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Macroeconomics with a Thick Pen

Abstract

This paper introduces two co-movement measures based on the Thick Pen Transform into the macroeconomic literature: the Thick Pen Measure of Association (TPMA) as well as Multi-Thickness Thick Pen Measure of Association (MTTPMA). Both measures are non-parametric, time-varying, and flexible. These methods are used to analyse the co-movement of, first, US long- and short-term interest rates, and, second, growth rates of per capita GDP and consumption. As methodological benchmark, this paper also applies the recently pro-posed measure of long-run covariability. The paper finds, first, the co-movement of all series to be stronger the more long-term the components of the time series are. Second, the co-movement of GDP and consumption growth rates is not only generally higher, it also fluctuates considerably less over time than that of the interest rates. Third, the co-movement of the interest rates is sensitive to choosing how long-term the components are. This is attributable to the different extents to which the interest rates exhibit cyclical behaviour. The benchmark method confirms this pattern of the results.

JEL-Codes: C140, E300, G150.

Keywords: co-movement, macroeconomics, Thick Pen, covariability.

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1 Introduction

This paper does macroeconomics with a thick pen. It applies two measures of association based on the so-called Thick Pen approach: first, Fryzlewicz and Oh's (2011) Thick Pen Measure of Association (TPMA) as well the Multi-Thickness Thick Pen Measure of Association (MTTPMA), proposed by Jach (2021). These measures have the following features: First, they are applicable to stationary as well as non-stationary time series. Second, they are applicable in both bivariate and multivariate situations. Third, they are time-varying; thus, changes in the extent of co-movement can be captured. Fourth, they are capable of capturing co-movement with respect to a given time scale, for a range of time scales from small to large.[1] Finally, they are capable of quantifying co-dependence across different time scales.

These measures of association are used to analyse the following two macroeconomic relationships: first, 10-years US Treasury Bonds and 3-month US treasury Bills, and, second, growth rates of per capita GDP and consumption. As methodological benchmark, this paper uses Mueller and Watson's (2018) long-run covariability, who use the same data. This method is similar insofar as it also measures long-run co-movement; however, it is time-invariant and does not allow a cross-scale analysis. This paper initially replicates Mueller and Watson's (2018) results, and, subsequently, analyses how sensitive the results are to additional settings of q, a key parameter in Mueller and Watson (2018) which captures "what a researcher considers to be the long-run".

Figures 1 displays the interest rate data. The observations are at quarterly frequency, period of observation is 1953Q2 - 2016Q4.[2] This results in 252 observations. It is evident that a strong co-movement is present. Mueller and Watson (2018) find the long-run correlation coefficient to be 0.96.[3] A careful inspection of Figure 1, however, shows that the extent of

[1]The term "time scale" is conceptually similar to the term "period" used in Mueller and Watson (2018) in specific and in frequency-domain statistics in general.

[2]Note that the data frequency of the original series is monthly. This paper follows Mueller and Watson (2018) who aggregate the original data from monthly to quarterly frequency.

[3]See Table II, p790, in Mueller and Watson (2018). This is the result obtained from

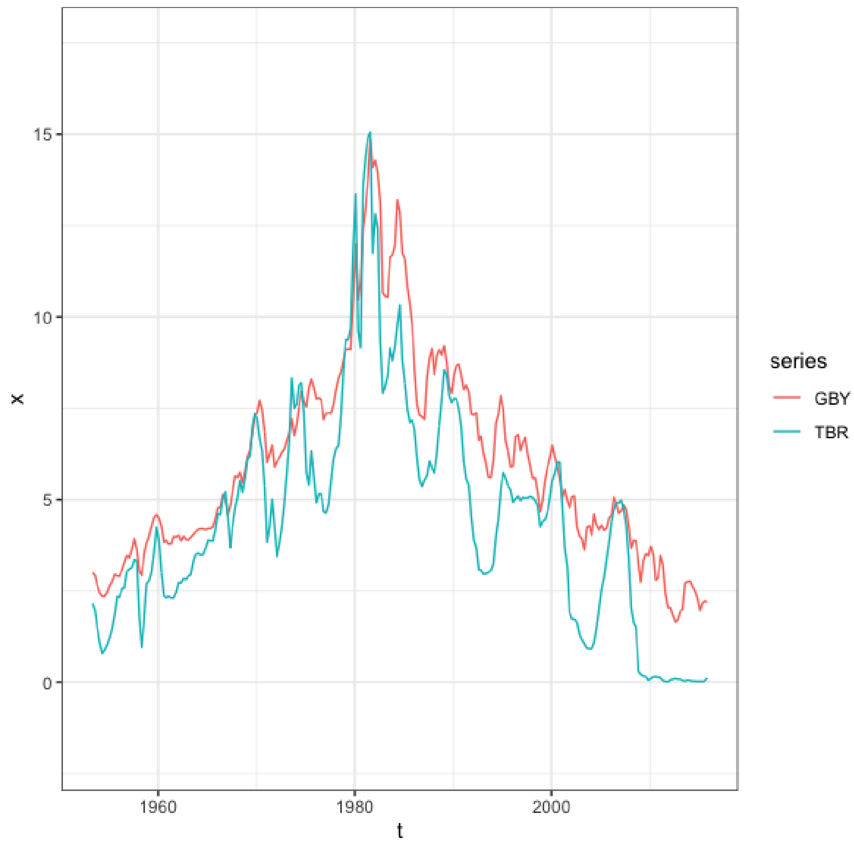


Figure 1: 10 years U.S. Treasury Bonds (GBY) and 3 month U.S. Treasury Bills (TBR)

co-movement is changing over time. First, the Treasury Bill rates exhibit a strong degree of cyclicality. This property can also be found in the Treasury Bond rates, but to a smaller extent. Second, prior to the turning point in 1980, the peaks of the interest rate cycles overlap; otherwise the behaviour of the two rates seem to differ from each other. An example for this is the period in the early 1960s when the Treasury Bond rate moves horizontally while the Treasury Bill rate drastically increases. After the turning point, the peaks still overlap; but otherwise the behaviour of the two series is more similar. Finally, from 2010 onwards, Treasury Bill rates are close to zero

the application of the so-called A,B,c,d-Model. See Mueller and Watson (2018) for details.

