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### The Value of Information in Water Quality Monitoring and Management

#### **Abstract**

Environmental managers face substantial uncertainty when deciding on management actions. To reduce this uncertainty prior to decision-making, collecting new data may help arrive at more informed decisions. Whether any resulting improvement in the decision will outweigh the cost of collecting the data, and thus make investing in the acquisition of the information worthwhile, is an intricate question. The concept of the value of information (VoI) is a convenient tool to address this problem. We use the VoI framework to analyse a decision problem in water quality management. Based on real-world monitoring data, we calculate the VoI of monitoring nitrogen, which is used as an indicator of the ecological state of water body. We find that the VoI is significant in our case and we further investigate the dependency of the VoI in a similar setting on the management cost, the assumed value of a good state and on the level of uncertainty regarding the ecological state. In addition, we observe a negative relation between the relative management cost and the prior probability that maximises VoI. These insights may help decide on information acquisition in the presence of substantial uncertainties and sparse data.

JEL-Codes: C110, C610, D810, Q250, Q570.

Keywords: value of information, decision analysis, uncertainty, environmental management, monitoring.

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#### 1 Introduction

Eutrophication is one of the main problems in the North Sea's coastal waters (OSPAR, 2017). It is caused by increased enrichment of the water with nutrients and can disturb the composition of organisms and eventually reduce the overall quality of the water. Managing aquatic systems threatened by eutrophication is challenging, since there are many inherent uncertainties about its exact causes and effects. Consequently, environmental managers face a high degree of uncertainty when deciding on management actions, but interventions often do not take these uncertainties into account. They may therefore be ineffective or even counterproductive (Cook et al., 2010; Bennett et al., 2018). To reduce anthropogenic stressors and to mitigate eutrophication, legislation, such as the European Marine Strategy Framework Directive (MSFD) and the European Water Framework Directive (WFD), has been enacted (European Parliament, 2000, 2008; Desmit et al., 2020). The WFD requires EU member states to obtain and maintain a "good ecological status" (GES) by 2027, based on a range of biological quality elements that are used to classify the state of a water body as either high, good, moderate, poor or bad. Although the GES target was initially set to be achieved by 2015, only about 40% of European water bodies reached that goal by 2018 (Carvalho et al., 2019; European Environment Agency, 2018). For the coastal waters of the North Sea, the riverine nutrient influx is seen as a reason for eutrophication (Desmit et al., 2020) and hence a cause for the qualities of water bodies falling short of the GES target. These high riverine nutrient concentrations are predominantly due to non-point sources of pollution, from agricultural and other land use activities, or derive from uncontrolled and untreated discharge from sealed surfaces after storm events or heavy rainfall (Carvalho et al., 2019).

In this study, we evaluate the need for monitoring or taking direct actions to manage the water quality in the Weser River basin in Northern Germany. As most of Germany's water bodies still fail to reach the GES, many de-eutrophication measures focus on nitrogen reduction. For rivers entering the North Sea, a special target for nitrogen concentrations has been established in the limnic-marine transition zone to reduce eutrophication in coastal waters and therefore meet the GES targets (BLMP, 2011). Although the ecological and chemical developments of German rivers are closely monitored, few of these rivers have met the GES targets. A thorough assessment of the ecological state is the prerequisite for any recommendation and implementation of restoration measures. However, such an assessment requires reliable data (Koski et al., 2020). The acquisition of a sufficient amount of information through monitoring is therefore essential to evaluate the system's state and to decide whether interventions are necessary or the desired good state of the ecosystem has already been reached. Monitoring activities are at the core of understanding the state of the system and its response to stressors (Nygård et al., 2016). Although monitoring data do not directly solve any environmental problem, they may help facilitate targeted management and policy interventions (Bouma et al., 2011). At the same time, monitoring and data collection involve many resources, while conservation budgets are often limited (Bennett et al., 2018). Additionally, postponing the decision to act may result in missed opportunities for management (Martin et al., 2012) and could result in further degradation of the ecosystem. WFD regulations require extensive monitoring programs, which in turn require significant financial resources, for which governments must find cost-effective, yet qualitatively sufficient solutions (Carvalho et al., 2019). In this context, acquiring new information is only worthwhile if it can be expected to change the choice of the decision maker and, in this way, lead to more effective management. It is therefore mandatory to carefully evaluate whether or not,

and if so, to what extent, monitoring – or more broadly, an information service – will be useful for providing valuable information. For this, we can use the Value of information (VoI) analysis. Vol is a decision-analytic tool to determine the value of additional information for decision-making: it computes how much a (rational) decision maker's expected payoff would increase if uncertainty is, at least partially, reduced before the decision is made. The uncertainty here is represented by a probability distribution over possible states of the system (Pannell and Glenn, 2000). Vol gives the value of an information service, i.e. the expected value of acquiring information before any specific information or data have been received. That is, Vol represents the willingness-to-pay of the decision maker for the acquisition of new data, while not yet knowing what this data will look like. This implies that before the decision-making, more data will only be collected if it is expected to be beneficial. In this way, Vol helps the decision maker enhance their decision through means of a well-judged acquisition of information. Specifically, the expected value of perfect information gives the payoff when uncertainty is entirely eliminated, i.e. when complete knowledge about the true state of the world (clairvoyance) is achieved; while in contrast, the expected value of sample (or imperfect) information gives the increase in the payoff on obtaining some, even though imperfect, information.

