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Robert Lehmann, Ida Wikman

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Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
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Quarterly GDP Estimates for the German States: New Data for Business Cycle Analyses and Long-Run Dynamics

Abstract

To date, only annual information on economic activity is published for the 16 German states. In this paper, we calculate quarterly regional GDP estimates for the period between 1995 to 2021, thereby improving the regional database for Germany. The new data set will regularly be updated when quarterly economic growth for Germany becomes available. We use the new data for an in-depth business cycle analysis and to estimate long-run growth dynamics. The business cycle analysis reveals large heterogeneities in the duration and amplitudes of state-specific fluctuations as well as in the degrees of cyclical concordance. Long-run trends are found to vary tremendously, with positive developments in economically strong regions and flat or even negative trends for economically much weaker states.

JEL-Codes: C320, C530, E320, R110.

Keywords: regional economic activity, mixed-frequency Vector Autoregression, regional business cycles, concordance, Bayesian methods.

*Robert Lehmann**
ifo Institute – Leibniz Institute for Economic Research at the University of Munich / Germany
lehmann@ifo.de

Ida Wikman
Seminar for Macroeconomics
LMU Munich / Germany
ida.wikman@econ.lmu.de

*corresponding author

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1 Introduction

The sharp decline in economic activity due to the Corona-pandemic led one aspect appear crystal clear: policymaker are in need of quasi real-time information on the economic stance to formulate appropriate policy instruments. Whereas data availability is quite satisfactory at the national level, the regional database is lacking important macroeconomic variables. The assessment of regional economic conditions in a timely manner is therefore not possible.

Macroeconomic aggregates such as gross domestic product (GDP) are released on a quarterly basis at the national level and become available shortly after the respective quarter ends. For most advanced economies, quarterly regional GDP is nonexistent. One exemplary exception is the U.S., where quarterly state-level GDP flash estimates are released approximately three months after the respective quarter ends. For large European economies such as Germany only annual information on state-specific GDP is published. In our paper, we fill this gap and provide quarterly GDP estimates for each of the 16 German states, together with an in-depth business cycle analysis and a discussion on long-run growth dynamics.

In Germany, annual GDP at the state level (NUTS-1 in the Nomenclature of Territorial Units for Statistics) is calculated by a specific working group and released approximately one quarter after the specific year ends. Annual assessments on the regional economic stance can thus only be formulated with a substantial delay and more timely or even real-time assessments are not possible with these data. Koop *et al.* (2020c) developed an econometric framework and published quarterly GDP data for the regions in the United Kingdom since 1970. In their conclusion they state (Koop *et al.*, 2020c, p. 195): "We hope that the methodology we propose will be useful in applications beyond the UK that seeks to improve the regional database." We do so for the German case as it is an economically interesting one. First, the 16 German states are characterized by having quite heterogeneous economic structures. On the one hand, Germany consists of highly industrialized states, mainly in the south, with manufacturing shares of total output exceeding one fourth. On the other hand, strongly service-oriented states exist that, for example, focus on tourism activities or communication technologies. Second, structural change challenges current business models across the states. Regions such as the Ruhr area have to develop new economic ideas as rather old technologies or industrial clusters are no longer supported. On the opposite, large and economic prosperous regions exist that host headquarters of large German firms. Third, the demographic composition of the population is far from being equal and varies noticeably across the states. This leads to large productivity disparities within Germany that will get even more pronounced in the second half of the 2020s.

Annual data are not well-suited for a number of macroeconomic questions. With our paper, we enrich the regional database in Germany and contribute to the literature on both regional business cycles and long-run growth dynamics. We provide quarterly real GDP growth estimates for each of the 16 German states for the period from 1995 to 2021 based on the methodology by Koop *et al.* (2020c). They formulated a mixed-frequency Vector Autoregressive model with stochastic volatility in the error term (MF-VAR-SV) for the UK. In a state space representation, unobserved quarterly regional GDP growth is linked to official annual information together with both macroeconomic and regional indicators. As macroeconomic indicators we add—next to quarterly German GDP growth—consumer price inflation, the bank rate, the exchange rate, and the oil price. To date, no comprehensive and long time series on regional indicators such as industrial production are available in Germany. Therefore, we use the regional business survey results of the German ifo Institute as an alternative source. The most important feature in the specification of the MF-VAR-SV is its ability to ensure that the quarterly state-specific GDP estimates fulfill two essential criteria. First, the intra-annual estimates have to add up to the official annual values for regional GDP growth (temporal constraint). Second, the sum of quarterly state GDP has to be consistent with the quarterly value for Germany which is published by the Federal Statistical Office (cross-sectional restriction). Both conditions ensure maximal consistency to official statistics. The MF-VAR-SV is estimated with Bayesian Markov Chain Monte Carlo (MCMC) algorithms. On a regular basis—namely the publication of quarterly German GDP—the regional data will be updated and made available online to the general public.

Our paper adds to the existing literature on the provision or creation of regional data. Cuevas *et al.* (2015) introduce a time series approach together with national accounting standards to estimate quarterly GDP for the Spanish regions. Their methodology ensures that temporal and cross-sectional constraints are met, thereby taking into account the issues that arise from chain-linking. The previously mentioned UK applications by Koop *et al.* (2020b,c) are in the same vein. Baumeister *et al.* (2022) go one step further in terms of frequency. Based on rather unconventional data such as electricity consumption, they develop a weekly indicator to track U.S. state-level economic activity. Their chosen methodology is a dynamic factor model with mixed-frequencies to bring together weekly, monthly, and quarterly observations. Bokun *et al.* (2023) instead compiled a data set with real-time indicators for the U.S. states and use this information for regional and national forecasting experiments. As the data situation at the German state level is definitely expandable, we provide one piece of that puzzle—namely quarterly real GDP growth—to stimulate further research activities with a particular macroeconomic focus.

The distinctive differences in state-level economic characteristics call for both a business cycle diagnosis and an investigation of long-run growth dynamics. Business cycle synchronization is intensively discussed from a rather global perspective such as countries' convergence of economic fluctuations being part of a monetary union (e.g., the EMU). The literature for sub-national or regional economies and their business cycle dynamics is rather scarce (for a literature overview, see Bandrés *et al.*, 2017). Artis *et al.* (2011) argue that trade can shape the synchronization of regional business cycle dynamics. A higher trade intensity might lead to larger co-movements across cycles due to higher degrees of economic integration. More trade openness, on the opposite, might lead to higher specialization of a region and thus less synchronization due to asymmetric effects of economic shocks. Furthermore, not only trade relations lead to heterogeneity in regional business cycle dynamics but also migration flows or regional interconnections (e.g., input-output relations). Most of the existing studies rely on annual information only that are likely to suppress intra-annual business cycle dynamics. Two prominent studies for the U.S., however, build on data with higher frequencies. Owyang *et al.* (2005) apply the monthly coincident index of Crone and Clayton-Matthews (2005) and find large differences in business cycle concordance across U.S. states. These differences seem to be driven by the cyclical timing, with Hamilton and Owyang (2012) documenting time-shifted regional business cycle profiles. For Germany, Gießler *et al.* (2021) find that the business cycles of Eastern and Western Germany have synchronized over time. Ferreira-Lopes and Sequeira (2011) document a stronger synchronization either across Western or Eastern German states. Whereas the first study only relies on quarterly data for regional aggregates, the latter one is based on annual information only. With our data we can document business cycle dynamics for all 16 German states. Our analysis reveals large heterogeneities in state-specific duration and amplitudes of up- and downswing phases for the period 1995 to 2021. Downswing phases range, on average, from 2.6 to 7.4 quarters with decreases in economic activity spanning from -8.5% to -3.4%. Upswing phases last, on average, longer with a span from 7.2 to 21.0 quarters and an increase in GDP running from 5.6% to 14.1%. The degree of concordance with the German business cycle strongly varies across the states hinting to the fact that rather regional factors shape local business cycle dynamics. On the contrary, regional economic fluctuations might have valuable information for German-wide developments, as it was shown by Beraja *et al.* (2019) for the U.S. case.

