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Imperfect Signals

Abstract

A pre-condition for employer learning is that signals at labor market entry do not fully reveal graduates' productivity. I model various distinct sources of signal imperfection—such as noise and multi-dimensional types—and characterize their implications for the private return to skill acquisition. Structural estimates using NLSY data suggest an important role for noise, pushing the private return below the social return. This induces substantial under-investment and causes output losses of up to 22 percent. Value-added-based evidence from Swedish high school graduates also points to noise and under-investment. Highlighting the distinction between schooling duration and skills acquired, I conclude that individuals likely spend too much time in school, but learn too little.

JEL-Codes: D820, I260, J240, J310.

Keywords: human capital, signalling, employer learning, returns to schooling.

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1 Introduction

Any graduate entering the labor market faces the problem that employers have incomplete knowledge of her productive abilities. In theory, her educational choices may serve as a signal and help resolve the information asymmetry (Spence, 1973). However, in reality such signals are always imperfect. Some signals are coarse (college graduation, years of schooling), others are more varied but noisy (a grade point average), yet others are precise but narrow (the GRE math score). Clearly, there are many aspects of schooling—effort, content, quality of instruction—that cannot be perfectly observed by employers. The employer learning literature supports such casual observations by establishing that it takes time for the market to learn a worker's productivity (Farber and Gibbons, 1996; Altonji and Pierret, 2001). This shows that observed educational choices do not perfectly reveal graduates' abilities—as signals, they are imperfect. The goal of this paper is to explore the sources of signal imperfection and to examine their implications for the efficiency of educational choices. In doing so, I shed new light on the roles of human capital and signaling motives in education (Weiss, 1995; Lange and Topel, 2006).

I build a theoretical model featuring human capital accumulation, signaling, and employer learning. Workers' productivity is a function of 'talent' (traits that predate schooling) and 'acquired skills' (abilities obtained in school), as in Spence (1974). In contrast to prior literature, workers choose both how long to stay in school (whether to attend college) and the amount of skills to acquire. This is an important distinction in relation to information frictions: Employers perfectly observe years of schooling, but the signal they receive about a workers' acquired skills (for instance, a grade point average) may be noisy.[1] I further assume that an individual's type has two dimensions: talent, as well as 'taste' for schooling. Talent and taste are both inversely related to skill acquisition costs, but unlike talent, taste has no effect on productivity. Employers form beliefs about graduates' productivity based on the skill acquisition signal as well as observed years of schooling. Once workers enter the labor market, further signals about their productivity are received by all employers each period, giving rise to employer learning.

The model serves three purposes. First, to highlight that signal imperfection can arise for multiple reasons; they are not mutually exclusive, but each by itself is a sufficient pre-condition for employer learning. Second, to establish that different sources of signal imperfection can have opposing effects on the efficiency of schooling choices. And third, to help characterize inefficiency in schooling choices in data from the US and Sweden.

The basic insights can be understood by considering two polar cases. First, suppose acquired skills are perfectly observed. Employers still do not fully know workers' produc-

[1] For instance, grades may contain a luck component (Landaud, Maurin, Willage, and Willén, 2022), or degree classifications may be coarse (Feng and Graetz, 2017).

tivity at labor market entry, because taste heterogeneity implies heterogeneity in talent for each level of acquired skills. However, skill acquisition will be inefficiently high, provided the correlation between talent and acquired skills is positive. As in the classical signaling model, observing acquired skills leads employers to update about both a worker's acquired skills and her talent, pushing the private return to skill acquisition above the social return. Second, suppose the skill acquisition signal is infinitely noisy. Skill acquisition will be inefficiently low due to the delay with which the market rewards it.

To the best of my knowledge, this distinction between different sources of signal imperfection is not made explicit in existing literature. A more familiar feature of the model is that college attendance tends to be too high, since college graduation is perfectly observed and may thus be used to compensate for noisy skill acquisition signals. Here, the novelty of my analysis lies in highlighting that this may coincide with inefficiently low skill acquisition, and thus insufficient human capital accumulation in the aggregate.

A regression of log wages on talent and skill acquisition, separately by years of schooling and labor market experience, reveals the direction of inefficiency in skill acquisition. The coefficient on acquired skill measures the private return to skill acquisition. At high levels of experience, however, the private return is very close to the social return, since employer learning is essentially complete. The test for the direction of inefficiency is to check whether the coefficients approach their limit from above or below, indicating over-investment and under-investment, respectively.

I conduct an empirical exercise inspired by this test using data on Swedish high school graduates. I use parental background and compulsory school GPA to proxy for talent and prior skill acquisition. Conditional on those variables, high school GPA should capture additional skill acquisition ('value added') during the last three years prior to labor market entry. I find that the returns to the standardized high school GPA are initially zero but quickly grow and converge to around two percent for experienced workers. This suggests that due to information frictions, Swedish high school graduates early in their career face a wage return to skill acquisition that is below the social return, which in the theoretical model implies under-investment.

Beyond this qualitative conclusion based on the Swedish evidence, I offer structural estimates of my model and use these to explore the counterfactual scenario of full information. For this, I return to the NLSY data used by Altonji and Pierret (2001), Lange (2007), and Arcidiacono, Bayer, and Hizmo (2010). The test for the presence of employer learning suggested by Altonji and Pierret (2001) is to check that wage returns to years of schooling decrease with experience while returns to AFQT increase. My model is able to match this pattern and other moments documented in the afore-mentioned articles, but these on their own do not reveal the direction of inefficiency.[2] My own test described above is infeasible

[2]The model is also consistent with instrumental variables estimates of the effect of college on wages

because the NLSY data do not contain direct measures of talent and skill acquisition. Instead, they contain the Armed Forces Qualification Test (AFQT) score, which I treat as potentially containing information about both talent and skill acquisition. In line with the literature, I assume that the AFQT score is not directly observed by employers, but they instead observe a productivity correlate (the skill acquisition signal) that is not observed by the econometrician. In order to characterize inefficiency in schooling choices, I need to estimate all model parameters, requiring additional assumptions about the aggregate production function, the type distribution, and the average effect of college on productivity.

Across a range of assumptions, several robust patterns emerge. The output elasticity of acquired skill exceeds that of pre-existing talent in most cases. The AFQT score appears to mainly reflect talent, not acquired skills. Most importantly, I find that acquired skills are indeed highly imperfectly observed at labor market entry, especially so among high school graduates. This conclusion arises from a prominent feature of the data, namely that the wage return to the AFQT score for inexperienced high school graduates is zero. Even when the AFQT score does not reflect acquired skills but only talent, it will be positively correlated with acquired skills since they are an increasing function of talent. A zero wage return at labor market entry therefore strongly suggests that employers know very little about high school graduates' acquired skills (or their talent).

I calculate that under-investment in productive skills leads to output losses of up to 22 percent, relative to a perfect-information counterfactual scenario. Moreover, the fraction of college graduates is typically inefficiently high, though the extent of this depends critically on the substitution elasticity between high school and college workers in the aggregate production function. Under perfect substitutes, the observed college share of 26 percent would drop to 1 percent in the counterfactual of full information. However, assuming a more conventional value for the substitution elasticity of 1.5, the counterfactual college share would range from 21-25 percent.

The paper proceeds as follows. Section 1.1 discusses related literature. For the sake of expositional clarity, I present the model in stages. Section 2 introduces imperfectly observed skill acquisition, holding the length of education fixed, into an otherwise standard employer learning framework, establishes that the direction of inefficiency is ambiguous, and suggests an empirical test. Section 3 completes the model by allowing for a choice between a high school and a college track, and clarifies the connection between econometric employer learning models and the test suggested in Section 2. Section 4 estimates the parameters of the two-track model using NLSY data, and performs a counterfactual exercise. Section 5 estimates and interprets the returns to high school GPA for Swedish workers over the course of their careers. Section 6 offers a concluding discussion.

that decline with experience, as in Aryal, Bhuller, and Lange (2022). Again, this is true regardless of the direction of inefficiency in skill acquisition.

1.1 Related literature

This paper speaks to an ongoing debate among researchers and policy makers about the importance of signaling versus human capital motives in educational choices (Oreopoulos, 2021). It is often asserted that if signaling motives are important, then education is largely wasteful (Caplan, 2018). On the other hand, fast employer learning may limit the importance of signaling motives (Lange, 2007; Lange and Topel, 2006). This paper contributes to the debate by exploring multiple sources of signal imperfection; by showing that they have distinct effects on the direction of inefficiency in schooling choices; and by presenting evidence that under-investment is an empirically relevant phenomenon. And unlike prior literature, I highlight the distinction between amount of time spent in school and amount of skills acquired. My results suggest that the former tends to be too high but the latter too low, implying more nuanced policy conclusions than would follow from traditional signaling arguments (see Section 6).

The paper is closely related to the employer learning literature (Farber and Gibbons, 1996; Altonji and Pierret, 2001; Arcidiacono, Bayer, and Hizmo, 2010; Ablay and Lange, 2022). This literature finds that private returns to easily observed characteristics, such as years of schooling, typically exceed the social returns early in workers' careers, which is interpreted as a signaling premium (Lange, 2007; Aryal, Bhuller, and Lange, 2022). I show that such evidence is consistent with the presence of under-investment, since the productive skills that students acquire can remain poorly observed. Kahn and Lange (2014) present evidence that information frictions depress the return to on-the-job skill acquisition particularly among older workers. My findings instead concern skill acquisition prior to labor market entry, when welfare losses are potentially larger, given the longer investment horizon. Finally, I extend the standard employer learning model by including measures of school performance.[3]

I contribute to the theoretical analysis of signaling games with employer learning that was initiated by Alós-Ferrer and Prat (2012). They consider a precise signal stemming from a perfectly observed, costly educational action, and point out that a separating equilibrium leaves no room for employer learning. Alós-Ferrer and Prat (2012) thus restrict their attention to pooling equilibria involving mixed strategies. In contrast, I focus on separating equilibria throughout, allowing for employer learning by assuming that signals are incomplete in various ways.

The possibility that employer learning can cause under-investment is highlighted by

[3]Throughout the paper I focus on symmetric employer learning whereby information about workers is revealed to all employers at the same time, as do the papers cited in this paragraph (see Schönberg, 2007; Kahn, 2013, for tests of this assumption). Also in common with those papers, I abstract from match-specificity and search frictions (Jovanovic, 1979; Fredriksson, Hensvik, and Skans, 2018), differential on-the-job learning, as well as time-varying skills and the possibility that employer learning remains incomplete even for very experienced workers (Kahn and Lange, 2014; Caplan, 2018).

Craig (forthcoming), who derives optimal tax rates that correct for this inefficiency. Craig (forthcoming) assumes a noisy signal as the only source of initial uncertainty about workers' productivity. As I show here, employer learning may however also arise due to multidimensional types, which can imply over-investment. Moreover, I suggest an empirical test for the direction of inefficiency.[4] In the context of statistical discrimination, Coate and Loury (1993) show that under-investment may arise as one of several possible equilibria. However, they do not incorporate employer learning—doing so would make it more difficult to sustain a self-fulfilling equilibrium.[5]

2 Baseline model of signaling and employer learning with an imperfect signal

The theoretical analysis proceeds in three steps. First, I introduce a noisy signal and multidimensional types into an employer learning framework in Section 2.1. Second, I model workers' optimal skill acquisition within this framework and characterize the equilibrium of the resulting signaling game in Section 2.2. Third, I introduce duration of schooling as an additional choice variable in Section 3.

2.1 A labor market with asymmetric information, noisy signals, and learning

I begin by describing production, markets, and information, taking acquired skills as given. The results in this section thus hold regardless of the way skill acquisition is determined.

Worker i is characterized by pre-existing talent θ_i and acquired skills s_i. 'Pre-existing talent' refers to the productive traits that individuals possess before they begin schooling (which may result from both nature and nurture), while 'acquired skills' refers to potentially productive capabilities picked up at school. Worker i produces log output

$$y_i = a_s s_i + a_\theta \theta_i, \qquad a_s \geq 0, a_\theta \geq 0, a_s + a_\theta > 0. \tag{1}$$

Let t denote years since graduation. The market does not observe output directly, but

[4] Craig (forthcoming) also does not allow for a direct impact of talent on productivity in his main analysis, but shows in an appendix that relaxing this assumption may lead to over-investment.

[5] This paper is also related to the literature that employs quasi-experimental approaches to estimate the wage returns to a degree receipt or degree quality. See for instance Tyler, Murnane, and Willett (2000), Jepsen, Mueser, and Troske (2016), Clark and Martorell (2014), Freier, Schumann, and Siedler (2015), Feng and Graetz (2017), and Khoo and Ost (2018). A common identification strategy in that literature is the regression discontinuity design. For degrees to have an effect on wages, the underlying running variable—such as GPA—must be at least partially hidden from employers. At the same time, the running variable may both reflect talent as well as acquired skills. In Graetz (2021), I show that under these conditions, information frictions may lead to under-investment. However, the analysis in Graetz (2021) is specific to the context of regression discontinuity designs, and the paper provides no evidence on under-investment.

instead observes the output signal

$$\tilde{y}_{it} = y_i + \varepsilon_{it}, \tag{2}$$

where ε_{it} is independently drawn from a normal distribution with zero mean and variance σ_ε^2. I denote the worker's history of output signals at the start of period t by $\tilde{\mathbf{y}}_{it} = (\tilde{y}_{i0}, \tilde{y}_{i1}, ..., \tilde{y}_{i,t-1})'$ and the average of the signals by $\overline{y}_{it} = (1/t)\sum_{t'=0}^{t-1} \tilde{y}_{it'}$ for $t > 0$. I define $\tilde{\mathbf{y}}_{i0} = \emptyset$ and $\overline{y}_{i0} = 0$. I sometimes use the words 'time' or 'period' when referring to t. It should be clear that I mean years since graduation and labor market experience, not calendar time.

The market also observes the skill acquisition signal

$$\tilde{s}_i = s_i + u_i, \tag{3}$$

where the disturbance term u_i is drawn independently from a mean-zero normal distribution with variance σ_u^2. Acquired skills are thus observed with error unless $\sigma_u^2 = 0$. I further assume that talent θ_i is normally distributed with mean μ_θ and variance σ_θ^2. Let the mean and variance of acquired skills be denoted by μ_s and σ_s^2. Finally, let the covariance and correlation coefficient of acquired skills and talent be denoted by $\sigma_{s\theta}$ and $\rho_{s\theta}$.

In a competitive market, workers receive a wage equal to their expected productivity, conditional on publicly available information,

$$W_{it} = \mathrm{E}\left[\exp\{y_i\}|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right]. \tag{4}$$

Suppose that y_i is conditionally normally distributed, to be verified later. Then $\exp\{y_i\}$ is conditionally log-normally distributed, and therefore

$$\mathrm{E}\left[\exp\{y_i\}|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right] = \exp\left\{\mathrm{E}[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}] + \frac{1}{2}\mathrm{Var}[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}]\right\}, \tag{5}$$

with

$$\mathrm{E}\left[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right] = (1-\lambda_t)\left(\mu_y - b_0^{y\tilde{s}}\mu_s\right) + (1-\lambda_t)b_0^{y\tilde{s}}\tilde{s}_i + \lambda_t\overline{y}_{it}, \tag{6}$$

and

$$\mathrm{Var}\left(y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right) = (1-\lambda_t)\sigma_{y|\tilde{s},0}^2, \tag{7}$$

where

$$b_0^{y\tilde{s}} = \frac{\sigma_s^2}{\sigma_s^2+\sigma_u^2}a_s + \frac{\sigma_{s\theta}}{\sigma_s^2+\sigma_u^2}a_\theta, \qquad \sigma_{y|\tilde{s},0}^2 = \frac{\sigma_s^2}{\sigma_s^2+\sigma_u^2}a_\theta^2\sigma_\theta^2(1-\rho_{s\theta}^2) + \frac{\sigma_u^2}{\sigma_s^2+\sigma_u^2}\sigma_y^2 \tag{8}$$

and

$$\lambda_t = \frac{\lambda_1 t}{1 + \lambda_1(t-1)}, \qquad \lambda_1 \equiv \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_{y|\tilde{s},0} + \sigma^2_{\varepsilon}}. \tag{9}$$

The preceding expressions are derived in Appendix A. They have intuitive interpretations. At labor market entry, the skill acquisition signal contains all information available to employers. The informativeness of the signal is captured by $b_0^{y\tilde{s}}$, which is simply the best linear prediction of log productivity given the signal. Similarly, $\sigma^2_{y|\tilde{s},0}$ is the variance of productivity conditional on the signal. As a worker gains experience, employers' beliefs about her productivity are increasingly influenced by her history of output signals as opposed to her initial skill acquisition signal, as captured by the 'learning weight' λ_t. In particular, $\partial\lambda_t/\partial t > 0$ and $\lim_{t\to\infty} \lambda_t = 1$. For highly experienced workers, only the output signals matter. The rate at which employers' attention shifts from the initial signal to the subsequent signals is captured by $\lambda_1 \in (0, 1)$, which Lange (2007) refers to as the speed of employer learning. The speed is high when the subsequent signals are more informative than the initial signal.[6] While these insights are well known, I deviate from the literature by allowing $\sigma_u^2 > 0$. As it turns out, this small modification has important implications for the relationship between private and social returns to skill acquisition.