Initially formulated by economists (Raiffa and Schlaifer, 1961; Hirshleifer and Riley, 1979), Vol has been widely applied in health economics (Yokota and Thompson, 2004; Fenwick et al., 2020), engineering (Bratvold et al., 2009), fishery management (Clark and Kirkwood, 1986; Costello et al., 2010; Kuikka et al., 2011) and invasive species management (Li et al., 2021; Johnson et al., 2017; Moore and Runge, 2012). In spite of its apparent benefits, Vol has been only sparsely applied for conservation planning (Runge et al., 2011; Williams et al., 2011; Moore and Runge, 2012), water management (Borisova et al., 2005) or environmental monitoring (Nygård et al., 2016; Koski et al., 2020; Venus and Sauer, 2022). The reasons for this lack of application may include the difficulty of quantifying the value of an ecological system (Koski et al., 2020) or the high computational costs with the increasing complexity of the decision problem (Canessa et al., 2015; Bolam et al., 2019). Furthermore, the calculation of Vol requires explicitly defining a decision framework: the probabilities of the states of the world, the set of available management actions, and the consequences of each management action, all of which may represent challenging tasks for environmental decision problems.

For our specific context, we use a Vol framework similar to the one used by Koski et al. (2020) to solve this ecological management problem with available real-world monitoring data. Building on this, we extend the analysis by performing a sensitivity analysis and showing the interaction between the management cost and the probability distribution of the ecological state. Specifically, we identify the prior probabilities for which Vol is a maximum over a range of management costs. Lastly, transcending our concrete case study, we provide generic results on Vol for all two-state, two-action decision problems under uncertainty with respect to two crucial determinants of Vol: the prior probability distribution and the management costs in relation to the good state.

The remainder of this article is structured as follows: In the next section, we provide information regarding the data and methods used in our investigation. Section 3 provides the results of our Vol analysis along with a detailed sensitivity analysis showing how the Vol depends on the management cost and the prior distribution in Section 4. This is followed by a discussion of the results in Section 5 and a conclusion in Section 6.

#### 2 Data and Methods

#### 2.1 Decision problem and data

According to the WFD, the state of a water body is determined by several elements of biological quality and supporting chemical-physical parameters. In the case of Germany, coastal waters are prone to high riverine input of nutrients, leading to eutrophication (BLMP, 2011; Desmit et al., 2020) and thus leading to a failure to meet the GES target (BLMP, 2011). Due to a correlation between nitrogen and chlorophyll-a, it is frequently hypothesised that the overall nitrogen concentration in the water body affects the biological quality element phytoplankton (BLMP, 2011). Consequently, water quality management predominantly targets a reduction of nitrogen concentrations to reach the GES in coastal waters. In accordance with this policy focus, we restrict our assessment to total nitrogen because it serves as an indicator of the state of a water body. Our goal is to assess the Vol of monitoring nitrogen data for rivers of the Weser River basin that enter the German Wadden Sea. We use the official and open-source monitoring data provided by Niedersächsischer Landesbetrieb für Wasserwirtschaft, Küsten- und Naturschutz (NLWKN) $^1$  and of the Flussgebietsgemeinschaft Weser (FGG Weser) $^2$ .

We consider a sample of water bodies within the case study area and differentiate between water bodies within the target state, i.e. fulfilling the criteria of the GES according to the WFD, and those that fail to meet the target state. We consider data for the WFD assessment periods 2000-2018. Since little data is available on water bodies in a good state, we used the raw data and disregarded temporal or spatial differentiation. We acknowledge that in this way, the analysis is biased towards water bodies with a high frequency of measurements or with many measurement stations; also, spatial differences, as well as different river types, cannot be taken into account. However, this approach is still suitable for highlighting the value of monitoring data for environmental management. To base the Vol analysis on empirical data, we assume that total nitrogen is a proxy for the state of the water body. Since the main target of the WFD is that water bodies either maintain or reach the GES, the threshold between the categories GES and non-GES becomes essential; at the same time, subcategories within GES and non-GES are inessential. Consequently, the threshold between GES and non-GES determines whether management interventions must be taken. We, therefore, disregard the original division of the state of a body of water into five categories and consider only two: those that meet the target state (GES) and those that do not (non-GES). We will refer to the latter as bad state  $(x_0)$ and the former as good state  $(x_1)$ . Accordingly, the state X of a water body takes either of two values:  $X \in \Omega = \{x_0, x_1\}$ , with a prior probability  $p_X(x)$  for state x being true. We assume that for any section of a river, two management alternatives  $a \in A = \{a_0, a_1\}$  can be considered: either no action is taken  $a=a_0$  (default), or a specified action is taken  $a=a_1$ . The resulting payoff then depends on both the action and the state:  $v: A \times \Omega \to \mathbb{R}$ , as shown in Table 1. We next determine the value of actions, costs and prior probabilities.

The estimated cost of action  $a_1$  is retrieved from reports by LAWA (2020) and Flussgebietsgemeinschaft Weser (FGG) (2021) (section "cost for management of pollution from diffuse sources") and is set to EUR 90 million per year. The cost of action  $a_0$  is set to zero. The value of a water body in good state is estimated from a report by the European Commission

<sup>&</sup>lt;sup>1</sup>Lower Saxony Water Management, Coastal Protection and Nature Conservation Agency.

<sup>&</sup>lt;sup>2</sup>River Basin District Weser.

Table 1: Payoff matrix for the river management problem.

	actio	prior belief	
ecological state $X$	$a_0$	$a_1$	$p_X$
$X=x_0$ : bad state	$v(a_0,x_0)$	$v(a_1,x_0)$	$p_X(x_0)$
$X = x_1$ : good state	$v(a_0,x_1)$	$v(a_1,x_1)$	$p_X(x_1)$

(2019). The cost of not reaching GES for Germany, i.e. the benefit forgone, is estimated to range between EUR 820–3304 million per year. Scaled down to the area of the Weser River basin area, this results in a value within the range of roughly EUR 115–450 million per year. We set the value at EUR 200 million per year for our initial analysis. Therefore, the value of a river in good state  $(x_1)$ , without management cost, is set to EUR 200 million per year.