Long-run growth dynamics or trend estimations are closely connected to business cycle questions. González-Astudillo (2019), for example, applies an unobserved components model to estimate the U.S. output gap and exploits the variation in state-level output growth. To be more precise, he assumes that the U.S. states are characterized by both common and idiosyncratic trends and cycles. These cross-sectional information are highly valuable for the output gap estimation. In our paper, we approximate long-run growth dynamics for the 16 German states by the approach of Müller *et al.* (2022). We attribute all fluctuations larger than 10 years to long-run dynamics. Our results suggest that economically strong states still exhibit positive GDP trends, whereas weaker regions' trends are either flat or already negative. This finding is quite alarming as the demographic conditions in Germany are starting to change and will worsen quite tremendously in the near future.

Finally, our data offers paths for regional forecasting analyses. The academic literature on regional now- and forecasting has developed fast in recent years and we observe a growing interest in this field by the public and academic community.[1] Newer articles either exploit a factor model structure on the data (Chernis *et al.*, 2020; Gil *et al.*, 2019) or apply Vector

[1] Lehmann and Wohlrabe (2014) provide an early survey on the articles published until the mid 2010s.

Autoregressions with mixed-frequencies (Koop *et al.*, 2020a,c). The regional forecasting literature for Germany has also developed in the last decade. Earlier articles either rely on panel data models for annual information due to missing quarterly observations (Kholodilin *et al.*, 2008) or on simple time series and indicator approaches applied to semi-official quarterly estimates for a small subset of German states (Henzel *et al.*, 2015; Lehmann and Wohlrabe, 2015). Newer articles apply more sophisticated approaches such as boosting (Lehmann and Wohlrabe, 2017), use mixed-frequency approaches such as mixed-data sampling (Claudio *et al.*, 2020) and compare the latter to dynamic factor models (Kuck and Schweikert, 2021). All these articles, however, have in common that they either focus on one single state, state aggregates (for example, Eastern Germany) or on a rather small subset of regional entities. Our estimates make it possible to study the performance for all 16 states simultaneously and to explore the role of inter-regional variation as a new source for economic forecasting.

The paper is organized as follows. Section 2 introduces the publication scheme for German GDP together with a timeline of national and regional accounts. In Section 3 we describe the methodology and the applied data. Our results are presented in Section 4 along with the business cycle diagnosis and the analysis of long-run growth dynamics. This section also contains a multitude of robustness as well as stability checks. Section 5 concludes.

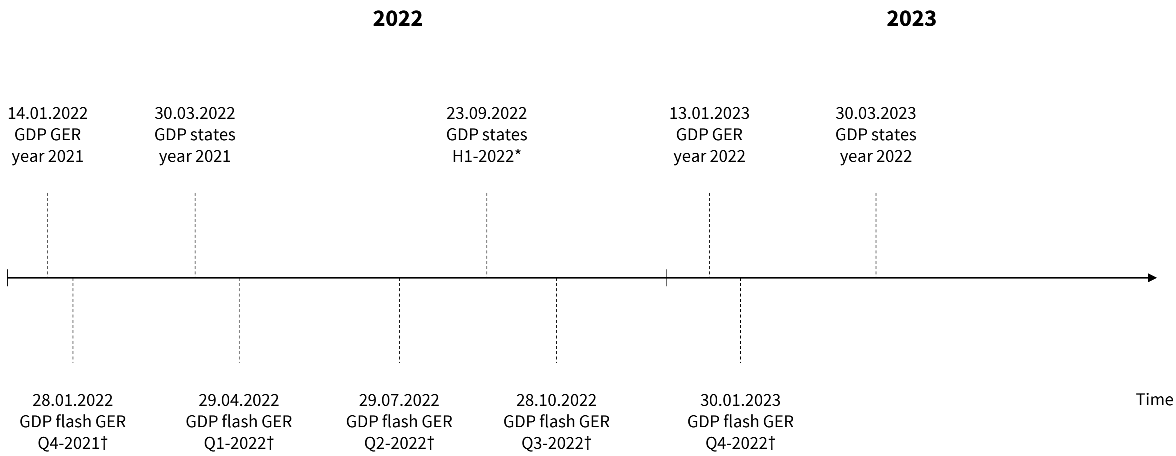
2 Publication Scheme of National Accounts in Germany

In Germany, national accounts data on a regional level are provided by the Working Group Regional Accounts (`https://www.statistikportal.de/de/vgrdl`). This Working Group consists of the federal states' Statistical Offices, the Federal Statistical Office, and the Association of German Cities; the Statistical Office of Baden-Wuerttemberg is responsible for leading the Working Group. Our application focuses on the 16 German federal states which correspond with the official NUTS-1-level to classify homogeneous economic units in Europe (see the Supplementary Material for an overview of the German regional classification).

The regional accounts data essentially include the complete production, expenditure and income approach of state GDP together with selected aggregates at the district-level. One single component is coordinated and calculated by one German state for each regional entity. Consistency with the German values is then achieved by fixing the top aggregate. This means that, for example, German GDP is not calculated as the sum of all state values. State GDPs are, on the contrary, calculated by breaking down the German value. This is either done by a bottom-up approach or by a top-down method. The first approach is characterized by a proportional allocation to the state aggregates of the delta between the state sum of GDP and the fixed German benchmark. The second method is characterized by an application of regional key indicators to break down German GDP to the regional unities. All data calculations are based on the current European System on National Accounts.

Figure 1 shows the publication timeline of national and regional accounts in Germany for the year 2022 and the first quarter of 2023. In comparison to the release schedule for German GDP figures, the publication of state-level GDP has two drawbacks. First, the regional data are only available on an annual basis, while Germany-wide GDP is released every quarter. Specifically, there is a flash estimate published 30 days after the end of the relevant quarter.[2] The quarterly publication cycle for German GDP is crucial for our application and defines the date at which new quarterly estimates for state-level GDP will be calculated. We elaborate more on this timing issue in the following sections. Second, the annual state-level values are only available with a significant delay. In comparison to the annual German aggregate, for which the Federal Statistical Office publishes a first estimate already in mid-January, the values for the past year for the states are only available at the end of March. Both the previously described coordination process across regional layers and a longer publication delay of regional key statistics seem to be the main contributors for these discrepancies. It can be argued that this sparsity and delay of data prevents users from formulating timely statements on the states' current economic stance.

Figure 1: Publication Timeline of National and Regional Accounts in Germany



Notes: Germany (GER) is officially classified as NUTS-0. The NUTS-1-level is represented by the 16 German states. Quarters and half-years are abbreviated by Q and H, respectively. State-level GDP values for the first half of the year are not revised afterwards (∗) and are not comparable with official annual values. New state-level estimates will be calculated at the release date for quarterly German GDP (†). *Sources:* Federal Statistical Office, Working Group Regional Accounts.

Currently, only a handful of state statistical offices publish quarterly GDP figures with which we can compare our estimates. For example, Baden-Wuerttemberg and Rhineland-Palatinate regularly update quarterly GDP figures on their homepages. Additionally, the Halle Institute for Economic Research publishes a non-official quarterly GDP series for Eastern Germany and the ifo Institute Munich calculated quarterly GDP estimates for Saxony. However, these examples have in common that they either base their estimates on univariate

[2]It has to be noted that the Working Group Regional Accounts publishes GDP growth rates for the first half of a specific year at the end of September. However, these values are not revised afterwards and are therefore not comparable with upcoming publications of annual values.

approaches or publish a quarterly series for one single state. Our approach, on the contrary, has a multivariate structure and produces consistent estimates for all 16 German states simultaneously. The following section outlines this multivariate approach, together with a description of the data.

3 Methodology

3.1 A Model with Mixed-Frequencies

As was outlined in Section 2, there is a diverse mix of data frequencies and publication schemes. This setting calls for an empirical model that can handle these features of the data. A very popular approach is the Vector Autoregressive model with mixed-frequencies (MF-VAR). We follow the article by Koop *et al.* (2020c) that introduces a MF-VAR to estimate or interpolate GDP for the various regions of the United Kingdom. The main idea of the model is to link low frequency variables to observables measured at a higher frequency, given that there is an existing relationship between the two groups. In the vein of Mariano and Murasawa (2010) and Schorfheide and Song (2015), the model is set out in state space form. The state equations are given by a standard VAR at the quarterly frequency and the measurement equations ensure that the accounting rules are met. Put differently, for each quarter the estimated state-level GDPs need to sum up to the German value and the four values within a year have to be consistent with the observed annual values. Finally, the Kalman Filter is applied to fill in missing values.

