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Abstract

The incorporation of increasing returns and imperfect competition into applied general-equilibrium (AGE) models, beginning with Harris (1984), led to much larger welfare effects from changes such as trade liberalization. But the imperfect competition side of these IO developments has often failed to incorporate meaningful strategic behavior, largely ruling out firm-level productivity and scale effects. I show here that the incorporation of theory-based endogenous markups into AGE models is not difficult in spite of the added simultaneity of the system. I first derive the optimal markup equations for Nash Cournot and Nash Bertrand competition in a CES environment with free entry and exit. Then I code a simple numerical model using non-linear complementarity. Three alternatives are considered: large-group monopolistic competition (LGMC), small-group Cournot (SGC) and small-group Bertrand (SGB). Growth in the economy is the experiment used to compare these specifications. While the overall effects of growth on welfare are qualitatively similar, the gains to initially small economies are much larger under either small-group assumption relative to LGMC, but diminish relative to LGMC as economies grow large. Secondly I show how the contributions of variety (entry), firm scale (productivity), and markups (distortions) to welfare changes differ substantially among the three alternatives.

JEL-Codes: F120, D580.

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Preliminary: additional references to work on theory-based markups into general-equilibrium models, whether analytical theory or numerical simulation, are welcome.

1. Introduction

In its first couple of decades, applied general-equilibrium modeling relied on the assumptions of perfect competition, constant returns to scale, and Armington product differentiation in trade models, the latter ensuring interior solutions to systems of equations. Many important trade distortions such as quantitative restrictions and real trade costs were converted into ad valorem tax equivalents which, in my opinion, biased the results of counter-factual experiments. I addressed some of these limitations in an earlier paper (Markusen 2021), showing how modern tools and software, particularly non-linear complementarity, allow us to build more realistic models without much added complexity.

This is, in a sense, a companion paper that heads further down that road. It is also a pedagogic work that takes existing theory seriously and shows how AGE models can be made more realistic. The specific target of this paper is the incorporation of theory-consistent imperfect competition in an environment of increasing returns to scale. The incorporation of increasing returns and imperfect competition into applied general-equilibrium (AGE) models, beginning with Harris (1984) led to much larger welfare effects from simulated changes such as trade liberalization. But the imperfect competition side of these IO developments has generally failed to incorporate meaningful strategic behavior, largely ruling out firm-level scale and pro-competitive effects.

Specifically, a large, perhaps overwhelming share of papers involving imperfect competition use the "large-group" monopolistic competition specification (LGMC), which assumes that firms are so small that they cannot affect their industry's price index, and hence have constant and exogenous markups. It is bafflingly inconsistent to assume that firms produce with increasing returns to scale, yet have no mass. This has remained true in almost all papers modeling heterogeneous firms, where the most productive firms are very large relative to their industry average. A newly published paper by Balistreri and Tarr (2022) is a major step forward in incorporating heterogeneous firms into AGE models, but it continues to use LGMC. Perhaps my paper can also be seen as a companion paper to Balistreri and Tarr, with the two together pointing the way to modeling heterogeneous firms with non-zero market shares and variable markups added.

There are a number of problem with LGMC that leave our AGE models detached from important realities. (1) trade liberalization (or increased protection) creates no firm-scale effects, no increase (decrease) in productivity. There are no pro-competitive effects, no fall in markups, no strategic behavior. (2) In heterogeneous firm models, all firms charge the same markups. Price ratios are exactly proportional to marginal cost ratios across firms. (3) In models with endogenous multinationals where firms chose between exporting and foreign affiliate production, their choices are not affect by the size of the foreign market. By making firm size constant, growth only adds more varieties at constant scale. No firm will bear the fixed costs of switching to a foreign plant as the foreign market size grows. (4) To emphasize the point that LGMC is devoid of any of the key features of industrial organization economics, large group monopolistic competition with constant markups is equivalent to a simple formulation of production with industry-level external economies of scale. A brief literature review is provided at the end of this section.

Why then are endogenous markups and firm scale effects avoided? Endogenous markups create an added complexity for analytical (algebraic) solutions to models. Under typical formulations such as Bertrand and Cournot, the markup depends on a firm's market share, which is of course an endogenous variable. Even with identical firms as in typical monopolistic-competition models, the market share depends on the number of firms in equilibrium, and this in turn depends on total income. These dependencies means that the markup must be solved for simultaneously with all other endogenous variables.

There have been a few attempts that I know of (and likely some I don't) to incorporate endogenous markups into analytical general-equilibrium models. (A) allow the output of firms in an industry to be perfect substitutes. As I will show, this greatly simplifies the markup formulation, at least for Cournot conjectures. (B) use quasi-linear preferences for consumers. This typically means that the number of firms and hence markups in equilibrium do not depend on total income, and this in turn removes a key interdependency in the simultaneous-equation general-equilibrium model. An important and highly-cited paper that takes this route is Melitz and Ottaviano (2008). As is well understood, these preferences imply that the income elasticity of demand for the industry is zero (borderline inferior good). Yet the industries named to motivate the imperfectly competitive sectors are typically ones that we would conjecture have income elasticities greater than one (superior goods). (C) forsake analytical solutions for numerical simulation. The computer doesn't care about the number of equations/inequalities and variables, nor about the simultaneity among firm scale, markups and total income. Since applied general-equilibrium modeling focuses on numerical simulation, the incorporating of variable markups should or could be a standard feature of modeling.

As noted above, the purpose of the paper is to demonstrate how our basic AGE models can be extended to incorporate Nash competition in increasing-returns sectors. The first objective is to derive markup up formulae under Cournot versus Bertrand conjectures and identify clearly the limiting assumption that leads to constant markups, the LGMC case. Second, I will present a simple general-equilibrium model with endogenous markups and code it into GAMS. Third, I will discuss and identify some awkward calibration and interpretation issues when comparing counter-factual results under Cournot, Bertrand and LGMC alternatives.

Fourth, I will compare simulations under the three alternatives SGC, SGB, and LGMC. The experiment is growth in the economy, a parable for trade among similar economies first exploited by Krugman (1979). While the overall effects of growth on welfare are qualitatively similar, the gains to initially small economies are much larger under either small-group assumption relative to LGMC, but diminish relative to LGMC as economies grow large.

The drivers of the welfare gains under the three cases are quite different. There is a tension between added varieties and increased firm scale, and therefore productivity. The LGMC model has the largest expansion of firm (variety) numbers, but no change in firm scale, markups or efficiency. SGC under the limiting assumption that varieties are perfect substitutes derives its welfare gains from growth entirely the other way around. There is no variety effect, but firm scale increases, moving firms down their average cost curve and to higher productivity. SGB with differentiated products lies in between. The intuition for the result that initially small economies get a bigger boost from growth under Cournot is because their firms' small outputs are on the steep downward sloping section of their average cost curves. For large economies, the firms are down on the flatter sections of their average cost curves, so further growth generates smaller productivity gains than the variety gains under LGMC.

2. Related literature

Perhaps the paper closed to mine is Francois, Manchin and Martin (2013). They have the similar overall goal of considering alternative market structures in AGE models, and indeed it is a broader paper than mine. They derive a markup formula in CES environment that is equivalent to my Bertrand markup. They do not appear to derive the perceived demand elasticity for SGC except for the case of perfect substitutes, a special case of my more general formula. Francois et. al. also don't provide a bridge as to how to incorporate this into a computational model, a major goal here, although they do report simulation results. Finally, they use a iterative (solve and update) procedure that is not necessary in the non-linear complementarity tools I present here.

There are a number of other related papers, that I will mention by publication date. As important early paper is Levinsohn (1993). He hypothesizes that trade liberalization causes firms to behave more competitively and finds that this is supported in the data. Levinsohn's markup formula is the same as my SGC equation if firm's products are perfect substitutes, but also adds in a multiplicative conjectural variable parameter. Bernard, Eaton, Jensen and Kortum (2003) features variable markups, which they describe as having a Bertrand foundation. This is however a different concept than the one I have here, and I comment more on this a little further down. These two papers are important for documenting that constant markups and firm scale are not consistent with empirical evidence, and that the latter in turn require variable markups.

Atkeson an Burstein (2008) use a Cournot-type assumption about firm behavior in a CES environment. Markup equations quite similar to my Cournot formula and the same as mine if preferences are Cobb-Douglas across sectors. This paper also relies on an iterative procedure which is not needed in non-linear complementarity. Melitz and Ottaviano (2008) introduce variable markups into their heterogeneous firm model as noted above. But a drawback, in my view, is that they use quasi-linear preferences which, as I also noted earlier, remove any income effects from demand for the sector's output. This seems counter-empirical, but it does remove the simultaneity among firm scale, markups, and income.