Using (4), (5), (6), and (7), I obtain the log wage as

$$w_{it} = (1-\lambda_t)\left(\mu_y - b_0^{y\tilde{s}}\mu_s + \frac{1}{2}\sigma^2_{y|\tilde{s},0}\right) + (1-\lambda_t)b_0^{y\tilde{s}}\tilde{s}_i + \lambda_t\overline{y}_{it}.$$

Furthermore, using (1), (2) and (3), the log wage can be expressed as a function of a worker's acquired skills and her talent,

$$w_{it} = \beta_t^0 + \beta_t^s s_i + \beta_t^\theta \theta_i + e_{it}, \qquad \begin{pmatrix} \beta_t^0 \\ \beta_t^s \\ \beta_t^\theta \\ e_{it} \end{pmatrix} \equiv \begin{pmatrix} (1-\lambda_t)\left(\mu_y - b_0^{y\tilde{s}}\mu_s + \frac{1}{2}\sigma^2_{y|\tilde{s},0}\right) \\ (1-\lambda_t)b_0^{y\tilde{s}} + \lambda_t a_s \\ \lambda_t a_\theta \\ (1-\lambda_t)b_0^{y\tilde{s}}u_i + \lambda_t\overline{\varepsilon}_{it} \end{pmatrix}. \tag{10}$$

The private return to skill acquisition is given by β_t^s. It is characterized in relation to the social return a_s (and similarly for the returns to talent) by the following result.

Proposition 1. *Suppose that $\sigma_u^2 > 0$ or $\rho_{s\theta} < 1$, so that $\lambda_1 > 0$.*

[6] The noise in the initial signal is measured by $\sigma^2_{y|\tilde{s},0}$, the variance in log output that remains after observing the skill acquisition signal. As seen in (8), this unexplained variance is a weighted average of the unexplained variance conditional on observing acquired skills perfectly and the unconditional variance of productivity, with the weights depending on the signal-to-noise ratio in the skill signal, σ_s^2/σ_u^2.

(a) *The wedge between the private and social returns to skill acquisition is given by*

$$\beta_t^s - a_s = \frac{1}{\sigma_s^2 + \sigma_u^2}(1-\lambda_t)\left(a_\theta \rho_{s\theta}\sigma_s\sigma_\theta - a_s\sigma_u^2\right). \tag{11}$$

Hence, the private return approaches the social return over time,

$$\lim_{t\to\infty} \beta_t^s = a_s,$$

monotonically from above or below depending on parameters,

$$\beta_t^s \gtreqless a_s \quad \Leftrightarrow \quad \frac{\partial \beta_t^s}{\partial t} \lesseqgtr 0 \quad \Leftrightarrow \quad \rho_{s\theta}\frac{\sigma_s\sigma_\theta}{\sigma_u^2} \gtreqless \frac{a_s}{a_\theta}.$$

(b) *The private return to talent approaches the social return over time monotonically from below,* $\partial\beta_\theta/\partial t > 0$, $\lim_{t\to\infty}\beta_t^\theta = a_\theta$.

Suppose instead that $\sigma_u^2 = 0$ *and* $\rho_{s\theta} = 1$, *so that* $\lambda_1 = 0$.

(c) *The private returns to skill acquisition and talent are constant at* $\beta_t^s = b_0^{y\tilde{s}} > a_s$ *and* $\beta_t^\theta = 0$.

Part (a) implies that the private return to skill acquisition exceeds the social return when the correlation between acquired skills and talent is positive and the skill acquisition signal is precise. In particular, under a perfectly informative signal, $\sigma_u^2 = 0$, observing acquired skills affects employers' beliefs about a worker's log productivity due to the direct effect of acquired skill, as well as the effect of talent. This can be seen from (8) which implies $b_0^{y\tilde{s}} = a_s + \frac{\sigma_\theta}{\sigma_s}\rho_{s\theta}a_\theta$. Thus, when $\rho_{s\theta} > 0$, workers are incentivized to acquire skill not only because of its productive effects, but also because it allows them to signal their talent. In contrast, when $\sigma_u^2 > 0$, the social return to acquired skill a_s is attenuated. The private return could thus fall below the social return even if $\rho_{s\theta} = 1$.

Part (a) also highlights that, if an econometrician were able to observe wages, acquired skills, and talent at various levels of labor market experience, she would be able to tell the direction of inefficiency from the way the coefficient on acquired skills changes with experience.

Parts (b) and (c) show that an increasing wage return to an initially hard-to-observe productivity component such as talent—which is typically interpreted as evidence of employer learning—requires that the skill acquisition signal does not reveal all information, in contrast to the canonical signaling model. Information may be only partially revealed due to noise, or simply due to an imperfect correlation of acquired skills and talent. In Section 3.3 I show that these insights also apply to the familiar setting where there is variation in years of schooling, perfectly observed by employers, and where the econometrician only observes years of schooling and a variable that is imperfectly correlated with acquired skills and talent.

2.2 Closing the model: Optimal skill acquisition

I now turn to the determination of skill acquisition. Workers are infinitely lived expected utility maximizers with time-additive log preferences over consumption and discount factor δ. They have no access to savings and thus consume their wage income each period. They spend the first τ_s years of their lives in school and afterwards supply labor inelastically each year t. To avoid confusion, I write $\tau \in \{1, 2, ..., \tau_s\}$ to denote years in school and $t \in \{0, 1, ...\}$ to denote time in the labor market (years of experience). The length of schooling τ_s is fixed, though in Section 3 I allow for a choice between two different tracks, high school and college, which are of different fixed lengths.

During each year in school, workers acquire skills $s_{i\tau}$, so they enter the labor market with a total amount $s_i = \sum_{\tau=1}^{\tau_s} s_{i\tau}$.[7] The period utility cost of acquiring skills is $z(\tau)\exp\{s_{i\tau} - \kappa(\theta_i + \gamma_i)\}$, where γ_i is a normally distributed taste parameter and $z(\tau) > 0$ describes how skill acquisition costs vary over time.[8] The parameter $\kappa > 0$ is unimportant at the moment but will play a crucial role in the two-track model below.

The worker's value function can thus be written as

$$V_i = \max_{\{s_{i\tau}\}_{\tau=1}^{\tau_s}} \left\{ -\sum_{\tau=1}^{\tau_s} \delta^\tau z(\tau)\exp\{s_{i\tau} - \kappa(\theta_i + \gamma_i)\} + \delta^{\tau_s} \sum_{t=0}^{\infty} \delta^t \left(\beta_t^0 + \beta_t^s \sum_{\tau=1}^{\tau_s} s_{i\tau} + \beta_t^\theta \theta_i \right) \right\}.$$

There is no need to include an expectation operator because the only source of uncertainty—the error term in the wage equation (10)—enters additively into the utility function.

The set of first-order conditions is

$$\delta^\tau z(\tau)\exp\{s_{i\tau} - \kappa(\theta_i + \gamma_i)\} = \delta^{\tau_s} \sum_{t=0}^{\infty} \delta^t \beta_t^s, \qquad \tau \in \{1, 2, ..., \tau_s\}. \tag{12}$$

The amount of skills acquired per period generally varies over time, both due to impatience and because of time-varying costs. One may imagine that the costs of acquiring skills decrease with time spent in education, at least at conventional durations. For simplicity, I set $z(\tau) = \zeta\delta^{-\tau}$ so that the two effects cancel out.[9]

[7] Instead of 'acquired skills', the term 'acquired human capital' may also be appropriate, but in contrast to the conventional assumption of human capital theory, in this model s_i may not be perfectly observed by employers.

[8] The cost function is similar to that in Hendricks and Schoellman (2014), which also features talent and tastes as well as the exponential form.

[9] Given the simplifying assumption, one could drop the explicit sub-division of the schooling period, and simply state the utility cost as $\tau_s \zeta \exp\{s_i/\tau_s - \kappa(\theta_i + \gamma_i)\}$. This yields the same solution.

Optimal skill acquisition is thus characterized by

$$s_i = s_0 + \tau_s \kappa \left(\theta_i + \gamma_i\right), \qquad s_0 \equiv \tau_s \log(B_s/\zeta), \; B_s \equiv \delta^{\tau_s} \sum_{t=0}^{\infty} \delta^t \beta_t^s. \tag{13}$$

Acquired skills depend positively on talent, taste, and the present discounted value of returns B_s, and negatively on the cost shifter ζ. The effect of the parameter κ is positive (negative) if the sum of talent and taste is positive (negative). The effect of schooling duration is similarly ambiguous. The linearity of acquired skills allows for a complete characterization of equilibrium.

Proposition 2. *There exists exactly one pure-strategy Perfect Bayesian Equilibrium featuring beliefs as in (6) and (7). In this equilibrium, log wages and skill acquisition choices are given by (10) and (13), respectively.*

To see that employers' expectations are correct, note that the normality assumption made in Section 2.1 is ensured given linearity of acquired skills (13) in talent and tastes. Furthermore, (13) shows that the endogenous moments σ_s^2 and $\sigma_{s\theta}$ only enter the intercept s_0 of the skill acquisition function, implying that σ_s^2 and $\sigma_{s\theta}$ depend on parameters only: $\sigma_s^2 = (\tau_s\kappa)^2 \left(\sigma_\theta^2 + \sigma_\gamma^2 + 2\sigma_{\theta\gamma}\right)$ and $\sigma_{s\theta} = \tau_s\kappa\left(\sigma_\theta^2 + \sigma_{\theta\gamma}\right)$. This means in turn that s_0 is uniquely pinned down by parameters, ensuring equilibrium existence and uniqueness.

The equilibrium is a 'separating equilibrium' in the sense that higher talent implies a larger amount of acquired skills, holding tastes constant (and greater tastes imply more skill acquisition, holding talent constant). However, if the correlation between talent and tastes is less than unity, then skill acquisition does not reveal talent perfectly, since a given skill level is associated with a continuum of talent levels. In this sense, the equilibrium also features 'pooling'. If talent and tastes are perfectly correlated, then the equilibrium may be fully separating in terms of skill acquisition, but a noisy skill signal will induce 'pooling' in terms of observed skill acquisition. As implied by Proposition 1, 'pooling' of some sort is needed for the model to produce the employer learning patterns observed in the data.[10]

Finally, a useful property of the worker's optimal skill acquisition choice is that the

[10] A traditional pooling equilibrium (featuring a degenerate distribution of skill acquisition) does not exist in this model if $\sigma_u^2 > 0$: With an imprecise skill signal, there is no notion of out-of-equilibrium actions. Employer learning rewards skill acquisition at least to some extent, so that uniform skill acquisition cannot be sustained. While Proposition 2 highlights existence and uniqueness of an equilibrium featuring log-linear beliefs and wage schedules, it is silent on the existence of equilibria in which beliefs are non-linear in the signals. This issue is left for future research.

value function is linear,

$$V_i = \nu_0 + \nu_\theta \theta_i + \nu_\gamma \gamma_i, \qquad \begin{pmatrix} \nu_0 \\ \nu_\theta \\ \nu_\gamma \end{pmatrix} \equiv \begin{pmatrix} B_0 + (s_0 - \tau_s) B_s \\ B_\theta + \tau_s B_s \\ \tau_s B_s \end{pmatrix}, \tag{14}$$

where $B_x \equiv \delta^{\tau_s} \sum_{t=0}^{\infty} \delta^t \beta_t^x$.

2.3 The social planner problem

I next solve the social planner problem—equivalent to a decentralized economy under perfect information—to determine whether the decentralized equilibrium of Proposition 2 is socially efficient. The planner's value function is

$$V_i^{\mathrm{SP}} = \max_{\{s_{i\tau}\}_{\tau=1}^{\tau_s}} \left\{ -\sum_{\tau=1}^{\tau_s} \zeta \exp\{s_{i\tau} - \kappa(\theta_i + \gamma_i)\} + \delta^{\tau_s} \sum_{t=0}^{\infty} \delta^t \left(a_s \sum_{\tau=1}^{\tau_s} s_{i\tau} + a_\theta \theta_i \right) \right\}.$$

Defining $A_x \equiv \frac{\delta^{\tau_s}}{1-\delta} a_x$, the first-order conditions lead to

$$s_i^{\mathrm{SP}} = s_0^{\mathrm{SP}} + \tau_s \kappa \left(\theta_i + \gamma_i \right), \qquad s_0^{\mathrm{SP}} \equiv \tau_s \log(A_s/\zeta), \tag{15}$$

and

$$V_i^{\mathrm{SP}} = \nu_0^{\mathrm{SP}} + \nu_\theta^{\mathrm{SP}} \theta_i + \nu_\gamma^{\mathrm{SP}} \gamma_i, \qquad \begin{pmatrix} \nu_0^{\mathrm{SP}} \\ \nu_\theta^{\mathrm{SP}} \\ \nu_\gamma^{\mathrm{SP}} \end{pmatrix} \equiv \begin{pmatrix} \left(s_0^{\mathrm{SP}} - \tau_s \right) A_s \\ A_\theta + \tau_s A_s \\ \tau_s A_s \end{pmatrix}. \tag{16}$$

The comparison of (13) and (15) reveals that the distribution of acquired skills in the decentralized equilibrium can differ from the socially optimally one only in terms of location, so that the direction of inefficiency will be the same for all types. This is of course a consequence of the functional form assumptions. By noting that $s_0^{\mathrm{SP}} = s_0 + \tau_s \log(A_s/B_s)$, I verify that the condition for the direction of inefficiency is the same as that concerning the private return to skill acquisition in Proposition 1, so that the same empirical test applies.[11] To summarize:

Proposition 3. *The equilibrium described in Proposition 2 features too much (too little, the right amount of) skill acquisition compared to the social optimum if and only if* $\rho_{s\theta}(\sigma_s \sigma_\theta)/\sigma_u^2 \gtreqless a_s/a_\theta$*, where* σ_s *and* $\rho_{s\theta}$ *are determined by (13). The direction of inefficiency can be tested for empirically based on the relationship of the private return to skill acquisition with labor market experience, as described in Proposition 1.*

[11] Note that when $\rho_{\theta\gamma} = 1$ and hence $\rho_{s\theta} = 1$, and at the same time $\sigma_u^2 = 0$, the model boils down to a version of Spence (1974). Skill acquisition is inefficiently high, and no EL takes place.

