a deep subconversation on *B*, but has a superficial one before.

<u>2. Step</u>: Suppose that the equilibrium conversation ends with two consecutive deep subconversations on the different topics. Without loss of generality, we can assume that the whole conversation consists of a deep subconversation of length  $\ell$  on B followed by a deep subconversation of length  $T - \ell \equiv k$  on A. This corresponds to the following sequence of pieces:

$$\underbrace{B}_{\text{in }t=1}, \underbrace{2B}_{\text{in }t=2}, \ldots, \underbrace{(\ell-1)B}_{\text{in }t=T-k-1}, \underbrace{\ell B}_{\text{in }t=T-k}, \underbrace{A}_{\text{in }t=T-k+1}, \underbrace{2A}_{\text{in }t=T-k+2}, \ldots, \underbrace{kA}_{\text{in }t=T}.$$

Without loss of generality, we can again assume that it is Agent 2 who talks last about B. For the above to be an equilibrium, it has to be true that Agent 1 weakly prefers talking about B in period t = T - k - 1. As we argue below, if the above sequence was indeed an equilibrium, then by deviating to A in t = T - k - 1 Agent 1 could induce the sequence of pieces

$$\underbrace{B}_{\text{in }t=1}, \underbrace{2B}_{\text{in }t=2}, \ldots, \underbrace{(\ell-2)B}_{\text{in }t=T-k-2}, \underbrace{A}_{\text{in }t=T-k-1}, \underbrace{2A}_{\text{in }t=T-k}, \underbrace{3A}_{\text{in }t=T-k+1}, \underbrace{4A}_{\text{in }t=T-k+2}, \ldots, \underbrace{(k+2)A}_{\text{in }t=T-k+2}, \underbrace{(k+2)A}_{\text{in }t=T-k$$

that is, she could initiate a longer deep subconversation on her preferred topic A. But then Agent 1 clearly has an incentive to deviate, which gives us a contradiction.

To obtain the contradiction, we first observe that in our conjectured equilibrium Agent 2 weakly prefers

$$\underbrace{2A}_{\text{in }t=T-k+2},\ldots,\underbrace{kA}_{\text{in }t=T}$$

to any feasible sequence that starts with B in period t = T - k + 2. By the exact same arguments as in Step 1, this requires

$$\alpha \leq \begin{cases} \frac{k+1}{k+2} & \text{if } k \text{ is odd,} \\ \frac{k^2+1}{k^2+k+1} & \text{if } k \text{ is even.} \end{cases}$$
(7)

So, from now on, we assume that the relevant of these two inequalities holds.

Second, suppose that Agent 1 deviates to A in t = T - k - 1. If Agent 2 talks about B in t = T - k, the sequence of pieces going forward is

$$\underbrace{B}_{\text{in } t=T-k}, \underbrace{A}_{\text{in } t=T-k+1}, \underbrace{2A}_{\text{in } t=T-k+2}, \dots, \underbrace{kA}_{\text{in } t=T}.$$

This follows from the fact that — compared to the conjectured equilibrium — such a deviation makes it even more attractive for Agent 1 to talk about A in t = T - k + 1; given that Agent 1 does so, and given our conjectured equilibrium, the deep subconversation on A continues until the end. If Agent 2 instead talks about A in t = T - k, the sequence of pieces going forward is

$$\underbrace{A}_{\text{in } t=T-k}, \underbrace{2A}_{\text{in } t=T-k+1}, \underbrace{3A}_{\text{in } t=T-k+2}, \dots, \underbrace{(k+2)A}_{\text{in } t=T},$$

because talking about A in t = T - k makes talking about A more attractive in any future period. By the same arguments as in Step 1, in t = T - k, Agent 2 talks about B if and only if

$$\alpha + (1 - \alpha) \ge (k + 1 + k + 2)(1 - \alpha)$$
 or, equivalently,  $\alpha \ge \frac{2k + 2}{2k + 3}$ .

Because (7) holds by revealed preferences, however, the above cannot hold; a contradiction. We conclude that there cannot be an equilibrium that consists of a deep subconversation on B followed

by a deep subconversation on A. Similarly, we cannot have an equilibrium that consists of a deep subconversation on A followed by a deep subconversation on B.