Feenstra (2010) identifies an important role of reduced markups in the overall gains from trade. But in arguing for his alternative approach, he seems to suggest that CES functions imply constant markups. My formulation here makes it clear that constant markups and fixed firm scale are not a characteristic of CES preferences, but are due to the *added assumption* that firms have no mass, or alternatively that a firms' market share is zero, even for the largest firms. Behrens and Urata (2012) use a variable elasticity of substitution formulation to address similar issues in a model that includes variable markups and pro-competitive effects. However, the paper also makes the incorrect assertion that a CES formulation has no firm-scale and pro-competitive effects. Again, this is due to the zero market-share assumption not to CES.

Amiti, Itskhoki and Konings (2014) have variable markups in a CES framework. I had some difficulty in understanding how the markup rule is derived from underlying imperfectly competitive behavior, but the role of market shares is quite similar to what I derive. Atkin, Chaudhry, Chaudry, Khanderwal and Verhoogen (2015) show that larger firms charge higher markups, and that the elasticity of markups with respect to firm size is significantly greater than the elasticity of costs. De Loecker, Goldberg, Khandelwal and Pavcnik (2016) examine how prices, markups, and marginal costs respond to trade liberalization. Markups are estimated empirically, with no theoretical concept imposed on the data. They also show that constant markups are not consistent with the data.

Hsu, Lu and Wu (2020) have a variable markup mechanism in an environment of heterogeneous firms that seems closely related to the mechanism in Bernard et. al. (2003). As noted above, they describe behavior as Bertrand, but this concept of Bertrand is not consistent with the what I am deriving as the classic Nash Cournot and Bertrand mechanisms. In both their papers, the most productive firm in a sector prices at the minimum of either its (Bertrand) monopoly price, or the marginal cost of the second most productive firm. I would characterize this as more a limit pricing or preemption strategy. Hsu et. al. do find that pro-competitive gains do account for a sizeable proportion of the gains from trade-cost reduction which is the important motivation for my paper.[1]

3. The CES Marshallian demand function

An appendix to the paper derives the Marshallian (uncompensated) CES demand function, which will be a review for most readers, or an asset for your students to exploit. I do not derive a general case, but stick with a special case which is popular in the extensive theoretical and empirical literatures. This special case involves a two-sector economy, with Cobb-Douglas preferences between the two sectors. One sector, *Y*, is a homogeneous good produced with constant returns to scale by a competitive industry. The other sector is composed of an endogenous number of symmetric but imperfectly substitutable products, *X*, with an elasticity of substitution $\sigma > 1$ among the varieties.

Utility of the representative consumer between sectors, and the symmetry of varieties within a group of goods allows us to write utility as follows ($0 < \alpha < 1$).

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[1] I define a Bertrand equilibrium as the solution to Nash best-response behavior where firms view the other firms' prices as fixed. Especially if the goods are poor substitutes, it will generally not be optimal for the most productive firm to lower its price to the marginal cost of the next most productive firm to block entry, and that is not a Nash equilibrium in any case. Markusen (2002) has a chapter entitled "Incumbency, preemption and persistence".

Excuse me ending the review with a couple of self-citations. I believe that Markusen (1981) was the first trade paper to show how pro-competitive gains from trade occur with Cournot competition. Markusen (1990) was, I think, the first to show the equivalence of LGMC and industry-level external economies of scale, emphasizing the lack of strategic behavior in LGMC. Carr, Markusen and Maskus (2001) show empirically that foreign affiliate production increases more than in proportion to country size, indicating a switch from exporting to foreign production as markets grow, a result that is not consistent with the theoretical implication of LGMC.

$$U = X_c^{\beta} Y^{1-\beta}, \qquad X_c \equiv \left[\sum_i^N (X_i)^{\alpha}\right]^{1/\alpha} \qquad \sigma = \frac{1}{1-\alpha} \tag{1}$$

where the number of varieties N is endogenous and X_c is often referred to as a composite commodity. X_c is a utility value, not the sum over varieties of the total units produced. σ is the elasticity of substitution among the X goods.

This function permits the use of two-stage budgeting, in which the consumer first allocates total income (M) between Y and X_c. Let e denote the minimum cost of buying one unit of X_c at prices p_i for the individual varieties (i.e., e is the unit expenditure function for X_c). Y is numeraire. First-stage budgeting yields:

$$Y = (1-\beta)M \qquad X_c = \beta M/e \qquad e(p) = \min(X_i) \sum_i p_i X_i \quad st \quad X_c = 1 \tag{2}$$

A convenient feature of the Cobb-Douglas upper nest is that the share of expenditure on each sector is a constant. Let $M_x = \beta M$ be the expenditure on X in aggregate. The appendix solves for the demand for a given X variety and for the price index e. These are given by

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad X_i \equiv p_i^{-\sigma} e^{\sigma-1} M_x \tag{3}$$

$$e = N^{\frac{1}{1-\sigma}} p \qquad \text{if all prices equal} \tag{4}$$

An increase in the *range* of goods lowers the cost e of buy a unit of (sub)utility X_c. As one quick check, note that (3) is homogeneous of degree zero in prices and income as it should be.

4. Bertrand price elasticity of demand for an individual good (holds *prices* of other goods constant)

We can now derive an individual firm's perceived price elasticity of demand under the Bertrand assumption that the firm views the prices of other firms (varieties) as constant, and also views total income as constant. The latter also implies the firm views income spent on X goods, M_x, as constant under the assumption that preferences are Cobb-Douglas between X_c and Y. Using (3), we can derive the share of X sector expenditure on variety X_i, denoted s_i.

$$X_i = p_i^{-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1} M_x \qquad s_i \equiv \frac{p_i X_i}{M_x} = p_i^{1-\sigma}\left(\sum p_j^{1-\sigma}\right)^{-1} \tag{5}$$

To visually simplify the algebra a bit, we will us the following shorthand

$$(...) \equiv \left(\sum p_j^{1-\sigma}\right) \tag{6}$$

The response of demand to an increase in the firm's own price, holding other prices constant and expenditure on the sector constant, is given by

$$\frac{\partial X_i}{\partial p_i} = -\sigma p_i^{-\sigma-1}(...)^{-1}M_x - (1-\sigma)p_i^{-\sigma}(...)^{-2}p^{-\sigma}M_x$$

$$= -\sigma p_i^{-\sigma-1}(...)^{-1}M_x + (\sigma-1)p_i^{-2\sigma}(...)^{-2}M_x \qquad (7)$$

We can then derive the perceived Bertrand elasticity.

$$p_i\frac{\partial X_i}{\partial p_i} = -\sigma p_i^{-\sigma}(...)^{-1}M_x + (\sigma-1)p_i^{-2\sigma+1}(...)^{-2}M_x$$

$$\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = -\sigma + (\sigma-1)p_i^{-\sigma+1}(...)^{-1} = -\sigma + s_i(\sigma-1) \qquad (8)$$

A convention is to define the Marshallian price elasticity as positive so in order to aid memory: all variables and parameters in the model are positive.

$$\eta_b \equiv -\frac{p_i}{X_i}\frac{\partial X_i}{\partial p_i} = \sigma - (\sigma-1)p_i^{-\sigma+1}(...)^{-1} = \sigma - s_i(\sigma-1) \qquad \text{Bertrand} \qquad (9)$$

There are several things to note about this elasticity. (a) as the firm's market share goes to zero, the demand elasticity converges to σ, the elasticity of substitution among the X goods. This is precisely the case of *large group monopolistic competition* so widely used in the literature: bizarrely, even though firms have increasing returns to scale, they all have zero market share. But this assumption, though it defies logic, is immensely useful in that it reduces the simultaneity of a model by making the firm's markup exogenous to all other variables in the model. (b) the firm's Bertrand perceived elasticity is decreasing in the firm's market share, and becomes equal to one when the firm is a monopolist in the X sector: $s_i = 1$.