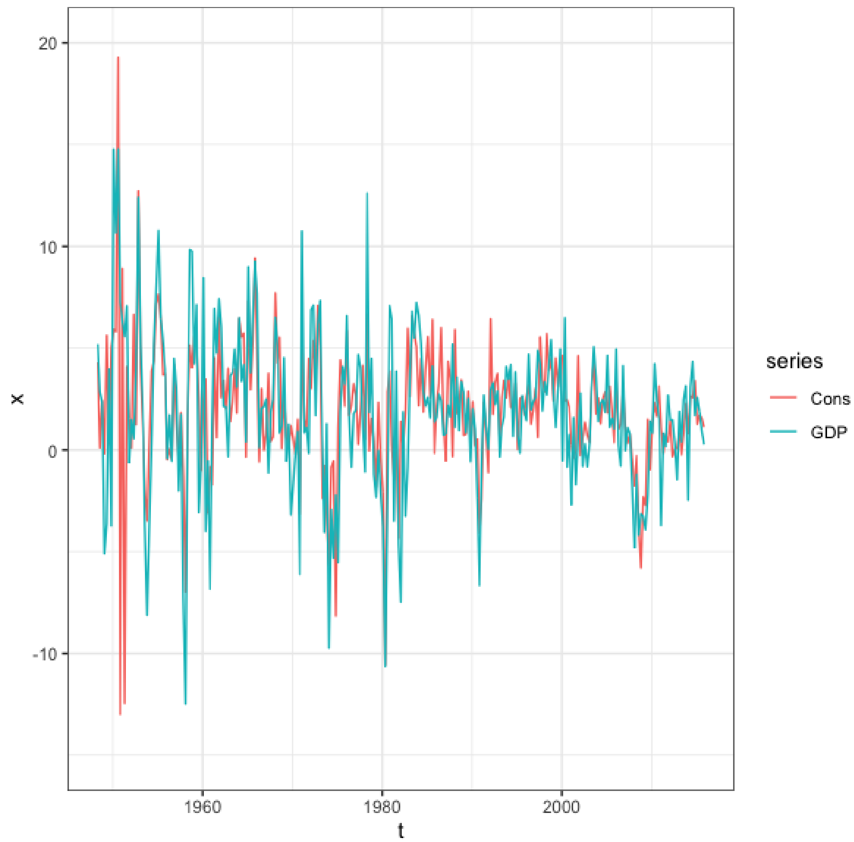


Figure 2: Growth rate of per capita GDP and per capita consumption

and exhibit very little fluctuation. Thus, this relationship is well-suited for being analysed using TPMA as well as MTTPMA.

Figure 2 displays the second macroeconomic relationship analysed: the growth rates of GDP per capita as well as consumption per capita. Mueller and Watson (2018) find the long-run correlation coefficient of these two time series to be 0.91. These two time series generally move closer together than the two interest rates displayed in Figure 1. Also in this example a change in the behavior of the two series is apparent: the variation of both important macroeconomic measures is decreasing over time. The effect of the financial crisis is also clearly visible as this is the only period in the second half of the sample where both series exhibit negative growth rates. The period of observation in this case is 1948Q1 to 2015Q4 which means that there are 272 quarterly observations.

The application of the two Thick Pen measures yields the following results: the co-movement of the long-term features of the two interest rates is found to be very high, but their short-term features are related to a much lower extent. This result is consistent with the results obtained from applying the benchmark method by Mueller and Watson (2018); it is largely attributable to the different extents to which the two series exhibit cyclical behaviour. In addition, the co-movement of the long-term features does not vary over time while that of the shorter-term features does so. Finally, the cross-scale analysis shows that the overlap between the long-term feature of Treasury Bond rates and the medium-term feature of Treasury Bill rates is relatively high. It is worth highlighting that Mueller and Watson's (2018) covariability does not allow one to make statements about time variation of co-movement and cross-scale relationships. The co-movement of both the long-term and short-term features of GDP and consumption growth rates, in contrast, fluctuates considerably less than the co-movement of the two interest rates. This implies that the relationship between these two macro time series is more stable. In addition, proposed Thick Pen measure is not sensitive to the observed decline in volatility of these two time series. Mueller and Watson's (2018) method would not allow one to make statements of this type. Thus, this paper demonstrates that the Thick Pen measures of Association are useful methods for the analysis of macroeconomic time series. Another contribution this paper makes is to show how sensitive results from applying Mueller and Watson's (2018) method are to choosing the period when constructing the long-run projections and the role cyclicality of the time series plays in this context.

