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A Monetary Equilibrium with the Lender of Last Resort

Abstract

This paper studies the role of a lender of last resort (LLR) in a monetary model where a shortage of a bank's monetary reserves (a liquidity crisis) occurs endogenously. We show that discount window lending by the LLR is welfare-improving but reduces banks' ex-ante incentive to hold monetary reserves, which increases the probability of a liquidity crisis, and can cause moral hazard in capital investment. We also analyze the combined effects of monetary and extensive LLR policies, such as a nominal interest rate, a lending rate, and a haircut.

JEL-Codes: E400.

Keywords: monetary equilibrium, liquidity crisis, lender of last resort, moral hazard.

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1 Introduction

Since financial crises re-emerged in the 1970s, central banks have provided emergency liquidity assistance to troubled financial institutions as a lender of last resort (LLR). Central banks in several developed countries conducted large-scale monetary interventions beyond the traditional scope of open market operations during the 2007-08 financial crisis. Another recent example is that two days after the collapse of Silicon Valley Bank on March 10th, 2023, the Federal Reserve created the Bank Term Funding Program (BTFP) to make additional funds available to troubled banks. While the importance of the LLR has been stressed by many economists and policymakers, there is much less consensus on the nature of its role. For example, Fischer (1999, p.86) put it: "While there is considerable agreement on the need for a domestic lender of last resort, some disagreements persist about what the lender of last resort should do." The main concern about the LLR policy is a moral hazard problem. In fact, many observers and critics caution that the introduction of the BTFP may promote moral hazard in banking.[1] The conventional view is that the existence of a credible LLR will give financial institutions an incentive to take risks ex-ante because they will expect ex-post liquidity provision by the LLR in the event of a crisis.

This paper studies the economic role and consequence of the LLR. We develop a monetary model in which individual agents face idiosyncratic uncertainty about the payment method, that is, whether money or credit can be used, and therefore banks are beneficial as liquidity insurance providers. Given the aggregate uncertainty of money demand, banks are sometimes short of monetary reserves and fail to satisfy their depositors' liquidity needs. In such an illiquidity situation, which shall be referred to as a liquidity crisis, there is room for emergency liquidity assistance by the LLR.

In our model, banks collect deposits and allocate them between monetary reserves and capital. In the presence of the LLR, banks with a high liquidity demand can borrow money

[1]For example, U.S. Congressional Research Service (March 31, 2023) put it: "The favorable BTFP terms, notably collateral valuation at par, reduces the incentive for banks to manage interest rate risk if they believe the Fed will lend them money regardless of the market value of the securities pledged." See also the *Economist* (March 16, 2023).

from the discount window by using their capital as collateral. This implies that with the LLR, pledged capital has a liquidity value during a liquidity crisis, which induces banks to over-invest in the capital. In addition, the existence of the LLR leads to a lower level of monetary reserves and, thus, to a higher ex-ante probability of a liquidity crisis relative to the case without the LLR. Nevertheless, introducing the LLR improves welfare because money and capital are substitutes during crises, and it mitigates losses associated with a liquidity crisis.

To examine a moral hazard problem, we consider a continuum types of capital investments, which differ in risk. The safe capital is risk-free. The risky capital yields a higher return if the gamble succeeds but a lower return if not. Importantly, we allow the LLR to lend more than their collateral value, i.e., with negative haircut rates. This corresponds to the recent implementation of BTFP, where troubled banks pledge collateral assets, such as U.S. Treasuries, but these assets will be valued at par. That is, the LLR will provide loans to troubled banks more than their collateral values at the time of lending.[2] Combining it with limited liability, which frees their payment responsibility in the case of default, we show that banks are induced to invest in risky capital rather than safe capital in the presence of the LLR. Hence, the LLR can cause a moral hazard in capital investments, that is, banks take more financial risk in terms of capital, resulting in a default on their discount window loans with a positive probability. We will refer to such an insolvent situation as a banking default.

To be clear, unlike in Diamond and Dybvig's (1983) model, the depletion of a bank's monetary reserves does not cause banking insolvency and bankruptcy in our model because banks can distinguish between two types of depositors (money and credit users) and refuse to allow them to withdraw after their reserves are exhausted (suspensions of convertibility). That is, no self-fulfilling bank runs occur in our model. The distinction between a bank's illiquidity (liquidity crisis) and insolvency (banking default) makes our model rich and makes it possible to study the important problems associated with the LLR.

Our model differs from the related banking models in the way we incorporate monetary factors. In fact, money matters in our model for several reasons. First, money serves as

[2]See the Federal Reserve's "Bank Term Funding Program" (FOMC, 2023).

a medium of exchange and overcomes trading frictions, such as a lack of commitment and imperfect monitoring in some decentralized transactions. Second, inflation positively impacts the likelihood and extent of a liquidity crisis because they depend on the amount of monetary reserves banks hold, and inflation increases the cost of money holdings. Third, inflation reduces welfare as in standard monetary models. Fourth, inflation affects the demand for central bank loans during a crisis. Finally, inflation may create a moral hazard problem associated with LLR.

Our paper points to the tension in public debate among economists and policymakers between the classical doctrine (or the Bagehot principle) versus the moral hazard problem. The former suggests that the LLR should give liquid loans to illiquid but solvent banks at a high-interest rate (or a "penalty" rate) against their good collateral (Thornton, 1802; Bagehot, 1873), while the latter concerns high financial risks taken by illiquid banks. This is one of the central issues of the LLR policy debates. The conventional view is that a high loan rate on the discount window prevents not only borrowing unnecessary amounts of liquidity but also taking excessive risks. For example, Solow (1982) states that "the penalty rate is a way of reducing moral hazard (p.247)," and Fischer (1999) comments that "the lender of last resort should seek to limit moral hazard by imposing costs on those who have made mistakes. Lending at a penalty rate is one way to impose such costs (p.93)."[3] Our results provide a partial vindication of this conventional view, but the mechanism is unconventional. We show that the penalty rate increases a bank's monetary reserves for the reason of self-guarding, which reduces the probability of a liquidity crisis and the amount of borrowing from an LLR. This implies that excessive borrowing, which causes moral hazard, does not occur even for higher inflation rates. However, we also show that a penalty rate reduces welfare. These are all new insights from our model.

Our paper also explores the implications of an extensive LLR based on the 'too-big-to-fail' doctrine for the moral hazard problem. In practice, central banks in many countries have expanded their LLR function beyond the classical Bagehot rule since the 1970s.[4] For example,

[3] See also Sheng (1991) and Summers (1991).

[4] According to Bordo (1990), major central banks in European countries generally followed the classical

Bordo (2014) points out that since the bailout of Franklin National Bank in 1974, the Fed's LLR policy has adopted the too-big-to-fail doctrine to prevent systemic risk and contagion irrespective of the classical doctrine. In addition, Giannini (1999) claims that most LLR policies adopted a non-penalty rate or even a subsidized rate without having stated it clearly in advance. Undoubtedly, the moral hazard associated with these LLR policies is a serious concern. However, no theory exists in the existing literature that takes into account monetary liquidity supports and moral hazard. We propose a new theory to fill this gap. The description of breadth (liquidity crises) and depth (banking defaults) of financial fragility is possible in the presence of the LLR only in a framework where money and the choice of investment risks are made explicit. To the best of our knowledge, our paper is the first to point out this possibility.

1.1 Related Literature

The LLR policy has a long history; its concept was elaborated in the 19th century by Thornton (1802) and Bagehot (1873). The classical doctrine has been criticized on two grounds. First, Goodfriend and King (1988), Kaufman (1991), and Schwartz (1992) argue that with efficient interbank markets, central banks should not lend to individual banks but instead provide liquidity via open market operations. However, others argue that interbank markets may fail to allocate liquidity efficiently due to asymmetric information (Flannery, 1996; Freixas and Jorge, 2008; Heider, Hoerova, and Holthausen, 2015), free-riding (Bhattacharya and Gale, 1987), coordination failures (Freixas, Parigi, and Rochet, 2000), incomplete network (Allen and Gale, 2000), incomplete contracts (Allen, Carletti, and Gale, 2009), or market power (Acharya, Gromb, and Yorulmazer, 2010), which can justify the role of the LLR. In this paper, we do not model interbank markets explicitly but consider a situation where a shortage of liquidity in a whole banking system occurs endogenously due to aggregate demand shocks that market capacity cannot satisfy. Second, Goodhart (1987, 1999) argues that there is no clear-cut distinction between illiquidity and insolvency during a crisis, and banks that require

doctrine between 1870 and 1970. In contrast, the Bank of Japan provided liquidity support to large illiquid and insolvent banks at a non-penalty rate based on the too-big-to-fail doctrine in response to the financial panic of 1927 (Yokoyama, 2018).

LLR assistance are already under suspicion of insolvency.[5] Our model captures Goodhart's emphasis well because, in our setup, the central bank must lend money to illiquid banks without knowing whether they would be insolvent.

Our study is related to the following three strands of literature. The first strand focuses on financial crises and the role of the LLR in a standard non-monetary banking model, for example, Allen and Gale (1998), Freixas, Parigi, and Rochet (2000), Rochet and Vives (2004), Repullo (2005), Martin (2006, 2009), Allen, Carletti, and Gale (2009), and Acharya, Gromb, and Yorulmazer (2010). Some of them regard LLR policies as real tax-transfer schemes without monetary considerations, while others consider monetary transfers but treat nominal assets as an exogenous restriction. The most crucial difference is that all the existing papers do not allow for the possibility that LLRs can lend more than their collateral value, which is the main driving force of our results. Furthermore, our approach is to take monetary factors into account, because we believe that traditional banking crises should represent a widespread attempt by the public to convert their deposits into cash and a suspension of convertibility (Calomiris and Gorton, 1991; Champ, Smith, and Williamson, 1996) and that the abilities to create high-powered money and distributing it quickly authorize a central bank to act as a lender of last resort (Schwartz, 2002).

The second strand examines the monetary factors of the LLR in an overlapping-generations model with random relocation along the lines of Champ, Smith, and Williamson (1996) and Smith (2002). See also Antinolfi, Huybens and Keister (2001), Antinolfi and Keister (2006), and Matsuoka (2012). Unlike our model, these models do not consider risky investment technologies and a moral hazard associated with the LLR. Williamson (1998) is only an exception in the literature. He develops a banking model with discount window lending and deposit insurance and shows that the introduction of these policies decreases a bank's effort to screen borrowers in the loan market. However, his analysis does not consider haircut policies explicitly as we pursue in our study.

The third strand considers banking with a New Monetarist approach along the lines of

[5] See Solow (1982) and Schwartz (1992) for more related issues.

Lagos and Wright (2005) and Rocheteau and Wright (2005). This strand includes Andolfatto, Berentsen, and Martin (2019), Berentsen, Camera, and Waller (2007), Ferraris and Watanabe (2008, 2011), Bencivenga and Camera (2011), Williamson (2012, 2016), Gu et al. (2013), Gu et al. (2019), Sanches (2018), and Matsuoka and Watanabe (2019). Most of the studies do not consider the economic role and consequences of the LLR, with the exception of Andolfatto, Berentsen, and Martin (2019). They explain how the combination of nominal deposit contracts and an LLR eliminates a bank run, but their model abstracts moral hazard. We offer a new and simple monetary general equilibrium approach to make the moral hazard problem, which is potentially very complicated, tractable.