5. Cournot price elasticity of demand for an individual good (holds *quantities* of other goods constant).

The appendix shows that we can solve for the inverse demand functions using the same procedure. The inverse demand function is

$$p_i = X_i^{-\delta}\left[\sum X_j^{1-\delta}\right]^{-1}M_x \qquad \delta \equiv \frac{1}{\sigma} \qquad (10)$$

which is homogeneous of degree zero in all quantities and income. The expenditure share on good i given by

$$s_i \equiv \frac{p_i X_i}{M_x} = X_i^{1-\delta}\left(\sum X_j^{1-\delta}\right)^{-1} \qquad (\ldots) \equiv \left(\sum X_j^{1-\delta}\right) \tag{11}$$

We can now use the same procedure as in the previous section to get the inverse Cournot (holding *quantities* of other goods constant) perceived elasticity of demand.

$$\frac{\partial p_i}{\partial X_i} = -\delta X_i^{-\delta-1}(\ldots)^{-1} M_x - (1-\delta) X_i^{-\delta}(\ldots)^{-2} X_i^{-\delta} M_x \tag{12}$$

$$= -\delta X_i^{-\delta-1}(\ldots)^{-1} M_x - (1-\delta) X_i^{-2\delta}(\ldots)^{-2} M_x$$

$$X_i \frac{\partial p_i}{\partial X_i} = -\delta X_i^{-\delta}(\ldots)^{-1} M_x - (1-\delta) X_i^{-2\delta+1}(\ldots)^{-2} M_x$$

$$-\frac{X_i}{p_i}\frac{\partial p_i}{\partial X_i} = \delta - (\delta-1) X_i^{-\delta+1}(\ldots)^{-1} = \delta - (\delta-1) s_1 \tag{13}$$

$$\frac{1}{\eta_c} = \frac{1}{\sigma} - \left(\frac{1}{\sigma} - 1\right) s_i = s_i + (1-s_i)\frac{1}{\sigma} \tag{14}$$

Invert this to get the Cournot perceived elasticity

$$\eta_c = \frac{\sigma}{\sigma s_i + (1-s_i)} = \frac{1}{s_i + (1-s_i)\frac{1}{\sigma}} \qquad \text{Cournot} \tag{15}$$

Although the Cournot elasticity seems quite different from the Bertrand formula in (9), they have the same values at the extremes $s_i = 0$ and $s_i = 1$. Here is the comparison of (9) and (15):

At	$s = 0$,	$\eta_c = \eta_b = \sigma$	(LGMC)
At	$s = 1$,	$\eta_c = \eta_b = 1$	(monopoly)
For	$0 < s < 1$,	$\eta_c < \eta_b < \sigma$	(Cournot is less elastic)
For	$0 < s < 1$ and $\sigma = \infty$,	$\eta_c = \frac{1}{s}$, $\eta_b = \infty$	(perfect substitutes)

For this last case of perfect substitutes, the Bertrand elasticity is infinite, which implies perfect competition. However, this cannot be supported in equilibrium with increasing returns to scale. The Cournot perceived becomes simply the firm's inverse market share.

The relationship between the firm's perceived elasticity of demand and it's market share are graphed for Bertrand and Cournot in Figures 1a (perceived elasticity) and 1b (markup) using the common elasticity of substitution $\sigma = 5$. As just noted, they have the same values at the extremes of market share, but the perceived elasticity is much less under Cournot for intermediate values. It is probably well understood that the firm's markup is related to the inverse of the demand elasticity, so this in turn tell us that Cournot will be a less competitive form of behavior.

Below, I will assume that all firms have the same technology (marginal and fixed costs), which implies that all active firms have the same market share ($s = 1/n$ where n is the number of firms). But note that our elasticity formulas do not require identical firms or rather permits heterogeneous firms with different productivities, thus having different (and endogenous) market shares and prices in general equilibrium.

6. Markup formulae

Consider first profits for a Cournot competitor using (10) above for the inverse demand function. Π denotes profit, *C* the firm's total cost, and *mc* denotes marginal cost. Firm i maximizes profits with respect to output holding the outputs of other firms constant.

$$\Pi_{ic} = p_i(X_i, \bar{X}_j)X_i - C(X_i) \qquad j \neq i \tag{16}$$

Dropping the i subscript for clarity, the first-order condition is given by

$$\frac{\partial \Pi}{\partial X} = p + X\frac{\partial p}{\partial X} - \frac{dC}{dX} = p + p\left[\frac{X}{p}\frac{\partial p}{\partial X}\right] - \frac{dC}{dX} = p\left[1 - \frac{1}{\eta_c}\right] - mc = 0 \tag{17}$$

The markup *(mk)*, is then simply $mk = 1/\eta_c$. Profits and the first-order condition in the Bertrand case are given by

$$\Pi_{ib} = X_i(p_i, \bar{p}_j)p_i - C(X_i(p_i, \bar{p}_j)) \qquad j \neq i \tag{18}$$

$$\frac{\partial \Pi}{\partial p} = X + p\frac{\partial X}{\partial p} - \frac{dC}{dX}\frac{\partial X}{\partial p} = 0 \tag{19}$$

Multiply through by p/X (again defining η as positive)

$$\frac{\partial \Pi}{\partial p}\frac{p}{X} = p + p\left[\frac{p}{X}\frac{\partial X}{\partial p}\right] - \frac{dC}{dX}\left[\frac{p}{X}\frac{\partial X}{\partial p}\right] = 0$$

$$\frac{\partial \Pi}{\partial p}\frac{p}{X} = p - p\eta_b + mc\eta_b = p - \left[\frac{\eta_b}{\eta_b - 1}\right]mc = p\left[1 - \frac{1}{\eta_b}\right] - mc = 0 \qquad (20)$$

Summarizing, the markups in the Bertrand and Cournot cases are the same in terms of the firm's (inverse) perceived demand elasticity, but the formula for this elasticity differs in the two cases.

$$p\left[1 - \frac{1}{\eta_b}\right] = mc \quad \text{Bertrand} \qquad\qquad p\left[1 - \frac{1}{\eta_c}\right] = mc \quad \text{Cournot} \qquad (21)$$

Beware of differences in the international trade literature as to what is being defined as the "markup". The markups using the output price as a basis in (21), $mk = 1/\eta$, are often flipped around and the markup is defined as follows.

$$p = \left[\frac{\eta}{\eta - 1}\right]mc \qquad (22)$$

Further confusion can occur with the markup (*mk*)defined as the whole term in brackets in (22), so that the markup is greater than one: $mk = p/mc$, or the term in brackets is interpreted as [1 + *mk*]. For the remainder of this paper, "markup" will be defined as those in (21), so that the markup is just the inverse of the firm's perceived elasticity of demand. I like this version since the left-hand side of the equations in (21) are just marginal revenue as usually defined.

7. Calibration subtleties

There are four parameter in the market formulae, some of which but usually not all of which may be in your data set: p, mc, σ, and s. With identical firms, s is just the inverse of the number of active firms. In a general-equilibrium model, three of these things are variables while σ is always treated as a parameter. The problem is that, if you have data or estimates of all four, then it is almost inconceivable that they will satisfy either the Cournot or Bertrand equations in (29). If you have three of the four, it may still well be the case that the fourth cannot be set at a permissible value that satisfied either markup rule. For example, if you have p, mc and σ, there may not be a value of s between 0 and 1 that satisfied one or both markup rules. Indeed, I will give an example of this later on where the only value of σ consistent with an s between 0 and 1in the Cournot formula is $\sigma = \infty$, perfect substitutes.

Suppose that you have p and mc, and therefore the markup, or the markup is calculated independently by usual methods giving the wedge between price and marginal cost. Then for either Cournot or Bertrand, the elasticity of substitution, σ, and market concentration s cannot be chosen independently. You can pick an s as an average firm market share across a distribution of firms (thereby implying the calibrated number of firms = $1/s$), but there is no freedom to choose σ. If instead you have a value for σ, then s is determinant. Does it matter how you divide a calibration between s and σ in this example? Yes it does, and I will try to show this in a later example. But to preview the point, if data is calibrated to a small s (large number of firms), then especially under Cournot competition an increase in market size (as in trade integration) will have weak pro-competitive effects relative to calibrating to a large *s* (small number of firms).

Now suppose that we want to run scenarios, such as trade liberalization or protection, and compare results under alternative assumptions such as Cournot, Bertrand, and large-group monopolistic competition. Further, imagine that we wish to run these three options by calibrating each case to the initial benchmark data. Finally, suppose that we have estimates of an industry's markup and therefore the wedge between price and marginal cost and we wish to hold this the same in the three calibrations. The problem is that this will require either changing the benchmark *s* in the three cases ($s = 0$ in large-group mc by definition) and/or changing σ, the elasticity of substitution in consumption. Refer back to the Bertrand formula in (14), and note that for an *s* of 0.25 (four firms), then we have to use a different σ in the Bertrand and LG cases to make $\eta_b = \eta_{lg}$ where the subscript lg refers to LG monopolistic competition.

Let the observed markup by 0.2 on a output-price basis in (21) (1.25 on a (gross) marginal cost basis (22)). Then the LG elasticity of substitution calibrates to $\eta_{lg} = \sigma = 5$. The Bertrand σ (equation (14)) would have to be $\sigma = 6.333$ if it is calibrated to four firms ($s = 0.25$) to get $\eta_b = 5$. But the dilemma is now that, in comparing counter-factual experiments under LG versus Bertrand, we are using two different consumer preference specifications. So the differences between the counter-factual results are going to be partly due to the pro- (or anti-) competitive effects in Bertrand and partly due to the difference preference elasticity.

