CESifo WORKING PAPERS
10392
2023
April 2023

Tax Uncertainty and Welfare-Improving Tax Disputes

Nigar Hashimzade

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp

An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 10392*

Tax Uncertainty and Welfare-Improving Tax Disputes

Abstract

Tax law is often uncertain. In particular, the use of tax shelters tends to be in the "grey area" between illegal tax evasion and legal tax avoidance. In this paper I show that uncertainty in tax law can help achieve higher efficiency than allowing or disallowing a tax shelter with certainty. Furthermore, a tax dispute can lead to a net welfare gain despite the litigation costs. Thus, tax uncertainty and tax disputes can be socially desirable.

JEL-Codes: K340, H260, D720.

Keywords: tax uncertainty, tax shelter, tax avoidance, rent-seeking.

Nigar Hashimzade
Department of Economics and Finance &
Research Centre for Law, Economics and Finance
Brunel University London / United Kingdom
nigar.hashimzade@brunel.ac.uk

20 April 2023

Tax uncertainty and welfare-improving tax disputes

Nigar Hashimzade*

20 April 2023

1 Introduction

Recently, attention has been growing in policy debate and public discourse concerning the tax sheltering activities of companies and private investors. The binary distinction between tax evasion as being illegal and tax avoidance as being legal has evolved into a continuum between a tax-planning activity characterisation as definitely illegal at the one end, definitely legal at the other end, and a "grey area" in between. Increasingly, tax planning, or business decisions leading to the reduction of tax obligations, bear uncertain implications for compliance with tax law. In particular, investments in low-tax jurisdictions or specially devised low-tax schemes, called tax shelters, tend to be in that grey area, and the legitimacy of such investment can be questioned by a tax authority. Proving or disproving the legitimacy of a tax shelter is costly for taxpayers and tax authorities. It is not obvious, however, that removing uncertainty in tax law is socially desirable. Indeed, a number of theoretical studies in the economic literature suggest that there are situations in which introducing a degree of uncertainty in tax rates can lead to an increase in tax revenues, or in social welfare, or both (see, for example, Weiss, 1976; Stiglitz, 1982; Bizer and Judd, 1989; Hines and Keen, 2021). At the same time, while tax disputes are costly and divert private resources from productive activities, they may be beneficial from the social viewpoint, if the parties' litigation efforts provide the court with additional information helping to make the correct decision (Posner, 2014). In the context of uncertain tax treatment of profits, the anticipated outcome of a tax dispute can lead to a more efficient allocation of investment resulting in the net welfare gains.

In this paper I investigate the economic implications of uncertainty in the tax law concerning tax shelters in a model of investment allocation between two sectors, a shelter and a non-shelter, in a setting where the investor, or the taxpayer, and the tax authority can dispute the tax bill in court. A tax differential between two sectors distorts the investment decision towards the sector with the lower tax rate. Uncertainty over the tax rates in the shelter changes this distortion through its effect on the investment allocation decision. Thus, uncertainty can be socially desirable when it helps reduce this distortion. I show that allowing for uncertainty can dominate declaring tax shelters legal or illegal with certainty and calculate the welfare-maximising and the tax revenue-maximising degree of uncertainty. In particular, the model predicts that maximal uncertainty (a 50-50 chance of the shelter being legal) can be optimal.

Next, I introduce in the model a possibility of a tax dispute, an institutional arrangement which exists in many jurisdictions. This additional assumption allows investigating welfare effects of costly dispute efforts, or litigation expenditures, in a rent-seeking framework. I demonstrate that in this framework litigation can increase welfare. This result provides an economic rationale for tax disputes.

Uncertainty in tax law is an example of a more general concept of law uncertainty that is the subject of both academic and policy interest. D'Amato (1983) argues that there is a tendency for law to become less certain over time. He defines legal certainty as the predictability of the official reaction to an agent's action: for example, if the reaction is the court decision 'win' or 'lose', the law is the most uncertain when the probability of 'winning' (or 'losing') is 0.5, and legal certainty is higher, the closer is this probability to either zero or one. According to D'Amato, a law, or a 'rule', becomes more uncertain over time in two ways, in statute and in applications. Statutes can be construed in different ways, with each interpretation becoming part of the statute; various exceptions and exemptions can be created in the legislative process. D'Amato uses the Internal Revenue Code as an example of statutes made increasingly more complex and convoluted by 'innumerable tax provisions and regulations' (p. 2). In applications, the uncertainty in law increases when agents 'disadvantaged' by the law modify their behaviour in such a way that the rules apply less clearly to their actions. We can see a parallel here with the description of tax avoidance behaviour as complying with the letter but not the spirit of the law, or as formally complying with the law but not with the legislative intent. That is, when the rules were developed the legislators could not anticipate the behavioural response of the agents 'disadvantaged' by these rules.

Braithwaite (2002) argues that rules work when the relevant environment is simple and stable. In a complex and changing environment, an attempt to anticipate every possible event by creating a specific rule for that event can result in greater imprecision in the law and even lead to worsening compliance. Numerous discrete rules might make it easier to spot a 'loophole' in a tax code, for example. In such an environment, a broad principle or a set of principles can work better. Furthermore, according to Braithwaite (2002), a mix of non-binding rules backed by binding principles is likely to work better in a complex environment than a mix of binding rules interpreted by non-binding principles.

One can formalise the description of uncertainty in law using D'Amato's interpretation, by considering the probability, denoted p_0, that a particular economic activity or a transaction is allowed under the law. The law is certain if it unambiguously states whether or not the activity is allowed, that is, if p_0 is either zero or one. This can be interpreted as a discrete rule. Uncertainty in law corresponds to p_0 between zero and one, and the closer is p_0 to 0.5, the higher is the uncertainty about the legality of this activity. A law with uncertainty can be interpreted as a broad principle. The closer is p_0 to either zero or to one, the more the law resembles a rule. In this paper the legal uncertainty is treated in this particular sense.

Is legal uncertainty desirable, and, if yes, in which circumstances? The usual argument is that in criminal law legal uncertainty might be desirable because it can deter agents from possibly legal but socially objectionable activities. In other areas, such as commerce or enterprise, however, legal uncertainty is undesirable because it can deter agents from the activities a society would like to encourage. Another negative effect of legal uncertainty is the susceptibility of 'official discretion', relied upon when the probability of an action being legal approaches 0.5, to 'extralegal influences', such as prejudice or corruption (D'Amato 1983, p. 3). '*...The more certainty there is, the less effort individuals and companies will have to divert to establishing how the tax system will affect them, and the less likely it is that taxpayers and the revenue authority will become involved in disputes about the tax effects of transactions and need to resort to the appeals system.*' (Treasury Committee 2011, p. 15)

Weisbach (2013) takes a different view, arguing that, generally, it is desirable that an agent knows the law prior to taking action, with tax law being an exception. The reason, according to Weisbach, is the purpose of the law: while criminal law, commercial law, environmental law, etc. aim at guiding behaviour, tax law aims at raising and redistributing revenue, and so changes in behaviour induced by tax law would undermine its purpose (assuming that a natural reaction of private agents is to attempt to pay less tax and receive more benefits). Therefore, not knowing tax law prior to taking action is socially desirable, as it prevents private agents from changing their behaviour in an adverse way. In an earlier work, Weisbach (1999) argues in favour of uncertain standards, as opposed to certain rules, in tax law as such uncertainty can reduce sheltering activities. However, a reduction in sheltering activities does not necessarily translate into higher revenues and higher social welfare, – for example, when a non-sheltered activity is less productive and thus generates less tax.