This paper contributes to a fast growing literature which proposes and applies flexible, innovative co-movement measures. As already mentioned, Mueller and Watson (2018) propose a measure for long-run covariability. Centre stage in this approach takes a so-called low-pass transformation of a univariate time series. The purpose of this transformation is the isolation of the variation in the series which exceeds a certain period. The outcome of this transformation is also referred to as long-run projection of a time series. In order to analyse the long-run covariability of two variables, the relationship of the respective long-run projections is evaluated. Mueller and Watson's (2018) method allows one to calculate correlation as well as linear regression coefficients for the long-run projections of time series. Papell and Prodan (2020) employ this method in their analysis of long-run purchasing power parity. Worth mentioning is also Baruník and Kley (2019) who propose a general measure for dependence between cyclical economic variables referred to as quantile coherency. Schüler et al. (2020) propose a measure referred to as power cohesion in order to accurately analyse financial cycles across countries. Mentioned should also be cohesion, a measure proposed by Croux et al. (2001). A common feature of these methods is that they are based on frequency-domain techniques and, thus, allow one to study the short-run and long-run dynamic properties of multiple time series. Finally, Lindman et al. (2020) conduct a cross-quantilogram analysis to examine quantile dependence between the conditional stock return distributions of several countries. Fryzlewicz and Oh (2011), in their original paper, use stock market indices to illustrate their method. Jach (2017) analyse co-movement of international stock markets and returns; Wadud et al. (2023) deal with the relationship between commodity and equity markets. In other words, these applications fall into the area of empirical finance; to the best knowledge of the author, there is to date no application in macroeconomics. These approaches generally offer an alternative perspective compared to more rigid cointegration model and are also more flexible with regard to the time series properties of the individual series.

The remainder of the paper is organised as follows: Section 2 explains the methods used in this paper, followed by a presentation of the results in Section 3. Section 4 offers some concluding remarks.

2 Methods

2.1 Long-run covariability

The first method this paper employs is the measure of long-run covariability proposed by Mueller and Watson (2018).[4] The main idea of this approach can be summarised as follows: centre stage takes a so-called low-pass transformation of a univariate time series $x_t, t = 1, \ldots, T$. The purpose of this transformation is the isolation of the variation in the series which exceeds a certain period. The length of this period is controlled by a parameter q. Cosine functions are used to capture these periodic functions: $\Psi_j(s) = \sqrt{2}\cos(js\pi)$ denotes these functions with period $2/j$.[5]

The outcome of this transformation is also referred to as low-frequency projection of x, denoted by $\hat{x}$. In order to analyse the long-run covariability of two variables (x, y), the relationship of the respective long-run projections $(\hat{x}, \hat{y})$ is evaluated. Ω_T denotes the average covariance matrix of those long-run projections in a sample of T. This (2x2) matrix summarises their variability and covariability. From that, the long-run correlation and long-run linear regression coefficient can be derived as follows:

$$\begin{aligned}
\rho_T &= \Omega_{xy,T}/\sqrt{\Omega_{xx,T}\Omega_{yy,T}}, \\
\beta_T &= \Omega_{xy,T}/\Omega_{xx,T}, \\
\sigma_{y|x,T} &= \Omega_{yy,T} - (\Omega_{xy,T})^2/\Omega_{xx,T},
\end{aligned} \tag{1}$$

where $(\Omega_{xx,T}, \Omega_{xy,T}, \Omega_{yy,T})$ are elements of Ω_T.

[4] This paper only descibes the essence of this method. Readers with interest in all methodlolgical details are referred to their original paper.

[5] $\Psi(s) = [\Psi_1(s), \Psi_2(s), \ldots, \Psi_q(s)]'$ denotes a vector of these functions with periods 2 through $2/q$, and Ψ_T denote the $T \times q$ matrix with tth row given by $\Psi((t-1/2)T)'$, so the jth column of Ψ has period $2T/j$.

2.2 Thick Pen measure of association

The Thick Pen Transform and the Thick Pen Measure of Association goes back to Fryzlewicz and Oh (2011). To describe this method in an intuitive way, recall that plotting a time-series by hand on a piece of paper is essentially not more than, first, making a scatterplot of the points over time, and, second, connecting them using a pen (Jach, 2017). The key idea behind the Thick Pen method is to re-do this exercise using pens of different thickness. This procedure allows one to capture different features of the data: a small-thickness pen mainly captures high-frequency movements whereas a thicker pen captures low-frequency ones. To express this more formally, let $X = (X_t)_{t=1}^{T}$ be a univariate time series. Note that there is no stationarity requirement. Furthermore, let $\mathcal{T} = \tau_1, \ldots, \tau_n$ be a set of n positive, constant thickness parameters. Based on what has been described, the following two random variables L and U are introduced:

$$L_t^{\tau_i} = \min(X_t, X_{t+1}, \ldots, X_{t+\tau_i})$$

and

$$U_t^{\tau_i} = \max(X_t, X_{t+1}, \ldots, X_{t+\tau_i}).$$

They describe the lower and upper boundaries of the area marked by a (square) pen of a given thickness τ_i. Changing τ_i yields a multiscale representation of the data. The Thick Pen Transformation (TPT) is a collection of n pairs of these boundaries and is denoted as follows:

$$TP_{\mathcal{T}}(X) = \{(L_t^{\tau_i}(X), U_t^{\tau_i}(X))_{t=1}^{T}\}_{i=1}^{n} \tag{2}$$

The total number of random variables this comprises is $2 \times n \times T$. The TPT forms the basis of the co-movement measure Thick Pen Measure of Association (TPMA) which has been proposed by Fryzlewicz and Oh (2011). TPMA measures the overlap between the areas formed by the TPTs of two (or more) time series. Note that these time series have to be standardised prior to the application of this method. To express this more formally, let $\underline{X} = (X^{(1)}, \ldots, X^{(K)})$ denote a vector of K standardised time series with