<u>3. Step</u>: Suppose that, in equilibrium, the conversation ends with a superficial subconversation, but also contains a deep subconversation on either topic. Because we can think of the conversation as the sum of deep subconversations, it is without loss of generality to assume that the conversation starts deeply. Specifically, consider the following sequence of pieces as a candidate equilibrium:

$$\underbrace{A}_{\text{in }t=1}, \underbrace{2A}_{\text{in }t=2}, \underbrace{3A}_{\text{in }t=3}, \ldots, \underbrace{kA}_{\text{in }t=k}, \underbrace{B}_{\text{in }t=k+1}, \underbrace{A}_{\text{in }t=k+2}, \ldots$$

It is without loss of generality to assume that it is Agent 2 — who is more interested in B — who switches topics. For this to be an equilibrium, it has to be true that Agent 2 indeed weakly prefers talking about B in period t = k + 1. Hence, she weakly prefers the above sequence of pieces to any feasible sequence in which the agents talk about A in t = k + 1. Clearly, Agent 2 could decide that, from t = k + 3 onward, she will only talk about her more preferred topic B. To give our candidate equilibrium the best shot, we assume that Agent 1 would respond to this by always talking about A. (This rules out delving deeply into B, the preferred topic of Agent 2, which would make deviating even more attractive.) This alternative sequence, starting in t = k + 1, is

$$\underbrace{(k+1)A}_{\text{in }t=k+1}, \underbrace{(k+2)A}_{\text{in }t=k+2}, \underbrace{B}_{\text{in }t=k+3}, \underbrace{A}_{\text{in }t=k+2}, \dots$$

Hence, Agent 2 does not want to deviate in t = k + 1 only if

$$\alpha + (1 - \alpha) \ge (1 - \alpha) \cdot (k + 1 + k + 2)$$
 or, equivalently  $\alpha \ge \frac{2k + 2}{2k + 3}$ .

From now on, let  $\alpha \ge (2k+2)/(2k+3)$ . For the above to be an equilibrium, Agent 2 also has to prefer talking about A in period t = k - 1. We look at the same kind of deviation, in which Agent 2 already talks about B in t = k - 1 (as well as in every future period), and Agent 1 responds by always talking about A. This again gives our candidate equilibrium the best shot. It follows that Agent 2 does not want to deviate in t = k - 1 only if

$$(1-\alpha)(k-1+k) \ge \alpha + 1 - \alpha$$
 or, equivalently  $\alpha \le \frac{2k-2}{2k-1}$ .

This cannot hold, because  $\alpha \geq (2k+2)/(2k+3)$  by revealed preferences. By similar arguments, we cannot have a conversation that starts by delving deeply into *B* but ends on a superficial sequence.

**Parts III and IV**  $(1/2 < \alpha < 2/3)$ . Consider  $\alpha \in (1/2, 2/3)$ , and let  $T \ge 3$ . Recall that, by Lemma 1, for any  $\alpha \in (1/2, (T+2)/2(T+1))$ , the equilibrium conversation is deep about A. We now gradually increase the misalignment of interests by successively looking at

$$\alpha \in \left(\frac{T - (k - 1) + 2}{2(T - (k - 1) + 1)}, \frac{T - k + 2}{2(T - k + 1)}\right) \quad \text{for} \quad k \in \{1, \dots, T - 2\}.$$

We establish that, by Lemma 1, the agent speaking in period k+1 can induce a deep subconversation (starting in this period) about her preferred topic for the relevant  $\alpha$ . If the conversation up to period k + 1 has been superficial, this agent will also have an incentive to do so. We will use this observation to argue that, in equilibrium, the conversation indeed "unravels" to be deep about exactly this topic. We distinguish two cases, depending on whether (i) k is odd or (ii) k is even.

<u>1. Case</u>: If k is odd, Agent 2 talks in period k + 1. Suppose for a moment that up to period k + 1 the conversation has been superficial. Then, by Lemma 1, for any  $T \ge k + 2$  and any

$$\alpha \in \left(\frac{1}{2}, \frac{(T-k)+2}{2(T-k)+2}\right) = \left(\frac{1}{2}, \frac{T+2-k}{2T+2-2k}\right),$$

this subconversation of length T - k is deep and — because Agent 2 has the first word — about B. Moreover, because  $\alpha > (T+2-(k-1))/(2T+2-2(k-1))$ , Agent 2 would indeed talk about B in this case.