This tension, and the caution needed to interpret counter-factual comparisons, also occurs with Cournot versus Bertrand. In our example of the (output price) markup 0.20 in (21), if we wish to calibrate Cournot to four firms to match Bertrand ($s = 0.25$), then there is no σ large enough to produce a perceived elasticity of $\eta_c = 5$. If instead we set σ equal to its limiting value of $\sigma = \infty$, then calibrating to five firms (s = 0.20) does produce a perceived elasticity $\eta_c = 5$ and a markup of 0.20. The Bertrand-Cournot comparison is then $\sigma = 6.333$, $s = 0.25$ (Bertrand) versus $\sigma = \infty$, s = 0.20 (Cournot).

There is at least one other way of calibrating Cournot and Bertrand to the same σ and same *s* (or number of firms $1/s$). This is to use a kluge or fudge factor, which is an exogenous parameter multiplied on the Cournot markup so as to give it the same initial value as Bertrand.[2] As noted above at the end of section 4, $\eta_c < \eta_b$ for any value $0 < s < 1$, so the Cournot markup will be greater than the Bertrand markup for the same values of σ and *s*. The kluge parameter multiplied on the Cournot markup will thus be less than one to give it the same value as the Bertrand markup. While a pure theorist might object to this, it does mean that counter-factuals are compared with the same consumer preferences and same initial degree of competition as measured by a representative firm's market share.

[2]Kluge (or spelled kludge) is a term from engineering, computer science and math programming. It means a workaround or quick-and-dirty solution that, while expedient, is clumsy and inelegant. A closely related technique under the term "conjectural variation" is to introduce a multiplicative parameter in the markup equation so as to make the theoretical markup such as the ones derived above be consistent with the empirical measure of markup characterizing the initial data (e.g., Levinsohn 1993).

8. Translating theory into GAMS using non-linear complementarity

In this section, I will show how to translate theory plus an initial set of micro-consistent data into GAMS. I will keep it brief, because I go though much of the process in my earlier JGEA paper, Markusen (2021). Please refer to that if the exposition here leaves too many gaps. I will go through just one of the three calibrations, choosing SGB as the example. There other programs can be obtained from me directly. After developing the SGB case, we will run and compare this with LGMC and SGC.

Please refer to the GAMS file at the end of the paper. The top of the file gives a small data matrix, with production activities and income levels the columns, and markets as the rows. Markets are labeled with their prices, which are complementary to market clearing equations. A column is a list of output (positive) and inputs (negative) in value terms. A row lists supply (positive) and demand (negative) in that market. Thus micro-consistency requires that both row and column sums are zero.

There is a single factor of production, labor. The first column is the technology for the total units of X produced, with inputs (costs or expenditures) being 160 units of labor and 40 units markup. We will choose the price of labor , PL =1 as numeraire, with the 200 units of X interpreted as 160 units at a price PX = 1.25. Fixed costs are treated as a production activity with the (scaled) output being N, the number of firms/varieties. 40 units of labor produce 40 units of fixed costs, and this will be interpreted as 4 firms with a fixed costs of 10 (parameter FC) per firm. The benchmark markup MK will then be 0.20 (40/200), using the output-basis formula in (21) to define our markup equation.

The Y sector is perfectly competitive, using 200 units of labor to produce 200 units of Y, so PY =1 with L being the numeraire. Welfare (W) is treated as a produced good: inputs of 200 each of X and Y in value produce 400 units of welfare (utility).

The final two columns are consumer (CONS) income and entre (ENTRE) income. The consumer is endowed with 400 units of labor and spends it all on “buying” utility W. The entrepreneur is sort of a dummy agent who receive markup revenues as income and spends all of that income buying fixed costs (starting firms). This is the technique I developed a very long time ago for getting free entry and exit into the model. If markup revenues go up, there is an increased demand for fixed costs, which translates into an increased number of firms. The number of firms then translates into a change in the markup, which feedback to the other variables including firm scale in general equilibrium.

There are three parameters in the model, with SI being the elasticity of substitution in consumption, set at SI = 6.333 as discussed above. Fixed costs per firm are FC = 10, and the labor endowment is ENDOWL = 400. Then there are the lists of equations and variables, and I have listed them in the same order according to their complementarity relationship.

Then there is the weak inequalities themselves, using GAMS notation which writes them as greater-than-or-equal to (=G=) expressions. Please refer to my earlier paper if a better understanding is needed. Rather than go through them in details, I will just comment on those relating to the industrial-organization aspects of the model. The first of four pricing equations uses the markup on the output basis definition. INDEX is just a definitional equation for the price index for the differentiated X goods.

There are five market-clearing equations, written as supply ≥ demand. DN is the new equation: the supply of fixed costs is production for the N firms times FC units per firm. The demand is entrepreneur income divided by the price of fixed costs, PN. Further down, the labor-market-clearing equation DL includes resources for fixed costs, and the income balance equation for the entrepreneur IENTRE gives markup revenues on the right-hand side. The markup equation MARKUPB uses the inverse of the perceived elasticity of demand for the Bertrand cases in (16).

Final steps are the declaration of a model, with each inequality associated with its correct complementary variable (the "dot" notation in GAMS). Then starting values for all variables are given, important in all non-linear problems (the "dot" L for "level value" after the variable). The last step is then the solve statement (MCP is for mixed complementarity problem, designating the solver to be used).

As noted above, I have also prepared three other versions of the model calibrated to the same data. These are SGC, LGMC. In all case, I run the same experiment to see how the results differ from one another. The experiment I report here is to loop over size of the economy, specifically 25 values of the labor endowment, considering values both smaller and larger than the benchmark value. Here again are the values used in the first three cases in order to calibrate to the same data (N is the number of symmetric firms implied by the calibrated value of s).

LGMC:	$\sigma = 5$	$s = 0$	$(N = \infty)$	
SGB:	$\sigma = 6.3333$	$s = 0.25$	$(N = 4)$	(32)
SGC:	$\sigma = \infty$	$s = 0.20$	$(N = 5)$	

Results for the three alternatives in (32) are shown in Figures 2-5. The common calibration point is size = 2, with size running from 0.2 to 5.0 on the horizontal axis. I will first describe all four set of results and then offer some interpretations about their similarities and differences.

Figure 2 gives the result for welfare per capita. All curves are particularly steep for small economies, but especially for the two small-group cases. For Cournot, doubling the size of the economy from 0.2 to 0.4 increases welfare by 23 percent. But at the upper end, welfare for the Cournot case runs out of steam, while LGMC still keeps chugging alone. Again for Cournot, doubling the size of the economy from 2.5 to 5.0 increases welfare by 3 percent.

Figures 3-5 are closely intertwined and simultaneously determined. Figure 3 gives an index of firm numbers. For LGMC, the number of firms is just linear in the size of the economy: double the size, double the number of firms (varieties). SGB is a slightly flatter curve, but it is also linear. SGC is concave: for small economies entry response significantly to an increase in size, but this effect runs out of steam for large economies. A major difference among the three cases is firm size/scale (which is also productivity) as shown in Figure 4. As I'm sure is well known (and which I have complained about), LGMC produces a constant output per firm so that there is no firm-scale effect and no firm-level productivity increase from a larger economy. SGC produces a large firm-scale effect. Figure 5 compares the firm markups in the three cases. The markup is constant for LGMC as is well know. As noted before, this is the major attraction of LGMC since it removes an important endogeneity in the general-equilibrium model. As in Figures 2-4, SGB lies between LGMC and SGC in Figure 5, the latter having the biggest effect.

Although the models capture the general-equilibrium simultaneity among firm numbers, firm scale, and markups, it is still a stylized model. The important properties in Figures 2-5 should be viewed as qualitative, and the quantitative differences shown should not be given too much emphasis. We now turn to interpreting these differences in Figure 2-5. I will stick to comparing the LGMC and SGC cases, since SGB generally lies between the two.

The three per-capita welfare curves in Figure 2 are qualitatively similar, but note that there is a very substantial difference for a very small economy (e.g., size 0.2-0.4), a point I will return to shortly. The basic tension between LGMC and SGC is the number of varieties (firms) versus firm scale, the latter corresponding to productivity. A larger number of varieties raises consumer welfare as is very well understood: half as much each of twice as many varieties raises utility. But there are no production efficiency effects with LGMC. SGC, with the calibrated value of $\sigma = \infty$ (perfect substitutes), has no variety effect, but has a strong productivity effect in the growing economy. Firms move down their average cost curve. One approach generates direct utility gains, the other direct production efficiency gains.[3] But although these very different effects result in similar per-capita welfare effects in Figure 2, I do not make any claim that this would hold in more complex models.