The payoff for each action is then calculated by subtracting the management cost  $-c_0 = c(a_0) = 0$  in case of action  $a_0$ , and  $c_1 = c(a_1) = 90$  in case of action  $a_1$  – from the value of the water body after the action became effective, which is either 0 or 200. Therefore, the value of the ecological state after management, which we refer to as the payoff, is given by

$$v(a,x) = \begin{cases} 0 & \text{if } (a,x) = (a_0, x_0) \\ 200 & \text{if } (a,x) = (a_0, x_1) \\ 200 - c(a_1) & \text{if } (a,x) = (a_1, x_0) \\ 200 - c(a_1) & \text{if } (a,x) = (a_1, x_1) \end{cases}$$

for  $c(a_1) = 90$  we obtain

$$v(a,x) = \begin{cases} 0 & \text{if } (a,x) = (a_0, x_0) \\ 200 & \text{if } (a,x) = (a_0, x_1) \\ 110 & \text{else} \end{cases}$$

We assume that after performing the action  $a_1$ , the water body will be in a good state; intuitively,  $a_1$  serves as a perfect hedge against a possible bad state of the water body.

The prior probabilities for each state are derived from a recent report, highlighting that less than 10% of German water bodies are currently in a good state (Bundesministerium für Umwelt, Naturschutz und nukleare Sicherheit, 2017). Hence, we set the prior belief for a water body's being in a good state at  $p_X(x_1)=0.1$  and for a water body being's in a bad state at  $p_X(x_0)=0.9$ . The four possible situations are summarised in Table 2 (costs are given in million Euros per year).

#### 2.2 Concept of the value of information

In this section, we outline the theory behind the Vol at a more abstract level, to present the general idea behind our approach. As we mentioned already, Vol is used in the case of revisiting a decision via determining whether it is worth investing in more information to reduce the

Table 2: The value of the ecological state after action  $a_0$  or  $a_1$  without additional information.

		payoff $v(a,x)$	
actions	cost of actions	$x_0$	$x_1$
$a_0$	$c_0 = 0$	0	200
$a_1$	$c_1 = 90$	110	110
prior belief	$p_X(x)$	0.9	0.1

uncertainty or whether the decision should be based on the current information. This uncertainty about the true state of the system is modelled by the random variable  $X:\Omega\to\mathbb{R}^+$ , with  $\Omega$  being the state space, which we assume to be discrete, and corresponding probability measures  $p_X$  on  $\Omega$ . The decision maker can choose any action  $a\in A$ . The payoff (profit or utility) of the decision maker resulting from state  $x\in\Omega$  and action  $a\in A$  is denoted by  $v:A\times\Omega\to\mathbb{R}$ :  $(a,x)\mapsto v(a,x)$ .

One of the key measurements of Vol, the value of perfect information or the value of *clairvoyance* about the true state of the world, is calculated by

$$V^{\circ} := PoV^{\circ} - PV$$

where the prior value (PV) describes the maximum expected outcome under current information; i.e. the expected utility resulting from adopting the action which produces the highest expected utility:

$$PV = \max_{a \in A} \, \mathbb{E} \left[ v(a,x) \right] = \max_{a \in A} \, \left[ \sum_{x \in \Omega} v(a,x) p_X(x) \right],$$

where the expectation is taken with respect to X. In our case, we explicitly calculate the PV by

$$PV = \max_{a \in A} [v(a, x_0)(1 - p) + v(a, x_1)p]$$
  
=  $\max [v(a_0, x_0)(1 - p) + v(a_0, x_1)p, v(a_1, x_0)(1 - p) + v(a_1, x_1)p]$   
=  $\max(200p, 200 - c(a_1))$ 

$$PV = \begin{cases} 200 - c(a_1) & \text{if } p < \frac{200 - c(a_1)}{200} \\ 200p & \text{if } p \ge \frac{200 - c(a_1)}{200} \end{cases}$$

Note that PV is not differentiable at the point  $p=\frac{200-c(a_1)}{200}$ . This lack of differentiability in the function will impact the behavior of the variable of interest, which will be introduced later, and can be visually observed in Figure 4 and Figure 7.

On the other hand, the posterior value (PoV) represents the expected utility after being informed about the realisation of X; it gives the expected utility when taking the optimal action for each state of the world  $x \in \Omega$  (Yokota and Thompson, 2004):

$$PoV^{\circ} = \mathbb{E}\left[\max_{a \in A} v(a, x)\right] = \sum_{x \in \Omega} p_X(x) \max_{a \in A} v(a, x).$$

Here,  $PoV^{\circ}$  represents the probability-weighted sum of the utilities of the optimal actions. Then, the difference between the expected utility under perfect information and under current information gives the *value of perfect information* (Yokota and Thompson, 2004). If perfect information can be obtained, and the value of the perfect information exceeds the cost of acquiring it, then it is worthwhile to acquire this information prior to making a decision.

Calculating the expected value of perfect information is useful for exploring the upper bound of the value of eliminating uncertainty. However, in real-world problems, obtaining perfect information about the state of the world (here, the state of the water body) is almost always impossible (Canessa et al., 2015). Therefore, instead of obtaining perfect information on the realisation of X, the decision maker can reduce, but not entirely eliminate, uncertainty by observing some information (or message) y, which may thus be viewed as specific information about the probability distribution of X. Since the information being received is not known in advance, it represents a realisation of a (continuous) random variable Y with probability distribution  $p_Y$ . In this way, any realisation of Y provides some specific indication of the probability distribution of X; we denote this conditional probability distribution of X by  $p_{X|Y}$ , and specifically, write  $p_{X|Y}(\cdot|y)$  if Y=y. Intuitively, we may interpret the probability distribution of the possible message  $p_Y$  as an information service, which induces the conditional information  $p_{X|Y}$  on the distribution of X. It is the acquisition of this information service about which the decision maker has to decide before deciding on the action itself.

