CESifo WORKING PAPERS

10399
2023

April 2023

Monetary Policy Interactions: The Policy Rate, Asset Purchases, and Optimal Policy with an Interest Rate Peg

Isabel Gödl-Hanisch, Ronald Mau, Jonathan Rawls

CESifo

Impressum:

CESifo Working Papers
ISSN 2364-1428 (electronic version)
Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH
The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute
Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de
Editor: Clemens Fuest
https://www.cesifo.org/en/wp
An electronic version of the paper may be downloaded
· from the SSRN website: www.SSRN.com
· from the RePEc website: www.RePEc.org
· from the CESifo website: https://www.cesifo.org/en/wp

CESifo *Working Paper No. 10399*

Monetary Policy Interactions: The Policy Rate, Asset Purchases, and Optimal Policy with an Interest Rate Peg

Abstract

We study monetary policy in a New Keynesian model with a variable credit spread and scope for central bank asset purchases to matter. A novel financial and labor market interaction generates an endogenous cost-push channel in the Phillips curve and a credit wedge in the IS curve. The "divine coincidence" holds with the nominal short-term rate and central bank balance sheet available as policy tools. Credit spread-targeting balance sheet policy provides a determinate equilibrium with a fixed policy rate. This policy induces similar welfare losses relative to dual-instrument policy as inflation-targeting interest rate policy with a fixed balance sheet.

JEL-Codes: E430, E520, E580.

Keywords: unconventional monetary policy, optimal monetary policy, New Keynesian model, policy rate lower bound, interest rate peg.

Isabel Gödl-Hanisch
LMU Munich / Germany
isabel.goedl-hanisch@econ.lmu.de

Ronald Mau
University of Mississippi / USA
rrmau@olemiss.edu

Jonathan Rawls
Bank of America
jonarawls@gmail.com

April 27, 2023
Thank you to John Conlon, Robert Lester, Eric Sims, and Mirko Wiederholt for comments that substantially improved the paper. We received helpful comments from seminar participants at the Bank of Canada, Midwest Macro Spring 2022, ASSA 2022 Annual Meeting, University of Notre Dame, LMU Munich, and Spring 2023 I-85 Macroeconomics Workshop.

1 Introduction

Balance sheet policy, or quantitative easing (QE), has become a staple tool for central bankers.[1] QE programs have varied in the scope and types of assets purchased but have largely been conducted with policy rates at the effective lower bound (ELB). In 2019, however, the Federal Reserve expanded its balance sheet by $400 billion with the federal funds rate above the ELB and *before any news of Covid-19*.[2] Balance sheet expansion away from the ELB is at odds with the idea that balance sheet policy is a substitute for interest rate policy at the ELB. For this reason, this paper revisits optimal monetary policy – both interest rate and balance sheet policy – asking whether active balance sheet policy away from the ELB can be optimal.

We point out that limits to central bank interest rate policy arise and balance sheet policy matters in a world where indebted households alter their labor supply in response to changing financial conditions. We then ask two questions. First, can dual-instrument policy, meaning simultaneous interest rate and balance sheet policies, overcome these limitations? Second, is balance sheet policy effective if interest rate policy is unavailable to the central bank? Given our answers to these questions, we investigate the welfare costs of various single-instrument monetary policies, e.g., inflation-targeting interest rate or balance sheet policy, relative to the optimal dual-instrument policy.

We analyze monetary policies in a model where households finance a part of their consumption bundles with debt and financial markets are incomplete. The model adds two features to a standard New Keynesian (NK) model framework. First, a representative household simultaneously saves and borrows, issuing debt to finance a second expenditure type in addition to conventional consumption, dubbed debt-financed expenditure (e.g., housing, education, or automobiles).[3] The household finances debt-financed expenditure and debt repayment with a chosen fraction of labor income and new debt issuance, while financing conventional consumption and saving in a separate account with the remainder of its labor income and non-labor income.

Second, incomplete financial markets arise from constrained private lending due to an agency problem between banks and depositors, as in Gertler and Karadi (2011, 2013). Banks in the Gertler and Karadi setup stochastically exit and face an endogenous lever-

1. In the aftermath of the financial crisis, the Federal Reserve implemented various rounds of quantitative easing (e.g., QE 1, 2, 3). The Bank of England, the European Central Bank, and other major central banks followed almost in lockstep.

2. The Federal Reserve's balance sheet expanded from $3.76 trillion to $4.18 trillion between September 2019 and February 2020 (Federal Reserve Board: Credit and Liquidity Programs and the Balance Sheet).

3. Our household setup is a consolidated version of Guerrieri and Iacoviello (2017), a generalization of Sims et al. (2021), and a simplification of Cúrdia and Woodford (2011, 2016).

age constraint. We introduce bank reserve holdings that are not subject to the leverage constraint.[4] Central bank reserve issuance in exchange for debt securities relaxes the leverage constraint, bringing the economy closer to the complete markets case. Focusing on the deterministic bank exit limit maintains the endogenous nature of the leverage constraint while generating analytical tractability.[5]

The combination of the household borrowing setup and constrained financial sector cause financial conditions to vary with non-financial economic conditions, providing a new propagation mechanism for shocks originating outside the financial sector. Conventional consumption and debt-financed expenditure have the same price. For this reason, the household desires to equate the marginal utilities from both expenditure types. However, incomplete financial markets limit the household's ability to do so, generating a wedge between these marginal utilities, which we call the *liquidity premium*. The liquidity premium is time-varying due to fluctuations in household borrowing costs relative to the deposit rate, micro-founded by the constrained banking problem. In response to liquidity premium fluctuations, the household alters its labor income allocation between the expenditure accounts and labor supply. The interaction of changes in financial conditions, labor supply, and the household's labor income allocation delivers a source and propagation force of shocks emerging in the financial and non-financial sectors. Furthermore, we show that this new propagation mechanism limits the ability of interest rate policy to stabilize inflation and the output gap simultaneously.

An approximation of the model simplifies to eight structural equations. The first two equations are the Phillips and IS curves. The liquidity premium acts as an *endogenous* cost-push channel in the Phillips curve and credit wedge in the IS curve because a fraction of aggregate expenditures vary with the lending rate.[6] A third equation shows that the liquidity premium equals the ex-ante forward-looking path of the loan to deposit rate, or credit, spread. This causes the output gap to vary with a weighted average of the real deposit and lending rate paths. The remaining equations define the financial block in the economy. These equations can be combined into one equation defining the liquidity premium in terms of the output gap, inflation, policy instruments, and exogenous economic shocks. The policy instruments include the interest rate on reserves, which the nominal deposit rate equals, and the quantity of reserve issuance.

4. For a bank that holds non-reserve assets and reserves, backed by deposits and equity, we define bank leverage as the ratio of non-reserve asset holdings to equity.

5. This is a more complex banking setup than Sims et al. (2021) but simplification, in the form of a limiting case, of Gertler and Karadi (2011, 2013).

6. For certain parameter restrictions, our model nests the standard Phillips and IS curves in the 3-equation NK model.

To first order, the general equilibrium output level in this economy varies with derived labor demand and supply shifts. The labor supply shift depends on both the real deposit and lending rate paths. Optimal monetary policy must simultaneously balance the impact of both rates on labor supply. The so-called "divine coincidence", coined by Blanchard and Galí (2007), reflects the ability of a monetary authority to simultaneously target inflation and the output gap in the textbook NK model. In that case, inflation-targeting interest rate policy eliminates inefficient labor demand variability due to the markup and provides the optimal labor supply response with the policy rate equal to the natural rate. This result fails to hold in our model due to the labor supply dependency on the lending rate. Output variability stemming from variation in the credit spread generates welfare losses with inflation-targeting interest rate policy.

Balance sheet policy that targets the credit spread restores the divine coincidence. Balance sheet expansion in response to tightening financial conditions, or vice versa, stabilizes the credit spread. Under this policy, the labor supply shift now only varies with the real deposit rate. Inflation-targeting interest rate policy eliminates any inefficient labor demand shifts and, in combination with this balance sheet policy, restores the optimal labor supply shift in response to the natural rate shock. Both the deposit and lending rates equal the natural rate. In summary, targeting the credit spread with balance sheet policy causes inflation-targeting interest rate policy to simultaneously stabilize the output gap, providing the welfare-maximizing outcome. This result points to the potential need for policymakers to use balance sheet policy much more. The key lesson is that: In a credit crunch, expand the central bank balance sheet *independent* of the source of financial market variability or the policy rate level relative to the ELB.

A natural question arises in our modeling context as to how well balance sheet policy performs independent of interest rate policy — a somewhat extreme but theoretically interesting question. A policy rate peg typically leads to nominal indeterminacy in macroeconomic models (Sargent and Wallace, 1975), a point not lost on NK models. In our model, a policy rate peg does not necessarily lead to nominal indeterminacy when considering active balance sheet policy. We prove that balance sheet policy that targets the credit spread path via the liquidity premium or inflation in the face of a policy rate peg leads to a determinate linear rational expectations equilibrium. However, the inflation-targeting policy introduces non-stationarity in the central bank balance sheet size. Given this, liquidity premium-targeting balance sheet policy results in a smaller welfare loss than inflation-targeting balance sheet policy with a policy rate peg. The welfare loss of this policy relative to dual-instrument policy is similar in magnitude to the loss from inflation-targeting interest rate policy with no balance sheet policy.

This paper contributes to the literature on the "financial accelerator" (see Bernanke et al., 1996, 1999) by characterizing this mechanism through the labor supply decision. Consider a deflationary natural rate shock that raises conventional consumption. From an accounting perspective, the household must save less, work more, or retain more labor income for conventional consumption. Retaining more labor income for conventional consumption crowds out debt-financed expenditure, but the household desires to equate marginal utilities from both expenditures. Thus, the household issues more debt or works more to counteract the crowding-out effect. However, incomplete financial markets restrict the household's ability to issue more debt, potentially dampening the debt-financed expenditure and, thus, output responses relative to a model with complete financial markets. The household internalizes this effect and further increases its labor supply, generating an amplified output response due to the amplified labor supply shift.

This paper also relates to the broader literature on general equilibrium macroeconomic modeling. We provide an alternative micro-foundation for an endogenous cost channel to input cost borrowing as in Christiano et al. (2005) or Ravenna and Walsh (2006), endogenizing the "cost-push" shock introduced in Clarida et al. (1999). In addition, we micro-found an endogenous credit wedge in the IS curve (similar to Cúrdia and Woodford, 2011, 2016) without assuming utility over safe and liquid security holdings as in Fisher (2015), endogenizing the "risk premium" shock from Smets and Wouters (2007). These mechanisms provide a unified framework for analyzing the demand- and supply-side effects of financial market variability on the economy.

Finally, this paper adds to the literature on optimal QE policy (e.g., Sims et al., 2021 or Mau, 2022), outside of the positive analysis of QE policies (e.g., Gertler and Karadi, 2011, 2013; Carlstrom et al., 2017; or Boehl et al., 2022). Sims et al. (2021) prescribe balance sheet policy in response to shocks originating in the financial sector only. We prescribe the use of balance sheet policy in response to all shocks as in Mau (2022) in the context of a tractable model as in Sims et al. (2021). Balance sheet policy alone neutralizes the effects of financial shocks as in Sims et al. (2021), but non-financial shocks require a combination of interest rate and balance sheet policy responses.

The remainder of this paper is structured as follows: Section 2 describes the key parts of the model and shows how shock transmission differs in our model compared to flexible- and sticky-price models with complete financial markets. Section 3 highlights the properties of single- and dual-instrument interest rate and balance sheet policies. Section 4 quantifies the welfare losses to single-instrument monetary policies relative to optimal dual-instrument policy. Section 5 concludes. The appendices provide all micro-founded model details, derivations of results, and accompanying proofs.

2 The Model

We consider an NK model with financial frictions and a role for central bank asset purchases to matter. The economy consists of a representative household, financial and production sectors, and a central bank. In this section, we discuss the household, financial sector, and central bank setups. We then define a linear approximation of the equilibrium. Finally, we outline monetary policy transmission in the model. The production sector is standard to NK models with Calvo (1983) type nominal price rigidity. Appendix A derives the micro-founded nonlinear model and a linear approximation of this model with further micro-founded banking sector details provided in Appendix B.

The household setup nests a model in which two types of workers, patient versus impatient, provide labor to the production sector. The patient worker is the residual claimant in the economy and saves via financial sector deposits. The impatient worker supplements labor income with long-term debt issuance held by the financial sector or the central bank. That is, financial markets are segmented in the sense that the patient worker cannot hold long-term debt issued by the impatient worker. Long-term debt takes the form of a perpetuity with geometrically decaying coupon payments.

Mau et al. (2023) show that the patient/impatient worker setup consolidates to a representative household framework, illustrated by Figure 1. The representative household has utility over two expenditure types — conventional consumption (henceforth consumption) and debt-financed expenditure. The household exhibits relative impatience over utility flows from debt-financed expenditure and is the residual claimant in the economy. The household allocates labor income across a conventional goods account and a financial account, highlighted by red in the model illustration. Debt-financed expenditure and debt repayment are financed by the labor income allocation to the financial account and new debt issuance. The representative household consolidation shows that the aggregate labor preference shifter in the economy is endogenous and varies with the state of financial conditions. Aggregate labor supply varies with the wage, marginal utility from consumption, and state of financial conditions in the economy.

The financial sector follows Gertler and Karadi (2011, 2013), with the addition of central bank reserve issuance to financial intermediaries as in Sims and Wu (2021).[7] We consider the deterministic bank exit limit to the Gertler and Karadi financial sector setup for analytical tractability. Banks face a modified leverage constraint over private debt holdings, which we assume always binds. Bank private debt holdings cannot exceed a multiple of accumulated bank equity. We refer to this multiple as the modified leverage

7. Sims and Wu (2021) consider reserve requirements on the banking sector, which we abstract from.

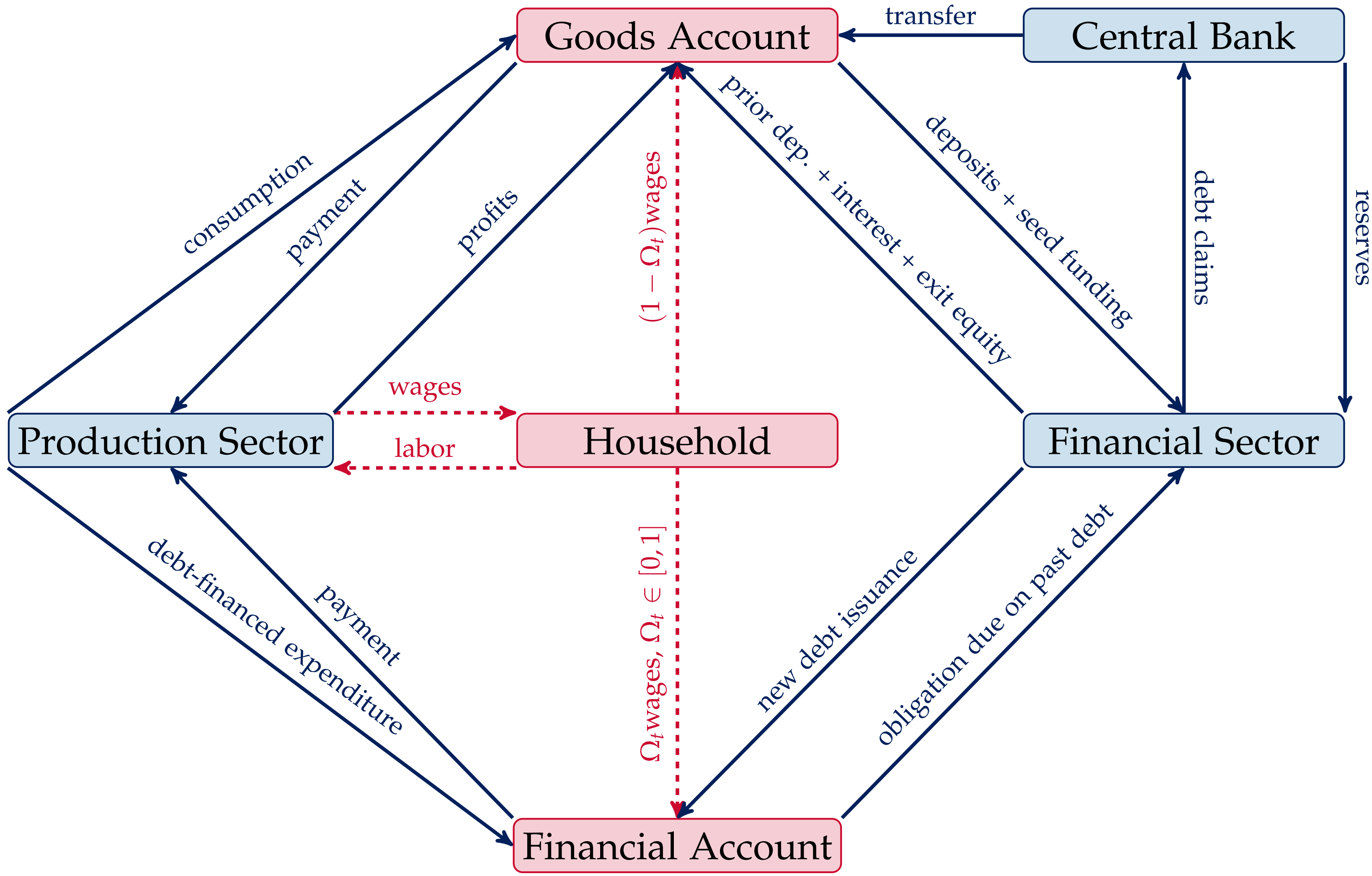


Figure 1: The Model Economy

ratio. The equilibrium conditions from the financial sector include a definition of the modified leverage ratio and the modified leverage constraint. The modified leverage constraint defines the private debt holdings of the financial sector.