The rest of the paper is organized as follows. Section 2 describes the basic environment. Section 3 analyzes a monetary equilibrium with an LLR policy, while Section 4 examines welfare. Section 5 provides discussions, and Section 6 concludes. All mathematical proofs are provided in the Appendix.

2 Environment

The model builds on a version of Lagos and Wright (2005). Time is discrete and continues forever. Each period is divided into two subperiods, called day and night, and different markets open sequentially. A decentralized market (DM) opens during the day, and a Walrasian centralized market (CM) opens during the night. There are two types of $[0,1]$ continuum of infinitely-lived agents. Agents of the same type are homogeneous. One type of agent, called sellers, has production technology during the day, which allows them to produce perishable and divisible goods, referred to as the special good. The other type of agent, called buyers, does not have the production technology during the day but wants to consume the special goods. Other divisible goods, referred to as the general good, are produced and consumed by both types of agents during the night. Agents discount future payoffs at a rate $\beta \in (0,1)$ across periods, but there is no discounting between the two subperiods.

The instantaneous utility functions for buyers and sellers are given by $u(q^b) + U(x) - h$ and $-q^s + U(x) - h$, respectively, where q^b is the amount of the special good consumed by the buyer, q^s is the amount of the special good produced by the seller, x is the amount of the general good consumed, and h is the nighttime hours of work. Marginal production costs of both goods are constant and normalized to one. The utility function $u(q)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u(0) = 0$ and $u'(0) = \infty$. Let q^* denote the efficient quantity of the special good, which solves $u'(q^*) = 1$. For analytical tractability, we assume $\xi \equiv -\frac{qu''(q)}{u'(q)}$ is a positive constant. The utility function of the general good, $U(x)$, is also strictly increasing, concave, and twice continuously differentiable. Without loss of generality, we normalize $U(x^*) - x^* = 0$, where x^* solves $U'(x^*) = 1$.

There is an intrinsically worthless object, which is perfectly divisible and storable, called fiat money. Let ϕ denote the price of money in terms of the general good. The total supply of fiat money, denoted by M, grows (or shrinks) at a constant rate $\pi > \beta$, that is, $M_+ = \pi M$, through injection to (or withdrawing it from) buyers in a lump-sum manner in the CM at night, where the subscript "+" stands for the next period. In a stationary monetary equilibrium, where the real money balances are constant over time, the rate of return on money must be equal to the inverse of the money growth rate, that is, $\frac{\phi_+}{\phi} = \frac{1}{\pi}$.

During the day, buyers and sellers can trade the special goods bilaterally in the DM. Just like in Sanches and Williamson (2010) and Williamson (2012, 2016), we assume that in the DM, there is a fraction $\alpha \in (0, 1)$ of sellers who are engaged in a *non-monitored* exchange and a fraction $1 - \alpha$ of sellers who are engaged in a *monitored* exchange. At the beginning of the day, sellers meet with their counterparts, and buyers learn whether they will trade with sellers in non-monitored or monitored meetings. In the non-monitored meetings, exchanges are anonymous, and trading histories are private knowledge. Thus, given the random meeting, sellers must receive money for immediate compensation for their products. In contrast, there is a record-keeping technology in the monitored meetings, and perfect commitment is possible so that buyers can promise credibly that they will make a payment to sellers later during the night. Those individual buyers face randomness in different requirements of the medium of exchange, playing the role of a "liquidity preference shock." This is similar in spirit to Diamond and Dybvig (1983) to motivate the banks' risk-sharing role. In contrast to the Diamond and Dybvig model, we assume that an individual buyer's type, non-monitored or monitored, is public information, implying that there is no self-fulfilling bank run in our model. For simplicity, we assume that buyers make a take-it-or-leave-it offer to sellers in any meeting.

The fraction α of monitored/non-monitored meetings is a random variable. It is publicly observable and identically distributed over time. Let $G(\alpha)$ represent the cumulative distribution function, which is assumed to be continuous, differentiable, and strictly increasing, and $g(\alpha) > 0$ is the associated density function. This randomness will play a key role in our model. Let us define $E(\alpha) \equiv \int_0^1 \alpha g(\alpha) d\alpha$ as the expected value of α.

One can imagine several interpretations of the stochastic fluctuations of α. First, it is typically thought of as a seasonal variation in the demand for money. Historically, large seasonal pressures, mostly in the spring planting season and fall crop moving season, have caused banking panics in agricultural economies (e.g., Sprague, 1910; Miron, 1986; Calomiris and Gorton, 1991). Second, small changes in the cost of information acquisition about counterparty or asset quality used as collateral in an imperfect credit system would have large effects on credit transactions (Lester, Postlewaite, and Wright, 2012). Finally, unexpected events such as large-scale natural disasters, blackouts, and September 11, 2001, would damage social communication tools necessary for credit transactions and suddenly increase the aggregate demand for money. As all of them seem to be potentially relevant, we are agnostic here about the exact nature of stochastic fluctuations.

During the night, buyers have access to a production technology that transforms k units of the general good, called capital, into $A(\eta)f(k)$ units of the general good at the beginning of the CM in the next period. Capital depreciates at a rate $\delta \in (0, 1)$ after one period. We assume that $f(0) = 0$, $f'(k) > 0 > f''(k)$, $f'(0) = \infty$, and $A(\eta)$ has a two-point structure:

$$A(\eta) = \begin{cases} \eta^{\sigma-1} & \text{with prob.} \quad \eta, \\ 0 & \text{with prob.} \quad 1-\eta, \end{cases}$$

with $\sigma \in (0, 1)$ and $\eta \in [0, 1]$. Observe that with a lower value of η, $A(\eta)$ takes a higher value if successful, which occurs with probability η, but a lower expected value, $E[A(\eta)] = \eta^{\sigma} \leq 1$. As σ decreases away from unity, the investment becomes more productive since $E[A(\eta)]$ decreases with σ. As in Martin (2006), we assume that a capital investor can choose a value of $\eta \in [0, 1]$, which represents the quality of capital investment, at the initiation phase. If the investor chooses $\eta = 1$, the investor is said to be *prudent*, and the expected return on capital is deterministic (safe) and maximized. Moral hazard is said to occur if the socially inefficient risky choice, $\eta < 1$, is made in equilibrium.[6] The quality choice is neither observable nor verifiable by outsiders (e.g., the central bank).

At the beginning of the DM, but after the realization of α, the central bank opens a discount window as a lender of last resort (LLR). The LLR offers private banks, which are formed by buyers, an intra-day monetary loan with a gross real interest rate of $R \geq 1$. A bank that borrows money from the LLR during the day must repay the loan in the CM the following night. Since private banks operate subject to limited liability, the LLR needs a guarantee for the loan's repayment, given their possible default. That is, the LLR's loans must be collateralized as in current practice. Further, since the LLR can not force a defaulting bank (and its depositors) to work, and outputs are not verifiable and pledgeable, the LLR can seize only undepreciated capital in the event of a default. Therefore, if a bank holding k capital is willing to borrow b real balances, then it faces the following borrowing constraint,

$$Rb \leq \lambda(1 - \delta)k, \tag{1}$$

where $\lambda \geq 0$ describes the haircut on the collateral. We assume that the revenue (or loss) earned by the central bank through the LLR policy is rebated to (or taxed on) buyers in a lump sum manner.

There are two types of LLR policies to be considered. According to the classical view based on the ideas of Bagehot (1873), an LLR should lend to illiquid but solvent banks freely against good collateral at a high (penalty) rate. In our model, an LLR policy with $\lambda \in (0, 1]$ and $R > 1$ captures this view. On the other hand, according to Bordo (1990, 2014), the Fed has expanded its reach beyond the classical view and adopted the "too big to fail" doctrine since the 1970s. This type of extensive policy can be captured in our model by setting $\lambda > 1$,

[6] The investment in the risky capital can be interpreted broadly to include low efforts for screening projects, monitoring projects, or management of financial risks. Typically, these efforts are costly and not observable by outsiders, implying that there is a private benefit from shirking as long as the project succeeds.

implying that an LLR can lend liquidity more than a bank's collateral values. Under this policy, there is a possibility that a bank that borrows from the LLR defaults partially on the loan. We refer to the former as a classical LLR policy, while the latter as an extensive LLR policy hereafter.

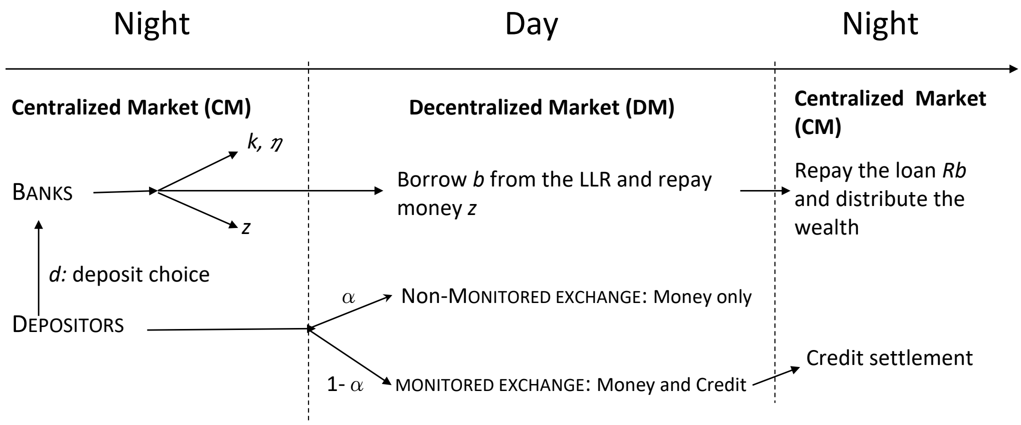

Fig 1: Timing of Events

The timing of events is illustrated in Figure 1. At the end of a night, in a CM, buyers form private banks, and the banks offer a contract to each of its depositors (buyers), which stipulates a repayment plan as specified below. Then, the banks collect deposits (d), invest them in a portfolio of real monetary reserves (z) and capital (k), and choose the capital investment quality (η). At the beginning of the next day, agents observe the realized value of α, and buyers learn their individual types of meetings. Then, buyers can receive money from their banks and use it for their consumption in the DM. The buyers who trade using money in a non-monitored exchange, referred to as *non-monitored buyers*, consume q^n, while the buyers who trade using credit in a monitored exchange, referred to as *monitored buyers*, consume q^m. The banks can access the discount window and decide how much money to borrow from the LLR. Finally, at the beginning of the night, the banks repay the discount loans (if they borrowed), distribute their wealth among their depositors, and dissolve. The monitored buyers settle private debts in the CM.

The first-best solution in our economy is straightforward. The socially optimal levels of capital and its quality are given by the solution to maximize the net expected gain from the capital, $-k + \beta\{\eta^\sigma f(k) + (1-\delta)k\}$. The solutions are $k = k^*$ and $\eta = 1$, where k^* solves $\beta\{f'(k^*) + (1-\delta)\} = 1$. The socially optimal levels of consumption are given by $q^m = q^n = q^* \equiv u^{-1\prime}(1)$, that is, the marginal utility of special goods consumption ($= u'(q^*)$) is equal to the marginal cost ($= 1$), and $x = x^* \equiv U^{-1\prime}(1)$, that is, the marginal utility of general goods consumption ($= U'(x^*)$) is equal to the marginal cost ($= 1$).