$X^{(k)} = \{X_t^{(k)}\}_{t=1}^T, k = 1, \dots, K$. Furthermore, let $TP_{\mathfrak{T}}(X^{(k)}), k = 1, \dots, K$ denote their corresponding TPTs for a given set of n thickness parameters $\mathfrak{T} = \tau_1, \dots, \tau_n$. The TPMA between them, for all t and τ_i is defined as follows:

$$\rho_t^\tau(X^{(1)}, \dots, X^{(K)}) = \frac{\min(U^\tau(X)^{(k)}) - \max(L_t^\tau(X)^{(k)})}{\max(U_t^\tau(X)^{(k)}) - \min(L_t^\tau(X)^{(k)})} \tag{3}$$

Note that this random variable is bounded: $\rho_t^{\tau_i}(X^{(1)}, X^{(2)}) \in (-1, 1]$ This feature makes the interpretation of this measure straightforward: it measures the overlap between the TPTs. If two time series move together in a very general sense, their TPTs (for a given t and τ_i) will overlap and, thus, TPMA will be close to 1. If, however, the two series are out of sync, their TPTs will not overlap and, thus, TPMA will become negative.[6] It is important to note that all time series are transformed using the same thickness value τ_1. A generalisation of this measure leads to the Multi-thickness TPMA (MTTPMA). This measure has been proposed by Jach (2021).

The key difference between TPMA and MTTPMA is that the latter uses not only one, but k different thickness values. Thus, $\underline{\tau} = (\tau^{(1)}, \tau^{(K)})$ denoted a K-dimensional vector of thickness values and $\tau^{(k)}$ is the thickness value used for transforming the k-th time series $X^{(k)}$. MTTPMA is then defined as follows:

$$\rho_t^{\tau^{(1)}, \dots, \tau^{(K)}}(X^{(1)}, \dots, X^{(K)}) = \frac{\min(U^{\tau^{(k)}}(X)^{(k)}) - \max(L_t^{\tau^{(k)}}(X)^{(k)})}{\max(U_t^{\tau^{(k)}}(X)^{(k)}) - \min(L_t^{\tau^{(k)}}(X)^{(k)})} \tag{4}$$

MTTPMA has all features of TPMA, but it also allows one to to measure cross-scale dependence between time series via the overlap of areas marked by pens of different thickness. This paper analyses bivariate relationships ($K = 2$) and uses three different thickness values ($n = 3$).

[6] It should also be noted that the TPMA for independent time series can be large if a sufficiently thick pen is used. See Jach (2017, 2021) for more detailed discussions of the method as well as various useful illustrations.