Central bank private debt holdings are financed via reserve issuance to the financial sector. Central bank reserves pay interest. Profit maximization in the financial sector implies that the interest rate on reserves equals the deposit rate in the economy. The policy instruments of the central bank include the interest rate on reserves and the size of its balance sheet. Debt market clearing implies that the market value of long-term debt outstanding equals the debt holdings from the financial sector plus holdings by the central bank. Central bank debt holdings equal the reserve level in the economy.

A log-linear approximation of the nonlinear model around the zero net inflation steady state simplifies to eight structural equations. The first two equations are the Phillips and IS curves:

$$\pi_t = \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \tag{2.1}$$

$$gap_t = \mathbb{E}_t gap_{t+1} - \left(\left(1-\overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r^L_{t+1} - \mathbb{E}_t \pi_{t+1} - r^n_t\right) \tag{2.2}$$

where π_t is net inflation, gap_t is the output gap, ξ_t is a liquidity premium in the long-term debt market, r_t is the policy rate, r^L_t is the nominal lending rate, and r^n_t is the natural rate. The interest rates are in level-deviations from steady state. The liquidity premium is in log-deviations from steady state. The output gap is defined as the log-difference between the output level in the underlying economy and a benchmark economy with a fixed markup and liquidity premium. That is, the benchmark economy includes steady-state distortions due to monopolistic competition and incomplete financial markets but no time-variation in these wedges.

γ is the product of the output gap elasticity of marginal cost and the marginal cost semi-elasticity of inflation. η is the inverse wage elasticity of labor supply. $\overline{C}_b$ is the steady-state debt-financed expenditure share of aggregate expenditures. Ω is the steady-state labor income allocation to the financial account. $\overline{C}_b - \Omega$ is the liquidity premium elasticity of marginal cost. The natural rate is exogenous and follows an AR(1) process with AR term ρ_n and standard deviation of an i.i.d. mean-zero disturbance σ_n.

The Phillips curve is standard absent the liquidity premium term. The liquidity premium acts as an endogenous cost channel in the model. The IS curve reflects the fact that a fraction of aggregate expenditures, $1 - \overline{C}_b$, vary with the ex-ante real deposit rate, whereas the remaining fraction, $\overline{C}_b$, are debt-financed and vary with the ex-ante real lending rate. For $\overline{C}_b = \Omega = 0$, these equations nest the standard Phillips and IS curves in the 3-equation NK model. That is, if there is no debt-financed expenditure, then no wage income is allocated to the financial account, and the output gap only varies with ex-ante real deposit rate deviations from the natural rate.

Asset pricing conditions from the household problem define the nominal lending rate and the liquidity premium in the economy:

$$r_t^L = \kappa\beta\zeta q_t - q_{t-1} \tag{2.3}$$

$$\xi_t = \mathbb{E}_t\xi_{t+1} + \mathbb{E}_t r_{t+1}^L - r_t \tag{2.4}$$

where q_t is the current price of long-term debt. κ is the coupon decay rate for the long-term debt perpetuity. The implied duration of debt equals $1/(1-\kappa)$. ζ measures the relative impatience over debt-financed expenditure and defines the steady-state lending to deposit rate, or credit, spread. The liquidity premium equals the forward-looking path of the credit spread. With complete financial markets or credit spread stabilizing monetary policy, the liquidity premium is fixed, $\xi_t \equiv 0$,. In this case, the IS and Phillips curves again nest the standard versions from the 3-equation NK model.

The model includes a financial block:

$$c_{b,t} = gap_t - \left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n}r_t^n \tag{2.5}$$

$$\begin{aligned}\overline{C}_b c_{b,t} = \Omega\frac{\varepsilon-1}{\varepsilon}\Bigg(\bigg(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\bigg)\xi_t + (2+\eta)gap_t - \frac{1}{1-\rho_n}r_t^n\Bigg)&\\ + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r_t^L - \pi_t\right)\right)&\end{aligned} \tag{2.6}$$

$$q_t + b_t = \left(1-\overline{RE}\right)\phi_t + \overline{RE}re_t \tag{2.7}$$

$$\phi_t = \frac{\Phi}{\zeta}\left(\mathbb{E}_t r_{t+1}^L - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \tag{2.8}$$

The financial block consolidates to define the liquidity premium in terms of the output gap, inflation, policy instruments, and the exogenous shocks to the economy.

Equation (2.5) is the aggregate resource constraint, written in terms of debt-financed expenditure, $c_{b,t}$, the output gap, the liquidity premium, and the natural rate. Equation (2.6) defines debt-financed expenditure in terms of the labor income allocation to the financial account, written in terms of the liquidity premium, output gap, and natural rate, and the current market value of debt net of obligations due on past debt. ε is the goods elasticity of substitution. As ε increases, the steady-state wage rises. As Ω increases, a larger share of labor income goes to the financial account. $q_t + b_t$ is the equilibrium level of debt in the economy. A higher steady-state relative size of the financial sector, QB/Y, amplifies the effect of the debt level and debt obligations, $q_{t-1} + b_{t-1} + r_t^L - \pi_t$, on the equilibrium level of debt-financed expenditure. A higher steady-state real lending rate, $1/\beta\zeta$, amplifies the effects of the household debt obligation.

Equation (2.7) defines the equilibrium market value of debt. This varies with variability in private financial sector debt holdings and the central bank balance sheet size, each weighted by the steady-state share of debt held by the private financial market, $1 - \overline{RE}$, or the central bank, $\overline{RE}$. Private debt holdings only vary with modified leverage defined by equation (2.8). The modified leverage increases with the credit spread where Φ is the steady-state modified leverage ratio. We assume that there is exogenous variation in modified leverage which we call a financial shock, θ_t. We assume that the financial shock follows an AR(1) process with AR term ρ_θ and standard deviation of an i.i.d. mean-zero disturbance σ_θ.

An equilibrium is defined by sequences of quantities, $\{gap_t,\ c_{b,t},\ b_t,\ re_t,\ \phi_t\}$, and prices, $\{\pi_t,\ r_t,\ r_t^L,\ \xi_t,\ q_t\}$, such that equations (2.1) – (2.8) hold given: (i) policy decisions for the interest rate on reserves and balance sheet size, $\{r_t,\ re_t\}$; and (ii) sequences of the natural rate and financial shocks, $\{r_t^n,\ \theta_t\}$. In this section, we consider rules-based interest rate policy, $r_t = 1.5\pi_t$, holding the relative size of the central bank balance sheet to output fixed, $qe_t = re_t - y_t = 0$, where $y_t = gap_t - r_t^n/(1-\rho_n)$.[8] Note that output equals the output gap absent natural rate variability.

Table 1 provides the model parameterization. The discount rate, β, implies a 2% annualized steady-state real deposit rate. The Frisch wage elasticity of labor supply, $1/\eta$, is one-third. The goods elasticity of substitution, ε, implies a 15% steady-state goods price markup. The relative impatience over debt-financed expenditures, ζ, implies a 1.5% annualized steady-state credit spread. The steady-state modified leverage is set to 4. We calibrate the steady-state labor income allocation to the financial account, Ω, such that steady-state debt-financed expenditure equals steady-state new debt issuance. The steady-state debt-financed expenditure share is 20%, $\overline{C}_b = 0.2$. We calibrate the steady-state share of long-term debt held by the central bank, $\overline{RE}$, to the pre-2008 relative size of the central bank reserve stock to quarterly GDP flows. The steady-state relative size of the financial sector to output is 8. This is about double the pre-2008 size of the private depository institution and Federal Reserve financial asset holdings to GDP.

8. We hold the relative size of the balance sheet to output fixed, initially, as a proxy for the pre-ELB balance sheet policy. From 1984q1 to 2007q4, the relative size of the stock of depository institution reserves (FRED code: $MADIRL$) to the quarterly GDP flow (FRED code: GDP, quarterly flow meaning $GDP/4$) averaged 1.6% with a persistent decline from a peak of 4.1% to 0.6% in 2007q4. From 2000q1 to 2007q4 this ranged from 0.5% to 1.0%, whereas by 2008q4 the relative size of reserves to GDP was 21.7%, with a pre-Covid peak in 2014q1 at 56.6%.

Table 1: Calibration

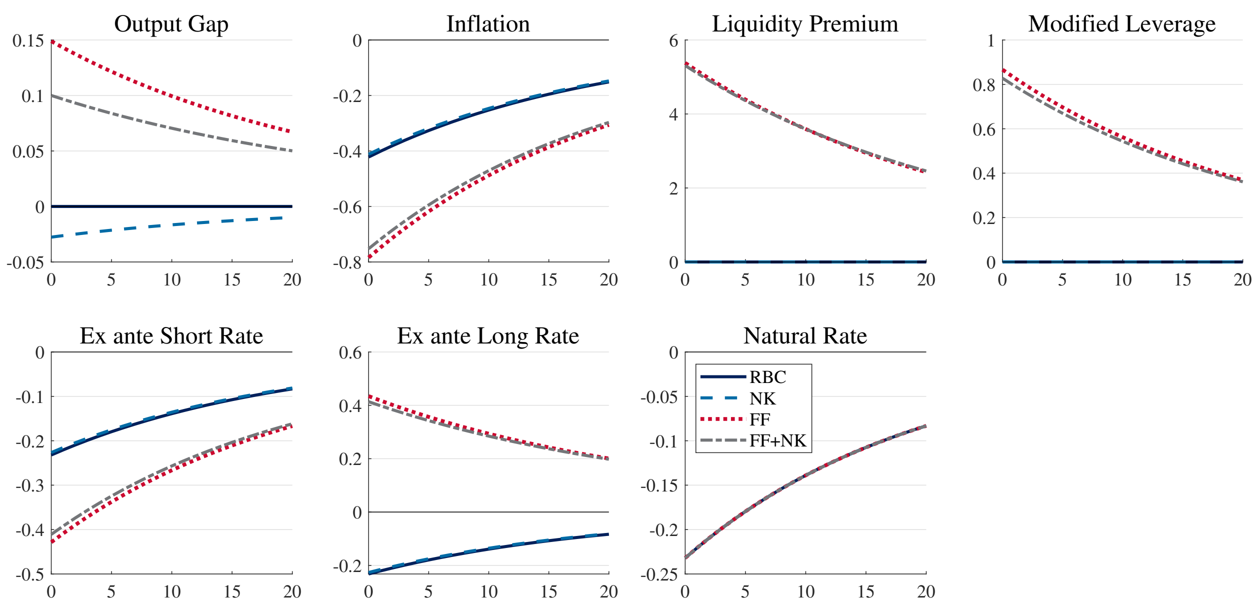
Parameter	Description	Target/Value
β	Discount factor	0.995
η	Inverse wage elasticity of labor supply	3
ε	Goods elasticity of substitution	7.667
ζ	Relative impatience over debt-financed expenditure	0.996
Φ	Steady-state modified leverage ratio	4
Ω	Steady-state labor income allocation to the financial account	0.311
$\overline{C}_b$	Steady-state share of credit good expenditure	0.2
$\overline{RE}$	Steady-state share of debt held by central bank	0.005
QB/Y	Relative size of financial sector to output	8
ρ_n	Natural rate shock autoregressive parameter	0.95
ρ_θ	Financial shock autoregressive parameter	0.95
σ_n	Natural rate shock standard deviation	0.012
σ_θ	Financial shock standard deviation	0.020

Figure 2 shows the impulse responses to a natural rate shock. The shock is scaled such that the impact output gap response in our model, labeled "FF+NK", is 0.1%. We present the responses in the corresponding real business cycle (RBC) model ($\gamma \to \infty$ and $\xi_t \equiv 0$), NK model ($\gamma = 0.204$ and $\xi_t \equiv 0$), and a financial frictions (FF) model with our constrained financial sector ($\gamma \to \infty$ and ξ_t time-varying). The NK model responses highlight the dampening effect of nominal price rigidity relative to the RBC model, whereas the FF responses highlight a financial accelerator-type mechanism present due to the constrained financial sector. By financial accelerator-type mechanism, we allude to the words of Ben Bernanke, "an economic upswing tends to improve the financial conditions of...banks, which in turn encourages greater lending..." see Bernanke (2022, p. 375). Taken together, our model exhibits a dampened financial accelerator due to the combination of nominal price rigidity and financial frictions.

The financial accelerator mechanism in our model is quite different. Tightening financial conditions (a rising liquidity premium) generate an amplified output response. The negative natural rate shock is deflationary and the policy rate falls.[9] Desired consumption *and* debt-financed expenditure levels increase. Debt-financed expenditure can only increase if the household allocates a higher fraction of labor income to the financial account, works more, or issues more debt. Financial sector demand for debt is constrained while the debt supply increases. The equilibrium lending rate rises. Given these dynamics, the partial equilibrium response of debt-financed expenditure to the natural rate

9. If it helps with intuition, consider a negative natural rate shock due to rising productivity.

Figure 2: Impulse Responses to a Natural Rate Shock



Notes: Solid lines: RBC model, $\gamma \to \infty$ and equations (2.5) – (2.8) replaced by $\xi_t \equiv 0$; dashed lines: NK model, $\gamma = 0.204$ and equations (2.5) – (2.8) replaced by $\xi_t \equiv 0$; dotted lines: a financial frictions (FF) model described by equations (2.1) – (2.8) with flexible prices, $\gamma \to \infty$; dashed-dotted lines: the complete model described by equations (2.1) – (2.8) with nominal price rigidity (FF+NK), $\gamma = 0.204$. The natural rate shock is scaled such that the output gap response in the FF+NK model is 0.1%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

shock is dampened. In general equilibrium, the household allocates more income to the financial account and works more, amplifying the natural rate shock effects.