2.4 Baseline model: Summary and discussion

I briefly summarize the insights gained thus far, and discuss their limitations, to be addressed in the following sections. Recall from the discussion of Proposition 1 above that private returns to skill acquisition are always greater than social returns whenever acquired skills are perfectly observed, $\sigma_u^2 = 0$, and the correlation between talent and acquired skills is positive, $\rho_{s\theta} > 0$. In contrast, private returns are attenuated when acquired skills are observed with noise, $\sigma_u^2 > 0$, and even more so when in addition $\rho_{s\theta}$ is low. Employer learning takes place whenever $\sigma_u^2 > 0$ or $|\rho_{s\theta}| < 1$, or both. While the presence of employer learning on its own does not reveal the nature of inefficiency, the time pattern of the coefficients on acquired skills in a log wage regression does so. This result inspires the empirical exercise of Section 5. However, in the EL setting that is standard in existing literature, acquired skills and talent are typically not observed directly. Instead, a proxy variable for productivity is available, in addition to years of schooling (here, such a proxy would presumably be correlated with both talent and acquired skill). Therefore, in the next section I introduce the duration of schooling as an additional choice variable, allowing for a precise mapping between my theoretical model and the existing EL literature.

3 Two-track model: Choosing how much skill to acquire, and when to graduate

3.1 Introducing a choice between high school and college

I distinguish between just two types of education, high school (h) and college (c). I assume that the choice between high school and college is made at the beginning of life, akin to tracking in high school.[12] High school and college are of fixed lengths τ_h and τ_c, respectively, with $\tau_c > \tau_h$. Within each track, individuals choose how much skills to acquire, denoted by s_i^h and s_i^c. Let the educational track be denoted by $j \in \{h, c\}$.

Employers observe track attendance perfectly, in addition to the skill acquisition and output signals introduced in Section 2. For tractability, I assume that employers believe skill acquisition and talent to be jointly normally distributed. I further discuss this assumption below. Given normality, the wage equation (10) continues to hold, but parameters and moments must be indexed by track. Let $\mu_{x|j} \equiv \mathrm{E}[x|j]$, $\sigma_{x|j}^2 \equiv \mathrm{Var}(x|j)$ etc.

[12]This simplification is justified given that here I am not interested in the adjustment of educational choices in response to new information. Alternatively, I could assume that everyone first completes high school and then chooses between college and direct labor market entry. But this would not add any insight unless I also assume that individuals are uncertain about their type or the environment, and obtain relevant information in the course of their studies.

We have

$$w_{it} = \beta_t^{0j} + \beta_t^{sj} s_i + \beta_t^{\theta j} \theta_i + e_{it}^j, \qquad \begin{pmatrix} \beta_t^{0j} \\ \beta_t^{sj} \\ \beta_t^{\theta j} \\ e_{it}^j \end{pmatrix} \equiv \begin{pmatrix} (1-\lambda_t^j)\left(\mu_{y|j} - b_0^{y\tilde{s}|j}\mu_{s|j} + \frac{1}{2}\sigma^2_{y|\tilde{s},0,j}\right) \\ (1-\lambda_t^j) b_0^{y\tilde{s}|j} + \lambda_t^j a_s \\ \lambda_t^j a_\theta \\ (1-\lambda_t^j) b_0^{y\tilde{s}|j} u_i + \lambda_t^j \bar{\varepsilon}_{it} \end{pmatrix}, \tag{17}$$

where

$$\begin{pmatrix} b_0^{y\tilde{s}|j} \\ \lambda_t^j \end{pmatrix} \equiv \begin{pmatrix} b_0^{y\tilde{s}|j}\left(\sigma^2_{s|j}, \sigma_{s\theta|j}, \sigma^2_{uj}, \sigma^2_{\varepsilon j};\ a_s, a_\theta\right) \\ \lambda_t^j\left(t, \sigma^2_{s|j}, \sigma^2_{\theta|j}, \sigma_{s\theta|j}, \sigma^2_{uj}, \sigma^2_{\varepsilon j};\ a_s, a_\theta\right) \end{pmatrix}, \tag{18}$$

with the respective functional relationships given by (8) and (9). I allow the noise terms to have different variances across the tracks, which will turn out to be important when taking the model to the data.

Turning to optimal educational choices, I proceed by backward induction, first solving for optimal skill acquisition within each track. The value function of pursuing either track can now be written as

$$V_i^j = \max_{\{s_{i\tau}^j\}_{\tau=1}^{\tau_j}} \left\{ -\sum_{\tau=1}^{\tau_j} \zeta_j \exp\{s_{i\tau}^j - \kappa_j(\theta_i + \gamma_i)\} + \delta^{\tau_j} \sum_{t=0}^{\infty} \delta^t \left(\beta_t^{0j} + \beta_t^{sj} \sum_{\tau=1}^{\tau_j} s_{i\tau}^j + \beta_t^{\theta j} \theta_i \right) \right\}.$$

I let the parameters ζ_j and κ_j differ by track. They determine the tracks' overall attractiveness and complementarity with talent (as well as taste), respectively.

Following the same steps as above, we obtain

$$s_i^j = s_0^j + \tau_j \kappa_j (\theta_i + \gamma_i), \qquad s_0^j \equiv \tau_j \log(B_s^j / \zeta_j) \tag{19}$$

and

$$V_i^j = \nu_0^j + \nu_\theta^j \theta_i + \nu_\gamma^j \gamma_i, \qquad \begin{pmatrix} \nu_0^j \\ \nu_\theta^j \\ \nu_\gamma^j \end{pmatrix} \equiv \begin{pmatrix} B_0^j + \left(s_0^j - \tau_j\right) B_s^j \\ B_\theta^j + \tau_j \kappa_j B_s^j \\ \tau_j \kappa_j B_s^j \end{pmatrix}. \tag{20}$$

Breaking indifference in favor of attending college, we see that worker i attends college if and only if

$$\theta_i \geq x_0 + x_1 \gamma_i, \qquad \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \equiv \frac{1}{\nu_\theta^c - \nu_\theta^h} \begin{pmatrix} \nu_0^h - \nu_0^c \\ -\left(\nu_\gamma^c - \nu_\gamma^h\right) \end{pmatrix}. \tag{21}$$

College attracts individuals with greater ability and greater taste for acquiring skills ('positive selection') whenever $\nu_\theta^c > \nu_\theta^h$ and $\nu_\gamma^c > \nu_\gamma^h$. Expressing these conditions in terms of deep parameters is only possible in the social planner (or full information) solution, as shown below. In all numerical exercises, I verify that the conditions hold both for the social planner and the decentralized solutions.

The selection rule (21) implies that the joint distributions of acquired skills and talent conditional on track follow a truncated bi-variate normal distribution. This means that the normality assumption underlying employers' beliefs, and thus the log wage equation (17), is incorrect, though it may be a good approximation. Maintaining this assumption greatly facilitates connection of model and data. However, it does necessitate a specific definition of equilibrium.

Definition 1. *A decentralized equilibrium in the model presented in this section consists of employers' beliefs about individual workers given by (6) and (7), and further conditioned on observed educational track; wages specified by (17); a within-track skill acquisition given by (19); a selection rule for the college track given by (21); and employers' beliefs about the first and second moments of acquired skills and talent that are consistent with workers' choice of track and within-track skill acquisition.*

Thus, I do impose that employers' beliefs are correct with respect to first and second moments conditional on observing educational track. The normality assumption however means that employers do not make optimal use of additional information—skill acquisition and output signals—when updating their beliefs.[13]

It is difficult to further characterize the model analytically, or to investigate the uniqueness of an interior equilibrium where the fraction choosing college lies strictly between zero and one. However, the model can easily be solved numerically, as explained in Appendix D. The solution method consists of guessing and verifying the selection rule (21). It is thus feasible to perform a search over a two-dimensional grid to verify that there is only one interior equilibrium. I have checked that this is the case for all estimated parameter configurations that I will present in Section 4.

3.2 The social planner problem in the two-track model

I next solve the social planner problem, equivalent to a perfect information economy. This is useful not only for an eventual welfare analysis but also to derive parameter restrictions making it more likely that there is positive selection into college, as is empirically plausible.

I again proceed by backward induction. For each worker i, I first determine optimal

[13] Note that due to unbounded support of the type distribution, and under the selection rule (21), the distributions of types and acquired skills conditional on track also feature unbounded support.

skill acquisition within each track. Let $j \in \{h, c\}$. The value functions are

$$V_i^{j,\text{SP}} = \max_{\{s_{i\tau}\}_{\tau=1}^{\tau_j}} \left\{ -\sum_{\tau=1}^{\tau_j} \zeta_j \exp\{s_{i\tau}^j - \kappa_j(\theta_i + \gamma_i)\} + \delta^{\tau_j} \sum_{t=0}^{\infty} \delta^t \left(a_s \sum_{\tau=1}^{\tau_s} s_{i\tau} + a_\theta \theta_i \right) \right\}.$$

The first-order conditions lead to

$$s_i^{j,\text{SP}} = s_0^{j,\text{SP}} + \tau_j \kappa_j \left(\theta_i + \gamma_i\right), \qquad s_0^{j,\text{SP}} \equiv \tau_j \log(A_s^j / \zeta_j), \tag{22}$$

and

$$V_i^{j,\text{SP}} = \nu_0^{j,\text{SP}} + \nu_\theta^{j,\text{SP}} \theta_i + \nu_\gamma^{j,\text{SP}} \gamma_i, \qquad \begin{pmatrix} \nu_0^{j,\text{SP}} \\ \nu_\theta^{j,\text{SP}} \\ \nu_\gamma^{j,\text{SP}} \end{pmatrix} \equiv \begin{pmatrix} \left(s_0^{j,\text{SP}} - \tau_j\right) A_s^j \\ A_\theta^j + \tau_j \kappa_j A_s^j \\ \tau_j \kappa_j A_s^j \end{pmatrix}.$$

The social planner allocates individual i to the college track if and only if

$$\left(\nu_\theta^{c,\text{SP}} - \nu_\theta^{h,\text{SP}}\right) \theta_i \geq \nu_0^{h,\text{SP}} - \nu_0^{c,\text{SP}} - \left(\nu_\gamma^{c,\text{SP}} - \nu_\gamma^{h,\text{SP}}\right) \gamma_i. \tag{23}$$

Whether it is optimal to allocate more-talented individuals, as well as those with a greater taste for acquiring skills, to college, depends on parameters. In particular,

$$\nu_\gamma^{c,\text{SP}} > \nu_\gamma^{h,\text{SP}} \Leftrightarrow \tau_c \kappa_c \delta^{\tau_c} > \tau_h \kappa_h \delta^{\tau_h}, \qquad \nu_\theta^{c,\text{SP}} > \nu_\theta^{h,\text{SP}} \Leftrightarrow \frac{a_s}{a_\theta} > \frac{\delta^{\tau_h} - \delta^{\tau_c}}{\tau_c \kappa_c \delta^{\tau_c} - \tau_h \kappa_h \delta^{\tau_h}}. \tag{24}$$

Intuitively, talent-and-taste complementarity in skill acquisition costs, as captured by κ_c, needs to be sufficiently strong to offset the delayed payoff period, and at the same time acquired skills need to be sufficiently important in production relative to pre-existing talent. I will impose these restrictions in all numerical exercises. While they do not guarantee positive selection also in the decentralized case, they nevertheless help to narrow down the parameter space.

Comparing the social planner (perfect information) solution to the decentralized equilibrium, two differences are apparent. First, the fraction of college attendants, as well as their types, may differ, since the selection rules (21) and (23) generally do not coincide. Second, within-track skill acquisition will differ, as is apparent from comparing (19) and (22). For a given individual who chooses the same track under both scenarios, skill acquisition can differ only due to divergence in the intercept of the skill acquisition function, and Proposition 1 applies with regard to efficiency properties. But aggregate skill acquisition for each track can also differ due to selection, via the part of the function that depends on talent and tastes.

A key question is to what extent existing evidence from the employer learning literature can shed light on the way in which college attendance and within-track skill acquisition deviate from the efficient benchmark. I discuss this next.

3.3 Connection with the econometric employer learning model

The ideal data set for estimating the model would contain information on college graduation, denoted by $D_i \in \{0,1\}$, log wages at different experience levels, as well as skill acquisition s_i and talent θ_i. Running the regression

$$w_{it} = \phi_t^{0j} + \phi_t^{sj} s_i + \phi_t^{\theta j}\theta_i + \eta_{it}$$

separately by track and experience would allow for direct estimation of the log wage equation (17). At high levels of experience this would allow for identification of the output elasticities, as in the probability limit and letting experience become arbitrarily large, $\phi_\infty^{sj} = \beta_\infty^{sj} = a_s$ and $\phi_\infty^{\theta j} = \beta_\infty^{\theta j} = a_\theta$.

However, data on s_i and θ_i are typically not available. Instead, we may observe a proxy of productive ability z_i, such as the AFQT score. This allows us to run the regressions

$$w_{it} = \varphi_t^0 + \varphi_t^D D_i + \varphi_t^z z_i + \eta_{it}, \tag{25}$$

which are very similar to existing literature, except that the college dummy replaces years of schooling. The coefficients from these regressions can be expressed as

$$\varphi_t^z = \frac{(1-p)\sigma_{zw|h,t} + p\sigma_{zw|c,t}}{(1-p)\sigma_{z|h}^2 + p\sigma_{z|c}^2}, \qquad \varphi_t^D = \mu_{w|c,t} - \mu_{w|h,t} - \left(\mu_{z|c} - \mu_{z|h}\right)\varphi_t^z, \tag{26}$$

where $p = \mathrm{E}[D_i]$ denotes the fraction of workers who graduated college.[14] It will turn out to be useful to express the coefficient on z from regression (25) as a weighted sum of bivariate regression coefficients from regressing log wages on z separately by college status,

$$\varphi_t^z = \omega\varphi_t^{zh} + (1-\omega)\varphi_t^{zc}, \qquad \omega \equiv \frac{(1-p)\sigma_{z|h}^2}{(1-p)\sigma_{z|h}^2 + p\sigma_{z|c}^2}.$$

To characterize the coefficients φ_t^{zh} and φ_t^{zc}, an assumption about the data-generating process behind z_i is needed. I assume

$$z_i = \pi_s s_i + \pi_\theta\theta_i + \xi_i, \qquad \pi_s \geq 0, \pi_\theta \geq 0, \pi_s + \pi_\theta > 0. \tag{27}$$

where ξ_i is independently mean-zero normally distributed with variance σ_ξ^2.

I further assume that z_i is only observed by the econometrician, not employers. The

[14] The results stated in this section are proved in Appendix B.

signal $\tilde{s}_s$ is observed by employers but not the econometrician, and actual skill acquisition s_i and types (θ_i, γ_i) are observed by neither. This gives a setting identical to that of Altonji and Pierret (2001) and Lange (2007).

Given these assumptions, as well as the log wage equation (17), the coefficients φ_t^{zh} and φ_t^{zc} are characterized by the following result.

Proposition 4. *Running a regression of log wages on z_i conditional on track $j \in \{h, c\}$ and labor market experience t yields coefficients, in the probability limit,*

$$\varphi_t^{zj} = \frac{\sigma_{zw|j,t}}{\sigma_{z|j,t}^2} = \chi_s^j \beta_t^{sj} + \chi_\theta^j \beta_t^{\theta j} = \left(1 - \lambda_t^j\right) \chi_s^j b_0^{y\tilde{s}|j} + \lambda_t^j \left(\chi_s^j a_s + \chi_\theta^j a_\theta\right), \tag{28}$$

where $\begin{pmatrix} \chi_s^j \\ \chi_\theta^j \end{pmatrix} \equiv \frac{1}{\sigma_{z|j}^2} \begin{pmatrix} \pi_s \sigma_{s|j}^2 + \pi_\theta \sigma_{s\theta|j} \\ \pi_s \sigma_{s\theta|j} + \pi_\theta \sigma_{\theta|j}^2 \end{pmatrix}$. *If the conditional covariances between log productivity and z_i are positive, implying $\varphi_\infty^{zj} = \chi_s^j a_s + \chi_\theta^j a_\theta > 0$, and if $\lambda_1^j \in (0, 1)$, then the coefficients are strictly increasing in labor market experience, $\partial \varphi_t^{zj} / \partial t > 0$.*

Thus, the empirical test for the direction of inefficiency suggested by Proposition (1) does not apply to regressions of log wages on the productivity correlate z_i. Due to omitted variables bias, φ_t^{zj} always increases with experience, regardless of whether β_t^{sj} increases or decreases with experience. Notice also that the conditions for employer learning to take place are the same as in the simpler model in Section 2, namely that $\lambda_1^j > 0$, which will be the case if $\rho_{s\theta|j}^2 < 1$ or $\sigma_{uj}^2 > 0$ or both. As discussed above, the two conditions have different implications for efficiency. Again, observing employer learning in itself says nothing about the direction of inefficiency in skill acquisition.