Givati (2009) reports empirical evidence of tax law uncertainty and investigates the role of advanced tax ruling (ATR) as a tool of eliminating this uncertainty. ATR is a procedure by which a taxpayer can approach the tax authority before entering into a transaction, so as to obtain a binding decision on the tax implications of that transaction. Although ATR eliminates uncertainty, as Givati (2009) shows, it is used rather infrequently. Givati argues that the costs of ATR often outweigh its benefits for the taxpayer: among other reasons, an ATR application essentially "red-flags" the transaction for inspection by the tax authority, potentially with a negative outcome. At the same time, an ATR may lead to a revenue loss for the tax authority: if the ruling is negative the taxpayer can choose an alternative transaction that results in less tax being paid. Thus, elimination of tax law uncertainty is likely not to benefit either side.

The rational behavioural response of a taxpayer to tax law uncertainty changes simultaneously the tax base and the effective tax rate applied to the taxpayer's activity. In this paper I develop a model showing that legal uncertainty can be socially desirable when it counteracts a distortion caused by differential taxation. By equalising net-of-tax returns, uncertainty improves efficiency, and it can also increase tax revenues. Furthermore, in a setting where the taxpayer and the tax authority can dispute the tax payment in court, I show that welfare gains resulting from improved efficiency of investment allocation can outweigh the cost of the parties' dispute efforts. Thus, a legal system that allows costly tax disputes can be justified from the viewpoint of economic efficiency.

My findings echo the results of Bizer and Judd (1989) obtained in a different framework. They construct a dynamic stochastic general equilibrium model with two sources of uncertainty, stochastic shocks to production technology and stochastic tax policies. The rates of the capital income tax and the investment tax credit are exogenous and are assumed to follow a Markov process. Bizer and Judd show that in this economy uncertainty in tax rates increases tax revenue at a relatively low efficiency cost, thus increasing social welfare. This undermines an argument of desirability of stable tax policies. Intuitively, uncertainty in future tax rates encourages investment and reduces the distortion created by the capital income tax.[1] This is also akin to the result obtained by Weiss (1976) and, independently, Stiglitz (1982): random labour income taxation can increase ex-ante expected utility of risk-averse agents through its effect on labour supply.

In another setting, Hines and Keen (2021) demonstrate that a mean-preserving spread in input taxes can raise producers' profits and improve the efficiency of the tax system. In their model, the tax uncertainty, as opposed to a single, certain tax rate, effectively widens the range of policy instruments for the government and works as a screening device, so that higher (lower) tax rates apply to the activities less (respectively, more) responsive to the tax. This reduces the deadweight loss created by the distortionary taxation of inputs. In my model, taxation distorts the investment allocation decision, and uncertainty can reduce this distortion, thus improving efficiency and increasing tax revenues. In addition, as this paper demonstrates, a further increase in efficiency can be achieved by the means of a tax dispute.

The system of tax disputes exists in many jurisdictions, including the UK and the USA. While costly tax disputes divert resources from productive activities, they are an important part of maintaining an effective tax system. One compelling argument for tax disputes is that they can lead to court rulings or settlements that clarify the meaning of complex or ambiguous tax laws, making them easier to understand

[1] This result, however, does not hold for the uncertainty in the investment tax credit, because it generates fluctuations in the current investment behaviour. This reduces welfare as the agents are assumed to be risk-averse.

and follow in the future. The model presented in this paper offers an efficiency rationale for tax disputes. It shows that, in the presense of distortion of the tax base and uncertainty about tax rates, a costly tax dispute can lead to net welfare gains by changing the degree of uncertainty which, in turn, reduces the distortion.

My paper contributes to two strands of literature. First, it contributes to the literature on tax avoidance by embedding an investment allocation decision in a setting with uncertaint tax treatment and tax disputes. This framework reflects the contemporary environment where uncertainty about tax treatment of profits is one of the most important factors of investment decisions by large businesses (Deveraux, 2016; Brock, 2019). Second, it contributes to the literature on rent-seeking contests in the context of litigation by modelling an environment where the prize, the sharing rule, and the productivity of efforts are all endogenous. In the standard model of contest with productive efforts there is always a deadweight loss in equilibrium: the welfare gains in a productive contest do not fully compensate the parties for their effort exerted in the competition for the prize (Chung, 1996; Baye and Hoppe, 2003). The endogeneity of the effect of the parties' efforts on the size of the prize and the possibility of net welfare gains demostrated in this paper are novel results. In the model, depending on the initial uncertainty, the privately optimal efforts can be either productive or destructive in equilibrium. Productive efforts can move the uncertainty closer towards the efficient level in such a way that the resulting increase in the size of the price outweighs the aggregate cost of efforts.

In the remainder of the paper I set out and analyse a model of investment allocation under uncertainty over tax rates in various settings. Section 2 presents a benchmark framework in which tax returns cannot be disputed. Section 3 presents an extended framework in which the taxpayer and the tax authority can dispute in court the tax treatment of a profitable investment. I calculate the degree of uncertainty that maximises welfare and tax revenues in two different scenarios: when the taxpayer's investment decision precedes the decision on the litigation expenditures, and when the two decisions are made simultaneously. Section 5 compares the welfare and revenue implications of raising the relative cost of litigation for taxpayers and those of increasing the responsiveness of the court to the evidence presented by the parties in the tax dispute. Section 6 illustrates that costly litigation can lead to net welfare gains. Section 7 concludes. All proofs and codes used to produce numerical illustrations are available upon request.

2 Optimal uncertainty without tax disputes

A risk-neutral taxpayer, an individual investor or a firm, decides how to allocate one unit of resources between two sectors (these can be two jurisdictions). Investment in either sector returns a certain level of profit liable to profit tax. One sector is a tax shelter, with an effective tax rate of τ_L, possibly after any exemptions or reliefs are applied. The tax law is uncertain: the definition of a legitimate tax shelter is vague. The ex-ante probability that τ_L will apply to the taxpayer's investment is p_0. With probability $(1-p_0)$ the effective tax rate is $\tau_H > \tau_L$; this higher effective tax rate may reflect a situation in which exemptions and reliefs are not applicable. It can also include any penalties that might be imposed if the tax shelter is deemed illegitimate, or is disallowed. The second investment opportunity is a non-sheltered activity taxed with certainty at rate τ_0, such that $\tau_H > \tau_0 > \tau_L$.[2] There are no effects on third parties, and the purpose of taxation is purely to raise revenue.

As a benchmark, I consider first the situation in which the actions of the taxpayer and the tax authority have no effect on tax uncertainty. Let $\lambda \in [0,1]$ be the proportion of resource invested in the tax shelter. The profit-maximising allocation of investment by the taxpayer solves

$$\lambda^o = \arg\max_{[0,1]} (1-\tau_0)\,\pi_0\,(1-\lambda) + [p_0\,(1-\tau_L) + (1-p_0)\,(1-\tau_H)]\,\pi\,(\lambda)\,,$$

[2] Since the taxpayer and the tax authority know the probability distribution, this is not a Knightian uncertainty. Strictly speaking, the described situation is one of "tax risk". Following Bizer and Judd (1989) and Hines and Keen (2021), I use "uncertainty" as it is commonly referred to in the legal literature and in information theory.

where π_0 is the profit function of the non-sheltered activity and π is the profit function of the sheltered activity.