The Vol concept can be adapted to this situation as well: Yokota and Thompson (2004) define the value of information, more precisely the value of an information service, as the difference between the expected payoff under current information and the expected payoff when new information is obtained. Specifically, the expected value of imperfect information is the difference between the expected value of the best action based on the posterior probability distribution on X induced by the, ex-ante unknown, information Y, and the PV:

$$V := PoV - PV$$
.

Here, a realisation of the random variable Y and the associated probability density  $p_Y$  represents some, yet imperfect, information about the state  $p_{X|Y}$ . This information might be obtained, for example, by means of monitoring or buying monitoring data. Given the probability density  $p_Y$ , PoV can be written as

$$\operatorname{PoV} = \int \max_{a \in A} \, \mathbb{E} \left[ v(a,x) | y \right] p_Y(y) \, \mathrm{d}y = \int \max_{a \in A} \bigg( \sum_{x \in \Omega} v(a,x) p_{X|Y}(x|y) \bigg) p_Y(y) \, \mathrm{d}y,$$

where the expected value of the best outcome is taken over all possible messages (or monitoring results) y weighted by their probabilities of observing  $p_Y(y)$ .

Since any received message (or information) y provides information on the distribution of X, the probabilities for realisations of X need to be updated accordingly. Bayesian updating reflects the belief-updating process of the probability of X for all possible sample information, y:

$$p_{X|Y}(x|y) = \frac{p_X(x) \, p_{Y|X}(y|x)}{p_Y(y)},$$

with  $p_{Y|X}(y|x)$  representing the likelihood function of observing y when the state of the world is x, and  $p_Y(y)$  representing the marginal density of y:

$$p_Y(y) = \sum_{x \in \Omega} p_X(x) p_{Y|X}(y|x).$$

#### 3 Vol analysis for the Weser River basin

We now continue with the Vol analysis for our management problem described in Section 2 where we consider two states of a water body  $X \in \Omega = \{x_0, x_1\}$  and two actions  $a \in A = \{a_0, a_1\}$ . For this simplified case, the (prior) probability distribution  $p_X$  can be represented by a single probability  $p := p_X(x_1) = 1 - p_X(x_0)$ . Our initial analysis exemplifies the value of monitoring information based on the prior p and the management cost c.

#### 3.1 Computing conditional and posterior distributions

Vol analysis depends on Bayesian updating, therefore one key aspect is to determine the likelihood of the data. In our case, we fit distributions to the empirical data to simulate monitoring activity by randomly sampling values from these distributions. To choose the best fit for the data, we first compute the descriptive parameters of the empirical data. We use the Cullen and Frey plot – a skewness-kurtosis plot – for a visualisation of the possible best distribution. We then choose from the proposed theoretical distribution consistent with the skewness and kurtosis of the empirical data and conduct a goodness-of-fit analysis. We choose the best fit by comparing the maximum likelihood estimators (MLE), log-likelihood, Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). For the bad state data, the Cullen and Frey graph, in addition to the MLE, suggests a gamma distribution as the best fitting distribution, while the best fit for good state data based on the same criteria is a beta distribution. However, for the fitting process, the data has to be re-scaled to the support of a beta distribution, i.e. rescaled to [0,1]. This is problematic, as there is no way to "scale back" after conducting the Vol analysis. We avoid the need to scale the data by choosing a four-parameter beta distribution, a highly flexible bounded distribution, where the lower and upper limits can be set based on the data. Fitting the best possible distribution to the data is an important part of our Vol analysis as it requires sampling from the distribution and refitting the sampled values.

In order to estimate the posterior value of imperfect information from the available data, that is from sampling values for Y, we estimate  $p_{Y|X}\left(y_i|x\right)$  from the distributions fitted to the empirical data using a Monte Carlo approach. Random samples (n=10000) are drawn from the fitted distribution and the distributions are refitted to the random samples. Then, using the estimator  $\widehat{p}_{Y|X}\left(y_i|x\right)$ , we calculate

$$\widehat{PoV} = \frac{1}{n} \sum_{i=1}^{n} \max_{a \in A} \mathbb{E}\left[v\left(x, a\right) | y_i\right] = \frac{1}{n} \sum_{i=1}^{n} \max_{a \in A} \left(\sum_{x \in \Omega} v(x, a) \widehat{p}_{X|Y}\left(x | y_i\right)\right),$$

with n being the number of observations. The corresponding confidence intervals (CI) for  $\widehat{PoV}$  are estimated using a Monte Carlo bootstrapping approach, for which the procedure is repeated

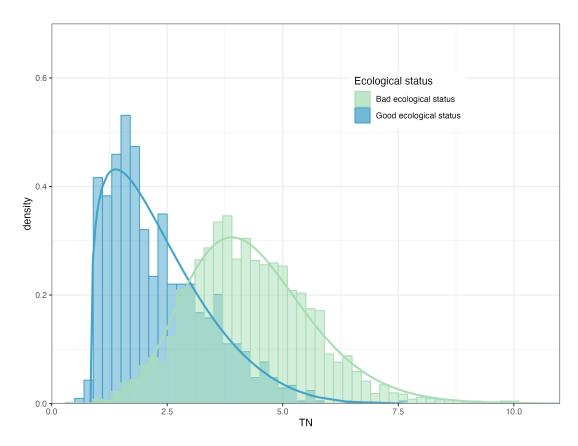


Figure 1: Histograms of the empirical data (total nitrogen in mg/I = TN) with fitted four-parameter-beta and gamma distributions. The empirical data is divided into two categories for the ecological state: good ecological status and bad ecological status

1000 times and the confidence intervals are obtained by subtracting the value of PV from the calculated PoV in each step.