In our free-entry zero-profit models, firm scale and firm markup are inextricably linked. But conceptually, there are two separate welfare effects which could show up separately in a model without free entry. First, there is a pure production efficiency effect: when a firm increases output, it moves down its average cost curve, increasing productivity. Second, there is a distortion-reduction gain as the wedge between price and marginal cost is reduced. This can be termed a pro-competitive effect. In this paper with zero profits in equilibrium, these cannot be separated: the lower markup effect on profits is absorbed by lower average cost. But consider, for example, a model with a monopolist in sector *X* (the elasticity of substitution between *X* and *Y* will now need to be greater than one, Cobb-Douglas won't work). As the economy expands, *X* output will expand lowering average cost, leading to a per-capita welfare gain. But there will be no change in the markup. There is no pro-competitive effect in this case.

I do think that there is some intuition from theory as to why LGMC generates a larger welfare gain relative to SGC starting at the calibration point size = 2, and a smaller gain starting at the left of Figure 2 with a very small economy. Figures 2-5 indicate that for small-group cases, particularly SGC, the effect of growth on per-capita welfare diminishes as the number of firms gets much beyond 5. If a model is calibrated with 10 firms, for example, then increases in size will show very little welfare effects in the SGC while effects in the LGMC case will

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[3]A little historical note with a bit of a sweeping generalization. In the 1980s, positive theory concentrated on LGMC (e.g., Krugman 1979, 1980, Ethier 1982), while strategic trade policy with free entry and exit concentrated on SGC (e.g., Venables 1985, Horstmann and Markusen 1986).

continue to be strong. For SGC, the effect of growth on the crucial variable output per firm (falling average cost) diminishes as output grows. The average cost curve is a rectangular hyperbole. The per-capita effect of LGMC due to added varieties is more sustained as the economy grows.

To illustrate the importance of falling average costs and pro-competitive gains for a small economy, we can renormalize welfare in Figure 2, giving each case a welfare value of one at size 0.2. Let's think of these as three different economies, have three different initial welfare levels. What we are doing is looking at the proportional gain in each of those welfare levels starting at size = 0.2. The renormalization of the same data gives us Figure 6. The gains from growth are much larger for SGC and SGB starting at a small size. This is, I believe, due to the fact that a given expansion in firm scale as shown in Figure 4 generates a large decline in average cost (increase in productivity). Or we can put it another way, which is that the expansion in firm scale causes a sharp decline in the wedge between price, equal to average cost in a free-entry model, and (constant) marginal cost, which is the steep fall in the markup as shown in Figure 5. On the right-hand section of Figure 6, the Cournot economy runs out of steam because the average cost curve is now very flat and there is no variety effect. Bertrand continues to chug along, in spite of the diminished firm-scale effect since it does have a variety effect.

9. Summary

My motivation for writing this paper has both a "carrot" and a "stick" aspect. The stick motive is a very long dissatisfaction with the use of the large-group monopolistic-competition idea in both analytical and applied (numerical) general-equilibrium models. This is used in an environment of increasing returns to scale and imperfect competition, but it implies that firms have zero market shares under Nash Cournot or Bertrand conjectures. Yet the industries used to motivate the models and even more so in heterogeneous firm models are typically dominated by a small number of large firms. My assertion is that the LGMC assumption is a kluge to avoid the added simultaneity that a proper theoretical approach would entail.

The carrot is to show that introducing positive firm size and market share, along with Cournot and/or Bertrand behavior by these firms is not a big challenge in numerical modeling. And it can be done without resorting to alternative kluges such as quasi-linear preferences in which the relative industries are borderline inferior goods. I show that the markup rules for Bertrand and Cournot involve only the elasticity of substitution (parameter) among the industries varieties and the equilibrium market shares of the active firms (endogenous variable). Although I treat a special case of symmetric firms, it is clear that the markup rules apply perfectly in a world of heterogeneous firms with differing market shares..

The next section of the paper shows how to code such a generalization into a numerical model using GAMS. The three forms of behavior, LGMC, SGB, and SGC are then compared via a simulation in which the key parameter is change in market size, a parable for trade liberalization among similar economies first exploited by Krugman (1979). While we cannot be sure that the results generalize, the simulation show that, for initially quite small economies the positive effects of growth on welfare are much stronger under the small-group assumptions than under LGMC. But conversely, for initially quite large economies a corresponding proportion growth (e.g., doubling in size), the effect of growth on welfare is much smaller under small-group, especially Cournot, than under LGMC. The reason is that, for small economies, growth results in a steep fall in firm average cost (increase in productivity) that dominates the increase in variety in LGMC. But for large economies, the initial firm size is much larger under Cournot and thus the average cost curve is much flatter. The increase in variety under LGMC then dominates any further increase in productivity under small-group Cournot.

These results suggest a policy implication for modelers, which is that the us of LGMC may significantly underestimate the gains from liberalization for small countries. And these are likely precisely the economies that often have highly concentrated section under protective policies.

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APPENDIX

solve for the demand for a given X variety, and for the price index e. The consumer's sub-problem maximizing the utility from X goods subject to an expenditure constraint (using λ as a Lagrangean multiplier) and first-order conditions are:

$$\max X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} + \lambda\left(M_x - \sum p_i X_i\right) \quad \Rightarrow \quad \frac{1}{\alpha}\left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}-1}\alpha X_i^{\alpha-1} - \lambda p_i = 0 \qquad \text{(A1)}$$

Let σ denote the elasticity of substitution among varieties. Dividing the first-order condition for variety i by the one for variety j,

$$\left[\frac{X_i}{X_j}\right]^{\alpha-1} = \frac{p_i}{p_j} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j}\right]^{\frac{1}{\alpha-1}} = \left[\frac{p_i}{p_j}\right]^{-\sigma} \qquad \textit{since} \quad \sigma = \frac{1}{1-\alpha} \qquad \text{(A2)}$$

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \qquad p_j X_j = p_j p_j^{-\sigma} p_i^{\sigma} X_i \qquad \sum p_j X_j = M_x = \left[\sum p_j^{1-\sigma}\right] p_i^{\sigma} X_i \qquad \text{(A3)}$$

Inverting this last equation, the demand for an individual variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma-1}{\sigma} \qquad \text{(A4)}$$

Use X_i to construct X_c and then solve for e, noting the relationship between α and σ.

$$X_i^{\alpha} = X_i^{\frac{\sigma-1}{\sigma}} = p_i^{1-\sigma}\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha}$$

$$\sum X_i^{\alpha} = \left[\sum p_i^{1-\sigma}\right]\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} M_x^{\alpha}$$

$$X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} = \left[\sum X_i^{\alpha}\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} M_x \qquad \text{(A5)}$$

$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad \text{if all prices equal: } e = N^{\frac{1}{1-\sigma}} p \qquad \text{(A6)}$$

An increase in the *range* of goods lowers the cost of buy a unit of (sub)utility. Having derived e, we can then use equation (A6) in (A4) to get the demand for an individual variety using e.

$$X_i \equiv p_i^{-\sigma} e^{\sigma - 1} M_x \qquad since \qquad e^{\sigma - 1} = \left[\sum p_j^{1-\sigma}\right]^{-1} \tag{A7}$$

A similar procedure allows us to derive the inverse demand function which is used in the Cournot markup. Using the same first-order condition (A2), this is as follow

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \quad p_j^{\sigma} = p_i^{\sigma} X_i X_j^{-1} \quad p_j = p_i X_i^{1/\sigma} X_j^{-1/\sigma} = p_i X_i^{\delta} X_j^{-\delta} \quad \delta \equiv \frac{1}{\sigma} \tag{A8}$$

$$\sum p_j X_j = M_x = p_i X_i^{\delta}\left[\sum X_j^{1-\delta}\right]$$

This gives the inverse demand function as

$$p_i = X_i^{-\delta}\left[\sum X_j^{1-\delta}\right]^{-1} M_x \tag{A9}$$

with the expenditure share on good i given by

$$s_i \equiv \frac{p_i X_i}{M_x} = X_i^{1-\delta}\left(\sum X_j^{1-\delta}\right)^{-1} \qquad (\ldots) \equiv \left(\sum X_j^{1-\delta}\right) \tag{A10}$$