We can derive the labor supply shift analytically. Combining the Phillips and IS curves, equations (2.1) and (2.2), defined in terms of output instead of the output gap, $gap_t = y_t + r_t^n/(1-\rho_n)$, iterating the IS curve forward, and substituting out the liquidity premium solved forward, defines the equilibrium level of output:

$$y_t = \underbrace{-\frac{r_t^n}{1-\rho_n}}_{\text{productivity}} + \underbrace{\frac{1}{\eta}\left[\underbrace{-\frac{r_t^n}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}(\pi_t - \beta\mathbb{E}_t\pi_{t+1})}_{\text{labor demand shift}} + \underbrace{\mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\Omega)r_{t+j} + \Omega r_{t+j+1}^L - \pi_{t+j+1}\right\}}_{\text{labor supply shift}}\right]}_{\text{equilibrium labor level}} \tag{2.9}$$

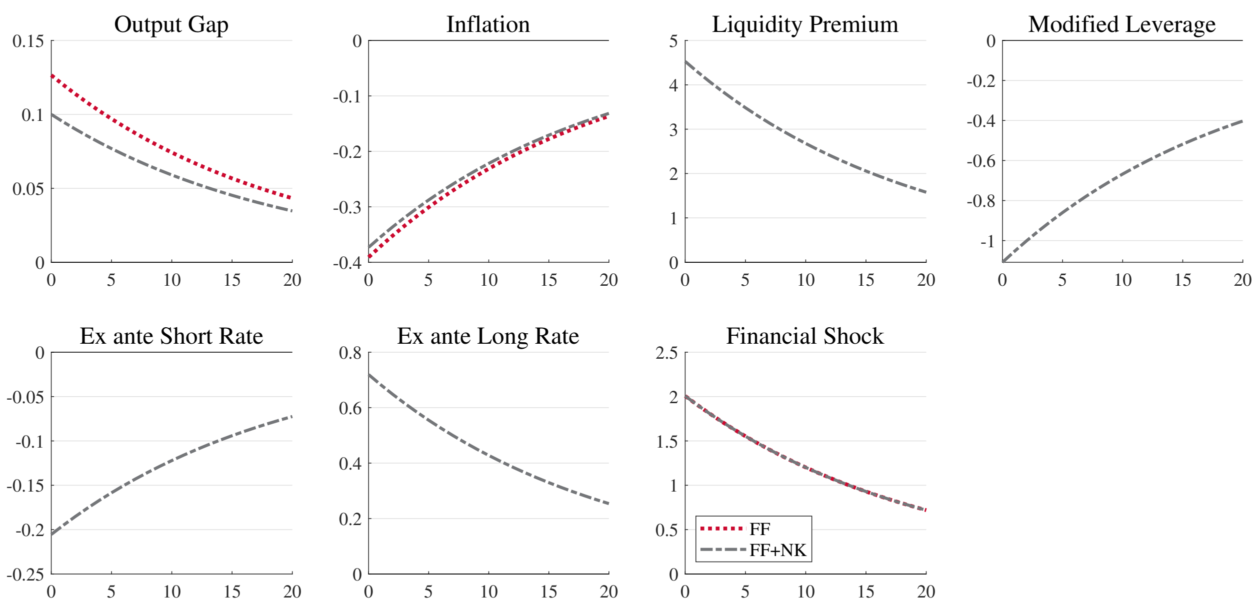
where $\widetilde{\gamma}$ is the marginal cost semi-elasticity of inflation, $\widetilde{\gamma} = \gamma/(1+\eta)$.

Equation (2.9) is the production function. The first term is the productivity level. Output varies one-for-one with the productivity level. The second term is the equilibrium labor response, which decomposes into labor demand and supply shifts. With inelastically supplied labor, $\eta \to \infty$, labor is fixed. Labor demand varies with the productivity level and an inefficient labor wedge due to nominal price rigidity. With flexible prices, $\widetilde{\gamma} \to \infty$, the labor wedge drops out. Labor supply varies with a weighted average of the forward-looking paths of the ex-ante real deposit and lending rates.

As the real deposit rate path falls in response to the natural rate shock, labor supply shifts in. However, worsening financial conditions cause the credit spread to rise. As the real lending rate path rises, labor supply shifts out. This is the financial accelerator mechanism in our model. Endogenously tightening financial conditions dampen the labor supply response to the natural rate shock relative to a model without debt-financed expenditure ($\Omega = 0$) or complete financial markets ($\mathbb{E}_t r_{t+1}^L \equiv r_t$ for all t).

Figure 3 shows the impulse responses to a financial shock. The financial shock is scaled such that the impact output gap response in our model is 0.1% as in the natural rate shock case. We do not show the RBC and NK responses. In these cases, complete financial markets ($\xi_t \equiv 0$) cause the financial shock not to affect the non-financial block of the economy. A positive financial shock corresponds to a reduction in the modified leverage ratio and tightening financial conditions. That is, the liquidity premium rises, raising the ex-ante real credit spread. The financial shock acts as a supply shock in the sense that output rises as inflation falls. This is due to a shift in aggregate labor supply in response to tightening financial conditions in the economy, similar to the response to the natural rate shock described above.

Figure 3: Impulse Responses to a Financial Shock



Notes: Dotted lines: a financial frictions (FF) model described by equations (2.1) – (2.8) with flexible prices, $\gamma \to \infty$; dashed-dotted lines: the complete model described by equations (2.1) – (2.8) with nominal price rigidity (FF+NK), $\gamma = 0.204$. The financial shock is scaled such that the output gap response in the FF+NK model is 0.1%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

3 Properties of Endogenous Balance Sheet Policy

This section highlights the properties of single- and dual-instrument monetary policy in the linear model described by equations (2.1) – (2.8). More specifically, we study the cases where: (i) the central bank has both the balance sheet and policy rate as policy instruments available, or (ii) the central bank only uses its balance sheet to support a permanent interest rate peg. Appendix C provides all proofs to the propositions and corollaries below.

Dual-instrument Policy Ensures that the Divine Coincidence Holds. The divine coincidence is a canonical result in the textbook NK model, absent the effects of an ELB. Targeting inflation simultaneously stabilizes the output gap with the policy rate equal to the natural rate. In our model, this result does not hold with interest rate policy alone.

The liquidity premium acts as an endogenous cost channel in the Phillips curve due to the financial frictions in the economy. This means that targeting inflation leads to the output gap varying proportionally to the liquidity premium:

$$x_t = \frac{\Omega - \overline{C}_b}{1+\eta}\zeta_t$$

Proposition 1. *Absent endogenous balance sheet policy, the divine coincidence fails due to liquidity premium variability.*

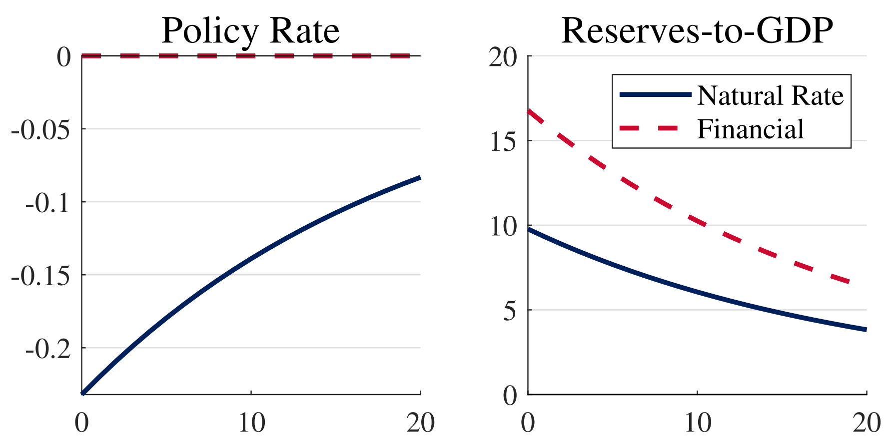
Inflation-targeting interest rate policy *does not* simultaneously stabilize the liquidity premium. This also implies that the output gap is time-varying and that the policy rate does not equal the natural rate. However, handing the central bank an additional policy tool — the central bank balance sheet — restores a version of the divine coincidence.

Proposition 2. *There exists endogenous balance sheet policy, re_t^*, that stabilizes the output gap, inflation, and the liquidity premium, the equivalent of the "divine coincidence" in this economy:*

$$\text{(DC)} \qquad re_t^* = \frac{1}{\beta\zeta}re_{t-1}^* + \frac{1-\overline{RE}}{\overline{\overline{RE}}}\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\left(\theta_t - \frac{1}{\beta\zeta}\theta_{t-1}\right) - \frac{1}{\overline{\overline{RE}}}\left(\frac{\kappa(1-\rho_n)}{1-\kappa\beta\zeta\rho_n} - \frac{Y}{QB}\left(\Omega\frac{\varepsilon-1}{\varepsilon} - \overline{C}_b\right)\right)\frac{r_t^n}{1-\rho_n} + \frac{1}{\overline{\overline{RE}}}\frac{1}{\beta\zeta}\frac{r_{t-1}^n}{1-\kappa\beta\zeta\rho_n}$$

The endogenous balance sheet policy that instills the divine coincidence responds to financial and natural rate shocks. Under this policy, the credit spread is fixed. A fixed credit spread implies that modified leverage, ϕ_t, only varies with the financial

Figure 4: Impulse Responses to a Financial and Natural Rate Shock



Notes: Central bank balance sheet policy, re_t, follows (DC). The policy rate equals the natural rate and is reported in devations from steady state in annualized percentage units. The reserves-to-GDP ratio is in level percentages. The financial and natural rate shocks are scaled to generate a 0.1% output gap response in our model as shown in Figures 2 and 3.

shock. Furthermore, balance sheet policy alone is sufficient to stabilize the economy in response to financial shocks.

Corollary 2.1. *The policy rate, r_t, equals the natural rate when balance sheet policy supports liquidity premium stabilization.*

That is, the policy rate *only* responds to the natural rate as shown in Figure 4. In contrast, the central bank balance sheet expands in response to *both* a negative natural rate shock and tightening financial conditions, $\theta \uparrow$.

Our balance sheet policy prescription differs from previous studies of central bank balance sheet policy using small-scale NK models. For example, Sims et al. (2021) find that the balance sheet policy only responds to financial shocks, whereas we show that this is insufficient in an environment with a robust banking sector and endogenously time-varying modified leverage ratio. Our result provides a clear policy prescription in a tractable model. This policy prescription is consistent with optimal balance sheet policy in a medium-scale model with a similar financial sector setup such as Mau (2022). An important message for policymakers is to employ balance sheet policy much more often, and even away from the ELB. Section 4 discusses how to conduct balance sheet policy in practice. We show that liquidity premium-targeting balance sheet policy, which is an implication of our policy prescription, is achieved through *yield curve control*.

Balance Sheet Policy Supports Model Determinacy with a Policy Rate Peg. A natural question related to central bank balance sheet management is: is balance sheet policy sufficient to conduct monetary policy, independent of interest rate policy? More concretely, do balance sheet policies exist such that the model is determined with a fixed policy rate? The answer is yes! We view this exercise as an extreme proxy for the ELB. At the ELB, the policy rate is fixed for an (ex-ante) indefinite period of time. Here, we push that to the limit by fixing the policy rate forever.

Proposition 3. *Endogenous balance sheet policy can provide a determinate rational expectations equilibrium with a permanent policy rate peg.*

Under a permanent policy rate peg, liquidity premium targeting balance sheet policy leads to a determinate linear rational expectations equilibrium. In addition, inflation-targeting balance sheet policy leads to a determinate solution.

Corollary 3.1. *Inflation-targeting balance sheet policy, re_t^{π}, leads to a determinate linear rational expectations equilibrium, even with a permanent policy rate peg.*

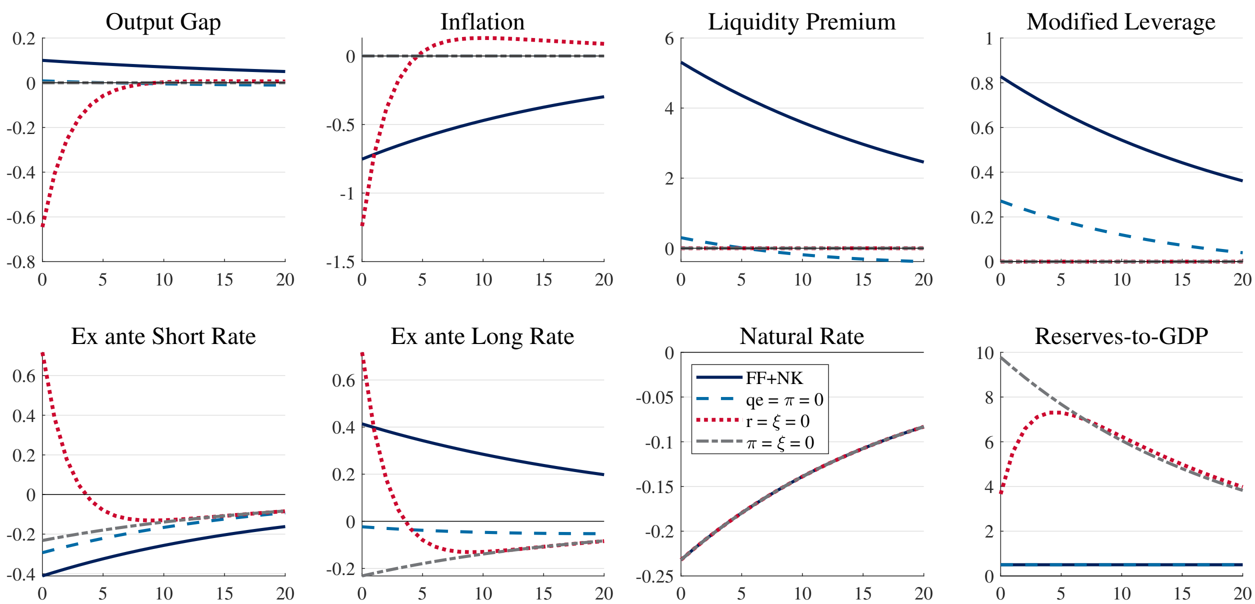
However, inflation-targeting policy is non-stationary, converging to a permanently higher balance sheet size in response to shocks, whereas the liquidity premium-targeting policy is stationary. Conversely, output gap-targeting balance sheet policy under an interest rate peg results in model indeterminacy.

Corollary 3.2. *Balance sheet policy that targets the output gap, re_t^{gap}, with a permanent policy rate peg results in model indeterminacy.*

With output gap-targeting balance sheet policy, the financial block simplifies to one equation with two unknowns - the liquidity premium and endogenous balance sheet policy - and thus results in model indeterminacy.