Assumption 0. $\pi_0(0) = \pi(0) = 0$, $\pi_0'(x) > 0$, $\pi_0''(x) < 0$, $\pi'(x) > 0$, $\pi''(x) < 0$ for $x \in [0, 1]$.

Assumption 0 states that both profit functions are strictly increasing and strictly concave over the relevant domain; in general, the technology and the cost of production in these two activities can be different, so that $\pi(x) \neq \pi_0(x)$.

I am interested in the interior solutions, which are described by the following first-order condition:

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)} = \frac{1-\tau_0}{p(1-\tau_L) + (1-p)(1-\tau_H)}.$$

In this model, I interpret welfare as the total surplus generated by the investment. In this framework it is the pre-tax profit,

$$W(\lambda) = \pi_0(1-\lambda) + \pi(\lambda).$$

and it is divided between the investor, or the taxpayer, whose payoff is the after-tax profit, and the state represented by the tax authority, whose payoff is the tax revenue.

The efficient allocation of investment is the allocation which maximises the total surplus. In an interior equilibrium it is determined by the solution to the equation

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)} = 1. \tag{1}$$

Let p^e denote the value of p_0 which induces the efficient allocation of investment. Condition (1) requires

$$\frac{1-\tau_0}{p^e(1-\tau_L) + (1-p^e)(1-\tau_H)} = 1$$

which gives

$$p^e = \frac{\tau_H - \tau_0}{\tau_H - \tau_L}.$$

Expected tax revenue,

$$T = \tau_0 \pi_0(1-\lambda) + [p\tau_L + (1-p)\tau_H]\,\pi(\lambda)$$

is maximised when

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)} = \frac{\tau_0}{p\tau_L + (1-p)\tau_H}.$$

Thus, the investment allocation maximises tax revenue when $p = p^T$ which solves

$$\frac{\tau_0}{p^T\tau_L + (1-p)\tau_H} = \frac{1-\tau_0}{p^T(1-\tau_L) + (1-p^T)(1-\tau_H)}.$$

This gives

$$p^T = \frac{\tau_H - \tau_0}{\tau_H - \tau_L} = p^e.$$

Efficient uncertainty equalises the net marginal benefits of the two investment opportunities by equating the expected tax rate in the tax shelter to the certain tax rate in the non-shelter:

$$p^{e,T}\tau_L + \left(1 - p^{e,T}\right)\tau_H = \tau_0.$$

This eliminated investment distortion caused by the tax differential. The efficient uncertainty maximises the tax base (the aggregate pre-tax profit), and, since the marginal tax rates are equalised between two sectors, it also maximising the tax revenue. In particular, maximal uncertainty, $p = 1/2$, is efficient, and raises maximal expected revenue for $\tau_0 = [\tau_H + \tau_L]/2$.

3 Optimal uncertainty with tax disputes

In many jurisdictions, the tax authority can dispute the legitimacy of a tax shelter used by the taxpayer and demand application of a higher tax rate and, sometimes, a penalty for underpayment of tax. At the same time, a taxpayer has the right to challenge the decision of the tax authority. Either side can bring the case to the court, along with additional information, or evidence in support of their claims. The balance of evidence can change the likelihood of the taxpayer winning the case. In other words, the efforts of the two sides in a tax dispute can change the uncertainty about the tax shelter. This, in turn, will change the equilibrium welfare.

To analyse the tax uncertainty and welfare implications of the tax disputes I extend the model presented in the previous section. The tax authority is aware of the investment options available to the taxpayer and can challenge the legitimacy of the tax shelter in court. Knowing about this possibility, the taxpayer can choose to acquire additional information or to hire a tax advisor, at cost I_T, before entering into tax shelter scheme. The tax authority can also acquire additional information or hire a tax expert, at cost I_R, if it decides to take the case to court. The efforts of the parties affect the probability of the court's decision in favour of the taxpayer. With probability $p \in [0,1]$ the court will declare the tax shelter with the effective tax rate of τ_L as legal, and with probability $(1-p)$ tax shelter will be declared illegal, in which case the effective tax rate of τ_H will apply. As in the previous section, $\tau_H > \tau_0 > \tau_L$, where τ_0 is the tax rate applied with certainty to the non-sheltered part of investment. Depending on the resources spent by the parties on presenting their case, the ex post probability p of the tax shelter being deemed legitimate, can be higher or lower than p_0, the ex ante probability.

Formally, the taxpayer invests fraction λ of her capital in the tax shelter and the rest . in a non-sheltered activity. The tax authority challenges the legality of the tax shelter at cost I_R of hiring a tax advisor or collecting evidence against the sheltering of investment to be presented at the court. The taxpayer pays cost I_T of information and legal advice on the tax shelter opportunity. I will refer to I_R and I_T as the litigation costs or the tax dispute costs, i.e. any information obtained by the parties about the tax shelter contributes to the evidence they present to the judge. If the two sides take the case to court, the judge reviews evidence from the taxpayer and the tax authority and makes a decision. Regardless of the outcome, each party pays only its own litigations costs.[3]

The probability p that the judge's decision will be in favour of the taxpayer depends on the ex-ante probability p_0 that the tax law allows the tax shelter, and on the litigation efforts of the parties in the court, measured by the cost of evidence, I_T and I_R, acquired by the taxpayer and the tax authority:

$$\begin{aligned} p &= p\left(p_0, I_T, I_R\right), \\ \frac{\partial p}{\partial p_0} &> 0, \frac{\partial p}{\partial I_T} > 0, \frac{\partial p}{\partial I_R} < 0. \end{aligned}$$

To illustrate the properties of the equilibrium, I further assume that the effect of the evidence presented in court on the probability of the taxpayer winning the case is given by the functions

$$p\left(p_0, I_T, I_R\right) = (1-\theta)\, p_0 + \theta q\left(I_T, I_R\right), \ \theta \in (0,1); \tag{2}$$

$$q\left(I_T, I_R\right) = \frac{\delta I_T}{\delta I_T + I_R}, \ \delta > 0. \tag{3}$$

Here $q\left(I_T, I_R\right)$, modelled as in Tullock (1975), is an adjustment in the probability of the tax shelter being allowed, and it is increasing in the taxpayer's effort and decreasing in the tax authority's effort. The parameter δ measures the effectiveness of the taxpayer's efforts on defending her case against the tax

[3] This is referred to as the American rule. Baye et al. (2005) discuss the implications of the various divisions of legal costs between the parties induced by different legal systems.

authority. This can also be interpreted as the judge's relative bias in the favour of the taxpayer. A value of $\delta > 1$ then means that the taxpayer (or the taxpayer's legal representative in the court) is more effective in persuading the judge, or that the judge favours the taxpayer more than the tax authority. The parameter θ reflects the responsiveness of the judge to the evidence brought forward by the parties; higher (lower) θ means higher (lower) responsiveness. There two parameters can also be viewed as the characteristics or the design parameters of the tax dispute system. Thus, an increase (decrease) in δ can be interpreted as making it easier (harder) for the taxpayer to justify their use of a tax shelter. Similarly, an increase (decrease) in θ could mean a higher (lower) degree of evidence-based discretion exercised by the judge in the tax dispute case.

I am interested in the situation in which the filed tax return is taken to court. I further assume that an interior equilibrium ($p \in (0,1)$, $\lambda \in (0,1)$, $I_T > 0$, $I_R > 0$) exists and focus on the analysis of its properties.