#### 3.2 Value of perfect and imperfect information

We conduct the initial Vol analysis with the estimated prior probabilities and monetary values as given in Table 2. We consider the prior belief  $p:=p_X(x_1)=0.1$  (see subsection 2.1) for the water body being in a good state, meaning that, a priori, the decision maker is fairly certain that the water body is not meeting the desired state  $X=x_1$ . In view of the prevailing uncertainty and without additional information, the strategy with the highest expected benefit would be to choose the specified action  $a_1$  for the water body. Under current information, this action would result in a maximum expected payoff of 110 million EUR/year. In contrast, the value of perfect information yields a maximum expected value of 119 million EUR/year. If the decision maker could obtain perfect information, it would be worthwhile to pay up to 9 million EUR/year and postpone the decision-making until after additional information is acquired. Lastly, the value of imperfect information, meaning that new information may reduce but not eliminate completely the uncertainty, is 112.21 million EUR/year. In this case, the decision maker is willing to pay up to 2.21 million EUR/year (with a 95% CI [2.06, 2.84]) for acquiring information through

Table 3: Value of perfect and imperfect information for the case of the Weser River

Prior	$p_X(x)$	Prior value	Perfect i	information	Imperfe	ct information
$x_1$	$x_0$	PV	$PoV^{\circ}$	$V^{\circ}$	PoV	V
0.1	0.9	110	119	9	112.21	2.21
						CI(2.06, 2.84)

monitoring in order to be more certain about the true state of the water body, see Table 3.

#### 4 Dependence on costs and prior probabilities

In real-world applications, the monetary values, management costs and prior probabilities are estimates and are thus themselves subject to uncertainty. A careful sensitivity analysis may help to reduce the uncertainty incorporated in these parameters and to examine the robustness of the Vol analysis with respect to these data. In this section, we, therefore, compute V for different management costs c and prior probabilities (for the good state)  $p:=p_X(x_1)$ , and explore the sensitivity of V to these two crucial parameters. Naturally, the management costs are assumed to be non-negative and to not exceed the value of the water body's being in the good state, i.e.  $c \in [0,v]$ , while  $p \in [0,1]$ . Formally

$$V:[0,1]\times[0,v]\to\mathbb{R}:(p,c)\mapsto V(p,c).$$

Among other things, this formalisation helps us to find the priors for which  $\,V\,$  is maximised in relation to management costs. Since in the course of our analysis, we vary (p,c) over its domain, we will provide qualitatively generic results for all two-state, two-action decision problems under uncertainty.

Before we present and discuss the properties of V for its full parameter range, we begin with computing V for specific values of the management cost. Fig. 2 displays the values of perfect and imperfect information for low (c=50), medium (c=100), and high (c=150) management costs (all in million EUR/year), along with 95% CI. If the action has a medium cost, V reaches its maximum when uncertainty is highest, i.e. at a prior probability of p=0.5, see Fig. 2b. In this case, the value of perfect information reaches up to 50 million EUR/year, and the value of imperfect information is up to 30 million EUR/year. In contrast, in the absence of uncertainty, i.e. for either p=0 or p=1, the values of perfect and imperfect information are both zero, as the decision maker already has full knowledge about the true state of the water body.

For low management costs (50 million EUR/year), see Fig. 2a, V is highest when the ecological state is believed to be likely to meet the target (p=0.75), and the decision maker is therefore relatively confident that there is no need for any action. Intuitively, if the management cost is low, the decision maker is willing to undertake the action  $a_1$  even if the water body is quite likely to be in a good state; only if this probability is sufficiently high does the decision maker omit taking action. It follows that there is a (relatively high) level of this probability at which the decision maker is indifferent between undertaking the action (because its cost is low) and omitting it (because it is seemingly not necessary). But V reaches its maximum exactly at the level of p where the decision maker is indifferent between actions  $a_0$  and  $a_1$ , because any

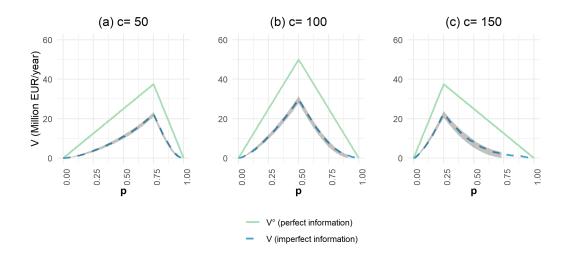


Figure 2: The value of perfect and imperfect information (with 95 % confidence intervals) for c = 50, 100, 150 and  $p \in [0, 1]$ .

additional piece of information may flip the decision to either side. Specifically, for  $c=50,\ V$  is maximised at p=0.75 with perfect information attaining a value of more than 40 million EUR/year and imperfect information more than 20 million EUR/year. In this case, it is worth getting more information to either confirm or reject the hypothesis that the water body is in good state so that an action can either be justifiably disregarded or undertaken. In this way, the decision maker avoids the risk that either an unnecessary action will be performed, or a beneficial and relatively cheap action will be omitted.

The reverse line of argument holds if the cost of the action is high (here 150 million EUR/year). Then, the action will not be undertaken unless the probability of the water body's being in good state is quite low. The value of p at which the decision maker is indifferent between actions  $a_0$  and  $a_1$  is therefore relatively low – and it is here that V reaches its maximum, for any additional indication of the water body's being in the good or in the bad state means changing the decision to one side or the other. Specifically, for c=150, V reaches its maximum at p=0.25, see Fig. 2c.