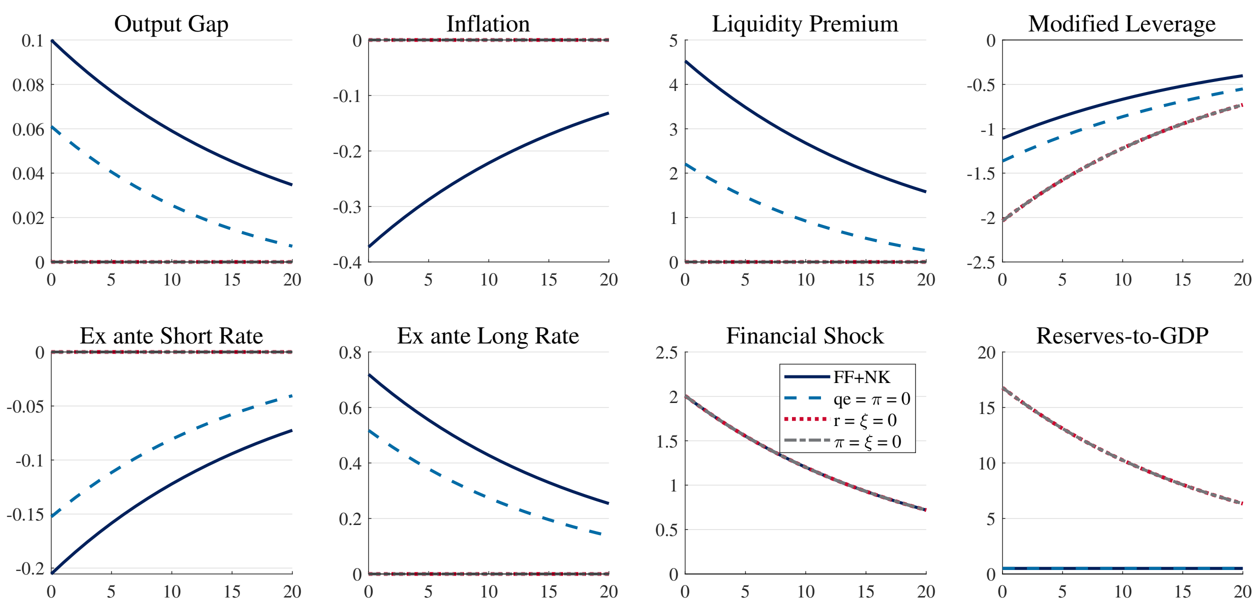
The result that active balance sheet policy suffices for supporting model determinacy is novel to the literature. So far, the literature has argued that only interest rate policy satisfying a generalized Taylor principle renders model determinacy, meaning that permanently fixing the policy rate is not possible. Here, we have shown that this is not the case with the balance sheet available as a policy tool. Figures 5 and 6 compare the model responses to natural rate and financial shocks, respectively, across monetary policy prescriptions. These figures plot the "FF+NK" responses from Figures 2 and 3, along with: (i) inflation-targeting interest rate policy with no balance sheet policy, $\pi_t = qe_t = 0$; (ii) liquidity premium-targeting balance sheet policy with an interest rate peg, $r_t = \xi_t = 0$; and (iii) the optimal dual-instrument policy, $\pi_t = \xi_t = 0$. Inflation-targeting interest rate policy results in inefficient output gap and liquidity premium variability in response to financial shocks. Liquidity premium-targeting balance sheet policy results in inefficient output gap and inflation variability in response to natural rate shocks.

Figure 5: Impulse Responses to a Natural Rate Shock: Varying Policy Instruments/Prescriptions



Notes: Solid lines: the complete model described by equations (2.1) – (2.8) with nominal price rigidity (FF+NK), $\gamma = 0.204$, flexible inflation-targeting interest rate policy $r_t = 1.5\pi_t$, and no balance sheet policy, $qe_t = 0$; dashed lines: strict inflation-targeting interest rate policy, $\pi_t = 0$, with no balance sheet policy, $qe_t = 0$; dotted lines: liquidity premium-targeting balance sheet policy, $\xi_t = 0$, with a fixed policy rate, $r_t = 0$; dashed-dotted lines: optimal dual-instrument policy with strict inflation and liquidity premium targeting, $\pi_t = \xi_t = 0$. The natural rate shock is scaled such that the output gap response in the FF+NK model is 0.1%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

Figure 6: Impulse Responses to a Financial Shock: Varying Policy Instruments/Prescriptions



Notes: Solid lines: the complete model described by equations (2.1) – (2.8) with nominal price rigidity (FF+NK), $\gamma = 0.204$, flexible inflation-targeting interest rate policy $r_t = 1.5\pi_t$, and no balance sheet policy, $qe_t = 0$; dashed lines: strict inflation-targeting interest rate policy, $\pi_t = 0$, with no balance sheet policy, $qe_t = 0$; dotted lines: liquidity premium-targeting balance sheet policy, $\xi_t = 0$, with a fixed policy rate, $r_t = 0$; dashed-dotted lines: optimal dual-instrument policy with strict inflation and liquidity premium targeting, $\pi_t = \xi_t = 0$. The financial shock is scaled such that the output gap response in the FF+NK model is 0.1%. All variables are in terms of percentage deviations from steady state outside of the inflation and interest rates which are deviations from steady state in annualized percentage units.

4 Policy Evaluation

In this section, we quantify the welfare losses to various monetary policy specifications related to our theoretical results. We report the relative losses under these policies to dual-instrument policy which targets inflation and the liquidity premium via the combination of interest rate and balance sheet policy — the divine coincidence policy discussed above.

To compute the welfare losses, we consider a second-order approximation of the underlying non-linear model, defined by equations (NL.1) – (NL.22) in Appendix A, under varying monetary policies. A second-order approximation of the model allows us to compute the stochastic steady-state welfare level in the economy, W_m^s, or the steady-state welfare level accounting for the risk of future shocks, for a given monetary policy specification, m. The deterministic steady-state welfare level, W, is constant across monetary policy specifications. The deterministic steady-state welfare level is a function of the steady-state levels of consumption, C_p, debt-financed expenditure, C_b, aggregate labor, N, and the labor income allocation to the financial account, Ω:[10]

$$W = W^d\left(C_p, C_b, N, \Omega\right)$$

We define the expenditure- and labor-equivalent welfare losses for a given monetary policy specification, λ_m^e and λ_m^n, respectively, similar to the consumption-equivalent welfare measures in Mau (2022):

$$W_m^s = W^d\left(\left(1-\lambda_m^e\right)C_p, \left(1-\lambda_m^e\right)C_b, N, \Omega\right) = W^d\left(C_p, C_b, \left(1+\lambda_m^n\right)N, \Omega\right) \tag{4.1}$$

These welfare loss measures quantify either how much each deterministic steady-state expenditure level must fall or aggregate labor must rise such that the deterministic steady-state welfare level equals the stochastic steady-state level. Comparing equivalent welfare loss measures across various monetary policy specifications, for example,

10. Appendix A clearly defines the household welfare function which decomposes into:

$$W_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{p,t+j} - \psi \frac{\left((1-\Omega_{t+j})N_{t+j}\right)^{1+\eta}}{1+\eta} \right\} + \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left\{ \Gamma \ln C_{b,t+j} - \psi \frac{\left(\Omega_{t+j}N_{t+j}\right)^{1+\eta}}{1+\eta} \right\}$$

allowing us to define the deterministic steady-state welfare level:

$$W = \frac{1}{1-\beta}\left[\ln C_p - \psi \frac{\left((1-\Omega)N\right)^{1+\eta}}{1+\eta}\right] + \frac{1}{1-\beta\zeta}\left[\Gamma \ln C_b - \psi \frac{\left(\Omega N\right)^{1+\eta}}{1+\eta}\right] = W^d\left(C_p, C_b, N, \Omega\right)$$

Table 2: Welfare Results

Policy, m	Equivalent Loss λ_m^e	λ_m^n
$\pi_t = \xi_t = 0$	0%	0%
$\pi_t = re_t = 0$	0.086%	0.114%
$\pi_t = r_t = 0$	0.469%	0.615%
$\xi_t = r_t = 0$	0.129%	0.170%

Notes: Expenditure- and labor-equivalent welfare costs as defined by equation (4.1) across varying monetary policy specifications under the model calibration from Table 1. $\pi_t = \xi_t = 0$ corresponds to inflation- ($\pi_t = 0$) and liquidity premium-targeting ($\xi_t = 0$) dual-instrument monetary policy (r_t and re_t time-varying). $\pi_t = re_t = 0$ corresponds to single-instrument ($re_t = 0$) inflation-targeting ($\pi_t = 0$) interest rate policy (r_t time-varying). $\pi_t = r_t = 0$ corresponds to single-instrument ($r_t = 0$) inflation-targeting ($\pi_t = 0$) balance sheet policy (re_t time-varying). $\xi_t = r_t = 0$ corresponds to single-instrument ($r_t = 0$) liquidity premium-targeting ($\xi_t = 0$) balance sheet policy (re_t time-varying).

$m \in \{1, 2\}$, quantifies the relative performance of each specification:

$$\lambda_1^e - \lambda_2^e \quad \text{or} \quad \lambda_1^n - \lambda_2^n$$

Given these definitions, $\lambda_1^e - \lambda_2^e > 0 \iff \lambda_1^n - \lambda_2^n > 0$, providing a consistent welfare performance ordering independent of the equivalent welfare loss measure considered.

Table 2 presents the quantitative welfare results under the calibration from Table 1. Under this parameterization, the deterministic steady-state level of output is normalized to one, resulting in a mean level of output of one. The shock sizes imply a standard deviation of output of 4.1% with 99.4% of output variability generated by the natural rate shock in the baseline model, FF+NK.[11] That is, the financial shock has almost no effect on output under the baseline calibration. Even so, welfare costs to inflation-targeting interest rate policy with no balance sheet policy arise. Note, there are no welfare costs to inflation- and liquidity premium-targeting dual-instrument policy as this results in dynamics consistent with the friction-less RBC model — the efficient outcome.

In terms of a 50-week work-year across a 40 year career, the welfare cost to inflation-targeting interest rate policy with a fixed balance sheet is equivalent to *every worker* working two extra weeks over their career ($0.00114 \times 50 \times 40 = 2.28$). Another way

11. Mau et al. (2023) show that financial shocks in this economy lead to substantial substitution between consumption and debt-financed expenditure. For this reason, the aggregate output response to financial shocks is small, explaining why output variability is dominantly driven by natural rate shocks under the baseline calibration.

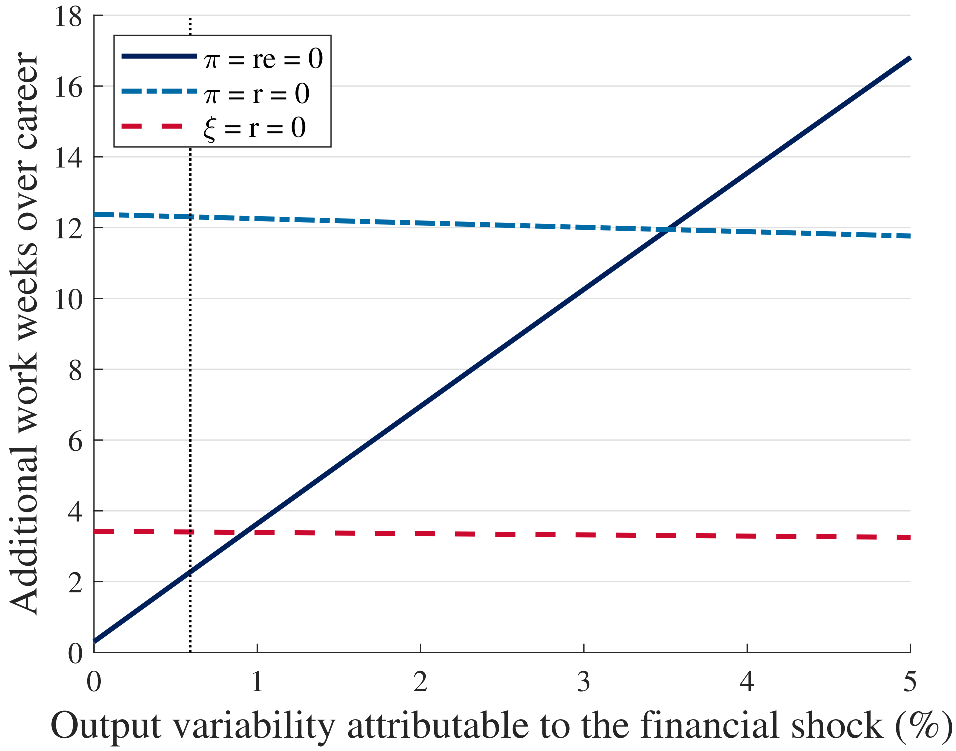
of conceptualizing this is to consider an economy with 200 million workers. This welfare cost is equivalent to needing to *permanently* add 228,000 workers to the workforce ($0.00114 \times 200,000,000 = 228,000$), abstracting from population growth. The welfare cost to liquidity premium-targeting balance sheet policy with no interest rate policy is equivalent to every individual working three extra weeks over their career (permanently adding 340,000 workers). Inflation-targeting balance sheet policy with no interest rate policy generates a welfare cost equivalent to every individual working approximately 12 extra weeks over their career (permanently adding 1,230,000 workers).

Nearly all of the welfare losses associated with inflation-targeting interest rate policy with a fixed balance sheet are due to the presence of the financial shock in the economy, even if it is small. If the importance of the financial shock in the economy rises, these costs rise, whereas the performance of liquidity premium-targeting balance sheet policy with a fixed policy rate is unaffected. With active balance sheet policy, the financial shock requires no interest rate response. Thus, the welfare cost to liquidity premium-targeting balance sheet policy with a fixed interest rate does not vary with the relative importance of the financial shock in the economy, conditional on the size of the natural rate shock.

Figure 7 plots the welfare costs to these two policies (along with inflation-targeting balance sheet policy with a fixed policy rate), varying the relative importance of the financial shock in the economy while holding the standard deviation of output fixed. That is, as the financial shock volatility varies, the natural rate shock volatility adjusts to hold the standard deviation of output fixed. This results in a degree of variability in the welfare costs to liquidity premium-targeting balance sheet with a fixed interest rate, albeit modest, due to the changing natural rate shock variability. We plot the labor-equivalent welfare cost in terms of additional weeks worked across every workers career in the economy under the same career definition as above.

Even if 1% of output variability is attributable to the financial shock, the welfare cost to inflation-targeting interest rate policy with a fixed balance sheet exceeds that from liquidity premium-targeting balance sheet policy with a fixed policy rate. As the output variability attributable to the financial shock rises to 5%, inflation-targeting interest rate policy with no balance sheet policy performs worst among the policy specifications considered. Again, the notion of balance sheet policy with no interest rate policy is purely theoretical. We are not advocating for this policy prescription. Instead, our results provide clear evidence that simply prioritizing inflation variability via interest rate policy in a world with incomplete financial markets and exogenous variability stemming from the financial sector, such as the model studied here, leads to significant welfare costs. Significant enough, that extreme theoretical policy prescriptions start to look reasonable.

Figure 7: Welfare Costs and Output Variance Decomposition



Notes: Solid lines: inflation-targeting interest rate policy with no balance sheet policy; dashed-dotted lines: inflation-targeting balance sheet policy with a fixed interest rate; dashed lines: liquidity premium-targeting balance sheet policy with a fixed interest rate; dotted vertical line: baseline results presented in Table 2. Computed labor-equivalent welfare costs, λ_m^n, follow equation (4.1) and are presented in terms of additional work weeks required over every workers career ($\lambda_m^n \times 50 \times 40$).

5 Conclusion

This paper studies interest rate and balance sheet policy in a tractable NK model with incomplete financial markets. A leverage constraint in the financial sector constrains private lending capacity in the economy. A household that simultaneously saves and borrows attempts to equate its marginal utility from consumption and debt-financed expenditure. Due to the incomplete financial markets, a time-varying wedge referred to as the liquidity premium arises between these two marginal utilities. The liquidity premium acts as an endogenous cost channel in the Phillips curve and an endogenous credit wedge in the IS curve. The endogenous nature of the financial sector leverage constraint causes the liquidity premium to vary with non-financial variables, acting as a propagation mechanism for non-financial shocks in the economy. We show that the model setup considered alters the labor supply response to monetary policy.