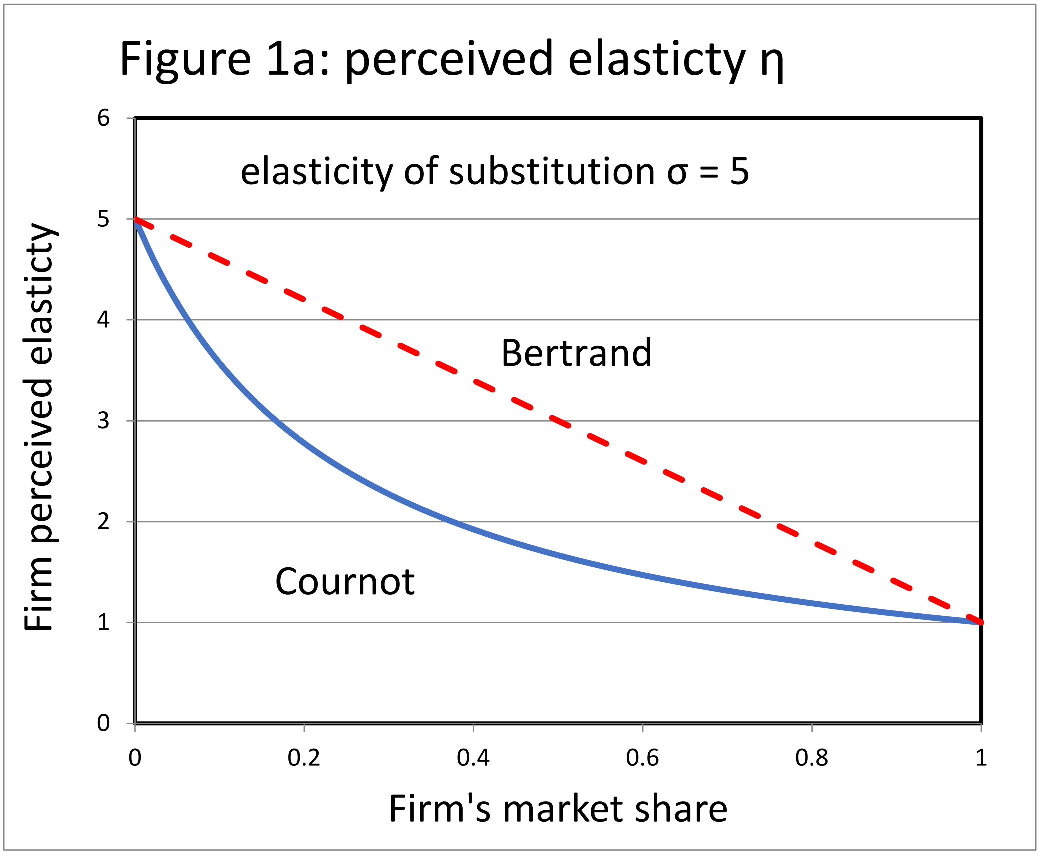


Figure 1a: perceived elasticty η
elasticity of substitution σ = 5
Bertrand
Cournot
Firm perceived elasticty
Firm's market share
0
1
2
3
4
5
6
0
0.2
0.4
0.6
0.8
1

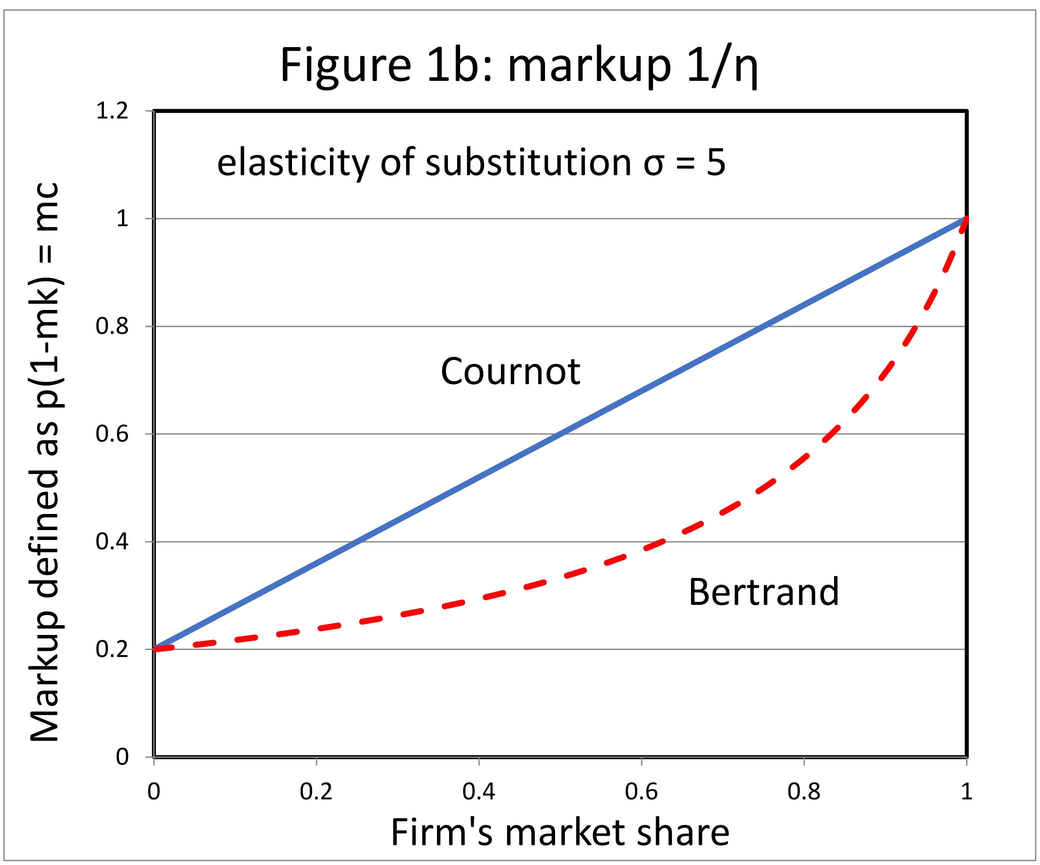


Figure 1b: markup 1/η
elasticity of substitution σ = 5
Cournot
Bertrand
Markup defined as p(1-mk) = mc
Firm's market share
0
0.2
0.4
0.6
0.8
1
1.2
0
0.2
0.4
0.6
0.8
1

Figure 2: Welfare per capita

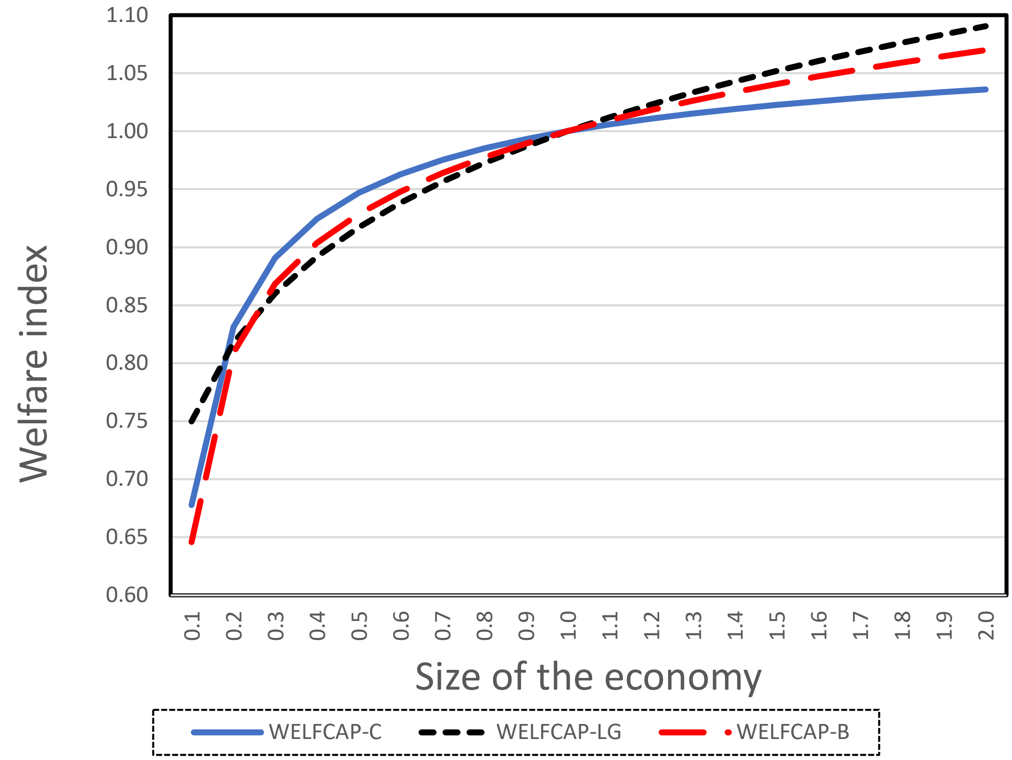
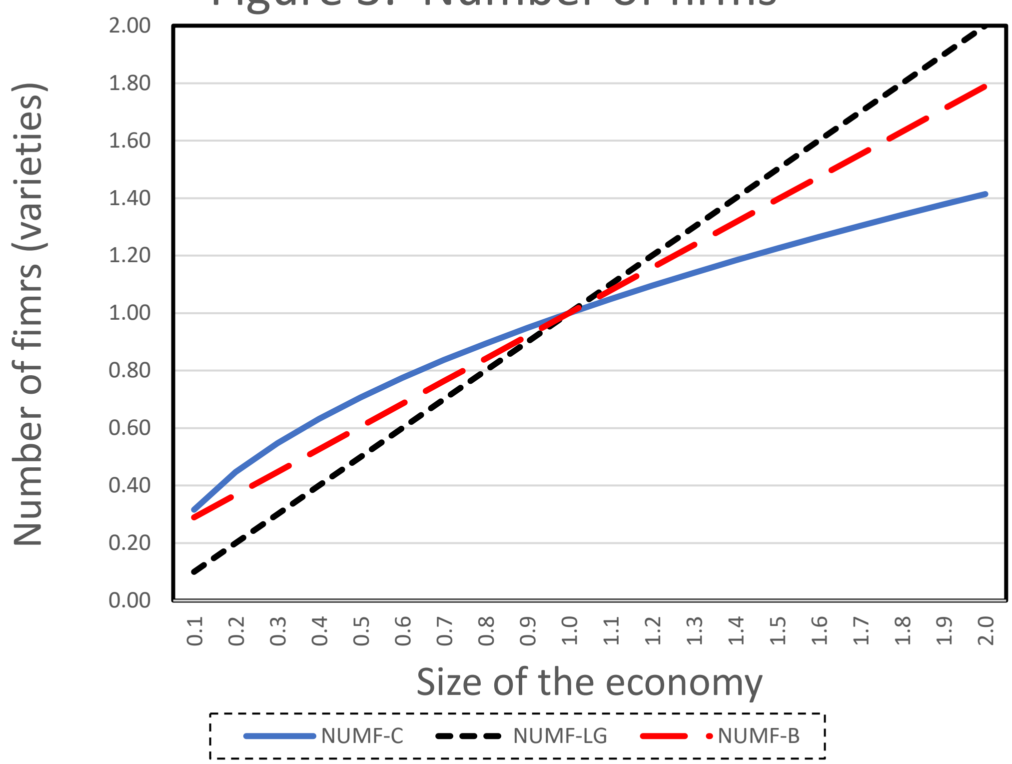