3 Monetary Equilibrium with LLR

We derive a stationary monetary equilibrium in the presence of an activated LLR, where the private banks borrow money from the discount window in some states. In particular, we are interested in a situation where a banking default can occur only when a bank's capital investment fails, i.e., when $A(\eta) = 0$, but not when the investment succeeds, i.e., when $A(\eta) = \eta^{\sigma-1}$. To prevent a bank from defaulting strategically, the following incentive constraint must hold:

$$-Rb + \eta^{\sigma-1} f(k) + (1-\delta)k \geq \chi \eta^{\sigma-1} f(k). \tag{2}$$

The left-hand side of the constraint is what a bank gets if it decides to pay back the LLR duly when the project succeeds, and the right-hand side is what it gets if it defaults strategically on the discount loan. If a strategic default occurs, the capital used as collateral is confiscated by the LLR, and a defaulting bank gets only a fraction $\chi \in [0, 1)$ of its production, where $1 - \chi$ represents a fraction of defaulting costs. χ is an exogenous parameter. Given the borrowing constraint (1), if

$$(1-\chi)\eta^{\sigma-1} f(k) + (1-\delta)k \geq \lambda(1-\delta)k \tag{3}$$

holds true, then the incentive constraint (2) is satisfied automatically. Hence, we assume not too high λ $\left(\leq 1 + \frac{(1-\chi)\eta^{\sigma-1}f(k)}{(1-\delta)k}\right)$ hereafter so that (3) is satisfied.

When the investment fails, i.e., when $A(\eta) = 0$, given the limited liability, a bank that borrows a discount loan more than the collateral values, i.e., $Rb > (1-\delta)k$, which is possible when $\lambda > 1$, should default on the loan amount $Rb - (1-\delta)k$. If default happens, then the LLR confiscates a defaulting bank's undepreciated capital used as collateral, $(1-\delta)k$.

We also assume that

$$\eta R \geq 1, \tag{4}$$

because if $\eta R < 1$, a bank would not hold real monetary balances over time (i.e., $z = 0$) when it chooses a risky investment. Since $\eta \leq 1$, the assumption (4) implies $R \geq 1$. Finally, we make a tie-breaking assumption that if indifferent, banks do not borrow from the LLR. Below, we attempt to examine whether there exists a parameter space in which this can happen in a monetary equilibrium.

We solve a bank's problem backward. At the beginning of a day, after buyers find out the type of their meetings, a bank's payment is made to buyers given its portfolio, z and k, and the capital quality, η, selected in the previous CM (see below). Note that a buyer in a monitored exchange can purchase by using credit (or equivalently, issuing a personal IOU) and consume any quantity he/she wishes to, irrespective of their daytime money holdings since the payment can be surely made later at night. Hence, in each period, a buyer in the monitored exchange consumes the first best quantity, that is, $q^m(\alpha) = q^*$, for any α. As the buyers in the monitored meetings do not need money during the day, their banks do not allocate money to them.

A bank's repayment schedule should determine how much money to allocate to each non-monitored buyer and how much money to borrow from the LLR. The payment can be contingent on the realized aggregate state. We assume competitive banks with free entry so that each maximizes the expected value of its representative depositor (i.e., buyer). Denote by $q^n = q^n(\alpha)$ the consumption of a non-monitored buyer, by $b = b(\alpha)$ the amount of real balances borrowed from the LLR, and by $\theta = \theta(\alpha)$ the fraction of its monetary reserves that a bank pays out to non-monitored buyers during a day. Without loss of generality, we assume that the remaining reserves and the proceeds from the capital are distributed uniformly among buyers after the DM closes. For each realized value $\alpha \in (0,1)$, dropping the constant terms, a bank's maximization problem in the DM can be written as:

$$\max_{\theta\in[0,1],b\geq 0} \alpha u(q^n) + (1-\theta)z - [\eta Rb + (1-\eta)\min\{Rb,(1-\delta)k\}],$$

subject to

$$\alpha q^n = \theta z + b, \tag{5}$$

and the borrowing constraint (1). The first term in the objective function represents the daytime utility of non-monitored buyers, who need money for the daytime trade, the second term represents the nighttime real value of the remaining monetary reserves, and the third term represents the expected loan repayment to the LLR. If the investment succeeds, the bank can repay the loan, Rb, by selling output from the capital investment. But if it fails, the bank can repay the loan fully only when the repayment value is less than or equal to the remaining capital value, that is, $Rb \leq (1-\delta)k$. Otherwise, the bank must default on the loan partially, and the LLR confiscates the bank's capital. Constraint (5) states that each individual non-monitored buyer receives $\frac{\theta z+b}{\phi\alpha}$ units of money from his/her bank and, given take-it-or-leave-it offers, exchanges it with the matched seller for $\frac{\theta z+b}{\alpha}$ units of the special good.

The first-order conditions are

$$z\left\{u'(q^n)-1\right\} \geq 0, \tag{6}$$

with equality if $\theta < 1$ and

$$u'(q^n) + \mu_b = [\eta + (1-\eta)\mathbb{1}_{\{Rb\leq(1-\delta)k\}} + \mu_k]R, \tag{7}$$

where $\mu_b \geq 0$ and $\mu_k \geq 0$ are the Lagrange multipliers of the non-negativity constraint $b \geq 0$ and the borrowing constraint (1), respectively, and $\mathbb{1}_{\{Rb\leq(1-\delta)k\}}$ is the indicator function that takes value one if $Rb \leq (1-\delta)k$ and zero otherwise. The condition (6) shows that two situations are possible in a monetary equilibrium (i.e., $z > 0$). The first case, $\theta < 1$, implies

$q^n = q^m = q^*$, i.e., perfect risk sharing. The other case, $\theta = 1$, implies that the bank exhausts all its monetary reserves and fails to achieve perfect risk sharing, that is, $q^n < q^m = q^*$. We refer to such an event as *a liquidity crisis*. This notation captures the situation where a significant number of depositors suddenly demand to redeem bank debt for cash, leading to a shortage of the overall amount of monetary reserves in the banking system and a suspension of convertibility. Note that the suspension of convertibility is embedded in our setup in the sense that a bank refuses to liquidate real assets prematurely and only pays out reserves selected in the previous CM. The condition (7) determines the borrowing quantity. The right-hand side of (7) is no less than $\eta R \geq 1$, which implies, together with the tie-breaking assumption, that $\mu_b = 0$ only in the case of a liquidity crisis. This guarantees the "pecking order" that the bank first uses its monetary reserves and then borrows from the LLR. Further, (7) shows that irrespective of the value of λ, the borrowing constraint is binding, i.e., $\mu_k > 0$, only when the crisis is sufficiently severe, i.e., when q^n is sufficiently low.

Lemma 1 (Bank's Optimal Payment Plan) *Given (z, k, η), the optimal payment plan of a bank is characterized by four critical values, α_θ, α_b, α_o, and α_k, such that:*

- *A liquidity crisis (i.e., $q^n < q^*$) occurs if and only if $\alpha > \alpha_\theta$;*
- *The LLR is activated (i.e., $b > 0$) if and only if $\alpha > \alpha_b$;*
- *Excessive borrowing (i.e., $Rb > (1-\delta)k$) occurs if and only if $\lambda > 1$ and $\alpha > \alpha_o$;*
- *The borrowing constraint (1) is binding if and only if $\alpha \geq \alpha_k$.*

If $\lambda \leq 1$, then $\alpha_\theta < \alpha_b < \alpha_k$. If $\lambda > 1$, then $\alpha_\theta < \alpha_b < \alpha_o < \alpha_k$ for $\eta > \eta_o$, with some $\eta_o \in (0, 1)$.

The lemma shows that the DM outcomes can be stated in terms of realized values of α. For low values of $\alpha < \alpha_\theta$, the realized aggregate demand for money in the DM is relatively low so that the bank's monetary reserves are sufficient to cover the needs of the non-monitored buyers, leading to $q^n = q^*$ and $\theta < 1$. In this case, the bank does not need to borrow from the discount window, $b = 0$. For high values of $\alpha \geq \alpha_\theta$, the realized aggregate demand for money is relatively high, and thus, the bank's reserves are not enough to cover the needs of non-monitored buyers, leading to $q^n \leq q^*$ and $\theta = 1$. This results in a liquidity crisis, and the bank exhausts its monetary reserves. For $\alpha \in [\alpha_\theta, \alpha_b]$, the bank exhausts its monetary reserves but does not borrow, leading to $q^n = \frac{z}{\alpha} < q^*$, because borrowing is relatively expensive. For $\alpha \in (\alpha_b, \alpha_k)$, the bank can borrow money as much as it wants because the borrowing constraint (1) is slack, leading to $q^n = u^{-1\prime}(R) \geq \frac{z}{\alpha}$. For $\alpha \in [\alpha_k, 1)$, the borrowing constraint is binding, and the bank borrows the maximum level, $\frac{\lambda(1-\delta)k}{R}$. The consumption level is reduced to $q^n = \frac{Rz+\lambda(1-\delta)k}{R\alpha} \leq u^{-1\prime}(R)$. Note, however, that for $\alpha \in (\alpha_o, 1)$, which can be non-empty only when $\lambda > 1$, the bank borrows more than the collateral value, and so the consumption is increased to $q^n = u^{-1\prime}(\eta R) \geq u^{-1\prime}(R)$. In what follows, we assume high enough values of η, i.e., $\eta > \eta_o$, so that the critical values satisfy $\alpha_\theta < \alpha_b < \alpha_o < \alpha_k$.

A broader implication of Lemma 1 is that the existence of a credible LLR can make a financial system unstable, but it is potentially welfare-improving. In this sense, there is a trade-off between efficiency and financial stability.

It is worth mentioning that one of the important criticisms of Bagehot's rule is that there is no clear-cut distinction between illiquidity and insolvency during a crisis (e.g., Goodhart, 1987; 1999). We capture this point well because, in our model, the timings of a bank's illiquidity and insolvency are different; illiquidity can occur during the day, while insolvency can occur at the beginning of the night. Thus, the LLR must decide whether to lend its funds to illiquid banks before their capital returns are realized. In such an environment, the banks may have the incentive to borrow and default.