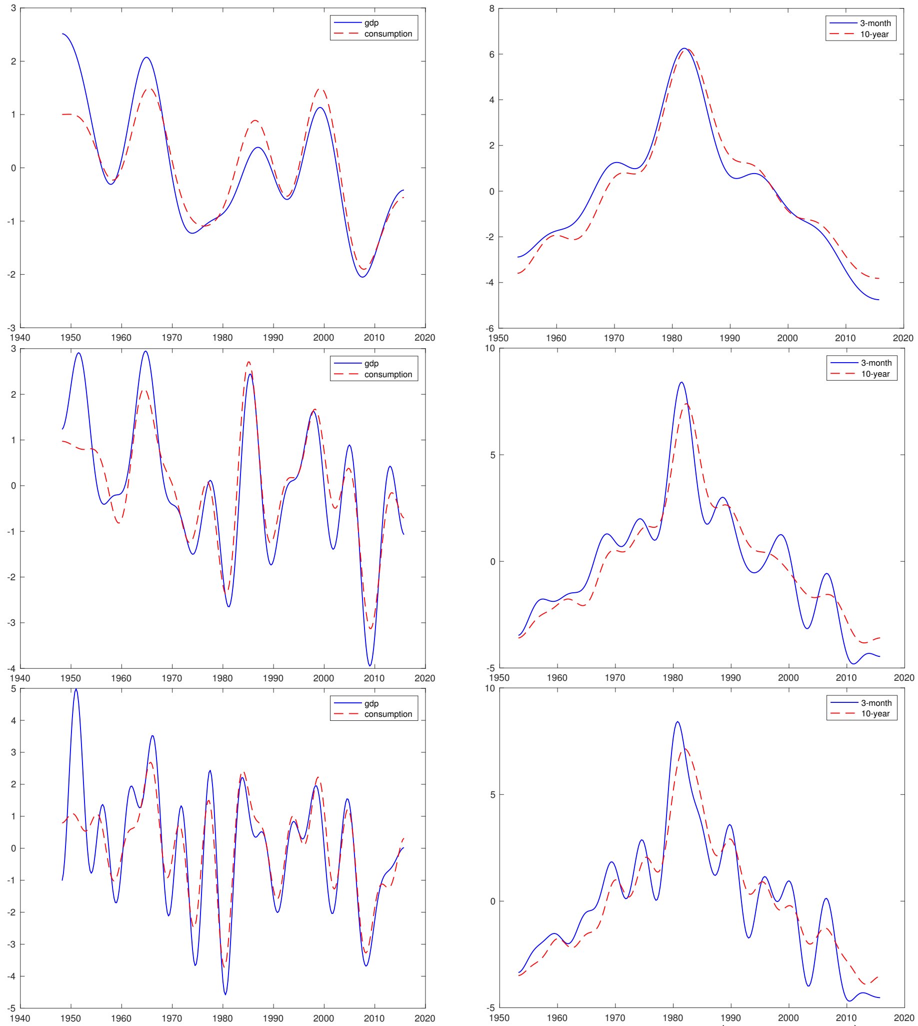
3 Results

The presentation of the results begins with those obtained from applying the methodological benchmark, Mueller and Watson's (2018) long-run covariability.[7] As stated above, their original paper uses the same data sets also used here. Mueller and Watson (2018) extensively discuss the setting of q as this controls the period of the long-run projections of the data. The choice of this parameter reflects what the researcher considers to be the "long run". In their original application, they select the parameter in such way that the period of consideration is longer than approximately 11 years: They use $q = 11$ and $q = 12$; the period this corresponds to can be calculated using the expression $\frac{2T}{q}$, with T for the number of observations. This paper initially uses the same parameter values; but also analyses how sensitive the results are to changes in this parameter. Note that varying this parameter q is methodologically equivalent to using different pen thickness values. In this paper, also $q = 20$ and $q = 26$ is used; this corresponds to peridos of approximately 6.5 years and 5 years, respectively.

Figure 3 illustrates how the choice of q affects the long-run projection of the data which are used for the calculation of the long-run covariability measures. The upper panel shows those projections for $q = 11$ and $q = 12$, respectively; in other words, a replication of Mueller and Watson's (2018) original results. The long-run projections are fairly smooth time series; all short-run fluctuation has been smoothed out. This explains the large long-run correlation coefficients obtained in the original paper. The middle panel shows the long-run projections for $q = 20$, which corresponds to periods longer than approximately 6.5 years. To express this differently, under this setting, the "long-run" is considered to be shorter. As a result, a larger extent of short-run fluctuation remains in those long-run projections of the series. Worth highlighting, however, is that the two growth rate series are affected by this change in q in the same way; both long-run projections seem to follow the same overall pattern. A different picture emerges for the case of the two interest rates: the long-run projection of the Treasury

[7] This paper uses the original replication code provided by the authors.

Figure 3: Long-run projections, different q



Note: The following values for q have been used: original values (upper panel), 20 (middle panel), 26 (lower panel).

Bond rate is similarly smooth as the one obtained for $q = 12$, but this is not the case for the Treasury Bill rate where a larger extent of short-run fluctuation remains in the data. This is attributable to the stronger cyclical behaviour Treasury Bill rates exhibit. As stated above, q is selected according to what a researcher considers the "long-run". This illustration indicates that the properties of the data seem to influence how sensitive the results of this transformation are to changes in the parameter q. To what extent this affects the estimated long-run correlation and linear regression coefficient will be discussed later. This effect becomes even more apparent in the bottom panel: the long-run projection of Treasury Bond rate does not change much when $q = 20$ is changed to $q = 26$ (which corresponds to periods longer than 5 years), but long-run projection of Treasury Bill rate fluctuates considerably as an even larger extent of short-run fluctuation remains in the data.

Tables 1 to 3 present the long-run covariability estimates for three different choices of q: the original choice used in Mueller and Watson (2018) in Table 1, followed by $q = 20$ and $q = 26$ in Tables 2 and 3, respectively. The discussion of the results begins with the two growth rate series. It is evident that the long-term correlation coefficient decreases with an increase in q, see the left panels in Tables 1 to 3. The same applies to the long-run linear regression coefficient. Quantitatively, however, these changes are small and not statistically significant. Thus, the long-run covariability of GDP and consumption growth rates is not very sensitive to changes in q - or in other words, what a researcher considers to be the "long-run". The visual inspection of the data highlighted that there is a decline in variability of two series; this, however, does not change the long-run relationship as both series are affected by this transformation in a similar way. A different picture emerges for the case of the relationship between the two interest rates. The overall pattern is similar; both correlation and linear regression coefficient decrease when q increases. Quantitatively, this is more pronounced than in the previous case; but also qualitatively as now there is a significant statistical difference. This result is attributable to the different extents the two time series exhibit cyclicality: a larger q results in a larger extent of cyclical

Table 1: Long-run covariability estimates, original choice for q

	ρ	β	$\sigma_{y\|x}$
Estimate	0.91	0.77	0.41
67% CI	0.83, 0.96	0.66, 0.87	0.33, 0.53
90% CI	0.71, 0.97	0.48, 0.96	0.29, 0.66
67% Bayes CS	0.83, 0.96	0.66, 0.87	0.33, 0.53
90% Bayes CS	0.71, 0.97	0.58, 0.96	0.29, 0.66

(a) GDP and consumption

	ρ	β	$\sigma_{y\|x}$
Estimate	0.96	0.95	0.63
67% CI	0.92, 0.98	0.87, 1.07	0.49, 0.97
90% CI	0.89, 0.99	0.76, 1.16	0.42, 1.27
67% Bayes CS	0.92, 0.98	0.87, 1.03	0.49, 0.82
90% Bayes CS	0.89, 0.99	0.81, 1.09	0.42, 1.02

(b) Interest rates

Table 2: Long-run covariability estimates, $q = 20$

	ρ	β	$\sigma_{y\|x}$
Estimate	0.90	0.74	0.53
67% CI	0.87, 0.94	0.66, 0.81	0.43, 0.63
90% CI	0.80, 0.96	0.60, 0.87	0.40, 0.73
67% Bayes CS	0.87, 0.94	0.66, 0.81	0.45, 0.63
90% Bayes CS	0.80, 0.96	0.60, 0.87	0.41, 0.73

(a) GDP and consumption

	ρ	β	$\sigma_{y\|x}$
Estimate	0.93	0.83	0.94
67% CI	0.90, 0.97	0.72, 1.01	0.79, 1.19
90% CI	0.80, 0.97	0.63, 1.10	0.71, 1.39
67% Bayes CS	0.90, 0.97	0.73, 0.92	0.79, 1.19
90% Bayes CS	0.81, 0.97	0.65, 0.98	0.71, 1.39