Figure 3: Number of firms



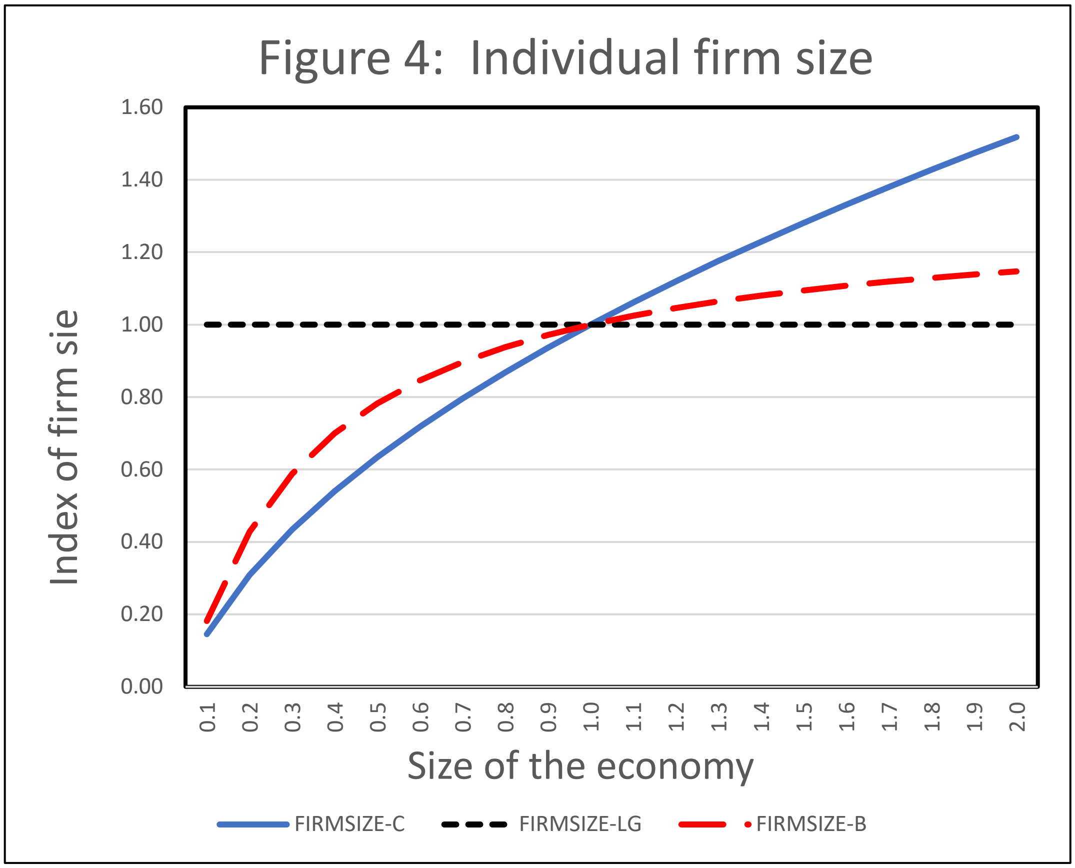


Figure 4: Individual firm size
Index of firm sie
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
Size of the economy
FIRMSIZE-C
FIRMSIZE-LG
FIRMSIZE-B

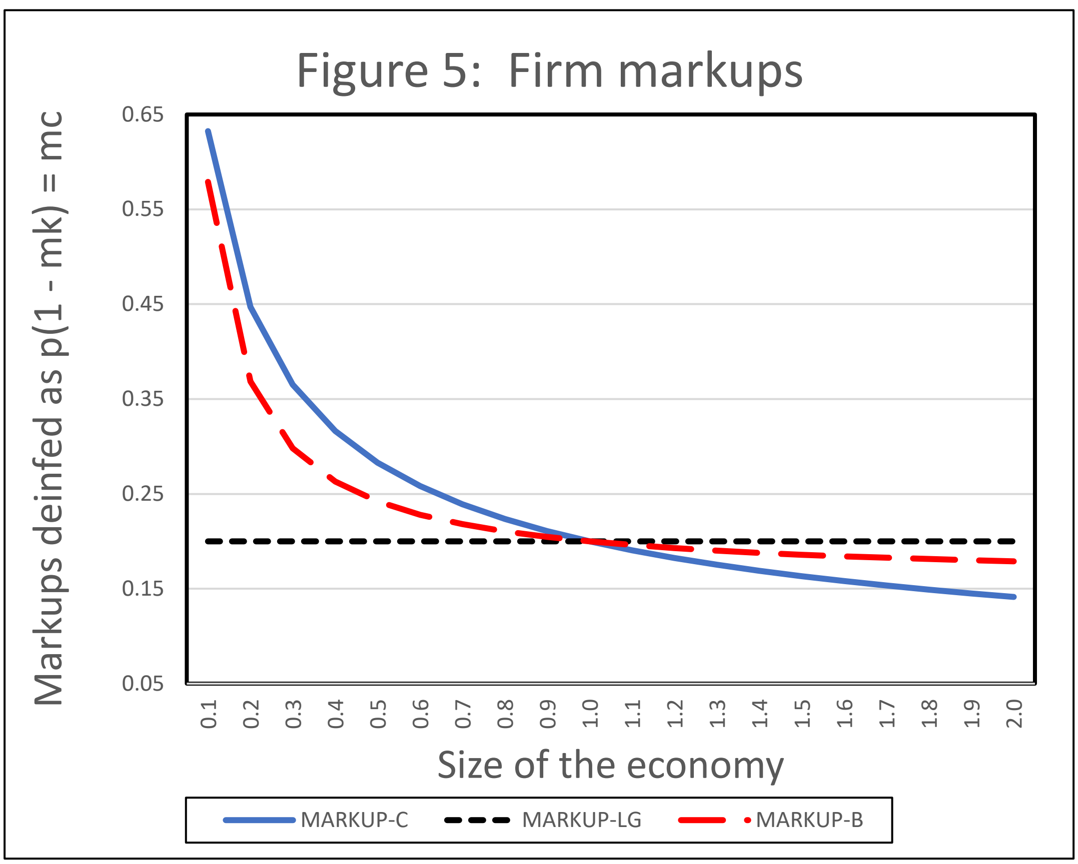


Figure 5: Firm markups
Markups deinfed as p(1 - mk) = mc
0.65
0.55
0.45
0.35
0.25
0.15
0.05
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
Size of the economy
MARKUP-C
MARKUP-LG
MARKUP-B