If Agent 1 instead talks about B in t = k, it becomes even more attractive to talk about B in any later period. Hence, when assuming a superficial conversation up to period t = k, Agent 1 chooses between the following two sequences of pieces going forward:

$$\underbrace{2B}_{\text{in }t=k}, \underbrace{3B}_{\text{in }t=k+1}, \underbrace{4B}_{\text{in }t=k+2}, \dots, \underbrace{(T-k+2)B}_{\text{in }t=T},$$

and

$$\underbrace{A}_{\text{in }t=k}, \underbrace{B}_{\text{in }t=k+1}, \underbrace{2B}_{\text{in }t=k+2}, \dots, \underbrace{(T-k)B}_{\text{in }t=T}$$

Here, Agent 1 prefers to talk about B in t = k if and only if

$$(1-\alpha)\left(\frac{(T-k+2)(T-k+3)}{2}-1\right) > \alpha + (1-\alpha)\frac{(T-k)(T-k+1)}{2}$$

or, equivalently,

$$\alpha < \frac{2(T-k)+2}{2(T-k)+3}.$$

For any  $T \ge k+2$ , we have (2(T-k)+2)/(2(T-k)+3) > (T-k+2)/(2(T-k)+2). This means in turn that, for any  $\alpha < (T-k+2)/(2(T-k)+2)$ , Agent 1 talks about B in t = k. Hence, if Agent 2 talks about B in t = k - 1, the remaining conversation will be deep about B. Clearly, Agent 2 does so.

We can iterate the exact same argument until we reach the first period. More specifically, assuming a superficial conversation up to period t = k - l for  $l \in \{4, ..., k - 1\}$ , Agent 1 prefers to talk about B in this period if and only if

$$\alpha < \frac{2(T - (k - l)) + 2}{2(T - (k - l)) + 3}.$$

Since the right-hand side above is strictly increasing in l, this holds for any  $\alpha < (T-k+2)/(2(T-k)+2)$ . We conclude that for any  $\alpha \in ((T+2-(k-1))/(2T+2-2(k-1)), (T+2-k)/(2T+2-2k))$  with  $T \ge k+2$  and k odd, the equilibrium conversation is deep about B.

<u>2. Case</u>: If k is even, Agent 1 talks in period k + 1. Again by Lemma 1, the subconversation starting in period k + 1 is deep about A. And by the exact same arguments as in the first case, the conversation then "unravels" to be deep about A. We conclude that, for any  $\alpha \in ((T+2-(k-1))/(2T+2-2(k-1)), (T+2-k)/(2T+2-2k))$  with  $T \ge k+2$  and k being even, the equilibrium conversation is deep and about A.

**Parts III and IV** ( $\alpha > 2/3$ ). We start by giving a full proof of Lemma 2. Afterwards we will use Lemma 2 to iteratively characterize the equilibrium conversation for any  $\alpha > 2/3$ .

Proof of Lemma 2. First, let  $T \ge 3$  be odd, so that Agent 1 has the last word,<sup>20</sup> and let  $\alpha > (T-1)/T$ . We start by arguing that, if Agent 1 starts on A, the conversation has to be either deep about A or superficial. To see why, suppose that Agent 2 switches to B in t = 2. This is as if a new conversation of length T - 1 starts. By Part I, this subconversation is either deep about B or superficial. A deep conversation about B requires Agent 1 to follow through in all remaining periods. But, even if the agents only talk about B in-between, Agent 1 does so in t = T if and only if

$$(1-\alpha)(T-1) \ge \alpha$$
 or, equivalently,  $\alpha \le \frac{T-1}{T}$ .

<sup>&</sup>lt;sup>20</sup> For T = 2, whenever  $\alpha > 2/3$ , both agents value a superficial conversation more than a deep conversation about their less preferred topic. Hence, the equilibrium conversation is deep if and only if  $\alpha \leq 2/3$ .

When starting on A, Agent 1 can thus credibly threaten to talk about A also in the last period.