Given the repayment plan $(q^n(\alpha), q^m(\alpha))$, $\theta = \theta(\alpha)$, and $b = b(\alpha)$, described in Lemma 1, the banks solicit deposits d from its customers to accumulate real monetary reserves z and capital k and choose its quality η in order to maximize a depositor's expected utility in the CM. Given the balance sheet constraint, $d = \pi z + k$, the banks' portfolio choice problem in the CM can be described by

$$\max_{z,k,\eta} \; -(\pi z + k) + \beta V(z, k, \eta)$$

where

$$V(z,k,\eta) \equiv \int_0^1 \left[\alpha u(q^n) + (1-\alpha)u(q^*) + W\left((1-\theta)z, k, (1-\alpha)q^*, \eta, b\right)\right] g(\alpha) d\alpha, \tag{8}$$

represents the expected value of a buyer in the DM. Here, $W(z,k,c,\eta,b)$ is the expected value of a buyer who holds z real balances, k capital with the quality η, c debts issued in the previous DM, and b borrowings from the LLR, evaluated at the beginning of the CM. Note here that the remaining reserves and the proceeds from the capital are distributed uniformly among depositors in the CM. This CM value function is given by

$$W(z,k,c,\eta,b) = \max_{x,h,z_+,k_+,\eta_+} U(x) - h + \beta V(z_+,k_+,\eta_+) \tag{9}$$

subject to

$$x + d_+ + c = h + z + \eta\left\{\eta^{\sigma-1} f(k) + (1-\delta)k - Rb\right\} + (1-\eta)\max\{(1-\delta)k - Rb, 0\} + E(T), \tag{10}$$

and the usual non-negativity constraints, where $E(T)$ denotes the expected transfers (or taxes if negative) from the government consisting of the expected revenue of the LLR activities and the seigniorage, that is, $E(T) = \eta Rb + (1-\eta)\min\{Rb, (1-\delta)k\} - b + \left(1 - \frac{1}{\pi}\right)\phi M$. Substituting h from (10) into (9) and using $d_+ = \pi z_+ + k_+$, we have

$$\begin{aligned} W(z,k,c,\eta,b) = {} & z + \eta\left\{\eta^{\sigma-1} f(k) + (1-\delta)k - Rb\right\} + (1-\eta)\max\{(1-\delta)k - Rb, 0\} \\ & + E(T) - c + \max_{x \geq 0}\{U(x) - x\} + \max_{z_+,k_+,\eta_+}\{-(\pi z_+ + k_+) + \beta V(z_+,k_+,\eta_+)\}, \end{aligned}$$

implying that the choice of z_+ and k_+ is independent of the wealth as usual in the Lagos and Wright model. It is obvious that $x = x^* \equiv U^{-1\prime}(1)$.

Based on the above value functions, we can formalize a bank's problem as the optimal choice of z, k, and η. The following optimality conditions, evaluated at η close to unity, are quite intuitive, and so we will leave their derivation to the Appendix. The Euler equation for money holdings z is:

$$i = \int_{\alpha \in \Omega_m} \left\{u'(q^n) - 1\right\} g(\alpha) d\alpha + (R-1)\left\{G(\min\{\alpha_k, 1\}) - G(\min\{\alpha_b, 1\})\right\}. \tag{11}$$

This equation states that the opportunity cost of an additional unit of money holdings, which is the nominal interest rate, $i \equiv \frac{\pi-\beta}{\beta}$, should equal its marginal benefit, which consists of the net marginal utility $u'(q^n) - 1$ if $\alpha \in \Omega_m \equiv [\alpha_\theta, \min\{\alpha_b, 1\}] \cup [\min\{\alpha_k, 1\}, 1]$, and the interest saving on the discount window loan, $R-1$, which is generated with probability $G(\min\{\alpha_k, 1\}) - G(\min\{\alpha_b, 1\})$ where money substitutes the LLR borrowing.

Similarly, the Euler equation for capital k is:

$$\frac{1}{\beta} = f'(k) + 1 - \delta + \frac{\lambda(1-\delta)}{R} \int_{\min\{\alpha_k,1\}}^{1} \left\{ u'(q^n) - R \right\} g(\alpha) d\alpha, \tag{12}$$

which states that the opportunity cost of an additional unit of capital, $\frac{1}{\beta}$, should equal its marginal benefit, which consists of the expected marginal product, which is $f'(k)$ when η is close to unity, plus the undepreciated unit $1-\delta$, and the "liquidity premium" of relaxing the borrowing constraint. We see from (12) that whenever the liquidity premium is positive, over-accumulation of capital $k > k^*$ occurs.

Finally, the optimal choice of η, evaluated at η close to unity, is described as follows: $\eta < 1$ if and only if

$$\int_{\min\{\alpha_o,1\}}^{1} \{Rb(\alpha) - (1-\delta)k\} g(\alpha) d\alpha > \sigma f(k). \tag{13}$$

The right-hand side represents the opportunity cost of a risky investment ($\eta < 1$), which is the marginal product decrease, measured by $\sigma f(k)$, while the left-hand side represents the expected net gain, which is the difference between the avoided repayment Rb and the loss of capital as is confiscated by the LLR, $(1-\delta)k$, which happens with probability $1 - G(\min\{\alpha_o, 1\})$. Note that with a classical LLR, excessive borrowing, i.e., borrowing more than the collateral value, never happens, and so there is no gain from making a risky investment, i.e., the left-hand side of (13) is zero when $\lambda \leq 1$.

Imposing the market-clearing condition, $z = \phi M$, at all periods, we can construct a stationary monetary equilibrium as follows:

Proposition 1 (Monetary Equilibrium with LLR) *There exists a unique stationary monetary equilibrium with LLR, satisfying (11), (12), and (13). In particular, moral hazard occurs* *if and only if the LLR policy is expansive, i.e.,* $\lambda > 1$*, with high enough nominal interest rates (i.e.,* $i > i_o$ *with some* $i_o \in (0, \infty)$*) and high enough expected returns of risky investments (i.e.,* $\sigma < \sigma^*$ *with some* $\sigma^* \in (0, 1)$*).*

Corollary 1 *The monetary equilibrium with LLR described in Proposition 1 satisfies:* $\alpha_k \leq 1$ *for* $i \in [i_k, \infty)$*;* $\alpha_o \leq 1 < \alpha_k$ *for* $i \in [i_o, i_k)$*;* $\alpha_b \leq 1 < \alpha_o$ *for* $i \in [i_b, i_o)$*;* $1 < \alpha_b$ *for* $i \in (0, i_b)$*, with some* $0 < i_b < i_o < i_k < \infty$*.*

There are four regimes in the monetary equilibrium with LLR. When the nominal interest rate is low, $i \in (0, i_b)$, money holding cost is low so that banks hold enough reserves and do not need to use the LLR in any states of the world, i.e., $\alpha_b > 1$, even with a liquidity crisis, $\alpha_\theta < 1$. Notice that in our model, a crisis can be avoided only by the Friedman rule, $i \to 0$. Banks reduce money holdings as the nominal interest rate rises. The LLR will be activated when $i > i_b$ because banks will borrow money from the LLR when the realized aggregate money demand is high enough, $\alpha \in (\alpha_b, 1]$. In particular, when the nominal interest rate is high enough, $i > i_o$, the LLR borrowing can be excessive, $\alpha_o < 1$, which is possible only when the LLR policy is expansive, $\lambda > 1$. In this case, banks will default when their investment project fails. Finally, with high enough $i > i_k$, a crisis will be so severe that the borrowing constraint (1) will be binding when $\alpha \in [\alpha_k, 1)$.

As the condition (13) indicates, moral hazard can occur when the LLR policy is expansive, $\lambda > 1$, and excessive borrowing is possible, i.e., $\alpha_o < 1$. Further, the risky investment has to be productive enough, i.e., σ is low enough because otherwise, the expected loss would be large enough to make risky investment an unattractive option.

Historically, discount window loans made during banking crises are often defaulted partially (sometimes totally), or their payback dates are extended since it is difficult for the LLR to distinguish between an illiquid and an insolvent bank. For example, the Bank of Japan provided emergency special loans (called *toku-yu*) to 114 selected banks in response to the panic of 1927, but about half of the rescued banks had been insolvent and were overdue in their repayments in 1933. Furthermore, the Bank of Japan could not collect more than 52 million yen in loans even in 1952 (see Yokoyama, 2018). Since, in the model, the timings of illiquidity and insolvency are different and there is asymmetric information about the quality of a bank's portfolio, our model captures some of the important elements of the LLR policy.

An LLR policy can be described by two parameters R and λ given $i > 0$.[7] The absence of an LLR corresponds to the case of $R \to \infty$ and/or $\lambda = 0$, where a bank must insure itself against a liquidity shock by holding enough reserves. So, in this case, a bank's reserves are larger relative to that in the presence of an LLR, leading to a low probability of a liquidity crisis, and capital bears no liquidity premium, that is $k = k^*$. In the presence of an active LLR, we refer to the case where $R = 1$ and $\lambda = 1$ as a benchmark. To analyzes policy implications for moral hazard, we now examine the effects of changing these policy parameters on the thresholds, $i_o \in (0, \infty)$ and $\sigma^* \in (0, 1)$, that determine the occurrence of moral hazard.[8] In general, these effects are ambiguous, but focusing on the neighborhood around the benchmark delivers the following results:

Proposition 2 (Policy Implications for Moral Hazard) *Suppose the economy is in the neighborhood of the benchmark, i.e., $R \approx 1$ and $\lambda \approx 1$. Then, the effects of LLR policies on moral hazard can be described as follows:*

$$\frac{\partial i_o}{\partial R} > 0, \quad \frac{\partial \sigma^*}{\partial R} \approx 0, \quad \frac{\partial i_o}{\partial \lambda} = 0 \quad \text{and} \quad \frac{\partial \sigma^*}{\partial \lambda} > 0, \tag{14}$$

where the first inequality holds if $\xi < \xi_o$, with some $\xi_o > 0$.

Proposition 2 states that an increase in the penalty rate has an effect of reducing moral hazard in the sense of narrowing the range of i in which moral hazard can occur — as shown in Proposition 1, remember that a banking default can occur if $i \geq i_o$. We find this is the case as long as $\xi < \xi_o$. When $\xi \geq \xi_o$, this may or may not occur (see the proof of Proposition 2).

[7] Since an LLR policy makes sense only when a liquidity crisis occurs, we fix some $i > 0$.

[8] Comparative statics with respect to other parameters on other variables are in order. For instance, as i increases, the probability of a crisis increases since z decreases with i. In the region where the borrowing constraint is binding, i.e., for $i > i_k$, $k > k^*$ increases with i since money and capital are substitutes during the crisis. The proof of this and other statements are available upon request.

This occurs because if $R > 1$ is increased, a bank's monetary reserves are increased so that the necessity of excessive borrowing is reduced. This implies that excessive borrowing, which causes moral hazard, does not occur even for higher nominal interest rates with higher values of R. On the other hand, the threshold σ^* does not change by increasing R. Intuitively, in the neighborhood of the benchmark, the marginal net benefit of strategic default is negligible, i.e., $Rb - (1-\delta)k \approx 0$, so that marginal changes in R will not affect σ^*.

On the contrary, the haircut policy has no influence on the critical nominal interest rate i_o because the expected interest saving on the discount window loan does not depend on λ. On the other hand, the haircut policy will matter for the critical productivity σ^* because a higher λ implies a larger amount of borrowing when the borrowing constraint is binding. The increased amount of borrowing raises the expected net benefit of strategic default, i.e., the left-hand side of (13). These policy implications are new and have not been reported in the previous literature.

4 Welfare

In this section, we examine the welfare properties of the monetary equilibrium and policy implications. In a steady state, the welfare measure $\mathcal{W}$ is defined by

$$\mathcal{W} \equiv \int_0^1 \left[\alpha \left\{u(q^n(\alpha)) - q^n(\alpha)\right\} + (1-\alpha)\left\{u(q^m(\alpha)) - q^m(\alpha)\right\}\right] g(\alpha) d\alpha \\ + U(x) - x - \frac{k}{\beta} + \eta^\sigma f(k) + (1-\delta)k, \quad (15)$$

which consists of the weighted sum of the expected net surplus in the DM and the expected net surpluses from working and capital investment in the CM. As stated before, the first-best allocations satisfy $q^n(\alpha) = q^m(\alpha) = q^*$ for all $\alpha \in (0,1)$, $x = x^*$, $\eta = 1$, and $k = k^*$, which can be achieved as $i \to 0$.