Inflation-targeting interest rate policy fails to generate the efficient labor supply response counter to the textbook NK model. The divine coincidence fails due to a financial accelerator mechanism present in the model. Inflation-targeting interest rate policy does not simultaneously stabilize the financial accelerator. Introducing balance sheet policy that stabilizes the credit spread path neutralizes the financial accelerator effect. That is, dual-instrument policy restores the divine coincidence. Balance sheet policy can provide a determinate equilibrium with a fixed policy rate. Welfare calculations show that simply prioritizing inflation variability via interest rate policy absent balance sheet policy in the model considered leads to welfare costs.

Our results hinge on the binding leverage constraint in the financial sector, a point addressed by Karadi and Nakov (2021). If this constraint is occasionally binding, then economic states arise for which balance sheet policy is useless (both at and away from the ELB). This criticism applies to any paper that relies on financial sector constraints to generate incomplete financial markets and a role for central bank balance sheet policy such as Gertler and Kiyotaki (2010), Gertler and Karadi (2011, 2013), Carlstrom et al. (2017), Sims and Wu (2021), Boehl et al. (2022), or Mau (2022). However, alternative financial sector setups generate incomplete financial markets and a role for balance sheet policy absent any type of financial sector constraint. For instance, Cúrdia and Woodford (2011, 2016) do so by introducing resource costs to lending and incomplete intermediary information on loan types. This financial sector setup would not affect the structural changes to the IS and Phillips curves due to incomplete financial markets as presented in the current paper. That is, our main result — balance sheet policy should respond to all types of shocks in the economy — is robust to the financial sector model considered.

References

Bernanke, B. (2022). *21st Century Monetary Policy: The Federal Reserve from the Great Inflation to COVID-19*. W. W. Norton.

Bernanke, B., Gertler, M., and Gilchrist, S. (1996). The Financial Accelerator and the Flight to Quality. *The Review of Economics and Statistics*, 78(1):1–15.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.

Blanchard, O. and Galí, J. (2007). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, 39(s1):35–65.

Boehl, G., Goy, G., and Strobel, F. (2022). A Structural Investigation of Quantitative Easing. *The Review of Economics and Statistics*, pages 1–45.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Carlstrom, C. T., Fuerst, T. S., and Paustian, M. (2017). Targeting Long Rates in a Model with Segmented Markets. *American Economic Journal: Macroeconomics*, 9(1):205–242.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45.

Clarida, R., Galí, J., and Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Cúrdia, V. and Woodford, M. (2011). The central-bank balance sheet as an instrument of monetarypolicy. *Journal of Monetary Economics*, 58(1):54–79.

Cúrdia, V. and Woodford, M. (2016). Credit Frictions and Optimal Monetary Policy. *Journal of Monetary Economics*, 84(C):30–65.

Fisher, J. D. (2015). On the Structural Interpretation of the Smets–Wouters "Risk Premium" Shock. *Journal of Money, Credit and Banking*, 47(2-3):511–516.

Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34.

Gertler, M. and Karadi, P. (2013). QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, 9(1):5–53.

Gertler, M. and Kiyotaki, N. (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. In *Handbook of Monetary Economics*, volume 3, chapter 11, pages 547–599. Elsevier.

Guerrieri, L. and Iacoviello, M. (2017). Collateral constraints and macroeconomic asymmetries. *Journal of Monetary Economics*, 90(C):28–49.

Karadi, P. and Nakov, A. (2021). Effectiveness and addictiveness of quantitative easing. *Journal of Monetary Economics*, 117(C):1096–1117.

Mau, R. (2022). What is in a name? Purchases and Sales of Financial Assets as a Monetary Policy Instrument. *Journal of Money, Credit and Banking*, Forthcoming.

Mau, R., Richter, A. W., and Throckmorton, N. A. (2023). QE Transmission: Spend More or Work Less. *Working Paper*.

Ravenna, F. and Walsh, C. E. (2006). Optimal monetary policy with the cost channel. *Journal of Monetary Economics*, 53(2):199–216.

Sargent, T. J. and Wallace, N. (1975). "Rational" Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule. *Journal of Political Economy*, 83(2):241–254.

Sims, E. and Wu, J. C. (2021). Evaluating Central Banks' tool kit: Past, present, and future. *Journal of Monetary Economics*, 118(C):135–160.

Sims, E., Wu, J. C., and Zhang, J. (2021). The Four Equation New Keynesian Model. *The Review of Economics and Statistics*, pages 1–45.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

ONLINE APPENDIX

Monetary Policy Interactions: The Policy Rate, Asset Purchases, and Optimal Policy with an Interest Rate Peg by Isabel Gödl-Hanisch, Ronald Mau, and Jonathan Rawls

Contents

Appendix References

Gertler, M. and Karadi, P. (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics*, 58(1):17–34.

Gertler, M. and Karadi, P. (2013). QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, 9(1):5–53.

Mau, R., Richter, A. W., and Throckmorton, N. A. (2022). QE Transmission: Spend More or Work Less. *Working Paper*.

Sims, E. and Wu, J. C. (2021). Evaluating Central Banks' tool kit: Past, present, and future. *Journal of Monetary Economics*, 118(C):135–160.

Sims, E., Wu, J. C., and Zhang, J. (2021). The Four Equation New Keynesian Model. *The Review of Economics and Statistics*, pages 1–45.

A Model and Equilibrium Definitions

The household setup in our model nests a problem with patient and impatient workers. The patient worker maximizes lifetime utility over consumption, $C_{p,t}$, and labor, $N_{p,t}$, choosing consumption, hours worked, and deposit savings with the financial sector, S_t, and is the residual claimant to all profits within the economy, D_t:

$$\max_{C_{p,t},N_{p,t},S_t} \quad \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{p,t+j} - \frac{N_{p,t+j}^{1+\eta}}{1+\eta} \right\}$$
$$\text{s.t.} \quad C_{p,t+j} = W_{t+j}N_{p,t+j} + R_{t+j-1}\Pi_{t+j}^{-1}S_{t+j-1} + D_{t+j} - S_{t+j}$$

where β is the patient worker's discount factor and $R_t\Pi_t^{-1}$ is the real interest rate on savings. The impatient worker maximizes lifetime utility over consumption, $C_{b,t}$, and labor, $N_{b,t}$, choosing consumption, hours worked, and the outstanding debt stock, B_t:

$$\max_{C_{b,t},N_{b,t},B_t} \quad \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\zeta)^j \left\{ \ln C_{b,t+j} - \frac{N_{b,t+j}^{1+\eta}}{1+\eta} \right\}$$
$$\text{s.t.} \quad C_{b,t+j} = W_{t+j}N_{b,t+j} + Q_{t+j}B_{t+j} - \left(1+\kappa Q_{t+j}\right)\Pi_{t+j}^{-1}B_{t+j-1}$$

where ζ is the degree of relative impatience of the impatient worker and Q_t is the current price of debt. $\kappa \in [0,1)$ defines the rate of decay of a geometric coupon payment paid each period for each debt obligation. $1+\kappa Q_t$ is the cum coupon price of past debt issuance.

The individual worker lifetime utility objectives add together to define economy-wide lifetime utility:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{p,t+j} - \frac{N_{p,t+j}^{1+\eta}}{1+\eta} - \zeta^j \frac{N_{b,t+j}^{1+\eta}}{1+\eta} + \zeta^j \ln C_{b,t+j} \right\}$$

Defining the impatient worker labor share, $\Omega_t = N_{b,t}/\left(N_{b,t}+N_{p,t}\right)$, allows this to be written as:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{p,t+j} - \psi \left[(1-\Omega_{t+j})^{1+\eta} + \zeta^j \Omega_{t+j}^{1+\eta} \right] \frac{N_{t+j}^{1+\eta}}{1+\eta} + \zeta^j \ln C_{b,t+j} \right\}$$

and the budget constraints to be written as:

$$C_{p,t+j} = (1-\Omega_{t+j})W_{t+j}N_{t+j} + R_{t+j-1}\Pi_{t+j}^{-1}S_{t+j-1} + D_{t+j} - S_{t+j}$$
$$C_{b,t+j} = \Omega_{t+j}W_{t+j}N_{t+j} + Q_{t+j}\left(B_{t+j} - \kappa\Pi_{t+j}^{-1}B_{t+j-1}\right) - \Pi_{t+j}^{-1}B_{t+j-1}$$

Mau et al. (2022) show that maximizing economy-wide lifetime utility subject to the two budget constraints choosing Ω_t and N_t rather than $N_{p,t}$ and $N_{b,t}$ is equivalent to the patient/impatient worker setup. In our paper, we introduce a preference shifter Γ that is constant across time on the utility flow from debt-financed expenditure. This allows for separate calibration of the steady-state impatient household labor and consumption shares, or labor income allocation to the financial account and debt-financed expenditure share in the language of the main text.

A.1 Household Problem

Preferences:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln C_{p,t+j} - \Psi_j\left(\Omega_{t+j}\right) \frac{N_{t+j}^{1+\eta}}{1+\eta} + \zeta^j \Gamma \ln C_{b,t+j} \right\}$$

where:

$$\Psi_j\left(\Omega_{t+j}\right) = \psi\left[(1-\Omega_{t+j})^{1+\eta} + \zeta^j \Omega_{t+j}^{1+\eta}\right]$$

Constraints:

$$\begin{aligned} C_{p,t+j} &= (1-\Omega_{t+j})W_{t+j}N_{t+j} + R_{t+j-1}\Pi_{t+j}^{-1}S_{t+j-1} + D_{t+j} - S_{t+j} \\ C_{b,t+j} &= \Omega_{t+j}W_{t+j}N_{t+j} + Q_{t+j}\left(B_{t+j} - \kappa\Pi_{t+j}^{-1}B_{t+j-1}\right) - \Pi_{t+j}^{-1}B_{t+j-1} \end{aligned}$$

First-order conditions:

$$\begin{aligned} C_{p,t+j}: &\quad & \mu_{p,t+j} &= \beta^j C_{p,t+j}^{-1} \\ C_{b,t+j}: &\quad & \mu_{b,t+j} &= (\zeta\beta)^j \Gamma C_{b,t+j}^{-1} \\ N_{t+j}: &\quad & \beta^j \Psi_j\left(\Omega_{t+j}\right) N_{t+j}^{\eta} &= W_{t+j}\left[(1-\Omega_{t+j})\mu_{p,t+j} + \Omega_{t+j}\mu_{b,t+j}\right] \\ \Omega_{t+j}: &\quad & \beta^j \Psi_j'(\Omega_{t+j}) N_{t+j}^{1+\eta} &= (1+\eta)W_{t+j}N_{t+j}\left(\mu_{b,t+j} - \mu_{p,t+j}\right) \\ S_{t+j}: &\quad & \mu_{p,t+j} &= \mu_{p,t+j+1}R_{t+j}\Pi_{t+j+1}^{-1} \\ B_{t+j}: &\quad & \mu_{b,t+j}Q_{t+j} &= \mu_{b,t+j+1}\left(1+\kappa Q_{t+j+1}\right)\Pi_{t+j+1}^{-1} \end{aligned}$$

Rewritten at $j = 0$, taking expectations, and imposing a change of variables:

$$\begin{aligned} \Xi_t &= \Gamma \frac{C_{p,t}}{C_{b,t}} \\ \Psi_0\left(\Omega_t\right) N_t^{\eta} &= W_t C_{p,t}^{-1}\left[1 - \Omega_t + \Omega_t \Xi_t\right] \\ \Psi_0'(\Omega_t) N_t^{1+\eta} &= (1+\eta) W_t N_t C_{p,t}^{-1}\left(\Xi_t - 1\right) \\ C_{p,t}^{-1} &= \beta \mathbb{E}_t C_{p,t+1}^{-1} R_t \Pi_{t+1}^{-1} \\ C_{p,t}^{-1}\Xi_t Q_t &= \zeta\beta \mathbb{E}_t C_{p,t+1}^{-1}\Xi_{t+1}\left(1+\kappa Q_{t+1}\right)\Pi_{t+1}^{-1} \end{aligned}$$

Dividing the optimal labor income allocation condition by the labor supply condition implies:

$$\Xi_t = \left(\frac{\Omega_t}{1-\Omega_t}\right)^{\eta}$$

Substituting this from the labor supply condition implies:

$$W_t = \psi\left((1-\Omega_t)N_t\right)^{\eta} C_{p,t}$$

Household equilibrium conditions:

$$\begin{aligned}
\Xi_t &= \Gamma \frac{C_{p,t}}{C_{b,t}} \\
\Lambda^N_{t-1,t} &= \beta \frac{C_{p,t-1}}{C_{p,t}} \Pi_t^{-1} \\
W_t &= \psi \left((1-\Omega_t) N_t \right)^\eta C_{p,t} \\
\Xi_t &= \left(\frac{\Omega_t}{1-\Omega_t} \right)^\eta \\
1 &= \mathbb{E}_t \Lambda^N_{t,t+1} R_t \\
1 &= \zeta \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\Xi_{t+1}}{\Xi_t} R^L_{t+1} \\
C_{b,t} &= \Omega_t W_t N_t + Q_t B_t - \frac{R^L_t}{\Pi_t} Q_{t-1} B_{t-1} \\
R^L_t &= \frac{1+\kappa Q_t}{Q_{t-1}}
\end{aligned}$$

A.2 Financial and Production Sectors

The financial sector details are presented in Appendix B. The relevant equilibrium conditions follow:

$$\begin{aligned}
Q_t B^{FI}_t &= \Phi_t X^s \\
1 &= \Phi_t \left[\Theta_t - \left(\mathbb{E}_t \Lambda^N_{t,t+1} R^L_{t+1} - 1 \right) \right]
\end{aligned}$$

Central bank reserve issuance is backed by long-term debt holdings:

$$Q_t B^{cb}_t = RE_t$$

The profit maximization problem of the current intermediary implies that the interest rate on reserves and the deposit rate are equivalent. The central bank controls both the real size of its balance sheet and the interest rate on reserves. Debt market clearing with a binding financial sector risk-weighted leverage constraint implies:

$$Q_t B_t = \Phi_t X^s + RE_t$$

Define balance sheet policy in terms of the relative size of the balance sheet to output:

$$QE_t = \frac{RE_t}{Y_t}$$

The production sector is standard to NK models with Calvo pricing. Labor demand follows:

$$W_t = \mathrm{MC}_t A_t$$

For price-resetting firms, price-setting follows:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta\phi_p\right)^s \frac{C_{p,t}}{C_{p,t+s}} \left[\frac{P_t(j)}{P_{t+s}} - \text{mc}_{t+s}\right] \left(\frac{P_t(j)}{P_{t+s}}\right)^{-\varepsilon_{t+s}} Y_{t+s}$$

With the first-order condition:

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta\phi_p\right)^s C_{p,t+s}^{-1} \left[\left(1-\varepsilon_{t+s}\right) P_{t+s}^{-1} + \varepsilon_{t+s}\text{mc}_{t+s} P_t(j)^{-1}\right] \left(\frac{P_t(j)}{P_{t+s}}\right)^{-\varepsilon_{t+s}} Y_{t+s}$$

With a fixed elasticity of substitution, $\varepsilon_t = \varepsilon$ for all t, the optimal reset price, $P_t(j) = P_{\#,t}$, follows:

$$P_{\#,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta\phi_p\right)^s C_{p,t+s}^{-1} \text{mc}_{t+s} P_{t+s}^{\varepsilon} Y_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta\phi_p\right)^s C_{p,t+s}^{-1} P_{t+s}^{\varepsilon-1} Y_{t+s}}$$