Given the probability p that the judge will decide in favour of the taxpayer, the expected payoff V_R of the tax authority is the expected tax revenue less the litigation cost,

$$V_R = -I_R + \tau_0 \pi_0 (1-\lambda) + [p\tau_L + (1-p)\tau_H]\, \pi(\lambda).$$

The expected payoff V_T of the taxpayer is the expected net of tax profit less the litigation cost,

$$V_T = -I_T + (1-\tau_0)\pi_0 (1-\lambda) + [p(1-\tau_L) + (1-p)(1-\tau_H)]\, \pi(\lambda)$$

Even though tax returns are typically audited some time after the investment activity has taken place (for example, in the UK the typical lag is up to 18 months), it is plausible to assume that, when the case is taken to the court, the taxpayer and the tax authority do not observe each other's efforts in acquiring evidence to support their positions in the possible tax dispute. Thus, I assume that the decisions by the parties on their litigation expenditures are taken simultaneously. For the timing of the investment decision I analyse two possible scenarios: when the taxpayer makes it simultaneously with the litigation expenditure decision or prior to that. I refer to the former situation as the simultaneous choice and to the latter as the sequential choice.

3.1 Simultaneous decisions

In this section I assume that all decisions are made at the same time.

The tax authority chooses I_R to maximise its expected payoff, taking the taxpayer's litigation cost, I_T, and investment choice, λ, as given. Setting $\frac{dV_R}{dI_R} = 0$ gives the necessary optimisation condition for an interior equilibrium:

$$[\tau_H - \tau_L]\, \pi(\lambda) \frac{\partial p}{\partial I_R} = -1. \tag{4}$$

The taxpayer chooses the investment allocation, λ, and the information expenditure, I_T, simultaneously to maximise V_T, taking I_R as given. The necessary conditions for an interior equilibrium are given by

$$(\tau_H - \tau_L)\, \pi(\lambda) \frac{\partial p}{\partial I_T} = 1, \tag{5}$$

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)} = \frac{1-\tau_0}{p(1-\tau_L) + (1-p)(1-\tau_H)}. \tag{6}$$

Solving equations (4)-(6) simultaneously gives I_T, I_R, and λ as functions of the ex ante uncertainty and the tax rates.

The welfare equals the surplus from the productive activity net of litigation costs:

$$TS = V_T + V_R = \pi_0 (1-\lambda) + \pi(\lambda) - I_T - I_R. \tag{7}$$

From (2)-(3),

$$\frac{\partial p}{\partial I_R} = -\frac{\theta \delta I_T}{\left[\delta I_T + I_R\right]^2}, \frac{\partial p}{\partial I_T} = \frac{\theta \delta I_R}{\left[\delta I_T + I_R\right]^2}, \tag{8}$$

and in the interior equilibrium, using (4) and (5),

$$\begin{aligned} I_T &= I_R, \\ p &= (1-\theta) p_0 + \frac{\theta \delta}{1+\delta}. \end{aligned}$$

Finally, using (8) in (4),

$$I_T = I_R = (\tau_H - \tau_L) \pi (\lambda) \frac{\theta \delta}{\left[1+\delta\right]^2}. \tag{9}$$

Observe that the equilibrium efforts increase in p_0:

$$\begin{aligned} \frac{\partial I_T}{\partial p_0} &= \frac{\partial I_R}{\partial p_0} = \left[\tau_H - \tau_L\right] \frac{\left[1-\theta\right] \theta \delta}{\left[1+\delta\right]^2} \pi' \left(\lambda (p)\right) \lambda' (p) \\ &= \left[\tau_H - \tau_L\right]^2 \frac{\left[1-\theta\right] \theta \delta}{\left[1+\delta\right]^2} \frac{\left[\pi' \left(\lambda (p)\right)\right]^2}{-H} > 0. \end{aligned}$$

where

$$H \equiv \pi'' (\lambda) \left[p (1-\tau_L) + (1-p)(1-\tau_H)\right] + \pi_0'' (1-\lambda) \left[1-\tau_0\right] < 0.$$

Finally, using (9) in (7) gives for the equilibrium welfare

$$W = \pi_0 (1-\lambda) + \pi (\lambda) \left[1 - 2\frac{\theta \delta}{\left[1+\delta\right]^2} (\tau_H - \tau_L)\right]. \tag{10}$$

Thus, the efficient allocation of investment requires[4]

$$\frac{\pi' (\lambda)}{\pi_0' (1-\lambda)} = \frac{1}{1 - 2\frac{\theta \delta}{\left[1+\delta\right]^2} (\tau_H - \tau_L)}. \tag{11}$$

Comparing (11) with (6) one can see that the investment allocation maximises welfare when the optimal ex-post probability is given by

$$p^W = \min \left\{1, \max \left\{0, \frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{2\theta \delta}{\left[1+\delta\right]^2} \left[1-\tau_0\right]\right\}\right\} \tag{12}$$

and the optimal ex-ante probability is then given by

$$p_0^W = \min \left\{1, \max \left\{0, \frac{1}{1-\theta} \frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta} \frac{\delta}{1+\delta} \left[1 + \frac{2\left[1-\tau_0\right]}{1+\delta}\right]\right\}\right\} \tag{13}$$

The maximisation condition for the tax revenues net of litigation cost of the tax authority is

$$\frac{\pi' (\lambda)}{\pi_0' (1-\lambda)} = \frac{\tau_0}{\left[\left[p \tau_L + (1-p) \tau_H\right] - (\tau_H - \tau_L) \frac{\theta \delta}{\left[1+\delta\right]^2}\right]}$$

[4] The ratio on the right-hand side of (11) is positive if, and only if $\theta < \frac{[1+\delta]^2}{2\delta(\tau_H - \tau_L)}$.It is straightforward to check that the expression in the right-hand side has a global minimum at $\delta = 1$, where it is equal to $\frac{2}{\tau_H - \tau_L} > 0$ for any τ_H and τ_L between 0 and 1. Hence, this condition hold for any $\theta \in [0, 1]$.

Thus, the investment allocation maximises the net tax revenue when the ex post probability is equal to

$$p^R = \min\left\{1, \max\left\{0, \frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{[1+\delta]^2}[1-\tau_0]\right\}\right\} \tag{14}$$

and the corresponding ex ante probability is equal to

$$p_0^R = \min\left\{1, \max\left\{0, \frac{1}{1-\theta}\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1+\frac{2-\tau_0}{\delta}\right]\right\}\right\} \tag{15}$$

3.2 Sequential decisions

It is plausible to assume that the investment decision is made before the tax authority decides to challenge the legitimacy of the tax shelter. In this case the parties move sequentially. In the first stage the taxpayer makes the investment decision, and in the second stage the taxpayer and the tax authority choose their litigation expenditures. Thus, the investment decision will anticipate the equilibrium probability of the tax shelter deemed legitimate. The game is solved by backward induction.