Given $i > 0$, so that the economy is away from the Friedman rule, we obtain the following results irrespective of whether the LLR policy is classic or extensive:

Proposition 3 (LLR Policy Effects on Welfare)

$$\frac{\partial \mathcal{W}}{\partial R} \leq 0, \quad and \quad \frac{\partial \mathcal{W}}{\partial \lambda} \geq 0. \tag{16}$$

The intuitions of these results are simple. First, an increase in the lending rate, R, increases the cost of borrowing from the LLR and reduces the amount of a bank's borrowing, which reduces the consumption in the DM. $\frac{\partial \mathcal{W}}{\partial R} > 0$ as long as a bank borrows from the LLR in some states of nature, i.e., for $\alpha_b < 1$. If R is high enough to discourage a bank from borrowing, i.e., for $\alpha_b \geq 1$, the effect on welfare is zero, that is, $\frac{\partial \mathcal{W}}{\partial R} = 0$. Second, an increase in λ increases the amount of borrowing and consumption in the DM when the borrowing constraint is binding. This effect is absent, leading to $\frac{\partial \mathcal{W}}{\partial \lambda} = 0$, when the borrowing constraint is slack, i.e., for $\alpha_k \geq 1$. The main policy implication of this Proposition is that the extensive LLR policy with a low lending rate is desirable from a welfare perspective, even when it causes a moral hazard. However, this message needs to be interpreted with caution because we do not model a negative externality caused by a bank's default on the discount loan for the whole banking system.

5 Discussions

We can summarize the implications of our analysis as follows:

(i) The existence of an LLR reduces a bank's monetary reserves and increases collateral capital, increasing the likelihood of its reserves' depletion. However, the magnitude of a liquidity crisis is mitigated. See Lemma 1.

(ii) An LLR policy based on the classical Bagehot's doctrine will not create moral hazard, and a low lending rate is desirable. See Proposition 1 and 3.

(iii) Moral hazard is driven by an extensive LLR policy that lends money more than a bank's collateral value and the productivity of capital investment, providing asymmetric information about the quality of collateral. See Proposition 1.

(iv) Increasing a lending rate and/or a haircut rate will have an effect of reducing moral hazard. See Proposition 2.

(v) An LLR policy beyond the scope of the classical view is desirable from the welfare point of view even when it causes a moral hazard. See Proposition 3.

Results (i), (iii), (iv), and (v) are related to the recent Bank Term Funding Program (BTFP), which was introduced by the Federal Reserve on March 12th, 2023. In particular, troubled banks can borrow more than their collateral values at the time of lending. This corresponds to our setup of $\lambda > 1$. Results (iii) and (iv) then suggest that this type of LLR lending has a risk of generating moral hazard.

Result (iii) was driven based on the assumption that LLR policies cannot make the lending contingent on asset qualities. What's if an LLR could observe the quality of collateralized assets, e.g., by eliminating asymmetric information about the quality of their assets (i.e., audit and stress test)? Naturally, it would increase the banks' incentives to behave prudently if the policy can be asset-quality contingent. However, there is an important time inconsistency problem, as discussed in Kydland and Prescott (1977) and Ennis and Keister (2009). That is, the central bank would like banks to believe that it will accept only safe assets as collateral. However, if a crisis actually occurred, the central bank would find it optimal to accept risky assets. Then, banks will be willing to hold risky assets in advance. In other words, the lack of commitment leads to moral hazard and banking defaults, similarly as in result (iii).

Our results are also related to some existing literature on the LLR policy and moral hazard. The most crucial difference is that all the existing papers do not allow for the possibility that $\lambda > 1$, which is the main driving force of our results (i), (ii), and (iii). For instance, Martin (2006) shows that a liquidity provision policy by the central bank can prevent panic without creating moral hazard. In contrast to our model, risk-averse depositors in his model, which is based on a standard non-monetary banking model, prefer the safe asset to the risky asset because borrowing strategies and portfolio choices are dichotomized. Repullo (2005) shows that the existence of the LLR does not increase banks' risk-taking incentives but simply reduces their liquid reserves. Although his and our paper share a common result when $\lambda \leq 1$, i.e., with the traditional LLR (result (ii)), the collateralized LLR lending in our model creates a liquidity premium of capital and a bank's incentive to take a risk only when $\lambda > 1$ (result (iii)). In addition, the implication of the penalty rate is different — a higher penalty rate will increase moral hazard in his model as opposed to our result (iv).

Finally, we have abstracted from a reputation effect by assuming that a bank lives only for one period. This assumption makes the analysis simple and allows us to highlight a bank's risk-taking investment behavior in the presence of an LLR. If a bank would live infinitely, an LLR could implement a history-dependent lending policy that has a positive effect on a bank's incentives for prudent behavior. However, once the LLR adopts the too-big-to-fail doctrine or fails to make a strong commitment, the reputation effect would be weakened so that result (iii) remains relevant.

6 Conclusion

We developed a monetary model of a liquidity crisis that allows us to investigate the economic role and consequence of the LLR. Given that private banks operate subject to limited liability, collateralized assets have liquidity values so that money and capital become substitutes during a crisis in the presence of the LLR. We showed that the LLR's liquidity provision will diminish banks' incentive to hold liquid assets, which in turn increases the probability of a liquidity crisis. Despite this unpleasant side effect, the LLR will mitigate the loss from a liquidity crisis and is beneficial. Importantly, we examined the possibility of negative haircut rates, which all the existing studies have not considered, and showed that private banks can be induced to invest in risky assets rather than safe assets. That is, the extensive LLR can create moral hazard in an investment, i.e., private banks take more financial risk in terms of long-term assets.

Our results point to the public debate on a moral hazard problem associated with LLR policies. To the best of our knowledge, our paper is the first to study the effect of the LLR's liquidity provision on banks' portfolio decisions, which eventually increases the ex-ante probability of a liquidity crisis and causes the moral hazard problem that increases financial risks in the capital. We have done this in a monetary framework where the role of liquidity is made explicit in the occurrence of a liquidity crisis. Our results provide a partial vindication of the conventional view that a high rate on the discount window can prevent banks from taking excessive risks, but the mechanism is unconventional. Our analysis implies that controlling lending and haircut rates influences the banks' choice of risk in different ways.

There are several directions to which our model could be extended. First, our model would be used to evaluate the impact of a capital requirement on the choice of investments, as discussed in Repullo (2004, 2005). This issue could be addressed by adding risk-neutral investors who provide equity capital to the banks. Second, it would be interesting to consider constructive ambiguity, defined as not declaring in advance and being ambiguous about which banks would be regarded as large enough to fail and be rescued. This issue could be addressed by introducing the possibility that banks' access to the LLR is limited. Third, we could consider an open economy to assess the desirability and design of an international LLR. We leave these important issues for future research.

Appendix

Proof of Lemma 1

We build the following Lagrangian:

$$\mathcal{L} = \alpha u\left(\frac{\theta z + b}{\alpha}\right) + (1-\theta)z - \left[\eta Rb + (1-\eta)Rb\mathbb{1}_{\{Rb\leq(1-\delta)k\}} + (1-\eta)\left(1-\mathbb{1}_{\{Rb\leq(1-\delta)k\}}\right)(1-\delta)k\right]$$
$$+ \mu_\theta(1-\theta) + \mu_b b + \mu_k\left[\lambda(1-\delta)k - Rb\right],$$

where $\mu_\theta \geq 0$, $\mu_b \geq 0$, and $\mu_k \geq 0$ are the Lagrange multipliers of $\theta \leq 1$, the non-negativity constraint $b \geq 0$, and the borrowing constraint (1), respectively. In addition, the indicator function $\mathbb{1}_{\{Rb\leq(1-\delta)k\}}$ takes unity if $Rb \leq (1-\delta)k$, and zero otherwise.

The first-order conditions for a maximum with respect to θ and b are:

$$u'\left(\frac{\theta z + b}{\alpha}\right) - 1 = \frac{\mu_\theta}{z}, \tag{A.1}$$

$$u'\left(\frac{\theta z + b}{\alpha}\right) + \mu_b = R\left[\eta + (1-\eta)\mathbb{1}_{\{Rb\leq(1-\delta)k\}} + \mu_k\right], \tag{A.2}$$

with complementary slackness conditions.

<u>Case 1: $\theta < 1$ and $b = 0$.</u> Since $\theta < 1$ implies $\mu_\theta = 0$, we have in (A.1) with $b = 0$,

$$u'\left(\frac{\theta z}{\alpha}\right) = 1 \iff \theta = \frac{\alpha q^*}{z},$$

where $\theta < 1 \iff \alpha < \alpha_\theta \equiv \frac{z}{q^*}$. Further, since $b = 0$ implies $\mu_b \geq 0 = \mu_k$ and $\mathbb{1}_{\{Rb\leq(1-\delta)k\}} = 1$, we have $\mu_b = R - 1 \geq 0$ from (A.2), which is consistent with $\mu_b \geq 0$. Hence, for $\alpha \in (0, \alpha_\theta)$, the solutions are $\theta < 1$, $q^n = q^*$ and $b = 0$.

<u>Case 2: $\theta = 1$ and $b = 0$.</u> When $\theta = 1$ and $b = 0$, we have $\mu_\theta \geq 0$, $\mu_b \geq 0 = \mu_k$ and $\mathbb{1}_{\{Rb\leq(1-\delta)k\}} = 1$, and so

$$u'\left(\frac{z}{\alpha}\right) \geq 1 \iff \alpha \geq \alpha_\theta,$$
$$u'\left(\frac{z}{\alpha}\right) \leq R \iff \alpha \leq \alpha_b,$$

where $\alpha_b \equiv \frac{z}{u^{-1\prime}(R)} > \frac{z}{q^*} \equiv \alpha_\theta$. Hence, for $\alpha \in [\alpha_\theta, \alpha_b]$, the solutions are $\theta = 1$, $q^n = \frac{z}{\alpha}$ and $b = 0$.

<u>Case 3: $\theta = 1$ and $b \in \left(0, \frac{\lambda(1-\delta)k}{R}\right)$.</u> Since $b \in \left(0, \frac{\lambda(1-\delta)k}{R}\right)$ implies $\mu_k = \mu_b = 0$, we have from (A.2)

$$u'\left(\frac{z+b}{\alpha}\right) = R\left[\eta + (1-\eta)\mathbb{1}_{\{Rb\leq(1-\delta)k\}}\right] \iff b = \alpha u^{-1\prime}\left(R\left[\eta + (1-\eta)\mathbb{1}_{\{Rb\leq(1-\delta)k\}}\right]\right) - z.$$

There are two cases. Suppose $\lambda \leq 1$. Then, the borrowing constraint (1) implies $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 1$ and so $b > 0 \iff \alpha > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)}$ and $Rb < \lambda(1-\delta)k \iff \alpha < \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(R)} \equiv \alpha_{k_1}$. Hence, when $\lambda \leq 1$, for $\alpha \in (\alpha_b, \alpha_{k_1})$, the solutions are $\theta = 1$, $q^n = u^{-1\prime}(R)$ and $b = \alpha u^{-1\prime}(R) - z \in \left(0, \frac{\lambda(1-\delta)k}{R}\right)$.