State space form. We strictly follow Koop *et al.* (2020c) and apply their empirical setup. The vector of unobserved quarterly GDP growth for all $S = 16$ German states, $y_t^Q = \left(y_t^{1'} \dots y_t^{16'}\right)$, together with quarterly German GDP growth, y_t^{GER}, is modeled by a VAR. The model is further augmented by additional exogenous predictors.[3] GDP growth is measured in log-differences. The total vector of observed German and unobserved state-level GDP, $y_t = \left(y_t^{GER}, y_t^Q\right)'$, with a dimension of $n = S + 1$ is assumed to evolve as:

$$y_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i y_{t-i} + u_t\,, \quad u_t \overset{iid}{\sim} N(0, \Sigma_t)\,. \tag{1}$$

This state equation assumes intertemporal connectedness between state-level GDP and implies that quarterly German GDP growth has valuable information for the economic development of each German state and vice versa. u_t denotes the Gaussian error term with the variance-covariance-matrix Σ_t, on which we elaborate at the end of this section. Similar to Koop *et al.* (2020c), we choose a lag length of $p = 7$ quarters.

[3]Following Koop *et al.* (2020c), we also add several German and state-level predictors to the model. For a better readability, we skip the exogenous variables from the notation and only show the relationships across GDP figures.

Next to the state equation (1), we need to impose three further restrictions on the system—so called measurement equations—that have to be met when estimating the unobserved vector y_t^Q. To begin with, we need a temporal constraint that links the observed annual values of state-level GDP to the unobserved quarterly values. Similarly, the second constraint matches the quarterly and annual growth values for Germany. Lastly, we have to ensure that the (weighted) average of quarterly state-level GDP growth meets the German benchmark.

According to Mariano and Murasawa (2003, 2010), Mitchell *et al.* (2005) and Schorfheide and Song (2015) the annual growth rate of state-level GDP that is observed only in the fourth quarter of each year, $y_t^{s,A}$, can be expressed as a weighted sum of the contemporaneous and lagged values of the unobserved quarterly growth rates y_t^s:

$$y_t^{s,A} = \frac{1}{4}y_t^s + \frac{1}{2}y_{t-1}^s + \frac{3}{4}y_{t-2}^s + y_{t-3}^s + \frac{3}{4}y_{t-4}^s + \frac{1}{2}y_{t-5}^s + \frac{1}{4}y_{t-6}^s \,. \tag{2}$$

Evidently, the first two quarters in a given year as well as the last quarter of the previous year get the highest weight for annual growth. This intertemporal restriction is also consistent with the chosen lag length. Given this linear relationship, we define the first measurement equation for the German states in style of Koop *et al.* (2020c):

$$y_t^A = M_t^A \Lambda^A z_t \,, \tag{3}$$

with $y_t^A = \left(y_t^{1,A'} \dots y_t^{16,A'}\right)'$ and $z_t = \left(y_t' \dots y_{t-6}'\right)'$. The matrix Λ^A contains the weights introduced in equation (2). With the matrix M_t^A we can control state-level observables and unobservables. As $y_t^{s,A}$ is only available in the fourth quarter of each year, $M_t^A = 1$ if $t = 4$ and $M_t^A = 0$ otherwise. As emphasized by Koop *et al.* (2020c), M_t^A has an important role for real-time now- and forecasting purposes and the treatment of missing observations at the end of the data set.

The second measurement equation handles the German data structure. As we observe German GDP growth each quarter, the matrices are much simpler. The link between annual and quarterly growth for Germany is expressed by:

$$y_t^{GER} = M_t^{GER} \Lambda^{GER} y_t \,, \tag{4}$$

with Λ^{GER} only grabbing the quarterly German values from y_t. M_t^{GER} is constructed as M_t^A, with $M_t^{GER} = 1$ if the value is observed or $M_t^{GER} = 0$ if publication delays exist.

Both measurement equations (3) and (4) impose the temporal nature of the data. In addition, the consistency between national and regional accounts has to be achieved. Put differently, weighted state-level GDP growth has to equal the German value. Therefore, we add a cross-sectional restriction as a third measurement equation:

$$y_t^{GER} = \frac{1}{S}\sum_{s=1}^{S} y_t^s + \eta_t \,, \quad \eta_t \sim N(0, \sigma_{cs}^2) \,. \tag{5}$$

As we apply log-differenced data, Koop *et al.* (2020c) show that this first order approximation holds and German GDP growth, y_t^{GER}, can be expressed as a simple average of the state-level growth rates, y_t^s. However, this relationship is not perfect. The error term η_t is added to capture this imperfection in a stochastic sense. This stochastic nature also covers an accounting feature of the system. Since 2005, price-adjustment in Germany is based on previous year prices instead of fixed prices from a given year. Due to the chain-linking nature of the data, the sum of price-adjusted volumes does not equal the values of higher aggregates such as GDP (see, for example, IMF, 2018). This issue is called additive inconsistency. Therefore, the sum of price-adjusted state-level GDP does not equal price-adjusted German GDP. The error η_t grabs both inconsistencies.

Stochastic volatility. The last step is to set a definition on how the variance-covariance-matrix of the VAR, Σ_t, looks like. The recent literature on the dynamics of the German business cycle is strongly in favor of changes in the volatility and thus allowing for a heteroscedastic error structure (Reif, 2022). We follow Koop *et al.* (2020c) and apply the stochastic volatility specification of Cogley and Sargent (2005) and Carriero *et al.* (2016):

$$\Sigma_t^{-1} = \mathbf{L}'\mathbf{D}_t^{-1}\mathbf{L}\,, \quad \text{with} \quad \mathbf{L} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{1,1} & 1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ a_{n,1} & \cdots & a_{n,n-1} & 1 \end{bmatrix}_{n\times n} \tag{6}$$

and $\mathbf{D}_t = \text{diag}[\exp(h_{1,t})\ldots\exp(h_{n,t})]^{-1}$. It grabs the log-volatilities $\mathbf{h}_t = (h_{1,t}\ldots h_{n,t})'$ that follow a Random Walk specification:

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \nu_t\,, \quad \nu_t \sim N(0, \Sigma_h)\,, \tag{7}$$

with $\Sigma_h = \text{diag}(\omega_{h_1}^2 \ldots \omega_{h_n}^2)$. The complete model specification is labeled as MF-VAR-SV. In the following, we discuss the priors set and how the posterior is simulated.

3.2 Prior Setting and Posterior Simulation

The MF-VAR-SV is clearly over-parameterized. Even without exogenous variables, the number of endogenous variables is $n = S + 1 = 17$, meaning that 16 latent variables have to be estimated based on a few annual observations only. On top, we need estimates for the volatilities. We achieve the attenuation of the parameter problem by efficiently shrinking the priors to zero and follow Koop *et al.* (2020c). They apply the Dirichlet-Laplace hierarchical prior that induces a theoretical-optimal shrinkage (Bhattacharya *et al.*, 2015).

VAR parameter. Our VAR from equation (1) can be expressed as a multivariate regression problem with the k-dimensional coefficient vector $\beta = \text{vec}([\Phi_0\Phi_1 \dots \Phi_p]')$ to be estimated. With $\beta = (\beta_1 \dots \beta_k)'$, the prior for each coefficient is (Bhattacharya *et al.*, 2015):

$$\beta_j \sim N(0, \psi_j^\beta \vartheta_{j,\beta}^2 \tau_\beta^2)\,, \tag{8}$$

$$\psi_j^\beta \sim \text{Exp}\left(\frac{1}{2}\right)\,, \tag{9}$$

$$\vartheta_{j,\beta} \sim \text{Dir}\left(\alpha_\beta, \dots, \alpha_\beta\right)\,, \tag{10}$$

$$\tau_\beta \sim \text{G}\left(k\alpha_\beta, \frac{1}{2}\right)\,. \tag{11}$$

The unknown variance parameters of the coefficients have to be estimated and are automatically chosen by the algorithm. Thus, the algorithm decides how much shrinkage on the parameters is allowed. If the variance is close to zero, it is likely that the coefficient β_j is set to zero. The Dirichlet-Laplace prior is a global-local prior with only one hyperparamter α_β. One part of the coefficient variance is global (τ_β), meaning that this term applies similarly to all coefficients. Another part is local (ψ_j^β), meaning that it applies individually to each coefficient β_j. The last term, $\vartheta_{j,\beta}$, leads the Dirichlet-Laplace prior to produce a posterior that optimally contracts to its true value (Bhattacharya *et al.*, 2015).