(b) Interest rates

Table 3: Long-run covariability estimates, $q = 26$

	ρ	β	$\sigma_{y\|x}$
Estimate	0.88	0.66	0.70
67% CI	0.78, 0.92	0.59, 0.74	0.58, 0.80
90% CI	0.72, 0.94	0.51, 0.78	0.56, 0.91
67% Bayes CS	0.78, 0.92	0.59, 0.73	0.61, 0.80
90% Bayes CS	0.72, 0.93	0.54, 0.78	0.56, 0.91

(a) GDP and consumption

	ρ	β	$\sigma_{y\|x}$
Estimate	0.92	0.79	1.03
67% CI	0.87, 0.97	0.69, 0.98	0.86, 1.24
90% CI	0.80, 0.97	0.62, 1.03	0.77, 1.68
67% Bayes CS	0.87, 0.97	0.71, 0.88	0.86, 1.24
90% Bayes CS	0.81, 0.97	0.63, 0.93	0.77, 1.42

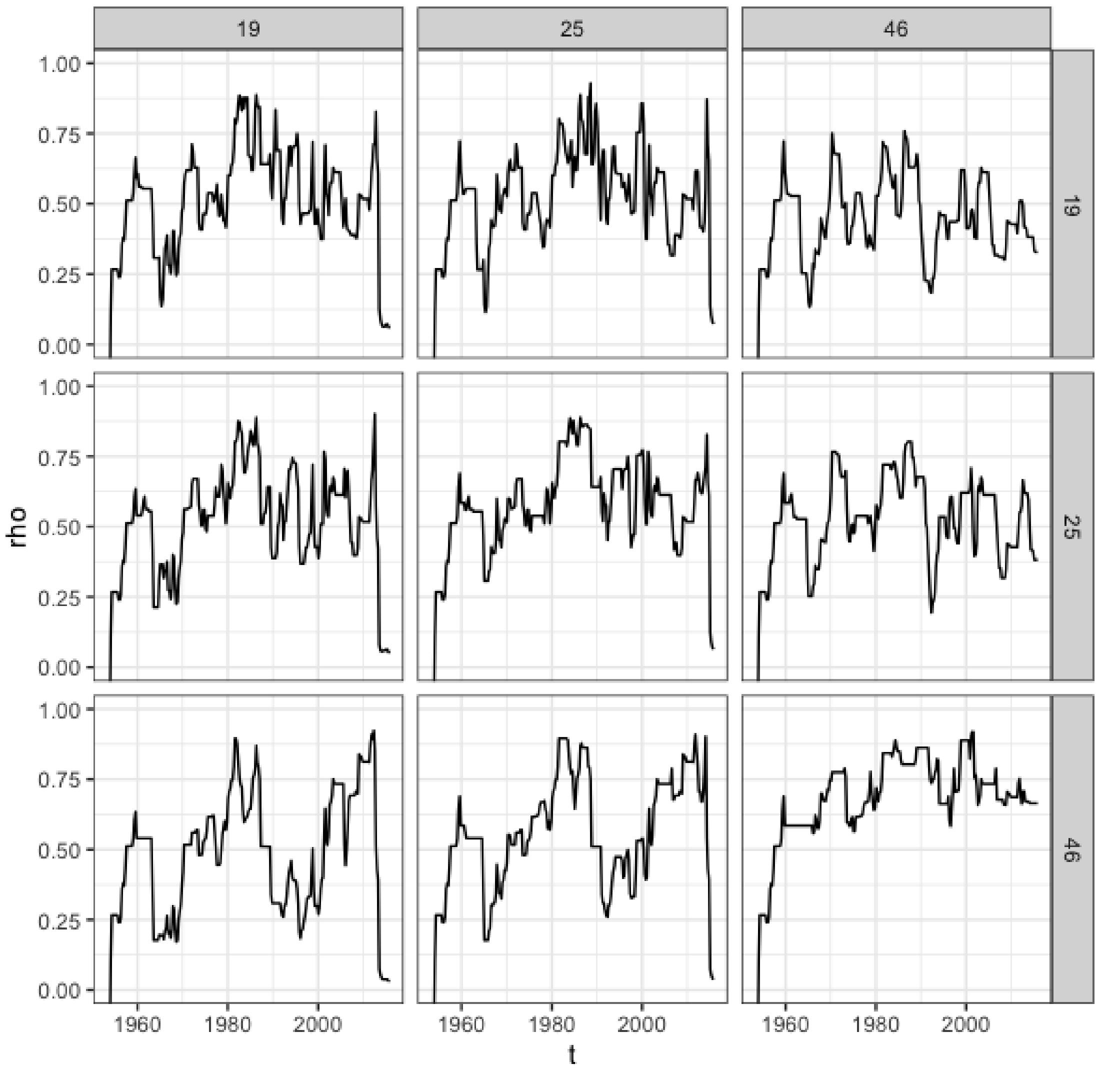
(b) Interest rates

fluctuation in Treasury Bill rates remaining in the data while this cyclical behaviour is not present in the Treasury Bond rates. Thus, the relationship between these long-run projections is weaker. To summarize, the results obtained from applying Mueller and Watson's (2018) method may depend on what the researchers believes the "long run" is. In this application, the properties of the data is to blame. Thus, this decision needs to be made very carefully.