We now show that this indeed allows Agent 1 to steer the conversation toward her preferred topic. Suppose that up to period  $t \in \{2, 4, ..., T-1\}$  the two agents have only talked about A, and that it is Agent 2's turn to talk in period t. If Agent 2 talks about B in period t, it is as if a new conversation of length T - (t - 1) starts. Again by Part I, the subconversation following such a switch of topics would be either deep or superficial. Moreover, for any  $\alpha > (T-1)/T$ , Agent 1 talks about A in the last period in any case. Hence, if Agent 2 indeed talks about B in period t, the remaining conversation will be superficial. Thus, for any  $\alpha > (T-1)/T$ , the conversation is deep about A if and only if, for any  $t \in \{2, 4, ..., T - 1\}$ , Agent 2 does not switch topics when anticipating a superficial conversation afterwards. This is the case if and only if, for all such t,

$$(1-\alpha)\left(\frac{T(T+1)}{2} - \sum_{i=1}^{t-1}i\right) \ge \alpha \frac{T - (t-1)}{2} + (1-\alpha)\frac{T - (t-1)}{2}$$

or, equivalently,

$$\alpha \le \frac{T^2 - t^2 + 2t - 1}{T^2 - t^2 + T + t}.$$
(8)

The right-hand side of Eq. (8) monotonically increases in t, so that Agent 2's incentive to switch topics is strongest early on in the conversation. Hence, a deep conversation about A is indeed feasible if and only if

$$\alpha \le \left. \frac{T^2 - t^2 + 2t - 1}{T^2 - t^2 + T + t} \right|_{t=2} = \left. \frac{T + 1}{T + 2} \right|_{t=2} = \bar{\alpha}(T).$$

We conclude that, for any odd number of periods and any  $\alpha \in ((T-1)/T, (T+1)/(T+2))$ , the equilibrium conversation is deep and about A. For any odd T and any  $\alpha > (T+1)/(T+2)$ , it is superficial.

Next, consider an even number of periods  $T \ge 4$ , so that Agent 2 has the last word. To start, suppose again that Agent 1 talks about A in the first period. If Agent 2 then talks about B in t = 2, it is as if a new conversation of length  $T - 1 \ge 3$  starts. Because this subconversation has an odd number of periods and Agent 2 (still) has the last word, for any

$$\alpha \in \left(\frac{(T-1)}{(T-1)+1}, \frac{(T-1)+1}{(T-1)+2}\right] = \left(\frac{T-1}{T}, \frac{T}{T+1}\right],$$

the remaining conversation following such a switch of topics would be deep about B. As a consequence, for any such  $\alpha$ , a deep conversation about A can arise in equilibrium only if, from the perspective of period t = 2, Agent 2 prefers it to the alternative sequence of signals

$$\underbrace{B}_{\text{in }t=2}, \underbrace{2B}_{\text{in }t=2}, \ldots, \underbrace{(T-1)B}_{\text{in }t=T}.$$

This is indeed the case if and only if

$$(1-\alpha)\left(\frac{T(T+1)}{2}-1\right) \ge \alpha \frac{(T-1)T}{2}$$
 or, equivalently  $\alpha \le \frac{T+2}{2T+2}$ 

Note that (T+2)/(2T+2) < (T-1)/T for any  $T \ge 4$ . Hence, for any  $\alpha > (T-1)/T$ , a deep conversation about A is not feasible. Thus, by Part I, the conversation is either deep about B or superficial.

From the perspective of the first period, Agent 1 prefers a deep conversation about B over a superficial one if and only if

$$(1-\alpha)\frac{T(T+1)}{2} \ge \alpha \frac{T}{2} + (1-\alpha)\frac{T}{2}$$
 or, equivalently,  $\alpha \le \frac{T}{T+1}$ .