Stochastic volatility. For the parameters that control the error covariances in the $\mathbf{L}$ matrix, $\mathbf{a} = (a_{1,1} \dots a_{n,n-1})'$, we also apply a Dirichlet-Laplace prior in a similar fashion to the VAR coefficients. The terms are $\psi_i^a, \vartheta_{i,a}$ and τ_a, with one hyperparameter α_a. For the ω_{h_j}, we assume: $\omega_{h_j}^2 \sim \text{IG}(\nu_{h_j}, S_{h_j})$.

Cross-section restriction. Due to the approximate nature of equation (5) and the accounting standards, the average of state-level GDP growth does not necessarily equal German GDP growth. For this cross-sectional error we assume: $\eta_t \sim N(0, \sigma_{cs}^2)$. The variance term is modeled in such a way that the prior mean is close to zero, using the following tight prior: $\sigma_{cs}^2 \sim \text{IG}(1000, 0.001)$.

Hyperparameter choices. Like in Koop *et al.* (2020c), we set the hyperparameter to the following. For the Dirichlet-Laplace prior, we choose similar hyperparameter for both the coefficients and the stochastic volatility: $\alpha_\beta = \alpha_a = 0.5$. To draw the initial conditions of the stochastic volatilities, $\mathbf{h}_0$, we follow Chan and Eisenstat (2018) and set: $\mathbf{a}_h = \mathbf{0}$, $\mathbf{V}_h = 10$, $\nu_i = \nu_{h_j} = 5$, and $S_i = S_{h_j} = 0.01$.

Start values and algorithm. As the starting values for the Dirichlet-Laplace prior we set: $\psi_j^\beta = \vartheta_{j,\beta} = \tau_\beta = \psi_i^a = \vartheta_{i,a} = \tau_a = 0.1$. For the cross-section restriction, we initialize the error with: $\eta_0 = 0.0001$. Our MCMC algorithm is similar to Koop *et al.* (2020c) with a total of 20,000 draws, whereby the first 10,000 draws are discarded.

3.3 National and State-Level Data

Gross domestic product. We have to rely on the latest vintage of regional accounts data (March 2022) as currently no real-time database is available. To ensure consistency, we use the comparable quarterly data from German national accounts. Quarterly price-, seasonal- and calendar-adjusted German GDP is calculated by the Federal Statistical Office for the period 1991 to 2021. For the same period, the Working Group Regional Accounts publishes annual chain-linked real GDP figures at the state-level. These figures are consistent with currently valid national accounting standards, coordinated on the annual German values and in delimitation of the 16 German states after reunification. As we focus on log-differences, our data set comprises the years 1992 to 2021, starting with the first quarter of 1992 (t_0).

Macroeconomic and state-level indicators. It seems reasonable to augment the MF-VAR-SV by additional macroeconomic and state-level variables that might explain quarterly GDP growth. In line with Koop *et al.* (2020c); Reif (2022); Schorfheide and Song (2015) we select the following four macroeconomic variables for Germany: the seasonally-adjusted consumer price index, the bank rate, the exchange rate, and the oil price (see the Supplementary Material for more details on the indicators). With the exception of the bank rate that enters the model in quarterly first differences, all other macroeconomic series are transformed into quarterly log-differences.

We tried to follow Cuevas *et al.* (2015) and wanted to add a number of important state-level indicators to our model. However, consistent and long time series for all 16 German states are not available from official sources. Either changes in statistical standards prevent the timely comparability of economic indicators (e.g., new industrial classifications), the time series is too short for our purposes (e.g., total employment) or economic time series are not publicly available for all states (e.g., industrial production). The only exception is the number of unemployed people that is available from the Federal Employment Agency on a monthly basis starting in December 1991. For our purpose, the seasonally-adjusted unemployment figures also enter the model in log-differences after the monthly values were averaged to meet the quarterly frequency.

Unemployment as one single indicator at the state-level alone might not be sufficient enough due to several other influences. Next to business cycle fluctuations, the number of unemployed people is also driven by large labor market reforms (for an evaluation of the Hartz-reforms, see Hochmuth *et al.*, 2021), policy instruments such as short-time work (Balleer *et al.*, 2016), or a shrinking labor force because of demographic changes. Therefore, we want to add indicators that closer track aggregate economic fluctuations. We do so by relying on qualitative survey information that are found to track and forecast economic activity quite well (see, for example, Angelini *et al.*, 2011; Basselier *et al.*, 2018). In Germany, the ifo Institute Munich is the largest survey provider with the ifo Business Climate Germany as its most famous survey-based indicator. Next to the forecasting power of the ifo Business

Climate Germany (Lehmann and Reif, 2021), the survey has proved to have high forecasting power in several dimensions (see, for a recent literature survey, Lehmann, 2022). So it does for the German states, for which the ifo Institute provides a large subset of its main indicators. We apply the ifo Business Climate Industry and Trade for each of the German states or state aggregates which are available on a monthly basis since January 1991.[4] The seasonally-adjusted survey indicators enter the model in quarterly first differences after they have been averaged to meet the quarterly frequency.

The state-level indicators enter the model equation-wise as exogenous regressors, thus, the indicator for a state s only explains movements in state-specific GDP growth, y_t^s. Taking all variables into account, we estimate a 21 dimensional MF-VAR-SV, where two exogenous indicators additionally explain economic activity for each state. As previously stated, we specify the VAR to have a lag length of $p = 7$.

4 Quarterly State-Level GDP Estimates and Applications

In this section, we present the time series of GDP estimates for all 16 German states from 1995 to 2021 together with a comparison to the official data for Germany. Based on these estimates we analyze the business cycle characteristics across the German states and discuss long-run growth dynamics. The section closes with some discussions on the robustness of the quarterly estimates.

4.1 Estimates from 1995 to 2021

Figure 2 shows the quarterly annualized GDP growth rates for all 16 German states (thick black lines) together with the rate for Germany (thin blue lines). The series run from the second quarter 1995 to the fourth quarter of 2021. The plot demonstrates a large heterogeneity in growth rate patterns across states and in comparison to Germany. The annualized growth rates for Germany seem to be mainly driven by the most populous and economically relevant—in terms of share in German GDP—states: Baden-Wuerttemberg, Bavaria, and North Rhine-Westphalia. These three states combined represent more than the half of German GDP in 2021 according to the figures from the Working Group Regional Accounts. The correlation coefficients in Table 1 underpin this observation (Baden-Wuerttemberg: 0.97, Bavaria: 0.94, North Rhine-Westphalia: 0.95). The state Hesse also shows a large homogeneity with the development of German economic activity; the annualized growth rates