Nonetheless, the coefficients φ_t^{zh} can help recover important moments of the model. As inLange (2007), I can treat the estimated sequences $\left\{\varphi_t^{zj}\right\}_{t=0}^T$ as data points, and estimate (28) by non-linear least squares. This recovers the initial value $\chi_s^j b_0^{y\tilde{s}|j}$, the terminal value $\chi_s^j a_s + \chi_\theta^j a_\theta$, and the speed of learning λ_1^j. Together with further (strong) assumptions, all model parameters can be recovered from the data, as I show in Section 4.

The positive relationship between φ_t^{zj} and experience also means that the coefficient on z_i in the pooled regression (25) is increasing in experience. If the college premium is stable, then (26) implies that the coefficient on the college dummy must decrease with experience, as in Altonji and Pierret (2001).[15]

[15] Farber and Gibbons (1996) emphasize that the college premium in *levels*, in their specification, must be constant with experience if employers' unconditional beliefs are consistent. In the model here, the college premium in logs can be expressed, using (17) as

$$\mu_{w|c,t} - \mu_{w|h,t} = \mu_{y|c,t} - \mu_{y|h,t} + \frac{1 - \lambda_t^c}{2} \sigma_{y|\tilde{s},0,c}^2 - \frac{1 - \lambda_t^h}{2} \sigma_{y|\tilde{s},0,h}^2.$$

Due to the variance terms, the college premium in logs is generally not constant in experience.

3.4 Imperfect substitutability of high school and college workers

To conclude the exposition of the two-track model, I discuss how aggregate output is produced. So far, I have implicitly assumed that aggregate output is simply the sum of all workers' individual output,

$$Y = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\{y_i\} f(\theta, \gamma) d\theta d\gamma,$$

where $f(\theta, \gamma)$ is the joint density function of talent and taste (assumed to be bi-variate normal).

Alternatively, it may be the case that high school and college tracks teach skills that are specific to separate sectors of the economy, which are imperfect substitutes in final production,

$$Y = \left(\alpha^{\frac{1}{\sigma}} Y_h^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} Y_c^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

with sectoral production functions

$$\begin{aligned} Y_h &= \int_{-\infty}^{\infty} \int_{-\infty}^{x_0 + x_1 \gamma} \exp\{a_s s_i + a_\theta \theta_i\} f(\theta, \gamma) d\theta d\gamma, \\ Y_c &= \int_{-\infty}^{\infty} \int_{x_0 + x_1 \gamma}^{\infty} \exp\{a_s s_i + a_\theta \theta_i\} f(\theta, \gamma) d\theta d\gamma, \end{aligned}$$

where (x_0, x_1) denote intercept and slope of the selection rule (21).

Perfectly competitive final good producers buy sectoral outputs at prices Ψ_h and Ψ_c, while sectoral production takes place under the assumptions on market structure outlined above. The wage now equals the expected revenue product,

$$W_{it}^j = \Psi_j \mathrm{E}\left[\exp\{y_i\} | j, \tilde{s}_i, \tilde{\mathbf{y}}_{it}\right].$$

Sectoral prices are solved for in standard fashion, and log sectoral prices now enter the value functions. See Appendix C for details.

Imperfect substitutability is a common interpretation of a negative relationship, all else equal, between the relative supply and wages of college workers (see for instance Ciccone and Peri, 2005). In the present model, such a negative relationship could in principle arise even with perfect substitutes, due to compositional changes which affect expected productivities. Moreover, assuming imperfect substitutability immediately rules out a pure signaling effect of college. In the quantitative exercise of Section 4, I will therefore consider both the case $\sigma = 1.5$ (Ciccone and Peri, 2005) as well as $\sigma \to \infty$.

4 Estimating the model on NLSY data

In this section I use NLSY data to estimate the parameters of the two-track model of Section 3. Estimating this model is not an easy task given the well-known problems of empirically distinguishing between human capital and signaling models. Moreover, apart from the output elasticities of acquired skill and talent, I also need to estimate the mapping from model variables to the AFQT score, the type correlation $\rho_{\theta\gamma}$, and production function parameters in the case of imperfect substitutes. The data are not sufficiently rich to identify all parameters without imposing further assumptions. My approach will be to present a range of estimates, based on varying the assumptions. As it turns out, due to certain prominent features of the data, robust patterns do emerge.

4.1 Description of NLSY data

I obtain NLSY data from the replication files of Arcidiacono, Bayer, and Hizmo (2010), and apply sample selection criteria as Lange (2007), except restricting the data to individuals with 12 (high school) or 16 (college) years of schooling. These data contain $32,793$ observations during years 1979-1998 on $3,592$ individuals born 1957-1965. The key variables are the log wage and the AFQT score.[16]

4.2 Assumption on parameters

I fix several parameters at the outset. I set the discount factor $\delta = 0.95$ and the lengths of high school and college $(\tau_h, \tau_c) = (6, 10)$.[17] I normalize the cost parameters for the high school track such that $\zeta_h = 1$ and $\tau_h \kappa_h = 1$. I further normalize the type distributions to have zero mean and unit variance, $\left(\mu_\theta, \mu_\gamma, \sigma_\theta^2, \sigma_\gamma^2\right) = (0, 0, 1, 1)$. I vary the type correlation $\rho_{\theta\gamma}$ over a set of values, usually 0.2, 0.5, and 0.8.

I estimate the model both under the assumption that $\sigma \to \infty$ and assuming $\sigma = 1.5$ as in Ciccone and Peri (2005). The pure signaling view implies $\sigma \to \infty$, so it is informative to seek estimates consistent with this assumption, as well as for the more conventional scenario of imperfect substitutability. In the latter case, I also need to fix the share parameter α. I choose $\alpha = 0.5$ or $\alpha = 0.825$ depending on the value assumed for the average treatment effect of college on log productivity (see the discussion in the next subsection).[18] See panel A of Table 1 for the full list of pre-set parameters.

[16] For continuity with prior literature, I use the 1979 wave of the NLSY. Ablay and Lange (2022) present evidence for employer learning in the 1997 wave, as well.

[17] I thus exclude primary schooling, meaning that θ_i (and γ_i) can be interpreted as partially capturing the impact of earlier schooling (as well as parental inputs), rather than being purely innate.

[18] In both cases there is a range of values that fit the data, but the results do not vary much. Estimating this parameter is challenging because it essentially requires measuring the price of a unit of skill (Bowlus and Robinson, 2012).

The 12 parameters left to be estimated are listed in panel B of Table 1. I allow the noise parameters to differ by track. This is motivated by evidence that the speed of employer learning differs between high school and college graduates (Arcidiacono, Bayer, and Hizmo, 2010).

4.3 Targeted moments

I propose to estimate the parameters by targeting 13 moments following a minimum-distance approach. The moments are listed in panel C of Table 1. They can be computed from the data as described shortly, with the exception of $\Delta\psi + a_s\Delta s_0$. This is the average treatment effect of attending college on log (revenue) productivity, which I will also denote by ATE_y. Since estimating an average treatment effect is highly demanding, I instead fix the value of ATE_y. This is practical, given the important role this moment plays in the model equations. But note also that the value of ATE_y speaks directly to the human capital versus signaling debate. $\text{ATE}_y = 0$ is an implication of a pure signaling view of education, $\Delta\psi = 0$ (perfect substitutes in aggregate production) and $a_s = 0$ (no productive effect of skills acquired in school). But $\text{ATE}_y = 0$ may also arise from $\Delta\psi = 0$, $\Delta s_0 = 0$ (the average worker obtains the same amount of skills in both high school and college), and $a_s > 0$, consistent with human capital models. In estimating the model parameters imposing that $\text{ATE}_y = 0$, I thus do not rule out a pure signaling view at the outset. A second set of estimations however imposes $\text{ATE}_y = 0.16$, so that a year of college raises productivity by about four percent on average. This is somewhat lower than the 6.7 percent estimated by Carneiro, Heckman, and Vytlacil (2011).[19]

The fraction college is directly observed in the data. As log productivity is not observed, I use wage data for workers with potential experience ranging from 13-17 years. For these workers, employer learning has sufficiently progressed so that the influence of the initial schooling signals (the skill signal $\tilde{s}_i$ and college completion D_i) has faded. I regress log wages on year dummies and worker fixed effects, and obtain first and second moments from the estimated distribution of fixed effects. This yields the moments $\Delta\mu_y$ and $\sigma^2_{y|h}/\sigma^2_{y|c}$. The reason I do not use the variance of log wages itself is that I want to eliminate the impact of transitory shocks, which are not part of my model.[20] The AFQT gap and conditional variances are directly observed. The covariances of AFQT and productivity are estimated as part of fitting the employer learning curves, which I turn to next.

[19] Carneiro, Heckman, and Vytlacil (2011) also include wage observations early in workers' careers, so that their estimate could include a signaling component. Here I am instead interested in the purely productive effect of schooling, so I assume a smaller effect.

[20] I target the ratio of variances rather than the levels. Matching the productivity gap and conditional variances at the same time is infeasible given the current specification. However, variance levels could be easily matched by letting ζ_j vary across individuals, as this would increase dispersion in skill acquisition without necessarily affecting the conditional means, see (19).

First, for continuity with previous literature, I estimate pooled regressions of log wages on a college dummy and AFQT at different levels of experience, as in (25). The results are shown in panel (a) of Figure 1, and they are very similar to the results of Lange (2007), who uses years of schooling instead of the college dummy.

Next, I estimate bi-variate regressions of log wages on AFQT separately for high school and college graduates, obtaining estimates for φ_t^{zj}. The results are shown in panel (b) of Figure 1. High school graduates face a zero return to AFQT initially, and fast-rising returns subsequently. College graduates see sizeable returns already at labor market entry, with somewhat slower subsequent growth. These patterns were first documented by Arcidiacono, Bayer, and Hizmo (2010).

As in Lange (2007), the estimated φ_t^{zj} can be treated as a sequence of data points. Using these, non-linear least squares estimation based on (28) identifies the initial values $\varphi_0^{zj} = \chi_s^j \beta_t^{sj}$, the terminal values $\varphi_\infty^{zj} = \chi_s^j a_s + \chi_\theta^j a_\theta$, and the speeds of learning λ_1^j. Panel C of Table 1 displays estimated initial values and learning speeds, confirming a lower initial return but faster learning for high school graduates compared to college graduates. I use the estimated terminal values to calculate the covariances of AFQT and productivity displayed in panel C of Table 1, using (28). Estimated learning speeds and initial returns are displayed in the final four rows. The fitted curves from the non-linear least squares estimation are shown in panel (b) of Figure 1.[21]

4.4 Estimation approach

To estimate the model, I pursue essentially an indirect inference approach in that I search for parameter values that minimize the distance between theoretical and empirical moments (Gourieroux, Monfort, and Renault, 1993). To speed up estimation, the minimization is conditional on an initial guess which guarantees that the fraction choosing college in the estimated model is equal to that in the data.[22]

I start by guessing the slope of the selection rule (21) and a value for κ_c. Given the type distribution and the observed fraction of college graduates, p, I calculate the implied intercept in the selection rule. Given the selection rule and the guess for κ_c, I calculate the conditional distributions of talent θ_i and taste γ_i, as well as the conditional covariance matrix of θ_i and acquired skills s_i using (19).

Next, I find values a_s and a_θ to target the moments $\Delta\mu_y$ and $\sigma^2_{y|h}/\sigma^2_{y|c}$ from the data,

[21] I also fitted non-linear least squares models to the pooled regression estimates, shown in panel (a) of Figure 1. These estimates can in principle be used as an alternative source for the log productivity gap, which equals the terminal value φ_∞^D as seen in (26). However, this terminal value is estimated highly imprecisely.

[22] The estimation procedure estimates and numerically solves the model simultaneously, which speeds up estimation. Minimizing a single objective function would involve solving the model numerically for each guess of the parameter vector, which can be slow, especially for configurations that imply a very low or very high fraction choosing college. See Appendix E for more details on the estimation procedure.

using (1). Here I make use of the assumed value for ATE_y. Having estimates for a_s and a_θ in hand, I estimate the parameters of the AFQT score equation (27). This together with learning speeds and initial returns lets me recover the noise parameters for skill and output signals, using (18), (8), and (9). I can now compute B_s^j, and thus ζ_c using (19), given that I know Δs_0 from the assumption about ATE_y and the estimated a_s. Finally, I compute the selection rule implied by the estimated type distributions and parameters.

I repeat the procedure over a grid of guesses for the slope of the selection rule and κ_c, searching for fixed points where the implied selection rule equals the one guessed. In the cases I consider, there is either none or just one such point. Even when guessed and implied selection rules agree, not all moments are perfectly matched. This is because I restrict all parameters to be non-negative, so that corner solutions may occur, and because the estimation of the parameters generating the AFQT score is over-identified. For the latter component I report the value of the loss function.

4.5 Estimation results

Table 2 shows how well the moments are matched. Here and in subsequent tables, empty rows indicate that for the given set of assumptions the estimation algorithm did not find a fixed point. Table 2 shows that the log productivity gap, the learning speeds, and initial returns are always matched perfectly (conditional on a fixed point having been found). The variance ratio of log productivity is perfectly matched except in one case. The remaining moments involving the AFQT score are less well matched. As Table 3 shows, problems with fitting the moments are mostly due to corner solutions, where the output elasticity of talent or the effect of acquired skills on the AFQT score are estimated to be zero.[23]

Table 3 displays the estimated parameters, and reveals several robust patterns. First, the output elasticity of acquired skill is always positive, and in most cases well above the output elasticity of talent. This contradicts a pure signaling view of schooling, even when the average treatment effect of college is assumed to be zero. Second, the AFQT score is estimated to reflect mostly talent, not acquired skills (Carlsson, Dahl, Öckert, and Rooth, 2015, document a positive but small effect of schooling duration on cognitive skills measured at Swedish military enlistment).

Third, the skill signal sent by high school graduates is estimated to be completely uninformative in all cases. In fact, this is a direct consequence of the estimated zero initial return φ_0^{zh}. Equations (8) and (28) reveal that the only way this can occur is if $\sigma_{uh}^2 \to \infty$.[24] Intuitively, if employers had any information about a worker's acquired skills,

[23]Inspecting the values of the loss function from matching the AFQT-related moments, it is apparent that for each combination of assumptions on ATE_y and σ there is a value for the type correlation that fits the data reasonably well, but this value is not very stable.

[24]Strictly speaking, $\varphi_0^{zj} = 0$ also holds in the double-knife-edge cases of $\rho_{s\theta|j} = 0$ *and* at least one of

then a positive correlation between log wages and the AFQT score would arise regardless of whether the AFQT elicits talent or acquired skills—because talent and acquired skills are positively correlated by (19).

Fourth and finally, the output signal is usually, but not always, slightly noisier for college graduates. This means that the slower speed of learning for college graduates is due to both a more informative initial signal and noisier subsequent signals, though the difference related to the initial signal is of course more striking.