Written in terms of inflation, this simplifies to:

$$\begin{aligned}
\Pi_{\#,t} &= \frac{\varepsilon}{\varepsilon-1} \Pi_t \frac{G_t}{H_t} \\
G_t &= C_{p,t}^{-1} \text{mc}_t Y_t + \beta\phi_p \mathbb{E}_t \Pi_{t+1}^{\varepsilon} G_{t+1} \\
H_t &= C_{p,t}^{-1} Y_t + \beta\phi_p \mathbb{E}_t \Pi_{t+1}^{\varepsilon-1} H_{t+1}
\end{aligned}$$

with aggregate output, price dispersion, and the aggregate inflation rate given by:

$$\begin{aligned}
Y_t &= \frac{A_t N_t}{\Delta_t} \\
\Delta_t &= \left(1-\phi_p\right) \left(\frac{\Pi_t}{\Pi_{\#,t}}\right)^{\varepsilon} + \phi_p \Pi_t^{\varepsilon} \Delta_{t-1} \\
\Pi_t^{1-\varepsilon} &= (1-\phi_p) \Pi_{\#,t}^{1-\varepsilon} + \phi_p
\end{aligned}$$

For policy and analytical purposes, define a benchmark level of output, $Y_{*,t}$, by considering the labor market equilibrium in the flexible price, $\phi_p = 0$, and complete markets, $\Xi_t \equiv 1$, economy assuming that a constant fraction of labor income equal to the steady state value in the incomplete markets model, Ω, is paid to the financial account:

$$Y_t^* = \left[\frac{1}{\psi} \frac{1+\Gamma}{(1-\Omega)^{\eta}} \frac{\varepsilon - 1}{\varepsilon}\right]^{\frac{1}{1+\eta}} A_t$$

We define the output gap as the relative output percentage deviations from steady state in the baseline and benchmark economies:

$$Gap_t = \frac{Y_t}{Y_t^*} \frac{Y^*}{Y}$$

A.3 Nonlinear Equilibrium System

$$\Xi_t = \Gamma \frac{C_{p,t}}{C_{b,t}} \tag{NL.1}$$

$$\mathrm{MC}_t A_t = \psi\left((1-\Omega_t)N_t\right)^{\eta} C_{p,t} \tag{NL.2}$$

$$\Xi_t = \left(\frac{\Omega_t}{1-\Omega_t}\right)^{\eta} \tag{NL.3}$$

$$\Lambda^N_{t-1,t} = \beta \frac{C_{p,t-1}}{C_{p,t}} \Pi_t^{-1} \tag{NL.4}$$

$$1 = \mathbb{E}_t \Lambda^N_{t,t+1} R_t \tag{NL.5}$$

$$1 = \zeta \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\Xi_{t+1}}{\Xi_t} R^L_{t+1} \tag{NL.6}$$

$$C_{b,t} = \Omega_t \mathrm{MC}_t A_t N_t + Q_t B_t - \frac{R^L_t}{\Pi_t} Q_{t-1} B_{t-1} \tag{NL.7}$$

$$R^L_t = \frac{1+\kappa Q_t}{Q_{t-1}} \tag{NL.8}$$

$$1 = \Phi_t \left[\Theta_t - \left(\mathbb{E}_t \Lambda^N_{t,t+1} R^L_{t+1} - 1\right)\right] \tag{NL.9}$$

$$Q_t B_t = \Phi_t X^s + RE_t \tag{NL.10}$$

$$\Pi_{\#,t} = \frac{\varepsilon}{\varepsilon-1} \Pi_t \frac{G_t}{H_t} \tag{NL.11}$$

$$G_t = C_{p,t}^{-1} \mathrm{MC}_t Y_t + \beta \phi_p \mathbb{E}_t \Pi_{t+1}^{\varepsilon} G_{t+1} \tag{NL.12}$$

$$H_t = C_{p,t}^{-1} Y_t + \beta \phi_p \mathbb{E}_t \Pi_{t+1}^{\varepsilon-1} H_{t+1} \tag{NL.13}$$

$$\Pi_t^{1-\varepsilon} = (1-\phi_p)\Pi_{\#,t}^{1-\varepsilon} + \phi_p \tag{NL.14}$$

$$\Delta_t = (1-\phi_p)\left(\frac{\Pi_t}{\Pi_{\#,t}}\right)^{\varepsilon} + \phi_p \Pi_t^{\varepsilon} \Delta_{t-1} \tag{NL.15}$$

$$Y_t \Delta_t = A_t N_t \tag{NL.16}$$

$$Y_t = C_{p,t} + C_{b,t} \tag{NL.17}$$

$$QE_t = \frac{RE_t}{Y_t} \tag{NL.18}$$

$$Y_t^* = \left[\frac{1}{\psi} \frac{1+\Gamma}{(1-\Omega)^{\eta}} \frac{\varepsilon-1}{\varepsilon}\right]^{\frac{1}{1+\eta}} A_t \tag{NL.19}$$

$$GAP_t = \frac{Y_t}{Y_t^*} \frac{Y^*}{Y} \tag{NL.20}$$

$$\ln A_t = (1-\rho_a)\ln A + \rho_a \ln A_{t-1} + \sigma_a \epsilon_t^a \tag{NL.21}$$

$$\ln \Theta_t = (1-\rho_\theta)\ln \Theta + \rho_\theta \ln \Theta_{t-1} + \sigma_\theta \epsilon_t^\theta \tag{NL.22}$$

Endogenous Variables (20): C_p, C_b, N, Ω, Λ^N, Y, RE, Ξ, MC, Π, $\Pi_\#$, G, H, Δ, Q, R^L, B, Φ, Y_*, Gap; Exogenous variables (2): A, Θ; Policy Variables (2): R, QE

Steady state calculations: Given values for $\{\beta, \eta, \phi_p, \varepsilon\}$, and the calibration targets: ${}^{R^L}\!/_{R} = 1.015^{0.25}$; $\Pi = 1$; $QE = 0.05$; $Y = 1$; $N = 1$; $(1-\kappa)^{-1} = 40$; ${}^{C_b}\!/_{Y} = 0.2$; $\Phi = 4$, and $C_b = QB\left(1 - {}^{\kappa}\!/_{\Pi}\right)$ (debt-financed expenditure equals new issuance); solve the following:

$$R = \frac{\Pi}{\beta} \leftarrow \text{implies } R^L$$

$$Q = \frac{1}{R^L - \kappa}$$

$$\zeta = \frac{R}{R^L}$$

$$B = \frac{C_b}{Q(1 - {}^{\kappa}\!/_{\Pi})}$$

$$\Pi_{\#} = \left(\frac{\Pi^{1-\varepsilon} - \phi_p}{1 - \phi_p}\right)^{\frac{1}{1-\varepsilon}}$$

$$\Delta = \frac{\Pi^{\varepsilon}}{\Pi_{\#}^{\varepsilon}} \frac{1 - \phi_p}{1 - \phi_p \overline{\Pi}^{\varepsilon}}$$

$$A = \frac{Y\Delta}{N}$$

$$\mathrm{MC} = \frac{\varepsilon - 1}{\varepsilon} \frac{\Pi_{\#}}{\Pi} \frac{\left(1 - \beta\phi_p \Pi^{\varepsilon}\right)}{\left(1 - \beta\phi_p \Pi^{\varepsilon-1}\right)}$$

$$G = \frac{\mathrm{MC}Y}{C_p\left(1 - \beta\phi_p \Pi^{\varepsilon}\right)}$$

$$H = \frac{Y}{C_p\left(1 - \beta\phi\overline{\Pi}^{\varepsilon-1}\right)}$$

$$\Omega = \frac{C_b + \left(\frac{R^L}{\overline{\Pi}} - 1\right) QB}{\mathrm{MC}N}$$

$$\Xi = \left(\frac{\Omega}{1 - \Omega}\right)^{\eta}$$

$$B^{FI} = B - \frac{QE}{Q} Y$$

$$\psi = \frac{\mathrm{MC}}{\left((1 - \Omega) N\right)^{\eta} C_p}$$

$$\Gamma = \Xi \frac{C_b}{C_p}$$

$$Y^* = \left[\frac{1}{\psi} \frac{1 + \Gamma}{(1 - \Omega)^{\eta}} \frac{\varepsilon - 1}{\varepsilon}\right]^{\frac{1}{1+\eta}} A$$

$$\Theta = \Phi^{-1} + \frac{1 - \zeta}{\zeta}$$

$$X^s = \frac{QB - RE}{\Phi}$$

A.4 Log-linear Equilibrium System

Let $c_t = \ln C_t - \ln C$. Note, for the interest and inflation rates, we make use of the approximation $\ln R_t - \ln R \approx R_t - 1 - (R-1) = R_t - R$ allowing us to interpret these variables as net rate deviations from steady state.

$$\xi_t = c_{p,t} - c_{b,t} \tag{L.1}$$

$$\mathrm{mc}_t + a_t = \eta n_t + c_{p,t} - \eta \frac{\Omega}{1-\Omega}\omega_t \tag{L.2}$$

$$\xi_t = \frac{\eta}{1-\Omega}\omega_t \tag{L.3}$$

$$\lambda^N_{t-1,t} = c_{p,t-1} - c_{p,t} - \pi_t \tag{L.4}$$

$$0 = \mathbb{E}_t \lambda^N_{t,t+1} + r_t \tag{L.5}$$

$$0 = \mathbb{E}_t \xi_{t+1} - \xi_t + \mathbb{E}_t r^L_{t+1} - r_t \tag{L.6}$$

$$\overline{C}_b c_{b,t} = \Omega \mathrm{MC} \Delta \left(\omega_t + \mathrm{mc}_t + a_t + n_t\right) + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r^L_t - \pi_t\right)\right) \tag{L.7}$$

$$r^L_t = \frac{\kappa\beta\zeta}{\Pi} q_t - q_{t-1} \tag{L.8}$$

$$\phi_t = \frac{\Phi}{\zeta}\left(\mathbb{E}_t r^L_{t+1} - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \tag{L.9}$$

$$q_t + b_t = \left(1 - \overline{RE}\right)\phi_t + \overline{RE} re_t \tag{L.10}$$

$$\pi_{\#,t} = \pi_t + g_t - h_t \tag{L.11}$$

$$g_t = \left(1 - \beta\phi_p \Pi^{\varepsilon}\right)\left(\mathrm{mc}_t + y_t - c_{p,t}\right) + \beta\phi_p\Pi^{\varepsilon}\left(\varepsilon \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t g_{t+1}\right) \tag{L.12}$$

$$h_t = \left(1 - \beta\phi_p\Pi^{\varepsilon-1}\right)\left(y_t - c_{p,t}\right) + \beta\phi_p\Pi^{\varepsilon-1}\left((\varepsilon-1)\mathbb{E}_t\pi_{t+1} + \mathbb{E}_t h_{t+1}\right) \tag{L.13}$$

$$\pi_t = \left(1 - \phi_p\right)\left(\frac{\Pi}{\Pi_*}\right)^{\varepsilon-1}\pi_{*,t} \tag{L.14}$$

$$\delta_t = \varepsilon\pi_t - \varepsilon\left(1 - \phi_p\Pi^{\varepsilon}\right)\pi_{*,t} + \phi_p\Pi^{\varepsilon}\delta_{t-1} \tag{L.15}$$

$$y_t + \delta_t = a_t + n_t \tag{L.16}$$

$$y_t = \left(1 - \overline{C}_b\right)c_{p,t} + \overline{C}_b c_{b,t} \tag{L.17}$$

$$qe_t = re_t - y_t \tag{L.18}$$

$$y^*_t = a_t \tag{L.19}$$

$$gap_t = y_t - y^*_t \tag{L.20}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon^a_t \tag{L.21}$$

$$\theta_t = \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon^\theta_t \tag{L.22}$$

where $\overline{C}_b$ is the steady state debt-financed expenditure share of aggregate expenditures and $\overline{RE}$ is the steady state share of long-term debt held by the central bank.

Consider the above log-linear approximation of the model around a zero net inflation steady-state. The structural system consists of IS and Phillips curves, augmented by the interest rate spread and liquidity premium where the liquidity premium is equal to the forward-looking path of the interest rate spread:

$$
\begin{aligned}
\pi_t &= \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
gap_t &= \mathbb{E}_t gap_{t+1} - \left(\left(1-\overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r^L_{t+1} - \mathbb{E}_t \pi_{t+1} - r^n_t\right) \\
r^n_t &= -(1-\rho_a) a_t \\
r^L_t &= \kappa\beta\zeta q_t - q_{t-1} \\
\xi_t &= \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r^L_{t+1} - r_t
\end{aligned}
$$

where $\gamma = {}^{(1-\phi_p)(1-\phi_p\beta)(1+\eta)}/_{\phi_p}$. The model also includes a robust financial block:

$$
\begin{aligned}
c_{b,t} &= gap_t + a_t - \left(1-\overline{C}_b\right)\xi_t \\
\overline{C}_b c_{b,t} &= \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t + (2+\eta) gap_t + a_t\right) \\
&\quad + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r^L_t - \pi_t\right)\right) \\
\phi_t &= \frac{\Phi}{\zeta}\left(\mathbb{E}_t r^L_{t+1} - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1-\overline{RE}\right)\phi_t + \overline{RE} re_t
\end{aligned}
$$

Where exogenous shocks follow:

$$
\begin{aligned}
a_t &= \rho_a a_{t-1} + \sigma_a \epsilon^a_t \\
\theta_t &= \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon^\theta_t
\end{aligned}
$$

Simplify further by writing the productivity shock, a_t, in terms of the natural rate.

$$\pi_t = \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \tag{LZ.1}$$

$$gap_t = \mathbb{E}_t gap_{t+1} - \left(\left(1-\overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r^L_{t+1} - \mathbb{E}_t \pi_{t+1} - r^n_t\right) \tag{LZ.2}$$

$$r^L_t = \kappa\beta\zeta q_t - q_{t-1} \tag{LZ.3}$$

$$\xi_t = \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r^L_{t+1} - r_t \tag{LZ.4}$$

where $\gamma = {}^{(1-\phi_p)(1-\phi_p\beta)(1+\eta)}/_{\phi_p}$. The model also includes a robust financial block:

$$c_{b,t} = gap_t - \left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n} r^n_t \tag{LZ.5}$$

$$
\begin{aligned}
\overline{C}_b c_{b,t} &= \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t + (2+\eta) gap_t - \frac{1}{1-\rho_n} r^n_t\right) \\
&\quad + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r^L_t - \pi_t\right)\right)
\end{aligned} \tag{LZ.6}
$$

$$\phi_t = \frac{\Phi}{\zeta}\left(\mathbb{E}_t r^L_{t+1} - r_t\right) - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \tag{LZ.7}$$

$$q_t + b_t = \left(1 - \overline{RE}\right)\phi_t + \overline{RE}re_t \tag{LZ.8}$$

Where exogenous shocks follow:

$$r^n_t = \rho_n r^n_{t-1} + \sigma_n \epsilon^n_t \tag{LZ.9}$$

$$\theta_t = \rho_\theta \theta_{t-1} + \sigma_\theta \epsilon^\theta_t \tag{LZ.10}$$

Equations (LZ.1) – (LZ.8) correspond to equations (2.1) – (2.8) in the text. To derive equation (2.9), start by writing the output gap in terms of output, y_t:

$$gap_t = y_t - a_t = y_t + \frac{r^n_t}{1-\rho_n}$$

and substitute this from the IS and Phillips curves:

$$\begin{aligned}
\pi_t &= \widetilde{\gamma}(1+\eta)\left(y_t + \frac{r^n_t}{1-\rho_n}\right) + \beta\mathbb{E}_t\pi_{t+1} + \widetilde{\gamma}\left(\overline{C}_b - \Omega\right)\xi_t \\
y_t &= \mathbb{E}_t y_{t+1} - \left(\left(1 - \overline{C}_b\right) r_t + \overline{C}_b \mathbb{E}_t r^L_{t+1} - \mathbb{E}_t \pi_{t+1}\right)
\end{aligned}$$

where $\widetilde{\gamma}$ is the marginal cost semi-elasticity of inflation, $\gamma = \widetilde{\gamma}(1+\eta)$. Iterating forward the IS curve defines output in terms of the forward-paths of the ex ante real short- and long-term rates:

$$y_t = -\mathbb{E}_t \sum_{j=0}^{\infty}\left\{(1-\overline{C}_b)r_{t+j} + \overline{C}_b r^L_{t+j+1} - \pi_{t+j+1}\right\}$$