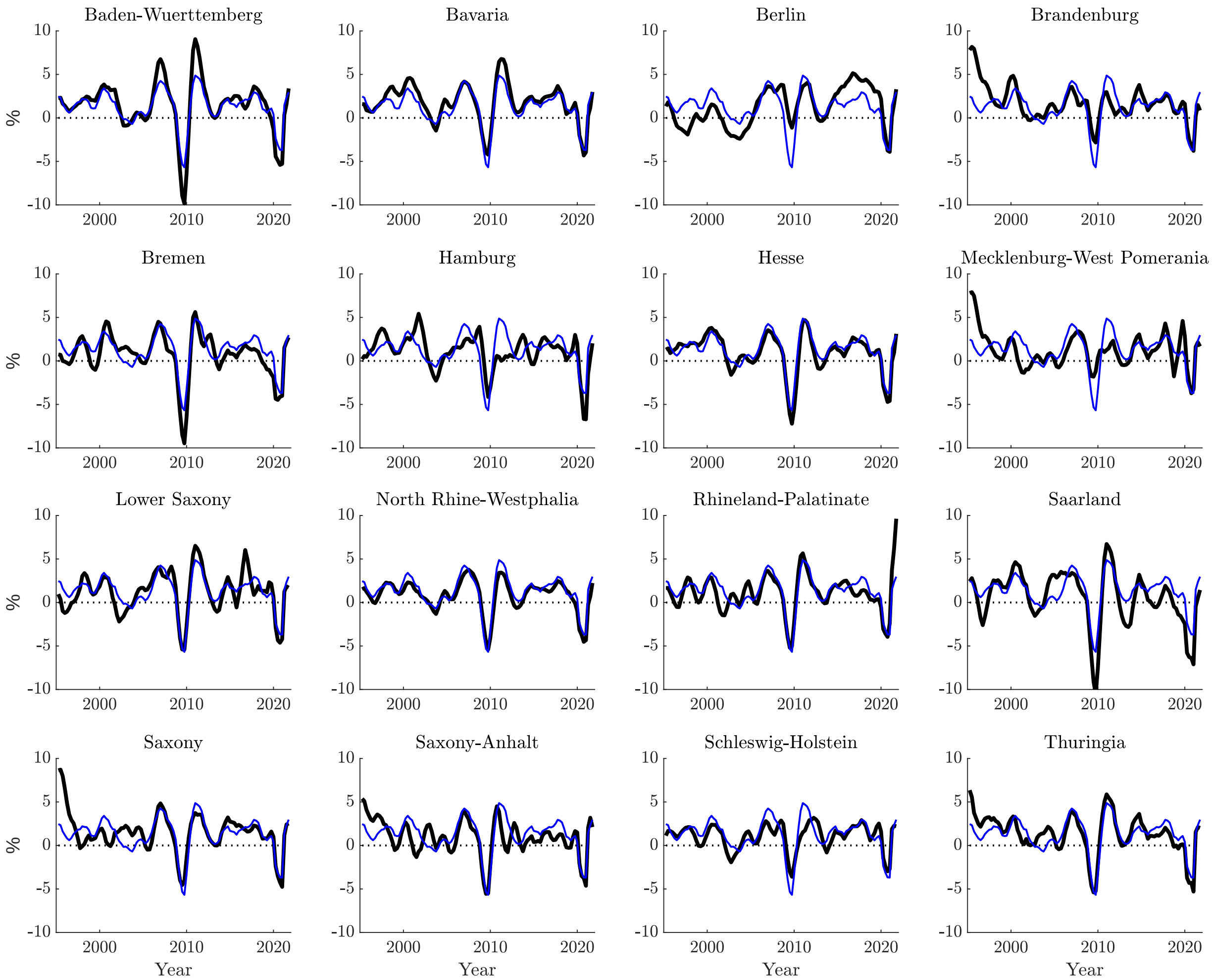
[4]Industry and Trade is the aggregation of manufacturing, construction, retail sales, and wholesale trade. Unfortunately, the survey indicators for the service sector are only available since January 2005 and thus not suited for our purposes. Due to representation issues, business climates are not available for all 16 German states separately. However, the ifo Institute provides state aggregates. These aggregates have been re-weighted by state-specific gross value added information, assuming that the industrial climates behave in the same way for all states representing a specific aggregate. In the end we come up with 16 regional ifo Business Climates Industry and Trade.

correlate by 0.95. The lowest correlation and thus the largest heterogeneity in economic growth compared to Germany is observed for Mecklenburg-West Pomerania (0.52) and the two city-states Berlin (0.58) and Hamburg (0.60). Common for these three states is the size and importance of the service sector. Mecklenburg-West Pomerania is mainly characterized by a large amount of touristic activity due to its location at the Baltic Sea and Berlin and Hamburg are the two states with the highest share of service activities in its total gross value added according to the working group's annual figures. The service sector of Berlin predominantly consists of large parts of the federal government (e.g., ministries), headquarters of large firms, a strong information and communication industry (such as international publisher), and the main representations of interest groups and political parties. Hamburg instead has a large share of transportation and logistic activities, which is not surprising as the largest German seaport is located there. On top of that, Hamburg also has a large information and communication industry with, for example, the production of Germany's most important newscast: the *Tagesschau*. A second commonality of these states is the lack of importance of manufacturing. In fact, the lowest shares of manufacturing in overall economic activity can be observed for these three states.

In terms of the largest variation in annualized growth, Baden-Wuerttemberg, Bremen and Saarland dominate the picture. Their respective standard deviations are 3.0%, 2.5% and 3.1% (Germany: 1.9%). The lowest variation in annualized growth can be observed for Schleswig-Holstein (1.5%). A large increase in economic activity in 2021 is observed for Rhineland-Palatinate. This is not an artefact of the data but attributed to the development of the COVID-19-vaccine by Biontech, whose headquarters is in the state capital Mainz.

A special focus should be put on the Eastern German states (Brandenburg, Mecklenburg-West Pomerania, Saxony, Saxony-Anhalt, and Thuringia). After reunification, Eastern Germany faced a fast and strong catch-up process to the Western German states until 1995 (Ragnitz, 2019). The large annualized growth rates at the beginning of the sample seen in Figure 2 are an indication of this progression. Since 1996, convergence in terms of GDP per capita more or less stopped (1996: 60.5% of Western German GDP per capita; 2021: 70.5% of the Western German value). The large catch-up at the beginning of the 1990s is the main reason why the correlation coefficients—except for Thuringia—are smaller compared to Western German states. If we look at the data after 2000, the correlations increase but are still smaller in comparison to Western Germany. This might be an expression that nowadays Eastern German Business Cycles are more synchronized to Western German ones, as argued by Gießler *et al.* (2021). We further investigate this point in the following section.

Figure 2: Annualized Growth Rates for all German States Compared to Germany



Notes: The thick black lines show the annualized quarterly growth rates for each state. The thin blue lines represent the values for Germany. The period ranges from the second quarter of 1995 to the fourth quarter of 2021. The large increase in economic activity in 2021 for Rhineland-Palatinate is attributed to the development of the COVID-19-vaccine by Biontech, whose headquarters is located in the state capital Mainz.

Table 1: Correlation of the State-specific and the German Annualized Rates

State	**Corr**	**State**	**Corr**
Baden-Wuerttemberg	0.97	Lower Saxony	0.87
Bavaria	0.94	North Rhine-Westphalia	0.95
Berlin	0.58	Rhineland-Palatinate	0.82
Brandenburg	0.62	Saarland	0.88
Bremen	0.88	Saxony	0.70
Hamburg	0.60	Saxony-Anhalt	0.72
Hesse	0.95	Schleswig-Holstein	0.81
Mecklenburg-West Pomerania	0.52	Thuringia	0.86

Notes: The correlations are calculated for the total sample ranging from 1995 to 2021.

4.2 Business Cycle Characteristics

The large heterogeneity in the variation of annualized growth rates calls for a deeper investigation of state-specific business cycles. We implement the well-known and accepted algorithm for monthly data by Bry and Boschan (1971), which has been extended to quarterly data by Harding and Pagan (2002). We choose this non-parametric dating algorithm as it is simple as well as easy to replicate for readers due to its high transparency. Harding and Pagan (2003) also show that non-parametric approaches are very robust to, for example, adding new observations, more so than parametric ones.

The Bry-Boschan-algorithm (henceforth: BBQ-algorithm for quarterly data) dates classical business cycles in economic activity. This type of cycle is detected in the level of the series, thus, a business cycle is defined as the movement around an unknown trend. By following Bry and Boschan (1971) and Harding and Pagan (2002), we apply the BBQ-algorithm to the quarterly levels of our estimated series. The levels are achieved by setting the first quarter of 1995 to 100 and multiplying this start value with our quarterly estimates.