For any  $\alpha > T/(T+1)$ , Agent 2 always talks about *B* in the *last* period. Hence, if Agent 1 talked about *A*, the remaining conversation would be superficial. By the same arguments as for an odd *T* (cf. Eq. (8)), Agent 1's incentive to talk about *A* is strongest early on in the conversation; namely, in t = 1. Hence, for  $\alpha \in ((T-2)/(T-1), T/(T+1))$ , the equilibrium conversation is deep about *B*.  $\Box$ 

We now use Lemma 2 to show that for any  $\alpha \in (2/3, \bar{\alpha}(T))$  the conversation is deep about the topic preferred by the last mover. Consider an odd number of periods  $T \geq 5$ , so that again Agent 1 talks last. (The argument for even  $T \geq 6$ , which means that Agent 2 talks last, is analogous.)<sup>21</sup>

In a first step, we assume that  $\alpha \in ((T-3)/(T-2), (T-1)/T)$ . For the sake of a contradiction, suppose that Agent 1 starts on A and Agent 2 switches topics in t = 2. If Agent 1 again switches topics in t = 3, it is as if a new conversation of length T - 2 starts. By Part I of Lemma 2, for any

$$\alpha \in \left(\frac{(T-2)-1}{(T-2)}, \frac{(T-2)+1}{(T-2)+2}\right) = \left(\frac{T-3}{T-2}, \frac{T-1}{T}\right),$$

this subconversation of length T - 2 is deep and about A. Clearly, if Agent 2 talks about A in t = 2, it becomes even more attractive to talk about A in later periods. Hence, by starting on A, Agent 1 can ensure that from t = 3 onward the conversation is deep about her preferred topic.

<sup>&</sup>lt;sup>21</sup> For T = 3 and T = 4, Lemmas 1 and 2 together fully characterize the topic of a deep conversation.

Thus, if Agent 1 starts on A, Agent 2 can choose between the following two sequences in t = 2:

$$\underbrace{2A}_{\text{in }t=2}, \underbrace{3A}_{\text{in }t=3}, \underbrace{4A}_{\text{in }t=4}, \dots, \underbrace{TA}_{\text{in }t=T}.$$

and

$$\underbrace{B}_{\text{in }t=2}, \underbrace{A}_{\text{in }t=3}, \underbrace{2A}_{\text{in }t=4}, \ldots, \underbrace{(T-2)A}_{\text{in }t=T}$$

As we have seen earlier, Agent 2 prefers the deep conversation about A if and only if

$$\alpha < \frac{2(T-2)+2}{2(T-2)+3}.$$

Because (2(T-2)+2)/(2(T-2)+3) > (T-1)/T for any  $T \ge 5$ , the above holds for any  $\alpha < (T-1)/T$ . As a consequence, if Agent 1 starts on A, the conversation will be deep about A. So, she clearly does so.

With an odd T, the second-to-last time that Agent 1 talks is in period t = T - 2. We can thus apply the same argument T - 2 times in total. Consider  $\alpha \in (((T-k)-1)/(T-k), ((T-k)+1)/((T-k)+2))$ for some even  $k \in \{2, 4, 6, ..., T - 3\}$  and  $T \ge k + 3$ . Furthermore, suppose for a moment that up to period t = k + 1 the conversation has been superficial. We now argue that in this case for any odd  $T \ge k + 3$  and any such  $\alpha$ , the conversation "unravels" to be deep about A. If Agent 1 talks about A in t = k + 1, it is as if a new conversation of length T - k starts. By Lemma 2, for any

$$\alpha \in \left(\frac{(T-k)-1}{(T-k)}, \frac{(T-k)+1}{(T-k)+2}\right)$$

this subconversation of length T - k is deep and about A. Moreover, if Agent 2 instead talks about A in t = k, it clearly becomes even more attractive to talk about A in later periods. Hence, if the conversation up to period t = k has been superficial, Agent 2 can choose between the sequences

$$\underbrace{2A}_{\text{in }t=k}, \underbrace{3A}_{\text{in }t=k+1}, \underbrace{4A}_{\text{in }t=k+2}, \dots, \underbrace{(T-k+2)A}_{\text{in }t=T},$$

and

$$\underbrace{B}_{\text{in }t=k}, \underbrace{A}_{\text{in }t=k+1}, \underbrace{2A}_{\text{in }t=k+2}, \ldots, \underbrace{(T-k)A}_{\text{in }t=T}.$$

Here, Agent 2 prefers the deep conversation about A if and only if

$$\alpha < \frac{2(T-k)+2}{2(T-k)+3}.$$

For any  $T \ge k+3$ , we have (2(T-k)+2)/(2(T-k)+3) > ((T-k)+1)/((T-k)+2), so that the above holds in the relevant range of  $\alpha$ . Hence, by talking about A in t = k - 1, Agent 1 can ensure that the remaining conversation is deep and about her preferred topic. So, clearly she does so.