4.6 Perfect-information counterfactual

The implications of signal imperfection for the private returns to acquired skills are shown, for a selection of cases, in Figure 2. Not surprisingly given $\sigma^2_{uh} \to \infty$, among high school graduates the private returns always approach the social return from below, suggesting under-investment (Proposition 1). The same is typically true for college graduates, but the differences are less pronounced. In one case, private and social returns essentially coincide at all experience levels, suggesting efficient skill acquisition for college graduates.

Understanding the broader implications of informational frictions requires simulating the counterfactual scenario of perfect information. With the estimated parameters in hand, I simulate the perfect-information (or social planner) model presented in Section 3.2. Table 4 displays the results from this exercise. In particular, for each set of assumptions, the table displays the difference in selected moments between the estimated model and the associated perfect-information counterfactual.

First, consider the fraction attending college p, which is 0.26 in the data. Table 4 shows that when high school and college workers are perfect substitutes, almost all college attendance can be attributed to information frictions. Many workers use the unambiguous if coarse signal of college graduation, combined with relatively precise signaling about acquired skills, to compensate for the impossibility of conveying to the market what they learn in high school. In the perfect-information counterfactual, only one percent would choose the college track. However, when the aggregate production function features imperfect subsitutability, the difference to the counterfactual is much less pronounced. Figure 3 plots the selection rules for selected cases. The perfect-information selection rule typically has a similar slope as the one consistent with data, while the intercept can differ substantially.

Second, consider skill acquisition. Given (19), differences between imperfect- and perfect-information scenarios may arise both due to the intercept s_0^j—which contains the present discounted value of returns B_s^j—as well as the term $\tau_j \kappa_j (\mu_{\theta|j} + \mu_{\gamma|j})$, which reflects selection. Table 4 shows the differences in these terms separately for the two tracks, but

$a_s = 0$, $\pi_s = 0$. These cases are however of no theoretical or practical relevance.

multiplied by a_s. This allows an interpretation in terms of log wages. For high school, skill acquisition falls substantially short of the perfect-information benchmark due to the intercept effect alone, with the gap ranging from 10-22 log points (compare this to the observed log productivity gap between college and high school of 43 log points). This is somewhat exacerbated by selection effects. For college, the intercept effect can be even larger, but is sensitive to assumptions about ATE_y. Selection effects are sensitive to assumptions about the substitution elasticity.

Third, note that the effects of under-investment and inefficient selection can be summarized by their effect on aggregate output. The output loss ranges from 4-25 log points, or 4-22 percent. Output losses tend to be larger when $\text{ATE}_y = 0$, as this assumption leads to higher estimated output elasticities of acquired skills. Fourth, welfare losses—expressed in terms of the difference in per-period log consumption—are modest in magnitude (at most three percent). This is due to the offsetting effect of the disutility of skill acquisition.

A broad conclusion from these results is that due to information frictions, individuals spend too much time on schooling, but learn too little (in the sense of acquiring skills that are productive in the labor market). Characterizing the policy mix that would implement the first-best outcome is beyond the scope of this paper. Simple intuition based on a traditional signaling view can be misleading: For instance, reducing college attendance by making college less affordable would likely depress human capital accumulation even more, since skill acquisition signals appear to be much noisier for high school graduates. Making signals more precise, say via better testing procedures, will induce greater skill acquisition, but perfectly precise signals will lead to over-investment.

4.7 The effects of college on log wages

To conclude the discussion of estimation results based on NLSY data, I explore how the effects of college on log wages vary with experience. Aryal, Bhuller, and Lange (2022) estimate the experience-varying earnings effects of an additional year of schooling in Norway, using a compulsory schooling reform for exogenous variation. For the sub-sample in which employers arguably do not observe the instrument, they document a pattern of declining private returns as workers gain labor market experience. Here I demonstrate that such a pattern is qualitatively consistent with my estimates based on NLSY data, despite the fact that returns to acquired skills increase with experience.

I define the treatment effect of college on log wages for individual i as

$$\text{TE}_{w,i} = \log\left(\Psi_c/\Psi_h\right) + \beta_t^{0c} - \beta_t^{0h} + \beta_t^{sc} s_i^c - \beta_t^{sh} s_i^h + \left(\beta_t^{\theta c} - \beta_t^{\theta h}\right)\theta_i. \tag{29}$$

This expression takes into account that a given individual would generally acquire different amounts of skill depending on the track she attends.

For each set of estimation results, I compute the average treatment effect (ATE), the average effect on the treated (TET), the average effect on the non-treated (TEN), and a local average treatment effect (LATE). While instrumental variables identify the LATE for the sub-population of compliers (Imbens and Angrist, 1994), this group may differ depending on the context. Here, I consider the individuals that are indifferent between the two tracks (individuals on the selection line), but this is of course just one example.

Figure 4 displays the results for selected sets of assumptions. In all cases there is a clear ranking, TET > ATE > TEN, typical of Roy-type models where individuals select on returns. The LATE considered here exceeds the ATE in all cases. All treatment effects are declining in experience. Due to employer learning, they converge to the social return to college, the effect of college on log productivity. For instance, the ATE for log wages converges to ATE_y.

In sum, a decomposition of the earnings effects of college into a positive signaling component and the social return, as in Aryal, Bhuller, and Lange (2022), is consistent with a model in which students under-invest in productive skills.

5 Evidence on signal imperfection from Swedish high school graduates

In this section, I use Swedish administrative data to get closer to estimating a wage equation like (17). The goal is to estimate the private returns to acquired skills as a function of experience, and to use Proposition 1 to learn about the nature of inefficiency in study choices. The approach I take here is based on a measure of value added during high school, namely the high school GPA conditional on the compulsory school GPA.

I use data on Swedish high school (HS) graduation cohorts 1993-2007, who I observe in the labor market 1993-2017. The data include high school GPA, compulsory school (CS) GPA, standard demographics, as well as parental education and parental region of birth. For a large sub-sample, I also observe father's cognitive and non-cognitive skills as measured at military enlistment (Lindqvist and Vestman, 2011). The data further include employment status and annual labor earnings. For the universe of public sector workers and a large sample of private sector workers, the data also contain wage rates. This is the outcome variable of interest. I use sampling weights in all regressions, though the results are robust to dropping the weights.

Since the data do not contain information on performance at university, I restrict the sample to individuals who enter the labor market directly after graduating high school, and who never enrolled in college. I require that individuals are employed during each of the first three years after high school and do not receive study grants from the government.[25]

[25] I do not require the panel to be balanced beyond the first three years after graduation. After this

I take HSGPA as a measure of recent skill acquisition when conditioning on CSGPA and parental background. This value-added measure should capture performance gains in the last three years of secondary education. CSGPA and parental background jointly control for prior skill acquisition as well as talent. The strategy is inspired by the wage equation (17). However, I cannot literally estimate that equation. While in the theoretical model there are skill acquisition choices at various instances in time—during each year in school—these are perfectly collinear. In reality, HSGPA is of course not perfectly correlated with CSGPA, which is precisely the feature of the data that I take advantage of. The model would produce this feature if tastes γ_i were allowed to vary over time, or if the study cost parameter ζ was allowed to vary across individuals and over time.

High school graduates in Sweden typically do not report the HSGPA on their CV (Adermon and Hensvik, 2022), though this information may be requested during the interview process. To what extent the HSGPA is observed by employers and reflected in wages is an empirical question that I aim to answer by running the regression

$$w_{ity} = \beta_t \text{HSGPA}_i + \varpi_t \mathbf{x}_{iy} + u_{ity} \tag{30}$$

separately by potential experience t. The vector of controls $\mathbf{x}_{iy}$ includes CSGPA, a female indicator, as well as sets of fixed effects for year (y), region of birth, parents' education and parents' region of birth.

The results are shown in Figure 5. The private returns to HSGPA are initially very close to zero but rise to about 2 percent per one standard deviation within four years of labor market entry. This suggests that employers do not observe productive skills obtained by high school graduates, but learn about them quickly. The finding is similar to the returns estimated based on NLSY data shown in Figure 2. In light of Propositions 1 and 3, these results suggest that Swedish high school students, too, acquire inefficiently little skills.

I perform several robustness checks, which are reported in Figure A1. The results are robust when restricting the sample to women, men, or immigrants.[26] A partial exception is that for women, the returns stay around zero for three years, and then converge to only about 1 percent. The results are also robust to controlling for fathers' cognitive and non-cognitive skills, and to controlling for high school fixed effects.[27]

cutoff, individuals are absent from the sample when they are not employed in a given year, but may re-join later.

[26]Here, immigrants are individuals born outside Sweden or Swedish-born but with both parents born outside Sweden.

[27]The latter robustness check is motivated by the possibility that grading policies vary across schools, and that schools are typically named on CVs.

6 Discussion

Motivated by evidence of employer learning, in this paper I incorporate imperfect signals into the analysis of educational choices. I highlight that signal imperfection may have various sources, such as multi-dimensional types or noise. Their comparative relevance has sharp implications for the direction of inefficiency in schooling choices. When signal imperfection results mainly from noise, students will under-invest in productive skills, as they face a private return that is below the social return. I present structural estimates using NLSY data, as well as evidence from Swedish high school graduates based on a more direct measure of skills acquired, both suggesting that noisy signals and under-investment are important in reality.

My analysis highlights the distinction between time spent in school and the amount of skills acquired. Indeed, I find that college attendance tends to be inefficiently high, as the college degree is used to compensate for other, noisier skill acquisition signals. In the absence of the distinction between schooling length and skills acquired, one might conclude that restricting access to college would increase welfare (Caplan, 2018). However, in my model such a policy would further depress human capital accumulation, which is inefficiently low to begin with. Instead, the focus should be on improving signal precision, especially for high school graduates.

Understanding the causes of signal imperfection is an important area for future research. Institutional features, or country-specific conventions, such as whether high school graduates typically report the GPA on their CV, could potentially make a difference. Another open question is why college graduates appear to be able to signal with greater precision. Promising explanations include the salience of college GPA, field of study, or the institution attended (Arcidiacono, Bayer, and Hizmo, 2010). However, one must keep in mind that perfectly precise signals are not desirable, either, as they will induce over-investment.

My analysis informs broader policy discussions related to study effort and skill acquisition. A large literature investigates how student effort responds to study incentives or information about educational wage premia (Leuven, Oosterbeek, and van der Klaauw, 2010; Fryer, 2011, 2016). Results are mixed, and are often attributed to behavioral aspects that are absent from my analysis (Gneezy, Meier, and Rey-Biel, 2011; Lavecchia, Liu, and Oreopoulos, 2016). A related literature highlights that students often spend remarkably little time studying, which could be interpreted as time-inconsistent behavior (Babcock and Marks, 2011; Oreopoulos, Patterson, Petronijevic, and Pope, 2022). I emphasize that the returns to study effort may actually be low, while returns to an additional year of schooling, or college graduation, can be large and typically do include a signaling component. If so, putting in the minimum effort required to graduate may be consistent with rational, forward-looking behavior. Relatedly, my results provide additional justification for some institutional features of education systems that are consistent with signaling theory but not a pure human capital view, such as exams.[28]

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[28]Under noisy signals, exams can induce greater skill acquisition and thus increase efficiency. This may include exams whose purpose is to sort students into the next stage of education, such as high school or university. Tilley (2021) finds that Swedish compulsory school students increased their study effort when a policy reform made the final GPA more important in high school admissions.

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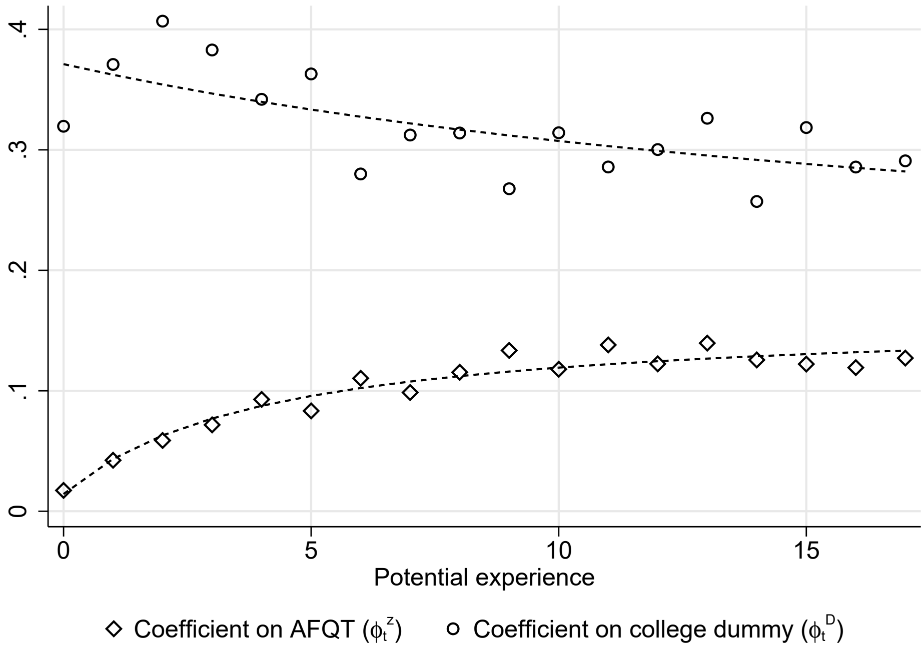
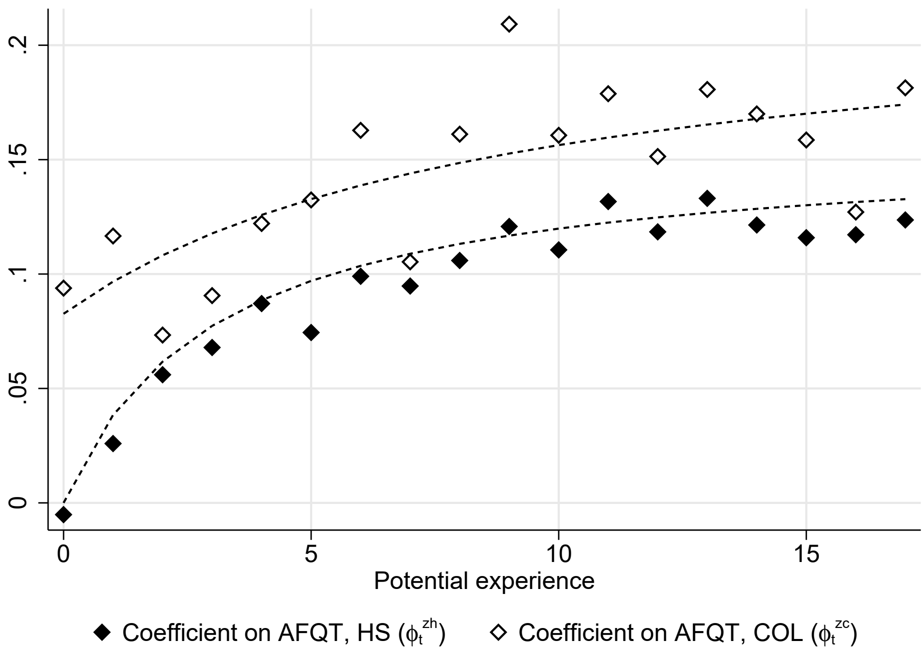