In its standard version, the BBQ-algorithm divides the business cycle into two phases—upswing and downswing—that follow each other based on predefined criteria. Upswings (downswings) are characterized by time periods with increasing (decreasing) economic activity. Both phases are connected by peaks and troughs, whereas the peak (trough) is the point in time where an upswing (downswing) ends. A complete cycle is the time period in which each phase has been passed once. In practice, the BBQ-algorithm identifies the peaks and troughs in the time series, allowing for dating the complete cycle.

According to Harding and Pagan (2002), a dating algorithm has to fulfill three requirements. First, the approach needs to identify at least a minimum number of peaks and troughs. Second, peaks and troughs have to differ from each other and should vary over time. Third, the identified phases must satisfy some minimum requirements for a cycle. A peak P_t (trough T_t) at quarter t occurs if the level of economic activity, Y_t, is lower (higher) in k periods before and after this point in time:

$$
\begin{aligned}
P_t &= (Y_{t-k}, \ldots, Y_{t-1}) < Y_t > (Y_{t+1}, \ldots, Y_{t+k}) \ , \\
T_t &= (Y_{t-k}, \ldots, Y_{t-1}) > Y_t < (Y_{t+1}, \ldots, Y_{t+k}) \ .
\end{aligned}
$$

For our state-specific business cycle dating we apply standard values from the literature on the U.S. and on Germany (Harding and Pagan, 2002, 2003; Schirwitz, 2009) as no specific discussion on regional economic activity exists. The time span that defines peaks and troughs is set to $k = 2$ quarter. Additionally, upswings and downswings have to last at least two quarters and a complete cycle is comprised of at least five consecutive quarter.

Table 2 summarizes the average duration and amplitude of the state-specific up- and downswing phases. As suggested, the Saarland exhibits the longest average duration in downswing phases with 7.4 quarter. The shortest downswings reveals Brandenburg (2.6 quarter). Such a heterogeneity can also be found for the upswing phases. The longest average upswing phases are found for Bavaria (21.0 quarter), Brandenburg (16.8 quarter), and Baden-Wuerttemberg (14.8 quarter). These phases are almost three times higher compared to Saxony-Anhalt, for which an upswing only lasts 7.2 quarter on average. The states Hamburg, Mecklenburg-West Pomerania, and Rhineland-Palatinate immediately follow with rather short average upswing phases of 10 quarter in duration.

Table 2: State-specific Business Cycle Characteristics

State	**Duration (# quarters)**		**Amplitude (in %)**		$\mathbf{CI_{s,GER}}$
	Down	**Up**	**Down**	**Up**	
Baden-Wuerttemberg	4.6	14.8	-5.9	11.3	90.7
Bavaria	3.3	21.0	-4.4	14.1	94.4
Berlin	5.0	13.2	-4.5	9.9	73.8
Brandenburg	2.6	16.8	-3.6	8.0	85.0
Bremen	5.4	12.3	-6.1	8.5	79.4
Hamburg	3.0	10.0	-3.9	7.0	77.6
Hesse	5.8	12.0	-4.8	6.7	79.4
Mecklenburg-West Pomerania	5.2	9.7	-4.0	7.0	79.4
Lower Saxony	5.2	11.5	-5.4	10.0	74.8
North Rhine-Westphalia	4.2	11.0	-3.4	5.6	85.0
Rhineland-Palatinate	3.9	10.0	-3.5	6.0	83.2
Saarland	7.4	12.0	-8.5	9.2	72.0
Saxony	2.8	14.4	-4.5	8.6	88.8
Saxony-Anhalt	2.9	7.2	-4.2	5.7	76.6
Schleswig-Holstein	3.8	10.9	-3.5	6.7	85.0
Thuringia	4.2	10.6	-5.3	9.0	81.3

Notes: An upswing (Up) is the time period between one trough and the following peak. The opposite holds true for a downswing (Down). The duration measures the average number of quarters that up- and downswings last. The amplitude measures the average percentage change in GDP in up- or downswing phases. The last column shows the concordance index (CI) to Germany.

The state-specific business cycles also significantly differ in their amplitudes, which are defined as the percentage change in the levels between a peak and a trough. The Saarland, Bremen and Baden-Wuerttemberg show the deepest downswings with an average change of -8.5%, -6.1%, and -5.9%, respectively. For North Rhine-Westphalia, Rhineland-Palatinate and Schleswig-Holstein, the recessions are only half as deep as for the three previous mentioned states (-3.4%, -3.5%, and -3.5%). Contrary, the strongest upswings are found for Bavaria (14.1%), Baden-Wuerttemberg (11.3%), and Lower Saxony (10.0%). Interestingly, the Saarland with the deepest recessions also exhibits relatively large upswings with 9.2%. The smallest upswing phases are experienced by North Rhine-Westphalia (5.6%), Saxony-Anhalt (5.7%), and Hesse (6.7%).

Given the large heterogeneities in state-specific business cycles, the question raises how strong they overlap with the German cycle. We express this with the concordance index (CI) of Harding and Pagan (2002). The CI can be interpreted as a measure for how large the co-movement between two business cycles is. It is defined as the ratio when both economies are in the same business cycle phase compared to the total number of observations:

$$\mathrm{CI}_{s,GER} = \frac{1}{T} \sum_{t=1}^{T} \left[U_{s,t} U_{GER,t} + (1 - U_{s,t})(1 - U_{GER,t}) \right] .$$

If state s is facing an upswing at time t, it applies that $U_{s,t} = 1$. The same holds true for Germany ($U_{GER,t} = 1$). A downswing is therefore assigned a value of zero. For the CI holds: $\mathrm{CI}_{s,GER} \in [0, 1]$. A value of one is observed if both business cycles overlap perfectly and all up- and downswings as well as peaks and troughs are identical between the specific state and Germany. The opposite holds true for a value of zero.

The largest overlap to the German business cylce is observed for Bavaria (94.4%), Baden-Wuerttemberg (90.7%), and Saxony (88.8%). Especially the results for the first two states is not surprising as both contribute to German GDP by 18% and 15%, respectively. The lowest concordance to the German business cycle shows Lower Saxony (74.8%), Berlin (73.8%), and the Saarland (72.0%). Especially Berlin is characterized by a large amount of public service activities for which one might assume that they follow other regularities than standard business cycle fluctuations with different degrees in capacity utilization. In the end, we observe large heterogeneities in business cycle dynamics. Future research might go in the direction of asking which underlying forces lead to these results (e.g., shock resilience).