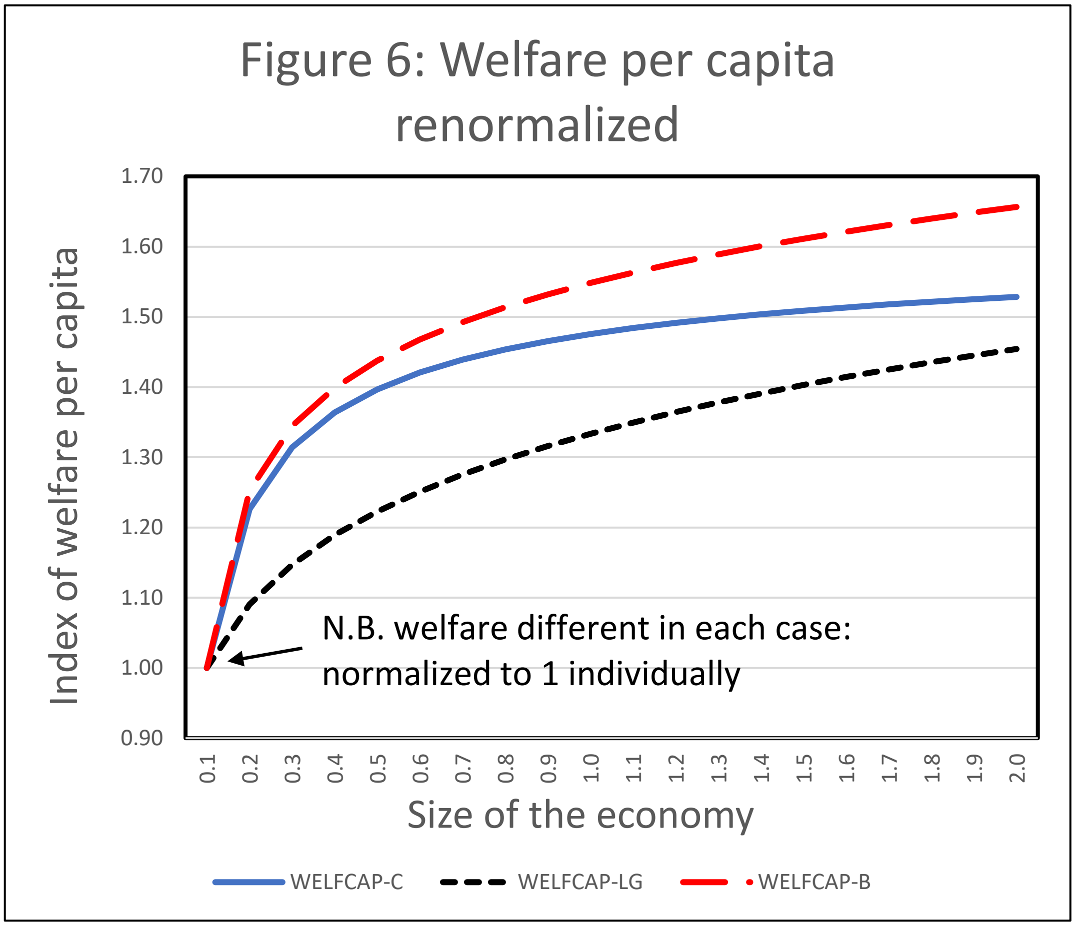


Figure 6: Welfare per capita renormalized
Index of welfare per capita
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
N.B. welfare different in each case: normalized to 1 individually
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
Size of the economy
WELFCAP-C
WELFCAP-LG
WELFCAP-B

```
$TITLE: M7-5n James R. Markusen, University of Colorado, Boulder
* Bertrand Small-Group Monopolistic Competition

$ONTEXT
  P(1 - MK) = MC, markup MK =  1/(SI - 1/N*(SI - 1))
  N  = endogenous number of firms,  1/N = firm market share
  SI = elasticity of substitution (exogenous)

  calibrate to the same data below as LGMC SI=5, MK=0.2;
  calibration for SGB is then  SI = 6.3333, N = 4,
  MK = 1/(sigma - (1/N)(sigma - 1) = 0.20

                      Production Sectors          Consumers
  Markets   |   X       N        Y        W    |  CONS    ENTR
--------------------------------------------------------------
       PX   |  200                      -200   |
       PY   |                  200      -200   |
       PN   |          40                      |           -40
       PW   |                            400   |  -400
       PL   | -160    -40     -200             |   400
       MK   |  -40                             |            40
$OFFTEXT

PARAMETERS
  SI          SIGMA: elasticity of substitution
  FC          parameter setting the level of fixed costs
  ENDOWL      endowment of labor;

SI = 6 + 1/3;
FC = 10;
ENDOWL = 400;

NONNEGATIVE VARIABLES

  X       Activity level for X (output per firm)
  N       Number of X sector firms (variety measure)
  Y       Activity level of Y output
  W       Activity level for welfare

  PE      Price index for X goods (unit expenditure function)

  PX      Price of an individual X variety
  PN      Price of fixed costs (price of entering)
  PY      Price of Y
  PW      Price index for utility (consumer price index)
  PL      Price of labor
```

```
   CONS      Income of the representative consumer
   ENTRE     Income of firm owners spent on fixed costs
   MK        Markup;
```

EQUATIONS

```
   PRICEX     MR = MC in X (associated with X output per firm)
   PRICEN     Zero profits - free entry in X (associated with N)
   PRICEY     Zero profit condition for Y (PY = MC)
   PRICEW     Zero profit condition for W (PW = MC of utility)

   INDEX      Definitional equation for the price index PE

   DX         Supply-demand balance for X (individual variety)
   DN         Supply-demand for firms N: markup rev = fexed cost
   DY         Supply-demand balance for Y
   DW         Supply-demand balance for utility W (welfare)
   DL         Supply-demand balance for labor

   ICONS      Consumer income
   IENTRE     Firm owner markup income
   MARKUPB    Bertrand markup equation;


PRICEX..    PL =G= PX*(1 - MK);

PRICEN..    PL =G= PN;

PRICEY..    PL =G= PY;

PRICEW..    (PE**0.5)*(PY**0.5) =G= PW;

INDEX..     ((N/4)*PX**(1-SI))**(1/(1-SI)) =G= PE;

DX..        X*80  =G=  PX**(-SI)*(PE**(SI-1))*CONS/2;

DN..        N*FC  =G= ENTRE/PN;

DY..        Y*100 =G= CONS/(2*PY);

DW..        200*W =G= (1.25**0.5)*CONS/PW;

DL..        ENDOWL =E= Y*100 + N*X*20 + N*FC;

ICONS..     CONS =E= PL*ENDOWL;
```

```
IENTRE..   ENTRE =E= MK*PX*X*20*N;

MARKUPB..  MK =E= 1/(SI - 1/N*(SI - 1));


MODEL M75 /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, INDEX.PE,
           DX.PX, DN.PN, DY.PY, DW.PW,
           DL.PL, ICONS.CONS, IENTRE.ENTRE, MARKUPB.MK/;

*    set initial values at benchmark for replication check

PE.L = 1.25;  CONS.L = 400;  ENTRE.L = 40;
X.L = 2;  Y.L = 2;  N.L = 4;  W.L = 2;
PX.L = 1.25; PN.L = 1; PY.L = 1; PW.L = 1.25**0.5; PL.L = 1;
MK.L = 0.20;

*    choose the price of good Y as numeraire
PY.FX = 1;

*    check for calibration, starting-value errors with ITERLIM=0
M75.ITERLIM = 0;
SOLVE M75 USING MCP;
*    free up iteration limit, set to 1000
M75.ITERLIM = 1000;
SOLVE M75 USING MCP;

* show welfare as a function of the economy's size

SETS I indexes 25 different size levels  /I1*I25/;

PARAMETERS
 SIZE(I)         benchmark I16 SIZE=2 ENDOWL=400)
 RESULTS(I, *)   summarizes results;

MK.L = 0.2;

LOOP(I,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOWL = 200*SIZE(I);

SOLVE M75 USING MCP;

RESULTS(I, "SIZE") = SIZE(I);
RESULTS(I, "WELFARE-B")  = W.L;
```

```
RESULTS(I, "WELFCAP-B")  = W.L/SIZE(I);
RESULTS(I, "FIRMSIZE-B") = X.L;
RESULTS(I, "NUMBERF-B")  = N.L;
RESULTS(I, "MARKUP-B")   = MK.L;

);

DISPLAY RESULTS;

* Write parameter RESULTS to an Excel file M7n.XLS,
* starting in Sheet3, cell A3

Execute_Unload 'M7n.gdx' RESULTS
execute 'gdxxrw.exe M7n.gdx par=RESULTS rng=SHEET3!A3:J29'
```