The solution for the optimal choice of litigation expenditure in the second stage is the same as in the model with simultaneous decisions:

$$I_T = I_R = (\tau_H - \tau_L)\,\pi(\lambda)\,\frac{\theta\delta}{[1+\delta]^2}, \tag{16}$$

$$p = (1-\theta)\,p_0 + \frac{\theta\delta}{1+\delta}. \tag{17}$$

The net payoff of the taxpayer in the first stage is then given by

$$V_T = (1-\tau_0)\,\pi_0\,(1-\lambda) + \left[\left[p - \frac{\theta\delta}{[1+\delta]^2}\right](\tau_H - \tau_L) + (1-\tau_H)\right]\pi(\lambda), \tag{18}$$

and the taxpayer's investment choice is described by

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)} = \frac{1-\tau_0}{\left[p - \frac{\theta\delta}{[1+\delta]^2}\right](\tau_H - \tau_L) + 1 - \tau_H}. \tag{19}$$

Comparing this with (11) gives the condition for the investment choice being efficient:

$$\frac{1-\tau_0}{\left[p - \frac{\theta\delta}{[1+\delta]^2}\right](\tau_H - \tau_L) + 1 - \tau_H} = \frac{1}{1 - 2\frac{\theta\delta}{[1+\delta]^2}(\tau_H - \tau_L)}.$$

This gives for the efficient ex-post probability

$$p^W = \min\left\{1, \max\left\{0, \frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta\,[1-2\tau_0]}{[1+\delta]^2}\right\}\right\} \tag{20}$$

and for the efficient ex-ante probability

$$p_0^W = \min\left\{1, \max\left\{0, \frac{1}{1-\theta}\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1+\frac{2}{\delta}[1-\tau_0]\right]\right\}\right\}. \tag{21}$$

$p^{W,T}$	**Maximal welfare**	**Maximal revenue**
No tax disputes	$\frac{\tau_H-\tau_0}{\tau_H-\tau_L}$	
Simultaneous choice	$\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{2\theta\delta[1-\tau_0]}{[1+\delta]^2}$	$\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta\delta[1-\tau_0]}{[1+\delta]^2}$
Sequential choice	$\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta\delta[1-2\tau_0]}{[1+\delta]^2}$	

Table 1: Ex-post interior welfare- and revenue-maximising probabilities

$p_0^{W,T}$	**Maximal welfare**	**Maximal revenue**
No tax disputes	$\frac{\tau_H-\tau_0}{\tau_H-\tau_L}$	
Simultaneous choice	$\frac{1}{1-\theta}\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta}{1-\theta}\frac{\delta}{1+\delta}\left[1+\frac{2[1-\tau_0]}{1+\delta}\right]$	$\frac{1}{1-\theta}\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1+\frac{2-\tau_0}{\delta}\right]$
Sequential choice	$\frac{1}{1-\theta}\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1+\frac{2}{\delta}\left[1-\tau_0\right]\right]$	

Table 2: Ex ante interior welfare- and revenue-maximising probabilities

The expected net payoff of the revenue authority is given by

$$V_R=\tau_0\pi_0(1-\lambda)+\left[\left[p\tau_L+(1-p)\tau_H\right]-(\tau_H-\tau_L)\frac{\theta\delta}{[1+\delta]^2}\right]\pi(\lambda), \tag{22}$$

and, hence, the maximisation condition for the tax revenues net of information cost of the tax authority is

$$\frac{\pi'(\lambda)}{\pi_0'(1-\lambda)}=\frac{\tau_0}{\left[\left[p\tau_L+(1-p)\tau_H\right]-(\tau_H-\tau_L)\frac{\theta\delta}{[1+\delta]^2}\right]} \tag{23}$$

Comparing (19) with (23), one can calculate the ex post and the ex ante probabilities that maximise the expected tax revenue net of litigation costs:

$$p^R = \min\left\{1,\max\left\{0,\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta\delta}{[1+\delta]^2}[1-2\tau_0]\right\}\right\} \tag{24}$$

$$p_0^R = \min\left\{1,\max\left\{0,\frac{1}{1-\theta}\frac{\tau_H-\tau_0}{\tau_H-\tau_L}-\frac{\theta}{1-\theta}\left[\frac{\delta}{1+\delta}\right]^2\left[1+\frac{2}{\delta}[1-\tau_0]\right]\right\}\right\} \tag{25}$$

One can see that the the efficient ex-ante probability also maximises revenue under the assumption of sequential moves, as was the case without tax disputes, although the probabilities differ between the two cases.

4 Optimal uncertainty

Tables 1 and 2 summarise the interior solutions for the welfare-maximising and for the revenue-maximising ex-ante and ex-post probabilities of the tax shelter deemed legitimate for the three cases analysed in the previous sections. The expressions allow direct calculation of the effect of the taxpayer's litigation cost and the court's responsiveness to the evidence on the optimal ex-ante probability, $\frac{\partial p_0^W}{\partial\theta}$ and $\frac{\partial p_0^W}{\partial\delta}$. This does not indicate, however, whether the optimal uncertainty becomes higher or lower when δ or θ change, since both low and high values of p_0^W describe low uncertainty, and the values close to 1/2 describe high uncertainty. A commonly used measure of the degree of uncertainty is Shannon's entropy (Shannon, 1948):

$$\sigma(p)=-p\log_2 p-[1-p]\log_2(1-p).$$

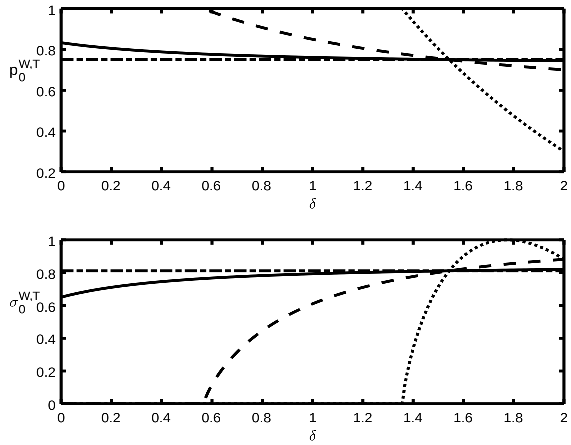

Figure 1: Welfare- and revenue-maximising probability and uncertainty, with disputes under sequential choice, for $\theta = 0.1$ (solid), 0.5 (dash), 0.9 (dot), and without disputes (dash-dot).

This measure achieves its maximum of 1 at $p = \frac{1}{2}$. Rather than presenting the cumbersome expressions for $\frac{\partial\sigma(p_0^W)}{\partial\theta}$ and $\frac{\partial\sigma(p_0^W)}{\partial\delta}$, I illustrate the general patterns using numerical examples.

Figures 1-3 show the optimal ex ante probabilities of a tax shelter deemed legitimate (top panels) and the optimal ex ante measures of tax law uncertainty (bottom panels) in the simultaneous-move and sequential-move frameworks, respectively, for $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.1$, 0.5, and 0.9. The no-dispute case is also shown for comparison.

5 Equilibrium welfare and revenues: a comparison

Given that in many countries taxpayers and tax authorities have a right to dispute tax treatment, and that uncertainty in tax law can incentivise both an investment in a tax shelter and an expenditure for tax dispute, an interesting question is what effect increasing the cost of the tax dispute for the taxpayer will have on optimal uncertainty in the tax law. Table 3 summarises the results for the interior solutions when the investment and litigation expenditure decisions are made sequentially and simultaneously.

WIth sequential moves, the same optimal probability maximises welfare and net tax revenue, and the effects of θ and δ on surplus and revenues are similar. Under simultaneous choice, the maximal welfare always falls with θ, holding other parameters constant. Intuitively, higher responsiveness of the court to the evidence provided by the parties incentivises unproductive expenses on tax disputes, which leads to the loss of surplus. The effect of δ on surplus is non-monotone. The net surplus falls with δ when $\delta < 1$ and increases with δ when $\delta > 1$. Recall that the smaller is δ, the higher is the per unit cost of legal advice or information about the tax shelter faced by a taxpayer relative to the cost faced by the tax authority (or the less effective is the taxpayer's effort in court relative to the tax authority's effort). Thus, an intervention in the market for legal advice that increases the cost for taxpayers has a negative effect on total surplus when the cost to the taxpayer is low relative to the cost of legal advice to the tax authority ($\delta > 1$). Importantly, these results do not depend on the timing of the moves or on the ex ante probability p_0.