4.3 Long-Run Growth Dynamics

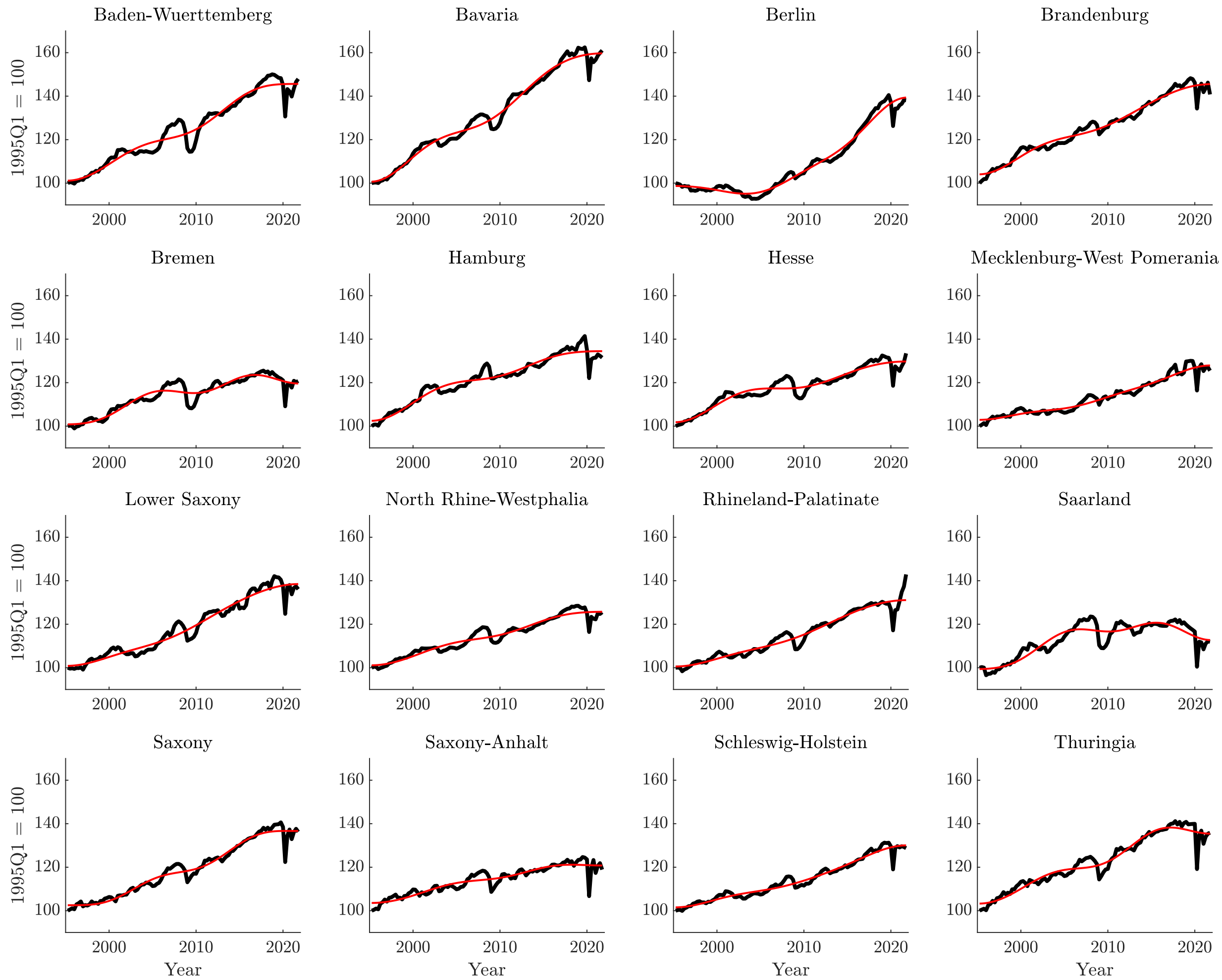
Besides large heterogeneities in business cycle characteristics, also long-run growth dynamics seem to vary tremendously across the German states. The average real GDP growth rates from 1996 to 2021 range between 1.8% for Bavaria and 0.5% for the Saarland. Given our quarterly estimates and the level transformation described in the previous section, we filter the long-run growth trends by methods introduced in Müller and Watson (2019) and applied to questions of economic convergence (Müller *et al.*, 2022) or the implications of changing sectoral trends for overall economic growth (Foerster *et al.*, 2022). The main idea is to fit a number of low-frequency functions that separate rather short-run fluctuations from long-run trends in the observed data. This is achieved by fitting a linear trend, a constant as well as $q - 1$ low-frequency cosine transforms as in Müller *et al.* (2022). We especially filter dynamics with periodicities that are longer than $2T/q$ quarters, with $T = 107$ as the number of observations and $q = 5$ the number of regressors. Given these parameter specifications, we filter long-run growth dynamics that comprise at least $2T/q = 42.8$ quarters or 10.7 years. Every movement in the data shorter than 10.7 years is labeled as short-run fluctuations. This makes the estimated trend less sensitive to end-of-sample values.

Figure 3 reveals the large differences in long-run growth across the German states. Economically strong states such as Baden-Wuerttemberg and Bavaria exhibit quite stable long-run trends, whereas economically weaker states show a flat trend (for example, Saxony-Anhalt) or even negative developments (e.g., Bremen and the Saarland). This finding is quite alarming as demographic (pre-)conditions also vary tremendously across the states. According to the latest coordinated population projection by the Federal Statistical Office, the number of inhabitants aged between 20 and 66 years will sharply decline until 2040. The two city-states Berlin and Hamburg might observe declines of -3.8% and -4.8%, respectively. Baden-Wuerttemberg follows with a projected decline of -10.9%, raising the question whether the local economy can compensate this decline to keep the stable trend growth rate of the last years alive. The picture gets even darker by focusing on the states that rank at the end of the distribution. These are: the Saarland (decline: -22.5%), Thuringia (decline: -23.6%), and Saxony-Anhalt (decline: -25.3%). Given these projections together with the observed flat or even negative trend rates raises the question whether it is possible to generate equal standards of living across the states and how economic policy can act in order to mitigate these sharp differences.

The filtering of the long-run growth dynamics comes with the by-product of deviations of observed economic growth from the estimated trends. These deviations might cautiously be interpreted as state-specific "output gaps".[5] The Supplementary Material shows a figure with these estimated output gaps. According to these estimates, six German states show positive output gaps at the end of 2021, thus, they already passed the preceding Corona shocks. The other ten states were still operating with underutilized capacities. Interestingly, the output gaps at the end of 2021 positively correlate with the states' economic share in manufacturing. Thus, the underutilized states are mainly those with a high share in service activities, which are the industries mainly hitted by the Corona crisis. Furthermore, the output gaps correlate to different degrees with each other across the states. With some exceptions, especially states that share a common border (for example, Bavaria and Baden-Wuerttemberg or Saxony and Thuringia) show correlation coefficients of their output gaps that are close to one. We also hypothesize that strong economic interconnections across states' firms and industries drive these correlations. Linkages across regions might be analyzed with the regional input-output data by Krebs (2020), but we leave such considerations for future research activities.

[5] The estimated output gaps strongly correlate with survey-based, demeaned capacity utilization measures introduced by Lehmann *et al.* (2022) for the German states. For the period from the second quarter 2011 to the fourth quarter of 2021, all correlation coefficients are greater or equal to 0.70.

Figure 3: Long-Run Growth Estimates in State-Level GDP



Notes: The black lines show the estimated quarterly growth rates transformed into levels with the first quarter of 1995 set to 100. The red lines represent the estimated long-run trends. The period ranges from the first quarter of 1995 to the fourth quarter of 2021. The large increase in economic activity in 2021 for Rhineland-Palatinate is attributed to the development of the COVID-19-vaccine by Biontech, whose headquarters is located in the state capital Mainz.

4.4 Stability and Robustness Checks

As estimations are fraught with uncertainty, we want to discuss their stability and robustness by three checks. First, given our data set we step-wise estimate the quarterly rates by different annual vintages and calculate different stability measures. Second, we compare our results to the few estimates that are available from official and non-official sources. Third, we vary the estimation procedure of the state-space model. Overall we can state that our estimates are quite robust and stable over time.

Step-wise estimation. The stability of the estimates is crucial for formulating meaningful conclusions from the data and to analyze quarterly state-level GDP growth in real-time. However, it has to be said beforehand that revisions will take place as the statistical material that enters our estimation (annual state figures our quarterly German GDP) changes over time. We therefore have to ask how reasonable the revisions of our estimates are and we do so by applying some measures to judge stability over time.

Instead of estimating the model on the whole data set, we start by estimating the period from 1995 to 2010 and subsequently repeat the estimation, sequentially adding one more year of data each time. This is done until the last observations of our data. We therefore end up with 12 vintages of quarterly and annualized growth rate estimates. For these vintages we calculate the difference of the minimum and maximum growth rate per quarter and state s, $\Delta_{s,Q}$. This procedure results in a range of growth rates for each point in time.[6] Afterwards, we take the mean of the range as a first stability measure: $\overline{\Delta}_{s,Q}$ for the quarterly growth rates and $\overline{\Delta}_{s,A}$ for the annualized rates. These averages are informative about the size of the delta between the estimated minimum and maximum growth rate. As a second stability measure we analyze how much the estimates vary per vintage. For that reason we calculate the standard deviations in the estimates per vintage, quarter and state, $\text{SD}_{s,Q}$. Once again, we take the mean over these standard deviations: $\overline{\text{SD}}_{s,Q}$ and $\overline{\text{SD}}_{s,A}$. Finally, we relate the mean standard deviation across vintages to the variation in quarterly economic growth by state, in order to calculate a measure in style of a Noise-to-Signal Ratio: $\text{NTS}_{s,Q}$. This ratio expresses how large the variation in estimated growth rates is compared to the volatility of the series. The smaller this ratio gets, the less noisy and the more informative are the estimates. Table 3 presents the three stability measures for each state and Germany.