To establish our claim, we repeat the exact same argument until we reach the second period. More specifically, if the conversation up to period t = k - l,  $l \in \{2, ..., k - 2\}$ , has been superficial, Agent 2 prefers to talk about A in period t = k - l if and only if

$$\alpha < \frac{2(T - (k - l)) + 2}{2(T - (k - l)) + 3}.$$

Since the right-hand side above is strictly increasing in l, this holds for any  $\alpha < ((T-k)+1)/((T-k)+2)$ . Hence, for any  $\alpha \in (((T-k)-1)/(T-k), ((T-k)+1)/((T-k)+2))$  with even  $k \in \{2, 4, \ldots, T-3\}$ , the equilibrium conversation is deep and about A. This, in turn, implies that for any odd  $T \ge 5$  and any  $\alpha \in (2/3, (T+1)/(T+2))$  the equilibrium conversation is deep about A. (An analogous argument yields that for any even  $T \ge 6$  and any  $\alpha \in (2/3, T/(T+1))$ , the conversation is deep about B.)

**Part II.** Follows immediately from the arguments given in the proof of Lemma 2.  $\Box$ 

# Appendix B: Derivations for a Random Ending

Proof of Proposition 1. We first argue that the equilibrium conversation is either deep or superficial. To see why, notice that, if one of the agents switches topics, it is as if a new conversation starts. Moreover, the incentives in this "new" conversation are exactly the same as in the original one. Hence, the remaining conversation following a switch of topics has the same set of Markov equilibria as the original conversation. By similar arguments as those for a fixed duration, the incentives to switch topics are strongest early on. Hence, if the conversation switches topics once, it has to be superficial. So, the equilibrium conversation is either deep or superficial.

But, in contrast to conversations with a fixed duration, a deep conversation is always about A the topic preferred by the first mover. This is because the expected duration conditional on reaching a given period, and thus the value of a superficial conversation, is constant over time. The value of a deep conversation, on the other hand, depends on the existing complementarity and (weakly) increases over time. As a consequence, the incentive to switch topics (weakly) decreases over time. This implies that, if the agent talking in the second period prefers a superficial conversation over a deep one on her less preferred topic, the agent talking in the first period does so too. Hence, if the conversation is deep, it has to be deep about the topic that the first mover prefers.

Finally, we determine the cutoff on the preference parameter  $\alpha$  above which the conversation turns superficial. Recall that incentives to switch topics are strongest early on in the conversation. So, we consider Agent 2's decision in the second period. As we have argued above, if Agent 2 indeed has an incentive to switch topics and talk about B, the remaining conversation will be superficial. Hence, if Agent 2 wants to deviate to B, her expected utility going forward is equal to

$$\alpha (1 + \delta^2 + \delta^4 + \ldots) + (1 - \alpha) (\delta + \delta^3 + \delta^5 + \ldots) = \frac{\alpha}{(1 - \delta)(1 + \delta)} + \frac{(1 - \alpha)\delta}{(1 - \delta)(1 + \delta)}.^{22}$$

On the other hand, from the perspective of the second period, Agent 2's expected utility from a deep conversation about A is given by

$$(1-\alpha)(2+3\delta+4\delta^2+5\delta^3+\ldots) = \frac{1-\alpha}{1-\delta}\left(1+\frac{1}{1-\delta}\right).^{23}$$

It follows that Agent 2 has no incentive to switch topics if and only if

$$\frac{1-\alpha}{1-\delta} \left( 1 + \frac{1}{1-\delta} \right) \ge \frac{\alpha}{(1-\delta)(1+\delta)} + \frac{(1-\alpha)\delta}{(1-\delta)(1+\delta)} \quad \text{or, equivalently,} \quad \alpha \le \frac{2}{3-\delta}.$$

This completes the proof.