The effects of θ and δ on net revenue depend on the type of equilibrium. In an equilibrium with arbitrary ex ante uncertainty, the effect of δ is always negative, whereas the effect of θ depends on the magnitude of δ. Keeping θ constant, a lower cost of tax advice or evidence collection for taxpayers (a

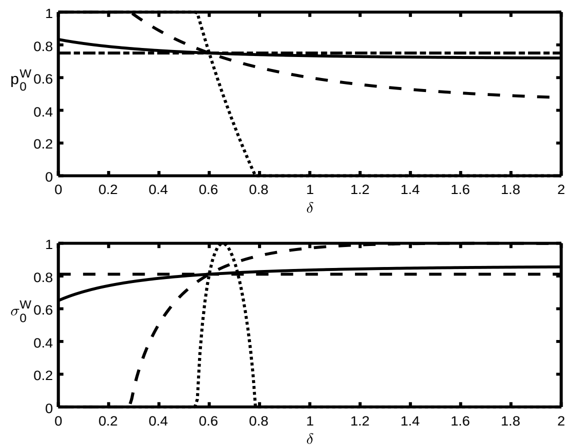

Figure 2: Welfare-maximising probability and uncertainty with disputes under simultaneous choice, for $\theta = 0.1$ (solid), 0.5 (dash), 0.9 (dot), and without disputes (dash-dot).

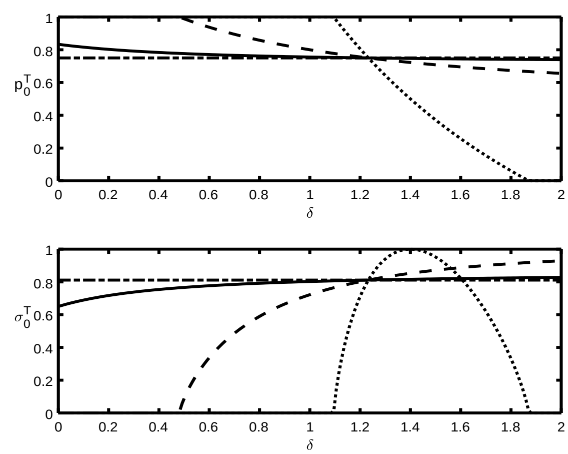

Figure 3: Revenue-maximising probability and uncertainty with disputes under simultaneous choice, for $\theta = 0.1$ (solid), 0.5 (dash), 0.9 (dot), and without disputes (dash-dot).

	Arbitrary uncertainty	Max welfare	Max revenue
Welfare			
θ	$-$		
δ	$+/-$ for $\delta \gtrless 1$		
Net revenues, sequential choice			
θ	$-$		
δ	$+/-$ for $\delta \gtrless 1$		
Net revenues, simultaneous choice			
θ	$+/-$ for $\delta \lessgtr \frac{1}{\sqrt{1-p^*}} - 1$	$+^{(*)}$	$-$
δ	$-$	$+/-$ for$^{(*)}$ $\delta \lessgtr 1$	$+/-0$ for $\delta \gtrless 1$

Table 3: Welfare and revenue effects of θ and δ. (*) This sign holds for $0 < \tau_0 < 1/2$ and is reversed for $1/2 < \tau_0 < 1$.

higher δ) makes it easier for the taxpayer to win the case and thus to lower her expected tax obligation. On the other hand, for a given δ, if the cost of information for the taxpayer relative to that for the tax authority is sufficiently high (δ is small), then increasing the court's responsiveness θ to the evidence will increase expected revenue. Intuitively, in this case the tax authority has a better chance of persuading the judge that the tax shelter should not be allowed.

In the welfare-maximising and in the revenue-maximising equilibria, the optimal probability depends on the timing of the moves and, in the simultaneous-move framework, it also depends on the objective.

Assuming $\tau_0 < \frac{1}{2}$ is empirically relevant for the profit tax rate, in the efficient equilibrium, expected tax revenue increases with the higher responsiveness of the court's decision to the evidence presented by the tax-dispute parties. This suggests that the tax authority benefits from the uncertainty in the tax law when tax payments can be disputed. At the same time, the non-monotone effect of δ on net tax revenue implies, in particular, that making it harder for the taxpayer to dispute the tax bill does not necessarily increase tax revenue. Namely, making it harder for the taxpayer to defend sheltering (decreasing δ) has a negative effect on net tax revenue when the taxpayer's efforts in the court are less effective than the tax authority's efforts ($\delta < 1$), and has a positive effect in the opposite case.

In the revenue-maximising equilibrium, expected tax revenue is higher, the less responsive is the court to the evidence. Intuitively, a deviation from ex ante probability that maximises revenue can only reduce revenue. The taxpayer's information cost has a non-monotone effect, but it is opposite to that in the welfare-maximising equilibrium (for $\tau_0 < \frac{1}{2}$): increasing the cost of information (decreasing δ) has a negative effect on expected tax revenue when the taxpayer's efforts in the court are more effective than the tax authority's efforts ($\delta > 1$), and has a positive effect otherwise.

6 Welfare gains of tax disputes

The tax dispute framework analysed above is, effectively, an extension of Tullock's (1975) model to a setting of contest with an endogenous shared prize. In the classical model of Chung (1996) it is assumed that size of the prize in increasing in the aggregate rent-seeking efforts. Thus, each party's effort exerts simultaneously a negative and a positive externality on the expected payoff of each other party. The negative externality comes from the effort diminishing the probability of other parties winning the prize. The positive externality comes from the effort increasing the size of the prize for everyone. One of the main results in Chung (1996) is that in equilibrium the aggregate efforts are always excessive. Thus, rent-seeking always leads to welfare loss even when the efforts are productive.

In this model, the prize is the pre-tax profit from investment, and it is shared between the taxpayer and the tax authority. The pre-tax profit depends on the parties' efforts, measured by their litigation costs, because they determine the effective tax rates and, thus, the equilibrium allocation of investment. Therefore, the size of the prize in this model is endogenous. The sharing rule is also endogenous, since the split of the prize between the tax bill and after-tax profits depends on the probability of the taxpayer winning the dispute. Furthermore, as shown below, in this framework the parties' efforts can be either productive or destructive in equilibrium. That is, the direction of the effect of the aggregate effort on the size of the prize is endogenous. Another interesting feature of this model is that when the parties' efforts are productive in equilibrium, they can be either higher or lower than the socially optimal level. Finally, when the efforts are productive, the equilibrium welfare can be higher than in the absense of rent-seeking, – in contrast with the results obtain in the model with exogenously productive efforts. Thus, the legal system that allows costly tax disputes can contribute to economic efficiency.

6.1 Productive and destructive equilibrium efforts

As before, let p_0 denote an arbitrary ex ante probability, let p denote the ex post probability, and let p^e be the probability that maximises the size of the prize.