Suppose next $\lambda > 1$. Consider first the region, $b \leq \frac{(1-\delta)k}{R}$, which implies $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 1$. Then, the borrowing constraint (1) is satisfied automatically with strict inequality, i.e., $\mu_k = 0$, and so $b = \alpha u^{-1\prime}(R) - z > 0 \iff \alpha > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)}$. Consider next the region, $b > \frac{(1-\delta)k}{R}$, which implies $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 0$. Then, $b = \alpha u^{-1\prime}(\eta R) - z > \frac{(1-\delta)k}{R} \iff \alpha > \frac{Rz+(1-\delta)k}{Ru^{-1\prime}(\eta R)} \equiv \alpha_o$ and $Rb < \lambda(1-\delta)k \iff \alpha < \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(\eta R)} \equiv \alpha_{k_\eta}$. Hence, when $\lambda > 1$ for $\alpha \in (\alpha_b, \alpha_o]$, the solutions are $\theta = 1$, $q^n = u^{-1\prime}(R)$ and $b = \alpha u^{-1\prime}(R) - z \in \left(0, \frac{(1-\delta)k}{R}\right)$, and for $\alpha \in (\alpha_o, \alpha_{k_\eta})$, the solutions are $\theta = 1$, $q^n = u^{-1\prime}(\eta R)$ and $b = \alpha u^{-1\prime}(\eta R) - z \in \left(\frac{(1-\delta)k}{R}, \frac{\lambda(1-\delta)k}{R}\right)$.

To sum up, we have shown that for $\alpha \in (\alpha_b, \alpha_k)$, where

$$\alpha_k = \begin{cases} \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(R)} \equiv \alpha_{k_1} & \text{if } \lambda \leq 1, \\ \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(\eta R)} \equiv \alpha_{k_\eta} & \text{if } \lambda > 1, \end{cases}$$

the solutions are $\theta = 1$, $q^n = u^{-1\prime}(R[\eta + (1-\eta)\mathbb{1}_{\{Rb \leq (1-\delta)k\}}])$ and $b = \alpha u^{-1\prime}(R[\eta + (1-\eta)\mathbb{1}_{\{Rb \leq (1-\delta)k\}}]) - z \in \left(0, \frac{\lambda(1-\delta)k}{R}\right)$. Further, when $\lambda > 1$, $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 0$ if and only if $\alpha > \alpha_o$.

Case 4: $\theta = 1$ and $b = \frac{\lambda(1-\delta)k}{R}$. The binding borrowing constraint, $Rb = \lambda(1-\delta)k$, leads to $\mu_b = 0 \leq \mu_k$. If $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 1$, which is the case with $\lambda \leq 1$, then (A.2), together with $\theta = 1$, implies that

$$u'\left(\frac{z}{\alpha} + \frac{\lambda(1-\delta)k}{\alpha R}\right) \geq R \iff \alpha \geq \frac{Rz + \lambda(1-\delta)k}{Ru^{-1\prime}(R)} \equiv \alpha_{k_1},$$

while, if $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 0$, which is the case with $\lambda > 1$, then

$$u'\left(\frac{z}{\alpha} + \frac{\lambda(1-\delta)k}{\alpha R}\right) \geq \eta R \iff \alpha \geq \frac{Rz + \lambda(1-\delta)k}{Ru^{-1\prime}(\eta R)} \equiv \alpha_{k_\eta},$$

Hence, for $\alpha \in [\alpha_k, 1)$, the solutions are $\theta = 1$, $q^n = \frac{z}{\alpha} + \frac{\lambda(1-\delta)k}{\alpha R}$ and $b = \frac{\lambda(1-\delta)k}{R}$.

The above covers all the possible cases. The ordering of the critical values is summarized as follows: If $\lambda \leq 1$, then $\alpha_k \equiv \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(R)} > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)} > \alpha_\theta \equiv \frac{z}{q^*}$; If $\lambda > 1$, then $\alpha_k \equiv \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(\eta R)} > \alpha_o \equiv \frac{Rz+(1-\delta)k}{Ru^{-1\prime}(\eta R)} > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)} > \alpha_\theta \equiv \frac{z}{q^*}$ for $\eta > \eta_o$, where $\eta_o \in (0,1)$ is a solution to $\alpha_o = \alpha_b$, or $Rz\left\{u^{-1\prime}(\eta R) - u^{-1\prime}(R)\right\} = (1-\delta)ku^{-1\prime}(R)$. This completes the proof of Lemma 1. ■

Derivation of the Optimal Conditions (11), (12) and (13)

Applying the optimal payment plan described in Lemma 1, the value function in the CM, $W(\cdot)$, and the balance sheet constraint, $d = \pi z + k$, the bank's portfolio choice problem can be reduced to

$$\max_{z,k,\eta} -(\pi z + k) + \beta\Bigg[\int_0^{\alpha_\theta} \left\{\alpha u(q^*) + \left(1 - \frac{\alpha}{\alpha_\theta}\right) z\right\} g(\alpha)d\alpha + \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \alpha u\left(\frac{z}{\alpha}\right) g(\alpha)d\alpha,$$
$$+ \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_o,\alpha_k,1\}} \left\{\alpha u\left(u^{-1\prime}(R)\right) - R\left(\alpha u^{-1\prime}(R) - z\right)\right\} g(\alpha)d\alpha$$
$$+ \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} \left\{\alpha u\left(u^{-1\prime}(R\eta)\right) - \eta R\left(\alpha u^{-1\prime}(R\eta) - z\right)\right\} g(\alpha)d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left\{\alpha u\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - [\eta + (1-\eta)\mathbb{1}_{\{Rb \le (1-\delta)k\}}]\lambda(1-\delta)k\right\} g(\alpha)d\alpha$$
$$+ \eta^\sigma f(k) + \left\{G(\min\{\alpha_o,1\}) + \left(1 - G(\min\{\alpha_o,1\})\right)\eta\right\}(1-\delta)k\Bigg],$$

where $\alpha_k \equiv \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(R)} > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)} > \alpha_\theta \equiv \frac{z}{u^{-1\prime}(1)}$ for $\lambda \le 1$, and $\alpha_k \equiv \frac{Rz+\lambda(1-\delta)k}{Ru^{-1\prime}(\eta R)} > \alpha_o \equiv \frac{Rz+(1-\delta)k}{Ru^{-1\prime}(\eta R)} > \alpha_b \equiv \frac{z}{u^{-1\prime}(R)} > \alpha_\theta \equiv \frac{z}{u^{-1\prime}(1)}$ for $\lambda > 1$ and $\eta > \eta_o \in (0,1)$.

⊙ Classical LLR (i.e., $\lambda \le 1$). Note that $\mathbb{1}_{\{Rb \le (1-\delta)k\}} = 1$ for all $\alpha \in (0,1)$, when $\lambda \le 1$. The bank's problem is

$$\max_{z,k,\eta} -(\pi z + k) + \beta\Bigg[\int_0^{\alpha_\theta} \left\{\alpha u(q^*) + \left(1 - \frac{\alpha}{\alpha_\theta}\right) z\right\} g(\alpha)d\alpha + \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \alpha u\left(\frac{z}{\alpha}\right) g(\alpha)d\alpha,$$
$$+ \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_k,1\}} \left\{\alpha u\left(u^{-1\prime}(R)\right) - R\left(\alpha u^{-1\prime}(R) - z\right)\right\} g(\alpha)d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left\{\alpha u\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \lambda(1-\delta)k\right\} g(\alpha)d\alpha + \eta^\sigma f(k) + (1-\delta)k\Bigg].$$

Obviously, the solution is $\eta = 1$ since $\eta^\sigma \le 1$ is increasing in η. The first-order conditions with respect to z and k yield

$$i \equiv \frac{\pi - \beta}{\beta} = \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \left\{u'\left(\frac{z}{\alpha}\right) - 1\right\} g(\alpha)d\alpha + (R-1)\int_{\min\{\alpha_b,1\}}^{\min\{\alpha_k,1\}} g(\alpha)d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left\{u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - 1\right\} g(\alpha)d\alpha, \quad \text{(A.3)}$$

and

$$\frac{1}{\beta} = f'(k) + 1 - \delta + \frac{\lambda(1-\delta)}{R}\int_{\min\{\alpha_k,1\}}^{1} \left\{u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - R\right\} g(\alpha)d\alpha. \quad \text{(A.4)}$$

$\odot$ Extensive LLR (i.e., $\lambda > 1$). Note that when $\lambda > 1$, $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 1$ if $\alpha \leq \alpha_o$ and $\mathbb{1}_{\{Rb \leq (1-\delta)k\}} = 0$ if $\alpha > \alpha_o$. The bank's problem can be written as

$$\max_{z,k,\eta} \; -(\pi z + k) + \beta \Bigg[\int_0^{\alpha_\theta} \left\{ \alpha u(q^*) + \left(1 - \frac{\alpha}{\alpha_\theta}\right) z \right\} g(\alpha) d\alpha + \int_{\alpha_\theta}^{\min\{\alpha_b, 1\}} \alpha u\left(\frac{z}{\alpha}\right) g(\alpha) d\alpha,$$
$$+ \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_o,1\}} \left\{ \alpha u\left(u^{-1\prime}(R)\right) - R\left(\alpha u^{-1\prime}(R) - z\right) \right\} g(\alpha) d\alpha$$
$$+ \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} \left\{ \alpha u\left(u^{-1\prime}(R\eta)\right) - \eta R\left(\alpha u^{-1\prime}(R\eta) - z\right) \right\} g(\alpha) d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left\{ \alpha u\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \eta\lambda(1-\delta)k \right\} g(\alpha) d\alpha$$
$$+ \eta^\sigma f(k) + \left\{ G(\min\{\alpha_o, 1\}) + (1 - G(\min\{\alpha_o, 1\}))\,\eta \right\} (1-\delta)k \Bigg].$$

The first-order conditions with respect to z, k and η yield

$$i = \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \left\{ u'\left(\frac{z}{\alpha}\right) - 1 \right\} g(\alpha) d\alpha + (R-1) \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_o,1\}} g(\alpha) d\alpha + (\eta R - 1) \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} g(\alpha) d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left\{ u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - 1 \right\} g(\alpha) d\alpha - \Upsilon(\eta), \quad \text{(A.5)}$$

$$\frac{1}{\beta} = \eta^\sigma f'(k) + (1-\delta) \left\{ 1 - (1-\eta) \int_{\min\{\alpha_o,1\}}^{1} g(\alpha) d\alpha \right\}$$
$$+ \frac{\lambda(1-\delta)}{R} \int_{\min\{\alpha_k,1\}}^{1} \left\{ u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \eta R \right\} g(\alpha) d\alpha - \frac{(1-\delta) u^{-1\prime}(R)}{R u^{-1\prime}(\eta R)} \Upsilon(\tilde{\eta}), \quad \text{(A.6)}$$

and

$$- \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} R\left(\alpha u^{-1\prime}(R\eta) - z\right) g(\alpha) d\alpha - \int_{\min\{\alpha_k,1\}}^{1} \lambda(1-\delta) k g(\alpha) d\alpha$$
$$+ \sigma \eta^{\sigma-1} f(k) + \left\{ 1 - G(\min\{\alpha_o, 1\}) - \frac{(1-\eta)\alpha_o g(\min\{\alpha_o, 1\})}{u^{-1\prime}(\eta R) u''(\eta R)} \right\} (1-\delta)k, \quad \text{(A.7)}$$

where

$$\Upsilon(\eta) \equiv \alpha_o g(\alpha_o) \left\{ \frac{u^{-1\prime}(\eta R) - u^{-1\prime}(R)}{u^{-1\prime}(R)} \right\} \left\{ \frac{u(u^{-1\prime}(\eta R)) - u(u^{-1\prime}(R))}{u^{-1\prime}(\eta R) - u^{-1\prime}(R)} - R \right\}.$$