*Proof of Corollary 1.* For a given  $\delta \in (0,1)$ , let  $\mathcal{T}$  be the (random) end of the conversation. This random end  $\mathcal{T}$  is distributed as follows:

$$\mathbb{P}[\mathcal{T} = t] = \delta^{t-1}(1-\delta) \quad \text{for any } t \ge 1.$$

The expected duration of the conversation is thus equal to

$$\mathbb{E}[\mathcal{T}] = (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} t = \frac{1}{1-\delta},$$

 $<sup>\</sup>boxed{\begin{array}{l} 2^{2} \text{ Define } S := 1 + \delta^{2} + \delta^{4} + \dots, \text{ and notice that } S + \delta S = \sum_{n=0}^{\infty} \delta^{n}. \text{ Thus, because } \sum_{n=0}^{\infty} \delta^{n} = \frac{1}{1-\delta}, S = \frac{1}{(1-\delta)(1+\delta)}. \\ \text{Moreover, defining } S' := \delta + \delta^{3} + \delta^{5} + \dots, \text{ we observe } S' = \delta S. \\ 2^{3} \text{ Define } S'' := 2 + 3\delta + 4\delta^{2} + 5\delta^{3} + \dots, \text{ and notice that } S'' - \delta S'' = 1 + \sum_{n=0}^{\infty} \delta^{n}. \text{ Hence, } (1-\delta)S'' = 1 + \frac{1}{1-\delta}. \end{aligned}$ 

where the second equality follows from the fact that

$$\sum_{t=1}^{\infty} \delta^{t-1}t - \delta \sum_{t=1}^{\infty} \delta^{t-1}t = \sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta} \quad \text{and, thus,} \quad \sum_{t=1}^{\infty} \delta^{t-1}t = \frac{1}{(1-\delta)^2}.$$

Suppose  $\mathbb{E}[\mathcal{T}] \in \mathbb{N}_{\geq 2}$ , which requires  $\delta = (n-1)/n$  for some  $n \in \mathbb{N}_{\geq 2}$ . By Part II of our main result, a conversation with fixed duration  $T = \mathbb{E}[\mathcal{T}]$  is efficient if and only if  $\alpha$  is smaller than

$$\bar{\alpha}(\mathbb{E}[\mathcal{T}]) = \begin{cases} \frac{1}{2-\delta} & \text{if } \mathbb{E}[\mathcal{T}] \text{ is even,} \\ \frac{2-\delta}{3-\delta} & \text{if } \mathbb{E}[\mathcal{T}] \text{ is odd.} \end{cases}$$

Notice that, for any such  $\delta$ , we have

$$0 < \frac{2}{3-\delta} - \bar{\alpha}(\mathbb{E}[\mathcal{T}]) = \begin{cases} \frac{1-\delta}{(2-\delta)(3-\delta)} & \text{if } \mathbb{E}[\mathcal{T}] \text{ is even,} \\ \frac{\delta}{3-\delta} & \text{if } \mathbb{E}[\mathcal{T}] \text{ is odd.} \end{cases}$$

Hence, fixing the expected duration of the conversation, a random end date improves efficiency.  $\Box$ 

# Appendix C: Decreasing Additional Utility

Suppose that the additional utility from receiving one more piece of information (or enjoyment) is decreasing. In particular, we want to allow for utility functions of the form

$$\hat{U}_1 = \alpha \cdot u (\text{total } \# \text{ of pieces on A}) + (1 - \alpha) \cdot u (\text{total } \# \text{ of pieces on B}),$$

and

$$\hat{U}_2 = (1 - \alpha) \cdot u (\text{total } \# \text{ of pieces on A}) + \alpha \cdot u (\text{total } \# \text{ of pieces on B}).$$

where  $u(\cdot)$  is strictly increasing and strictly concave. The structure of the equilibrium conversation will depend on the exact shape of this function.

To make this point precisely, suppose again that the number of generated pieces increases linearly as the agents stay on topic. The additional utility from staying on topic (for one more period) after talking about the same topic for k periods is then proportional to

$$u\left(\frac{(k+1)(k+2)}{2}\right) - u\left(\frac{k(k+1)}{2}\right).$$

The above expression is strictly increasing in k if and only if

$$\frac{2k+3}{2k+1} > \frac{u'(k(k+1)/2)}{u'(k(k+1)(k+2)/2)}.$$
(9)

Hence, if Eq. (9) holds for any  $k \in \{1, ..., T-1\}$ , the additional utility from staying on topic strictly increases over time and the equilibrium conversation has the same structure as in our main result. This requires, however, that the function  $u(\cdot)$  is not too concave.