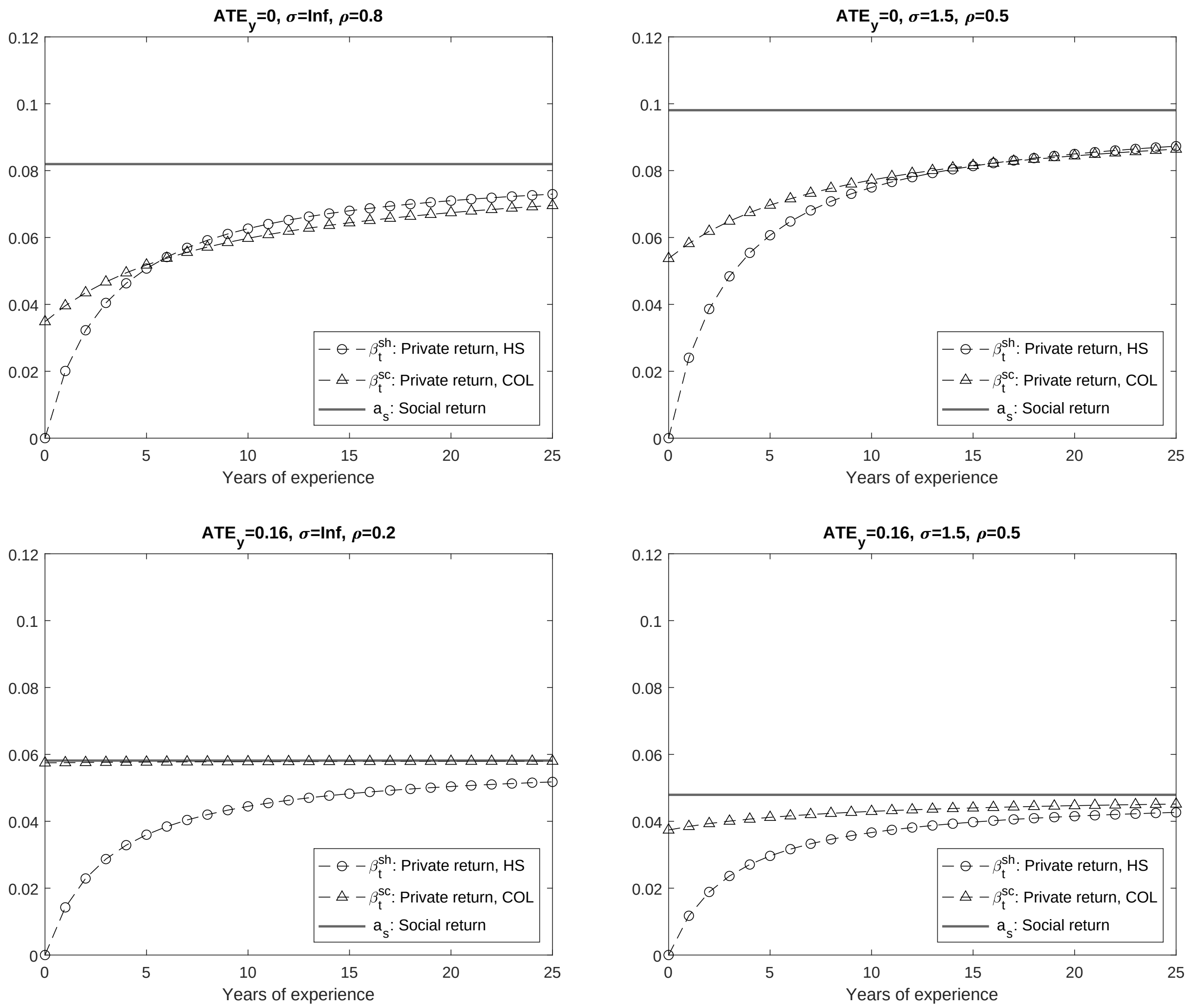
(a) Pooled regression



(b) Separate regressions

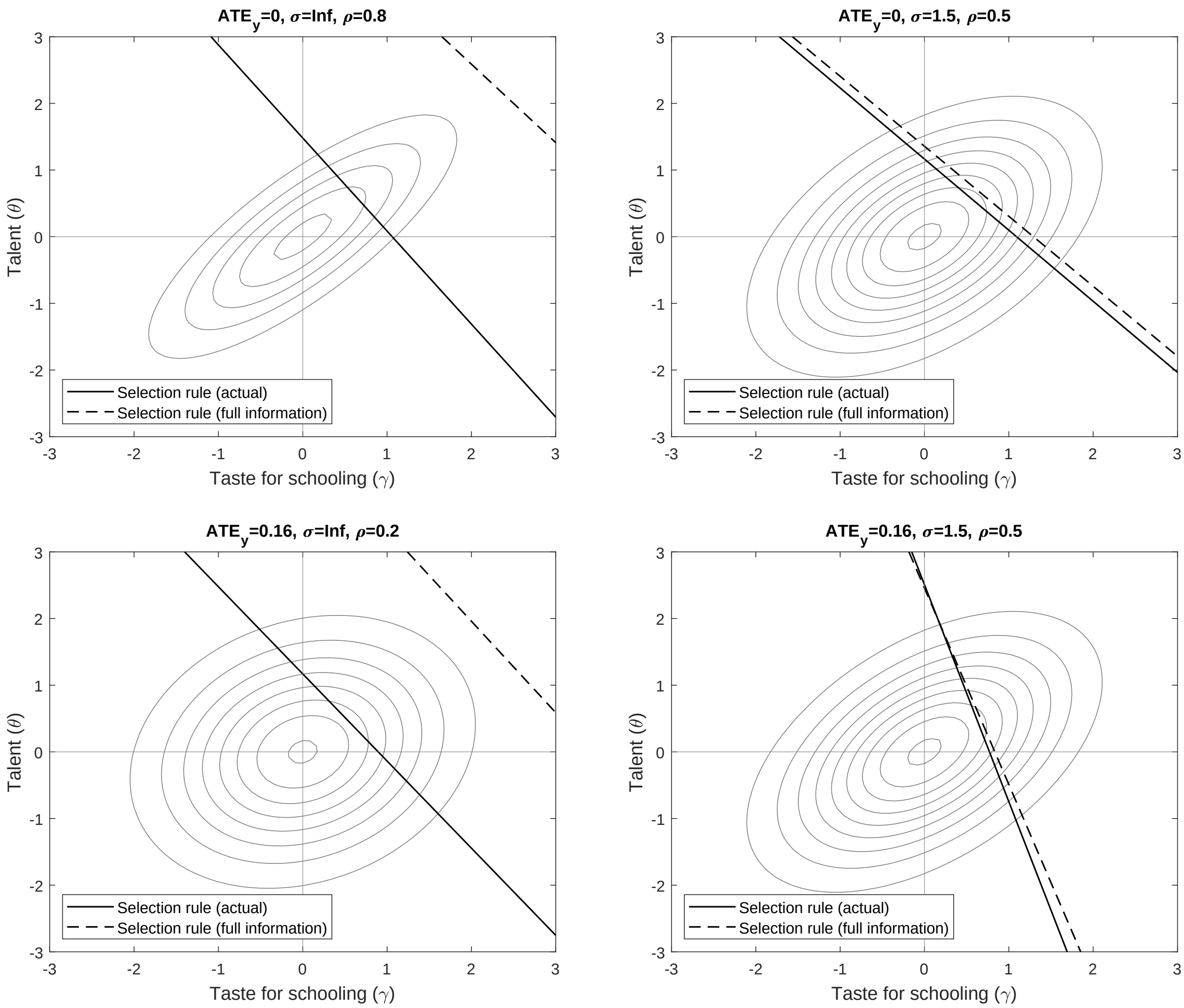
Notes: Panel (a) plots coefficients from estimating (25). Panel (b) plots estimates of (28). Dashed lines mark fitted values from non-linear least squares estimation of the underlying learning parameters.

Figure 1: Results from employer learning models estimated on NLSY data



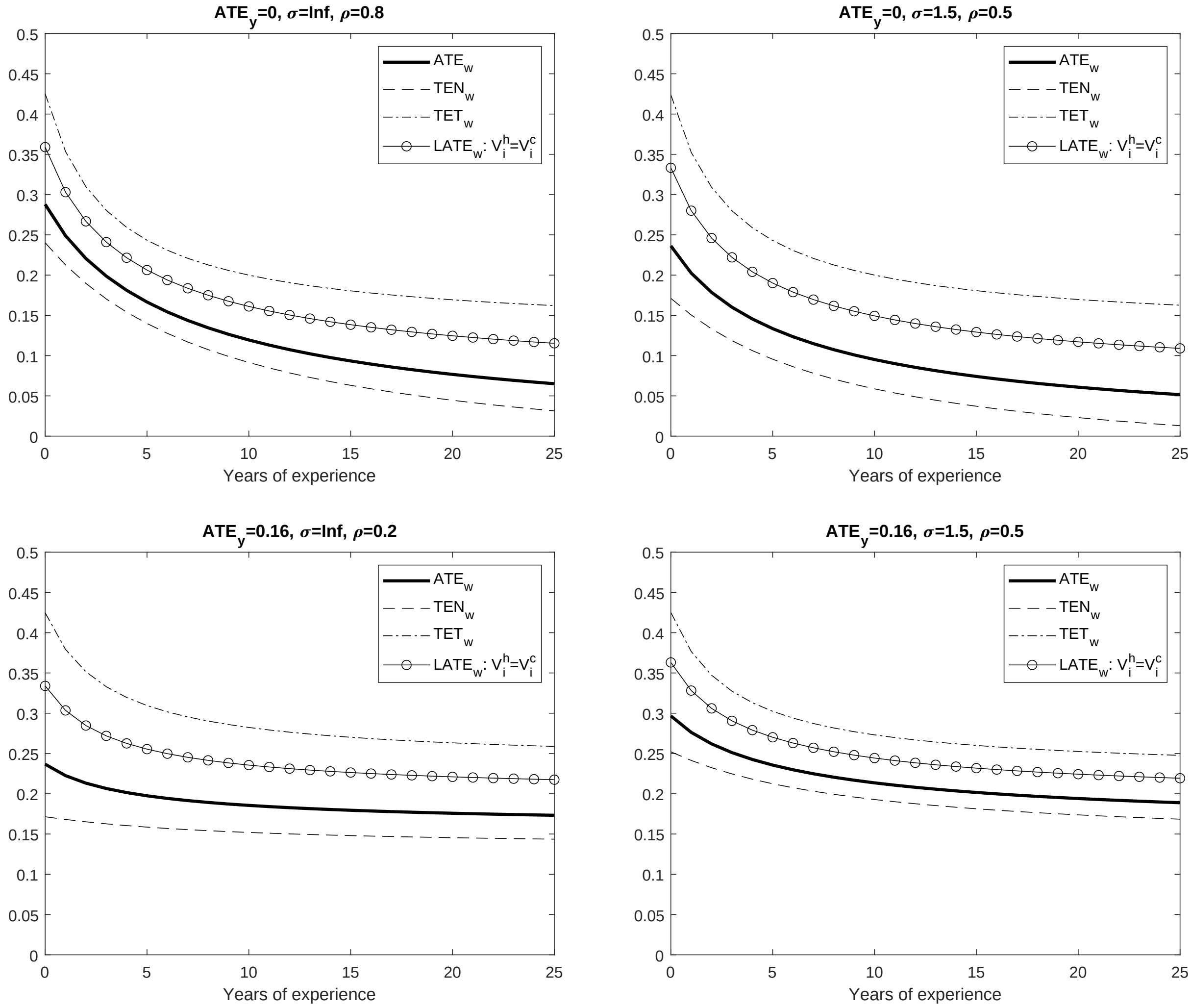
Notes: Private returns to acquired skills implied by the estimated parameters and the wage equation (17) are plotted, along with the estimated social return (the output elasticity of acquired skill), separately for high school and college graduates. Panel titles indicate assumptions about the average treatment effect of college on log productivity, the substitution elasticity, and the type correlation.

Figure 2: The private and social returns to acquired human capital



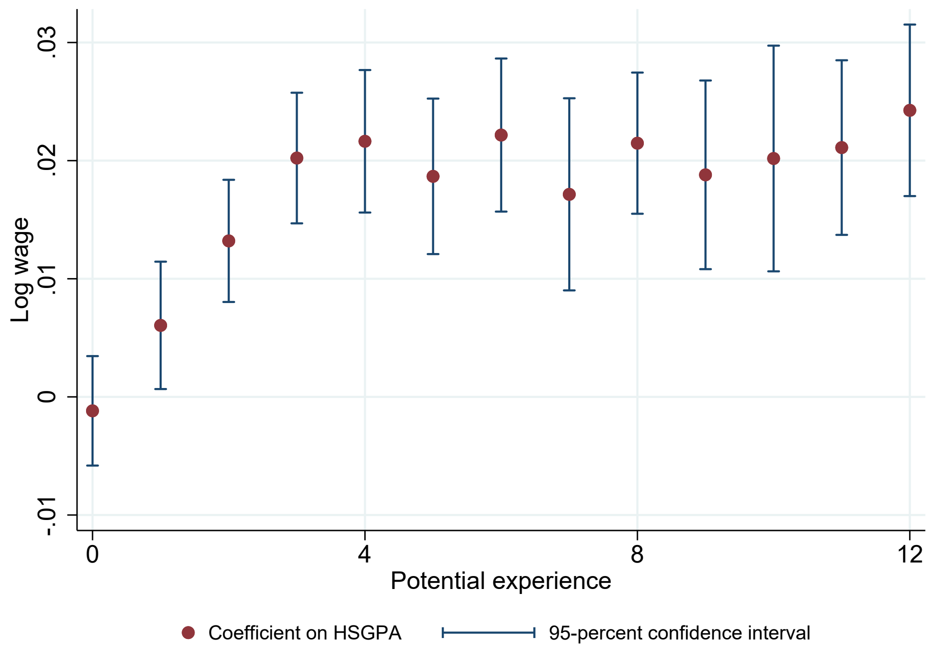
Notes: Solid lines represent the selection rule (21) implied by the estimated parameters. Dashed lines mark the selection rule (23) for the corresponding perfect-information counterfactual. Grey ellipses are contour lines of the joint PDF of talent and taste. Panel titles indicate assumptions about the average treatment effect of college on log productivity, the substitution elasticity, and the type correlation.

Figure 3: Selection rules for college attendance



Notes: Various conditional averages of the treatment effect of college on log wages (29) are plotted. See the text for details. Panel titles indicate assumptions about the average treatment effect of college on log productivity, the substitution elasticity, and the type correlation.

Figure 4: The effects of attending college on wages



Notes: Results are shown from regressions of log wages on the standardized high school GPA. The sample includes Swedish high school graduation cohorts 1993-2007 observed in the labor market 1993-2017. See Section 5 for more details. Controls include compulsory school GPA and sets of fixed effects for year, region of birth, and parental education and origin.

Figure 5: The returns to standardized high school GPA

Table 1: List of parameters and moments

Notation	Description	Value
	A. Parameters set pre-estimation	
δ	Discount factor	0.95
τ_h	Years spent in high school track	6
τ_c	Years spent in college track	10
κ_h	Complementarity of skill production, high school	$1/\tau_h$
ζ_h	Study cost shifter, high school	1.00
μ_θ	Mean of talent	0.00
μ_γ	Mean of taste	0.00
σ_θ^2	Variance of talent	1.00
σ_γ^2	Variance of taste	1.00
$\rho_{\theta\gamma}$	Correlation of talent and taste	$\{0.2, 0.25, 0.5, 0.8\}$
σ	Substitution elasticity	$\{1.5, \infty\}$
α	Share parameter in aggregate production function	$\{0.5, 0.825\}$
	B. Parameters to be estimated	
κ_c	Complementarity of skill production, college	
ζ_c	Study cost shifter, college	
a_s	Output elasticity of acquired skill	
a_θ	Output elasticity of talent	
π_s	Effect of acquired skills on AFQT score	
π_θ	Effect of talent on AFQT score	
$\sigma_{\xi h}^2, \sigma_{\xi c}^2$	Noise in AFQT score	
$\sigma_{uh}^2, \sigma_{uc}^2$	Noise in schooling signal	
$\sigma_{\varepsilon h}^2, \sigma_{\varepsilon c}^2$	Noise in output signal	
	C. Moments to be targeted	
p	Fraction college	0.26
$\Delta\psi + a_s\Delta s_0$	Average treatment effect of college on log productivity	$\{0.00, 0.16\}$
$\Delta\mu_y$	Log productivity gap	0.43
$\sigma_{y\|h}^2/\sigma_{y\|c}^2$	Ratio of log productivity variances	0.85
$\Delta\mu_z$	AFQT gap	1.00
$\sigma_{z\|h}^2$	AFQT variance, high school	0.77
$\sigma_{z\|c}^2$	AFQT variance, college	0.32
$\sigma_{zy\|h}$	Covariance of AFQT and productivity, high school	0.12
$\sigma_{zy\|c}$	Covariance of AFQT and productivity, college	0.07
λ_1^h	Speed of learning, high school	0.25
λ_1^c	Speed of learning, college	0.10
φ_0^{zh}	Initial return to AFQT, high school	0.00
φ_0^{zc}	Initial return to AFQT, college	0.08

Notes: See the text on the calculation of moments. The average treatment effect of college is assumed rather than estimated from data. 'gap' refers to the difference between college and high school.