Since $\Upsilon(\eta) \approx 0$ when η is sufficiently close to 1, applying $\eta \to 1$ to (A.5), (A.6), and (A.7) yields the Bellman equations (11) and (12), and the optimal choice of η, (13), given in the text. ■

Proof of Proposition 1

The analysis so far has established that the Bellman equations (11) and (12), and the optimality condition of η, (13), given in the text, describe a monetary equilibrium. All that remains here is to find a solution to these conditions. The proof takes the following steps. In Step 1, assuming $\alpha_k \le 1$, we find a unique solution $(z, k) > 0$ to the following system of equations (which are basically (A.5) and (A.6)),

$$
\begin{aligned}
i &= \int_{\alpha_\theta}^{\alpha_b} \left\{ u'\left(\frac{z}{\alpha}\right) - 1 \right\} g(\alpha) d\alpha + (R-1)\left\{G(\alpha_k) - G(\alpha_b)\right\} \\
&\quad + \int_{\alpha_k}^{1} \left\{ u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - 1 \right\} g(\alpha) d\alpha \equiv \Phi(z,k), \qquad \text{(A.8)} \\
\frac{1}{\beta} &= f'(k) + (1-\delta) + \frac{\lambda(1-\delta)}{R} \int_{\alpha_k}^{1} \left\{ u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - R \right\} g(\alpha) d\alpha \equiv \Psi(z,k). \qquad \text{(A.9)}
\end{aligned}
$$

Given this solution, we then identify a parameter space in which $\alpha_k \le 1$ indeed holds true. In Step 2, we use a similar approach to examine all the possible cases in which the borrowing constraint is not binding, i.e., $\alpha_o \le 1 < \alpha_k$, $\alpha_b \le 1 < \alpha_o$ and $1 < \alpha_b$. We also show that the parameter spaces that support these cases are all disjoint, i.e., the solution is unique. Given the obtained solution $(z, k) > 0$, we derive in Step 3 the optimal choice of η. These steps altogether prove the existence and uniqueness of a monetary equilibrium with LLR.

Step 1: There exists a unique solution, $z \in (0, q^*)$ and $k \in [k^*, \infty)$, to (A.8) and (A.9) for $i \ge i_k$, some $i_k \in (0, \infty)$.

Proof of Step 1: First of all, note that

$$
\alpha_k \equiv \frac{Rz + \lambda(1-\delta)k}{Ru^{-1\prime}(R)} \le 1 \iff z \le u^{-1\prime}(R) - \frac{\lambda(1-\delta)k}{R} \equiv \bar{z}(k) < q^*
$$

for all $k \in [k^*, \infty)$. Differentiation yields:

$$
\begin{aligned}
\frac{\partial \Phi(z,k)}{\partial z} &= \int_{\alpha_\theta}^{\alpha_b} u''\left(\frac{z}{\alpha}\right) \frac{g(\alpha)}{\alpha} d\alpha + \int_{\alpha_k}^{1} u''\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) \frac{g(\alpha)}{\alpha} d\alpha < 0, \\
\frac{\partial \Phi(z,k)}{\partial k} &= \frac{\lambda(1-\delta)}{R} \int_{\alpha_k}^{1} u''\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) \frac{g(\alpha)}{\alpha} d\alpha < 0.
\end{aligned}
$$

Observe in (A.8) that there exists a unique $\bar{z} \in (0, \bar{z}(k^*)]$ such that $\Phi(\bar{z}, k^*) = i$, and a unique $\underline{z} \equiv \bar{z}(\bar{k}) \in (0, \bar{z})$ such that $\Phi(\underline{z}, \bar{k}) = i$, where $\bar{k} \in (k^*, \infty)$ is given by $\alpha_k \equiv \frac{R\underline{z} + \lambda(1-\delta)\bar{k}}{Ru^{-1\prime}(R)} = 1$. Hence, (A.8) determines a function $z = z_\phi(k)$ that satisfies $z'_\phi(k) < 0$, $z_\phi(k^*) = \bar{z}$, and $z_\phi(\bar{k}) = \underline{z}$.

Similarly, differentiation yields

$$
\begin{aligned}
\frac{\partial \Psi(z,k)}{\partial z} &= \frac{\lambda(1-\delta)}{R} \int_{\alpha_k}^{1} u''\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) \frac{g(\alpha)}{\alpha} d\alpha < 0, \\
\frac{\partial \Psi(z,k)}{\partial k} &= f''(k) + \left\{\frac{\lambda(1-\delta)}{R}\right\}^2 \int_{\alpha_k}^{1} u''\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) \frac{g(\alpha)}{\alpha} d\alpha < 0.
\end{aligned}
$$

Observe in (A.9) that since the first best $k^* > 0$ is given by $f'(k^*) + 1 - \delta = \frac{1}{\beta}$, there exists a unique $\bar{z}(k^*) \in (0, q^*)$ given by $\alpha_k \equiv \frac{R\bar{z}(k^*)+\lambda(1-\delta)k^*}{Ru^{-1\prime}(R)} = 1$, leading to $\Psi(\bar{z}(k^*), k^*) = \frac{1}{\beta}$. Also, there exists a unique $\bar{k}_0 \in (k^*, \infty)$ satisfying $\Psi(0, \bar{k}_0) = \frac{1}{\beta}$. Hence, (A.9) determines a function $z = z_\psi(k)$ that satisfies $z'_\psi(k) < 0$, $z_\psi(k^*) = \bar{z}(k^*) = u^{-1\prime}(R) - \frac{\lambda(1-\delta)k^*}{R} \in (0, q^*)$, and $z_\psi(\bar{k}_0) = 0$.

We now combine the above analyses. The two curves, $z = z_\phi(k)$ and $z = z_\psi(k)$, will intersect at least once if

$$z_\psi(k^*) = \bar{z}(k^*) = u^{-1\prime}(R) - \frac{\lambda(1-\delta)k^*}{R} \geq z_\phi(k^*). \tag{A.10}$$

Further, since

$$\frac{\partial}{\partial k}\left[z_\psi(k) - z_\phi(k)\right] = -\frac{f''(k) \times \frac{\partial \Phi}{\partial z} + \frac{\lambda(1-\delta)}{R}\left[\int_{\alpha_\theta}^{\alpha_b} u''\left(\frac{z}{\alpha}\right)\frac{g(\alpha)}{\alpha}d\alpha\right] \times \frac{\partial \Phi}{\partial k}}{\frac{\partial \Phi}{\partial z} \times \frac{\partial \Psi}{\partial z}} < 0,$$

the intersection point is unique, implying the existence of a unique solution. See Figure 2.

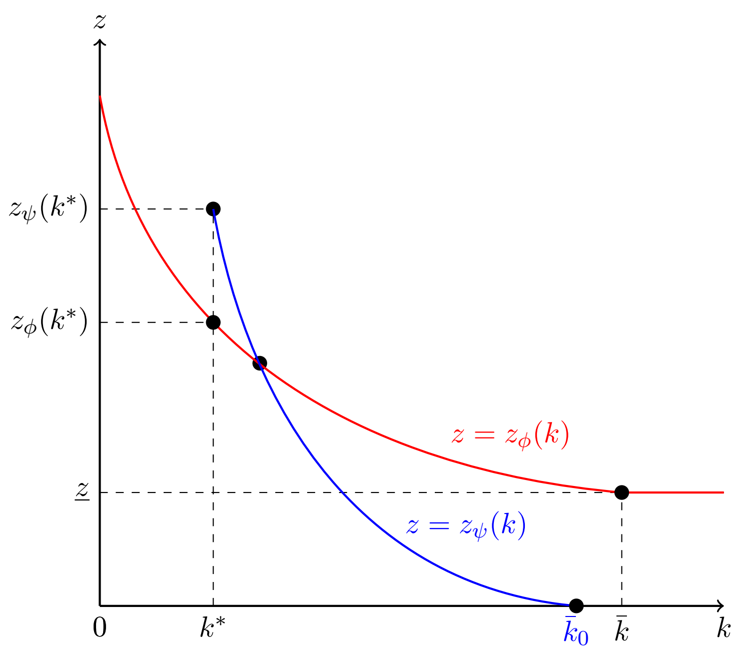

Fig 2: Existence of a Monetary Equilibrium

We now identify the parameter space in which the inequality (A.10) holds true. Note that

$z_{\phi}(k^*)$ is determined by

$$i = \int_{\frac{z_{\phi}(k^*)}{u^{-1\prime}(1)}}^{\frac{z_{\phi}(k^*)}{u^{-1\prime}(R)}} \left\{ u'\left(\frac{z_{\phi}(k^*)}{\alpha}\right) - 1 \right\} g(\alpha)d\alpha + (R-1)\left\{ G\left(\min\left\{\frac{Rz_{\phi}(k^*)+\lambda(1-\delta)k^*}{Ru^{-1\prime}(R)}, 1\right\}\right) - G\left(\frac{z_{\phi}(k^*)}{u^{-1\prime}(R)}\right)\right\}$$
$$+ \int_{\min\left\{\frac{Rz_{\phi}(k^*)+\lambda(1-\delta)k^*}{Ru^{-1\prime}(R)}, 1\right\}}^{1} \left\{ u'\left(\frac{Rz_{\phi}(k^*)+\lambda(1-\delta)k^*}{R\alpha}\right) - 1 \right\} g(\alpha)d\alpha = \Phi(z_{\phi}(k^*), k^*).$$

Observe that Φ is strictly decreasing in $z_{\phi}(k^*)$, and satisfies $\Phi(0, k^*) = \infty$. Also, there exists a unique critical value, $i_k \in (0, \infty)$, satisfying $\Phi(\bar{z}(k^*), k^*) = i_k$, such that (A.10) holds true if and only if $i \geq i_k$, where $\alpha_k \leq 1$, is guaranteed. Note that when $i = i_k$, we have $z = z_{\phi}(k^*) = z_{\psi}(k^*)$ and $k = k^*$ in equilibrium, leading to $\alpha_k = 1$.

Step 2: There exists a unique solution to (A.5) and (A.6), satisfying $\alpha_o \leq 1 < \alpha_k$ for $i \in [i_o, i_k)$, $\alpha_b \leq 1 < \alpha_o$ for $i \in [i_b, i_o)$ and $1 < \alpha_b$ for $i \in (0, i_b)$, some $i_b < i_o < i_k \in (0, \infty)$.