Example 1 (Power utility). If  $u(x) = x^{\theta}$  for some  $\theta \in (0, 1)$ , Eq. (9) holds for all  $k \in \{1, \dots, T-1\}$  if and only if  $\theta > \frac{\ln(9/5)}{\ln(3)} \approx 0.54$ .

Example 2 (Learning about standard-normal states). Consider agents who seek to minimize the  $\alpha$ -weighted posterior variances of  $\mu_A$  and  $\mu_B$ . Their prior is that both states are drawn independently from a standard normal distribution. They can generate and share signals  $s_j \sim \mathcal{N}(\mu_j, 1/\tau)$  for some  $\tau > 0$ . This corresponds to a utility function  $u(x) = -\frac{1}{1+\tau x}$ , where x denotes the number of signals.

Consider two conversations with a total number of signals  $(n_A, n_B)$  and  $(n'_A, n'_B)$ , respectively. Without loss of generality, we assume that neither conversation dominates the other (in the sense of having more signals on both,  $\mu_A$  and  $\mu_B$ ). Agent 1 prefers  $(n_A, n_B)$  over  $(n'_A, n'_B)$  if and only if

$$\alpha \frac{1}{1 + \tau n_A} + (1 - \alpha) \frac{1}{1 + \tau n_B} < \alpha \frac{1}{1 + \tau n'_A} + (1 - \alpha) \frac{1}{1 + \tau n'_B}$$

or, equivalently,

$$\frac{\alpha}{1-\alpha}\frac{n'_A - n_A}{n_B - n'_B} < \frac{(1+\tau n_A)(1+\tau n'_A)}{(1+\tau n_B)(1+\tau n'_B)}.$$
(10)

Now consider our model with constant additional utility, where Agent 1's utility function  $is^{24}$ 

 $U_1 = \alpha \cdot (\text{total } \# \text{ of signals on A}) + (1 - \alpha) \cdot (\text{total } \# \text{ of signals on B}).$ 

Whenever the utility function  $U_1$  implies a strict preference over two conversations with signals  $(n_A, n_B)$  and  $(n'_A, n'_B)$ , respectively, the left-hand side of Eq. (10) is different from 1. The righthand side of (10) approaches 1 as  $\tau \to 0$ . Now suppose that the left-hand side of Eq. (10) is less than 1, meaning that according to  $U_1$  the agent prefers  $(n_A, n_B)$ . Then there exists some  $\tau' > 0$ 

<sup>&</sup>lt;sup>24</sup> To be consistent with the notation used in this example, we refer in  $U_1$  and  $U_2$  to signals rather than pieces.

such that for any  $\tau < \tau'$  the inequality in (10) holds, meaning that also according to  $\hat{U}_1$  the agent prefers  $(n_A, n_B)$ . Similarly, if the left-hand side of Eq. (10) is larger than 1, meaning that  $U_1$ assigns higher utility to  $(n'_A, n'_B)$ , there exists some  $\tau'' > 0$  such that for any  $\tau < \tau''$ , we have

$$\frac{\alpha}{1-\alpha}\frac{n'_A - n_A}{n_B - n'_B} > \frac{(1+\tau n_A)(1+\tau n'_A)}{(1+\tau n_B)(1+\tau n'_B)}.$$

Thus, for any  $\tau < \tau''$ ,  $U_1$  and  $\hat{U}_1$  rank the two conversations in exactly the same way.

Because the number of periods is finite, there are only finitely many such binary comparisons l = 1, ..., L. Fixing  $\alpha \in (1/2, 1)$ , for any such binary comparison, we can find some  $\tau_l > 0$  such that for any  $\tau < \tau_l$ , the utility functions  $U_i$  and  $\hat{U}_i$  imply the same ranking over the two conversations. Fixing the number of periods  $T \ge 2$ , we can define  $\hat{\tau} := \min_l \tau_l$ , and conclude that for any  $\tau < \hat{\tau}$ , the utility functions  $U_i$  and  $\hat{U}_i$  imply the same behavior at every stage of the conversation. This formalizes the intuition that our reduced-form model captures discussions of complex matters (with a low  $\tau$ ) or conversations that are rather short.