Moreover, we infer from Fig. 2 that V is strictly quasi-concave in p. While V depends on p and c, it is true, by the construction of the Vol concept, that the value of perfect information exceeds the value of imperfect information, irrespective of p and p. Yet, for any fixed level of p0 the location of the maximum, i.e. the prior probability for which p1 is maximum, is the same for both perfect and imperfect information, again see Fig. 2. More formally, let us define

$$p^*(c) := \arg\max_{p} V(p, c).$$

Then, for any value of c, V has a maximum at  $p^*(c)$  with the value of V amounting to  $V^*(c) := V(p^*(c), c)$ .

We now construct the image of  $V^*$  step by step. In Fig. 2, we display  $V(\cdot, 50)$ ,  $V(\cdot, 100)$  and  $V(\cdot, 150)$ , identifying the corresponding maximisers  $p^*(50)$ ,  $p^*(100)$  and  $p^*(150)$ , and their

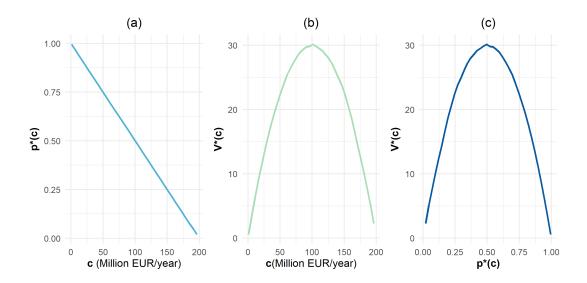


Figure 3: (a) Plot of maximum prior probability for which V has a maximum, i.e.  $p^*(c)$  versus the management cost c; (b) Maximum  $V^*(c)$  versus the management cost c; (c) Parametric plot of  $V^*(c)$  and  $p^*(c)$ .

respective values of  $V: V(p^*(50), 50), V(p^*(100), 100)$  and  $V(p^*(150), 150)$ . Proceeding in a similar way, we calculate  $p^*(c)$  and  $V^*(c)$  for all  $c \in [0, v]$ . The maximiser  $p^*(\cdot)$  is shown in Fig. 3a., while the maximised function  $V^*(\cdot)$  is shown in Fig. 3b. Finally, we display the graph of the mapping  $c \mapsto (p^*(c), V(p^*(c), c))$ , i.e. a parametric plot of c, in Fig. 3c. Fig. 3a shows that  $p^*(\cdot)$  decreases linearly, with  $p^*(0) = 1$  and  $p^*(200) = p^*(v) = 0$ , while Fig. 3b shows that  $V^*(\cdot)$  is strictly concave, with  $V^*(0) = 0 = V^*(v)$ . Lastly, along the path  $c \mapsto (p^*(c), V^*(c))$ ,  $V^*$  is maximum for  $(p^*(c), c) = (0.5, 100)$ , which can be seen from Fig. 3b and 3c. Intuitively, if management can be performed at zero cost, the decision maker will undertake the action in any case and is only indifferent between  $a_0$  and  $a_1$  if the water body will be in good state with probability 1. In contrast, if the management cost is equal to the value of the water body in the good state, which happens at c=v=200, the action will never be undertaken, and the decision maker is indifferent between  $a_0$  and  $a_1$  only if the probability of the water body's being in the good state is 0, i.e. the water body is in a bad state almost surely. Reversely, the value of reducing uncertainty as to which is the best decision,  $a_0$  or  $a_1$ , is highest when the monitoring costs are neither negligible nor excessive, and a prior uncertainty regarding the state of the water body is high (i.e. p = 0.5). In such a situation, any additional data that may give an indication as to what to do best is very valuable.

To summarise our findings, which are valid generically for all two-state, two-actions decision problems under uncertainty: When the cost of management is high, the decision maker does not undertake the action unless the prior probability is quite low (i.e. when the bad state is likely to hold). Therefore, when the cost and the probability of good state are both high, the arrival of new information is unlikely to reverse the decision maker's decision. Yet, when the prior probability is low, there is a significant risk that the actual state is bad, and thus the decision needs to be revised. Hence, given a high cost for management, V is largest when the prior probability is low, and therefore the prior probability for which V is maximised,  $p^*$ , is

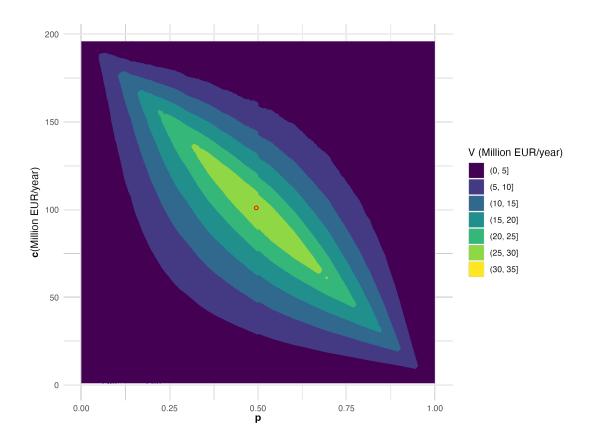


Figure 4: Contour plot of the value of imperfect (monitoring) information V as a function of the prior probability p and the management cost c.

small. Conversely, when the management cost is low, the decision maker is likely to undertake the protective action. This is especially the case when the prior probability is low, i.e. when the water body is likely to be in a bad state. When the prior probability is high, implying that good state is the probable result, undertaking a costly action, even if relatively cheap, may represent a waste of resources. Given a low value for the management cost, a high probability of good state tends to make the decision to undertake action needless. Consequently, for low management cost, V is the largest when the prior probability is high. This explains why there is a negative relation between c and  $p^*$ .