Proof of Step 2: For $i < i_k$, there is no equilibrium solution with $\alpha_k \leq 1$. Therefore, we must find a solution with $\alpha_k > 1$. Note first that when $\alpha_k > 1$, (A.9) implies that the capital level is fixed at the first best, k^*. Further, (A.8) is simplified to

$$i = \int_{\alpha_\theta}^{\alpha_b} \left\{ u'\left(\frac{z}{\alpha}\right) - 1 \right\} g(\alpha)d\alpha + (R-1)\left\{1 - G(\alpha_b)\right\}.$$

Since the right-hand side of this equation is strictly decreasing in z, there exists a unique solution, $z \in (\bar{z}(k^*), u^{-1\prime}(R) - \frac{(1-\delta)k^*}{R}]$, for $i \in [i_o, i_k)$, where $i_o \in (0, i_k)$ is a critical value that leads to $\alpha_o = \frac{Rz+(1-\delta)k^*}{Ru^{-1\prime}(R)} = 1$. Similarly, there exists a unique solution, $z \in (u^{-1\prime}(R) - \frac{(1-\delta)k^*}{R}, u^{-1\prime}(R)]$, for $i \in [i_b, i_o)$, where $i_b \in (0, i_o)$ is a critical value that leads to $\alpha_b = \frac{z}{u^{-1\prime}(R)} = 1$. For $i \in (0, i_b)$, the equilibrium condition is further simplified with $\alpha_b > 1$ to

$$i = \int_{\alpha_\theta}^{1} \left\{ u'\left(\frac{z}{\alpha}\right) - 1 \right\} g(\alpha)d\alpha,$$

which has a unique solution, $z \in (u^{-1\prime}(R), q^*)$.

Step 3: $\eta < 1$ if $\lambda > 1$, $i > i_o$, and $\sigma < \sigma^*$ with some $\sigma^* \in (0, 1)$. Otherwise, $\eta = 1$.

Proof of Step 3: Since $\eta = 1$ when $\lambda \leq 1$, we shall focus attention on the case of $\lambda > 1$. The optimality condition of η shows that $\eta < 1$ if and only if

$$\int_{\alpha_o}^{1} Rb(\alpha)g(\alpha)d\alpha > \sigma f(k) + \{1 - G(\alpha_o)\}(1-\delta)k.$$

Obviously, we need $\alpha_o = \frac{Rz+(1-\delta)k}{Ru^{-1\prime}(R)} < 1$ for this inequality to be true, which holds true if and only if $i > i_o$. Further, since $b(\alpha) = \alpha u^{-1\prime}(\eta R) - z \in \left(\frac{(1-\delta)k}{R}, \frac{\lambda(1-\delta)k}{R}\right)$ for $\alpha \in (\alpha_o, \alpha_k)$ and

$b(\alpha) = \frac{\lambda(1-\delta)k}{R}$ for $\alpha \in [\alpha_k, 1)$, we should have

$$\{1 - G(\alpha_o)\}(1-\delta)k < \int_{\alpha_o}^{1} Rb(\alpha)g(\alpha)d\alpha < \{1 - G(\alpha_o)\}\lambda(1-\delta)k.$$

and $\{1 - G(\alpha_o)\}\lambda(1-\delta)k < \{1 - G(\alpha_o)\}\{(1-\chi)f(k) + (1-\delta)k\}$ from (3). Therefore, there should exist a unique critical value,

$$\sigma^* \equiv \frac{\int_{\alpha_o}^{1}\{Rb(\alpha) - (1-\delta)k\}g(\alpha)d\alpha}{f(k)} \in (0, (1 - G(\alpha_o))(1-\chi)),$$

such that, given $\lambda > 1$ and $i > i_o$, the inequality in question holds if and only if $\sigma < \sigma^*$. This completes the proof of Proposition 1. ■

Proof of Proposition 2

Recall that in Step 2 in the proof of Proposition 1, i_o is determined implicitly by

$$i_o = \int_{\alpha_\theta}^{\alpha_b} \left\{ u'\left(\frac{z_{i_o}}{\alpha}\right) - 1 \right\} g(\alpha)d\alpha + (R-1)\{1 - G(\alpha_b)\}, \tag{A.11}$$

$$1 = \frac{Rz_{i_o} + (1-\delta)k^*}{Ru^{-1\prime}(R)}. \tag{A.12}$$

In (A.12), differentiating z_{i_o} with respect to R yields

$$R\frac{\partial z_{i_o}}{\partial R} = -\frac{u^{-1\prime}(R)}{\xi} + \frac{(1-\delta)k^*}{R}, \tag{A.13}$$

which is negative if $\xi < \frac{Ru^{-1\prime}(R)}{(1-\delta)k^*} \equiv \xi_o$. Then, in (A.11), differentiating i_o with respect to R, we have

$$\frac{\partial i_o}{\partial R} = \int_{\alpha_\theta}^{\alpha_b} \left\{ u''\left(\frac{z_{i_o}}{\alpha}\right) \frac{\partial z_{i_o}}{\partial R} \right\} g(\alpha)d\alpha + \{1 - G(\alpha_b)\} > 0, \tag{A.14}$$

if $\xi < \xi_o$. Since λ does not appear in (A.11) and (A.12), it is clear that $\frac{\partial i_o}{\partial \lambda} = 0$.

Next, differentiating σ^* with respect to R and λ and arranging it yield, respectively,

$$\frac{\partial \sigma^*}{\partial R} = \frac{1}{f(k)}\left[-\int_{\alpha_o}^{\alpha_k} \left\{ \alpha\xi^{-1}u^{-1\prime}(R) + R\frac{\partial z}{\partial R} - \left(1 - \kappa(k)\frac{R}{k}\frac{\partial k}{\partial R}\right)\left(\alpha u^{-1\prime}(R) - z\right) + (1-\delta)(1-\kappa(k))\frac{\partial k}{\partial R} \right\} g(\alpha)d\alpha \right.$$
$$\left. + (\lambda - 1)(1-\delta)\int_{\alpha_k}^{1}(1-\kappa(k))\frac{\partial k}{\partial R}g(\alpha)d\alpha \right], \tag{A.15}$$

$$\frac{\partial \sigma^*}{\partial \lambda} = \frac{1}{f(k)}\left[-\int_{\alpha_o}^{\alpha_k} \left\{ R\frac{\partial z}{\partial \lambda} + \frac{f'(k)}{f(k)}\frac{\partial k}{\partial \lambda}R(\alpha u^{-1\prime}(R) - z) + (1-\delta)(1-\kappa(k))\frac{\partial k}{\partial \lambda} \right\} g(\alpha)d\alpha \right.$$
$$\left. + (1-\delta)\int_{\alpha_k}^{1}\left\{ k + (\lambda-1)((1-\kappa(k))\frac{\partial k}{\partial \lambda} \right\} g(\alpha)d\alpha \right], \tag{A.16}$$

where $\kappa(k) \equiv \frac{kf'(k)}{f(k)} < 1$. In general, these effects are ambiguous, but, since $\alpha_k \to \alpha_o$ as $\lambda \to 1$, we have

$$\frac{\partial \sigma^*}{\partial R} \approx 0, \quad \text{and} \quad \frac{\partial \sigma^*}{\partial \lambda} \approx \frac{(1-\delta)k}{f(k)} \int_{\alpha_k}^{1} g(\alpha)d\alpha > 0, \tag{A.17}$$

if λ are sufficiently close to unity. ■

Proof of Proposition 3

We consider the two types of welfare depending on the types of the LLR policy in turn. In the monetary equilibrium with an extensive LLR policy, the welfare, denoted by $\mathcal{W}^E$, is

$$\begin{aligned}
\mathcal{W}^E =& \int_0^{\alpha_\theta} \left\{ \alpha u(q^*) + \left(1 - \frac{\alpha}{\alpha_\theta}\right) z \right\} g(\alpha)d\alpha + \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \alpha u\left(\frac{z}{\alpha}\right) g(\alpha)d\alpha \\
&+ \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_o,1\}} \left[\alpha u\left(u^{-1\prime}(R)\right) - R\{\alpha u^{-1\prime}(R) - z\}\right] g(\alpha)d\alpha \\
&+ \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} \left[\alpha u\left(u^{-1\prime}(\eta R)\right) - \{\eta R\{\alpha u^{-1\prime}(\eta R) - z\} + (1-\eta)(1-\delta)k\}\right] g(\alpha)d\alpha \\
&+ \int_{\min\{\alpha_k,1\}}^{1} \left[\alpha u\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \{\eta\lambda + (1-\eta)\}(1-\delta)k\right] g(\alpha)d\alpha \\
&+ [1 - E(\alpha)]\{u(q^*) - q^*\} - (1+i)z - \frac{k}{\beta} + \eta^\sigma f(k) + (1-\delta)k.
\end{aligned} \tag{A.18}$$

Differentiating (A.18) with respect to R and λ and using the first-order conditions yield

$$\begin{aligned}
\frac{\partial \mathcal{W}^E}{\partial R} =& -\int_{\min\{\alpha_b,1\}}^{\min\{\alpha_o,1\}} \{\alpha u^{-1\prime}(R) - z\} g(\alpha)d\alpha - \int_{\min\{\alpha_o,1\}}^{\min\{\alpha_k,1\}} \eta\{\alpha u^{-1\prime}(\eta R) - z\} g(\alpha)d\alpha \\
&- \frac{\lambda(1-\delta)k}{R^2} \int_{\min\{\alpha_k,1\}}^{1} u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) g(\alpha)d\alpha \leq 0,
\end{aligned} \tag{A.19}$$

with equality if $\alpha_b \geq 1$ (or $i \geq i_b$), and

$$\frac{\partial \mathcal{W}^E}{\partial \lambda} = \frac{(1-\delta)k}{R} \int_{\min\{\alpha_k,1\}}^{1} \left\{ u'\left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \eta R \right\} g(\alpha)d\alpha \geq 0, \tag{A.20}$$

with equality if $\alpha_k \geq 1$ (or $i \leq i_k$).

Similarly, the welfare in the monetary equilibrium with a classical LLR policy, denoted by

$\mathcal{W}^C$, is given by

$$\mathcal{W}^C = \int_0^{\alpha_\theta} \left\{ \alpha u(q^*) + \left(1 - \frac{\alpha}{\alpha_\theta}\right) z \right\} g(\alpha) d\alpha + \int_{\alpha_\theta}^{\min\{\alpha_b,1\}} \alpha u \left(\frac{z}{\alpha}\right) g(\alpha) d\alpha$$
$$+ \int_{\min\{\alpha_b,1\}}^{\min\{\alpha_k,1\}} \left[\alpha u \left(u^{-1\prime}(R)\right) - R\{\alpha u^{-1\prime}(R) - z\}\right] g(\alpha) d\alpha$$
$$+ \int_{\min\{\alpha_k,1\}}^{1} \left[\alpha u \left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - \lambda(1-\delta)k\right] g(\alpha) d\alpha$$
$$+ [1 - E(\alpha)]\{u(q^*) - q^*\} - (1+i)z - \frac{k}{\beta} + f(k) + (1-\delta)k, \tag{A.21}$$

and differentiating it with respect to R and λ, respectively, yields

$$\frac{\partial \mathcal{W}^C}{\partial R} = -\int_{\min\{\alpha_b,1\}}^{\min\{\alpha_k,1\}} \{\alpha u^{-1\prime}(R) - z\} g(\alpha) d\alpha - \frac{\lambda(1-\delta)k}{R^2} \int_{\min\{\alpha_k,1\}}^{1} u' \left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) g(\alpha) d\alpha \leq 0, \tag{A.22}$$

with equality if $\alpha_b \geq 1$ (or $i \leq i_b$), and

$$\frac{\partial \mathcal{W}^C}{\partial \lambda} = \frac{(1-\delta)k}{R} \int_{\min\{\alpha_k,1\}}^{1} \left\{u' \left(\frac{Rz + \lambda(1-\delta)k}{R\alpha}\right) - R\right\} g(\alpha) d\alpha \geq 0, \tag{A.23}$$

with equality if $\alpha_k \geq 1$ (or $i \leq i_k$). We obtained the same signs in the two cases, which completes the proof of this proposition. ■

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