Rearrange the Phillips curve:

$$y_t = -\frac{r^n_t}{1-\rho_n} + \frac{1}{\eta}\left[-\frac{r^n_t}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right) - y_t + \left(\Omega - \overline{C}_b\right)\xi_t\right]$$

and substitute out the forward-looking definitions of output and the liquidity premium from the right-hand side:

$$\begin{aligned}
y_t = -\frac{r^n_t}{1-\rho_n} + \frac{1}{\eta}\Bigg[&-\frac{r^n_t}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right) \\
&+\mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\overline{C}_b)r_{t+j} + \overline{C}_b r^L_{t+j+1} - \pi_{t+j+1}\right\} + \left(\Omega - \overline{C}_b\right)\mathbb{E}_t\sum_{j=0}^{\infty}\left\{r^L_{t+j+1} - r_{t+j}\right\}\Bigg]
\end{aligned}$$

which simplifies to equation (2.9):

$$y_t = -\frac{r^n_t}{1-\rho_n} + \frac{1}{\eta}\left[-\frac{r^n_t}{1-\rho_n} + \frac{1}{\widetilde{\gamma}}\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}\right) + \mathbb{E}_t\sum_{j=0}^{\infty}\left\{(1-\Omega)r_{t+j} + \Omega r^L_{t+j+1} - \pi_{t+j+1}\right\}\right]$$

B The Banking Model

The banking setup builds and extends on Sims and Wu (2021) and Gertler and Karadi (2013), integrating long-term debt and central bank reserve issuance into a banking model.[1] Banks indexed by j operate under perfect competition. Banks are financial intermediaries that originate bonds for financing debt-financed expenditures to households and hold reserves with funding from deposits and bank equity. Banks survive each period with probability σ and pay accumulated equity to the household upon exit. Consider a bank balance sheet with nominal private debt, $Q_t\widetilde{B}^{FI}_{jt}$, paying the interest rate R^L_{t+1} in the subsequent period and nominal reserves, $\widetilde{RE}_{jt}$, paying the interest rate R^{re}_t, backed by deposits, $\widetilde{S}_{jt}$, requiring interest payments, R_t, and accumulated bank equity, $\widetilde{X}_{jt}$:[2]

$$Q_t\widetilde{B}^{FI}_{jt} + \widetilde{RE}_{jt} = \widetilde{S}_{jt} + \widetilde{X}_{jt}$$

Implying bank equity accumulation follows:

$$\begin{aligned}
\widetilde{X}_{jt+1} &= R^L_{t+1}Q_t\widetilde{B}^{FI}_{jt} + R^{RE}_{jt}\widetilde{RE}_{jt} - R_t\widetilde{S}_{jt} \\
&= \left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}
\end{aligned}$$

with bank j's value function:

$$\begin{aligned}
\widetilde{V}_{jt} &= \mathbb{E}_t\sum_{i=1}^{\infty}(1-\sigma)\sigma^{i-1}\Lambda^N_{t,t+i}\widetilde{X}_{jt+i} = (1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{X}_{jt+1} + \sigma\mathbb{E}_t\Lambda^N_{t,t+1}V_{jt+1} \\
&= (1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\left[\left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}\right] + \sigma\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{V}_{jt+1}
\end{aligned}$$

In the event of bank default, banks can walk away with 100% of private debt holdings, $Q_t\widetilde{B}^{FI}_t$, with probability Θ_t.[3] To prevent this, depositors impose the following limited enforcement constraint on banks to ensure bank continuation:

$$\text{Expected value of default} = \Theta_t Q_t\widetilde{B}^{FI}_{jt} \leq \widetilde{V}_{jt} = \text{Continuation value}$$

Banks maximize the expected sum of future profits subject to the limited enforcement constraint. A Lagrangian, with $\varkappa_{jt}$ as the multiplier on the limited enforcement constraint, is given by:

$$\begin{aligned}
\mathcal{L}_t = &(1+\varkappa_{jt})(1-\sigma)\mathbb{E}_t\Lambda^N_{t,t+1}\left[\left(R^L_{t+1} - R_t\right)Q_t\widetilde{B}^{FI}_{jt} + \left(R^{re}_t - R_t\right)\widetilde{RE}_{jt} + R_t\widetilde{X}_{jt}\right] \\
&+ (1+\varkappa_{jt})\sigma\mathbb{E}_t\Lambda^N_{t,t+1}\widetilde{V}_{jt+1} - \varkappa_{jt}\Theta_t\left(Q_t\widetilde{B}^{FI}_{jt}\right)
\end{aligned}$$

Implying the first-order conditions:

$$\begin{aligned}
\widetilde{B}_{jt}: &\qquad \Theta_t\frac{\varkappa_{jt}}{1+\varkappa_{1,jt}} = \mathbb{E}_t\Lambda^N_{t,t+1}\left[1-\sigma+\sigma\frac{\partial\widetilde{V}_{jt+1}}{\partial\widetilde{X}_{jt+1}}\right]\left(R^L_{t+1} - R_t\right) \\
\widetilde{RE}_{jt}: &\qquad 0 = \mathbb{E}_t\Lambda^N_{t,t+1}\left[1-\sigma+\sigma\frac{\partial\widetilde{V}_{jt+1}}{\partial\widetilde{X}_{jt+1}}\right]\left(R^{re}_t - R_t\right)
\end{aligned}$$

1. In contrast to Sims and Wu (2021), we do not consider the effect of reserve requirements on the banking problem.
2. Variables, Z_t, with a tilde, $\widetilde{Z}_t$, reflect nominal quantities.
3. Reserves are fully recoverable in the case of bank default.

From the optimality condition for reserves, it follows that the reserve and deposit rates are equal. To solve for the envelope condition with respect to bank equity, $\partial \widetilde{V}_{jt+1}/\partial \widetilde{X}_{jt+1}$, start with expressing the continuation value of bank j as:

$$\widetilde{V}_{jt} = \chi_{b,jt} Q_t \widetilde{B}^{FI}_{jt} + \chi_{re,jt} \widetilde{RE}_{jt} + \chi_{x,jt} \widetilde{X}_{jt}$$

with:

$$\chi_{b,jt} = (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \left(R^L_{t+1} - R_t \right) + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{Q_{t+1} \widetilde{B}^{FI}_{jt+1}}{Q_t \widetilde{B}^{FI}_{jt}} \chi_{b,jt+1}$$

$$\chi_{re,jt} = (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \left(R^{re}_t - R_t \right) + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\widetilde{RE}_{jt+1}}{\widetilde{RE}_{jt}} \chi_{re,jt+1}$$

$$\chi_{x,jt} = 1 - \sigma + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \frac{\widetilde{X}_{jt+1}}{\widetilde{X}_{jt}} \chi_{x,jt+1}$$

Imposing the optimality conditions, $\chi_{RE,jt} \equiv 0$, implies that the limited enforcement constraint can be written as:

$$\begin{aligned} \Theta_t Q_t \widetilde{B}^{FI}_{jt} &\leq \overbrace{\chi_{b,jt} Q_t \widetilde{B}^{FI}_{jt} + \chi_{x,jt} \widetilde{X}_{jt}}^{\widetilde{V}_{jt}} \\ \Rightarrow Q_t \widetilde{B}^{FI}_{jt} &\leq \frac{\chi_{x,jt}}{\Theta_t - \chi_{b,jt}} \widetilde{X}_{jt} = \Phi_{jt} \widetilde{X}_{jt} = \frac{\widetilde{V}_{jt}}{\Theta_t} \end{aligned} \tag{B.1}$$

Assuming that the modified leverage constraint binds on average, $\varkappa_{jt} > 0$, and imposing that the reserve and deposit rates are equal, the bank capital accumulation can be written as follows:[4]

$$\Phi_{jt} = \frac{Q_t \widetilde{B}^{FI}_{jt}}{\widetilde{X}_{jt}} \Rightarrow \frac{\widetilde{X}_{jt+1}}{\widetilde{X}_{jt}} = \left(R^L_{t+1} - R_t \right) \frac{Q_t \widetilde{B}^{FI}_{jt}}{\widetilde{X}_{jt}} + R_t = \left(R^L_{t+1} - R_t \right) \Phi_{jt} + R_t \tag{B.2}$$

Rewrite the bank's continuation value, $\widetilde{V}_{jt}$, in terms of bank equity using the relationship in equation (B.1):

$$\begin{aligned} \Theta_t \Phi_{jt} \widetilde{X}_{jt} &= \mathbb{E}_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda^N_{t,t+i} \widetilde{X}_{jt+i} \\ &= (1-\sigma)\mathbb{E}_t \Lambda^N_{t,t+1} \widetilde{X}_{jt+1} + \sigma \mathbb{E}_t \Lambda^N_{t,t+1} \Theta_{t+1} \Phi_{jt+1} \widetilde{X}_{jt+1} \end{aligned}$$

which, with a binding leverage constraint and exploiting equation (B.2), can be written as:

$$\begin{aligned} &= \mathbb{E}_t \Lambda^N_{t,t+1} \left[1 - \sigma + \sigma \Theta_{t+1} \Phi_{jt+1} \right] \left[\left(R^L_{t+1} - R_t \right) \Phi_{jt} + R_{jt} \right] \widetilde{X}_{jt} \\ \Rightarrow \Theta_t \Phi_{jt} &= \mathbb{E}_t \Lambda^N_{t,t+1} \left[1 - \sigma + \sigma \Theta_{t+1} \Phi_{jt+1} \right] \left[\left(R^L_{t+1} - R_t \right) \Phi_{jt} + R_{jt} \right] \end{aligned}$$

4. The time-varying long- to short-term interest rate spread in the data suggests that the constraint binds on average.

Since no term on the right-hand side is bank-specific besides allowed modified leverage, the modified leverage ratio is the same across all banks and follows:

$$\Phi_{jt} = \Phi_t = \frac{\mathbb{E}_t \Lambda^N_{t,t+1} \left[1 - \sigma + \sigma \Theta_{t+1} \Phi_{t+1}\right] R_t}{\Theta_t - \mathbb{E}_t \Lambda^N_{t,t+1} \left[1 - \sigma + \sigma \Theta_{t+1} \Phi_{t+1}\right] \left(R^L_{t+1} - R_t\right)}$$

as in Gertler and Karadi (2011). With a binding leverage constraint, allowed modified leverage is constant across all banks and independent of the level of bank equity. This implies the envelope condition: ${\partial \widetilde{V}_{jt}}/{\partial \widetilde{X}_{jt}} = \Theta_t \Phi_t$; with the first-order condition over bond holdings given by:

$$\frac{\varkappa_{jt}}{1 + \varkappa_{jt}} \Theta_t = \mathbb{E}_t \Lambda^N_{t,t+1} \left[1 - \sigma + \sigma \Theta_{t+1} \Phi_{t+1}\right] \left(R^L_{t+1} - R_t\right)$$

Furthermore, bank equity and asset holdings integrate across all banks:

$$Q_t \widetilde{B}^{FI}_t = \Phi_t \widetilde{X}_t$$

and the law of motion for survivor bank equity can be written as:

$$\widetilde{X}_t = \left[\left(R^L_t - R_{t-1}\right) \Phi_{t-1} + R_{t-1}\right] \widetilde{X}_{t-1}$$

The law of motion for aggregate bank net worth includes the evolution of survivors' net worth and the net worth of new entrants. A fraction σ of bankers at $t-1$ survive until t with net worth evolution as described above. A fraction $1-\sigma$ of bankers at $t-1$ exit with the market value of end-of-life long-term assets $(1-\sigma)\kappa Q_t \widetilde{B}^{FI}_{t-1}$. Assume that each period, the household transfers $(1-\sigma)^{-1} P_t X^s$ to each new entrant to maintain the scale of the assets managed by the financial sector. Aggregate real net worth evolves according to:

$$X_t = \sigma \Pi_t^{-1} \left[\left(R^L_t - R_{t-1}\right) \Phi_{t-1} + R_{t-1}\right] X_{t-1} + X^s$$

with the real modified leverage constraint:

$$Q_t B^{FI}_t = \Phi_t X_t$$

Consider the relevant equations to the model equilibrium conditions under the deterministic exit limit, $\sigma = 0$, imposing $\mathbb{E}_t \Lambda^N_{t,t+1} R_t = 1$ from the household problem:

$$\begin{aligned} 1 + \frac{\varkappa_t}{1 + \varkappa_t} \Theta_t &= \mathbb{E}_t \Lambda^N_{t,t+1} R^L_{t+1} \\ Q_t B^{FI}_t &\leq \Phi_t X^s \\ 1 &= \Phi_t \left[\Theta_t - \left(\mathbb{E}_t \Lambda^N_{t,t+1} R^L_{t+1} - 1\right)\right] \end{aligned}$$

The modified leverage ratio applies over the entire net worth and is endogenous, differing Sims et al. (2021) who assume an exogenous leverage ratio. The net worth accumulation and long-term rate definitions are identical to Sims et al. (2021). Modified leverage rises for a given value of the financial shock as the long-term interest rate increases.

C Properties of Endogenous Balance Sheet Policy

C.1 Dual Instrument Policy Ensures that the Divine Coincidence Holds

Proposition 1. *Absent endogenous balance sheet policy, the divine coincidence fails due to liquidity premium variability.*

Let r_t^π be the policy rate that supports an inflation target. The IS and Phillips curves can be written as:

$$
\begin{aligned}
0 &= \gamma gap_t + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
gap_t &= \mathbb{E}_t gap_{t+1} - \left(r_t^\pi - \overline{C}_b \mathbb{E}_t\left(\xi_{t+1} - \xi_t\right) - r_t^n\right)
\end{aligned}
$$

From the Phillips curve, it is clear that under inflation-targeting interest rate policy the output gap co-varies with the liquidity premium. If the liquidity premium is not stabilized, the divine coincidence does not hold. The IS curve implies:

$$
\mathbb{E}_t\xi_{t+1} - \xi_t = \frac{1+\eta}{\eta\overline{C}_b + \Omega}\left(r_t^\pi - r_t^n\right)
$$

allowing the long-term rate to be written in terms of the inflation-targeting policy rate and the natural rate:

$$
\mathbb{E}_t r_{t+1}^L = \left(1 - \frac{1+\eta}{\eta\overline{C}_b + \Omega}\right) r_t^\pi + \frac{1+\eta}{\eta\overline{C}_b + \Omega} r_t^n
$$

Consider the financial block in the economy, substituting the output gap from the problem and instituting fixed balance sheet policy, $re_t \equiv 0$:

$$
\begin{aligned}
c_{b,t} &= \underbrace{-\frac{\overline{C}_b - \Omega + \left(1 - \overline{C}_b\right)\left(1+\eta\right)}{1+\eta}}_{\mathbb{A}}\xi_t - \frac{1}{1-\rho_n} r_t^n \\
\overline{C}_b c_{b,t} &= \Omega\frac{\varepsilon - 1}{\varepsilon}\left(\underbrace{\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right) - (2+\eta)\frac{\overline{C}_b - \Omega}{1+\eta}\right)}_{\mathbb{B}}\xi_t - \frac{1}{1-\rho_n} r_t^n\right. \\
&\quad \left. + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + \left(1 - \underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}}\right) r_{t-1}^\pi + \underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}} r_{t-1}^n\right)\right)\right) \\
q_t + b_t &= \left(1 - \overline{RE}\right)\left(\frac{\Phi}{\zeta}\underbrace{\frac{1+\eta}{\eta\overline{C}_b + \Omega}}_{\mathbb{C}}\left(r_t^\pi - r_t^n\right) - \underbrace{\left(1 + \Phi\frac{1-\zeta}{\zeta}\right)}_{\mathbb{D}}\theta_t\right)
\end{aligned}
$$

Combining the equations above and collecting terms, this simplifies to:

$$\left(\overline{C}_b\mathbb{A} - \Omega\frac{\varepsilon-1}{\varepsilon}\mathbb{B}\right)\xi_t = \left(\overline{C}_b - \Omega\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{1-\rho_n}r_t^n + \frac{QB}{Y}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C}\left(r_t^\pi - r_t^n\right)\right) -$$
$$\frac{QB}{Y}\frac{1}{\beta\zeta}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C} + (1-\mathbb{C})\right)r_{t-1}^\pi + \frac{QB}{Y}\frac{1}{\beta\zeta}\left(\left(1-\overline{RE}\right)\frac{\Phi}{\zeta}\mathbb{C} - \mathbb{C}\right)r_{t-1}^n$$
$$-\frac{QB}{Y}\mathbb{D}\left(\theta_t - \frac{1}{\beta\zeta}\theta_{t-1}\right)$$

The liquidity premium varies with natural rate and financial shocks, deviations of the policy rate from the natural rate, and lagged shocks and deviations.