With either simultaneous or sequential moves, in equilibrium

$$p = [1-\theta] p_0 + \frac{\theta\delta}{1+\delta}, \tag{26}$$

$$I_T = I_R = [\tau_H - \tau_L] \pi (\lambda (p)) \frac{\theta\delta}{[1+\delta]^2}. \tag{27}$$

Under Assumption 0, the expected pre-tax profit has a unique maximum at

$$p^e = \frac{\tau_H - \tau_0}{\tau_H - \tau_L}.$$

The efforts are productive in equilibrium if the ex post probability is closer to p^m than the ex ante probability,

$$p_0 < (1-\theta) p_0 + \frac{\theta\delta}{1+\delta} < \frac{\tau_H - \tau_0}{\tau_H - \tau_L},$$

which gives

$$p_0 < \min\left\{\frac{\delta}{1+\delta}, \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right]\right\}, \tag{28}$$

or

$$p_0 > (1-\theta) p_0 + \frac{\theta\delta}{1+\delta} > \frac{\tau_H - \tau_0}{\tau_H - \tau_L}$$

which gives

$$p_0 > \max\left\{\frac{\delta}{1+\delta}, \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right]\right\}. \tag{29}$$

The two remaining cases, where the ex ante probability is below and ex post probability is above p^m, or other way around, are inconclusive, and the outcome depends on the shape of the profit function. These cases correspond to

$$\frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right] < p_0 < \frac{\tau_H - \tau_0}{\tau_H - \tau_L}$$

or

$$\frac{\tau_H - \tau_0}{\tau_H - \tau_L} < p_0 < \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right]$$

Similarly, the efforts are destructive in equilibrium if the ex post probability is further away from p^m than the ex ante probability. The analysis of the welfare-maximising and revenue-maximising probabilities summarised in Table 2 shows that they can induce either productive or destructive efforts, depending on the model parameters. For example, in the parameterisation used for Fig. 1, in the sequential setting, the optimal ex ante probability induces productive efforts for any θ as long as $\delta \geq 1$ and for sufficiently large θ if $0 < \delta < 1$.

In the simultaneous-move case p_0^W is given by (13). Consider, first, case (28). The efforts are productive if

$$p_0 < \min\left\{\frac{\delta}{1+\delta}, \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right]\right\}.$$

Since the equilibrium efforts increase in p_0, the efforts are excessive if, and only if, $p_0 > p_0^W$, i.e. whenever the ex ante probability of the tax shelter being legitimate is above the efficient level.Clearly, if the model parameters are such that in equilibrium $p_0^W \in (0,1)$ and $\frac{1}{1-\theta}\left[\frac{\tau_H-\tau_0}{\tau_H-\tau_L} - \frac{\theta\delta}{1+\delta}\right] < \frac{\delta}{1+\delta}$, then the equilibrium efforts induced by ex ante probability, say, $\widehat{p}_0$, are productive but lower than socially optimal if

$$\widehat{p}_0 < p_0^W = \frac{1}{1-\theta}\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta}\frac{\delta}{1+\delta}\left[1 + \frac{2\left[1-\tau_0\right]}{1+\delta}\right] < \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right],$$

and productive but higher than socially optimal if

$$\frac{1}{1-\theta}\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta}\frac{\delta}{1+\delta}\left[1 + \frac{2\left[1-\tau_0\right]}{1+\delta}\right] = p_0^W < \widehat{p}_0 < \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right].$$

Next, consider case (29). The efforts are productive if

$$p_0 > \max\left\{\frac{\delta}{1+\delta}, \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right]\right\}.$$

If the model parameters are such that in equilibrium $p_0^W \in (0,1)$ and $\frac{1}{1-\theta}\left[\frac{\tau_H-\tau_0}{\tau_H-\tau_L} - \frac{\theta\delta}{1+\delta}\right] > \frac{\delta}{1+\delta}$, then the productive equilibrium efforts induced by ex ante probability $\widehat{p}_0$ are necessarily higher than socially optimal:

$$\widehat{p}_0 > \frac{1}{1-\theta}\left[\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta\delta}{1+\delta}\right] > \frac{1}{1-\theta}\frac{\tau_H - \tau_0}{\tau_H - \tau_L} - \frac{\theta}{1-\theta}\frac{\delta}{1+\delta}\left[1 + \frac{2\left[1-\tau_0\right]}{1+\delta}\right] = p_0^W.$$

It is straightforward to obtain a similar conclusion for the sequential-move case: here also, productive equilibrium efforts can be either higher or lower than the socially optimal level in case (28) and are necessarily higher than the socially optimal level in case (29).

Destructive efforts can be analysed in a similar way. The efforts are destructive in equilibrium if the ex ante probability is below p^m and the ex post probability is even lower:

$$\left(1-\theta\right)p_0 + \frac{\theta\delta}{1+\delta} < p_0 < \frac{\tau_H - \tau_0}{\tau_H - \tau_L}, \tag{30}$$

which gives

$$\frac{\delta}{1+\delta} < p_0 < \frac{\tau_H - \tau_0}{\tau_H - \tau_L},$$

or if the ex ante probability is above p^m and the ex post probability is even higher:

$$\left(1-\theta\right)p_0 + \frac{\theta\delta}{1+\delta} > p_0 > \frac{\tau_H - \tau_0}{\tau_H - \tau_L} \tag{31}$$

which gives

$$\frac{\tau_H - \tau_0}{\tau_H - \tau_L} < p_0 < \frac{\delta}{1+\delta}.$$

Again, destructive equilibrium efforts can be either higher or lower than the socially optimal level, depending on the model parameters.

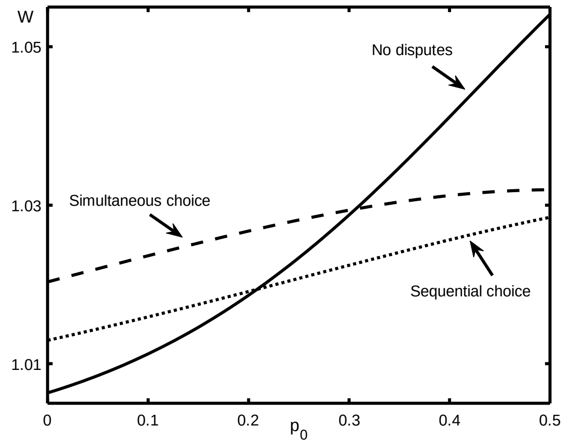

Figure 4: Welfare without and with tax disputes under simultaneous and sequential choices, for $\pi(x) = \pi_0(x) = x^{\gamma}$, evaluated at $\gamma = 0.9, \tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.5, \delta = 1.6$.

6.2 Welfare-improving tax disputes

Productive efforts increase total welfare for an arbitrary ex ante probability when an increase in the pre-tax profits is higher than the total cost of efforts in equilibrium:

$$\pi_0(1 - \lambda(p)) + \pi(\lambda(p)) \left[1 - 2(\tau_H - \tau_L)\frac{\theta\delta}{[1+\delta]^2}\right] - \pi_0(1 - \lambda(p_0)) + \pi(\lambda(p_0)) > 0.$$

The welfare effect depends on the shape of the profit function. For example, for $\pi(\lambda) = \pi_0(\lambda) = \lambda^{\gamma}$, $\gamma \in (0,1)$, numerical simulations show that productive efforts in tax dispute increase total welfare over a substantial range of the model parameters.