This negative relation between c and  $p^*$  is also shown in the contour plot in Fig. 4, displaying the iso-level curves of V for  $p \in [0,1]$  and  $c \in [0,v] = [0,200]$ . When the decision maker is a priori quite certain about the state of the water body, i.e. p is either close to 0 or to 1, the value of additional information is relatively low. Even more pronounced is the case when both p and p are simultaneously either low or high. In both of these cases, p is low, because of a low [high] probability of the good state, i.e. a high [low] probability of the bad state, together with low [high] management cost makes the decision maker perform [abandon] the action — and the arrival of new information is very unlikely to reverse this decision. In both of these polar cases, it is pretty evident that the action should be performed immediately (when both p and p are small), respectively that the action can be dispensed with (when both p and p are high), so that the arrival of new information is very unlikely to reverse this decision — and thus the value of

information is low. On the contrary, V is high when the management decision is close, which happens when the state of the water body is very unclear and management costs are moderate. Specifically, V is maximised when uncertainty is highest (p=0.5) and when at the same time the action costs are half of the gain in the value of the good over the bad state of the water body (c=v/2=100).

#### 5 Discussion and general insights

Acquiring more information through monitoring can have substantial value, as additional data may improve environmental decision-making. Vol analysis makes this economic benefit of data collection and monitoring activities explicit (Bouma et al., 2009). Decision makers may thereby improve the allocation of resources in monitoring and management and thus enhance returns on investments. Here, Vol represents the decision maker's willingness to pay for additional information. Our study aimed at demonstrating how to support an environmental decision problem by means of a Vol analysis. We applied the Vol framework using real-world monitoring data to a simple decision problem with two states of a water body and two decision options, using one variable (total nitrogen concentrations) as an indicator for the state of the water body. We calculated the value of additional monitoring data (or information) for a decision maker deciding on an environmental management action. Improved information, even when imperfect, yields a positive value and may lead to a higher payoff for the decision maker. Vol analysis can be a valuable tool in the light of monitoring being frequently criticised for being too expensive. The fact that these monitoring data may enhance decision making, and may thus have an additional value, is often ignored (Caughlan and Oakley, 2001; Lovett et al., 2007). Vol analysis focuses on this kind of extra value that data may have for environmental management, where investment decisions may be conditional on the collected data.

With our analyses, we obtained interesting methodological and general insights. From a methodological point of view, we see that it is especially difficult to calculate the value of imperfect information when the sample space is continuous. We showed that a Monte Carlo approach used in conjunction with Bayesian decision theory appears to be suitable for calculating an approximate value for imperfect information. To account for uncertainty incorporated in the estimated prior probabilities and the monetary values, we performed a sensitivity analysis. This method is also beneficial for providing further guidance to decision makers and environmental managers on the value of information for a range of combinations of prior probabilities and management costs. Moreover, this gives insight into the behaviour of Vol in relation to prior probabilities and management costs and highlights the importance of a sensitivity analysis.

Irrespective of the fact that the exact values that result from a Vol analysis are essentially case-specific, there are still some general findings that are worth emphasizing: Since V crucially depends on the prior probability p and the monitoring cost c, we investigate for which combinations of p and c V is maximum. To answer this, we calculate, for any value of c, the level of p for which V is maximal. Denoting this maximising prior by  $p^* = p^*(c)$ , we show that  $p^*$  is a decreasing function of c; moreover, V is, at least in our decision context, quasi-concave, which is illustrated in Fig. 4. We recognise the inherent uncertainties with regard to estimating prior probabilities, management costs, and the value of the state of the ecosystem. Improving these estimates leads to more confident estimates of the Vol. Intuitively, more informed priors, i.e. p

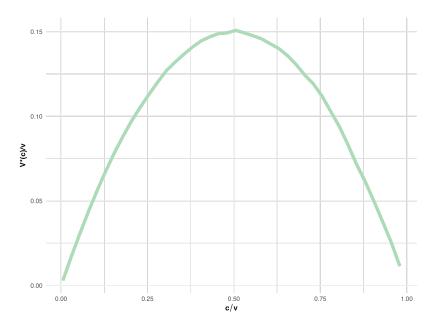


Figure 5: Effect of changing management cost on the maximum value of information V. The values of both the management cost and the V have been normalised with respect to the value of the good state (v).

close to either 0 or 1, results in a smaller Vol. The more certain the decision maker is about the state prior to making their choice, the lower the effect of additional information on their choice. However, the highest uncertainty (a prior probability p equal or close to 0.5) does not necessarily imply that V is maximal (Canessa et al., 2015). This is because  $p^*$  depends negatively on the management cost c.

To complement our analysis and to obtain more generic results, Fig. 5 shows  $V^*$  as a function of the ratio of the management cost c and the value of the good state v, i.e. on the relative management cost c/v; it shows that  $V^*$  is maximal when c=v/2. This generalisation provides us with some interesting and somewhat counter-intuitive insights: Let us assume that both c and p are fixed. If we now vary the value of the good state of the ecosystem, it turns out that increasing the value of the good state may lead to a decrease in the value of information. Intuitively, one might assume that the more relative value the ecosystem has, the more one would be willing to invest in monitoring. Yet, the analysis shows that a higher value leads to the fact that it is more useful to directly invest in actions instead of risking spending resources on monitoring and missing opportunities to act. Hence, the value of the information is relatively low.