Table 2: Goodness of fit

Assumptions				Moments											
ATE_y	σ	α	$\rho_{\theta\gamma}$	$\Delta\mu_y$	$\frac{\sigma^2_{y\|h}}{\sigma^2_{y\|c}}$	$\Delta\mu_z$	$\sigma^2_{z\|h}$	$\sigma^2_{z\|c}$	$\sigma_{zy\|h}$	$\sigma_{zy\|c}$	loss_z	λ^h_1	λ^c_1	φ^{zh}_0	φ^{zc}_0
			0.20												
0.00	∞		0.50												
			0.80	0.43	0.85	1.49	0.77	0.31	0.09	0.07	0.17	0.25	0.10	0.00	0.08
			0.25	0.43	0.85	0.83	0.77	0.36	0.08	0.08	0.15	0.25	0.10	0.00	0.08
0.00	1.50	0.50	0.50	0.43	0.85	1.29	0.77	0.36	0.07	0.06	0.24	0.25	0.10	0.00	0.08
			0.80	0.43	1.26	1.01	0.65	0.37	0.11	0.07	0.05	0.25	0.10	0.00	0.08
			0.20	0.43	0.85	1.10	0.77	0.38	0.05	0.05	0.48	0.25	0.10	0.00	0.08
0.16	∞		0.50	0.43	0.85	1.34	0.77	0.36	0.05	0.03	0.72	0.25	0.10	0.00	0.08
			0.80												
			0.20												
0.16	1.50	0.825	0.50	0.43	0.85	1.00	0.77	0.36	0.07	0.07	0.20	0.25	0.10	0.00	0.08
			0.80	0.43	0.85	1.61	0.77	0.34	0.06	0.04	0.58	0.25	0.10	0.00	0.08
Minimum				0.43	0.85	0.83	0.65	0.31	0.05	0.03	0.05	0.25	0.10	0.00	0.08
Maximum				0.43	1.26	1.61	0.77	0.38	0.11	0.08	0.72	0.25	0.10	0.00	0.08
Data				0.43	0.85	1.00	0.77	0.32	0.12	0.07		0.25	0.10	0.00	0.08

Notes: The table shows values of model-generated moments based on estimating the model parameters under the assumptions indicated in the first four columns. See Table 1 for a description of the moments, as well as parameter values common to all specifications. Empty rows indicate that the estimation algorithm did not find a fixed point.

Table 3: Parameter estimates based on NLSY data

Assumptions				Estimated parameters									
ATE_y	σ	α	$\rho_{\theta\gamma}$	$\tau_c\kappa_c$	ζ_c	a_s	a_θ	π_s	π_θ	σ^2_{uh}	σ^2_{uc}	$\sigma^2_{\varepsilon h}$	$\sigma^2_{\varepsilon c}$
			0.20										
0.00	∞		0.50										
			0.80	1.66	0.79	0.08	0.02	0.00	0.95	∞	3.85	0.05	0.11
			0.25	1.61	3.24	0.10	0.04	0.00	0.73	∞	2.13	0.07	0.13
0.00	1.50	0.50	0.50	1.61	3.27	0.10	0.01	0.00	0.89	∞	1.66	0.05	0.08
			0.80	1.31	2.83	0.11	0.00	0.07	0.95	∞	1.97	0.07	0.09
			0.20	1.70	0.71	0.06	0.03	0.03	0.74	∞	0.39	0.02	0.02
0.16	∞		0.50	1.59	0.71	0.06	0.00	0.00	0.92	∞	0.25	0.02	0.01
			0.80										
			0.20										
0.16	1.50	0.825	0.50	1.66	0.01	0.05	0.06	0.00	0.74	∞	2.42	0.04	0.07
			0.80	1.64	0.02	0.05	0.01	0.00	1.01	∞	1.21	0.02	0.02
Minimum				1.31	0.01	0.05	0.00	0.00	0.73	∞	0.25	0.02	0.01
Maximum				1.70	3.27	0.11	0.06	0.07	1.01	∞	3.85	0.07	0.13

Notes: The table shows estimated parameter values as functions of the assumptions indicated in the first four columns. See Table 1 for a description of the parameters, as well as values of pre-set parameters. Empty rows indicate that the estimation algorithm did not find a fixed point.

Table 4: Perfect-information counterfactual

Assumptions				Difference between estimated model and perfect-information counterfactual						
ATE_y	σ	α	$\rho_{\theta\gamma}$	p	$a_s s_0^h$	$a_s \mathrm{E}_h \left[s - s_0^h\right]$	$a_s s_0^c$	$a_s \mathrm{E}_c \left[s - s_0^c\right]$	$\log Y$	$(1-\delta)\mathrm{E}[V]$
			0.20							
0.00	∞		0.50							
			0.80	0.25	-0.16	-0.06	-0.25	-0.38	-0.12	-0.03
			0.25	0.04	-0.20	-0.01	-0.23	-0.03	-0.20	-0.01
0.00	1.50	0.50	0.50	0.04	-0.20	-0.01	-0.23	-0.02	-0.20	-0.01
			0.80	0.05	-0.22	-0.02	-0.33	-0.03	-0.25	-0.01
			0.20	0.25	-0.12	-0.04	-0.00	-0.24	-0.04	-0.02
0.16	∞		0.50	0.25	-0.13	-0.05	-0.04	-0.25	-0.05	-0.02
			0.80							
			0.20							
0.16	1.50	0.825	0.50	0.01	-0.10	-0.00	-0.05	-0.01	-0.08	-0.00
			0.80	0.01	-0.11	-0.00	-0.08	-0.01	-0.09	-0.00
Minimum				0.01	-0.22	-0.06	-0.33	-0.38	-0.25	-0.03
Maximum				0.25	-0.10	-0.00	-0.00	-0.01	-0.04	-0.00

Notes: The table shows the difference between moments generated by the estimated models and the corresponding perfect-information counterfactual moments. See the text for explanations of the column headings. Empty rows indicate that the estimation algorithm did not find a fixed point.

Appendices for online publication

A Deriving conditional expectations and variances for the theoretical employer learning model

Here I derive the worker's expected log output, as well as the variance of log output, conditional on her schooling signal and the history of output observations. Using standard results for multivariate normal distributions, the conditional expectation is given by

$$\mathrm{E}\left[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right] = \mu_y + \Sigma_{y\times(\tilde{s},\tilde{y}_t)}\Sigma^{-1}_{(\tilde{s},\tilde{y}_t)\times(\tilde{s},\tilde{y}_t)}\begin{pmatrix}\tilde{s}_i-\mu_s\\ \tilde{\mathbf{y}}_{it}-\mu_y\iota_t\end{pmatrix}, \tag{A.1}$$

where $\Sigma_{v\times z}$ is the covariance matrix of variables (potentially vectors) v and z, and ι_t is a size-t column vector of ones. The conditional variance is given by

$$\mathrm{Var}\left(y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right) = \sigma^2_y - \Sigma_{y\times(\tilde{s},\tilde{y}_t)}\Sigma^{-1}_{(\tilde{s},\tilde{y}_t)\times(\tilde{s},\tilde{y}_t)}\Sigma_{y\times(\tilde{s},\tilde{y}_t)}. \tag{A.2}$$

The covariance matrices are

$$\Sigma_{y\times(\tilde{s},\tilde{y}_t)} = \begin{pmatrix}\sigma_{sy} & \sigma^2_y\iota'_t\end{pmatrix}, \tag{A.3}$$

and

$$\Sigma_{(\tilde{s},\tilde{y}_t)\times(\tilde{s},\tilde{y}_t)} = \begin{pmatrix}\sigma^2_s+\sigma^2_u & \sigma_{sy}\iota'_t\\ \sigma_{sy}\iota_t & \sigma^2_\varepsilon I_t + \sigma^2_y\iota_t\iota'_t\end{pmatrix}, \tag{A.4}$$

where I_t is a size-t identity matrix. Given (A.3) and (A.4), one could solve (A.1) and (A.2) using block-wise matrix inversion and the Sherman–Morrison formula. However, it is easier to apply a two-step procedure.

The first step is to derive mean and variance of log productivity conditional on the schooling signal,

$$\mathrm{E}\left[y_i|\tilde{s}_i\right] = \mu_y + \frac{\sigma_{sy}}{\sigma^2_s+\sigma^2_u}\left(\tilde{s}_i-\mu_s\right) = \mu_y + b^{y\tilde{s}}_0\left(\tilde{s}_i-\mu_s\right) \tag{A.5}$$

and

$$\mathrm{Var}\left[y_i|\tilde{s}_i\right] = \sigma^2_y - \sigma_{sy}b^{y\tilde{s}}_0 = \sigma^2_{y|\tilde{s},0},$$

where $b^{y\tilde{s}}_0$ and $\sigma^2_{y|\tilde{s},0}$ are given by (8).

The second step is to further update these moments after observing $t > 0$ log output signals as specified in (2). Defining $\mu_{y|\tilde{s}} \equiv \mathrm{E}\left[y_i|\tilde{s}_i\right]$, we have

$$\begin{aligned}
\mathrm{E}\left[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right] &= \mu_{y|\tilde{s}} + \Sigma_{(y|\tilde{s})\times\tilde{y}_t}\Sigma^{-1}_{\tilde{y}_t\times\tilde{y}_t}\left(\tilde{\mathbf{y}}_{it} - \mu_{y|\tilde{s}}\iota_t\right) \\
&= \mu_{y|\tilde{s}} + \sigma^2_{y|\tilde{s},0}\iota_t'\left(\sigma^2_\varepsilon I_t + \sigma^2_{y|\tilde{s},0}\iota_t\iota_t'\right)^{-1}\left(\tilde{\mathbf{y}}_{it} - \mu_{y|\tilde{s}}\iota_t\right) \\
&= \mu_{y|\tilde{s}} + \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon}\iota_t'\left(I_t - \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon + \sigma^2_{y|\tilde{s},0}t}\iota_t\iota_t'\right)\left(\tilde{\mathbf{y}}_{it} - \mu_{y|\tilde{s}}\iota_t\right) \\
&= \mu_{y|\tilde{s}} + \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon}\frac{\sigma^2_\varepsilon t}{\sigma^2_\varepsilon + \sigma^2_{y|\tilde{s},0}}\left(\overline{y}_{it} - \mu_{y|\tilde{s}}\right) \\
&= \mu_{y|\tilde{s}} + \lambda_t\left(\overline{y}_{it} - \mu_{y|\tilde{s}}\right),
\end{aligned}$$

where the third line follows from the Sherman-Morrison formula and the last line uses (9). Substituting (A.5) into the last line, we obtain (6). Similarly, (7) is derived as

$$\begin{aligned}
\mathrm{Var}\left[y_i|\tilde{s}_i, \tilde{\mathbf{y}}_{it}\right] &= \sigma^2_{y|\tilde{s},0} - \Sigma_{(y|\tilde{s})\times\tilde{y}_t}\Sigma^{-1}_{\tilde{y}_t\times\tilde{y}_t}\Sigma'_{(y|\tilde{s})\times\tilde{y}_t} \\
&= \sigma^2_{y|\tilde{s},0} - \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon}\iota_t'\left(I_t - \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon + \sigma^2_{y|\tilde{s},0}t}\iota_t\iota_t'\right)\sigma^2_{y|\tilde{s},0}\iota_t \\
&= \sigma^2_{y|\tilde{s},0} - \frac{\sigma^2_{y|\tilde{s},0}}{\sigma^2_\varepsilon}\frac{\sigma^2_\varepsilon t}{\sigma^2_\varepsilon + \sigma^2_{y|\tilde{s},0}}\sigma^2_{y|\tilde{s},0} \\
&= (1-\lambda_t)\sigma^2_{y|\tilde{s},0}.
\end{aligned}$$

B Derivations for the econometric employer learning model

Here I prove the results stated in Section 3.3, in particular equation (26) and Proposition 4.

I start by stating some properties of second moments that will turn out to be helpful. Let p denote the mean of D_i, equal to the probability that $D_i = 1$. First, for any random variable x_i,

$$\mathrm{Cov}(x_i, D_i) = p(1-p)\left\{\mathrm{E}[x_i|D_i = 1] - \mathrm{E}[x_i|D_i = 0]\right\},$$

which can be expressed more succinctly using previously introduced notation as

$$\sigma_{xD} = p(1-p)\left(\mu_{x|c} - \mu_{x|h}\right). \tag{B.1}$$

Second, the Law of Total Covariance implies that for any two random variables x_i and u_i,

$$\sigma_{xu} = (1-p)\sigma_{xu|h} + p\sigma_{xu|c} + p(1-p)\left(\mu_{x|c} - \mu_{x|h}\right)\left(\mu_{u|c} - \mu_{u|h}\right), \tag{B.2}$$

and notice how this includes the case of the variance as well, since $\sigma_x^2 \equiv \sigma_{xx}$.

Next, by the properties of regression coefficients,

$$\varphi_t^z = \frac{\sigma_{\tilde{z}w,t}}{\sigma_{\tilde{z}}^2}, \qquad \tilde{z}_i = z_i - \frac{\sigma_{zD}}{\sigma_D^2} D_i.$$

Using (B.1), we obtain $\tilde{z}_i = z_i - \left(\mu_{z|c} - \mu_{z|h}\right) D_i$, and therefore

$$\varphi_t^z = \frac{\sigma_{zw,t} - p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)\left(\mu_{w|c,t} - \mu_{w|h,t}\right)}{\sigma_z^2 - p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)^2}.$$

Applying (B.1) again, as well as (B.2), yields

$$\varphi_t^z = \frac{(1-p)\sigma_{zw|h,t} + p\sigma_{zw|c,t}}{(1-p)\sigma_{z|h}^2 + p\sigma_{z|c}^2}.$$

Similarly,

$$\varphi_t^D = \frac{\sigma_{w\tilde{D},t}}{\sigma_{\tilde{D}}^2}, \qquad \tilde{D}_i = D_i - \frac{\sigma_{zD}}{\sigma_z^2} z_i = D_i - \frac{p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)}{\sigma_z^2} z_i,$$

and thus

$$\varphi_t^D = \frac{\sigma_{wD,t} - p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)\frac{\sigma_{zw,t}}{\sigma_z^2}}{p(1-p) - p^2(1-p)^2\left(\mu_{z|c} - \mu_{z|h}\right)^2 \frac{1}{\sigma_z^2}}.$$

Again using results (B.1) and (B.2), we obtain

$$\varphi_t^D = \frac{\sigma_z^2\left(\mu_{w|c,t} - \mu_{w|h,t}\right) - \left(\mu_{z|c} - \mu_{z|h}\right)\left\{(1-p)\sigma_{zw|h,t} + p\sigma_{zw|c,t} + p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)\left(\mu_{w|c,t} - \mu_{w|h,t}\right)\right\}}{\sigma_z^2 - p(1-p)\left(\mu_{z|c} - \mu_{z|h}\right)^2}$$

which further simplifies to

$$\varphi_t^D = \mu_{w|c,t} - \mu_{w|h,t} - \left(\mu_{z|c} - \mu_{z|h}\right)\varphi_t^z.$$

This completes the derivation of equation (26).