Having presented the long-run covariability results, now attention is turned to the TPMA as well as MTTPMA analysis. The presentation of the results continues with the case of the US long- and short-term interest rates; see. Figure 4. Three thickness values are used: $\underline{\tau} = (19, 25, 46)$. As quarterly data is used, this corresponds to 5-year, 6-year, and 11-year features of the data.[8] The diagonal subplots show the TPMA as in these cases the thickness value is identical for the two time series. The off-diagonal subplots show the MTTPMA. The latter measures cross-scale movement. The top-left panel measures the co-movement of the 5-year features of the two interest rates. Overall, the proportion of overlap fluctuates around 0.5, but there are also periods for which an overlap close to 0.25 is found, in particular early in the sample.[9] Noteworthy is that the overlap peaks above 0.75 on a few occasions. There is a stronger upward trend in the co-movement prior to 1980, followed by a downward trend. The stronger fluctuation in the co-movement is attributable to data properties highlighted above: the interest cycle peaks overlap, but otherwise the behaviour of the rates differs from each other, epitomised by the very low overlap found in the 1960s: while the Treasury Bond rate moved largely horizontally around 4%, the Treasury Bill rate considerably increased from about 2.5% to 4%. During this period, the co-movement between the two interest rates is low. The peaks

[8] Jach (2021) uses this terminology. As asserted above, the selection of q and the thickness values are methodologically equivalent. Note, however, that there is a sublte difference: q corresponds to "periods longer than a certain number of years"; a pen of a certain thickness yields exactly a feature of a given number of years.

[9] The very small overlap at the very beginning of the sample period is a consequence of the so-called boundary effect. This problem emerges because the calculation of the overlap is initally based on a very small number of observations, or very narrow chimneys.



Note: Rows vary the thickness value for Treasury Bond rates, columns those for Treasury Bill rates. Interpretation example: the bottom-right panel shows the overlap between the 10-year features of both interest rates; the bottom-left panel shows the overlap between the 11-year-feature of long-term and 5-year-feature of short-term interest rates.

Figure 4: TPMA (main diagonal) and MTTPMA (off-diagonal) for long- and short-run interest rates.

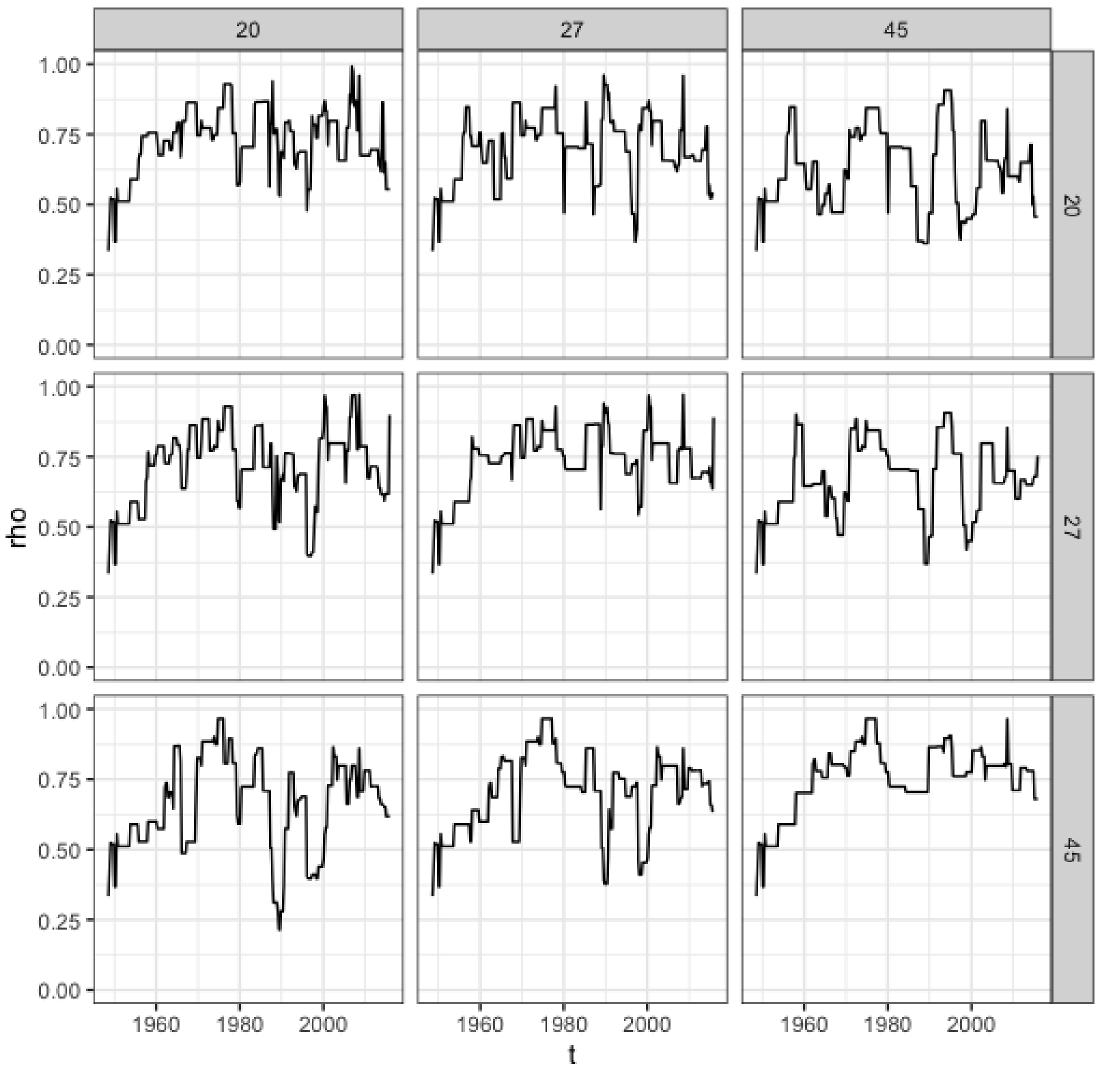


Figure 5: TPMA (main diagonal) and MTTPMA (off-diagonal) for growth rates of per capita GDP and consumption.