Figures 4 and 5 illustrate this possibility for $\gamma = 0.9$, $\tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.5$. This parametrisation allows for case (28), that is, the case in which a tax dispute can move the initial uncertainty towards the efficient level. Figure 4 depicts the welfare levels for $\delta = 1.6$, with and without tax disputes. One can see that in this parametrisation tax disputes achieve higher welfare when p_0 is relatively low (the maximal welfare without tax disputes is achieved at $p_0 = p^e = 0.75$). In Figure 5 the two curves trace the combinations of p_0 and δ for which the welfare levels with and without disputes are equal, under sequential choice and under simultanteous choice assumptions. In each case, tax disputes increase the total welfare over the range of p_0 and δ above the respective curve and decrease it below the curve. Thus, when p_0 is low, a tax dispute is more likely to achieve welfare gain when δ is sufficiently high. The numerical investigation of the effect of θ (the responsiveness of the judge to the parties' evidence) reveals that lower θ increases the range of $\{p_0, \delta\}$ where tax disputes achieve welfare gain, but this change is very small. Overall, the findings suggest that tax disputes are socially desirable when the ex ante likelihood of tax shelter being legitimate is low and that the welfare improvement is larger, the easier it is for the taxpayers to defend their case in court.

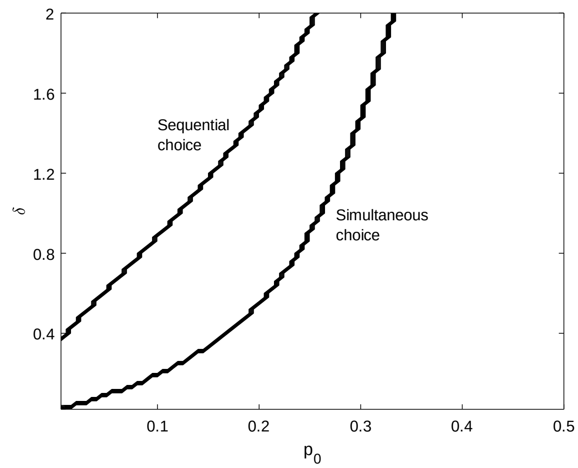

Figure 5: Welfare effect of tax disputes under simultaneous and sequential choices: net gains (above the respective curve) and losses (below the respective curve) relative to no-dispute case, for $\pi(x) = \pi_0(x) = x^\gamma$, evaluated at $\gamma = 0.9, \tau_0 = 0.2$, $\tau_H = 0.5$, $\tau_L = 0.1$, and $\theta = 0.5$.

7 Concluding remarks

As documented by Devereux (2016) in a survey of senior tax professionals in large businesses and tax advice firms, between 2010 and 2015 uncertainty in corporation tax law increased in 20 of 21 major countries analysed in the study (Japan being the notable exception). In their answers, '*uncertainty about the effective tax rate on profit'* was the third most important determinant of investment and location decisions, more important than the '*anticipated effective tax rate'* itself. Moreover, '*unpredictable or inconsistent treatment by tax authority*' appeared as the single most important factor of uncertainty. Brok (2019) has constructed a measure of legal uncertainty in corporate income tax law, based on legal literature and the outcomes of court cases in ten countries over seven years. Using this measure, he found evidence that legal uncertainty has significant effect on the financing and location decisions of companies. These findings suggest that tax law uncertainty can have wide economic implications, with questions then arising about welfare consequences.

I have shown that reducing uncertainty in tax law may or may not be socially desirable. Higher uncertainty can lead to higher welfare by reducing inefficiency caused by distortionary differential taxation. Furthermore, I show that tax litigation expense can more than offset this distortion. Thus, a tax dispute can lead to welfare gains, despite the litigation costs, by improving the efficiency of the investment decision of the taxpayer. The welfare effect of a tax dispute is more likely to be positive when tax uncertainty is not in taxpayerss favour, and the gains are higher, the easier it is for the taxpayers to gather evidence and defend their case.

The model is deliberately simple and focusses on a single taxpayer. For analytical tractability, I assumed that litigation cost does not reduce the amount of resources available for productive investment. This is consistent with the interpretation of the taxpayer as a profit-maximising firm, rather than a consumer facing budget constraint. Imposing a constraint on the total resource allocated between investment and tax dispute expenditures reduces the latter in the equilibrium. This does not change qualitatively the main results as long as the fraction of the total resource spent on tax dispute is sufficiently small.

The model can be further extended to explore the welfare properties of a continuum of fee-shifting rules, as in Baye et al. (2005) or to analyse redistributive consequences of tax uncertainty in a setting with heterogeneous taxpayers facing different costs of evidence in tax disputes (Kaplow, 1998; Kopczuk, 2001). This work is left for the future research.

Acknowlegements *I am grateful to Yulia Epifantseva, Wei Cui, and Damian Damianov for their comments on previous versions and to the participants in the Third Ariel Workshop on the Political Economy of Public Policy for constructive discussion.*

References

Baye, M. R., and Hoppe, H. C., 2003. The strategic equivalence of rent-seeking, innovation, and patent-race games. Games and economic behavior, 44(2), 217–226.

Baye, M. R., Kovenock, D., and De Vries, C. G., 2005. Comparative analysis of litigation systems: An auction-theoretic approach. The Economic Journal, 115(505), 583–601.

Bizer, D.S., and Judd, K.L., 1989. Taxation and uncertainty. American Economic Review 79(2), 331–336.

Braithwaite, J., 2002. Rules and principles: a theory of legal certainty. Australian Journal of Legal Philosophy 27, 47–82.

Brok, P., 2019. As uncertain as taxes, in: Proceedings. Annual Conference on Taxation and Minutes of the Annual Meeting of the National Tax Association, January 2019, 112, pp. 1–64. National Tax Association.

Chung, T.-Y., 1996. Rent-seeking contest when the prize increases with aggregate efforts. Public Choice, 87(1/2), 55–66.

D'Amato, A., 1983. Legal uncertainty. California Law Review 71, 1–55.

Devereux, M., 2016. Measuring corporation tax uncertainty across countries: Evidence from a cross-country survey. Oxford University Centre for Business Taxation Working Paper Series WP, 16, 13.

Givati, Y., 2009. Resolving legal uncertainty: The fulfilled promise of Advance Tax Rulings, Virginia Tax Review, 29, 137–175.

Hines Jr, J.R. and Keen, M., 2021. Certain effects of random taxes. Journal of Public Economics, 203, 104412.

Kaplow, L., 1998. Accuracy, complexity, and the income tax. Journal of Law, Economics and Organization 14, 61–83.

Kopczuk, W., 2001. Redistribution when avoidance behavior is heterogeneous. Journal of Public Economics 81, 51–71.

Posner, R. A., 2014. Economic analysis of law. Ninth Edition, Aspen Publishing, Wolters Kluwer Law & Business, New York, NY.

Shannon, C. E., 1948. A mathematical theory of communication. The Bell System Technical Journal 27(3), 379–423. DOI: 10.1002/j.1538-7305.1948.tb01338.x.

Stiglitz, J.E., 1982. Utilitarianism and horizontal equity: the case of random taxation. Journal of Public Economics 18, 1–33.

Treasury Committee, 2011. Principles of tax policy. House of Commons Treasury Committee Eighth report of session 2010-11, HC 753. London: The Stationery Office Limited.

Tullock, G., 1975. On the efficient organization of trials. Kyklos, 28(4), 745–762.

Weisbach, D.A., 1999. Formalism in the tax law. The University of Chicago Law Review 66(3), 860–886.

Weisbach, D.A., 2013. Is knowledge of the tax law socially desirable? American Law and Economics Review 15(1), 187–211.

Weiss, L., 1976. The desirability of cheating incentives and randomness in the optimal income tax. Journal of Political Economy 84(6), 1343–1352.