Germany is included for comparison and the measures are calculated from real-time vintages by the Deutsche Bundesbank and therefore represent the revision behavior of quarterly German GDP growth. Overall, the mean range of quarterly growth rate estimates ($\overline{\Delta}_{s,Q}$) ranges between 0.26 percentage points for North Rhine-Westphalia and 0.62 percentage points for Lower Saxony. The means for the annualized rates ($\overline{\Delta}_{s,A}$) are 0.10 percentage points and 0.37 percentage points. Especially the ranges for the annualized rates are small and all states but Lower Saxony have a value below the German one. Lower Saxony is also much in line with the average range in revisions for Germany (0.34 percentage points). The average ranges further drop by excluding the two large crises (2008/2009 and Corona).

The variation across vintages in quarterly growth rates ($\overline{\text{SD}}_{s,Q}$) is also quite small, ranging from 0.09 percentage points to 0.22 percentage points. For the average deviation in annualized growth rates ($\overline{\text{SD}}_{s,A}$) the span lies between 0.03 percentage points and 0.13 percentage points. Overall, these figures hint to very robust estimates and are close or even better

[6] As we currently do not have data extending beyond 2021 and therefore only have one estimation for quarterly growth in 2021, both figures coincide. We therefore exclude the year 2021 from the calculation of the stability measures.

than the stability measures for German GDP revisions (0.13 and 0.12 percentage points). Compared to the variation in quarterly state-level GDP growth, the NTS are close to zero, meaning that the variation in revisions are almost negligible compared to the underlying variation of the series. Based on these three stability measures, we conclude that the estimates are quite stable over time.

Table 3: Stability Measures for the Quarterly Estimates

State	$\overline{\boldsymbol{\Delta}}_{\mathbf{s,Q}}$ **(in p.p.)**	$\overline{\boldsymbol{\Delta}}_{\mathbf{s,A}}$ **(in p.p.)**	$\overline{\mathbf{SD}}_{\mathbf{s,Q}}$ **(in p.p.)**	$\overline{\mathbf{SD}}_{\mathbf{s,A}}$ **(in p.p.)**	$\overline{\mathbf{NTS}}_{\mathbf{s,Q}}$
Baden-Wuerttemberg	0.51	0.22	0.19	0.07	0.11
Bavaria	0.27	0.12	0.09	0.04	0.07
Berlin	0.36	0.12	0.13	0.04	0.10
Brandenburg	0.37	0.18	0.13	0.06	0.10
Bremen	0.37	0.23	0.13	0.07	0.07
Hamburg	0.42	0.20	0.14	0.07	0.09
Hesse	0.31	0.13	0.11	0.04	0.08
Mecklenburg-West Pomerania	0.39	0.19	0.15	0.07	0.09
Lower Saxony	0.62	0.37	0.22	0.13	0.13
North Rhine-Westphalia	0.26	0.10	0.09	0.03	0.08
Rhineland-Palatinate	0.30	0.13	0.11	0.04	0.08
Saarland	0.38	0.18	0.13	0.06	0.06
Saxony	0.36	0.14	0.12	0.04	0.07
Saxony-Anhalt	0.45	0.22	0.15	0.07	0.07
Schleswig-Holstein	0.39	0.18	0.14	0.06	0.10
Thuringia	0.41	0.19	0.14	0.06	0.06
Germany*	0.34	0.33	0.13	0.12	0.08

Notes: The table presents three stability measures for both quarterly (Q) and annualized (A) growth rates for each state s. The three measures are: i) the average range between the minimum and maximum estimate per quarter ($\overline{\Delta}_s$), ii) the average standard deviation in quarterly estimates per vintage ($\overline{\mathrm{SD}}_s$), iii) a ratio in style of the Noise-to-Signal Ratio between stability measure ii) and the standard deviation in quarterly growth rates ($\overline{\mathrm{NTS}}_{s,Q}$). For Germany, these measures are calculated based on the official real-time vintages by the Deutsche Bundesbank and therefore represent revision figures for German GDP growth ($*$).

Comparison to other estimates. Another robustness check is the comparison of our estimates to either official or non-official sources. For example, the Statistical Office Baden-Wuerttemberg publishes a long series for quarterly GDP and the ifo Institute provided a series for both Saxony and Saxony-Anhalt (for more details see the Supplementary Material). Our results are very similar to these univariate and independent estimates. The correlations between our annualized estimates and the official or non-official rates are 0.98 for Baden-Wuerttemerg, 0.99 for Saxony, and 0.97 for Saxony-Anhalt. Given these correlations, we are quite confident that we are close to the true quarterly values.

Estimation procedure. MCMC algorithms produce very accurate estimates but come with the price of large computational burdens. It took quite a long time to produce the quarterly estimates, especially in case of the step-wise procedure. Gefang *et al.* (2020) faced this issue and introduce Variation Bayes (VB) methods to their study of the United Kingdom. A potential drawback of VB methods is their approximative nature. Whereas MCMC methods simulate the posterior density, VB methods only approximate the true density by a much simpler density function, given that a certain criterion is met. Gefang *et al.* (2020) find that their VB-estimates—despite their approximate nature—are very accurate and close to the MCMC-estimates, but that VB methods are much more efficient and less time-consuming.

We transfer their idea to our case and prove whether this in-sample estimation produces robust quarterly estimates. We can confirm the results by Gefang *et al.* (2020) for the German case. The VB-estimates are very accurate and close to the MCMC-estimates (see the Supplementary Material for a detailed comparison). The correlations between annualized rates range from 0.94 to 1.00.

5 Conclusion

Regional macroeconomic aggregates such as GDP are only available at an annual frequency for most countries. This circumstance prevents, for example, policymaker from assessing the current state of the regional economy in a timely manner. This paper uses a modern time series framework to estimate regional quarterly real GDP for the period 1995 to 2021 based on nationwide developments and an under-explored source of regional information. Based on these estimates, the paper presents an in-depth business cycle analysis and discusses long-run growth dynamics.

Specifically, we transfer the methodology by Koop *et al.* (2020c) to the case of Germany and explore the only regional information for which long time series are available: comprehensive business survey results and the number of unemployed persons. The German states are characterized by large heterogeneities in their industrial mix and demographic (pre-)conditions, making them especially interesting for a business cycle analysis and the estimation of long-run growth dynamics. The business cycle analysis reveals large differences in the state-specific duration and amplitudes of upswing and downswing phases. Downswing phases last, on average, between a span of 2.6 to 7.4 quarter and the span for upswings ranges from 7.2 quarter to 21.0 quarter. The average loss in economic activity in a downswing ranges from -8.5% to -3.4%. For upswings, the average increase in GDP lies between 5.6% and 14.1%. We also observe large differences in the degrees of concordance to the German business cycle, breaking with the prevailing narrative that the German states behave like the national average in terms of business cycle dynamics.

Closely connected to the business cycle analysis is the estimation of long-run growth dynamics and trends. We find that economically strong German states exhibit quite stable trend growth rates, whereas economically much weaker states already show flat or even negative trends. This is particularly alarming as demographic conditions will change dramatically in the next two years, leading to even more pronounced differences across the states. This development puts much pressure on economic policy in the near future.

Whenever new quarterly GDP figures for Germany are released, we will update our estimates, thus, enriching the state-level database also in the future. We hope that the general public finds our results interesting enough and new research ideas will be initiated. The next steps could be the following. First, much effort might be undertaken to build up a comprehensive state-level data set with important economic indicators such as industrial production or turnover in the service industry. The presented time series framework might then be augmented by these indicators. Second, the interesting feature of a cross-section restriction might be extended to the single components of GDP together with standards in national accounting. This makes it possible to estimate state-level quarterly GDP not only directly but rather as the sum of industrial gross value added. Third, given the large heterogeneities in business cycle dynamics, future research might ask whether exogenous shocks hit state-specific economic activity with different intensities and how these shocks propagate across Germany. Finally, our data or the model introduced can be used for applied forecasting purposes by each interested external user.

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