in the overlap capture the period around 1980 where the Treasury Bill rate briefly increases to about the same level as the Treasury Bond rate and then gradually declines. In the second part of the sample, the two rates generally behave more similarly. After 2010, when the Treasury Bill rate exhibits no apparent fluctuation while the Treasury Bond rate still fluctuates, they are clearly out of sync: the overlap drops to 0. The 6-year-features of the data overall behave similar, but carefully inspecting the graph shows that, first, the extent of overlap is slightly higher, and, second, the fluctuation of the extent of overlap is slightly smaller than that of the 5-year-features.

Having discussed in detail the co-movement of the 5-year and 6-year features of the data, now the attention is shifted to the 11-year features. Is is evident that the extent of overlap between those is considerably higher; it fluctuates around 0.75. This finding reflects that the co-movement of the long-term components of the two interest rates is generally very high. The finding of a larger extent of co-movement between the 11-year features of the data is consistent with the findings of the benchmark analysis presented above: for smaller values of q, both long-run correlation coefficient and long-run regression parameter are found to be larger than for larger ones. Jach (2017) also describes this general feature of this method. It is worth emphasising again that Mueller and Watson's (2018) long-run covariability is time-invariant and, thus cannot capture change in co-movement over time. In this particular case, it also does not allow one to analyse which time period in particular drives the change in the results when changing q.

The off-diagonal subplots show the cross-scale overlap between the two interest rates. This overlap is generally found to be moderate as it fluctuates around 0.5. It is nevertheless worth highlighting, first, that there is evidence of asymmetry: the co-movement between the 5-year and 11-year features differs. While the overlap displayed in the bottom-left corner fluctuates stronger, the oscillation displayed in the top-right corner is less pronounced and slightly decreases towards the end of the sample. This type of analysis across time scales - co-movement between different features of the data which is displayed on the off-diagonal - is not possible using Mueller and Watson's (2018) method either.

Having discussed the co-movement of long- and short-term interest rates in detail, now the relationship between growth rates of per capita GDP and consumption is analysed. Recall that the application of Mueller and Watson's (2018) long-run covariability showed also here that there a very high long-run covariablity, but that that the results are less sensitive to varying q. The Thick Pen analysis also used three thickness values: $\underline{\tau} = (20, 27, 45)$; this corresponds to 5-year, 7-year, and 11-year features of the data. Note that the data frequency is quarterly as well. Figure 5 presents the results: the overlap is found to be around or above 0.75 - for all three features. Thus, not only the long-term features of the data exhibit a high extent of co-movement, also the shorter-term features. Also these results, in particular the small influence of what is considered "the long run", are consistent with the benchmark method. What is more, there is no apparent change in the extent of co-movement over time. This is remarkable in so far as the variation of the growth rates itself is decreasing considerably over time, but it does not affect the measurement of co-movement using TPMA.

4 Concluding remarks

Concerted research efforts have been undertaken in the past few decades into how to measure co-movement of economic time series. Cointegration is certainly among the most popular methods in this regard. However, for a number of reasons, more flexible approaches are required. First, empirical behaviour of individual time series is changing over time. Second, the relationship between certain economic series is changing as well: not only a particular relationship might get either stronger or weaker, on some occasions time series begin to co-move which have been essentially unrelated beforehand. The increased co-movement of various commodity prices during the Financial Crisis 2008/2009 is a good example in this regard.

This paper proposes to employ two measures of association based on the so-called Thick Pen Transform: Fryzlewicz and Oh's (2011) Thick Pen Measure of Association (TPMA) as well the Multi-Thickness Thick Pen Measure of Association (MTTPMA), proposed by Jach (2021). These are non-parametric, time-dependent, cross-scale/cross-frequency dependence measures for multivariate stationary and non-stationary time series which are visually interpretable and allow comparisons of co-movement of time series in different applications. As a methodological benchmark, this paper also uses Mueller and Watson's (2018) measure of long-run covariability. To illustrate this method, this paper analyses two macro relationship also considered by Mueller and Watson (2018): first, long- and short-term interest rates, and, second, growth rates of per capita GDP and consumption. The results obtained from applying the two methods are consistent: the extent of co-movement is larger the longer a researcher considers "the long-run" to be - determined by either the parameter q when estimating long-run covariability or the thickness parameter within the Thick Pen analysis. This, as such, means that the method produces trustworthy results. The extent to which the results are sensitive to this choice depends on properties of the data. In this paper, different extents of cyclical behaviour of the time series are found to be a crucial feature, as the analysis of the US long-term and short-term interest rates vividly demonstrated. Demonstrating that also the covariability results are sensitive to the choice of q is another contribution this paper makes. The mere change in volatility of time series, as present in the GDP and consumption growth rate series, does not have a considerable influence on the results. The advantage of the Thick Pen Measure of Association is that it allows one to capture, first, change in extent of co-movement over time, and, second, to conduct analysis of co-movement across time scales. In a nutshell, the method is a very flexible one, can capture change over time and can tell apart short-run and long-run movements. Macroeconomics, thus, is the ideal area of application of the Thick Pen Measure of Association.

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