So far, our discussion mostly focused on how the value of information is influenced by the key parameters c, v and p. Let us remember that from a decision-making perspective, whether or not the acquisition of additional data is actually worthwhile before a management decision is made, depends on the difference between the Vol and the cost of collecting the data (information acquisition). If the former exceeds the latter, new data should be collected before a decision is made. This is illustrated in Fig. 6. It shows the results of a Vol analysis for a case with two states of the world and two actions, similar to our previous example. The vertical axis gives the

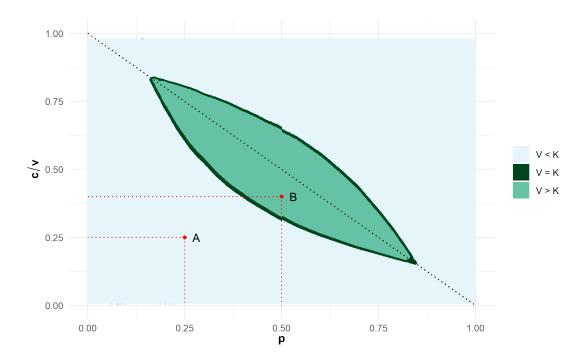


Figure 6: Value of information V in comparison to monitoring cost K. The black dotted line shows the maximising function  $p^*(c/v)$ . Example points A and B for parameter combinations where V is smaller than K (point A) and V is higher than K (point B). The decision maker would decide based on the results of the Vol analysis and the cost for monitoring if investing in additional information is worthwhile (V > K) or not (V < K).

ratio between the management cost c and the value v of the system. The horizontal axis is the prior probability p of the targeted state of the system. V is calculated over the full parameter range ([0,1]) and a fixed cost for monitoring (or information acquisition) K is given. This simple figure exemplifies under which conditions it is worthwhile for the decision maker to invest in monitoring. For a given constellation of parameters, such as in point A, the decision maker would decide against investing in information, as V is less than the cost K for monitoring. For another combination of values of the parameters, as in point B, V is larger than K and therefore the results suggest that investing in monitoring is welfare enhancing. This figure gives guidance to decision makers under which circumstances information acquisition is valuable. Further, it provides us with a certain amount of sensitivity information: As an example, since point B is relatively far in the interior of the green area, minor variations of parameter values c, v and p do not immediately change the decision to collect additional data.

Finally, we would like to emphasise that the generic results and insights from this discussion regarding the relation between the Vol and the management costs, the value of the good state and the prior probability are not restricted to our case study but apply to decision problems with the same structure. It should be noted, however, that the shape of the ellipse displayed in Figs. 4 and 6 not only depends on the parameters mentioned before but also on the posterior probability distributions which have to be fitted to the data of the specific decision problem under consideration (see Fig. 7 for an example).

#### 6 Conclusion

In our study, we demonstrated how value of information (VoI) analysis can serve as a valuable tool to enhance decision-making in environmental management as it may help to arrive at more welljudged decisions. We apply the Vol concept to a decision problem in water quality management in northern Germany. Our case study highlights that the Vol reaches substantial positive values. Even though acquiring data through monitoring may be costly, it may nevertheless be cost efficient to do so if the Vol outweighs the cost of monitoring. As the values and prior probabilities in our case study are estimates and are thus subject to uncertainty – which is the case for most decision problems - a careful and thorough sensitivity analysis is recommendable if not indispensable. Calculating the Vol for a suitable range of costs and prior probabilities enables the decision maker to place the results of the Vol analysis in the specific context and to highlight the specific conditions under which the collection of more data is, in fact, worthwhile. Our approach helps to expand the applications of Vol analysis to environmental management decision problems, especially to the value of imperfect information and monitoring. Even though the numerical results of the Vol analysis are case-specific, important general insights can still be obtained: The Vol has a maximum when the decision maker is indifferent between two alternative policies. In this case, a piece of new information may induce the optimal decision to switch from one action to another; the decision is sensitive to new information, so the Vol is high. Moreover, with a prior for which the maximum Vol is decreasing in the monitoring cost, the maximum Vol is reached when both the prior and the monitoring cost have moderate values. With our analysis, we arrive at qualitatively generic insights that are valid for all management decision problems under uncertainty with two states of the world and two actions.

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#### 7 Appendix

In our analysis we arrive at qualitatively generic results for all two-state and two-action decision problems. However, the shape of the ellipse (Fig. 4) not only depends on the management cost c and the prior probability p but also depends on the posterior probability. We obtain the posterior probability by sampling random values from distributions fitted to the empirical data. To display this change in shape, we calculate V using different distributions. We can see that the shape of the ellipse varies and becomes rounder or narrower depending on the posterior probability (see Fig. 7). The maximising function  $p^*$  and the structural components remain the same for all decision contexts with two states and two actions. Further, the results displayed in Fig. 7 can be interpreted the same way as explained in Section 4.

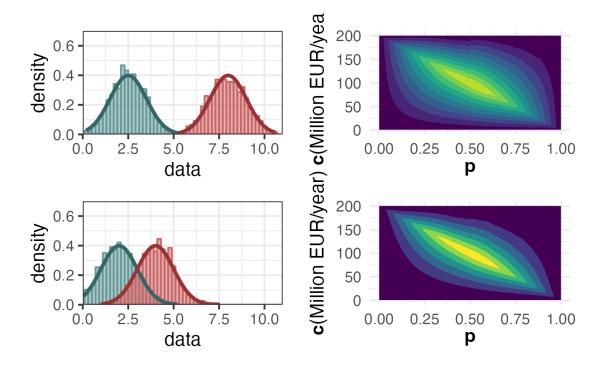


Figure 7: Results of two Vol analyses with different distributions of the data. The shape of the ellipse differs depending on the posterior probabilities which is obtained from sampled values from the fitted distributions. Qualitatively, the results of the analyses are generic.