Turning to the proof of Proposition 4, observe that

$$\sigma_{zw|j,t} = \mathrm{Cov}\left(\pi_s s_i + \pi_\theta \theta_i, \beta_t^{sj} s_i + \beta_t^{\theta j}\theta_i\right),$$

from which equation (28) follows. To prove that $\partial\varphi_t^{zj}/\partial t > 0$ if $\varphi_\infty^{zj} = \chi_s^j a_s + \chi_\theta^j a_\theta > 0$, start by observing that $\text{sign}\left\{\partial\varphi_t^{zj}/\partial t\right\} = \text{sign}\left\{\partial\varphi_t^{zj}/\partial\lambda_t^j\right\}$. Using equations (28) and (8),

$$\begin{aligned}
\frac{\partial\varphi_t^{zj}}{\partial\lambda_t^j} &= -\chi_s^j b_0^{y\tilde{s}|j} + \chi_s^j a_s + \chi_\theta^j a_\theta \\
&= -\chi_s^j \left(\frac{\sigma_{s|j}^2}{\sigma_{s|j}^2 + \sigma_{uj}^2} a_s + \frac{\sigma_{s\theta}|j}{\sigma_{s|j}^2 + \sigma_{uj}^2} a_\theta \right) + \chi_s^j a_s + \chi_\theta^j a_\theta \\
&= \frac{\sigma_{s|j}^2}{\sigma_{s|j}^2 + \sigma_{uj}^2} \left(\chi_s^j a_s + \chi_\theta^j a_\theta \right) + a_\theta \pi_\theta \frac{\sigma_{s|j}^2 \sigma_{\theta|j}^2}{\sigma_{z|j}^2} \left(1 - \rho_{s\theta|s}^2 \right),
\end{aligned}$$

which is positive if $\varphi_\infty^{zj} > 0$. This completes the proof of Proposition 4.

C Imperfect substitutes in the two-track model

Final output is produced in CES fashion using the intermediate goods made by high school and college workers, $Y = \left(\alpha^{\frac{1}{\sigma}} Y_h^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} Y_c^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$. Final good producers buy these intermediate goods at prices Ψ_h and Ψ_c, respectively. Profit maximization then implies

$$\frac{\Psi_c}{\Psi_h} = \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\sigma}} \left(\frac{Y_c}{Y_h} \right)^{-\frac{1}{\sigma}}.$$

The model is closed by the market clearing conditions

$$\begin{aligned}
Y_h &= e^{a_s s_0^h} \int_{-\infty}^{\infty} \int_{-\infty}^{x_0 + x_1\gamma} e^{\tau_h \kappa_h a_s(\theta+\gamma) + a_\theta\theta} f(\theta,\gamma) d\theta d\gamma, \\
Y_c &= e^{a_s s_0^c} \int_{-\infty}^{\infty} \int_{x_0 + x_1\gamma}^{\infty} e^{\tau_c \kappa_c a_s(\theta+\gamma) + a_\theta\theta} f(\theta,\gamma) d\theta d\gamma,
\end{aligned}$$

where $f(\theta, \gamma)$ is the joint probability density function of θ and γ, which I assume to be normal. Defining $\psi_j \equiv \log \Psi_j$, $\Delta x \equiv x_c - x_h$, and

$$\begin{pmatrix} \tilde{e}_h \\ \tilde{e}_c \\ \Delta\alpha \end{pmatrix} \equiv \begin{pmatrix} \log\left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{x_0+x_1\gamma} e^{\tau_h\kappa_h a_s(\theta+\gamma) + a_\theta\theta} f(\theta,\gamma) d\theta d\gamma \right\} \\ \log\left\{ \int_{-\infty}^{\infty} \int_{x_0+x_1\gamma}^{\infty} e^{\tau_c\kappa_c a_s(\theta+\gamma) + a_\theta\theta} f(\theta,\gamma) d\theta d\gamma \right\} \\ \log\{\alpha/(1-\alpha)\} \end{pmatrix}, \tag{C.1}$$

we obtain

$$\Delta\psi = -\frac{1}{\sigma} \left(\Delta\alpha + \Delta\tilde{e} + a_s \Delta s_0 \right). \tag{C.2}$$

The analysis of employer learning is unchanged. Optimal schooling choices, within track, are also unchanged. However, the intercepts of the value functions need to be adjusted, since now

$$B_0^j = \delta^{\tau_j} \sum_{t=0}^{\infty} \delta^t \left(\psi_j + \beta_t^{0j} \right).$$

Finally, a complication arises due to the different lengths of the two tracks. I have so far implicitly assumed that the fraction of college graduates in the labor market stays constant over time. This can be guaranteed by assuming a 'perpetual youth' setting where individuals are born and die at constant rates (the discount factor δ includes mortality risk in this case). My analysis then focuses on the steady state of such a setting, and the counterfactual exercises of Section 4.6 compare different steady states.

D Solution algorithm for the two-track model

The selection rule for college (21) can be rewritten as

$$\theta_i \geq x_0 + x_1 \gamma_i, \qquad \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \equiv \frac{1}{\nu_\theta^c - \nu_\theta^h} \begin{pmatrix} \nu_0^h - \nu_0^c \\ -\left(\nu_\gamma^c - \nu_\gamma^h\right) \end{pmatrix} \tag{D.1}$$

where (x_0, x_1) are equilibrium quantities to be solved for. In fact, the pair (x_0, x_1) constitutes a 'sufficient statistic' for employers' beliefs in the following sense. Given the linearity of the schooling equations, the variances and covariances of optimal schooling can be computed given knowledge of (x_0, x_1). This in turn allows one to compute the coefficients on schooling (and talent) in the wage equations, which yields the constant in the schooling equations. This in turn lets one compute the intercepts in the wage equations, and thus one can completely characterize the value functions. Hence, a simple procedure for numerically solving the two-step model is as follows.

1. Guess values (x_0, x_1).
2. Using this guess, the selection rule (D.1), and the assumption about joint normality of θ and γ, compute $\left(\mu_{\theta|j}, \mu_{\gamma|j}, \sigma^2_{\theta|j}, \sigma^2_{\gamma|j}, \sigma_{\theta\gamma|j}\right)$.
3. Using (19), compute $\left(\sigma^2_{s|j}, \sigma_{s\theta|j}\right)$.
4. Using (17) and (18), compute $\left(\beta_t^{sj}, \beta_t^{\theta j}\right)$ and $\left(B_\theta^j, B_s^j\right)$.
5. Using (19), compute s_0^j to obtain $\mu_{s|j}$ and hence β_t^{0j} and B_0^j.
6. Compute the implied (x_0', x_1').

For the grid search, I calculate and plot the loss function $\sqrt{(x_0' - x_0)^2 + (x_1' - x_1)^2}$. This lets me inspect whether there may be multiple equilibria. I use numerical optimization to find the precise location of the loss function's minimum and verify that the value is near zero within the specified tolerance.

E Estimation algorithm for the two-track model

E.1 Guessing the characteristics of high school and college graduates

I start by guessing the slope of the selection rule (21) and a value for κ_c. Given the type distribution and the observed fraction of college graduates, p, I calculate the implied intercept in the selection rule. Given the selection rule and the guess for κ_c, I calculate the conditional distributions of talent θ_i and taste γ_i, as well as the conditional covariance matrix of θ_i and schooling s_i using (19). Thus, I have almost completely characterized high school and college graduates in terms of their types and their schooling choices. The only missing moment is the term $\Delta s_0 \equiv s_0^c - s_0^h$.

E.2 Estimating the production function

With the characteristics of high school and college graduates in hand, it is possible to estimate the production function parameters a_s and a_θ, using the system

$$\begin{aligned} \Delta\mu_y &= \Delta\psi + a_s \Delta s_0 + a_s \left(\tau_c \kappa_c \left(\mu_{\theta|c} + \mu_{\gamma|c}\right) - \left(\mu_{\theta|h} + \mu_{\gamma|h}\right)\right) + a_\theta \Delta\mu_\theta, \\ \frac{\sigma^2_{y|c}}{\sigma^2_{y|h}} &= \frac{\sigma^2_{s|c} a_s^2 + 2\sigma_{s\theta|c} a_s a_\theta + \sigma^2_{\theta|c} a_\theta^2}{\sigma^2_{s|h} a_s^2 + 2\sigma_{s\theta|h} a_s a_\theta + \sigma^2_{\theta|h} a_\theta^2}, \end{aligned} \tag{E.1}$$

where $\Delta\mu_y \equiv \mu_{y|c} - \mu_{y|h}$ and so on. Recall that the average treatment effect of college, $\Delta\psi + a_s \Delta s_0$, has been guessed. The production function parameters are estimated by simply equalizing the model moments given by the right-hand side of (E.1) to their counterparts in the data.

E.3 Estimating Δs_0

In the case of perfect substitutes, the difference in the schooling intercept Δs_0 can be backed out from the assumption on ATE_y and the value of a_s estimated in the previous step. If $\sigma < \infty$, then Δs_0 is obtained from (C.2), after computing (C.1), and again using the assumption on ATE_y and the estimated a_s.

E.4 Estimating the data-generating process for the AFQT score

With estimates of a_s, a_θ, and Δs_0 in hand, it is possible to estimate the data-generating process for the AFQT score, as specified in (27). In particular, using the system

$$\begin{aligned}
\Delta\mu_z &= \pi_s \Delta\mu_s + \pi_\theta \Delta\mu_\theta, \\
\sigma^2_{z|h} &= \pi_s^2 \sigma^2_{s|h} + \pi_\theta^2 \sigma^2_{\theta|h} + 2\pi_s \pi_\theta \sigma_{s\theta|h} + \sigma^2_{\xi h}, \\
\sigma^2_{z|c} &= \pi_s^2 \sigma^2_{s|c} + \pi_\theta^2 \sigma^2_{\theta|c} + 2\pi_s \pi_\theta \sigma_{s\theta|c} + \sigma^2_{\xi c}, \\
\sigma_{yz|h} &= \sigma^2_{s|h} a_s \pi_s + \sigma_{s\theta|h} \left(a_s \pi_\theta + a_\theta \pi_s\right) + \sigma^2_{\theta|h} a_\theta \pi_\theta, \\
\sigma_{yz|c} &= \sigma^2_{s|c} a_s \pi_s + \sigma_{s\theta|c} \left(a_s \pi_\theta + a_\theta \pi_s\right) + \sigma^2_{\theta|c} a_\theta \pi_\theta,
\end{aligned}$$

I obtain estimates of π_s, π_θ, $\sigma^2_{\xi h}$, and $\sigma^2_{\xi c}$ by minimizing the distance (squared deviations) between model and data moments. This part of the estimation is over-identified, as there are five moments for four parameters to be estimated. I thus report the value of the loss function, which is the square of the Euclidean distance between model moments and data moments, with each component normalized by the relevant data moment.

E.5 Estimating the noise parameters

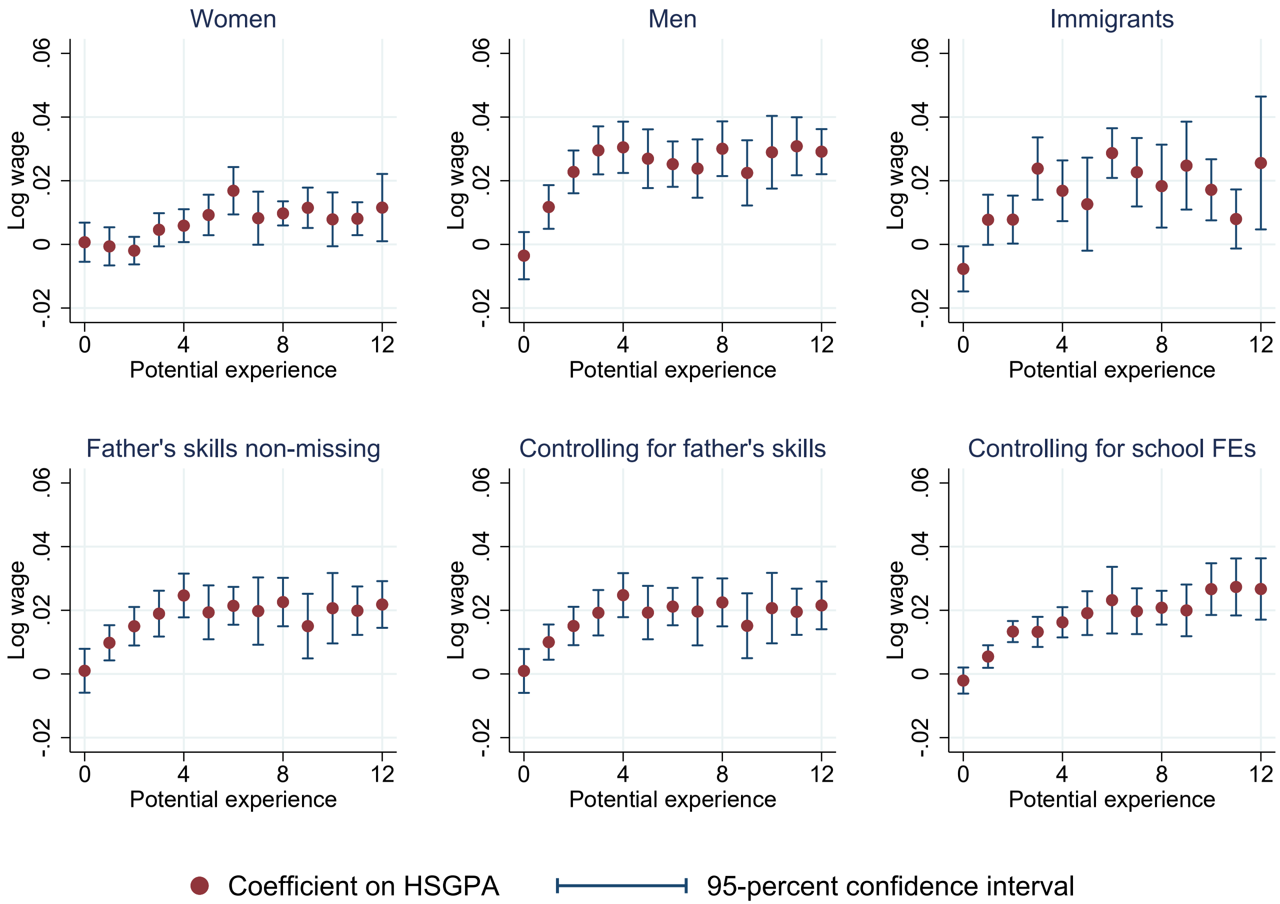
By this point I have estimated all the parameters needed to recover $\left(b_0^{y\tilde{s}|h}, b_0^{y\tilde{s}|c}\right)$ from the terminal value estimated by non-linear least squares, as described by (28). Using (8), I obtain estimates of the noise in the schooling signal, $(\sigma^2_{uh}, \sigma^2_{uc})$. Moreover, (8) yields the moments $\left(\sigma^2_{y|\tilde{s},0,h}, \sigma^2_{y|\tilde{s},0,c}\right)$, and together with (9) and the estimated learning rates, this yields estimates of the noise in the output signal, $(\sigma^2_{\varepsilon h}, \sigma^2_{\varepsilon c})$.

E.6 Estimating the study cost parameter, and verifying the guesses

I now have all the inputs needed for constructing B_s^h and B_s^c. Thus, using (19), I recover ζ_c.

Finally, I calculate the implied selection rule. I repeat the entire procedure, searching over a grid of initial guesses for x_1 and κ_c for fixed points where the implied selection rule equals the one that I guessed. In the cases I considered, there were either one or no fixed points.

F Appendix figures



Notes: Results are shown from regressions of log wages on the standardized high school GPA. The sample includes Swedish high school graduation cohorts 1993-2007 observed in the labor market 1993-2017. See Section 5 for more details. Controls include compulsory school GPA and sets of fixed effects for year, region of birth, and parental education and origin. Panel titles indicate further modifications to the sample or regression specification.

Figure A1: The returns to standardized high school GPA—robustness checks