Proposition 2. *There exists endogenous balance sheet policy, re_t^*, that stabilizes the output gap, inflation, and the liquidity premium, the equivalent of the "divine coincidence" in this economy.*

Corollary 2.1. *The policy rate, r_t, equals the natural rate when balance sheet policy supports liquidity premium stabilization.*

As shown above, output gap variability is proportional to liquidity premium variability when the central bank targets inflation. If both are equal to zero in all periods, the Phillips curve holds and the IS curve holds for the policy rate equal to the natural rate, $r_t = r_t^n$. Conjecture that there exists a balance sheet specification, re_t^*, for which this is true. The financial block in the economy under the divine coincidence simplifies to:

$$\left(\Omega\frac{\varepsilon-1}{\varepsilon} - \overline{C}_b\right)\frac{r_t^n}{1-\rho_n} = \frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta}b_{t-1}\right)$$
$$q_t + b_t = -\left(1-\overline{RE}\right)\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t + \overline{RE}re_t^*$$

With the liquidity premium fixed, the ex ante nominal long-term rate equals the natural rate. Using the definition of the long-term rate in terms of the bond price allows us to define the current bond price as a function of the natural rate:

$$\mathbb{E}_t r_{t+1}^L = \kappa\beta\zeta\mathbb{E}_t q_{t+1} - q_t = r_t^n \Rightarrow q_t = -\frac{r_t^n}{1-\kappa\beta\zeta\rho_n}$$

implying that the equilibrium debt level is given by:

$$b_t = \frac{r_t^n}{1-\kappa\beta\zeta\rho_n} - \left(1-\overline{RE}\right)\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\theta_t + \overline{RE}re_t^*$$

Modified leverage and the bond price are fully exogenous in this case. Thus, the equilibrium debt level only depends on exogenous terms and the policy variable, re_t^*. The equilibrium level of debt-financed expenditure is also fully exogenous and only varies with natural rate shocks. The financial account constraint defines the balance sheet policy that instills the divine coincidence:

$$re_t^* = \frac{1}{\beta\zeta}re_{t-1}^* + \frac{1-\overline{RE}}{\overline{RE}}\left(1+\Phi\frac{1-\zeta}{\zeta}\right)\left(\theta_t - \frac{1}{\beta\zeta}\theta_{t-1}\right)$$
$$-\frac{1}{\overline{RE}}\left(\frac{\kappa(1-\rho_n)}{1-\kappa\beta\zeta\rho_n} - \frac{Y}{QB}\left(\Omega\frac{\varepsilon-1}{\varepsilon} - \overline{C}_b\right)\right)\frac{r_t^n}{1-\rho_n} + \frac{1}{\overline{RE}}\frac{1}{\beta\zeta}\frac{r_{t-1}^n}{1-\kappa\beta\zeta\rho_n} \quad \text{(DC)}$$

C.2 *Balance Sheet Policy Supports Model Determinacy with a Policy Rate Peg*

Proposition 3. *Endogenous balance sheet policy can provide a determinate rational expectations equilibrium with a permanent policy rate peg.*

Consider the log-linear model and suppose the monetary authority credibly targets the liquidity premium, $\xi_t \equiv 0$ for all t, via reserve management, re_t^{ξ}, under a permanent policy rate peg, $r_t = 0$, implying:

$$\begin{aligned}
&\pi_t = \gamma gap_t + \beta \mathbb{E}_t \pi_{t+1} \\
&gap_t = \mathbb{E}_t gap_{t+1} + \mathbb{E}_t \pi_{t+1} + r_t^n \\
&\left[\frac{\overline{C}_b}{\Omega}\frac{\varepsilon}{\varepsilon - 1} - 1\right]\left(gap_t - \frac{1}{1-\rho_n} r_t^n\right) = (1+\eta) gap_t + \frac{1}{\Omega}\frac{\varepsilon}{\varepsilon - 1}\frac{QB}{Y}\left(b_t - \frac{1}{\beta\zeta}\left(b_{t-1} - \pi_t\right)\right) \\
&b_t = -\left(1 - \overline{RE}\right)\left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t + \overline{RE} re_t^{\xi}
\end{aligned}$$

Abstracting from shocks, the system can be written as:

$$\begin{bmatrix} \pi_t \\ gap_t \\ re_{t-1}^{\xi} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \chi_\pi & \chi_{gap} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \gamma + \beta & \gamma & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \zeta\beta \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} \pi_{t+1} \\ gap_{t+1} \\ re_t^{\xi} \end{bmatrix}$$

which is stable so long as the matrix $\mathbf{D}$ has two out of three eigenvalues inside the unit circle. The characteristic equation for matrix $\mathbf{D}$ is given by:

$$(\zeta\beta - e)\left[(\beta + \gamma - e)(1 - e) - \gamma\right] = 0$$

The eigenvalues are:

$$\begin{aligned}
0 < & e_1 = \zeta\beta < 1 \\
-1 < & e_2 = \frac{1 + \beta + \gamma - \sqrt{(1+\beta+\gamma)^2 - 4\beta}}{2} < 1 \\
1 < & e_3 = \frac{1 + \beta + \gamma + \sqrt{(1+\beta+\gamma)^2 - 4\beta}}{2}
\end{aligned}$$

Corollary 3.1. *Inflation-targeting balance sheet policy, re_t^{π}, provides a determinate linear rational expectations equilibrium with a permanent policy rate peg.*

Consider the log-linear model and suppose the monetary authority credibly targets inflation, $\pi_t \equiv 0$ for all t, via reserve management, re_t^{π}, under a permanent policy rate peg, $r_t = 0$:

$$\begin{aligned}
&0 = \gamma gap_t + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
&gap_t = \mathbb{E}_t gap_{t+1} + \overline{C}_b \mathbb{E}_t \Delta\xi_{t+1} + r_t^n \\
&r_t^L = \kappa\beta\zeta q_t - q_{t-1} \\
&\xi_t = \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r_{t+1}^L
\end{aligned}$$

$$
\begin{aligned}
c_{b,t} &= gap_t - \left(1-\overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n} r^n_t \\
\left(\overline{C}_b - \Omega\frac{\varepsilon-1}{\varepsilon}\right) c_{b,t} &= \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + 1 - \Omega\right)\xi_t + (1+\eta) gap_t\right) \\
&\quad + \frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta} b_{t-1}\right) \\
\phi_t &= \frac{\Phi}{\zeta}\mathbb{E}_t r^L_{t+1} - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1 - \overline{RE}\right)\phi_t + \overline{RE} re^\pi_t
\end{aligned}
$$

The Phillips curve implies that the output gap is proportional to the liquidity premium:

$$
gap_t = \frac{\Omega - \overline{C}_b}{1+\eta}\xi_t
$$

Substituting the output gap from the IS curve defines a first-order difference equation for liquidity premium dynamics:

$$
\xi_t = \mathbb{E}_t \xi_{t+1} + \frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t \tag{C.1}
$$

This implies from the liquidity premium definition that the expected future long-term rate is proportional to the natural rate:

$$
\mathbb{E}_t r^L_{t+1} = \frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t
$$

and, from the allowed modified leverage definition, allowed modified leverage is fully exogenous:

$$
\phi_t = \frac{\Phi}{\zeta}\frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t
$$

Rearranging the definition of the expected long-term rate defines the current bond price as a linear function of the natural rate:

$$
\kappa\zeta\beta\mathbb{E}_t q_{t+1} - q_t = \frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t \Rightarrow q_t = \kappa\zeta\beta\mathbb{E}_t q_{t+1} - \frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t = -\frac{1}{1-\zeta\beta\kappa\rho_n}\frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t
$$

The equilibrium debt level is a linear function of the natural rate, financial shock, and balance sheet policy:

$$
b_t = \left(1 - \overline{RE}\right)\left[\frac{\Phi}{\zeta}\frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t\right] + \overline{RE} re^\pi_t + \frac{1}{1-\zeta\beta\kappa\rho_n}\frac{1+\eta}{\Omega + \eta\overline{C}_b} r^n_t
$$

Given that the output gap is proportional to the liquidity premium, the two equations defining debt-financed expenditure can be written as:

$$
c_{b,t} = -\left[1 - \frac{\Omega + \eta\overline{C}_b}{1+\eta}\right]\xi_t - \frac{1}{1-\rho_n} r^n_t \tag{C.2}
$$

$$
\left(\frac{\overline{C}_b}{\Omega}\frac{\varepsilon}{\varepsilon-1} - 1\right) c_{b,t} = \left(\frac{1-\Omega}{\eta} + 1 - \overline{C}_b\right)\xi_t + \frac{1}{\Omega}\frac{\varepsilon}{\varepsilon-1}\frac{QB}{Y}\left((1-\kappa)q_t + b_t - \frac{1}{\beta\zeta} b_{t-1}\right) \tag{C.3}
$$

Equation (C.2) and (C.3) consolidate to define the current liquidity premium. We have shown that current bond price is a linear function of the current natural rate and that the equilibrium debt level is a linear function of the current natural rate, financial shock, and balance sheet policy. Given this, the current liquidity premium is a linear function of current and lagged values of the exogenous variables, $\{r_t^n, \theta_t\}$, and balance sheet policy, re_t^π:

$$\xi_t = \omega_1 r_t^n + \omega_2 r_{t-1}^n + \omega_3 \theta_t + \omega_4 \theta_{t-1} + \omega_5 re_t^\pi - \omega_6 re_{t-1}^\pi \tag{C.4}$$

Substituting equation (C.4) into equation (C.1) defines a first-order difference equation for balance sheet policy log-differences, $\Delta re_t^\pi = re_t^\pi - re_{t-1}^\pi$, as a function of exogenous terms:

$$\Delta re_t^\pi = \frac{\omega_5}{\omega_6}\mathbb{E}_t \Delta re_{t+1}^\pi + \left[\omega_2 - \omega_1(1-\rho_n) + \frac{1+\eta}{\Omega + \eta \overline{C}_b}\right]\frac{r_t^n}{\omega_6} + \left[\omega_4 - \omega_3(1-\rho_\theta)\right]\frac{\theta_t}{\omega_6} - \omega_2 \frac{r_{t-1}^n}{\omega_6} - \omega_4 \frac{\theta_{t-1}}{\omega_6}$$

This equation has a solution, and therefore completes the proof of model determinacy, if $|\omega_5| < |\omega_6|$. From equations (C.2) and (C.3) along with the definitions of the bond price, q_t, and debt level, b_t, in terms of the exogenous variables and the balance sheet policy:

$$\frac{\omega_5}{\omega_6} = \beta\zeta < 1 \Rightarrow \omega_5 < \omega_6$$

ω_5 and ω_6 have the same sign implying that the solution is non-oscillatory. Note, the inflation-targeting balance sheet policy under and interest rate peg is stationary in first-differences. This implies that the balance sheet level is non-stationary under this policy. Following shock innovations, the model converges to a new steady state over time.

Corollary 3.2. *Balance sheet policy that targets the output gap, re_t^{gap}, with a permanent policy rate peg results in model indeterminacy.*

Consider the log-linear model and suppose the monetary authority credibly targets the output gap, $gap_t \equiv 0$ for all t, via reserve management, re_t^{gap}, under a permanent policy rate peg, $r_t = 0$:

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t \pi_{t+1} + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t \\
0 &= \overline{C}_b \mathbb{E}_t \Delta\xi_{t+1} + \mathbb{E}_t \pi_{t+1} + r_t^n \\
r_t^L &= \kappa\beta\zeta q_t - q_{t-1} \\
\xi_t &= \mathbb{E}_t \xi_{t+1} + \mathbb{E}_t r_{t+1}^L - r_t \\
c_{b,t} &= -\left(1 - \overline{C}_b\right)\xi_t - \frac{1}{1-\rho_n} r_t^n \\
\overline{C}_b c_{b,t} &= \Omega \frac{\varepsilon - 1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t - \frac{1}{1-\rho_n} r_t^n\right) \\
&\qquad + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} + r_t^L - \pi_t\right)\right) \\
\phi_t &= \frac{\Phi}{\zeta}\mathbb{E}_t r_{t+1}^L - \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t \\
q_t + b_t &= \left(1 - \overline{RE}\right)\phi_t + \overline{RE}\, re_t^{gap}
\end{aligned}$$

The IS curve defines a relationship between inflation expectations and forward looking changes in the liquidity premium:

$$\mathbb{E}_t \pi_{t+1} = -\left(\overline{C}_b \mathbb{E}_t \Delta\xi_{t+1} + r_t^n\right)$$

Substituting inflation expectations from the Phillips curve defines the current inflation rate in terms of the liquidity premium and the natural rate:

$$\pi_t = -\beta\left(\overline{C}_b \mathbb{E}_t \Delta\xi_{t+1} + r_t^n\right) + \frac{\gamma}{1+\eta}\left(\overline{C}_b - \Omega\right)\xi_t$$

With output gap-targeting balance sheet policy under a permanent policy rate peg, the non-financial block of the model defines the current inflation rate in terms of the liquidity premium path and the natural rate. The financial block of the economy in this case is given by:

$$c_{b,t} = -\frac{1}{1-\rho_n} r_t^n - \left(1 - \overline{C}_b\right)\xi_t$$

$$\overline{C}_b c_{b,t} = \Omega\frac{\varepsilon-1}{\varepsilon}\left(\left(\frac{1-\Omega}{\eta} + \overline{C}_b - \Omega\right)\xi_t - \frac{1}{1-\rho_n} r_t^n\right) + \frac{QB}{Y}\left(q_t + b_t - \frac{1}{\beta\zeta}\left(q_{t-1} + b_{t-1} - \Delta\xi_t - \pi_t\right)\right)$$

$$q_t + b_t = -\left(1 - \overline{RE}\right)\left(\frac{\Phi}{\zeta}\mathbb{E}_t \Delta\xi_{t+1} + \left(1 + \Phi\frac{1-\zeta}{\zeta}\right)\theta_t\right) + \overline{RE} re_t^{gap}$$

Given the relationship between inflation and the liquidity premium from the non-financial model block, the financial block consolidates into a single equation relating the liquidity premium and endogenous balance sheet balance sheet policy to exogenous variables. Given that both the liquidity premium and balance sheet policy are unknown, this results in model indeterminacy.